Coordination in the fight against collusion*

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Abstract

While antitrust authorities strive to detect, prosecute, and thereby deter collusion, entities harmed by collusion are also encouraged to pursue their own strategies to deter collusion. The implications of such delegation of deterrence have largely been ignored, however. In a procurement context, we find that buyers may prefer to accommodate rather than deter collusion among their suppliers. In other circumstances, when they do deter collusion, the deterrence strategies may actually reduce social welfare below what it would be under collusion. The observation that the social cost of deterrence by market participants can exceed that of collusion emphasizes the importance of effective antitrust enforcement. We also show that a multi-market buyer, such as a centralized procurement authority, may optimally deter collusion when multiple independent buyers would not, consistent with the view that "large" buyers are less susceptible to collusion.

Keywords: collusion, cartel, auction, procurement, reserves, sustainability and initiation of collusion, coordinated effects

JEL Classification: D44, D82, H57, L41

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1 Introduction

Collusive conduct has long been recognized as harmful, both in the academic literature and in practice.[1] This has led most major jurisdictions to adopt antitrust laws and set up enforcement agencies dedicated to deterring collusion. In addition to these efforts, entities harmed by collusion have been encouraged to take matters into their own hands and pursue strategies to deter collusion.[2] However, the implications of such delegation of deterrence have largely been ignored. Prompted by this, we study the private costs and benefits of deterrence. We show that relying on market participants may result in socially insufficient deterrence, particularly because of free-riding problems among them. Conversely, given the limited tools available to participants to achieve deterrence,[3] even when they deter collusion, they may do so in such a way that society is better off with collusion than with market participants' deterring it. This suggests that antitrust enforcement by governments and legal authorities may be even more important than hitherto thought.

Specifically, we study a procurement context in which buyers face the possibility of collusion by suppliers, which is a setting of practical relevance for both public and private procurers.[4] We further assume that, if it occurs, collusion takes the form of a market allocation, whereby suppliers coordinate on who refrains from bidding below the reserve in a given market. Such market allocation schemes are among the most common collusive practices,[5] and have spurred the development of dedicated policy task forces (e.g., the U.S. Procurement Collusion Strike Force created in 2019) and detailed procurement guidelines.[6] The large and repeated public purchases prompted by the COVID pandemic, as well as recent evidence on the human cost of bid rotation schemes affecting public procurement,[7]

[1] See, e.g., Smith (1776) and Stigler (1964) and, e.g., the U.S. Federal Trade Commission's discussion of the Sherman Act of 1890 (https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/antitrust-laws).

[2] For example, U.S. DOJ (2015a) encourages procurement officials to, among other things, expand the list of bidders, maintain records, and press vendors to explain and justify their prices.

[3] See, e.g., Aubert et al. (2006) on the value of enhanced deterrence tools, such as incentives for whistleblowers, that require governmental support.

[4] Kawai and Nakabayashi (2021) document large-scale collusion by construction firms participating in Japanese government procurement auctions. Executives at optical disc drive manufacturer Hitachi-LG Data Storage pleaded guilty to felony charges that they "conspired with co-conspirators to suppress and eliminate competition by rigging bids for optical disk drives sold to Dell Inc. and Hewlett-Packard Company (HP) and/or fixing prices for optical disk drives sold to Microsoft Corporation" (U.S. Department of Justice Press Release, December 13, 2011, https://www.justice.gov/opa/pr/three-hitachi-lg-data-storage-executives-agree-plead-guilty-participating-bid-rigging-and).

[5] According to the U.S. DOJ's antitrust primer on price fixing, bid rigging, and market allocation schemes, "Most criminal antitrust prosecutions involve price fixing, bid rigging, or market division or allocation schemes" (U.S. Department of Justice, 2015b, pp. 1-2). See also U.S. DOJ (2015a).

[6] See, e.g., OECD (2009), U.S. DOJ (2015b), and U.K. CMA (2020).

[7] Barkley (2021) quantifies the cost in terms of human life of a bid rotation scheme used by a four-firm bidding ring in the Mexican insulin market, where disjoint procurement divisions used first-price

further emphasize the relevance of these issues.

When taking collusion or competition as a given, buyers find it optimal to set a more aggressive reserve in the case of collusion because they end up paying the reserve more often.[8] However, buyers can also *deter* collusion by setting a sufficiently aggressive reserve. As we show, this is best achieved with a differentiated treatment of markets because asymmetry impedes collusion. Still, deterrence comes at the cost of inefficiently low volumes of trade due to the lower reserves. Consequently, deterrence is optimal only when collusion is somewhat fragile, namely, if the discount factor is not too large; otherwise buyers are better off by accommodating collusion and setting the reserve accordingly.

The comparative statics of the buyers' optimal reserves with respect to the discount factor exhibit three regions. For small values of the discount factor, collusion is blockaded by setting the optimal reserve for competition, which is independent of the discount factor. However, as the discount factor increases, collusion is no longer blockaded by the competitive reserve, and deterrence becomes optimal. Because collusion becomes easier as the discount factor increases, the required deterrence reserves decrease with the discount factor. Eventually, accommodation becomes the best strategy for the buyer, at which point the optimal reserves increase discontinuously and no longer depend on the discount factor. Consequently, the optimal reserves vary non-monotonically with the discount factor.

We also show that buyers may fail to coordinate their deterrence strategies: they may fail to replicate an integrated buyer's optimal differentiated treatment across markets and even fail altogether to deter collusion when an integrated buyer would. By contrast, there is never a coordination failure in accommodating collusion. As a result, an integrated buyer deters collusion in more circumstances than would independent buyers. This result resonates with the concerns expressed by the European Commission that the prospect of delays in the award procedure, which often carry administrative, budgetary or even political consequences, dissuade procurement officers from dealing effectively with suspected cases of collusion (EC, 2021).

Our results further highlight that even though a market allocation misallocates resources whenever the designated supplier does not have the lowest cost, deterrence, which replaces inefficient production with insufficient trade, can be even worse for society than collusion. Indeed, we show that the buyer may find it optimal to deter collusion when

sealed-bid auctions with an identical reserve price. The ring's designated winner submitted a bid at (or just below) the reserve price, while the others submitted slightly higher bids. The government's successful deterrence strategy involved removing entry restrictions and consolidating procurement under a centralized authority.

[8]Specifically, trade takes place under the same circumstances under competition and collusion (namely, when a supplier has a cost below the reserve) but, conditional on trade taking place, the buyer always pays the reserve under collusion, whereas it can benefit from a lower price under competition.

society would be better off if collusion were accommodated.

In addition to studying the sustainability of collusion, our focus on market allocations enables us also to study the initiation of collusion. In our setup, a supplier can signal its intent to collude by bidding at the reserve—a credible signal that the supplier had a low enough cost to compete, but chose not to do so, thus indicating a natural allocation of the market. We find that, as long as bids are publicly observed, it is profitable to initiate collusion in this way whenever collusion is sustainable. Accounting for initiation enables us to revisit the common wisdom on classic defensive measures.

Bid rigging through the use of allocation schemes has received considerable attention in the economics literature (see, e.g., Harrington, 2006; Marshall and Marx, 2012).[9] Indeed, Stigler (1964, p. 46) recognizes the effectiveness of customer allocations as a collusive scheme, noting that, relative to fixing market shares, "Almost as efficient a method of eliminating secret price-cutting is to assign each buyer to a single seller." The possibility of deterring bidding rings through defensive measures has also has been extensively studied. Our findings related to auction formats and information disclosure are broadly consistent with the literature (e.g., Klemperer, 2002; Kovacic et al., 2006; Marshall and Marx, 2009; Kumar et al., 2015; Marshall et al., 2014; Marx, 2017), and those on the ability of aggressive reserves to deter bid rigging echo existing results (e.g., Graham and Marshall, 1987; Thomas, 2005; McAfee and McMillan, 1992; Kirkegaard, 2005; Larionov, 2021).[10] However, much less work has been done on the costs of implementing defensive measures and the optimal deployment of defensive measures taking into account both the benefits and costs of doing so, which is the focus of our paper.

An exception is Zhang (forth.), who studies optimal collusion in a repeated first-price auction setup, and finds that lower reserves can be used to deter collusion, and that the auctioneer may nevertheless optimally choose to accommodate rather than deter collusion. Our paper differs in that we focus on a setup with multiple markets and collusion based on a market allocation, allowing us to study, among other things, the value of asymmetric reserves across markets and the possibility of coordination (or miscoordination) of reserves across markets. Furthermore, we analyze the effects of deterrence strategies on social surplus and discuss policy implications.

Our finding that initiating collusion may sometimes be more difficult than sustaining it resonates with the discussion by Green et al. (2015) of the challenges that firms face

[9] Rey and Stiglitz (1995) show that exclusive territories, which are a type of market allocation, can reduce competition. Recent empirical evidence of bid rigging based on geographic market allocations is found in Barrus and Scott (2020). See Kawai et al. (forth.) on using bid rotation and incumbency to detect collusion.

[10] Abdulkadiroğlu and Chung (2003) consider auction design in the face of colluding bidders that optimize their collusive mechanism based on the auction design. In contrast, we take as given a collusive mechanism based on market allocation.

when trying to initiate collusion.[11] Finally, our analysis also relates to the literature on multimarket contact. In a setting in which firms can collude symmetrically in every market, Bernheim and Whinston (1990) show that multimarket contact helps collusion only if markets are asymmetric.[12] In contrast, we consider a procurement setting in which the winner takes all; as a result, multimarket contact supports collusion even when the markets are symmetric.

The remainder of the paper is organized as follows. Section 2 contains the baseline model. We analyze defensive measures in Section 3 and tradeoffs between accommodating and fighting collusion in Section 4. We consider extensions in Section 5 before discussing in Section 6 social welfare effects and implications for competition policy. Section 7 concludes the paper.

2 Baseline model

We are interested in firms' ability to collude by allocating markets among themselves, and in the factors that may affect their incentives to initiate and sustain such collusion. To this aim, we consider a discrete-time, infinite-horizon setting with two buyers operating in separate markets. A market could correspond to a distinct customer segment, product, service, or geographic area. Because each buyer corresponds to its own market, we use the terms buyer and market interchangeably.

2.1 Setup

There are two suppliers who, at the beginning of each period, draw their costs of serving any market in that period from a distribution, which is denoted by G and has finite, positive density g over the support $[\underline{c}, \overline{c}]$.

Whether a supplier's cost is the same for both markets or whether it is market specific does not affect the analysis. For the sake of exposition, we adopt the former interpretation. Cost draws are independent across suppliers and time, and are the suppliers' private information.

In every period, each buyer wishes to make a purchase, for which it has value $v > \underline{c}$. Buyers rely on first-price auctions, with a reserve equal to $r \in (\underline{c}, \overline{c}]$—reserves outside this range are dominated for the buyers. We assume that, in each period, suppliers bid simultaneously, both within and across procurements, and that bids are publicly observed before the next procurement. In a one-shot setting, the Bayes Nash equilibrium is unique

[11] Empirical evidence on initiation of collusion through price signals is also provided by, for example, Alé-Chilet (2017) and Byrne and de Roos (2019).

[12] Byford and Gans (2014) compare collusion within versus across markets in a similar setup.

(Lebrun, 1999). All agents are risk neutral with quasi-linear utility and discount the future according to the common discount factor $\delta \in [0,1)$. All of the above is common knowledge.

Let

$$\pi^m(c) \equiv \max\{0, r-c\}$$

denote a supplier's monopoly payoff per market given cost c, and let

$$\overline{\pi}^m \equiv \mathbb{E}_c\left[\pi^m(c)\right] = \int_{\underline{c}}^{r} G(c)dc$$

denote a supplier's expected monopoly payoff per market.

Similarly, let $\pi^c(c)$ denote the expected competitive payoff per market for a supplier with cost c, that is,

$$\pi^c(c) \equiv \mathbb{E}_{\tilde{c}}\left[\max\{0, \min\{r, \tilde{c}\} - c\}\right],$$

where $\tilde{c}$ denotes the rival's cost, and let

$$\overline{\pi}^c \equiv \mathbb{E}_c\left[\pi^c(c)\right] = \int_{\underline{c}}^{r} G(c)\left[1 - G\left(c\right)\right]dc$$

denote a supplier's expected competitive payoff per market. As is clear from these expressions, competitive payoffs are lower than monopoly payoffs.

The supplier's benefit from collusion is then

$$B \equiv \overline{\pi}^m - \overline{\pi}^c = \int_{\underline{c}}^{r} G^2(c)dc, \tag{1}$$

which is positive and increasing in r.

Figure 1 displays the suppliers' payoff and expected payoff functions when costs are uniformly distributed over $[0,1]$.

Remark: sales auctions. For the sake of exposition, we present our analysis in the context of procurement auctions; however, it applies equally to the case of sales auctions when suppliers repeatedly sell scarce resources, each one constituting a "market," at discrete points in time over an infinite horizon (e.g., annual wine auctions).[13]

[13]For instance, assume that two buyers privately and independently draw their values from distribution G over $[\underline{v}, \overline{v}]$, and two suppliers have a reservation value of $r \in (\underline{v}, \overline{v})$. In each market, for a buyer with value v the monopsony payoff is then $\pi^m(v) \equiv \min\{0, v-r\}$, whereas the competitive payoff is $\pi^c(v) \equiv \mathbb{E}_{\tilde{v}}\left[\max\{0, v - \max\{\tilde{v}, r\}\}\right]$.

(a) Monopoly and competitive payoffs (b) Monopoly and competitive expected payoffs

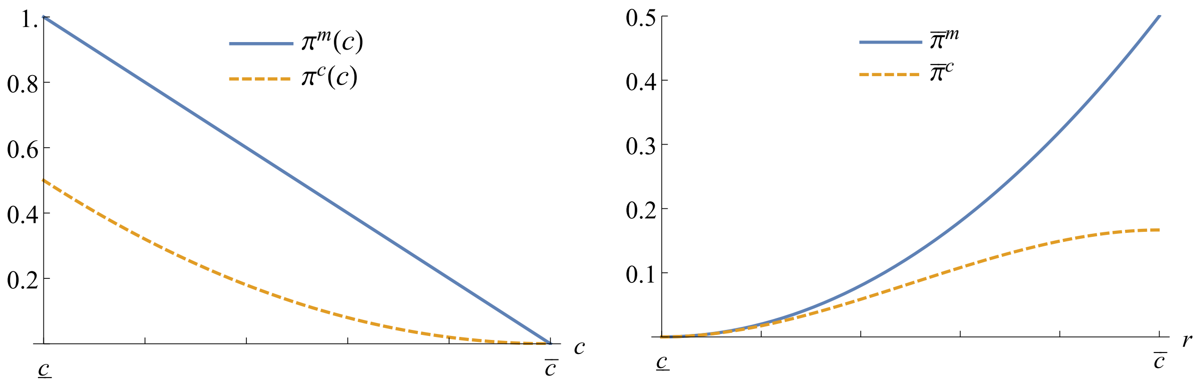


Figure 1: Expected payoffs under monopoly and competition when costs are uniformly distributed over $[0,1]$. Panel (a) assumes $r = \overline{c}$ and varies c from $\underline{c}$ to $\overline{c}$. Panel (b) takes expectations over c as r varies from $\underline{c}$ to $\overline{c}$.

2.2 Market allocation

A market allocation scheme assigns each market to a designated supplier and specifies that the other supplier should not bid less than the reserve in that market. When the reserve is less than $\overline{c}$, it is, however, both efficient and profitable to allow the other supplier to trade at the reserve when the designated supplier's own cost exceeds the reserve. Hence, in what follows, we consider the following market allocation scheme: (i) the designated supplier bids slightly below the reserve when its cost lies below it, and at cost otherwise; and (ii) the non-designated supplier bids the reserve whenever its cost lies below it, and bids at cost otherwise;[14] (iii) any deviation results in competitive conduct thereafter. We assume away the possibility of transfers between the suppliers. When considering attempts by suppliers to signal the initiation of collusion, we focus on signaling by bidding the reserve, as this provides a credible signal of the initiator's intent.

Under a market allocation, the designated supplier obtains the monopoly profit and the other supplier, when its cost is c, obtains an interim expected payoff equal to

$$\pi^n(c) \equiv [1 - G(r)]\, \pi^m(c)\,,$$

which accounts for the probability $1 - G(r)$ that the designated supplier's cost exceeds r.

[14]Specifying that the non-designated supplier should not bid at all (or, equivalently, bid above the reserve) would reduce expected profits, which also makes collusion more difficult to sustain, but would not affect our insights about the timing of purchases or the auction format. For further analysis of profitable tacit collusive schemes without explicit communication, see Skrzypacz and Hopenhayn (2004).

The non-designated supplier's expected payoff is therefore

$$\overline{\pi}^n \equiv \mathbb{E}_c\left[\pi^n\left(c\right)\right] = \left[1 - G(r)\right]\overline{\pi}^m,$$

which involves some sacrifice: the cost of collusion for the non-designated supplier is given by

$$C \equiv \overline{\pi}^c - \overline{\pi}^n = \int_{\underline{c}}^{r} [G(r) - G(c)]G(c)dc, \tag{2}$$

which is positive and increasing in r.

The market allocation is profitable for the suppliers if and only if:[15]

$$B > C. \tag{3}$$

Thus, we say that a market is always "at risk" for a market allocation if (3) holds for all $r \in (\underline{c}, \overline{c}]$. We have:

Lemma 1. *The market is always at risk for a market allocation if the distribution G has a monotone hazard rate (i.e., if $G(c)/g(c)$ is increasing in c).*

Proof. See Appendix A.1.

Because the designated bidder is not always the more efficient supplier, total profits may be greater under competition than under collusion. However, Lemma 1 shows that a market is at risk whenever the distribution $G(c)$ is log-concave.[16] The family of distributions with this property is large and includes most of the "standard" distributions such as the uniform, normal, exponential, power, and extreme value distribution.[17] From now on, we focus on markets that are indeed at risk.

