Optimal Allocations with Capacity Constrained Verification

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October 29, 2019

Introduction: Mechanism design & verifiable information

- How to design institutions and rules such that a desirable outcome can be reached, although information is privately held?
- Standard model assumes soft information, e.g. about preferences
- In some applications, private information is based on hard facts
- For example, "experts" with private information
  – Information is based on hard evidence
- Designer can verify experts' private information
- Examples: work permits, research grants, ...

Introduction: assigning working permits in Canada

- Recruit skilled workers:
  "*Canada is changing its economic immigration programs to provide more opportunities to prospective skilled immigrants*"
- Policy in place from 2015, called "express entry"
- Applicants are ordered in the pool according to a score
- Score is based on factors that are known to contribute to economic success

EXPRESS ENTRY
ENTRÉE EXPRESS

www.canada.ca/ExpressEntry

WHAT PROSPECTIVE CANDIDATES NEED TO KNOW

Canada is changing its economic immigration programs to provide more opportunities to prospective skilled immigrants.

As of January 2015, skilled foreign workers have access to Express Entry, which covers Canada's key economic immigration programs:

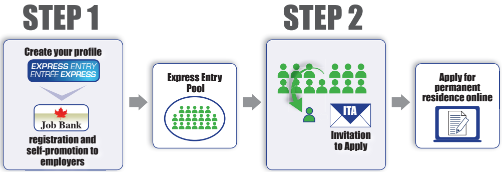
- the Federal Skilled Worker Program;
- the Federal Skilled Trades Program;
- the Canadian Experience Class; and
- a portion of the Provincial Nominee Program.

Candidates who are invited to apply for permanent residence under the Express Entry system will benefit from fast processing times of six months or less.

Express Entry also provides a pathway for skilled workers to connect with potential job opportunities in Canada prior to arrival.

Express Entry ensures that the candidates who are most likely to succeed economically – not simply those first in line – are able to immigrate to Canada.

How Express Entry Works



Step 1

Potential candidates create an online Express Entry profile

Express your interest in coming to Canada as a skilled foreign worker. Create an online Express Entry profile and tell us about your skills, work experience, language ability, education and other details. Before doing this, you will need to take a language test in English or French. If you were educated outside of Canada, you may also need to have your education assessed against Canadian standards. More information on language and education assessments is available online.

If you meet the criteria of one of the federal economic immigration programs subject to Express Entry, you will be placed in a pool of pre-screened candidates.

If you do not already have a Canadian job offer or a nomination from a province/territory, you must register with the Government of Canada's Job Bank. Job Bank is an easy, online search tool that will help you get matched with jobs in Canada based on your skills, knowledge and experience.

Express Entry Pool

You will be given a score to determine your place in the Express Entry pool using a Comprehensive Ranking System that includes factors known to contribute to economic success (such as language, education, and work experience).

There will be regular rounds of invitations issued to candidates from the Express Entry pool, inviting them to apply for permanent residence. Candidates with the highest scores, including those who have a valid job offer or a provincial/territorial nomination, will be invited to apply.

Your Express Entry profile will be valid for 12 months. During that time, you will need to update your profile if circumstances change such as your level of education or language test results.

Important: Filling out an online Express Entry profile is not a guarantee that you will qualify for permanent residence. If you are invited to apply for permanent residence, information provided in your Express Entry profile will be verified at that time.

Step 2

Selected candidates are invited to submit an electronic application for permanent residence

You will receive an Invitation to Apply for permanent residence if you:

- have a valid job offer from an employer in Canada
  (subject to the Labour Market Impact Assessment process in place at that time);
- have been nominated by a province or territory; or
- are among the top ranked in the pool based on your skills and experience.

Citizenship and Immigration Canada will process the majority of complete permanent residence applications received within six months or less.

Candidates in the Express Entry pool who do not receive an Invitation to Apply for permanent residence after 12 months can resubmit their profile and re-enter the pool if they still meet the criteria.

For more details on Express Entry, see www.canada.ca/ExpressEntry.

Introduction: verification and assigning working permits

- m working permits to assign among n applicants, $m < n$
- Allocate them to applicants with highest expected earnings
- An application consists of qualifications, degrees, ...
- Private information but it can be **verified**
- At most k applicants can be verified, $k < m$

What is the optimal way to allocate these permits, given the possibility of verifying agents?

