

# Passive internet search\*

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*(preliminary and incomplete)*

## Abstract

Advertisers compete in a market for horizontally differentiated products. Consumers are reached by unsolicited ads while surfing on the internet. Search is passive and searching through ads has no cost because consumers are online to enjoy some content. Delaying a purchase however involves some cost because future surplus is discounted. Clicking on an ad is costly and provides perfect information about the advertised product as well as an opportunity to buy. The market is either in a competitive regime or in a monopoly regime depending on whether click costs are low or high. In the competitive regime, firms advertise only product information, price increases with the click cost and it is lower if the advertised product information is more precise. In the monopoly regime, price is also advertised and decreases as the click cost become larger. Precise targeting may achieve a monopoly regime with complete surplus appropriation by firms even if click costs are low. If ads are solicited and search is active, the consumer's exclusive purpose on the internet is to purchase a product. As a result, searching through ads is costly. In the competitive regime, firms advertise price information in order to influence the consumers' search behavior. As a result, prices are lower and the competitive regime prevails for a larger range of click costs, but there is no monopoly regime and precise targeting leads to market unravelling because of a holdup problem.

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# 1 Introduction

The click technology has drastically modified the way firms interact with their customers through advertising. It provides consumers an immediate ability to learn more about the advertised product and possibly purchase it. Furthermore, although the internet provides numerous opportunities for active search, whereby consumers visit internet platforms (e.g. search engines, e-commerce platforms, price comparison web sites) for shopping purposes, much of the advertising activity on the web is generated by unsolicited advertising that reaches consumers while they are involved in online activities that are valued *per se* (e.g. e-mail, social networks, news, entertainment). When confronted with such advertising, a consumer trades off the costs and benefits from clicking on the ad and possibly buying, with the expected costs and benefits from waiting for future ads selling comparable products. My goal is to propose an analysis of such passive internet search and explore how it affects the informative content of advertising, the consumer's click and purchase behavior, firm pricing and the targeting of advertisements.

The core ingredient of my model is adapted from the monopoly analysis of advertising content in Anderson and Renault (2006). When consumers see an ad, they learn some information about how much they like the product and/or the price charged before deciding whether or not to click on the ad. Product information is merely a threshold match value such that each consumer when seeing the ad learns whether her match is above or below that threshold. Clicking involves some opportunity cost, reveals to consumers all relevant information (products are search goods) and allows them to make a purchase if they wish. This setting is embedded in a monopolistic competition sequential search framework adapted from Wolinsky (2006) and Anderson and Renault (1999), where sellers of horizontally differentiated products compete in prices.

In this passive search environment, consumers are exposed to unsolicited advertisements and there is no opportunity cost of staying online because consumers enjoy the services and

contents provided by the platform. If they decide not to click on an ad or not to purchase the product after clicking, they merely wait for the next ad concerning a competing product. The only cost of waiting is that the surplus from future purchases is discounted so that recalling a previously discarded purchase opportunity has an implicit cost (as compared to having seized that opportunity when it first came about). However, the expected utility from waiting cannot be negative (a consumer can always decide not to click on any ad).

It is shown that there is a competitive regime where the value of waiting is strictly positive and a monopoly regime where the value of waiting is zero, so that waiting is equivalent to the outside option of not buying. The former regime arises when the click cost is sufficiently low, in which case, each firm competes with the consumer's option of waiting for the next ad. Ads then contain only product information and price is increasing in the click cost because a higher click cost mitigates competition. For a given click cost, there is a continuum of equilibria depending on the level of the advertised threshold match. In equilibrium, a consumer clicks if and only if she finds out that her match exceeds the threshold. For the lowest such threshold, ads are completely uninformative and all consumers click on all ads. At the other extreme, the largest equilibrium threshold is such that only those consumers who end up buying the advertised product click on the ad. Price is decreasing in the threshold and it is highest when the threshold is so low that ads are completely uninformative and lowest when the threshold is such that all consumers who click buy the product. This is because, with a higher threshold, there are less wasted clicks (clicks without conversion), which increases search intensity and enhances competition. In this sense, a better product information makes consumers better off by reducing search costs. This contrasts with existing results in the literature where more product information hurts consumers by increasing perceived product differentiation and hence market power (see the discussion in Renault, 2015, section 3.2 and references therein)..

The monopoly regime arises when the click cost is large enough to deter consumers from clicking if they are not reassured that they will be charged a low enough price. Product

advertising with no price would then lead to a holdup problem and the market would unravel. Then, price is advertised along with product information. It is decreasing in the click cost. This decrease in price reflects the need for reassuring consumers with a lower price as the click cost becomes larger. This contrasts with the competitive regime where higher click costs induce higher prices because they mitigate competition.

Targeting is analyzed by assuming that advertisers can select to show an ad only to consumers whose match with the advertised product exceeds some threshold. If the targeting is loose (a low threshold) then the targeting outcome is similar to the outcome of product advertising with no advertised price in the competitive regime. However, in equilibrium, all consumers click on all ads. Furthermore, for a given threshold match, targeting consumers above that threshold induces a higher equilibrium price than partial product information with the same threshold so consumers unambiguously benefit and firms are hurt by targeting. A tighter targeting also induces a higher price. By contrast, if advertisers can achieve very tight targeting and use price advertising, the monopoly regime may arise in equilibrium even if click costs are low. Perfect targeting induces a perfect matching of products and consumers with firms capturing the entirety of social surplus.