2.3 Scope for collusion

We analyze here the critical discount factor required to initiate and sustain a market allocation when a market is at risk. We start with sustainability, and first derive the

[15] Although we focus on symmetric suppliers, our results are not knife edged in the sense that asymmetries between the suppliers' distributions can be accommodated. For example, for asymmetric suppliers, (3) becomes $\min\{\overline{\pi}_1^m + \overline{\pi}_2^n, \overline{\pi}_2^m + \overline{\pi}_1^n\} > \overline{\pi}_1^c + \overline{\pi}_2^c$, where, for $k \neq h \in \{1, 2\}$, $\overline{\pi}_k^m = \int_{\underline{c}}^{r} G_k(c)dc$, $\overline{\pi}_h^n = [1 - G_k(r)]\overline{\pi}_h^m$, and $\overline{\pi}_k^c = \int_{\underline{c}}^{r} [1 - G_h(c)]G_k(c)dc$.

[16] This follows from $[\ln G(\cdot)]' = g(\cdot)/G(\cdot)$, which implies that log-concavity of G amounts to G/g being increasing.

[17] See Bagnoli and Bergstrom (2005, Table 1) for a more comprehensive list.

collusive and deviation payoffs. In any given period and given its cost realization c, a supplier has an expected payoff from collusion of

$$\Pi\left(c\right) \equiv \pi^{m}\left(c\right) + \pi^{n}\left(c\right) + \frac{\delta}{1-\delta}(\overline{\pi}^{m} + \overline{\pi}^{n}).$$

Obviously, a supplier cannot gain from deviating when its cost exceeds the reserve. When instead the supplier's cost c lies below r, the optimal deviation consists of slightly undercutting the designated supplier's target price, and it yields an expected profit (almost) equal to the monopoly profit, $\pi^{m}(c)$. As the deviation triggers reversion to competitive bidding in both markets thereafter, the deviation yields a total expected payoff of

$$\tilde{\Pi}\left(c\right) \equiv 2\pi^{m}(c) + \frac{\delta}{1-\delta}2\overline{\pi}^{c}.$$

Collusion is sustainable when suppliers have no incentives to deviate, that is, $\Pi\left(c\right) \geq \tilde{\Pi}\left(c\right)$ for every cost realization c. These conditions are satisfied when the loss from reverting to competition in the future (which thus does not depend on the current cost c) offsets the short-term gain from a deviation, $\pi^{m}\left(c\right) - \pi^{n}\left(c\right)$. Because this gain decreases with the supplier's cost,[18] the most stringent sustainability condition can be expressed as

$$L\left(\delta\right) \geq S,$$

where

$$S \equiv \pi^{m}\left(\underline{c}\right) - \pi^{n}\left(\underline{c}\right) = G\left(r\right)\left(r - \underline{c}\right) \tag{4}$$

represents the *short-term stake*, namely, the gain from a deviation for a supplier that has the lowest possible cost, $\underline{c}$, whereas

$$L\left(\delta\right) \equiv \frac{\delta}{1-\delta}\left(\overline{\pi}^{m} + \overline{\pi}^{n} - 2\overline{\pi}^{c}\right), \tag{5}$$

represents the *long-term stake*, which is positive and increases from 0 to infinity as δ increases from 0 to 1. Hence, collusion is sustainable if and only if the discount factor is high enough, namely, $\delta \geq \hat{\delta} \equiv L^{-1}\left(S\right)$. From (4) and (5), the threshold $\hat{\delta}$ is characterized by

$$\frac{\hat{\delta}}{1-\hat{\delta}} = \frac{\pi^{m}\left(\underline{c}\right) - \pi^{n}\left(\underline{c}\right)}{\overline{\pi}^{m} + \overline{\pi}^{n} - 2\overline{\pi}^{c}}. \tag{6}$$

We now turn to the incentives to *initiate* a market allocation through a unilateral decision to stop competing for one of the buyers. A complete withdrawal fails to signal the

[18] For $c < r$, the derivative of $\pi^{m}(c) - \pi^{n}(c)$ with respect to c is equal to $-G(r) < 0$.

seller's willingness to initiate a market allocation because it is consistent with competitive bidding in the event that the supplier, facing a cost exceeding the reserve, cannot be profitably active anyway. By contrast, when its cost lies below the reserve, the initiator can credibly signal its intention by bidding the reserve. In this way, the initiator signals that it is prepared to supply at that price if the other supplier's cost happens to exceed the reserve, implying that it could have chosen to compete, but decided instead to confer market power to the other supplier.

Assuming that collusion is sustainable, bidding the reserve to initiate it involves a short-term sacrifice but increases future profits. The long-term stake amounts to switching from competition to collusion in future tenders, and is therefore again given by (5). The short-term sacrifice amounts to forgoing the competitive profit and instead obtaining the profit expected from bidding the reserve when the other supplier bids competitively; this sacrifice is therefore equal to $\pi^c(c) - \pi^n(c)$, which decreases with the initiator's cost. It follows that initiation is profitable whenever it is so for the lowest possible cost, which amounts to

$$L(\delta) \geq \hat{S},$$

where

$$\hat{S} \equiv \pi^c(\underline{c}) - \pi^n(\underline{c})$$

represents the *short-term stake for initiation*, namely, the initial profit sacrifice for a supplier that has the lowest possible cost, $\underline{c}$. Because $\pi^c(\underline{c}) < \pi^m(\underline{c})$, the short-term stake is lower for initiation:[19]

$$\hat{S} < S.$$

This establishes the following proposition:

Proposition 1. *Initiating collusion (by bidding the reserve in one market) is strictly profitable whenever it is sustainable, which is the case if and only if $\delta \geq \hat{\delta}$, given by* (6).

The intuition is as follows. A supplier faces a similar tradeoff when deciding whether to initiate a market allocation, or whether to stick to it: in both cases, the tradeoff involves higher future profits and a short-term sacrifice. The long-term stake corresponds to the difference between collusive and competitive profits and is thus the same for the two decisions. By contrast, the short-term sacrifice is lower for initiating collusion, where the supplier faces a competitive rival, than for sticking to collusion, where it faces a designated supplier that bids close to the reserve. Specifically, initiating or sticking to collusion

[19]Sustainability conditions must hold for *every* cost realization, including the lowest one. By contrast, collusion could be initiated for some cost realizations even if it could not be initiated for the lowest cost realization; insisting that initiation must be profitable for every cost realization is thus conservative and may overstate its difficulty.

yields the same payoff, $\pi^n(c)$, but *not initiating* collusion only yields the competitive profit, $\pi^n(c)$, whereas *deviating* from collusion yields the monopoly profit, $\pi^m(c)$. It follows that initiating collusion is profitable whenever it is possible to sustain it. Proposition 1 implies that, even when accounting for the need to initiate collusion, the key conditions determining the feasibility of collusion remain those driving sustainability; that is, collusion can both be profitably initiated and sustained if and only if $\delta \geq \hat{\delta}$.

2.4 Effects of collusion

When the suppliers bid competitively and face a reserve $r \in [\underline{c}, \overline{c}]$, a buyer's payoff is

$$U^{Comp}(v, r) \equiv 2(v - r)[1 - G(r)]G(r) + 2 \int_{\underline{c}}^{r} (v - c)G(c)g(c)dc, \tag{7}$$

where $2[1-G(r)]G(r)$ is the probability that exactly one cost draw is above and one below the reserve, and $2G(c)g(c)$ is the probability density function of the higher cost (with cumulative distribution function $G^2(c)$). Under collusive bidding, the buyer's expected payoff is instead

$$U^{Coll}(v, r) \equiv (v - r)\hat{G}(r), \tag{8}$$

where $\hat{G}(c) \equiv 1 - [1 - G(c)]^2$ is the cumulative distribution function of the lower cost.

The buyer is strictly better off with competitive rather than collusive bidding: in both cases, trade takes place under the same condition (namely, whenever at least one supplier's cost lies below the reserve); however, when it does take place, the buyer always pays the reserve in case of collusion, whereas it benefits from a lower price under competitive bidding. Integrating (7) by parts, the "competition benefit" can be expressed as:

$$U^{Comp}(v, r) - U^{Coll}(v, r) = \int_{\underline{c}}^{r} G^2(c)dc = B. \tag{9}$$

That is, the harm to the buyer coincides with the benefit for the designated supplier, defined by (1). The reason is that the designated supplier benefits from collusion if and only if both costs are below the reserve, which are precisely the instances in which the buyer benefits from competitive bidding. In other words, from the perspective of the designated supplier and the buyer, the market allocation scheme is merely a transfer. Because the market allocation scheme is inefficient, it follows that the reduction in social welfare that it causes is entirely borne by the non-designated supplier.[20]

[20]Formally, the cost of collusion for the non-designated supplier is C. The change in social welfare is equal to the change in the cost of production, which is $\int_{\underline{c}}^{r} cdG(c) + [1 - G(r)] \int_{\underline{c}}^{r} cdG(c)$ under collusion and $\int_{\underline{c}}^{r} cd\hat{G}(c)$ under competition. Integrating by parts and simplifying confirms that the difference in

Proposition 2. *A market allocation by suppliers harms the buyer and reduces social welfare.*

3 How to fight collusion

Market allocation schemes can be difficult to detect, especially given colluding agents' incentives to disguise their conduct. However, suspicions may be aroused by certain bidding patterns, such as bids that are consistently close to the reserve, or a supplier withdrawing from a market. This leads us consider now the various tools that a buyer can use to "fight back" when suspecting collusion—buyers may differ in their ability to use these tools based, for example, on their sophistication, commitment ability, and purchase volumes.

Three key tools for fighting collusion identified by the literature are auction format, timing of purchasing, and reserves.[21] With regard to auction format, the literature recognizes that first-price sealed-bid auctions are more robust to collusion than second-price or ascending-bid auctions, because in the latter cases, the designated supplier can bid at cost (and yet obtain the monopoly price), which reduces the gain from deviations. This continues to be true in our setup. To see this, suppose that buyers rely instead on second-price sealed-bid auctions and consider the same market allocation as before, except that the designated supplier now bids at cost. This limits the other supplier's gain from a deviation and reduces the short-term stake, from S to $\hat{S}$; as a result, collusion is sustainable whenever it is profitable to initiate it, leading to:

Proposition 3. *Compared with first-price auctions, second-price auctions make it easier for suppliers to sustain a market allocation.*

Proof. See Appendix A.2.

Consequently, our focus on first-price sealed-bid procurements, in addition to being motivated by common practice, is consistent with the buyer adopting the most effective auction format in the fight against collusion.

With regard to the timing of purchases, the literature recognizes that simultaneous procurements are less prone to collusion than sequential procurements, as deviations can be punished sooner in the latter case. Thus, our analysis focuses on the case in which the two buyers synchronize their procurements. To see that buyers indeed have an incentive to do so, suppose that in the second market tenders take place with a lag $\tau \in [0, 1)$; that

costs is indeed C.

[21] For a wider ranging discussion of tools for fighting bid rigging, see, e.g., Cassady (1967); Graham et al. (1996); Thomas (2005); Kovacic et al. (2006); Albano et al. (2006).

is, tenders take place at $t = 0, 1, ...$ in market 1 and at $t = \tau, \tau + 1, ...$ in market 2. The case of perfectly synchronous purchasing thus corresponds to $\tau = 0$, whereas the case of perfectly staggered purchasing corresponds to $\tau = 1/2$. Then we have:

Proposition 4. *A market allocation is most difficult to sustain with perfectly synchronous purchasing and easiest to sustain with perfectly staggered purchasing.*

Proof. See Appendix A.3.

The intuition for Proposition 4 is that maximizing the gap between *successive* tenders helps to fight collusion because it delays punishment in case of a deviation, and this is best achieved with synchronous purchases. Therefore, in what follows we maintain our focus on simultaneous procurement.

Turning to the tool of reserves, we first establish that optimal reserves in the face of collusion are more aggressive than optimal reserves under competition. To this end, we characterize the optimal reserve for a setting in which the buyers take suppliers' conduct as given. That is, the suppliers either collude or compete in every period, and anticipating the suppliers' conduct, the buyers optimally set their reserves. In either case, the buyers face a stationary context and their optimal policy is therefore also stationary.

Let $r^C(v)$ denote the optimal reserve in the absence of collusion and $r^A(v)$ denote the optimal reserve in the presence of collusion, which we refer to as the *competitive reserve* and *accommodation reserve*, respectively. The reserve $r^C(v)$ maximizes $U^{Comp}(v, r)$ and corresponds to the monopsony price that a buyer would charge, given its valuation v, if it were to face a single supplier with cost distribution G.[22] Perhaps more surprisingly, $r^A(v)$, which maximizes $U^{Coll}(v, r)$, is the reserve that would be optimal if the two suppliers had merged (Loertscher and Marx, 2019), a situation equivalent to perfect collusion. The market allocation scheme does not achieve perfect collusion because production does not necessarily occur at the lowest cost; however, this is immaterial for the buyer: what matters is whether production occurs, in which case the buyer pays the reserve, and it occurs in exactly the same instances as under perfect collusion, namely, when at least one supplier has a cost below the reserve. It follows that $r^A(v)$ is the monopsony price that a buyer would charge when facing a single supplier with the *enhanced* cost distribution $\hat{G}$, which corresponds to the lower of two draws. This distribution has a lower reverse hazard rate than the original distribution G, which makes the supply less elastic and leads to a more aggressive reserve.

The fact that the buyer always pays the reserve when it faces collusion suggests that it should set a more aggressive reserve in that case. A simple revealed preference argument

[22] As is well known, the optimal reserve does not depend on the number of suppliers; hence, it maximizes the monopsony payoff $G(r)(v - r)$.

confirms that intuition, by establishing that the competition benefit B must be greater for $r^C(v)$ than for $r^A(v)$ (see Appendix A.5 for a formal derivation). Because B increases with r, it follows that $r^C(v) \geq r^A(v)$. Inspection of the payoffs under collusion and competition shows that this ranking is strict:[23]

Proposition 5. *A buyer's optimal reserve is strictly more aggressive when facing colluding rather than competing suppliers:* $r^A(v) < r^C(v)$.

Proof. See Appendix A.5.

Proposition 5 implies that a buyer with concerns that its suppliers might engage in a market allocation may have an incentive to use a more aggressive reserve.[24] This is consistent with the advice given to practitioners that "[w]hen collusion among suppliers is suspected, the reserve price should be set at a lower value than in the absence of collusion. This simple policy forces the bidding ring to submit a lower bid" (Albano et al., 2006, p. 282). The optimality of more aggressive reserves in the face of collusion suggests that the inefficiency associated with collusive conduct includes not only possible inefficient allocation among ring members, but also the possibility that collusion induces buyers to modify their procurement practices to be less efficient.

Remark: on the nature of collusion. The collusive scheme that we consider enables the non-designated supplier to step in when trade would otherwise not occur. An alternative collusive scheme that requires the non-designated supplier to withdraw regardless of its realized costs amounts instead to a reduction in the number of suppliers. Because the optimal reserve is independent of the number of bidders, buyers' optimal reserves are then the same as under competition. Hence, the optimal reserve depends not only on whether suppliers collude, but also on the nature of the collusive scheme; it may, moreover, decrease as collusion becomes more efficient.

More aggressive reserves can also serve a buyer faced with the threat of collusion by making collusion less profitable for the suppliers and thereby unsustainable. Adding the reserve as an explicit argument to $\hat{\delta}$, B, C, and S, the discount factor threshold above

[23]It is interesting to note the similarity and subtle difference relative to Blume and Heidhues (2004). Their analysis implies that in a one-shot, second-price auction, any reserve below $\bar{c}$ eliminates supra-competitive Bayes Nash equilibria, whereas with $r \geq \bar{c}$, there are a continuum of noncooperative Bayes Nash equilibria. In our setting, the optimal reserve in the face of collusion is more aggressive than with competitive bidding—and always strictly lower than the reserve.

[24]Proposition 6 shows that for a range of reserves, the critical discount factor is decreasing in the reserve; however, that monotonicity need not hold for all reserves. For example, the Beta distribution with parameters $(1/2, 1/2)$ generates a critical discount factor that is increasing in r for r close to $\bar{c}$.

which collusion is profitable, $\hat{\delta}(r)$, is such that

$$\frac{\hat{\delta}(r)}{1-\hat{\delta}(r)} = \frac{S(r)}{B(r)-C(r)}. \tag{10}$$

While decreasing the reserve reduces suppliers' profits under both collusion and competition, for low enough reserves, the *expected* profitability of collusion, $B(r)-C(r)$, decreases at a faster rate than the deviation gain for an efficient supplier, $S(r)$; this is because the reserve is likely to prevent future trade, whereas an efficient supplier always trades. It follows that, for sufficiently low reserves, the threshold discount factors are decreasing in the reserve, and a sufficiently aggressive reserve can prevent a market allocation from being sustainable.[25]

Proposition 6. *Assuming $g(\underline{c}) > 0$, there exists $\overline{r} \in (\underline{c}, \overline{c}]$ such that in the range $r \in [\underline{c}, \overline{r}]$, the critical discount factor $\hat{\delta}(r)$ is decreasing in the reserve; furthermore, this threshold tends to 1 as r tends to $\underline{c}$.*

Proof. See Appendix A.4.

The result of Proposition 6 that sufficiently aggressive reserves can deter collusion by reducing its profitability resonates with the advice to practitioners in a sales auction context that: "In some auctions, the most effective way of overcoming a buyers' ring is to set a reserve price, prohibiting sale of the item below its estimated value and thus impairing the profitability of a collusive operation" (Cassady, 1967, p. 191).

4 To accommodate or fight collusion?

In the previous section, we have considered various tools that buyers can use to "fight back" when they suspect collusion. The costs of these fighting measures raise the question of whether it is worth using them to deter collusion. In particular, the tool of using a more aggressive reserve to respond to and potentially deter collusion clearly comes at a cost to the buyer because the reserve affects the probability of trade.