Introduction: allocations with capacity constrained verification

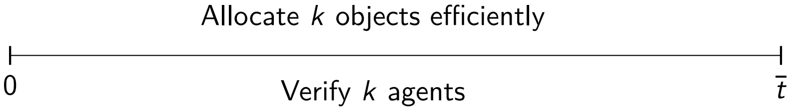
There is a group of n agents and a principal with m objects

The principal determines an allocation, and $m < n$

- Agents have information that is relevant for the principal
- Agents would like to have an object
- Principal can learn agents' types
- Learning is capacity constrained:
  at most k agents' information can be learned, where $k < m$
- Monetary transfers are not possible

Possible mechanisms

- **Mechanism 1:**



Possible mechanisms

- **Mechanism 1:**

Allocate k objects efficiently

0 Verify k agents $\bar{t}$

Improve by allocating remaining objects randomly:

- **Mechanism 2:**

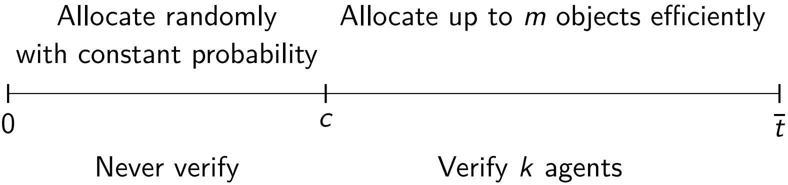
Allocate k objects efficiently and remaining
$m - k$ objects randomly with an equal probability

0 Verify k agents $\bar{t}$

Two possible mechanisms, both incentive compatible

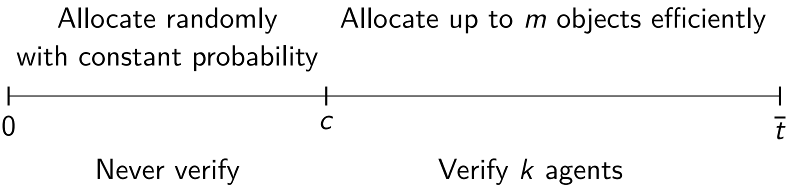
Possible mechanisms

- **Mechanism 3:**



Possible mechanisms

- **Mechanism 3:**



- Not ex-post incentive compatible:
  for example, if $t_i < c$ and m agents to the right of c,
  agent i could deviate and report $t_i' > c$
- Bayesian incentive compatible for sufficiently high cutoff c

Aside: Incentive constraints

Bayesian incentive compatibility:

$$\underbrace{Pr(\text{Getting an object}; t_i, t_i)}_{\textit{Truthtelling}} \geq \underbrace{Pr(\text{Getting an object}; t_i', t_i)}_{\textit{Lying}}$$

Aside: Incentive constraints

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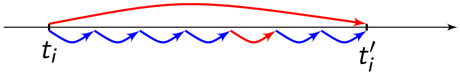
- In many settings local incentive constraints are sufficient



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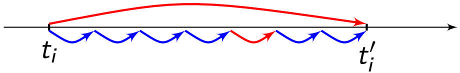
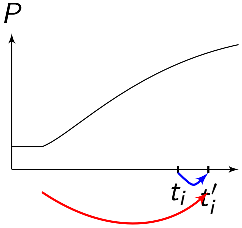


- In our model local constraints are not sufficient



Incentive compatibility

Local incentive constraints are not sufficient!

Let $P(t_i) = Pr(\text{Getting an object}; t_i, t_i)$

Incentive compatibility must hold for "worst-off" types

Incentive compatibility

A“worst-off” type $t_i' \in T_i$ is such that

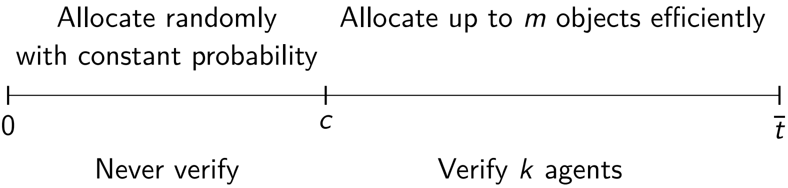
$$\underline{P} = P(t_i') \leq P(t_i) \text{ for all } t_i \in T_i$$

For a mechanism to be IC it follows that

- Types with higher probability of getting an object must be verified more often, but how often?
- Worst-off types do not want to mimic any other type

Possible mechanisms

- **Mechanism 3:**



- Bayesian incentive compatible for sufficiently high cutoff c

Possible mechanisms

There are many other mechanisms

1. How many objects to allocate efficiently?
2. More than two regions, if so how to allocate?
3. Where should the cutoffs be?
4. How to verify such that the mechanism is BIC?