The setting of this paper may also be modified to analyze active search where consumers go online to shop. Going from one ad to another then involves a surfing cost, which is the opportunity cost of remaining online. A consumer is reached by an additional ad only if she incurs that cost. Then the market does not survive in a monopoly regime, because consumers expecting to be held up, are not willing to incur the surfing cost to see any additional ad: hence they do not start search at all. In the analysis of the competitive regime, I take into account the possibility that, because consumers can see multiple ads simultaneously, they can modify their search behavior in favor of lower priced products, although they respond to price differences only imperfectly due to a limited ability to perceive these differences.

The competitive regime with active search differs in two important ways from the competitive regime with passive search. First, because prices affect search behavior, firms have

a strict incentive to advertise a price along with product information. Second, the impact of price on consumer search makes a firm's demand more elastic and hence, the equilibrium price is lower with active search.

The next section describes the passive search setting and provides an equilibrium characterization in the competitive and monopoly regimes. Section 3 introduces targeting and Section 4 presents the analysis of active search.

## 2 Passive search

A continuum of advertisers with measure 1 sell horizontally differentiated products to a continuum of buyers with measure 1. Production costs are zero. Each consumer is in the market for one unit. If she buys product  $i$  at price  $p_i$  her utility is:

$$u_i = r_i - p_i, \tag{1}$$

where  $r_i$  is a random term which is i.i.d. across consumers and products. If she does not buy, she has zero utility. The random valuation or match  $r_i$  has support  $[a,b]$ ,  $a \leq 0$  and  $b > 0$ : its distribution function is continuous and denoted  $F$  and it admits a density on  $(a,b)$  denoted  $f$ . the distribution function is assumed to satisfy the increasing hazard rate property.

In order to be able to sell, a firm needs to post an ad. Advertising is informative but not persuasive. It has no cost (or alternatively, a firm is charged per impression some amount that is assumed to be low enough so advertising remains profitable). An ad may contain: price information, and/or product information.

Product information may be partial (an informative signal about the true realization of the match). Each firm chooses the informative content of its ad. Ad content must be truthful. As in Anderson and Renault (2006), I assume that the firm has full control over that

joint distribution and consumers understand that advertised information is truthful so they update their beliefs in accordance with Bayes' rule. The monopoly analysis in Anderson and Renault (2006) shows that the firm cannot improve upon a product information disclosure scheme that tells each consumer whether her match is above or below some threshold  $\tilde{r}$  with no additional information. Here I assume that firms use this type of strategies without establishing that they would not wish to deviate to some alternative form of product information transmission. I also assume that a consumer clicks on an ad if and only if she infers from the ad that her match is above  $\tilde{r}$ . The latter assumption is without loss of generality as I explain below.

Each consumer is reached by one “unsolicited” ad at a time. She observes the information contained in the ad and then decides whether to click on the ad or not. If she clicks, she incurs a cost  $c > 0$ . If no price is advertised, then the consumer rationally anticipates the equilibrium price charged by the firm when deciding whether to click or not. Products are search goods so that, after clicking and before deciding whether to buy or not, the consumer finds out all the relevant information about the product as well as the price. If she chooses not to click or if she clicks and then chooses not to buy, she then waits at no cost until a new ad is shown to her. The wait is not costly because it is filled with surfing activity that the consumer values (doing e-mails, being involved in social networks, using the web to access news or entertainment). There is however discounting of any surplus enjoyed by the consumer from future ads. Let  $\delta \in (0, 1)$  be the discount factor that applies between two ads. The important point about unsolicited advertising is that whether the ad reaches the consumer or not does not depend on any decision made by the consumer. Although it is assumed, as in much of the search literature, that previous purchase opportunities can be recalled at no cost, recall is implicitly costly due to discounting: when giving up some utility  $u$  today a consumer anticipates that, if she waits for the next ad, she will only be able to retrieve the discounted utility  $\delta u$ .

Tie breaking rules for consumers are that if she is indifferent between clicking or not

she clicks and if she is indifferent between buying or not she buys. If the firm anticipated otherwise, it would lower its price slightly (or its advertised price in the case of clicking<sup>1</sup>) to change the consumer's decision. Tie breaking rule for the advertiser's choice to include additional information in its ad is that, when indifferent, it does not include the information. This captures the idea that there is some cost involved in disclosing more informative items.

I now turn to the equilibrium analysis starting with a characterization of the consumers' optimal search behavior..