While the literature has recognized the role of reserves to mitigate the effects of collusion and potentially deter collusion, little attention has been paid to the optimal deterrence reserves or the possibility that buyers aware of collusion may optimally choose not to deter it. To address this issue, we now study the optimal reserve policy, taking into

[25] Indeed, for $r = \underline{c}$ we have $S'(\underline{c}) = B'(\underline{c}) = C'(\underline{c}) = 0$ but $S''(\underline{c}) = g(\underline{c}) > 0 = B''(\underline{c}) = C''(\underline{c})$. The assumption that $g(\underline{c}) > 0$ can be dropped at the cost of an extended proof—because we assume that $g(c)$ is positive for $c \in (\underline{c}, \overline{c})$, there is some finite integer k such that the k-th derivative of $g(\underline{c})$ is positive.

account its impact on the scope for collusion. For the sake of exposition, we adopt the tie-breaking rule that collusion occurs whenever it is sustainable.

We begin in Section 4.1 by considering the case of an integrated buyer setting the same reserve in both markets. Then in Section 4.2, we allow that buyer to set different reserves across the two markets and show that optimal deterrence reserves are indeed asymmetric. Finally, in Section 4.3, we expand the analysis to allow for independent buyers and consider the Nash equilibrium of a simultaneous reserve-setting game.

4.1 Symmetric reserves

We begin by assuming that a single integrated buyer sets a common, stationary reserve across the two markets.[26]

From now on, we assume that a buyer's payoff is strictly quasiconcave in r for any v. A sufficient condition is that virtual costs are strictly increasing in c, where the *virtual costs* associated with the distributions G and $\hat{G}$ are

$$\Gamma(c) \equiv c + \frac{G(c)}{g(c)} \quad \text{and} \quad \hat{\Gamma}(c) \equiv c + \frac{\hat{G}(c)}{\hat{g}(c)} = c + \frac{G(c)}{g(c)} \frac{2 - G(c)}{2[1 - G(c)]},$$

where $\hat{g}$ is the density associated with $\hat{G}$. By construction, $\hat{\Gamma}(\underline{c}) = \Gamma(\underline{c}) = \underline{c}$ and $\hat{\Gamma}(c) > \Gamma(c) > c$ for $c > \underline{c}$. Because

$$\frac{\partial U^{Comp}}{\partial r}(v, r) = \hat{g}(r) \left[v - \Gamma(r)\right] \quad \text{and} \quad \frac{\partial U^{Coll}}{\partial r}(v, r) = \hat{g}(r)[v - \hat{\Gamma}(r)],$$

the optimal reserves are then given by

$$r^C(v) \equiv \min\left\{\Gamma^{-1}(v), \overline{c}\right\} \quad \text{and} \quad r^A(v) \equiv \hat{\Gamma}^{-1}(v) < \overline{c},$$

where the last inequality follows from $\hat{\Gamma}(\overline{c}) = \infty$.

The maximal value from competition or from accommodating collusion, respectively, is given by:

$$U^C(v) \equiv U^{Comp}(v, r^C(v)) \text{ and } U^A(v) \equiv U^{Coll}(v, r^A(v)).$$

By construction, $U^C(v) > U^A(v)$; it is therefore optimal to deter collusion using symmetric reserves if this can be achieved with a reserve that is close enough to $r^C(v)$.

[26] The assumption that the reserve, once set, is fixed forever is arguably restrictive. Applying the insights of Frezal (2006) to our setting, collusion can be deterred almost without cost if the buyer can commit to the nonstationary policy of setting the reserve $r^C(v)$ for a predetermined, large number of periods, and preventing trade subsequently for a sufficiently large number of periods. By choosing both numbers large enough, the discounted cost of no trade will be negligible. This nonstationary policy would, however, require a particularly strong form of commitment.

For the purpose of this analysis, we assume that the threshold $\hat{\delta}(r)$ is decreasing in the reserve.[27] Obviously, as long as:[28]

$$\delta < \delta^C(v) \equiv \hat{\delta}(r^C(v)) \in (0,1), \tag{11}$$

it is optimal for the buyer to set the competitive reserve. Using terminology from the literature on entry deterrence, collusion can be said to be *blockaded* because under condition (11), the optimal reserve absent collusion, $r^C(v)$, is sufficiently low to deter collusion.

When instead $\delta \geq \delta^C(v)$, deterring collusion requires a lower reserve than $r^C(v)$.[29] Because the competitive payoff $U^{Comp}(v,r)$ is strictly quasi-concave in the reserve r, the optimal deterrence strategy then consists of setting the reserve (slightly below) the *symmetric deterrence reserve* $r_S^D(\delta) \equiv \hat{\delta}^{-1}(\delta)$.[30] Deterring collusion remains optimal in the range

$$\delta^C(v) \leq \delta \leq \hat{\delta}(r^A(v)),$$

where $r_S^D(\delta)$ lies between $r^A(v)$ and $r^C(v)$, implying that the *symmetric deterrence payoff*, $U_S^D(v,\delta)$, satisfies:

$$U^A(v) = U^{Coll}(v, r^A(v)) < U^{Comp}(v, r^A(v)) \leq U^{Comp}(v, r_S^D(\delta)) \equiv U_S^D(v,\delta),$$

where the first inequality stems from the benefit of competition and the last one from the quasi-concavity of the buyer's competitive payoff.

When instead $\delta > \hat{\delta}(r^A(v))$ the symmetric deterrence reserve $r_S^D(\delta)$ sacrifices a substantial amount of trade and may not be in the buyer's interest. A buyer using symmetric reserves then faces a non-trivial tradeoff between *accommodating* collusion with $r^A(v)$ and *deterring* it with $r_S^D(\delta)$. By continuity, deterrence remains optimal as long as δ remains close to $\hat{\delta}(r^A(v))$; however, as δ further increases, deterring collusion eventually becomes too costly—in particular, as δ tends to 1, $r_S^D(\delta)$ tends to $\underline{c}$ and the deterrence payoff thus tends to vanish. It follows that for δ high enough, namely:

$$\delta \geq \delta_S^A(v) \equiv \inf\{\delta \mid U_S^D(v,\delta) \leq U^A(v)\} \in (\hat{\delta}(r^A(v)), 1), \tag{12}$$

[27] This holds if, for example, $G(c) = c^x$ for some $x > 0$; furthermore, by Proposition 6 it always holds for sufficiently low reserves.

[28] Recall that $\hat{\delta}(r) \in (0,1)$ for any $r \in (\underline{c}, \overline{c}]$.

[29] As always with deterrence, maintaining a threat begs the question of credibility; although we do not model this explicitly, the repeated-interaction setting under scrutiny may help the buyer to maintain a credible threat.

[30] When the reserve is set to the deterrence threshold, collusion is "barely" sustainable; to prevent it for sure, buyers should thus set a reserve below (but arbitrarily close to) the threshold. For the sake of exposition, we will simply say "set the symmetric deterrence reserve" when referring to this strategy.

the buyer will swallow the bitter pill of collusion and set the optimal reserve in the face of collusion. Summing up, we have:[31]

Proposition 7. *There exists $\delta_S^A(v) > \hat{\delta}(r^A(v))$—implying that $\delta_S^A(v) > \delta^C(v)$—such that an integrated buyer setting symmetric reserves optimally sets competitive reserves if $\delta < \delta^C(v)$, deterrence reserves if $\delta^C(v) \leq \delta < \delta_S^A(v)$, and accommodation reserves if $\delta_S^A(v) \leq \delta$.*

Proposition 7 implies that the buyer's optimal reserve is neither monotone nor continuous in the discount factor. As the discount factor increases from zero, the optimal reserve is initially constant at the competitive reserve, $r^C(v)$. When the discount factor becomes large enough (namely, when $\delta \geq \delta^C(v)$), collusion is no longer blockaded; it then becomes optimal to deter collusion by setting the reserve $r_S^D(\delta)$, which is *decreasing* in the discount factor. As the discount factor increases further, eventually deterring collusion becomes too costly (namely, when $\delta = \delta_S^A(v)$, where $U_S^D(v, \delta) = U^{Coll}(v, r^A(v))$), at which point the optimal reserve *jumps up* to the optimal collusive reserve. Figure 2 illustrates this for the case of costs uniformly distributed over $[0, 1]$ and a value $v = 1$.

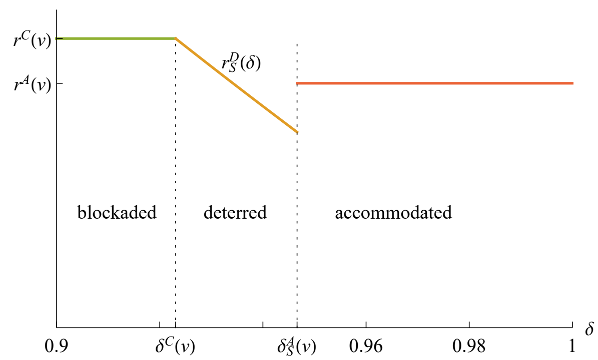


Figure 2: Optimal symmetric reserve as a function of the discount factor under synchronized purchasing and first-price auctions. Assumes that costs are uniformly distributed over $[0, 1]$ and that $v = 1$, implying that $\delta^C(v) = 0.9231$ and $\delta_S^A(v) = 0.9466$.

Remark: on second-price auctions. As already noted, collusion is easier to sustain under second-price auctions than under first-price auctions. Specifically, the long-term stake does not depend on the auction format but the short-term stake does, and is lower for second-price auctions. Because the long-term and short-term stakes are continuous in

[31] In principle, the buyer could also contemplate contingent strategies, such as setting the optimal reserve $r^C(v)$ in case of competitive bidding and using a "grim trigger" strategy of reverting to $r_S^D(\delta)$ forever if the suppliers are caught (or believed to be) colluding. If the suppliers were to consider this strategy as credible, they would refrain from colluding, which would yield the first-best outcome for the buyer.

the reserve, it follows that the symmetric deterrence reserve $r_S^D(\delta)$, which is the largest reserve below which the short-term stake exceeds the long-term one, is lower for a second-price auction than for a first-price auction.[32] In other words, not only are second-price auctions more susceptible to collusion than first-price auctions, in the sense that collusion is sustainable for a larger range of discount factors, but they also require a more aggressive reserve to deter collusion and, thus, a greater distortion of trade away from the efficient level. As a result, a buyer using second-price auctions is more likely to accommodate collusion than one using first-price auctions.

Remark: on secret reserves. Li and Perrigne (2003, p. 189) note that "The theoretic auction literature is still unclear on the rationale for using a random reserve price."[33] However, in our setup, opting for a *secret*, random reserve creates challenges for initiating and sustaining a market allocation. First, secret, random reserves prevent suppliers from signaling their wish to initiate collusion through a bid equal to the reserve, because they do not know what that reserve is. That said, if the reserve is drawn from a distribution with upper bound of the support $\overline{r} < \overline{c}$, then there remains the possibility of signaling initiation with a bid of $\overline{r}$. Second, secret, random reserves inhibit the ability of suppliers to maintain a market allocation while still having positive expected payoffs in their non-designated markets. To see this, note that with a secret, random reserve drawn from a distribution with upper bound of support $\overline{r}$, the only way for the non-designated supplier to ensure that it does not provide meaningful competition for the designated supplier is to bid $\overline{r}$ or above. But in this case, unless it bids exactly $\overline{r}$ and there is an atom in the distribution at that point, it wins with probability zero, and so it has an expected payoff of zero in its non-designated market. As these points suggest, secret, random reserves create challenges for colluding suppliers, and more so if those reserves are drawn from a distribution whose upper support is $\overline{r} = \overline{c}$. Further, the possibility of using secret, random reserves implies that when $r_S^D(\delta) < r^C(v)$ (which, as discussed above, holds for v sufficiently large), a buyer could deter collusion, and obtain a higher expected payoff than with a fixed reserve of $r_S^D(\delta)$, by using a secret reserve that is randomly chosen from $[r_S^D(\delta), r^C(v)]$. By increasing the profitability of deterrence, such a strategy can expand the range of values for which a buyer prefers to deter rather than accommodate collusion.

[32] For example, when costs are uniformly distributed on $[0, 1]$, symmetric reserves must be divided by two to maintain deterrence, from $r_S^D(\delta) = 6(1-\delta)/\delta$ to $r_{S,SPA}^D(\delta) = 3(1-\delta)/\delta$.

[33] Li and Perrigne (2003) estimate that the French forest service had lower profits as a result of using secret, random reserve prices at timber auctions; however, they assume noncooperative bidding. Given evidence of collusion at U.S. timber auctions (see, e.g., Baldwin et al., 1997; Athey and Levin, 2001), the secret, random reserve may have provided benefits in terms of deterring collusion not captured in an analysis based on noncooperative bidding.

4.2 Allowing asymmetric reserves

Although the optimal competitive and accommodation reserves are symmetric, we now show that the optimal deterrence reserves are not—asymmetric reserves provide a more effective way of deterring collusion than do symmetric reserves. If a supplier faces the reserve r_i in its designated market and r_j in the other market, then its short-term stake is given by $S(r_j)$, whereas its long-term stake becomes

$$L(\delta, r_i, r_j) \equiv \frac{\delta}{1-\delta}\left[B(r_i) - C(r_j)\right]. \tag{13}$$

A market allocation is sustainable if and only if

$$L(\delta, r_1, r_2) \geq S(r_2) \text{ and } L(\delta, r_2, r_1) \geq S(r_1).$$

Starting from optimal symmetric deterrence reserves, increasing r_2 while holding r_1 constant increases the short-term stake $S(r_2)$ as well as the cost of collusion $C(r_2)$.[34] This increases buyer's surplus and makes the first of the above conditions more stringent (and, so, the relevant one) and thus still deters the market allocation. It follows that, conditional on deterring a market allocation, it is optimal for an integrated buyer to use asymmetric reserves (see Proposition 9 below for a formal statement).

Of course, once asymmetric reserves are introduced, additional collusive strategies potentially become relevant. In particular, the suppliers may want to avoid having the same one systematically facing the lower reserve over time. A relevant and natural variant on the collusive strategy discussed so far involves rotation, whereby the suppliers switch markets in each round of purchasing.[35] As before, we assume that any deviation in either market triggers a reversal to competition in both markets. The long-term stake for the supplier designated for market i, who will rotate to market j in the next period and then back to market i in two periods, where $i \neq j \in \{1, 2\}$, is

$$L^R(\delta, r_i, r_j) \equiv \frac{\delta}{1-\delta^2}\left\{B(r_j) - C(r_i) + \delta\left[B(r_i) - C(r_j)\right]\right\}. \tag{14}$$

A rotation is sustainable if and only if:

$$L^R(\delta, r_1, r_2) \geq S(r_2) \text{ and } L^R(\delta, r_2, r_1) \geq S(r_1). \tag{15}$$

[34] Indeed, we have $S'(r) = G(r) + g(r)(r - \underline{c}) > 0$ and $C'(r) = \int_{\underline{c}}^{r} G(c)g(r)\,dc > 0$.

[35] Alternatively, the suppliers could randomize over the market assignment. It is however straightforward to check that a deterministic rotation is easier to sustain because the supplier currently assigned to the market with the lower reserve—and, thus, more prone to deviate—is then sure to be assigned to the more profitable market in the next tender.

If $r_1 < r_2$ (resp., $r_1 > r_2$), then the more stringent condition in (15) is the first (resp., second) one.[36] The set of deterrence reserves can therefore be described as

$$\mathcal{D}(\delta) \equiv \mathcal{D}_1(\delta) \cup \mathcal{D}_2(\delta),$$

where

$$\mathcal{D}_i(\delta) \equiv \{(r_1, r_2) \in [\underline{c}, \min\{v, \bar{c}\}]^2 \mid r_i \geq r_j \text{ and } S(r_i) > L^R(\delta, r_j, r_i)\}.$$

As δ increases, the long-term stake increases as well and deterrence thus becomes more difficult—that is, $\mathcal{D}(\delta)$ shrinks. For the sake of exposition, we assume further that $L^R(\delta, r_i, r_j)$ is increasing in r_i, which holds for δ sufficiently close to 1.[37] It follows that the boundary of $\mathcal{D}(\delta)$ is given by

$$\mathcal{B}(\delta) \equiv \mathcal{B}_1(\delta) \cup \mathcal{B}_2(\delta),$$

where

$$\mathcal{B}_i(\delta) \equiv \{(r_1, r_2) \in [\underline{c}, \min\{v, \bar{c}\}]^2 \mid r_i \geq r_j \text{ and } S(r_i) = L^R(\delta, r_j, r_i)\}.$$

The monotonicity assumption on the virtual cost function $\Gamma(\cdot)$ ensures that the buyer's average per-market payoff under competition,

$$\bar{U}^{Comp}(v, r_1, r_2) \equiv \frac{1}{2}\left[U^{Comp}(v, r_1) + U^{Comp}(v, r_2)\right],$$

is strictly quasi-concave. It follows that, whenever collusion is not blockaded, that is, $\mathbf{r}^C(v) \equiv (r^C(v), r^C(v)) \notin \mathcal{D}$, the optimal deterrence reserves lie on the boundary $\mathcal{B}(\delta)$.[38] Thus, conditional on deterring collusion in the region with $r_2 \geq r_1$, it is optimal for the buyer to set the reserves arbitrarily close to the *deterrence reserves* defined by:

$$\mathbf{r}^D(v, \delta) = \left(r_1^D(v, \delta), r_2^D(v, \delta)\right) \equiv \underset{(r_1, r_2) \in \mathcal{B}_2(\delta)}{\operatorname{argmax}} \bar{U}^{Comp}(v, r_1, r_2).$$

[36] See Appendix A.6.

[37] More precisely, it holds for $\delta > C'(r)/B'(r) = g(r)\int_{\underline{c}}^{r} G(c)dc/G^2(r)$. For example, with uniformly distributed costs, this condition is $\delta > 1/2$, which is satisfied for the entire relevant range of discount factors, i.e., discount factors such that collusion is not blockaded, $L^R(\delta, r^C(v), r^C(v)) \geq S(r^C(v))$; for $v = 1$ and uniform [0,1] costs, the relevant range is $\delta > 12/13$.

