Quality and Imperfect Competition*

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Abstract

We study the relationship between market power and quality supply. We develop a flexible approach to imperfect competition in product qualities and prices (or quantities), which encompasses several existing models. We characterize the equilibrium through a *quality parameter*, which can be estimated empirically. This approach allows us to extend the seminal analysis of quality distortion under monopoly to imperfect competition, and to analyze the effects of technology shocks and commodity taxes, as well as changes in market concentration, on the equilibrium allocation. Our approach accommodates various modelling choices related to firms' costs, the timing of the game, or multi-product firms.

Keywords: Quality, Imperfect competition, Distortions, Technology shock, Incidence, Market concentration, Mergers, Investment, Innovation.

JEL Codes: D43, H22, L13, L15, L41, O31, O33.

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1 Introduction

Understanding the relationship between market power and quality supply has been one of the main research questions in industrial organization and related fields since at least the work of Chamberlin (1933) and Robinson (1933).[1] This relationship still remains, in many aspects, a highly-relevant question nowadays.

For instance, recent studies of the sources and consequences of market power in digital markets call for such markets to be analyzed "using both price and quality,"[2] because quality is often the "relevant locus of competition,"[3] and because the "harms from insufficient competition appear in prices that are higher than competitive prices, [and] quality that is lower than competitive quality."[4] The interplay between quality supply and market power is also relevant for economic policies beyond antitrust, such as taxation.[5] Hence, it is important to develop a clear understanding of the effects of market power on quality supply, and also how quality is then affected by various factors such as market structure or taxation.

Under monopoly, following the contributions of Spence (1975) and Sheshinski (1976), it is well-known that the firm distorts the quality of its goods, compared to what a social planner would do, whenever inframarginal consumers' value of quality differs from that of marginal consumers. The underlying intuition is that a monopolist would select product quality in relation to its (average) value to marginal consumers, disregarding what would be optimal to inframarginal ones.

Pure monopoly markets, however, seldom exist. Under imperfect competition, a firm's choice of product quality is also dictated by its strategic interactions with competitors. Besides, marginal consumers can be indifferent between buying the firm's product, on the one hand, and either the outside option (as in the monopoly case) or buying a competitor's product, on the other hand. Hence, a suitable framework of analysis to study quality supply under imperfect competition should primarily allow for the following two elements: (i) flexible demand patterns at the

[1] For instance, Robinson (1933) explained that "[r]ival producers compete against each other in quality, in facilities, and in advertisement, as well as in price." See also the seminal work of Abbott (1953) and Dorfman and Steiner (1954) on the equilibrium analysis of quality choices by firms.

[2] Scott Morton et al. (2019).

[3] U.S. House of Representatives (2020); statement of T. Valletti at the Data and Privacy Hearing

[4] U.S. House of Representatives (2020); statement of F. Scott Morton at the Innovation and Entrepreneurship Hearing.

[5] See, e.g., the analysis of the optimal design of corporate taxation and research and development policies by Akcigit et al. (2019).

market level, and (ii) general substitution patterns between competitors. Standard models of oligopolistic competition do not provide such flexibility.[6]

In this paper, we introduce a flexible model of imperfect competition where firms compete in quality as well as in another instrument, either price or quantity. We maintain general demand and cost functions, and we express the equilibrium as a function of primitives of the model, such as the marketwide inverse demand as well as its derivatives with respect to quantity and quality. Moreover, in order to consider the strategic interactions between firms in an intelligible manner, we rely on the *conduct parameter* (see Weyl and Fabinger, 2013), and we introduce the *quality parameter*. The former captures, in our model, the intensity of competition resulting from (potential) horizontal product differentiation, while the latter plays a similar role, albeit with respect to the effect of competition in quality. Our approach encompasses commonly-used models of imperfect competition.

The strength of our approach mainly lies in maintaining a flexible demand-side in the model, in the spirit of Weyl and Fabinger (2013) and Mrázová and Neary (2017), that allows us to draw a connection between consumers' preferences, captured through demand primitives as well as the two parameters mentioned above, and relevant economic effects. We demonstrate this through three different applications. These applications also support the economic interpretation of our variables of interest. Our contribution is thus twofold. First, we provide a general model of imperfect competition with an intuitive interpretation of the relevant economic effects at play by distinguishing between effects that depend on properties of the marketwide demand (as in the monopoly case), and effects that relate to substitution patterns between competitors. Second, we apply our model to three different applications, generalizing several results of the literature.