Plan for rest of the talk

- Related literature
- Model
- Optimal mechanism
- Characterization of BIC mechanisms
- Feasible reduced forms

Related Literature

- Costly verification with transfers
  Townsend JET 1979; Gale & Hellwig RES 1985; Border & Sobel RES 1987
- Private good allocation with verification
  Ben-Porath, Dekel & Lipman AER 2014; Mylovanov & Zapechelnyuk AER 2017
- Mechanism design with evidence
  Green & Laffont RES 1986; Glazer and Rubinstein ECMA 2004; Ben-Porath, Dekel & Lipman ECMA 2019; ...
- Interim allocation rules
  Border ECMA 1991; Che et al. ECMA 2013

Model

- n **agents** with:

Types (symmetric)	$t_i \in T_i = [0,1]$, private information
Distribution	F, independent, and density f
Preferences	$u_i(t_i) > 0$ objects are desirable, private values and each agent need at most one object

- m homogeneous **objects**, where $n > m$

Model

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Types (symmetric)	$t_i \in T_i = [0, 1]$, private information
Distribution	F, independent, and density f
Preferences	$u_i(t_i) > 0$ objects are desirable, private values and each agent need at most one object

- m homogeneous **objects**, where $n > m$
- One **principal** with:

Payoff	$\sum_{i \in I'} t_i$, where I' is the set of agents that get an object
Action	whom to verify and then decide an allocation

- No monetary transfers

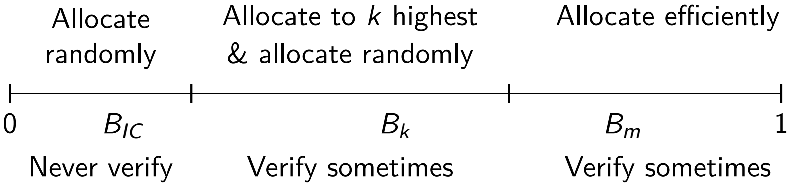
Verification

- Possible to randomly verify agents
- If agent i is verified the principal learns agent i's true type
- The penalty available is not to give an object to a lying agent

Verification Technology:

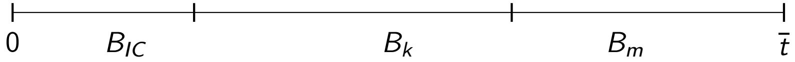
- At most k agents can be verified, and $k < m$

Optimal mechanism



Optimal mechanism

- Case 1: At least k reports in region B_m



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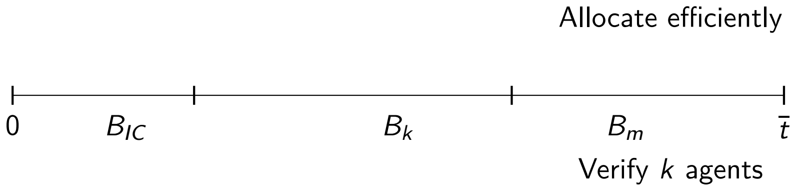

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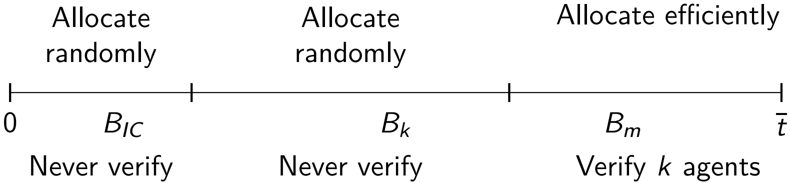
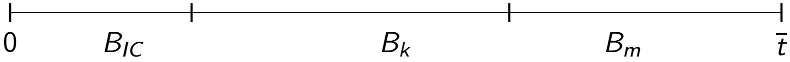


Optimal mechanism

- Case 2: Less than k reports in region B_m



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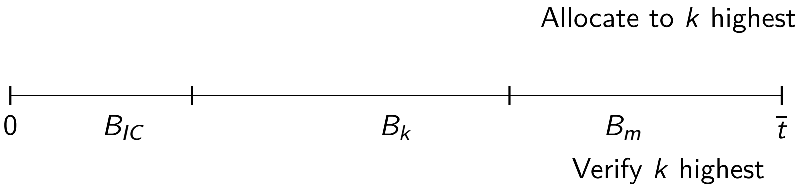

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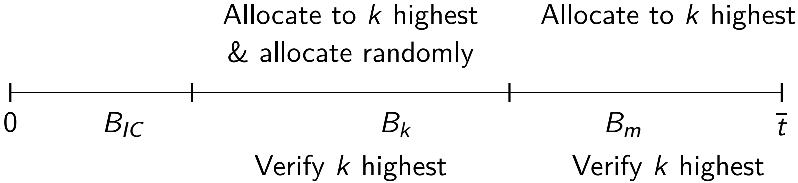