[38] Moving closer to the boundary of $\mathcal{D}(\delta)$ indeed enables the buyer to increase its payoff: for any $\mathbf{r} = (r_1, r_2) \in \mathcal{D}(\delta)$, there exists $\lambda \in (0, 1)$ such that $\mathbf{r}_\lambda \equiv \lambda\mathbf{r} + (1 - \lambda)\mathbf{r}^C(v) \in \mathcal{D}(\delta)$; because $\mathbf{r}^C(v)$ maximizes the buyer's payoff, the strict quasi-concavity of the buyer's payoff ensures that $\bar{U}^{Comp}(v, \mathbf{r}_\lambda) > \bar{U}^{Comp}(v, \mathbf{r})$.

For comparison, suppose that there is a unique market in which the suppliers try to collude by rotating. As in the case of multiple markets, assume further that such collusion is sustainable if and only if the reserve lies above a *unique-market deterrence* threshold, denoted by $r_U^D(\delta)$. As we show in the proof of Lemma 2, collusion is easier when the suppliers interact in two markets rather than in only one.[39] As a result, deterrence is easier when there is a unique market and can be achieved with a less aggressive reserve.[40]

Lemma 2. *All reserves in the deterrence set $\mathcal{D}(\delta)$ are less than the unique-market deterrence reserve $r_U^D(\delta)$. Furthermore, as δ increases, the set $\mathcal{D}(\delta)$ shrinks strictly and its boundary $\mathcal{B}(\delta)$ varies continuously.*

Proof. See Appendix A.7.

We illustrate $\mathcal{B}_1(\delta)$ and $\mathcal{B}_2(\delta)$ and the deterrence reserves in Figure 3.

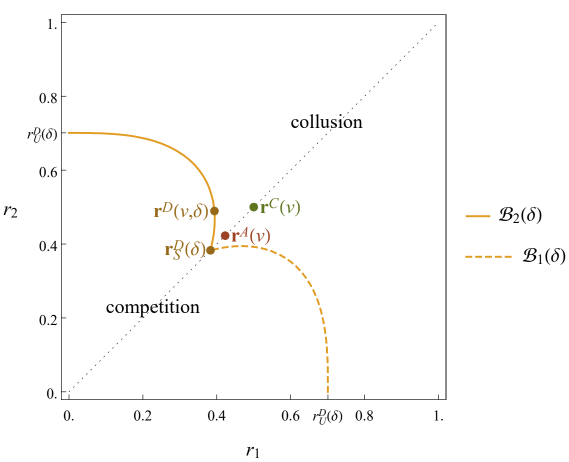


Figure 3: Deterrence boundary and relevant reserves. Assumes that costs are uniformly distributed over $[0, 1]$, $v = 1$, and $\delta = 0.94$.

While the joint profit of the suppliers and the payoff of the buyer are the same under a rotation and a market allocation,[41] as we now show, a rotation is easier to sustain when reserves are asymmetric.

[39] Hence, in our procurement setting where the winner takes all, interacting in multiple symmetric markets increases the scope for collusion. This contrasts with the result of Bernheim and Whinston (1990) that multimarket contact does not facilitate collusion in this case when firms can share each individual market.

[40] For the case of uniformly distributed costs and δ such that collusion is not blockaded, the unique-market deterrence reserve is $r_U^D(\delta) = \frac{6(1-\delta^2)}{\delta(2-\delta)}$. For example, in Figure 3, we have $r_U^D(0.94) = 0.7009$.

[41] The suppliers can moreover share ex ante the expected profit from collusion by randomizing over the initial market designation.

Proposition 8. *When $r_1 \neq r_2$, a rotation is easier for suppliers to sustain than a market allocation.*

Proof. See Appendix A.8.

A rotation does not eliminate the asymmetry among the long-term stakes but changes its direction, with the result that the supplier with the higher short-term stake (namely the supplier currently assigned to the less profitable market) has also the higher long-term stake (as it will be assigned to the more profitable market in the next period). By contrast, that supplier would also have the lower long-term stake in case of a fixed market allocation. Rotation therefore helps to render more symmetrical the incentives of suppliers to deviate but does not completely restore symmetry.

As the next proposition shows, regardless of whether suppliers use a market allocation or rotation, asymmetric reserves are optimal for deterring collusion.

Proposition 9. *Conditional on having to deter collusion, either in the form of a market allocation or a rotation, it is optimal for an integrated buyer to use asymmetric reserves.*

Proof. See Appendix A.9.

Intuitively, starting from the symmetric deterrence reserve, slightly increasing the reserve in one market while reducing the other reserve by the same amount has no impact on the buyer's expected payoff (as the effect of the two reserves offset each other), but improves deterrence by creating asymmetry in the suppliers' incentives to collude. It follows that there is a nearby (asymmetric) modification of the reserves that improves the buyer's expected payoff while preserving deterrence. Hence, when collusion is not blockaded, asymmetric reserves are more effective at deterring it:

Guided by Propositions 8 and 9, in what follows we focus on rotations and characterize the optimal reserves, allowing for asymmetric deterrence reserves. Recalling that $\delta^C(v)$ is the largest discount factor such that there is competition at reserves of $r_1 = r_2 = r^C(v)$, then for $\delta < \delta^C(v)$, setting both reserves equal to $r^C(v)$ is optimal. For $\delta \geq \delta^C(v)$, the buyer can either alter the reserves away from $r^C(v)$ so as to continue deterring collusion, or, alternatively, accommodate it. In the latter case, it is optimal for the buyer to set both reserves equal to $r^A(v)$, which yields a per-market accommodation payoff

$$U^A(v) \equiv U^{Coll}(v, r^A(v)).$$

The buyer's average per-market payoff under the (asymmetric) deterrence reserves is given by

$$U^D(v, \delta) \equiv \bar{U}^{Comp}\left(v, \mathbf{r}^D(v, \delta)\right).$$

It follows from Lemma 2 that $U^D(v,\delta)$ is decreasing in δ and goes to zero as δ goes to one.[42] Let

$$\delta^A(v) \equiv \inf\left\{\delta \in [0,1] \mid U^D(v,\delta) \leq U^A(v)\right\} \tag{16}$$

denote the discount factor threshold above which accommodation is optimal. We then have the following result:

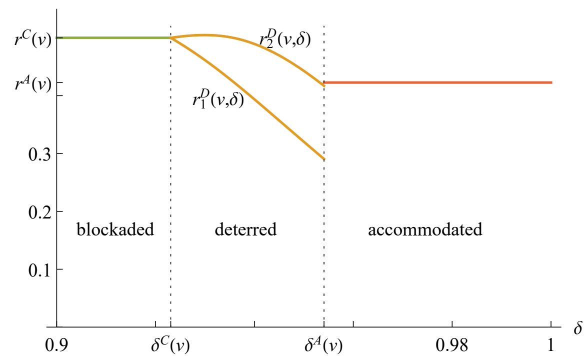
Proposition 10. *There exists $\delta^A(v) > \delta^A_S(v)$ such that an integrated buyer optimally sets competitive reserves if $\delta < \delta^C(v)$, asymmetric deterrence reserves if $\delta^C(v) \leq \delta < \delta^A(v)$, and accommodation reserves if $\delta^A(v) \leq \delta$.*

We illustrate the optimal reserves described in Proposition 10 in Figure 4. As highlighted by Figure 4(a), the optimal deterrence reserves are asymmetric and discontinuous in the discount factor as it increases from the region of deterrence to the region of accommodation.

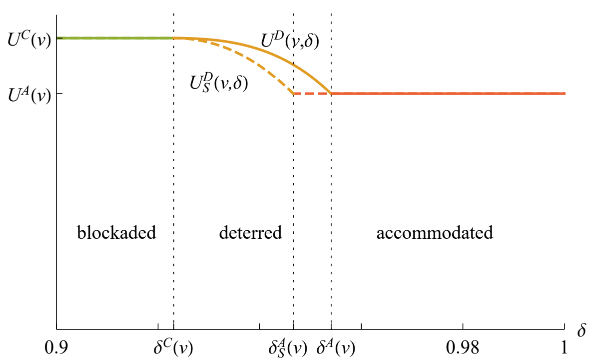
Further, as illustrated in panels (b) and (c) of Figure 4, asymmetric reserves make deterrence less costly for the buyer, who therefore finds it optimal to deter collusion for a greater range of discount factors. Thus, requiring the buyer to use symmetric reserves may induce the buyer to switch from deterrence to accommodation.

[42] As the discount factor goes to 1, the long-term stakes go to infinity for any given positive reserves; hence, to deter collusion, both reserves must go to zero.

(a) Optimal reserves



(b) Buyer payoff with symmetric and asymmetric reserves



(c) Social welfare with symmetric and asymmetric reserves

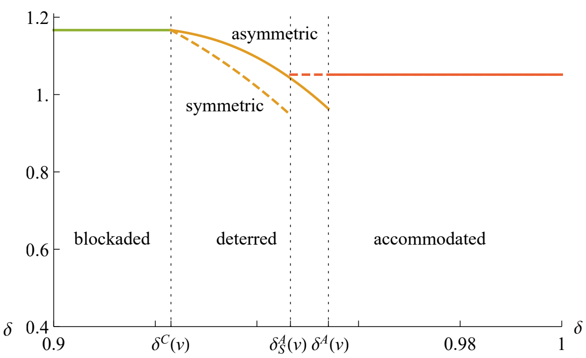


Figure 4: Optimal reserve, expected per-period buyer payoff, and expected per-period social welfare for an integrated buyer as functions of the discount factor under synchronized purchasing and first-price auctions. Dashed lines are for the case in which reserves are constrained to be the same in both markets. Assumes that costs are uniformly distributed over $[0,1]$ and that $v=1$, implying that $\delta^C(v)=0.9231$, $\delta_S^A(v)=0.9466$, and $\delta^A(v)=0.9541$.

Comparative statics. To assess the impact of demand and supply conditions on the optimal reserve policy, we consider here the case in which the suppliers' costs are distributed according to $G(c)=c^{1/s}$, where $s>0$ parameterizes the "strength" of the cost distribution: a larger s corresponds to a "better" cost distribution in the first-order stochastic dominance sense.[43] We study in this setting the effect of changes in the value v and in the strength s of the cost distribution on the optimal policy of an integrated buyer.

Figure 5(a) shows that buyers with higher values, who have more to lose from a failure to trade, are more likely to accommodate collusion; conversely, collusion is more likely to be blockaded if the buyer's value is low. In the range of buyer values where deterrence

[43] In this setup, $r^C(v)=\frac{v}{1+s}$, whereas $r^A(v)$ is the value of r that solves $v=r+sr\frac{2-r^{1/s}}{2-2r^{1/s}}$, implying that $\lim_{v\to\infty} r^A(v)=\bar{c}=1$. For the uniform distribution, that is, for $s=1$, $r^A(v)$ is given by $\frac{2+v-\sqrt{v^2-2v+4}}{3}$.

is optimal, the gap between the two asymmetric deterrence reserves increases with v. Figure 5(b) shows instead that, as suppliers become stronger, implying that collusion is more valuable to them, the reserves required to deter collusion decrease, until eventually the buyer prefers to accommodate. Weaker suppliers are thus more likely to be blockaded, whereas stronger suppliers are more likely to be accommodated.

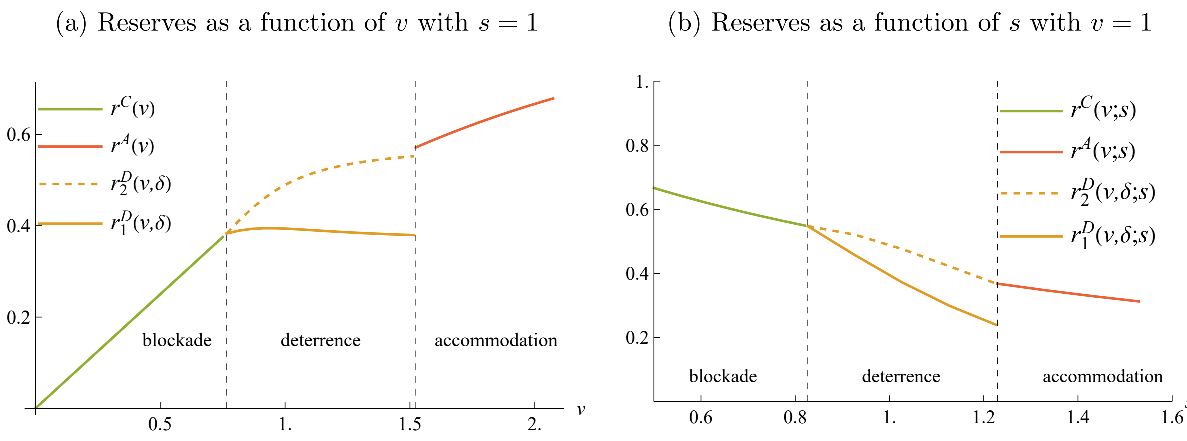


Figure 5: Optimal reserves for an integrated buyer assuming $G(c) = c^{1/s}$ for $s > 0$. (As s increases the cost distribution becomes "better".) Both panels assume that $\delta = 0.94$.

4.3 Independent buyers

The option of using asymmetric reserves raises the prospect of coordination issues when there are two different buyers, one in each market. To analyze this, in what follows we assume that two buyers simultaneously and independently set their reserves, which are then fixed for all time. The possibility of dynamic adjustments, of course, raises additional complexities.

To fix ideas, buyer 1 sets the reserve r_1 in market 1, and buyer 2 sets the reserve r_2 in market 2. Then the suppliers observe the reserves and engage in a collusive rotation whenever that is sustainable, and otherwise competition occurs. Thus, we analyze the Nash equilibrium reserves in the *reserve-setting game with independent buyers* in which the buyers choose reserves and the payoff of buyer i given strategy profile $\mathbf{r} = (r_1, r_2)$ is

$$U^{Comp}(v, r_i) \cdot \mathbf{1}_{\mathbf{r} \in \mathcal{D}(\delta)} + U^{Coll}(v, r_i) \cdot \mathbf{1}_{\mathbf{r} \notin \mathcal{D}(\delta)}.$$

Given any r_2, the best response for buyer 1 is either $r^C(v)$ (if that deters collusion), $r^A(v)$ (if accommodation is optimal), or a reserve arbitrarily close to r_1 such that $(r_1, r_2) \in$

$\mathcal{B}(\delta)$ (if deterrence is optimal, but $r^C(v)$ would allow collusion).[44] Building on this, the next proposition characterizes the equilibrium outcomes:

Proposition 11. *In the reserve-setting game with independent buyers:*

- *if collusion is blockaded (i.e., $\delta < \delta^C(v)$), then setting the competitive reserve (i.e., $r_1 = r_2 = r^C(v)$) is a Nash equilibrium;*
- *if instead an integrated buyer would optimally accommodate collusion (i.e., $\delta \geq \delta^A(v)$), then accommodating collusion (i.e., $r_1 = r_2 = r^A(v)$) is the unique Nash equilibrium;*
- *finally, in the intermediate range where an integrated buyer would optimally deter collusion (i.e., $\delta^C(v) \leq \delta < \delta^A(v)$), in equilibrium the buyers either accommodate collusion (i.e., $r_1 = r_2 = r^A(v)$) or set reserves on the deterrence boundary (i.e., $(r_1, r_2) \in \mathcal{B}(\delta)$).*

Proof. See Appendix A.10.

Proposition 11 first shows that independent buyers do not face any coordination problem when accommodation is optimal. To see why, suppose that an integrated buyer finds it optimal to accommodate collusion and set $r^A(v)$ in both markets—implying that, by playing with the reserves, it cannot obtain more than $U^A(v)$ per market. Setting $r_1 = r_2 = r^A(v)$ then constitutes the unique candidate equilibrium of the reserve-setting game, as any other configuration would give less than $U^A(v)$ to at least one buyer, giving that buyer an incentive to deviate to the optimal accommodation reserve, so as to obtain $U^A(v)$ if it induces collusion, and even more otherwise. Conversely, whenever an integrated buyer finds it optimal to set symmetric reserves, an independent buyer finds it optimal to stick to that reserve if it expects the other buyer to do so as well; hence, $r_1 = r_2 = r^A(v)$ does constitutes a Nash equilibrium of the reserve-setting game.

Further, if collusion is blockaded, implying that an integrated buyer would set the optimal competitive reserve in both markets and obtain the maximal possible payoff, independent buyers can do so as well; the only scope for coordination failure lies in the existence of additional equilibria. However, more serious coordination problems arise when an integrated buyer would optimally deter collusion, in which case it would do so using asymmetric reserves, with one market being more profitable than the other. Even when focusing on Pareto-efficient Nash equilibria, independent buyers may adopt sub-optimal deterrence reserves (i.e., $\mathbf{r}^D(v, \delta)$ may not be a Nash equilibrium of the reserve-setting

[44] In the latter case, any reserve lying in the interior of the deterrence region is dominated by another reserve that remains in the interior but is closer to $r^C(v)$; buyer 1's best response then consists in setting r_1 such that $\mathbf{r}$ lies slightly inside the boundary of $\mathcal{D}(\delta)$.

game) and even fail to deter collusion altogether (i.e., $r_1 = r_2 = r^A(v)$ may be the unique equilibrium).

From Proposition 11, with independent buyers a coordination failure may arise when (and only when) an integrated buyer would optimally deter collusion, in which case independent buyers may either adopt sub-optimal deterrence reserves, or fail to deter collusion altogether. Interestingly, in the latter case, coordination failure can potentially enhance social welfare because deterrence involves a greater reduction in trade than accommodation (see the discussion at the end of Section 4.2).

To illustrate the range of possibilities, Figures 6 and 7 consider the example in which costs are uniformly distributed over $[0, 1]$ and $v = 1$. The panels depict the deterrence boundary and the buyers' best-responses for different values of the discount factor. Interestingly, the scope for coordination failure is not monotonic in the discount factor.