Our first application relates to the analysis of quality distortions under imperfect competition. We find that our quality parameter determines whether quality is under- or over-supplied by the market, together with the change in marginal value of quality as the absolute willingness-to-pay varies (as identified by Spence (1975) and Sheshinski (1976) in the monopoly case). For instance, we find that the market

[6] As an example, consider a standard 'linear demand system' where each firm's demand is linear in its own strategic variables (price and quality) as well as in its competitors'. At symmetric prices and qualities, such a demand system induces a total market demand which is linear in the firms' prices and qualities, thus implying that consumers on the extensive margin (i.e., those indifferent between purchasing from one of the firms or not purchasing at all) have the same marginal valuation for quality as inframarginal consumers. We review this demand system as one of our examples.

under-supplies quality (at a given quantity) when our quality parameter lies below unity and consumers with greater willingness-to-pay value quality the most. The difference between our quality parameter and unity relates to the difference between the ('standard') *price* diversion ratio and the *quality* (or 'innovation') diversion ratio because the price diversion ratio is greater than the quality diversion ratio if and only if our quality parameter lies below unity.[7]

In a second application, we analyze the effects of technology shocks, which affect firms' costs, on the equilibrium outcome. We find that the direction and magnitude of the effects of such shocks on market quantity and quality are also determined by the change in marginal value of quality as the absolute willingness-to-pay falls and by the quality parameter. For instance, a technology shock which lowers the cost of a marginal increment in quality raises both the equilibrium quality and market quantity when our quality parameter lies below unity and consumers with greater willingness-to-pay value quality the least. Our results also apply to the analysis of tax incidence, and we find that commodity taxation, which always reduces quantity, decreases quality if and only if the valuation of quality is lower for the average consumer than for marginal ones.

In our third application, we study how the equilibrium outcome varies with market concentration. We find that, under a commonly satisfied monotonicity condition, demand systems can be sorted into three categories according to whether the corresponding quality parameter is smaller than, equal to, or greater than unity.[8] Our flexible approach allows us to explain why this categorization is informative about the direction of the effects of mergers or exogenous changes in market concentration on market quantity and quality, together with the change in marginal value of quality as the absolute willingness-to-pay varies.

The remainder of the paper is as follows. In Section 2 we present the related literature. In Section 3, we introduce our model, including our quality parameter, and, then, we discuss the equilibrium of the game. We apply our approach to the analysis of distortions in the supply of quality in Section 4, extending the work of Spence (1975) and Sheshinski (1976) to oligopolistic competition. Then, we analyze technology shocks and tax incidence in Section 5, and changes in

[7]See Katz and Shapiro (2003) and Farrell and Shapiro (2010) for the definitions of these ratios. The difference between both ratios was recently discussed by Bourreau, Jullien, and Lefouili (2021) in their analysis of the effects of horizontal mergers.

[8]Relying on a different approach, Bourreau, Jullien, and Lefouili (2021) also classify demand systems into three categories which are equivalent to ours.

market concentration in Section 6. In Section 7, we extend our baseline model to (i) general cost functions, (ii) two-stage timing, (iii) multi-product firms, and (iv) gross complements. Finally, Section 8 concludes.

2 Related Literature

Building on the seminal contributions of Dorfman and Steiner (1954), on the one hand, and Spence (1975) and Sheshinski (1976), on the other hand, we consider, in our baseline model, that each firm sells a single product with increasing marginal costs in product quality.[9] Related early contributions modelling imperfect competition between single-product firms include Shaked and Sutton (1982), with a model where firms set their product quality first and then prices,[10] and Moorthy (1985), with firms setting qualities and quantities simultaneously.[11]