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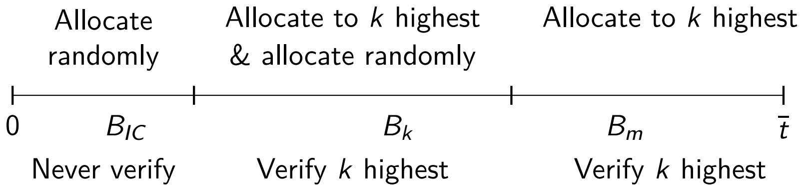
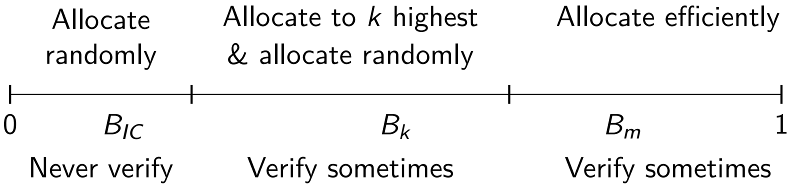


Optimal mechanism: Reduced forms



- Induces an expected probability of winning an object $P^*(t_i)$

Main result: optimal mechanism

Let $A^*(t_i) = P^*(t_i) - \inf_{t_i'} P^*(t_i')$.

Theorem

The mechanism (P^*, A^*) maximizes the expected utility of the principal and is therefore optimal among all BIC mechanisms.

Bayesian incentive compatibility

A **direct mechanism** is a pair (p, a)

- $p_i(t_i, t_{-i})$ is the probability that agent i gets an object conditional on not failing the audit
- $a_i(t_i, t_{-i})$ is the probability that agent i is verified

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Definition

A mechanism (p, a) is **Bayesian incentive compatible (BIC)** if, for all $t_i, t_i' \in [0, 1]$,

$$u_i(t_i) \cdot \mathbb{E}_{t_{-i}}[p_i(t_i, t_{-i})] \geq$$

$$u_i(t_i) \cdot \mathbb{E}_{t_{-i}}\Big[\underbrace{p_i(t_i', t_{-i})(1 - a_i(t_i', t_{-i}))}_{\text{no verification}} + \underbrace{0 \cdot a_i(t_i', t_{-i}, s)}_{\text{lie is detected}}\Big].$$

Revelation principle

Proposition

It is without loss of generality to consider direct mechanisms of the form (p, a)*, where*

$$p : T \to [0,1]^n \qquad \text{Allocation rule}$$
$$a : T \to [0,1]^n \qquad \text{Auditing rule}$$

in which truth-telling is a Bayesian equilibrium.

T denotes the product of types spaces $\times_{i=1}^{n} T_i$

Objective of the principal

$$\max_{p,a} \mathbb{E}_t\Big[\sum_i p_i(t_i, t_{-i})t_i\Big] \tag{P}$$

s.t. (p, a) is Bayesian incentive compatibility (BIC)

at most k agents are verified (k-feasibility)

at most m objects are allocated (m-feasibility)

- We allow for stochastic allocation and verification rules

Bayesian incentive compatibility

BIC implies that for all $t_i \in T_i, t_i' \in T_i$

$$u_i(t_i) \cdot \mathbb{E}_{t_{-i}}\left[p_i(t_i, t_{-i})\right] \geq$$

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We assume from now on that $a_i(t) > 0$ only if $p_i(t) = 1$.

$$\inf_{t_i \in T_i} P_i(t_i) \geq P_i(t_i') - A_i(t_i')$$

Characterization of BIC mechanisms

Let $P_i(t_i) = \mathbb{E}_{t_{-i}}[p_i(t_i, t_{-i})]$ and $A_i(t_i) = \mathbb{E}_{t_{-i}}[a_i(t_i, t_{-i})]$.

Lemma

A mechanism (p, a) is Bayesian incentive compatible (BIC) if and only if, for all $i \in \mathcal{I}$ and all $t_i' \in T_i$

$$\inf_{t_i \in T_i} P_i(t_i) \geq P_i(t_i') - A_i(t_i') \tag{BIC}$$

Characterization of BIC mechanisms

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$$\inf_{t_i \in T_i} P_i(t_i) \geq P_i(t_i') - A_i(t_i') \qquad \text{(BIC)}$$

- A **reduced form** allocation rule is a mapping $P_i : T_i \to [0, 1]$. Thus, P_i from above is a reduced form allocation rule.
- A **reduced form** verification rule is a mapping $A_i : T_i \to [0, 1]$. Thus, A_i from above is a reduced form verification rule.