As illustrated in Figure 6(a), when the discount is sufficiently high that an integrated buyer accommodates collusion, $\delta > \delta^A(v)$, then the unique Nash equilibrium of the reserve-setting game also involves accommodation, as established by Proposition 11. At the other extreme, as illustrated in Figure 6(b), when the discount factor is sufficiently low that collusion is blockaded, $\delta < \delta^C(v)$, the unique Nash equilibrium of the reserve-setting game has independent buyers both setting a reserve of $r^C(v)$, just as an integrated buyer would do.[45]

While Figure 6 focuses on cases in which there is no coordination failure as a result of having independent buyers, Figure 7 turns attention to cases in which deterrence is optimal for an integrated buyer, $\delta \in (\delta^C(v), \delta^A(v))$ and shows that coordination failure either necessarily arises or can arise because of multiplicity of Nash equilibria.

For instance, in panel (d), the optimal deterrence reserves constitute an Nash equilibrium of the reserve-setting game, but there is also a continuum of suboptimal deterrence equilibria; hence, while independent buyers deter collusion, they need not use the optimal deterrence reserves.[46] Furthermore, there exists a threshold discount factor $\delta^N(v) \in (\delta^C(v), \delta^A(v))$ such that coordination failure arises for sure, that is, optimal deterrence does not constitute a Nash equilibrium in the range $\delta \in (\delta^N(v), \delta^A(v))$.[47] For

[45]For $\delta \leq \delta^C$, consider whether there might be any other Nash equilibria besides $\mathbf{r}^C$in the setup of our illustration. Letting $\hat{r}_2$ be defined by $(r^C, \hat{r}_2) \in \mathcal{B}_2(\delta)$, then given any $r_2 < \hat{r}_2$, it follows that (r^C, r_2) is in the deterrence region and so r^C is the best response to r_2. Thus, for all $r_2 \leq \hat{r}_2$, $BR_1(r_2) = r^C$, to which buyer 2's own best-response is also r^C. Hence, in the range $r_2 < \hat{r}_2$, there is a unique Nash equilibrium in which both buyers set r^C. For $r_2 \geq \hat{r}_2$, $BR_1(r_2)$ is either on the deterrence boundary $\mathcal{B}_2(\delta)$ or equal to r^A. In both instances, $BR_1(r_2) < r^C$, and so $BR_2(BR_1(r_2)) = r^C < \hat{r}_2 \leq r_2$, a contradiction. It follows that the only possible Nash equilibrium is $\mathbf{r}^C$.

[46]The multiplicity of deterrence Nash equilibria raise nontrivial coordination problems because, for any suboptimal deterrence Nash equilibrium, at least one buyer prefers those reserves to the optimal ones.

[47]In the example of Figure 7, $\delta^C(v) = 0.9231$, $\delta^N(v) = 0.9475$, and $\delta^A(v) = 0.9540$.

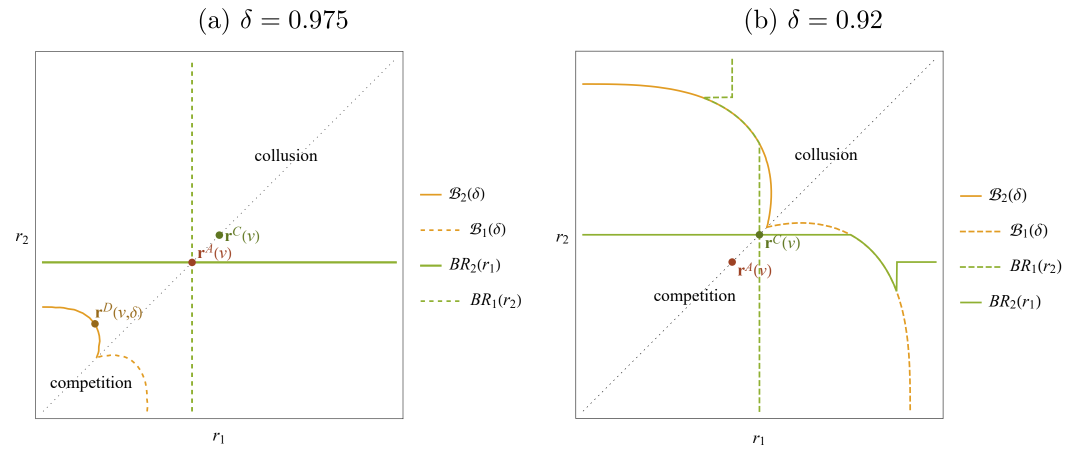


Figure 6: No coordination failure: an integrated buyer's optimal reserves, $\mathbf{r}^A(v)$ in the case of panel (a) and $\mathbf{r}^C(v)$ in the case of panel (b), are the unique Nash equilibrium of the reserve-setting game. The panels depict the deterrence boundaries and the buyers best-responses over the full relevant range $r_i \in [0, 1]$. Assumes that costs are uniformly distributed over $[0, 1]$, $v = 1$, and δ is as indicated. In this setup, $\delta^C(v) = 0.9231$ and $\delta^A(v) = 0.9540$, so an integrated buyer accommodates collusion in panel (a), and collusion is blockaded in panel (b).

instance, in panel (a) the only Nash equilibrium involves accommodation; in panel (b) there exists an accommodation equilibrium and a continuum of deterrence equilibria, all of which are suboptimal; and in panel (c), there only exist deterrence equilibria, all of which are suboptimal.[48]

[48] For sufficiently large δ in the range $\delta \in (\delta^N(v), \delta^A(v))$, accommodation is also an equilibrium.

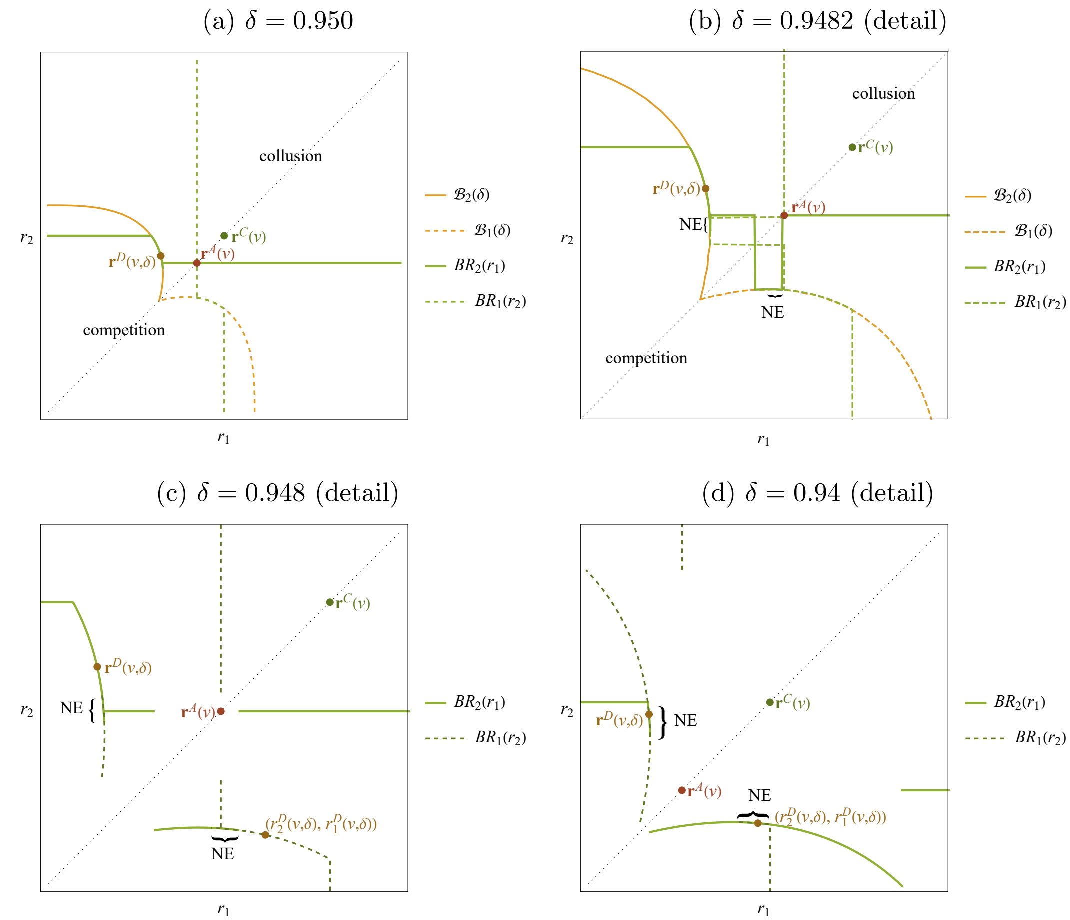


Figure 7: Coordination failure. In all panels, an integrated buyer deters collusion with optimal deterrence reserves $\mathbf{r}^D(v, \delta)$, but in the reserve-setting game with independent buyers, those optimal reserves are either not a Nash equilibrium (panels (a), (b), and (c)) or not the only Nash equilibrium (panel (d)). Panels (a) and (b) depict the deterrence boundaries, buyers' best-responses, and the diagonal; but to reduce clutter, panels (c) and (d) show only the best responses and diagonal. Panel (a) shows the full range $r_i \in [0, 1]$, but the other panels "zoom in." All panels assume that costs are uniformly distributed over $[0, 1]$ and $v = 1$. The discount factor δ is as indicated above the panels. In this setup, $\delta^C(v) = 0.9231$ and $\delta^A(v) = 0.9540$, so for all panels, we have $\delta \in (\delta^C(v), \delta^A(v))$.

Comparative statics. We now use the above setup to discuss how the scope for coordination failure is affected by the buyers' (common) value v and the strength s of the suppliers' cost distribution.

As shown in Figure 8, the range of discount factors in which independent buyers deter collusion only with suboptimal reserves, and the range of discount factors that fail to deter collusion, are both expanding in v and in s. Thus, coordination failure appears to be a greater concern when buyers have larger values and when suppliers draw their costs from better distributions.

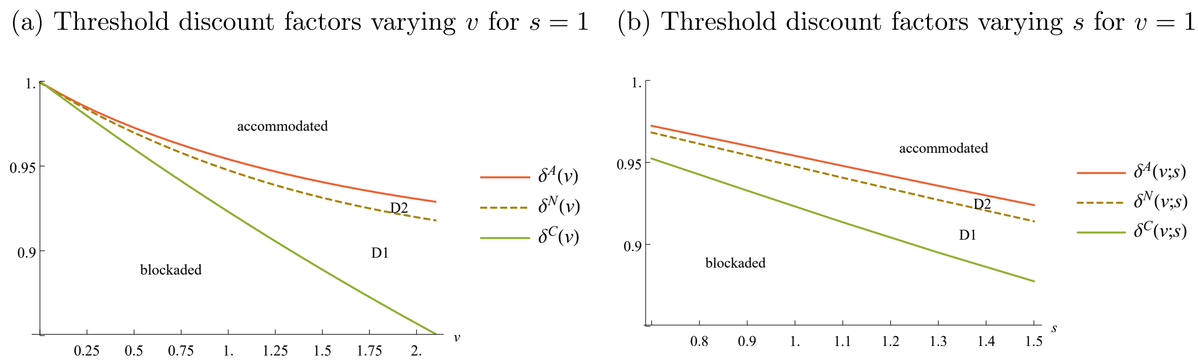


Figure 8: Effects of changes in the buyer's value and the suppliers' distributional strength on threshold discount factors. Assumes that costs are distributed over $[0, 1]$ with distribution $G(c) = c^{1/s}$. Panel (a) assumes $s = 1$ and varies v; panel (b) assumes $v = 1$ and varies s. In the region labeled "accommodated," both integrated and independent buyers accommodate collusion with the optimal collusive reserve $r^A(v)$. In the regions labeled D1 and D2, an integrated buyer would deter collusion. In D1, the optimal deterrence reserves are among the multiple Nash equilibria of the reserve-setting game with independent buyers, but in D2 they are not. In the region labeled "blockaded," both integrated and independent buyers deter collusion with the optimal competitive reserve $r^C(v)$.

To see that δ^A and δ^C are decreasing in v, as shown in Figure 8(a), note that the associated reserves, $r^A(v)$ and $r^C(v)$, increase with v, making collusion easier to sustain, and are not affected by the discount factor. As collusion is also easier to sustain with higher discount factors, it follows that as v increases, the threshold discount factors must fall. For changes in s, illustrated in Figure 8(b), more complex tradeoffs are involved because s also has a direct effect on the profitability of collusion.

Figure 8 shows that as v or s increases, collusion becomes easier—the range of discount factors for which collusion is blockaded shrinks. While some of this is offset by expanded deterrence (thereby increasing the scope for coordination failure), overall there is more collusion—the range of discount factors for which collusion is accommodated expands.

5 Extension to more than two markets

In this section, we consider an extension to a setup with n markets. We continue to assume that there are two suppliers and focus on the case with one integrated buyer.

5.1 Setting

Let $\mathcal{N} \equiv \{1, \ldots, n\}$ denote the set of markets and $\mathbf{r} = (r_1, \ldots, r_n)$ the vector of reserves in these markets, with the convention that markets are labeled by decreasing order of the reserves: that is, $r_1(n) \geq \cdots \geq r_n(n)$. As before, each market is characterized by the same value v for the buyer and the same distribution G over $[\underline{c}, \overline{c}]$ for the sellers' constant marginal costs, where cost draws are independent across suppliers and time.

Because symmetry facilitates collusion, for the sake of exposition we focus on market allocations that are as balanced as possible. Hence, if n is even, then each supplier is the designated winner in $n/2$ markets, alternating each period. For example, with $n = 4$, the markets might be divided up as $\{1, 2\}$ and $\{3, 4\}$, with supplier 1 designated for $\{1, 2\}$ in one period and for $\{3, 4\}$ in the next period. If n is odd, then suppliers alternate being the designated winner in $(n + 1)/2$ and in $(n - 1)/2$ markets. For example with $n = 3$, the markets might be divided up as $\{1, 2\}$ and $\{3\}$, with supplier 1 designated for $\{1, 2\}$ in one period and for $\{3\}$ in the next period.[49]

Consider a supplier facing the lowest cost and designated for the markets *other than* $\mathcal{M} \subset \mathcal{N}$. By deviating, the supplier can get the monopoly payoff rather than the non-designated supplier payoff in all markets in $\mathcal{M}$; the associated short-term stake is thus:

$$S_{\mathcal{M}}(\mathbf{r}) \equiv \sum_{j \in \mathcal{M}} S(r_j).$$

Although $S_{\mathcal{M}}(\mathbf{r})$ only depends on $(r_j)_{j \in \mathcal{M}}$, it is notationally convenient to write it as a function of the entire vector $\mathbf{r}$.

The long-term stake for a supplier that would forever be designated for the markets in $\mathcal{M}$ is instead given by

$$L_{\mathcal{M}}(\delta, \mathbf{r}) \equiv \frac{\delta}{1 - \delta} \left[\sum_{i \in \mathcal{M}} B(r_i) - \sum_{j \in \mathcal{N} \setminus \mathcal{M}} C(r_j) \right],$$

where $B(\cdot)$ and $C(\cdot)$ denote the benefit and cost of collusion, given by (1) and (2). Then the long-term stake for a supplier that is designated for the markets in $\mathcal{M}$ next period,

[49] It can be shown that these market allocations indeed maximize the scope for collusion for the optimal accommodation and deterrence reserves.

accounting for the rotation over the set of designated markets, is

$$L^R_{\mathcal{M}}(\delta, \mathbf{r}) \equiv \frac{1}{1+\delta} L_{\mathcal{M}}(\delta, \mathbf{r}) + \frac{\delta}{1+\delta} L_{\mathcal{N}\setminus\mathcal{M}}(\delta, \mathbf{r}).$$

Define k to be half the number of markets if there is an even number of markets and that number rounded up if there is an odd number of markets:

$$k \equiv \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

Collusion is not incentive compatible if a supplier has an incentive to deviate in k markets when it is designated for $n-k$ markets. Thus, given reserves $\mathbf{r}$, the suppliers are deterred from collusion if and only if for all $\mathcal{M} \in \mathcal{P}(\mathcal{N}, k)$, where $\mathcal{P}(\mathcal{N}, k)$ is the set of permutations of subsets of $\mathcal{N}$ containing k elements, either $L^R_{\mathcal{M}}(\delta, \mathbf{r}) \leq S_{\mathcal{M}}(\mathbf{r})$ or $L^R_{\mathcal{N}\setminus\mathcal{M}}(\delta, \mathbf{r}) \leq S_{\mathcal{N}\setminus\mathcal{M}}(\mathbf{r})$. For example, if $n = 4$, then a market allocation in which each supplier alternates between markets $\{1, 2\}$ and $\{3, 4\}$ is deterred if either

$$L^R_{\{1,2\}}(\delta, \mathbf{r}) \leq S_{\{1,2\}}(\mathbf{r}) \quad \text{or} \quad L^R_{\{3,4\}}(\delta, \mathbf{r}) \leq S_{\{3,4\}}(\mathbf{r}).$$

5.2 Optimal reserves

The buyer's optimal reserves conditional on deterrence satisfy:

$$\max_{\mathbf{r}} \sum_{i \in \mathcal{N}} U^{Comp}(r_i)$$

subject to, for all $\mathcal{M} \in \mathcal{P}(\mathcal{N}, k)$, either $L^R_{\mathcal{M}}(\delta, \mathbf{r}) \leq S_{\mathcal{M}}(\mathbf{r})$ or $L^R_{\mathcal{N}\setminus\mathcal{M}}(\delta, \mathbf{r}) \leq S_{\mathcal{N}\setminus\mathcal{M}}(\mathbf{r})$. The buyer then compares this payoff to $nU^{Coll}(r^{Coll})$ to determine whether to accommodate or deter collusion.

We first note that, as long as the number of markets remains even, asymmetric reserves still help to reduce the cost of deterrence:

Proposition 12. *If the number of markets is even and collusion is not blockaded, then an integrated buyer's optimal deterrence reserves are asymmetric.*

Proof: See Appendix A.11.

To go further, we now focus on the case in which $v = 1$ and costs are uniformly distributed over $[0, 1]$. Table 1 reports the buyer's optimal reserve policy for different numbers of markets (from $n = 1$ to $n = 6$) and a given value of the discount factor ($\delta = 0.94$).