Our modelling approach builds on recent contributions which aim at identifying sufficient statistics to many comparative statics questions within a rather flexible setting (Weyl and Fabinger, 2013; Mrázová and Neary, 2017), and extends these by considering firms' choice of product quality.[12] One of our main variables of interest, which captures whether the marginal value of quality falls or increases as the absolute willingness-to-pay falls, has been extensively discussed by Spence (1975) and Sheshinski (1976) in the monopoly case. Moreover, our quality parameter captures the relative weight between the extensive and switching margins related to changes in product quality, similarly to a parameter introduced by White and Weyl (2016) for platform markets. Also, as discussed in Section 4, our quality parameter relates to the difference between the 'standard' diversion ratio and the 'innovation' diversion ratio, which has been shown by Bourreau, Jullien, and Lefouili (2021) to be a relevant metric for the analysis of the effects of horizontal merger.

Our application to market distortions linked to the supply of quality extends the work of Spence (1975) and Sheshinski (1976) to the case of oligopolistic com-

[9]Following Mussa and Rosen (1978), a related literature focused, instead, on multi-product firms selling products of various qualities. Gal-Or (1983) introduced competition in quantity in this setting. Johnson and Myatt (2003, 2006) considered a more general 'upgrades' approach in order to analyze a multi-product incumbent's response to entry and quantity competition between multi-product oligopolists, respectively.

[10]We consider such two-stage game in one of our extensions, in Section 7.

[11]See also Spence (1977) and Dixit (1979) for models of monopolistic competition.

[12]See Anderson and de Palma (2020) for recent, related work on the CES model of monopolistic competition with a quality dimension.

petition.[13] Recent theoretical contributions have focused on analogous distortions in the context of platform markets (Weyl, 2010; White and Weyl, 2016) or selection markets (Veiga and Weyl, 2016).[14] On the empirical side, Crawford, Shcherbakov, and Shum (2019) have analyzed quality distortions in cable television markets.

In our second application, we discuss the effects of technology or cost shocks on equilibrium outcomes. This relates to a broad literature on tax incidence, absent any quality dimension. Recent contributions such as Weyl and Fabinger (2013) or Miklós-Thal and Shaffer (2021) have emphasized the role of the conduct parameter – which we rely on in order to capture the intensity of competition resulting from horizontal product differentiation – on the effects of commodity taxes on consumer prices. An extensive literature also compared the effects of per-unit vs. *ad valorem* commodity taxes on equilibrium quantity (or price) and quality, with rather specific models (see, e.g., Kay and Keen, 1983, 1991; Cremer and Thisse, 1994; Delipalla and Keen, 2006). Related results on the effects of cost shocks on quality supply can also be found in Zhou, Spencer, and Vertinsky (2002), Johnson and Myatt (2006) or the international trade literature (e.g., Eckel et al., 2015; Ludema and Yu, 2016).

Our final application relates to the effects of changes in market concentration and horizontal mergers on the equilibrium quality and quantity. There has been a long-standing debate about the impact of mergers on quality (OECD, 2018). Recent theoretical contributions on the topic include Motta and Tarantino (2016), Brekke, Siciliani, and Straume (2017a), Nocke and Schutz (2019), and Johnson and Rhodes (2021).[15] Moreover, Bourreau, Jullien, and Lefouili (2021) study mergers between single-product duopolists to a multi-product monopolist, when firms compete in prices and face fixed costs of investment in quality. They distinguish between environments in which the aggregate quality (or 'innovation') diversion ratio is greater than, equal to, or smaller than the aggregate price (or 'standard') diversion ratio. Their results, framed in terms of the economic effects at play, and ours, framed

[13]See also Leland (1977) and Leffler (1982).

[14]Studying the case of imperfect competition in a Hotelling model, Veiga and Weyl (2016) found that quality is under-provided by the market. In light of our own approach, their result can be explained by the fact that their model displays a quality parameter equals to unity as well as a decreasing marginal value of quality. We show that different results are possible; see Proposition 1.