Proof sketch: Principal's maximization problem

$$\max_{p,a} \sum_i \mathbb{E}_{t_i}\left[P_i(t_i)t_i\right] \tag{P}$$

s.t.

$$\inf_{t_i \in T_i} P_i(t_i) \geq P_i(t_i') - A_i(t_i') \text{ for all } t_i' \in T_i \tag{BIC}$$

$$\sum a_i(t) \leq k \tag{k-feasibility}$$

$$\sum p_i(t) \leq m \tag{m-feasibility}$$

- A maximization problem in reduced forms, or interim rules
- Need to ensure that P_i and A_i are feasible

Aside: Feasible reduced forms

There is one object, i.e., $m = 1$. An allocation rule is **feasible** if

$$\sum_{i=1}^{n} p_i(t) \leq 1, \text{ for all } t \in T$$

A reduced form $P : T_i \to [0, 1]$ is **implementable** if there exists a feasible allocation rule p such that

$$P(t_i) = \int_{T_{-i}} p(t_i, t_{-i}) dF^{n-1}(t_{-i})$$

- Which reduced forms are implementable?

Excursion: Feasible reduced forms

- $N = \{1, 2\}$ and $T = \{t_\ell, t_h\}$, $Pr(t_\ell) = Pr(t_h) = 1/2$
- Given P, find $p(t_\ell, t_\ell)$, $p(t_\ell, t_h)$, $p(t_h, t_\ell)$ and $p(t_h, t_h)$ s.t.:

$$P(t_\ell) = \frac{1}{2}p(t_\ell, t_\ell) + \frac{1}{2}p(t_\ell, t_h) \qquad (t_\ell)$$

$$P(t_h) = \frac{1}{2}p(t_h, t_\ell) + \frac{1}{2}p(t_h, t_h) \qquad (t_h)$$

Excursion: Feasible reduced forms

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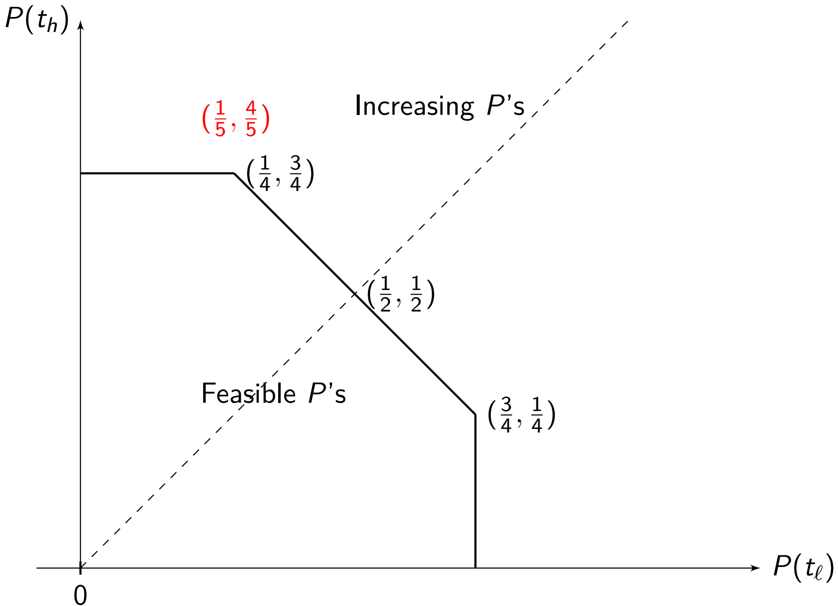
$$P(t_\ell) = \frac{1}{2}p(t_\ell, t_\ell) + \frac{1}{2}p(t_\ell, t_h) \qquad (t_\ell)$$

$$P(t_h) = \frac{1}{2}p(t_h, t_\ell) + \frac{1}{2}p(t_h, t_h) \qquad (t_h)$$

- Symmetry: $p(t_\ell, t_\ell) \leq 1/2$, $p(t_h, t_h) \leq 1/2$ and $p(t_\ell, t_h) + p(t_h, t_\ell) \leq 1$
- What about $(P(t_\ell), P(t_h)) = (\frac{1}{5}, \frac{4}{5})$, is that feasible?

Excursion: Feasible reduced forms



Aside: Feasible reduced forms

- For each $\alpha \in [0, 1]$, set $E_\alpha = \{t_i \in T_i | P(t_i) \geq \alpha\}$

Theorem (Border, 1991)

Let $P : T_i \to [0, 1]$*. Then* P *is implementable if and only if for each* $\alpha \in [0, 1]$

$$n \int_{E_\alpha} P(t_i) dF \leq 1 - F^n(E_\alpha^c) \qquad \text{(Border)}$$

Feasibility constraints for P

There is a feasibility constraint on P from $\sum p_i(t) \leq m$:

Lemma

P is implementable if and only if $\int_{E_\alpha} P(t_i)dF \leq CAP_m(E_\alpha)$.