Table 1: Optimal deterrence reserves $\mathbf{r}^D$ for $\delta = 0.94$

n	n even	n odd
1		$r_1 = 0.5$ (blockaded)
2	$r_1 = 0.4894$, $r_2 = 0.3937$	
3		$r_1 = \cdots = r_3 = 0.4953$
4	$r_1 = 0.4849$, $r_2 = \cdots = r_4 = 0.3894$	
5		$r_1 = \cdots = r_5 = 0.4512$
6	$r_1 = 0.4834$, $r_2 = \cdots = r_6 = 0.3875$	

Note: Assumes $v = 1$ and uniformly distributed costs.

Several features can be noted. First, for even numbers of markets, the asymmetry established by Proposition 12 takes a specific form, where a single reserve is set above the others. Intuitively, treating $n-1$ markets equally enhances the buyer's expected payoff because $U^{Comp}(r)$ is concave in r, and also limits the suppliers' ability to restore symmetry by optimizing over the composition of designated packages.[50] Second, for odd numbers of markets, the optimal reserve policy is instead symmetric.[51] This is because the market allocation itself is necessarily imbalanced (with one supplier designated for $(n+1)/2$ and the other for $(n-1)/2$ markets), to an extent such that there is no need to introduce further asymmetry.[52] Third, for each type of situation, collusion becomes easier as the number of markets increases, which in turn calls for more aggressive reserves. That is, letting $\mathbf{r}^D(n) = (r_1^D(n), \ldots, r_n^D(n))$ denote the optimal deterrence reserves, we have $\mathbf{r}^D(n+2) < \mathbf{r}^D(n)$. To see why, consider first the case of even numbers of markets. If the buyer were restricted to symmetric reserves, then increasing the number of markets would raise proportionally the short-term and long-term stakes, and thus have no impact on the scope for collusion.[53] However, the buyer finds it optimal to introduce an asymmetry by setting one reserve above the others and, given the optimal level of asymmetry, to adjust

[50]For instance, if $n = 4$ and $r_1 > r_2 > r_3 > r_4$, the suppliers can maintain some symmetry by designating one supplier for markets 1 and 4 and the other for markets 2 and 3. By contrast, with $r_1 > r_2 = r_3 = r_4$, one supplier necessarily ends up with a better designated packaged and the buyer moreover perfectly controls the level of asymmetry.

[51]It could therefore be implemented as well by independent buyers, the symmetry of the optimal reserves ensuring that the preference for accommodation versus deterrence is the same for all buyers, integrated or not—and conditional on deterrence, the optimal reserves constitute a Nash equilibrium of the reserve-setting game.

[52]As the number of markets increases, this imbalance however tends to become relatively small; asymmetric reserves may thus become again optimal.

[53]The optimal symmetric reserve is $\hat{\delta}^{-1}(\delta)$, where the condition determining the threshold $\hat{\delta}(\cdot)$ remains given by (10), as increasing the number of markets from 2 to $n = 2k$ leads to multiply both the numerator and denominator of the right-hand side by k.

the overall level of the reserves so as to ensure that the supplier not designated for that market has an incentive to deviate. As the number of markets increases, however, the long-term stake increases proportionally, which in turn calls for more aggressive reserves.

For odd numbers of markets, the optimal deterrence reserves are symmetric (at least for up to six markets), as the market allocation is itself sufficiently asymmetric. However, as the number of markets increases, the relative asymmetry of the market allocation is reduced, which calls again for more aggressive reserves.

Table 2: Threshold discount factor between deterrence and accommodation (δ^A)

n	n even	n odd
1		0.9714
2	0.9541	
3		0.9586
4	0.9501	
5		0.9545
6	0.9489	

Note: Assumes $v = 1$ and uniformly distributed costs.

It follows from the last observation that, for each type of situation (i.e., even or odd number of markets), deterrence becomes more costly as the number of markets increases, and is thus less likely to be optimal. This intuition is confirmed by Table 2, which reports, for the same numbers of markets as before, the discount factor threshold $\delta^A(n)$ above which accommodation dominates deterrence. We have:

$$\delta^A(n+2) < \delta^A(n).$$

Thus, as the number of markets increases, deterrence is optimal for a smaller range of discount factors.

In contrast, increasing the number of markets from an *even* to an *odd* number introduces an intrinsic asymmetry in the market allocation and can make collusion more fragile, and thus easier to deter. Indeed, deterrence is optimal for a wider range of discounts factors with $n = 3$ or even with $n = 5$ than with $n = 2$.

6 Discussion

Below we discuss the implications for social welfare, as well as for competition and procurement policy.

6.1 Social welfare analysis

We have so far considered buyers' optimal strategies. However, as buyers do not internalize the impact of their decisions on suppliers' profits, their strategies may not be socially optimal. For example, under competitive bidding, setting a reserve equal to $r \in [\underline{c}, \overline{c}]$ in a given market yields an expected welfare of

$$W^{Comp}(v,r) \equiv \mathbb{E}_{\mathbf{c}}[(v - \min\{c_1, c_2\}) \cdot \mathbf{1}_{\min\{c_1,c_2\} \leq r}] = \int_{\underline{c}}^{r} (v-c)\hat{g}(c)dc,$$

where $\hat{g}(c) = 2\left[1 - G\left(c\right)\right] g\left(c\right)$ is the probability density function of the lower cost (with cumulative distribution function $\hat{G}\left(c\right) = 1 - [1 - G(c)]^2$). It follows that social welfare is maximized by setting a reserve equal to $\min\{v, \overline{c}\}$.[54] Trade then occurs when and only when it is socially desirable, and competition ensures that the most efficient supplier is selected.

Likewise, under collusive bidding, expected welfare is equal to

$$W^{Coll}(v,r) \equiv \mathbb{E}_{\mathbf{c}}[(v - c_i) \cdot \mathbf{1}_{c_i \leq r} + (v - c_j) \cdot \mathbf{1}_{c_j \leq r \leq c_i}] = \left[2 - G\left(r\right)\right] \int_{\underline{c}}^{r} \left(v - c\right) dG\left(c\right),$$

where, in the first expression, i and j refer to the designated and non-designated suppliers, respectively, and the second expression follows from symmetry and integration by parts. It is therefore optimal to set the reserve below $\min\left\{v, \overline{c}\right\}$—starting from $r = \min\left\{v, \overline{c}\right\}$, a slight decrease in the reserve involves only a second-order loss from reduced trade, but a first-order gain of productive efficiency.[55] Yet, the socially optimal reserve will exceed the privately optimal one, $r^A\left(v\right)$, whenever suppliers' profit increase with the reserve, as one would expect.

For the same reason, a buyer's decision to deter or accommodate collusion may itself diverge from the socially desirable one. To explore this, we first consider the case of an integrated buyer, before studying the implications of coordination issues among independent buyers.

6.1.1 Integrated buyer

Focusing in symmetric reserves to start, we say that there is *over-deterrence* if an integrated buyer deters collusion (using $\mathbf{r}_S^D(v)$) when social welfare would be higher with accommodation (using $\mathbf{r}^A(v)$), and that there is *over-accommodation* if instead an integrated buyer accommodates collusion when social welfare would be higher with deterrence.

[54] If $\overline{c} < v$, then any reserve in $[\overline{c}, v]$ is also socially optimal.

[55] Formally, using integration by parts, we have $\left.\frac{\partial W^{Coll}(v,r)}{\partial r}\right|_{r=\min\{w,\overline{c}\}} = -g\left(\min\left\{v, \overline{c}\right\}\right) \int_{\underline{c}}^{\min\{v,\overline{c}\}} G\left(c\right) dc < 0.$

As shown in Figure 2 in Section 4.1, deterrence is optimal in the range $\delta \in (\delta^C(v), \delta_S^A(v))$, where the deterrence reserve is decreasing in δ and eventually falls below the accommodation reserve. It follows that, in the higher end of this range of discount factors, social welfare is greater when the buyer accommodates rather than deters collusion. To see this, note that at a discount factor of $\delta_S^A(v)$, where the buyer is indifferent between deterrence and accommodation, the suppliers strictly prefer accommodation, which involves both a larger reserve and a switch from competition to collusion. Thus, for discount factors close to $\delta_S^A(v)$, social welfare is greater when the buyer accommodates collusion than when the buyer deters collusion.

Proposition 13. *With an integrated buyer who uses symmetric reserves, there is never over-accommodation, but there is over-deterrence for an open range of discount factors.*

Proof. See Appendix A.12.

Loosely speaking, suppliers' total profits are always greater when the buyer accommodates rather than deters collusion. Because the buyer does not internalize suppliers' profits, it may deter collusion when social welfare would be larger if collusion were accommodated. The opposite instead cannot hold: the buyer accommodates collusion when fighting it would require excessively low reserves, but then the suppliers also prefer collusion to be accommodated.

The possibility of over-deterrence extends to asymmetric reserves, as shown in Figure 4(c), where the integrated buyer finds it optimal to deter collusion even though social welfare would be higher under accommodation for δ less than, but sufficiently close to, $\delta^A(v)$.

Remark: social welfare effects of a requirement of symmetric reserves. Interestingly, because social welfare is greater under accommodation than deterrence when the optimal deterrence reserves are particularly aggressive, a requirement that the buyer use symmetric reserves—e.g., a ban on price discrimination—can increase social welfare when it avoids over-deterrence, inducing the buyer to accommodate rather than deter collusion. In contrast, when a requirement that an integrated buyer use symmetric reserves does not lead to accommodation, it can *decrease* social surplus by causing the buyer to use less efficient deterrence reserves.[56]

[56] The first effect can be seen in Figure 4(c), where social welfare is greater under accommodation than deterrence for $\delta \in [\delta_S^A(v), \delta^A(v))$. In this range, deterrence (rather than accommodation) benefits the buyer but harms social surplus, implying that suppliers are harmed by deterrence—even though, as shown in Figure 4(a), one of the deterrence reserves is actually higher than the accommodation reserve in most of the range. The second effect can also be seen in Figure 4(c), where for $\delta \in (\delta^C(v), \delta_S^A(v))$, the buyer prefers deterrence even under a requirement of symmetric reserves, but social welfare is lower in that case.

6.1.2 Independent buyers

In the case of independent buyers, buyers' individual decisions fail to internalize not only suppliers' profits but also the impact on the other buyer. As a result, equilibria of the reserve-setting game may involve accommodation even though deterrence reserves exist that would make the buyers jointly better off, and equilibria with deterrence may involve reserves that differ from an integrated buyer's optimal deterrence reserves. In terms of social welfare, this can either generate an additional divergence between private and social objectives, or reduce the scope for over-deterrence that arises with an integrated buyer:

Proposition 14. *Relative to the outcome with an integrated buyer, an equilibrium of the reserve-setting game with independent buyers can:*
(i) increase social surplus by inducing a switch from deterrence to accommodation;
(ii) increase or decrease social surplus by inducing deterrence with different reserves.

Proof. See Appendix A.13.

6.2 Competition and procurement policy

The possibility that independent buyers accommodate collusion when an integrated buyer would have deterred it provides a rationale for the popular view that large buyers are less prone to be victims of collusion.[57] To the best of our knowledge, this is the first formalization of this notion. It apparently contrasts with Loertscher and Marx (2019), who show that endowing a buyer with buyer power makes covert collusion more attractive relative to a merger because the merger is a public event and causes the powerful buyer to react in a way that is detrimental to the merging suppliers. The way to reconcile these statements is that *powerful* and *large* buyers are distinct things. Our independent buyers are as powerful as an integrated buyer insofar as they can as well commit to a binding reserve forever. Yet, lacking size, they do not internalize the positive externality of deterring collusion in the other market. Conversely, a buyer cannot use reserves to fight collusion if it cannot credibly maintain them below $\min\{v, \bar{c}\}$. Hence, a buyer must be both powerful and large to fight collusion in an effective manner.

Demand aggregation, or centralized procurement, is already widely used for making purchases and awarding contracts, both in the public and in the private sector.[58] The existing policy and economic literature has however so far highlighted its potential benefit

[57] See, for example, Carlton and Israel (2011) for an expression of this view.

[58] For the public sector, examples of centralized units include the Government for Service Procurement in the United States, the Crown Commercial Service in the UK, and Consip in Italy. For the private sector, centralization is typically undertaken by creating a procurement department that purchases on behalf of multiple divisions.

in reducing the total cost of purchases via economies of scale, buyer power, professionalization, infrastructures (e-procurement tools), governance and transaction costs; the potential costs are those typical of delegation.[59] Our findings highlight the fighting of bid rigging as an additional benefit of centralization.[60]

Yet, creating centralized procurement units is unlikely to fully address bid-rigging problems. First, not all procurement can be centralized: for example, independent public and private buyers need to manage directly urgent or non-standardized purchases. Second, procurers, and especially centralized procurers, are not final users and their objective may underestimate the benefit of competition for total consumer surplus, thus failing to deter collusion even though doing so would be socially valuable. Third, as we have seen, delegating deterrence to an integrated buyer is not sufficient when the concerns of antitrust authorities is social welfare rather than consumer surplus. Over-deterrence may then arise, which is an issue that has so far not been part of the economic debate.

These considerations suggest that the fight of bid-rigging cannot be fully delegated to buyers: an effective antitrust enforcement remains desirable. In fact, our results reinforce the call for a close cooperation between public buyers and antitrust authorities, as advocated by the most recent international principles on fighting bid-rigging in public procurement (EC, 2021), and as recently implemented for example by the U.S. DOJ, with the setup of the Procurement Collusion Strike Force.

7 Conclusion

Actions that deter collusion impose costs of their own, implying that it can be optimal for buyers to accommodate collusion among their suppliers rather than deter it. We analyze the tradeoff between accommodation and deterrence in a procurement context in which bidders attempt to engage in market allocation. Although aggressive reserves can deter collusion, the use of such reserves comes at the cost of reduced trade. Indeed, for sufficiently high discount factors, collusion is so robust that excessively aggressive reserves would be required to deter it; buyers are then better off setting the optimal reserve in the face of colluding bidders. By contrast, for lower discount factors, buyers optimally adjust reserves to achieve deterrence. Interestingly, a buyer operating two markets optimally sets different reserves across the markets, so as to create an asymmetry that helps to impede collusion. Independent buyers do not internalize the benefit of this asymmetry, and, as a result, may fail to use the optimal deterrence reserves or even fail to deter collusion at all when an integrated buyer would.

[59] See e.g., Dimitri et al. (2006); Bandiera et al. (2009); Castellani et al. (2021).

[60] Related to this, our findings also add to the antitrust analysis of joint purchasing agreements, stressing the efficiency gain that they may generate in terms of fighting collusion among suppliers.

We also revisit a variety of defensive measures that have been discussed in the literature, including using first-price rather than second-price auctions, withholding information on bids and award prices, and avoiding supplier registration. We show in particular, that, contrary to some prevailing beliefs, synchronized purchasing makes collusion more difficult than staggered purchasing.[61] In addition, while the bid-rigging literature often focuses on sustainability, our focus on market allocations enables us to study initiation as well, and, as we show, limiting the information about bids or award prices may make initiation the key impediment to collusion.

The greater and more efficient deterrence of collusion by a multi-market buyer, as compared to independent buyers, suggests a benefit of centralized procurement agencies in terms of deterring collusion.[62] However, because the aggressive reserves associated with deterrence tend to limit trade, from a social welfare perspective, accommodation may be preferable to deterrence, implying that independent buyers may offer benefits in terms of social welfare even as they fail to deter collusion.

[61] In the context of the AT&T–Time Warner merger, the judge found that "the *staggered*, lengthy industry contracts would make [coordination] extremely risky" (emphasis added) because one party would have to "jump first" on the hope that the other would do the same later on, concluding that "putting such blind faith in one's chief competitor strikes this Court as exceedingly implausible!" (Leon, 2018, p. 163).

[62] In public procurement, this could be done by setting up national or regional procurement authorities that operate on behalf of local offices. In private procurement, it could be achieved by managing some purchases at a central rather than at a division level.