## 2.1 Optimal consumer search and firm demand

When deciding whether or not to buy after clicking on an ad or when deciding whether to click or not, a consumer solves an optimal sequential search problem. The nature of this problem depends on what the consumer anticipates to find in terms of price and ad content and what she anticipates her click behavior to be. I look for equilibria where all firms have identical pricing and disclosure strategies. Because there is an infinite number of firms to be searched that are *ex ante* identical and expected to behave identically, the search problem is stationary and a consumer always prefers searching on to going back to an ad she has not clicked on or buying a product she has previously chosen not to buy.<sup>2</sup> Let  $s$  denote the cost of one search, which is endogenously determined by the firms' equilibrium disclosure strategy and the consumers' optimal clicking behavior. Let  $v$  be a random variable that measures the utility a consumer can expect from a firm when incurring search cost  $s$ . It depends on the firms' equilibrium price as well as their equilibrium disclosure strategy. Let  $G$  denote the distribution function for  $v$ .

Suppose the best option the consumer holds so far has utility  $u$ . Because of the infinite number of firms and the i.i.d. distribution of matches across firms, in an equilibrium where

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<sup>1</sup>In particular, it was not advertising any price it would advertise a price slightly below the equilibrium price that consumers expect to find if they click.

<sup>2</sup>Here stationarity holds despite the implicit recall cost introduced by discounting discussed above. This would not be the case with a finite number of firms or with firms that are *ex ante* different from the consumers' perspective.

all firms have the same strategy the consumer's search problem is stationary. Let  $\hat{u}$  denote the expected utility from searching on. It is optimal for the consumer to keep searching as long as  $u < \hat{u}$  (the inequality is strict from the tie-breaking rule). Then  $\hat{u}$  solves

$$\hat{u} = \delta \left( -s + \int_{\hat{u}}^{\infty} (v - \hat{u}) dG(v) + \hat{u} \right). \quad (2)$$

The search cost,  $s$ , is discounted because it will be incurred only when the next ad comes along. The integral measures the expected utility gain in the event that the consumer prefers buying the next advertised product than searching on. Bringing the search cost to the right hand side, the above condition may be rewritten as

$$\int_{\hat{u}}^{\infty} (v - \hat{u}) dG(v) + \left(1 - \frac{1}{\delta}\right) \hat{u} = s. \quad (3)$$

This formula bears much resemblance to the standard condition that characterizes the reservation utility in an optimal sequential search problem. There is however an additional term  $(1 - \frac{1}{\delta})\hat{u}$  which reflects the cost of recalling  $\hat{u}$  after waiting for the next ad and therefore enjoying only a discounted utility level (this would be the interpretation for instance, if the consumer had to decide whether or not to search one firm with no further search options). Because the consumer can always choose not to click on any ad, the value of searching on cannot be strictly negative. If the click cost becomes too large, then a consumer never clicks on any ad so that the search cost is  $s = 0$  and the utility from seeing an ad,  $v$ , is zero with certainty: as a result  $\hat{u} = 0$ .

As in the sequential search problem with costless recall, the left-hand side is strictly decreasing in  $\hat{u}$ . However, because of the second term which is strictly negative if  $\hat{u} > 0$ , it reaches zero for some value of  $\hat{u}$  which may be strictly less than the maximum of the support of  $v$ : let  $\bar{v}$  denote that value. Then  $\hat{u}$  decreases from  $\bar{v}$  to  $-\infty$  as  $s$  increases from zero to  $+\infty$  ( $\bar{v}$  is the maximum of the support of  $v$  if and only if  $\hat{u} = 0$  with  $s = 0$ ).

Now consider a firm that has not advertised a price, deciding which price to charge to a

consumer who clicks on its ad. If the firm charges some price  $p$ , the consumer buys if and only if  $r - p \geq \hat{u}$ . Let  $p^*$  be the equilibrium price expected by the consumer. We necessarily have

$$p^* \geq \tilde{r} - \hat{u}, \quad (4)$$

where  $\tilde{r}$  is the common threshold used by firms in their match disclosure strategy, so that only consumer with a match above  $\tilde{r}$  are expected to click on the ad. If (4) was violated, the firm would find it profitable to deviate from  $p^*$  by increasing its price slightly and losing no consumer as long as the price is below  $\tilde{r} - \hat{u}$ . This is related to the holdup argument used by Anderson and Renault (2006) in the monopoly setting to show that the market unravels when advertising contains perfect match information but no price.<sup>3</sup> Now a firm charging a price  $p$  that satisfies (4) sells with unconditional probability  $\Pr\{r \geq \hat{u} + p\} = 1 - F(\hat{u} + p)$ <sup>4</sup>. Hence, in order for  $p^*$  to be the price selected by the firm, it must satisfy the standard first order condition

$$p^* \geq \frac{1 - F(\hat{u} + p^*)}{f(\hat{u} + p^*)}, \quad (5)$$

with equality if  $p^* > \tilde{r} - \hat{u}$ . The inequality in the first order condition results from the kink in demand at price  $\tilde{r} - \hat{u}$ .

Hence there are two regimes: the competitive regime where  $\hat{u} > 0$  and the firm is competing against ongoing search among competitors and the monopoly regime when  $\hat{u} = 0$  with firms competing against the outside option as would a monopolist. Because  $\hat{u}$  is decreasing in  $s$ , the latter regime arises if  $s$  is large enough. The competitive regime exists only if  $\bar{v} > 0$  so that  $\hat{u}$  is strictly positive for a low enough search cost.