[15]Empirical contributions on the link between competition and quality are numerous, in particular in the health care industry; see Gaynor and Town (2011), OECD (2013), and Gaynor et al. (2015) for surveys of the literature. Recent analyses cover the effects of competition on quality in the airline industry (Prince and Simon, 2017; Chen and Gayle, 2019), cable television (Chu, 2010; Crawford, Shcherbakov, and Shum, 2019), the supermarket industry (Matsa, 2011), the newspaper industry (Fan, 2013), pharmaceuticals (Haucap et al., 2019), or consumer goods (Sheen, 2014), among others.

in terms of the primitives of the demand system, can be seen as complementary for the analysis of horizontal mergers.[16]

3 Model and Equilibrium

In this section, we first present a model of competition between symmetric firms along two instruments: quality and price. Then, we derive the equilibrium and we introduce our quality parameter. We also derive the conditions for the existence, uniqueness and stability of the equilibrium, and, finally, we consider the case of competition in quality and quantity.

3.1 The Model

There are $n \geq 1$ symmetric, single-product firms in the market.[17] Firms sell substitute goods which are potentially differentiated according to two dimensions: horizontally and vertically.[18] Firm i, $\forall i \in \{1, ..., n\}$, sets two strategic variables, its product quality, s_i, and its price, p_i. (In Subsection 3.4, we show that our approach also encompasses the case where firms compete in qualities and quantities.)

Demands are symmetric and firm i's demand is given by $q_i(p_i, s_i, \mathbf{p_{-i}}, \mathbf{s_{-i}})$, where $\mathbf{p_{-i}}$ and $\mathbf{s_{-i}}$ correspond to the vectors of i's competitors' prices and qualities, respectively. We assume that the function q_i is twice continuously differentiable, with $\partial q_i/\partial p_i < 0$, $\partial q_i/\partial p_k \geq 0$, $\partial q_i/\partial s_i > 0$, and $\partial q_i/\partial s_k \leq 0$ everywhere over the relevant interval, $\forall i, k \neq i$. We make the natural assumption that own-effects dominate cross-effects: $\sum_k \partial q_i/\partial p_k < 0$ and $\sum_k \partial q_i/\partial s_k > 0$, $\forall i$. (This assumption is satisfied for all of our examples detailed in Appendix A.)

We denote the total market quantity by $Q \equiv \sum_i^n q_i$ and the marketwide inverse demand evaluated at symmetric quantities and qualities (i.e., $q_i = Q/n$ and $s_i = S$, $\forall i$) by $P(Q, S) \equiv P_i(q_i, s_i, \mathbf{q_{-i}}, \mathbf{s_{-i}})$, $\forall i$, where P_i corresponds to firm i's inverse demand function, obtained by inverting the demand system.

[16]This application to horizontal mergers also relates to a broader literature on competition and innovation (see, e.g., Aghion et al., 2001, 2005; Shapiro, 2012). The effects of competition on (stochastic) product innovation were analyzed, among others, by Letina (2016), Federico, Langus, and Valletti (2018), Marshall and Parra (2019), or Moraga-Gonzáles, Motchenkova, and Nevrekar (2019). See Federico, Scott Morton, and Shapiro (2020) and Kokkoris and Valletti (2020) for recent reviews with policy-relevant insights.

[17]In Section 7, we also consider the case of multi-product firms.

[18]In Section 7, we show that our approach also applies to the case where firms sells complements.

Costs. We assume that all firms face the same cost function. Firm i's costs only depend on its own production and quality. For clarity of exposition, we assume in the baseline model that costs are linear in quantity and that there is no fixed cost of production. That is, $c_i(s_i)$ is firm i's marginal cost of supplying products of quality s_i, and firm i's total cost thus equals $q_i\, c_i(s_i)$. We assume that the function c_i is twice continuously differentiable, and that marginal costs increase with product quality: $\partial c_i / \partial s_i > 0$, $\forall i$. Note that we also derive the equilibrium under more general cost functions in Section 7, where we allow for non-constant marginal costs as well as fixed costs of production.

Timing. In our baseline model, we assume that all firms set both of their strategic variables simultaneously. An alternative interpretation of this setting is that of a game where qualities are set in a first stage, and prices in a second stage, when firms are unable to observe their competitors' choice of quality. In Section 7, we also derive the equilibrium when firms first set their qualities simultaneously, and then, after having observed all other firms' qualities, set their other strategic variables.