A Proofs

A.1 Proof of Lemma 1

For any reserve $r \in [\underline{c}, \bar{c}]$, the impact of collusion on total profit is equal to:

$$\begin{aligned}\Delta(r) &\equiv \overline{\pi}^m(r) + \overline{\pi}^n(r) - 2\overline{\pi}^c(r) \\ &= \int_{\underline{c}}^{r} [2G(c) - G(r)]\, G(c)\, dc.\end{aligned}$$

The market is at risk (i.e., collusion is strictly profitable) if and only if $\Delta(r) > 0$. We have:

$$\begin{aligned}\Delta'(r) &= G^2(r) - \int_{\underline{c}}^{r} g(r)\, G(c)\, dc \\ &= G(r) \int_{\underline{c}}^{r} g(c)\, dc - \int_{\underline{c}}^{r} g(r)\, G(c)\, dc \\ &= \int_{\underline{c}}^{r} G(r)\, G(c) \left[\frac{g(c)}{G(c)} - \frac{g(r)}{G(r)}\right] dc \\ &> 0,\end{aligned}$$

where the inequality stems from the monotonicity of the hazard rate. Because $\Delta(\underline{c}) = 0$, it follows that $\Delta(r) > 0$ for any $r \in (\underline{c}, \bar{c}]$. ■

A.2 Proof of Proposition 3

In a second-price auction the monopoly and competitive payoffs remain given by $\pi^m(c)$ and $\pi^c(c)$, as in the baseline model, and the non-designated supplier's payoff is thus still given by $\pi^n(c) = [1 - G(r)]\, \pi^m(c)$. It follows that the long-term stake for initiation and sustainability remains given by $L(\delta)$, defined in (5). By contrast, because the designated supplier bids at cost as it would under competition, the non-designated supplier's payoff when deviating corresponds to the competitive payoff, $\pi^c(c)$. As a result, the short-term stake for sustainability, which was $S = \pi^m(\underline{c}) - \pi^n(\underline{c})$ under a first-price auction, becomes lower: $S_{SPA} \equiv \pi^c(\underline{c}) - \pi^n(\underline{c})$. Because $S_{SPA} < S$, a market allocation can be sustained for a broader range of discount factors, namely, if and only if $\delta \geq \hat{\delta}_{SPA}$, where

$$\hat{\delta}_{SPA} \equiv L^{-1}(S_{SPA}) < L^{-1}(S) = \hat{\delta}.$$

■

A.3 Proof of Proposition 4

Fix $\tau \in [0,1)$. If $\tau = 0$, purchases are perfectly synchronous and the analysis is the same as in the baseline model. The short-term stakes are thus given by S for sustainability and by $\hat{S}$ for initiation, and in both cases the long-term stake is given by $L(\delta)$. For asynchronous tenders (i.e., $\tau > 0$), the short-term stake is the same, whereas the long-term stake corresponds to the difference between collusive and competitive profits in all future tenders, *evaluated at the time of the initiation or deviation decision.* If a supplier deviates or initiates collusion in a tender for market 1, the next tender (for market 2, where the stake for that supplier is thus equal to $\overline{\pi}^m - \overline{\pi}^c$) comes with a lag τ and the following one (for market 1, where the stake for the supplier is thus equal to $\overline{\pi}^n - \overline{\pi}^c$) with a total lag of one period; the long-term stake for that supplier is thus equal to:

$$\begin{aligned}\hat{L}(\delta;\tau) &= \delta^{\tau}\left(\overline{\pi}^m - \overline{\pi}^c\right)(1+\delta+...) + \delta\left(\overline{\pi}^n - \overline{\pi}^c\right)(1+\delta+...) \\ &= \frac{\delta^{\tau}}{1-\delta}\left(\overline{\pi}^m - \overline{\pi}^c\right) - \frac{\delta}{1-\delta}\left(\overline{\pi}^c - \overline{\pi}^n\right).\end{aligned}$$

If instead the supplier's deviation or initiation takes place in market 2, the next tender comes with a lag $1-\tau$, and the long-term stake is thus equal to $\hat{L}(\delta;1-\tau)$. The condition for sustainability thus becomes:

$$S \leq \min\left\{\hat{L}(\delta;\tau), \hat{L}(\delta;1-\tau)\right\} = \hat{L}(\delta;\max\{\tau,1-\tau\}),$$

whereas the condition for initiation becomes $\hat{S} \leq \hat{L}(\delta;\max\{\tau,1-\tau\})$. Hence, collusion is easiest—to sustain or initiate—for the value of τ that minimizes $\max\{\tau,1-\tau\}$ in the range $\tau \in [0,1)$, which is obtained for $\tau = 1/2$, that is, for perfectly staggered tenders. By contrast, collusion is the most difficult to sustain or initiate for the value of τ that minimizes $\max\{\tau,1-\tau\}$ in the range $\tau \in [0,1)$, which is $\tau = 0$, that is, for perfectly synchronous tenders. ■

A.4 Proof of Proposition 6

It suffices to show that the right side of condition (6) is decreasing in r for r sufficiently close to $\underline{c}$. This right side can be written as

$$RHS\left(r\right) \equiv \frac{N(r)}{D\left(r\right)},$$

where

$$N\left(r\right) \equiv S = G(r)(r-\underline{c}),$$

and

$$D\left(r\right)\equiv\overline{\pi}^{m}-\overline{\pi}^{c}+\delta\left(\overline{\pi}^{n}-\overline{\pi}^{c}\right)=\int_{\underline{c}}^{r}\left[\left(1+\delta\right)G\left(c\right)-\delta G(r)\right]G(c)dc.$$

We thus have

$$RHS'\left(r\right)=\frac{\hat{N}(r)}{\hat{D}\left(r\right)},$$

where

$$\hat{N}\left(r\right)\equiv D\left(r\right)N_{a}'(r)-D'\left(r\right)N_{a}(r)\ \text{ and }\hat{D}\left(r\right)\equiv D^{2}\left(r\right).$$

Using

$$\begin{aligned}N'\left(r\right)&=G(r)+g(r)(r-\underline{c}),\\ N''\left(r\right)&=2g(r)+g'(r)(r-\underline{c}),\end{aligned}$$

and

$$\begin{aligned}D'\left(r\right)&=G^{2}(r)-\delta g(r)\int_{\underline{c}}^{r}G(c)dc,\\ D''\left(r\right)&=\left(2-\delta\right)G(r)g(r)-\delta g'(r)\int_{\underline{c}}^{r}G(c)dc,\\ D'''\left(r\right)&=\left(2-\delta\right)g^{2}(r)+2\left(1-\delta\right)G(r)g'(r)-\delta g''(r)\int_{\underline{c}}^{r}G(c)dc,\end{aligned}$$

yields

$$N(\underline{c})=N_{a}'(\underline{c})=0<N''(\underline{c})\ \text{ and }D\left(\underline{c}\right)=D'\left(\underline{c}\right)=D''\left(\underline{c}\right)=0<D'''\left(\underline{c}\right).$$

Building on this, we have:

$$\begin{aligned}\hat{N}'\left(r\right)&=D\left(r\right)N''(r)-D''\left(r\right)N(r),\\ \hat{N}''\left(r\right)&=D\left(r\right)N'''(r)+D'\left(r\right)N''(r)-D''\left(r\right)N'(r)-D'''\left(r\right)N(r),\\ \hat{N}'''\left(r\right)&=D\left(r\right)N'''''(r)+3D'\left(r\right)N''''(r)+2D''\left(r\right)N'''(r)-2D'''\left(r\right)N''(r)\\ &\quad-3D''''\left(r\right)N'(r)-D'''''\left(r\right)N(r),\end{aligned}$$

and

$$\begin{aligned}
\hat{D}'(r) &= 2D(r)\,D'(r)\,,\\
\hat{D}''(r) &= 2D(r)\,D''(r) + 2\left[D'(r)\right]^2,\\
\hat{D}'''(r) &= 2D(r)\,D'''(r) + 6D'(r)\,D''(r)\,,\\
\hat{D}''''(r) &= 2D(r)\,D''''(r) + 8D'(r)\,D'''(r) + 6\left[D''(r)\right]^2,\\
\hat{D}'''''(r) &= 2D(r)\,D'''''(r) + 10D'(r)\,D''''(r) + 20D''(r)\,D'''(r)\,,\\
\hat{D}''''''(r) &= 2D(r)\,D''''''(r) + 12D'(r)\,D'''''(r) + 30D''(r)\,D''''(r) + 20\left[D'''(r)\right]^2,
\end{aligned}$$

which yields:

$$\begin{aligned}
\hat{N}(\underline{c}) &= \hat{N}'(\underline{c}) = \hat{N}''(\underline{c}) = 0 > \hat{N}'''(\underline{c}) = -2D'''(\underline{c})\,N''(\underline{c}),\\
\hat{D}(\underline{c}) &= \hat{D}'(\underline{c}) = \hat{D}''(\underline{c}) = \hat{D}'''(\underline{c}) = \hat{D}''''(\underline{c}) = \hat{D}'''''(\underline{c}) = 0 < \hat{D}''''''(\underline{c}) = 20\left[D'''(\underline{c})\right]^2.
\end{aligned}$$

Using Taylor expansions then leads to:

$$\lim_{r\to\underline{c}} RHS'(r) = \lim_{r\to\underline{c}} \frac{\hat{N}(r)}{\hat{D}(r)} = \lim_{r\to\underline{c}} \frac{\hat{N}'''(\underline{c})\,\frac{(r-\underline{c})^3}{3!}}{\hat{D}''''''(\underline{c})\,\frac{(r-\underline{c})^6}{6!}} = \frac{6!}{3!}\frac{\hat{N}'''(\underline{c})}{\hat{D}''''''(\underline{c})} \lim_{r\to\underline{c}} \frac{1}{(r-\underline{c})^3} = -\infty.$$

Because $\lim_{r\to\underline{c}} RHS'(r) = -\infty$, there exists $\hat{r} > \underline{c}$ such that $RHS'(r) < 0$ for $r \leq [\underline{c}, \hat{r}]$.

To show that the critical discount factor tends to 1 as r tends to $\underline{c}$, we first recall that this threshold is determined by solving (6) with equality, where the left-hand side increases from 0 to $+\infty$ as δ increases from 0 to 1, whereas the right-hand side is given by $RHS(r)$. Using again Taylor expansions and noting that $D'''(\underline{c}) = g^2(\underline{c}) = N''_{SPA}(\underline{c}) \leq N''(\underline{c})$, this right-hand side satisfies:

$$\lim_{r\to\underline{c}} RHS(r) = \lim_{r\to\underline{c}} \frac{N(r)}{D(r)} = \lim_{r\to\underline{c}} \frac{N''(\underline{c})\,\frac{(r-\underline{c})^2}{2!}}{D'''(\underline{c})\,\frac{(r-\underline{c})^3}{3!}} \geq 3 \lim_{r\to\underline{c}} \frac{1}{r-\underline{c}} = +\infty.$$

It follows that the critical discount factor tends to 1 as r tends to $\underline{c}$. ■

A.5 Proof of Proposition 5

By virtue of the optimality of the reserves, we have

$$U^{Comp}(v, r^C(v)) \geq U^{Coll}(v, r^C(v)) \;\text{ and }\; U^{Comp}(v, r^A(v)) \leq U^{Coll}(v, r^A(v)).$$

Combining these with (9) yields

$$\int_{\underline{c}}^{r^A(v)} G^2(x)dx \leq \int_{\underline{c}}^{r^C(v)} G^2(x)dx,$$

which implies that

$$r^A(v) \leq r^C(v).$$

Thus, a revealed preference argument shows that the optimal reserve is weakly more aggressive when facing collusion. Furthermore, we have:

$$\left.\frac{\partial U^{Comp}}{\partial r}(v,r)\right|_{r=r^A(v)} = \left.\frac{\partial U^{Coll}}{\partial r}(v,r)\right|_{r=r^A(v)} + G^2(r^A(v)) = G^2(r^A(v)) > 0.$$

It follows that $r^A(v) < r^C(v)$. ■

A.6 Asymmetric deterrence reserves

The following lemma showing that reserves have more impact on short-term stakes than long-term stakes will be useful.

Lemma A.1. *For any $r > \underline{c}$:*

$$S(r) > B(r) + C(r) > 0, \tag{17}$$

and

$$S'(r) > B'(r) + C'(r) > 0. \tag{18}$$

Proof. We have:

$$B(r) + C(r) = \overline{\pi}^m(r) - \overline{\pi}^n(r) = G(r)\overline{\pi}^m(r),$$

where $G(r)$ and $\overline{\pi}^m(r)$ are both positive for $r > \underline{c}$, and strictly increasing; it follows that $B(r) + C(r)$ is also positive and strictly increasing.

Likewise, using $S(r) = \pi^m(\underline{c}) - \pi^n(\underline{c}) = G(r)\pi^m(\underline{c}; r)$, we have:

$$S(r) - B(r) - C(r) = G(r)\left[\pi^m(\underline{c}; r) - \overline{\pi}^m(r)\right],$$

where $G(r)$ and

$$\pi^m(\underline{c}; r) - \overline{\pi}^m(r) = (r - \underline{c}) - \int_{\underline{c}}^{r} G(c)dc = \int_{\underline{c}}^{r} [1 - G(c)]\, dc$$

are both positive for $r > \underline{c}$, and strictly increasing. The conclusion follows. ■

Building on this first lemma yields:

Lemma A.2. *If $r_1 < r_2$ (resp., $r_1 > r_2$), the more stringent condition in* (15) *is the first (resp., second) one.*

Proof. Let

$$\Delta(r_1, r_2) \equiv \left(1 - \delta^2\right) \left\{ \left[L^R(\delta, r_1, r_2) - S(r_2)\right] - \left[L^R(\delta, r_2, r_1) - S(r_1)\right] \right\}.$$

Straightforward manipulations yield:

$$\begin{aligned} \Delta(r_1, r_2) &= \left\{ \delta\left[B(r_2) - C(r_1)\right] + \delta^2\left[B(r_1) - C(r_2)\right] - \left(1 - \delta^2\right) S(r_2) \right\} \\ &\quad - \left\{ \delta\left[B(r_1) - C(r_2)\right] + \delta^2\left[B(r_2) - C(r_1)\right] - \left(1 - \delta^2\right) S(r_1) \right\} \\ &= (1 - \delta)\left\{ \delta\left[B(r_2) + C(r_2)\right] - (1 + \delta) S(r_2) \right\} \\ &\quad - (1 - \delta)\left\{ \delta\left[B(r_1) + C(r_1)\right] - (1 + \delta) S(r_1) \right\} \\ &= (1 - \delta)\left[\psi(r_2) - \psi(r_1)\right], \end{aligned}$$

where

$$\psi(r) \equiv \delta\left[B(r) + C(r)\right] - (1 + \delta) S(r)$$

is strictly decreasing in r:

$$\psi'(r) = \delta\left[B'(r) + C'(r)\right] - (1 + \delta) S'(r) < 0,$$

where the inequality follows from $\delta \geq 0$ and Lemma A.1 above. Therefore, if $r_1 < r_2$, then $\Delta(r_1, r_2) < 0$, implying that the first condition in (15) is strictly more stringent than the second one. The opposite holds if $r_1 > r_2$. ∎

A.7 Proof of Lemma 2

Recall that we assume that $L^R(\delta, r_j, r_i)$ is increasing in r_j (which holds for δ sufficiently close to 1 and for all relevant δ in some cases), and that there is a reserve threshold, $r_U^D(\delta)$, such that collusion in a unique market is sustainable if and only if the reserve is below $r_U^D(\delta)$. Because a reserve of $\underline{c}$ effectively eliminates a market, we can express the relevant long-term stake for a unique market with reserve r as $L^R(\delta, \underline{c}, r)$ and the condition for the sustainability of collusion as

$$L^R(\delta, \underline{c}, r) \geq S(r).$$

It follows from the monotonicity of $L^R(\delta, r_j, r_i)$ in r_j that collusion is therefore easier to sustain with two markets than with one; specifically, whenever collusion is sustainable in *one* market with reserve r on a stand-alone basis, it is sustainable as well when the suppliers interact in *two* markets with the same reserve r:

$$\forall r \in [\underline{c}, \max\{\overline{c}, v\}],\ L^R(\delta, \underline{c}, r) \geq S(r) \implies L^R(\delta, r, r) \geq S(r).$$

Accordingly, more aggressive reserves are required to deter collusion when suppliers interact in multiple markets. Indeed, any reserves $(r_1, r_2) \in \mathcal{D}_i$, for some $i \in \{1, 2\}$, satisfy $\underline{c} \leq r_j \leq r_i$ and

$$L^R(\delta, \underline{c}, r_i) \leq L^R(\delta, r_j, r_i) < S(r_i),$$

where the first inequality stems from the monotonicity of the long-term stake and the second one from the definition of $\mathcal{D}_i$. It then follows from the definition of the unique market deterrence reserve that $(r_j \leq) r_i < r_U^D(\delta)$.

We now turn to the second part of the lemma. It follows from the monotonicity of $L^R(\delta, r_j, r_i)$ in r_j that the boundary of $\mathcal{D}_i(\delta)$ is given by:

$$\mathcal{B}_i(\delta) \equiv \{(r_1, r_2) \mid r_i \geq r_j \text{ and } \phi(\delta, r_i, r_j) = 0\},$$

where

$$\begin{aligned} \phi(\delta, r_i, r_j) &\equiv \left(1 - \delta^2\right)\left[S(r_i) - L^R(\delta, r_j, r_i)\right] \\ &= \left(1 - \delta^2\right) S(r_i) - \delta\left[B(r_i) - C(r_j)\right] - \delta^2\left[B(r_j) - C(r_i)\right]. \end{aligned}$$

We have:

$$\begin{aligned} \left.\frac{\partial \phi}{\partial \delta}(\delta, r_i, r_j)\right|_{(r_i, r_j) \in \mathcal{B}_i} &= \left. -2\delta S(r_i) - \left[B(r_i) - C(r_j)\right] - 2\delta\left[B(r_j) - C(r_i)\right]\right|_{(r_i, r_j) \in \mathcal{B}_i(\delta)} \\ &= \frac{-\left(1 + \delta^2\right)\left[B(r_i) - C(r_j)\right] - 2\delta\left[B(r_j) - C(r_i)\right]}{1 - \delta^2} \\ &= -\frac{(1 - \delta)^2\left[B(r_i) - C(r_j)\right] + 2\delta\left[B(r_i) - C(r_i) + B(r_j) - C(r_j)\right]}{1 - \delta^2} \\ &< 0, \end{aligned}$$

where the second equality uses the definition of $\mathcal{B}_i(\delta)$, which implies that

$$S(r_i) = \frac{\delta}{1 - \delta^2}\left\{B(r_i) - C(r_j) + \delta\left[B(r_j) - C(r_i)\right]\right\},$$

the third equality rearranges, and the inequality follows from the assumption that the market is at risk, implying that $B(r_i) > C(r_i)$ and (because $B(\cdot)$ is increasing and

$r_i \geq r_j$) that $B(r_i) \geq B(r_j) > C(r_j)$. It follows that a slight increase in δ strictly reduces the set of deterrence reserves, $\mathcal{D}_i(\delta)$; because $\phi(\cdot)$ is continuous, the reduction in $\mathcal{D}_i(\delta)$ is also continuous. ■

A.8 Proof of Proposition 8

When $r_1 < r_2$, the relevant long-term stake for a rotation is $L^R(\delta, r_1, r_2)$, defined in (14), and for a market allocation is $L(\delta, r_1, r_2)$, defined in (13). Using the definitions of L^R, L, $\overline{\pi}^m$, and $\overline{\pi}^n$, we have:

$$L^R(\delta, r_1, r_2) - L(\delta, r_1, r_2) = \frac{\delta}{1-\delta^2}\left[B(r_2) + C(r_2) - B(r_1) - C(r_1)\right] > 0,$$

where the inequality stems from $r_1 < r_2$ and Lemma A.1 (in Appendix A.6). If follows that a rotation is easier to sustain than a market allocation. ■

A.9 Proof of Proposition 9

We show that, whenever collusion is not blockaded, asymmetric reserves are more effective at deterring it. We first consider the case where suppliers would rely on a market allocation when trying to collude. As discussed at the beginning of Section 4.2, such collusion is sustainable if and only if

$$L(\delta, r_1, r_2) \geq S(r_2) \quad \text{and} \quad L(\delta, r_2, r_1) \geq S(r_1),$$

where the long-term stake is given by

$$L(\delta, r_i, r_j) \equiv \frac{\delta}{1-\delta}\left[B(r_i) - C(r_j)\right].$$

If the buyer restricts attention to symmetric deterrence reserves, it finds it optimal to set $r_1 = r_2 = r^D$, where $r = r^D$ is the unique solution to $L(\delta, r, r) = S(r)$ and, by assumption, lies strictly below the optimal competitive reserve $r^C(v)$. However, starting from these reserves, increasing r_2 while holding r_1 constant still deters collusion (by increasing the short-term stake $S(r_2)$ and decreasing the long-term stake $L(\delta, r_1, r_2)$—see footnote 34) and enhances the buyer's payoff by bringing r_2 closer to the optimal competitive reserve $r^C(v)$ (recall that the buyer's payoff is strictly quasi-concave in the reserves). It follows that deterring a market allocation is optimally achieved with asymmetric reserves.