- Follows from Che et al. (2011)

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There is also a feasibility constraint on A from $\sum a_i(t) \leq k$:

Lemma

A is feasible if and only if $\int_E A(t_i)dF \leq CAP_k(E, p)$.

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- Follows from Che et al. (2011)

There is also a feasibility constraint on A from $\sum a_i(t) \leq k$:

Lemma

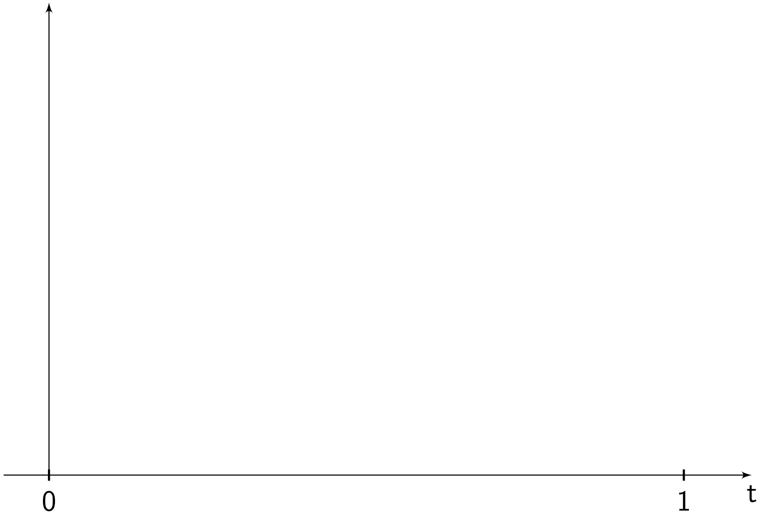
A is feasible if and only if $\int_E A(t_i)dF \leq CAP_k(E, p)$.

Since $P(t_i) - \inf_{t_i' \in T_i} P(t_i') = A(t_i)$, we get

$$\int_E P(t_i)dF(t_i) \leq CAP_m(E)$$

$$\int_E P(t_i) - \inf P \; dF(t_i) \leq CAP_k(E, p)$$

Intuition for optimal mechanism: Constraints on P



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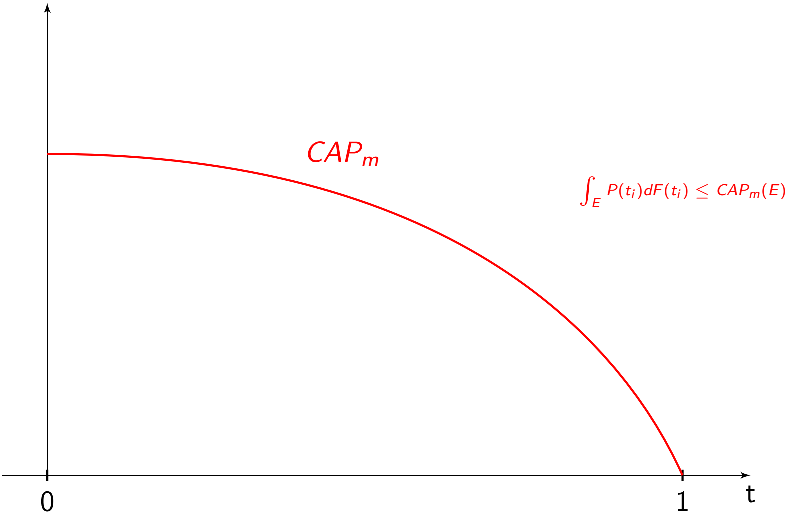

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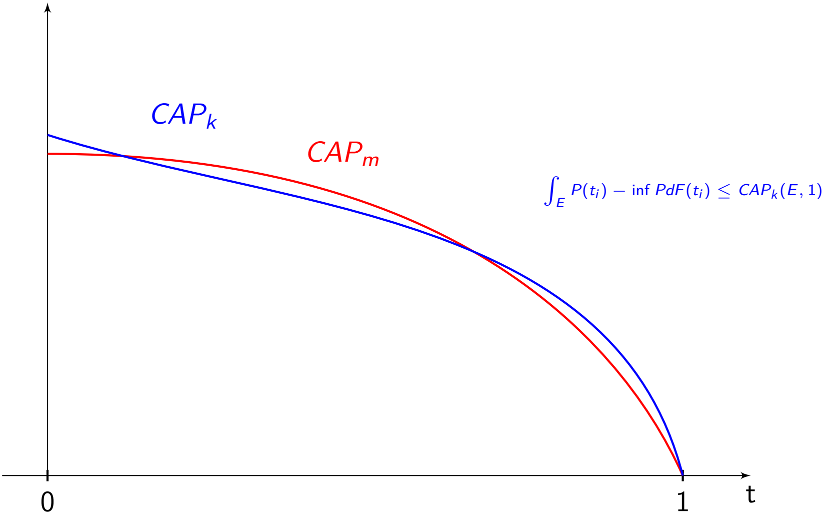