Much of the subsequent analysis focuses on situations where, for  $p^* = \tilde{r} - \hat{u}$ , the first order condition either holds with equality or is violated. It follows that the equilibrium price must satisfy the first order condition (5) with equality. The increasing hazard rate property guarantees existence. For  $\hat{u} > 0$ , the increasing hazard rate property also implies

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<sup>3</sup>see Stiglitz (1979) for an early formulation of the unraveling argument.

<sup>4</sup>This is the probability of a sale conditional on a click times the probability of a click.

that price is decreasing in  $\hat{u}$  and hence increasing in search cost  $s$ . Furthermore, for  $\hat{u} = 0$  the equilibrium price is merely the monopoly price, henceforth denoted  $p^m$ , because the firm then only competes with the outside option.

## 2.2 Product information and the competitive regime

Assume that ads contain no price. They merely provide product information and an opportunity for consumers to click and possibly buy the product. Provided that the click cost is strictly less than the expected surplus of a consumer when price equals marginal cost, there cannot be an equilibrium with no price advertising where firms have zero sales. A firm could deviate from such an equilibrium by advertising a low enough price, such that the expected consumer surplus from clicking is strictly positive, whereas the expected surplus from waiting for the next ad is zero because the consumer expects to buy nothing from the next firm.

In order for a firm to make some sales, there must be some range of match values for which a consumer weakly prefers to click on its ad rather than not. In equilibrium, either all the consumers click on all ads, in which case the disclosed product information has no impact on the outcome, or only those consumers who learn that their valuation exceeds  $\tilde{r}$  click. The former case is equivalent to a situation where ads are completely uninformative: it is then equivalent to the special case where  $\tilde{r} = a$  and all consumers click on all ads. In the latter case, the no holdup condition (4) must hold in equilibrium so that  $\tilde{r} \leq \hat{u} + p^*$ .

Let us start by analyzing the consumers' search behavior. Condition (4) may be written as

$$\tilde{r} \leq \hat{x}, \tag{6}$$

where  $\hat{x} \equiv \hat{u} + p^*$ . In order for the consumer to obtain a utility above  $\hat{u}$  with the next ad posted by some firm  $j$ , she must draw some  $r_j$  such that  $v = r_j - p^* \geq \hat{u}$ , and hence  $r_j \geq \hat{x} \geq \tilde{r}$ . Hence a consumer clicks on firm  $j$ 's ad if and only if  $r_j \geq \tilde{r}$  and buys firm  $j$ 's

product if and only if  $r_j \geq \hat{x}$ . When deciding to search, the consumer anticipates that she will click with probability  $[1 - F(\tilde{r})]$  so that her expected search cost is  $s = [1 - F(\tilde{r})]c$ . The optimal search condition (3) becomes

$$h(\hat{x}, p^*) \equiv \int_{\hat{x}}^b (r - \hat{x})dF(r) + \left(1 - \frac{1}{\delta}\right) (\hat{x} - P^*) = [1 - F(\tilde{r})]c. \quad (7)$$

Then a consumer searches if and only if she currently holds a utility level less than  $\hat{u} = \hat{x} - p^*$ . The function  $h$  being strictly decreasing in  $\hat{x}$  and strictly increasing in  $p^*$ ,  $\hat{x}$  is strictly decreasing in  $c$  and strictly increasing in  $p^*$  as well as in  $\tilde{r}$ . This means that consumers search less if the click cost is higher, price is lower or they expect to click on an ad with a higher probability. In order to have  $\hat{u} \geq 0$  we must have  $\hat{x} \geq p^*$ , which for a given price and threshold value requires that the click cost is not too large.

Let us now turn to pricing. Because  $\hat{u} \geq 0$ , the equilibrium price has a closed form solution given by

$$p^* = \frac{1 - F(\hat{x})}{f(\hat{x})}. \quad (8)$$

Because of the increasing hazard rate property,  $p^*$  is decreasing in  $\hat{x}$ . It is close to zero when  $\hat{x}$  is close to  $b$  and reaches the monopoly price  $p^m$  when  $\hat{x}$  falls to  $p^m$ . For lower values of  $\hat{x}$ , the requirement that  $\hat{x} \geq p^*$  would no more be satisfied. As a result we must have  $\hat{x} \geq p^m$ .

Finally, equilibrium also requires that when a consumer learns that her match with the advertised product exceeds the threshold, she prefers to click on the ad rather than waiting for the next ad. Because  $\hat{x} \geq p^*$ , a consumer who clicks on an ad expects to buy if and only if  $r \geq \hat{x}$  thus obtaining utility  $r - p^*$ . Her utility if she waits for the next ad rather than clicking is  $\hat{u} = \tilde{x} - p^*$ . Hence her expected gain from clicking is

$$\frac{\int_{\hat{x}}^b (r - \hat{x})dF(r)}{1 - F(\tilde{r})} > \frac{h(\tilde{x}, p^*)}{1 - F(\tilde{r})} = c, \quad (9)$$

where the inequality follows from (7). The optimal search condition (7) therefore ensures

that consumers strictly prefer clicking than waiting for the next ad.

From the analysis above, an equilibrium may be characterized as follows.