We now turn to rotations. For $r_i \geq r_j$ the relevant sustainability condition is $\phi(\delta, r_j, r_i) \geq$

0, where:

$$\begin{aligned}\phi(\delta, r_j, r_i) &\equiv \left(1-\delta^2\right)\left[L^R(\delta, r_j, r_i) - S\left(r_i\right)\right] \\ &= \delta\left[B\left(r_i\right) - C\left(r_j\right)\right] + \delta^2\left[B\left(r_j\right) - C\left(r_i\right)\right] - \left(1-\delta^2\right) S\left(r_i\right).\end{aligned}$$

An increase in r_i increases both the short-term stake $S\left(r_i\right)$ and the long-term stake $L^R(\delta, r_j, r_i)$; by contrast, an increase in r_j only affects the long-term stake. It follows from Lemma A.1 that, for symmetric reserves, the impact on short-term stakes prevails. Specifically:

Lemma A.3. *For any $r > \underline{c}$:*

$$\left.\frac{\partial\phi(\delta, r_j, r_i)}{\partial r_i}\right|_{r_i=r_j=r} < \left.\frac{\partial\phi(\delta, r_j, r_i)}{\partial r_j}\right|_{r_i=r_j=r}.$$

Proof. We have:

$$\begin{aligned}&\left.\frac{\partial\phi(\delta, r_j, r_i)}{\partial r_j}\right|_{r_i=r_j=r} - \left.\frac{\partial\phi(\delta, r_j, r_i)}{\partial r_i}\right|_{r_i=r_j=r} \\ =\;& \left[-\delta C'\left(r\right) + \delta^2 B'\left(r\right)\right] - \left[\delta B'\left(r\right) - \delta^2 C'\left(r\right) - \left(1-\delta^2\right) S'\left(r\right)\right] \\ =\;& \left(1-\delta^2\right) S'\left(r\right) - \delta\left(1-\delta\right)\left[C'\left(r\right) + B'\left(r\right)\right] \\ >\;& 0,\end{aligned}$$

where the inequality uses $1-\delta^2 > \delta\left(1-\delta\right)$ and Lemma A.1 (in Appendix A.6). ■

Building on these insights, suppose that, starting from $r_1 = r_2 = r^D$, the buyer increases r_i by $\varepsilon > 0$ and reduces r_j by the same amount; the change has no first-order effect on the buyer's expected payoff (starting from symmetric reserves, the impact of the two reserves offset each other), but strictly improves deterrence because

$$d\phi = \left(\left.\frac{\partial\phi(\delta, r_j, r_i)}{\partial r_i}\right|_{r_i=r_j=r^D(\delta)} - \left.\frac{\partial\phi(\delta, r_j, r_i)}{\partial r_j}\right|_{r_i=r_j=r^D(\delta)}\right)\varepsilon < 0,$$

where the inequality uses Lemma A.3. It follows that there exists a nearby (asymmetric) modification of the reserves that improves the buyer's expected payoff while preserving deterrence. ■

A.10 Proof of Proposition 11

Assume that two buyers, 1 and 2, play the reserve-setting game described in Section 4.3.

Proof of Bullet 1: If collusion is blockaded (i.e., $\delta < \delta^C(v)$), implying that $\mathbf{r}^C(v) = \left(r^C(v), r^C(v)\right) \in \mathcal{D}(\delta)$, then setting the competitive reserve $r^C(v)$ clearly constitutes a Nash equilibrium: if buyer i does so, then setting $r_j = r^C(v)$ successfully deters collusion and thus gives buyer j its maximal possible payoff, as:

$$U^{Comp}(r^C(v)) = \max_{r \in [\underline{c}, \min\{\bar{c}, v\}]} \left\{U^{Comp}(r)\right\} = \max_{r \in [\underline{c}, \min\{\bar{c}, v\}]} \left\{\max\left\{U^{Coll}(r), U^{Comp}(r)\right\}\right\}.$$

Proof of Bullet 2: Suppose now that an integrated buyer would optimally accommodate collusion (i.e., $\delta \geq \delta^A(v)$), implying:

$$2U^A(v) > U^{Comp}(v, r_1) + U^{Comp}(v, r_2) \text{ for any } \mathbf{r} = (r_1, r_2) \in \mathcal{D}, \tag{19}$$

where the strict inequality stems from the convention that suppliers collude whenever they can, including in the limit case $\delta = \delta^A(v)$ and $\mathbf{r} \in \mathcal{B}(\delta)$.

We first establish existence. Fix $r_i = r^A(v)$ and consider buyer j's best-response, for $i \neq j \in \{1, 2\}$. For any r_j satisfying $\left(r^A(v), r_j\right) \in \mathcal{D}$, we have:

$$\begin{aligned} U^{Comp}\left(v, r^A(v)\right) + U^A(v) &\geq 2U^A(v) \\ &> U^{Comp}\left(v, r^A(v)\right) + U^{Comp}(v, r_j), \end{aligned}$$

where the first inequality stems from $U^{Comp}(v, \cdot) \geq U^{Coll}(v, \cdot)$ and the second one from (19). Hence, $U^A(v) > U^{Comp}(v, r_j)$ for any r_j deterring collusion, implying that buyer j's best-response is to accommodate collusion and set $r_j = r^A(v)$.

We now turn to uniqueness. Suppose that there exists a different Nash equilibrium, $\left(r_1^N, r_2^N\right) \neq \left(r^A(v), r^A(v)\right)$. By deviating to $r^A(v)$, a buyer could obtain $U^A(v)$ if the deviation induces collusion, and $U^{Comp}\left(v, r^A(v)\right) \geq U^A(v)$ otherwise. Both buyers must therefore obtain at least $U^A(v) = \max_r U^{Coll}(v, r)$. It then follows from $\left(r_1^N, r_2^N\right) \neq \left(r^A(v), r^A(v)\right)$, together with the strict quasi-concavity of $U^{Coll}(v, r)$ in the reserve, that competition must arise in equilibrium. Hence, we have $\left(r_1^N, r_2^N\right) \in \mathcal{D}$ and $U^{Comp}\left(v, r_i^N\right) \geq U^A(v)$ for $i \in \{1, 2\}$, contradicting (19).

Proof of Bullet 3: Finally, we turn to the case in which collusion is not blockaded (i.e., $\delta \geq \delta^C(v)$ or $\mathbf{r}^C(v) \notin \mathcal{D}(\delta)$), but would be deterred by an integrated buyer (i.e., $\delta < \delta^A(v)$). Let $\mathbf{r}^N = (r_1^N, r_2^N)$ be a Nash equilibrium. If $\mathbf{r}^N \in \mathcal{D}(\delta)$, then each buyer could slightly adjust its reserve and still deter collusion. But then first-order optimality conditions imply $r_1^N = r_2^N = r^C(\delta)$, a contradiction. Hence, it must the case that $\mathbf{r}^N \notin$

$\mathcal{D}(\delta)$. Furthermore, if $\mathbf{r}^N \notin \mathcal{D}(\delta) \cup \mathcal{B}(\delta)$, then collusion arises and each buyer $i$obtains $U^{Coll}\left(r_i^N\right)$. But if $r_i \neq r^A(v)$, then buyer i could profitably deviate to $r^A(v)$, as this would give buyer i a payoff of $U^{Coll}\left(r^A(v)\right)$ if collusion is still sustainable, and an even higher payoff $U^{Comp}\left(r^A(v)\right)$ otherwise. Hence, either $\mathbf{r}^N = \mathbf{r}^A(v) \equiv \left(r^A(v), r^A(v)\right)$, which from the previous observation can occur only if $\mathbf{r}^A(v) \notin \mathcal{D}(\delta)$, or $\mathbf{r}^N \in \mathcal{B}(\delta)$ (interpreted as meaning "arbitrarily close to $\mathcal{B}(\delta)$"). Thus, in equilibrium, the buyers either accommodate collusion (i.e., $r_1 = r_2 = r^A(v)$) or set reserves on the deterrence boundary (i.e., $(r_1, r_2) \in \mathcal{B}(\delta)$). ■

A.11 Proof of Proposition 12

Suppose that there are $n = 2k$ markets. We first show that, for symmetric reserves, the scope for collusion is maximized when each supplier is designated for half of the markets. Let r denote the symmetric reserve and suppose without loss of generality that a supplier is currently designated for $n - h$ markets, for some $h \in \mathcal{N}$. The supplier's short-term stake from a deviation in the remaining h markets is then given by

$$S(r, h) \equiv hS(r),$$

whereas its long-term stake is:

$$L^R(\delta, r, h) \equiv \frac{\delta\left[hB(r) - (n-h)C(r)\right] + \delta^2\left[(n-h)B(r) - hC(r)\right]}{1-\delta^2}.$$

Hence, the supplier has no incentive to deviate if $\phi(\delta, r, h) \geq 0$, where

$$\begin{aligned}
\phi(\delta, r, h) &\equiv \left(1-\delta^2\right)\left[L^R(\delta, r, h) - S(r, h)\right] \\
&= \delta\left[hB(r) - (n-h)C(r)\right] + \delta^2\left[(n-h)B(r) - hC(r)\right] - \left(1-\delta^2\right)hS(r),
\end{aligned}$$

which is decreasing in h, that is,

$$\frac{\partial\phi(\delta, r, h)}{\partial h} = (1-\delta)\left[\delta B(r) + \delta C(r) - (1+\delta)S(r)\right] < 0,$$

where the inequality stems from (17). Collusion is sustainable if no supplier has an incentive to deviate, that is, if

$$\min\{\phi(\delta, r, h), \phi(\delta, r, n-h)\} \geq 0.$$

It follows that collusion is easiest to sustain when $h = n - h = k (= n/2)$. In particular, collusion is blockaded if $\phi\left(\delta, r^C(v), k\right) \geq 0$.

Suppose now that collusion is not blockaded. Because the buyer's payoff $U^{Comp}(v,r)$ is concave in r, the optimal symmetric deterrence reserve, $r_S^D(\delta)$, is then such that $\phi\left(\delta, r_S^D(\delta), k\right) = 0$. Starting from $\mathbf{r}_S^D(\delta) = \left(r_S^D(\delta), ..., r_S^D(\delta)\right)$, consider now a small change in reserves in which r_1 is slightly increased by $dr_1 = (n-1)\,dr > 0$, whereas all other reserves are reduced by dr. By construction, this small change in the reserves has no first-order effect on the buyer's overall payoff, as the net impact is given by

$$\sum_{i\in\mathcal{N}} \frac{\partial U^{Comp}}{\partial r_i}(v,r_i)dr_i\bigg|_{r_i=r_S^D(\delta)} = \frac{\partial U^{Comp}}{\partial r}(v,r)\bigg|_{r=r_S^D(\delta)} [dr_1 - (n-1)\,dr] = 0.$$

However, for the supplier currently *not* designated for market 1, the short-term stake becomes (where $r \equiv r_S^D(\delta) - dr$)

$$\hat{S}(\mathbf{r}) \equiv S(r_1) + (k-1)\,S(r),$$

whereas its long-term stake is

$$\hat{L}^R(\delta,\mathbf{r}) \equiv \frac{\delta\,[B(r_1) + (k-1)\,B(r) - kC(r)] + \delta^2\,[kB(r) - C(r_1) - (k-1)\,C(r)]}{1-\delta^2}.$$

The supplier thus has an incentive to deviate if $\hat{\phi}(\delta,\mathbf{r}) \equiv \left(1-\delta^2\right)\left[\hat{L}^R(\delta,\mathbf{r}) - \hat{S}(\mathbf{r})\right] < 0$. A first-order approximation yields:

$$\begin{aligned}
\hat{\phi}(\delta,\mathbf{r}) &\simeq \hat{\phi}\left(\delta,\mathbf{r}_S^D(\delta)\right) + \\
&\quad \delta\left\{B'\left(r_S^D(\delta)\right)[dr_1 - (k-1)\,dr] - kC'\left(r_S^D(\delta)\right)(-dr)\right\} \\
&\quad +\delta^2\left\{kB'\left(r_S^D(\delta)\right)(-dr) - C'\left(r_S^D(\delta)\right)[dr_1 - (k-1)\,dr]\right\} \\
&\quad -\left(1-\delta^2\right)S'\left(r_S^D(\delta)\right)[dr_1 - (k-1)\,dr] \\
&= \delta\left[B'\left(r_S^D(\delta)\right) + C'\left(r_S^D(\delta)\right)\right]kdr \\
&\quad -\delta^2\left[B'\left(r_S^D(\delta)\right) + C'\left(r_S^D(\delta)\right)\right]kdr \\
&\quad -\left(1-\delta^2\right)S'\left(r_S^D(\delta)\right)kdr \\
&= (1-\delta)\left\{\delta\left[B'\left(r_S^D(\delta)\right) + C'\left(r_S^D(\delta)\right) - (1+\delta)\,S'(r)\right]\right\}kdr \\
&< 0,
\end{aligned}$$

where the first equality follows from $\hat{\phi}\left(\delta,\mathbf{r}_S^D(\delta)\right) = \phi\left(\delta, r_S^D(\delta), k\right) = 0$ and $dr_1 = (2k-1)dr$, whereas the inequality stems from $\delta \in (0,1)$, $dr > 0$ and (18). It follows that the change in reserves strictly deters collusion while maintaining the buyer's total payoff. By continuity, there exists a neighboring change in reserves that keeps deterring collusion and enhances the buyer's payoff. ■

A.12 Proof of Proposition 13

Recall that given r, $U^{Comp}(v,r)$ and $U^{Coll}(v,r)$ are the buyer's expected payoff under deterrence and accommodation, respectively, and, similarly, denote by $\Pi^D(r)$ and $\Pi^A(r)$ the suppliers' expected joint payoff under deterrence and accommodation, respectively. Note that $U^{Coll}(v,r) < U^{Comp}(v,r)$ and $\Pi^D(r) < \Pi^A(r)$. Moreover, both $\Pi^D(r)$ and $\Pi^A(r)$ are increasing in r.

Given r, social welfare under deterrence is $U^{Comp}(v,r) + \Pi^D(r)$ and under accommodation it is $U^{Coll}(v,r) + \Pi^A(r)$. Social welfare under deterrence exceeding social welfare under accommodation is equivalent to

$$U^{Comp}(v, r^D(\delta)) + \Pi^D(r^D(\delta)) > U^{Coll}(v, r^A(\delta)) + \Pi^A(r^A(\delta)).$$

For the buyer to prefer accommodation, $U^{Comp}(v, r^D(\delta)) < U^{Coll}(v, r^A(\delta))$ has to hold, which requires that $r^A(\delta) > r^D(\delta)$. This implies that $\Pi^A(r^A(\delta)) > \Pi^D(r^D(\delta))$ and thereby that social welfare is higher under accommodation than under deterrence. That is, the buyer preferring accommodation implies that the social planner prefers accommodation.

That it is possible for the buyer to prefer deterrence while the planner prefers accommodation has been shown in the arguments preceding the proposition. This completes the proof. ■

A.13 Proof of Proposition 14

Part (i) of the proposition is established by noting that for $v = 1$ and uniformly distributed costs on $[0, 1]$, a discount factor of $\delta = 0.950$ is in the range of discount factors such that an integrated buyer deters collusion (see Figure 4(c)), but as shown in Figure 7(a), the only equilibrium of the reserve-setting game has accommodation, which increases social surplus as shown in Figure 4(c).

To establish part (ii), we first show that social welfare can be improved even when independent buyers continue to deter collusion, but with different reserves than $\mathbf{r}^D(v,\delta)$. Take the case of $v = 1$, $\delta = 0.94$, and uniformly distributed costs. As shown in Figure 7(d), an integrated buyer's optimal deterrence reserves $\mathbf{r}^D(v,\delta)$ (where $r_1^D(v,\delta) < r_2^D(v,\delta)$) belong to a range of Nash equilibrium deterrence reserves. Compared with the integrated buyer's optimal reserves $\mathbf{r}^D(v,\delta)$, an equilibrium based on a slightly lower r_1 involves a comparatively large increase in r_2 so as to stay on $\mathcal{B}_2(\delta)$, which increases the expected volume of trade and thus enhances social welfare.[63] A tweak to this example, in which

[63] As long as collusion is deterred, competition ensures that the more efficient supplier is selected; social welfare thus only depends on the expected volume of trade. In this example, where costs are uniformly distributed on $[0, 1]$, $v = 1$ and $\delta = 0.94$, $(r_1^D, r_2^D) = (0.393705, 0.489389)$ and, moving along

one increases r_1 and decreases r_2 shows that social welfare can decrease. ■

the deterrence boundary $\mathcal{B}_2$, reducing r_1 by 1×10^{-6} raises r_2 by 11×10^{-6} and increases the probability of trade, $\hat{G}(r_1^D) + \hat{G}(r_2^D)$, by 1×10^{-6}.

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