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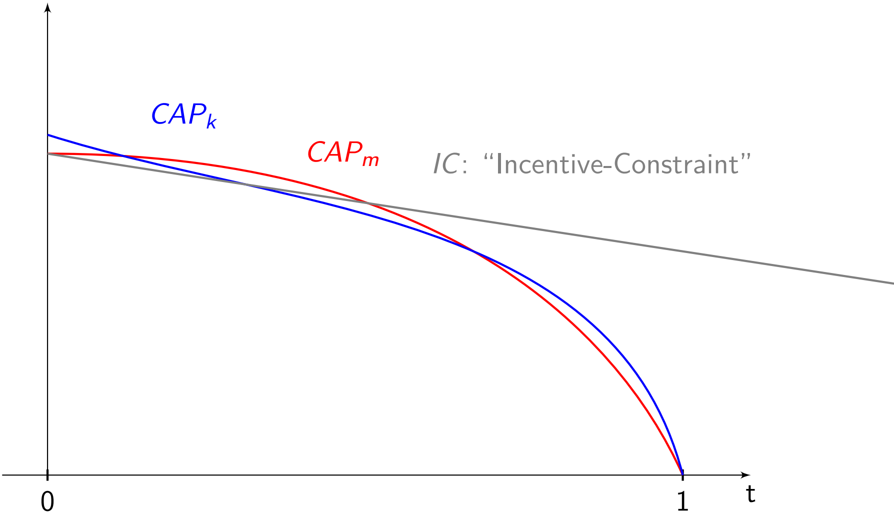

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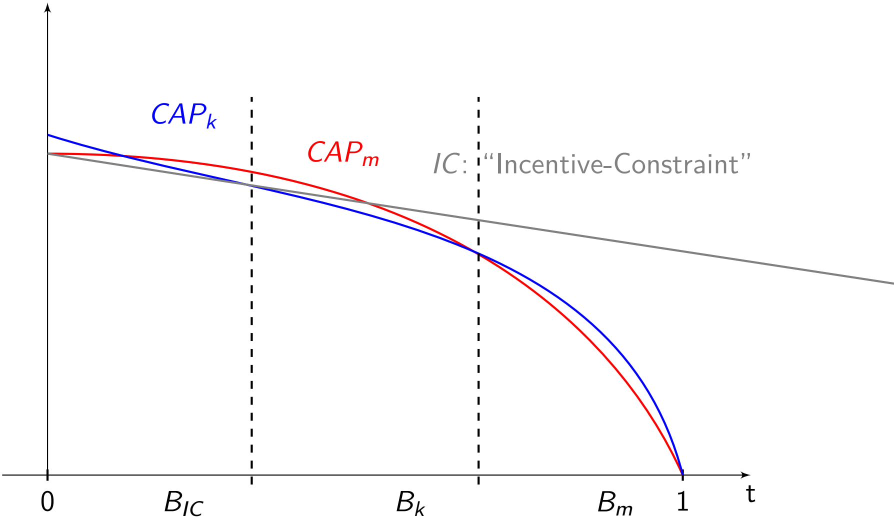

Main result: optimal mechanism

Theorem

There is $\varphi \in (0, m/n)$ such that the mechanism (P^*, A^*) with the reduced form allocation rule

$$P^*(t_i) = \begin{cases} -\frac{1}{n\ f(t_i)} CAP'_m(t_i) & \text{for } t_i \in B_m \\ -\frac{1}{n\ f(t_i)} CAP'_k(t_i) + \varphi & \text{for } t_i \in B_k \\ \varphi & \text{else.} \end{cases}$$

and verification rule $A^* = P^* - \varphi$ is optimal.

Optimal mechanism: Allocation rule (ex-post)

Given $t \in T$ let S be the set of agents such that

(1) $t_i \in B_m$ and among the m-highest reports or

(2) $t_i \in B_k$ and t_i among the k-highest reports

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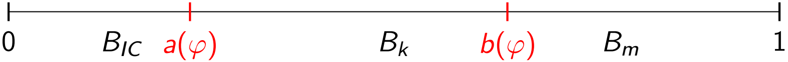
The optimal ex-post **allocation rule** p^* given a profile $t \in T$ is:

- all agents in S gets an object
- any remaining objects are allocated in a scramble randomly among agents that didn't get an object such that each type in $B_k \cup B_{IC}$ has the same interim probability of getting an object in the scramble.