Applications. Our approach applies to a wide range of models of product differentiation.[19] As will be made clear below, the main features that are required are (i) symmetry between firms' profit functions (more precisely, that profit functions are permutation-invariant), and (ii) existence of an extensive margin (i.e., the market is not 'covered'). Our modelling approach thus embodies standard representative consumer models and linear random utility models of product differentiation. It also covers address (or characteristics) models when there is *strong* gross substitution between all product varieties; that is, when the characteristic space is rich enough for all varieties to be 'neighbors' at all prices and qualities and, hence, that the effect of a change in price or quality of one variety is spread out over all the others (see Anderson, de Palma, and Thisse, 1989, 1992). However, our approach does not apply to models of product differentiation *à la* Mussa and Rosen (1978) or other address models with characteristics and locations such that not all product varieties are strong gross substitutes.

[19] In Appendix A, we introduce several examples our approach applies to. These include representative consumer models of price or quantity competition with a linear demand system, a quality-adjusted model of Cournot competition, or the logit model of price competition.

3.2 The Equilibrium

Under price competition, firm i's profit is given by $\pi_i \equiv (p_i - c_i)\, q_i$ and, assuming an interior solution, its first-order conditions by:

$$\begin{cases} \dfrac{\partial \pi_i}{\partial p_i} = 0 \Leftrightarrow p_i + \dfrac{q_i}{\partial q_i / \partial p_i} = c_i \\ \\ \dfrac{\partial \pi_i}{\partial s_i} = 0 \Leftrightarrow \left(\dfrac{p_i - c_i}{q_i}\right) \dfrac{\partial q_i}{\partial s_i} = \dfrac{\partial c_i}{\partial s_i} \end{cases} \tag{1}$$

We now disentangle the economic effects originating from the shape of market demand and from the strategic interactions between firms.

Reformulation of the equilibrium. Focusing on the symmetric equilibrium, where $q_i = Q/n$ and $s_i = S$, $\forall i$, we consider the marketwide inverse demand, $P(Q, S)$. Its derivatives, $P_Q \equiv \partial P/\partial Q$ and $P_S \equiv \partial P/\partial S$, represent the effects stemming from a marginal increase in the total market quantity (split evenly across firms) and from a marginal increase in all qualities, respectively. These are given by $P_Q = \left(\sum_i \sum_k \partial q_i/\partial p_k\right)^{-1}$ and $P_S = -\left(\sum_i \sum_k \partial q_i/\partial s_k\right)\left(\sum_i \sum_k \partial q_i/\partial p_k\right)^{-1}$, at symmetric prices and qualities.

Following Weyl and Fabinger (2013), the intensity of competition related to horizontal product differentiation can be captured by a conduct parameter, θ, equal to the 'elasticity-adjusted Lerner index,' $\left[(p_i - c_i)/p_i\right] \varepsilon_D$, for any i, where $\varepsilon_D \equiv -P/(Q\, P_Q)$ corresponds the price-elasticity of the marketwide demand. Under price competition, given the equation system (1), θ can thus be defined as follows.

Definition 1 (Conduct parameter under price competition)**.** *Under price competition, at the symmetric equilibrium, for any $i \in \{1, ..., n\}$, the conduct parameter is given by:*

$$\theta(Q, S) \equiv \frac{\sum_k (\partial q_k/\partial p_i)}{\partial q_i/\partial p_i}.$$

As products are gross substitutes, we have $\theta \in (0, 1]$. Under monopoly, or when firms' products are independent (implying $\partial q_i/\partial p_k = 0$, $\forall i, k \neq i$), $\theta = 1$. The conduct parameter decreases as competition becomes more intense, and tends towards 0 as the goods become less differentiated.[20] Although $\theta = 1$ when firms collude,

[20] In order for an equilibrium to exist, we rule out price competition in homogeneous goods, as

this parameter does not necessarily represent the "average collusiveness of conduct" (Bresnahan, 1989) or specific game-theoretical models of conduct (Genesove and Mullin, 1998). Instead, by allowing θ to depend on the total market quantity and quality, this modelling approach allows us to capture a wide range of forms of imperfect competition.[21]