**Lemma 1** *A Perfect Bayesian Equilibrium is characterized by a disclosure threshold,  $\tilde{r} \in [a, b]$ , a search threshold  $\hat{x} \in [p^m, b]$ , and a price  $p^* \in [0, p^m]$  that jointly satisfy the no hold up condition (6), the optimal search condition (7) and the optimal pricing condition (8).*

First consider the equilibria where ads are completely uninformative,  $\tilde{r} = a$ . The no holdup condition (6) then always holds because we must have  $\hat{x} \geq p^m > a$ . The optimal search rule is  $h(\hat{x}, p^*) = c$  which defines a strictly increasing relation between  $p^*$  and  $\hat{x}$ . The equilibrium pricing condition (8) defines a strictly decreasing relation between  $\hat{x}$  and  $p^*$ : as  $\hat{x}$  increases from  $p^m$  to  $b$ ,  $p^*$  decreases from  $p^m$  to 0. The two relations between  $\hat{x}$  and  $p^*$  cross at most once. I now show that for  $c$  small enough, such a crossing point exists and defines an equilibrium with uninformative ads.

For  $\hat{x} > p^*$ , the term  $(\frac{1}{\delta} - 1)(\hat{x} - p^*)$  is negative. hence, for any  $c > 0$ , in order to have  $h(\hat{x}, p^*) = c$  we must have  $\int_{\hat{x}}^b (r - \hat{x})f(r)dr > 0$ , which requires  $\hat{x} < b$ . Hence, in order to have a crossing point we only need to make sure that there exists a value of  $p^*$  such that the value of  $\hat{x}$  such that  $h(\hat{x}, p^*) = c$  is at least  $p^m$ . Because the value of  $\hat{x}$  that is a solution is increasing in  $p^*$ , a necessary and sufficient condition is that the value of  $\hat{x}$  solving  $h(\hat{x}, p^m) = c$  is at least  $p^m$ . However, since  $h$  is strictly decreasing in  $\hat{x}$ , this is equivalent to  $h(p^m, p^m) \geq c$ . Hence, the non informative ad equilibrium exists if and only if  $c \leq c_1 \equiv h(p^m, p^m)$ . Let  $\hat{x}_u$  and  $p_u^*$  respectively denote the search threshold and the price associated with this uninformative ad equilibrium. Standard comparative statics on equations (7) and (8) shows that as  $c$  increases from 0 to  $c_1$ ,  $\hat{x}_u$  decreases from some value strictly less than  $b$  to  $p^m$  and  $p_u^*$  increases from some strictly positive level to  $p^m$ .

Consider now an equilibrium where  $\tilde{r}$  has the largest value consistent with the no holdup condition, which is  $\tilde{r} = \hat{x}$ , so that all consumers who click on an ad end up purchasing the advertised product. Then the optimal search condition is  $\frac{h(\hat{x}, p^*)}{1 - F(\hat{x})} = c$ . Using arguments from

Anderson and Renault (2006), it is readily shown that, under the increasing hazard rate condition for the match distribution, the left-hand side is strictly decreasing in  $\hat{x}$ . Furthermore, in order for the term  $\frac{\int_{\hat{x}}^b (r-\hat{x})dF(r)}{1-F(\hat{x})}$  to be strictly positive,  $\hat{x}$  must be strictly below  $b$ . Then, analogous arguments to those used in the case  $\tilde{r} = a$  show that there exists an equilibrium if and only if  $c \leq c_2 \equiv \frac{h(p^m, p^m)}{1-F(p^m)} > c_1$ . Let  $\hat{x}_t$  and  $p_t^*$  be respectively the search threshold and the price associated with such an equilibrium where all consumers who click on an ad buy the product. Again, performing comparative statics on equations (7) and (8) yields that  $\hat{x}_t$  is strictly decreasing in  $c$  and is strictly below  $b$  while  $p_t^*$  is strictly increasing in  $c$  and is strictly positive, for  $c \in [0, c_2)$ , and they both equal the monopoly price  $p^m$  for  $c = c_2$ .

It is now straightforward to establish that, for  $c \in [0, c_1]$  and  $\tilde{r} \in (a, \hat{x}_t)$ , there exists a product advertising equilibrium at which,  $\hat{x}_u < \hat{x} < \hat{x}_t$  and  $p_t^* < p^* < p_u^*$ . First, for any  $\tilde{r} > a$ , the arguments used to prove the existence of the uninformative ad equilibrium can be applied to show that for  $c \leq c_1$ , equations (7) and (8) jointly determine a unique solution  $(\hat{x}, p^*)$ . Comparative statics shows that a larger  $\tilde{r}$  induces a higher  $\hat{x}$  and a lower price  $p^*$ . Furthermore,  $\hat{x}$  tends to  $\hat{x}_u$  as  $\tilde{r}$  tends to  $a$  and to  $\hat{x}_t$  as  $\tilde{r}$  tends to  $\hat{x}_t$  while  $p^*$  tends to  $p_u^*$  as  $\tilde{r}$  tends to  $a$  and to  $p_t^*$  as  $\tilde{r}$  tends to  $\hat{x}_t$ . These values characterize an equilibrium provided that they satisfy the no holdup condition  $\tilde{r} \leq \hat{x}$ . [Argument for this to be included later: basic idea is that because  $\tilde{r} < \hat{x}_t$ , if  $\tilde{r} = \hat{x}$  then the LHS of the optimal search condition is strictly larger than the RHS; then equality is achieved by picking  $\hat{x} > \tilde{r}$ ]. Hence, for  $c \in (0, c_1]$ , there is a continuum of equilibria characterized by different disclosure thresholds  $\tilde{r} \in [a, \hat{x}_t]$ , where a larger disclosure threshold induces more search ( $\hat{x}$  increases from  $\hat{x}_u$  to  $\hat{x}_t$ ) and a lower price ( $p^*$  decreases from  $p_u^*$  to  $p_t^*$ ).