Optimal cutoffs

Optimal mechanism is parametrized by one parameter:



- Increasing φ increases the allocation probability for lowest types
- Fewer audits necessary, CAP_k constraint relaxes
- But leaves fewer objects for efficient allocation
- Hence, $a'(\varphi) > 0$ and $b'(\varphi) < 0$

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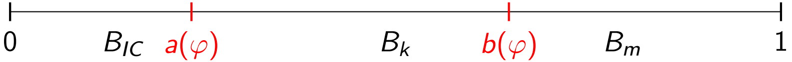
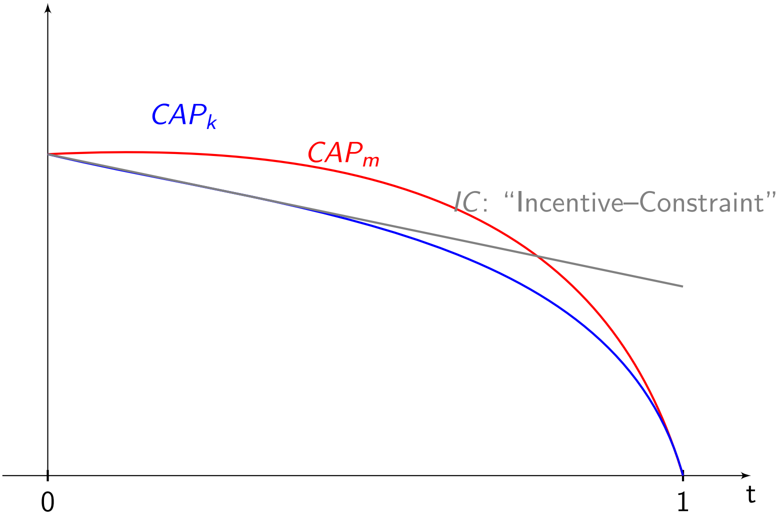


- Increasing φ increases the allocation probability for lowest types
- Fewer audits necessary, CAP_k constraint relaxes
- But leaves fewer objects for efficient allocation
- Hence, $a'(\varphi) > 0$ and $b'(\varphi) < 0$
- First-order condition: $\int_0^{b(\varphi)} tdF + [1 - F(b(\varphi))]b(\varphi) = a(\varphi)$

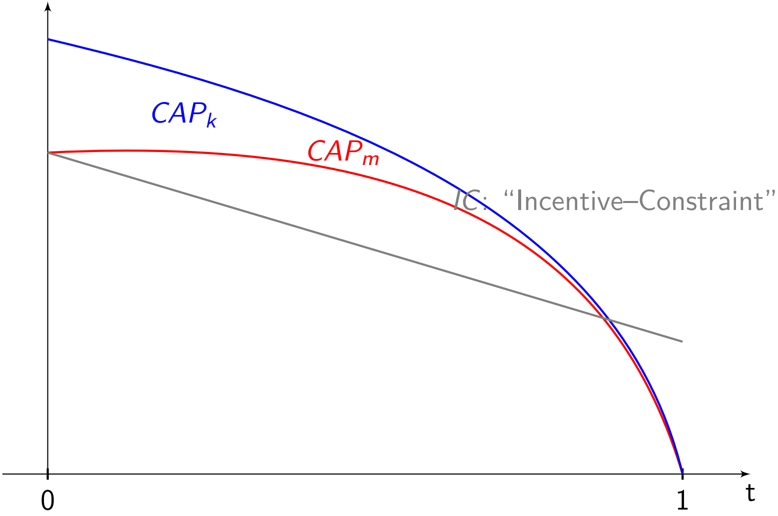
Optimal cutoffs

Mechanism 2: Allocate to k highest types, remainder randomly



Optimal cutoffs

Mechanism 3: Allocate efficient above some cutoff, remainder randomly



Optimal mechanism is not EPIC

Proof.

1. P is the essentially unique interim allocation rule. In particular, on B_m we allocate m objects efficiently.
2. Consider agent i and a type profile t such that m other agents have a type in B_m above t_i.
3. If agent i is truthful, he won't get an object.
4. If he claims to be a high type, he will get an object whenever he is not verified.
5. Not all m agents can get verified. □

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What is the optimal EPIC mechanism?

Summary

- Provide a model for the allocation of homogeneous objects with verifiable private information
- Characterize the optimal allocation rule
- Further questions:
  How does the optimal mechanism change when there is a shift in the distribution?
- How does the optimal mechanism change as we increase the number of objects?
- If there is a cost of increasing k: what is the optimal k^*?

Thank you for your attention!