For  $c \in (c_1, c_2]$ , analogous arguments show that there exists an equilibrium for any disclosure threshold  $\tilde{r} \leq \hat{x}_t$ , which is above some lower bound that strictly exceeds  $a$  and increases with  $c$ . Recall that  $c_1 = h(p^m, p^m)$ . The click cost  $c$  being above  $c_1$ , if the threshold  $\tilde{r}$  is too close to  $a$ , then  $(\hat{x}, p^*)$  are too close to  $(p^m, p^m)$ , and the optimal search condition cannot be satisfied. The lower bound on the disclosure threshold is given by  $h(p^m, p^m) = [1 - F(\tilde{r})]c$ .

For  $c > c_2$ , there is no solution to equations (7) and (8) such that  $\hat{x} \geq p^*$ , even if the disclosure threshold is  $\hat{x}$ . Then the market unravels if firms cannot advertise a price.

Results are summarized in the following proposition.

**Proposition 1** *A product advertising equilibrium exists if and only if  $c \leq c_2 \equiv \frac{h(p^m, p^m)}{1-F(p^m)}$ . Let  $(\hat{x}_t, p_t^*)$  be the solution to equations (7) and (8) for  $\tilde{r} = \hat{x}$ . Then for any  $c \leq c_2$ , there is a continuum of equilibria with  $\tilde{r} \leq \hat{x}_t$ : equilibrium search threshold is strictly increasing in the disclosure threshold and reaches  $\hat{x}_t$  for  $\tilde{r} = \hat{x}_t$  whereas the equilibrium price is strictly decreasing in the disclosure threshold and reaches  $p_t^*$  for  $\tilde{r} = \hat{x}_t$ .*

Let  $c_1 \equiv h(p^m, p^m) < c_2$  and let  $(\hat{x}_u, P_u^*)$  be the solution to equations (7) and (8) for  $\tilde{r} = a$ .

1. For  $c \in (0, c_1]$ , the disclosure threshold may be as low as the uninformative threshold  $\tilde{r} = a$ : the corresponding equilibrium involves the least equilibrium consumer search  $\hat{x} = \hat{x}_u$  and the highest price,  $p^* = p_u^*$ . As  $c$  increases,  $\hat{x}_u$  decreases and  $p_u^*$  increases and they are both  $p^m$ , for  $c = c_1$ .
2. For  $c \in (c_1, c_2]$ , the lowest value of the disclosure threshold satisfies  $h(p^m, p^m) = [1 - F(\tilde{r})]c$ : the corresponding equilibrium involves the least consumer search with  $\hat{x} = p^m$  and the highest price,  $p^* = p^m$ .

Product advertising equilibria are associated with the competitive regime unless  $c \geq c_1$  and the disclosure threshold has the lowest possible value, resulting in the monopoly price  $p^m$ .

I now turn to the role of price advertising.

### 2.3 Price advertising and the monopoly regime

First, let us show that given the tie breaking rule that a firm advertises an additional item only if it strictly benefits, if  $c \leq c_1$ , there is no equilibrium where price is advertised. To

see this, start with an equilibrium where the price is as in the previous analysis. If the firm chooses not to advertise this price, the consumer expects that she will be charged this price if she decides to click because it is profit maximizing for the firm. Besides, it can be shown that a firm could always profitably deviate from any advertised price that would not satisfy (5), in a way that would still induce any consumer with a match above the advertised threshold to click. The argument uses the quasiconcavity of profit with respect to price as well as equation (7).

Assume now that  $c > c_1$ . A critical intermediate result is that if there exists an equilibrium with an advertised price, then we necessarily have  $\hat{u} = 0$ . Suppose instead that  $\hat{u} > 0$ . First, the equilibrium price cannot be larger than  $p^*$  defined by (8). Because of the quasiconcavity of profit, the firm would always benefit from decreasing its price slightly and consumers with match values above the threshold would all the more be willing to click. Now suppose the equilibrium price is strictly below  $p^*$  and  $\hat{u} = \hat{x} - p > 0$ , where  $p$  is the equilibrium price. Then, equation (7) implies that the click constraint (9) is slack. It follows that, still using the quasiconcavity of profit with respect to price, a firm could increase its profit by slightly increasing its price and consumers would still click.

The result above implies that each firm behaves like a monopolist competing against the outside option and therefore the analysis in Anderson and Renault (2006) applies. It shows that for this range of click costs, a firm finds it optimal to advertise both a price and some product information. Price is decreasing in the click cost. Furthermore, there exists some click cost level  $c_3 > c_2$  such that for  $c \in (c_2, c_3]$  the advertised threshold match is equal to the price whereas for  $c \geq c_3$ , the advertised threshold match is equal to the click cost. Here, firms price below monopoly price to overcome large click costs rather than in response to competition from other firms. This is why price is decreasing in the click cost. In this monopoly regime, consumers with match values above the disclosure threshold are just indifferent between clicking or not and expected consumer surplus is zero.

As I now illustrate, the monopoly regime with price advertising may arise for click costs

blow  $c_2$  if the targeting of ads to consumers is sufficiently tight.

### 3 Targeted advertising

The modeling trick used to describe partial match information in ads can easily be adapted to analyze targeting. Suppose that instead of containing product information on the basis of which consumers determine whether their match exceeds some threshold  $\tilde{r}$ , ads are targeted so as to reach only consumers whose match exceeds  $\tilde{r}$ . In other words, advertisers are appropriately informed about consumers to determine whether their match exceeds the threshold. In his analysis of search advertising, de Cornière (forthcoming) provides a clever modeling of how such targeting might arise. He considers consumers and firms located on a Vickrey/Salop circle (Vickrey, 1964 and Salop, 1979). Consumers use keywords to indicate their precise location to a search engine and they are then matched with all firms whose product falls within a certain distance of their location. As a result, consumers expect to be shown only ads for products with which their match exceeds a certain threshold and firms know they are only dealing with consumers with a match above that same threshold.

In my passive search setting, targeting may be viewed as selecting for each ad the subset of consumers whose match is in some interval  $[\tilde{r}, b]$  with  $a < \tilde{r} \leq b$ . Consider now an equilibrium in Proposition 1 with  $\tilde{r} > a$  and suppose that each ad is targeted to consumers with a match above the threshold for the advertised product. Suppose that each consumer clicks on all the ads that are presented to her. Thanks to targeting, she anticipates that, for each new advertised product, her match distribution is now truncated to matches above  $\tilde{r}$  so the cumulative distribution function is now  $\frac{1}{1-F(\tilde{r})}F$ . Optimal search is then characterized by the following equation

$$\frac{\int_{\hat{x}}^b (r - \hat{x}) dF(r)}{1 - F(\tilde{r})} + \left(1 - \frac{1}{\delta}\right)(\hat{x} - p^*) = c, \quad (10)$$

or equivalently

$$h(\hat{x}, p^*) - F(\tilde{r})(1 - \frac{1}{\delta})(\hat{x} - p^*) = [1 - F(\tilde{r})]c. \quad (11)$$

This differs from equation (7) only because of the second term on the left-hand side which is positive. The left-hand side is therefore larger than in (7) and it is strictly decreasing in  $\hat{x}$ . Hence, for any  $p^*$  the corresponding value of  $\hat{x}$  is larger than without targeting, meaning that consumers search more. As a result, the equilibrium involves a lower price. Consumers therefore benefit from targeting in two ways. First, they get a better selection of products from any ad they are exposed to. Second, they pay a lower price. By contrast, firm profits are deteriorated by targeting. Furthermore, as targeting becomes more accurate, involving a higher threshold  $\tilde{r}$ , price goes down so that the consumers benefit and firms are hurt. De Cornière (forthcoming) finds a similar impact of improved targeting in his setting.

The impact of targeting however changes dramatically if firms may resort to price advertising. This may be illustrated with the following extreme equilibrium. Assume targeting is perfect, so consumers see only ads for products with which their valuation is  $b$ . Absent price advertising, this would yield an extreme form of the holdup problem whereby consumers would expect to be charged a price of  $b$  if they click and would therefore not click. However, if firms advertise a price of  $b - c$  then consumers are willing to click and firms may sell at that price. Note that this yields an expected utility of zero from searching on so we are in the monopoly regime even though the click cost may be strictly below  $c_1$ .

This argument may be generalized to less extreme forms of targeting. Take any targeting threshold above  $\hat{x}_t$ . With such a threshold, the no holdup constraint would be violated. This is why  $\hat{x}_t$  is the largest possible disclosure threshold consistent with product advertising. Nonetheless firms can resort to price advertising to commit to a low enough price so that a consumer expecting her threshold to be above  $\tilde{r}$  is willing to click [need to fill in the details].

## 4 Active search

### 4.1 Hold-up with active search

I call active search a situation where a consumer goes on the internet for the sole purpose of searching for and purchasing a product. This means that she goes on the internet and keeps on searching only as long as the value of search is positive. More specifically, suppose that staying on to see one more ad involves some small opportunity cost  $\gamma$ , henceforth called surfing cost. Because the consumer may see multiple ads in a very short period of time, there is however no discounting. As a result, the optimal search behavior for a given advertised threshold match  $\tilde{r}$  is now characterized by a critical match value  $\hat{x}$  solution to

$$j(\hat{x}) = \int_{\hat{x}}^b (r - \tilde{x}) dF(r) = \gamma + [1 - F(\tilde{r})]c, \quad (12)$$

where, for an equilibrium price  $\bar{p}^*$  the expected utility from searching is  $\hat{u} = \hat{x} - \bar{p}^*$ . The consumer prefers buying the product to searching on if and only if her match is at least  $\hat{x}$ .

With this specification, it is straightforward to show that the profit maximizing price in the competitive regime for a firm who has not advertised a price is  $\bar{p}^* = p^*$ , given by (8). As before, as the click cost increases starting from zero, price increases and  $\hat{x}$  decreases until they equal each other, which happens when  $\hat{x} = p^* = p^m$ . At this point, the utility of search is  $\hat{u} = 0$ . The largest click cost for which this may happen is obtained by focusing on the equilibrium where all consumers who click buy, that is when  $\tilde{r} = \hat{x} = \bar{x}$ . For a larger click cost, the expected utility from search becomes strictly negative and the profit maximizing price is  $p^m$  as long as consumers are still willing to click at that price. For the click cost at which  $\bar{x} = p^m$ , consumers still strictly prefer to click than to search if they learn from the ad that their match exceeds  $p^m$ . This can be seen from the optimal search characterization (12). However, when the click cost reaches  $\frac{j(p^m)}{1-F(p^m)}$ , then consumers would not want to click anymore unless they are reassured that the price is less than  $p^m$ .

Even though the firm commits to a lower price, it does so in a way that the expected surplus from clicking is always zero, as shown in Anderson and Renault (2006). As a result, the expected utility from searching is  $-s < 0$ . This quick analysis shows that when the click cost exceeds the value such that  $\bar{x} = p^m$ , the expected utility from search is strictly negative so that consumers would not start searching in the first place. Hence the monopoly regime is not viable because there is a holdup problem.

The analysis thus far assumes that consumers can only see the information in one ad at a time. This seems a bit unrealistic for search platforms, especially when it comes to price information.

## 4.2 Active search with pre search price comparisons

Accounting for the role of price advertising in sequential search models with product differentiation is quite challenging. Because all firms look *a priori* the same for consumers, in a model where they get to compare all prices before they start searching, the outcome is inevitably a mixed strategy equilibrium as in similar models of a market for a homogeneous product (e.g. Robert and Stahl, 1993). Accounting for these mixed strategies in prices greatly complicates the analysis of consumer search. One possible solution is to assume that consumers have some prior information about their match with the various products, which smoothes out the impact of price differences on the consumer's search behavior. Hahn, Morga and Petrikaite (2015), derive a pure strategy equilibrium with a unique price in a duopoly setting. The analysis is however quite involved, because each consumer follows a different order of search depending on her *ex ante* preference ordering of the different products.

Here I follow a different route, which relies on the idea that consumers differ in the attention they pay to price differences when they decide which firm to search first. This is inspired by the use of “just noticeable differences” by Anderson and de Palma (????) to partially smooth out the demand for a homogeneous product. Contrary to them, I assume that products are differentiated and that any price difference is completely accounted for in

a consumer's purchase decision. However, consumers only imperfectly take price differences into account when deciding which firm to search first. The specification is as follows.

In the base model used to analyze the holdup problem in active search, at each round of search, the expected number of consumers who search a given firm is equal to the expected number of consumers remaining at that round: because a consumer stops searching if and only if she finds a match above  $\hat{x}$ , the expected number of consumers still searching after  $k$  rounds is  $F(\hat{x})^k$  and hence this is the expected measure of consumers searching any given firm. This is because consumers search randomly and the initial measure of consumers per firm is one. Suppose now that, if a firm deviates to some price  $p$  different from the equilibrium price  $p^*$ , there is some possibility that some consumers realize this and as a result modify their search behavior: the expected number of searches for the firm at any given round is increased if  $p < p^*$  and decreased if  $p > p^*$ . The extent to which the number of searches is affected is smoothly increasing in the price difference. Formally, assume there is a differentiable and strictly increasing function  $h$ , such that  $h(0) = 1$  and the expected number of searches for the firm after  $k$  rounds is  $h(p^* - p)F(\hat{x})^k$ . The idea is that a consumer is more or less likely to notice price differences, but she is all the more likely to notice that the firm's price differs from the other firms' price that the price difference is larger.

A first important difference with the passive search setting is that, because price differences can influence the consumers' search behavior, firms always have a strict incentive to advertise a price. Hence, the competitive regime involves the advertising of both product information and price. This also modifies the profit maximization objective of the firm. It now seeks a price that maximizes its expected profit, taking into account its impact on the probability that it is searched early on. Firm  $j$  now selects a price  $p_j$  that maximizes profit

$$\frac{h(\bar{p}^* - p_j)}{1 - F(\hat{x})} [1 - F(\hat{s} + p_j - \bar{p}^*)] p_j, \quad (13)$$

where  $\bar{p}^*$  is the equilibrium price charged by the other firms.

The symmetric equilibrium price is therefore

$$\bar{p}^* = \frac{1 - F(\hat{x})}{f(\hat{x}) + h'(0)[1 - F(\hat{x})]}. \quad (14)$$

(Incomplete)

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