A novelty of this paper is to capture a firm's ability to compete through the quality of its product by the quality parameter, ϕ, equal to the 'Dorfman-Steiner ratio,' $(\varepsilon_D/\varepsilon_S)(c_i/p_i)$, for any i, where $\varepsilon_S \equiv (c_i/Q)[(-P_S/P_Q)/(\partial c_i/\partial s_i)]$ corresponds to the marketwide elasticity of demand with respect to quality variation.[22] Dorfman and Steiner (1954) demonstrated that $(\varepsilon_D/\varepsilon_S)(c_i/p_i) = 1$ under monopoly.[23] Under price competition, given the equation system (1), we define ϕ as follows.

Definition 2 (Quality parameter under price competition)**.** *Under price competition, at the symmetric equilibrium, for any $i \in \{1, ..., n\}$, the quality parameter is given by:*

$$\phi(Q, S) \equiv \theta \, \frac{\partial q_i/\partial s_i}{\sum_k (\partial q_k/\partial s_i)}.$$

With gross substitution, we have $\phi > 0$. When $n = 1$, or when products are independent (implying both $\theta = 1$ and $\partial q_k/\partial s_i = 0$, $\forall i, k \neq i$), we obtain $\phi = 1$. Moreover, when firms collude we also have $\phi = 1$.[24] Instead, under imperfect competition, the quality parameter can be lower than, equal to, or greater than unity, depending on the specific model and demand system.[25] As detailed below, the difference between our quality parameter and unity relates to the firms' diversion ratios and has important implications with respect to market-driven distortions in the supply of quality.

Given the definitions above, we can express the system of first-order conditions

explained below in Subsection 3.3. Hence, $\theta > 0$.

[21] As stated by Weyl and Fabinger (2013): "we are not aware of any commonly used complete information symmetric models it does not include."

[22] Denoting by $D(p, S)$ the total market demand at symmetric prices p and qualities S, we obtain $\varepsilon_S = (c_i/Q)[D_S/(\partial c_i/\partial s_i)]$, with $D_S \equiv \partial D/\partial S$.

[23] See their equation (7). See also Gaynor et al. (2015) for a discussion.

[24] See Dixit (1979) for an analysis of the effects of collusion when firms set qualities and quantities, under monopolistic competition.

[25] For instance, $\phi < 1$ for linear demand systems *à la* Sutton (1997), $\phi = 1$ for linear demand systems *à la* Häckner (2000), and $\phi > 1$ in the CES model of price competition. We provide various examples in Appendix A.

(1) as follows, for any i:

$$\begin{cases} P + Q\theta P_Q = c_i \\ \\ \phi P_S = \dfrac{\partial c_i}{\partial s_i} \end{cases} \tag{2}$$

The symmetric equilibrium outcome is thus expressed through variables related to the marketwide (inverse) demand (such as P, P_Q and P_S), the cost function, and variables which capture the strategic interactions between firms, θ and ϕ.

As stated in their respective definitions, both parameters are evaluated at the (symmetric) equilibrium. They could thus be affected by other primitives of the model, such as cost functions for instance. Because of that, the following two points are worth mentioning. First, for various standard models of imperfect competition, θ and ϕ are constant in the firms' strategic variables (e.g., $\partial\theta/\partial Q = 0$) and, hence, independent of other primitives; see, e.g., our Examples A.1, A.2 and A.3 in Appendix A. Second, although the exact value of our quality parameter could be affected by other primitives of the model, we demonstrate below that it is the sign of $1 - \phi$ that is particularly relevant to capture the effects of market power on quality. We are not aware of any commonly-used model of imperfect competition for which the sign of $1 - \phi$ would change with other primitives.

Interpretation of the quality parameter. We now discuss the role played by our quality parameter, ϕ. At given prices and qualities, a firm serves inframarginal and marginal consumers. Firm i's marginal consumers can be indifferent between purchasing its product or not purchasing at all (the 'extensive margin'), and/or indifferent between purchasing from firm i or from a competitor (the 'switching margin'). The quality parameter captures the relative importance of these two sets of marginal consumers, considering quality changes.[26] When a marginal change in quality affects consumers at the extensive margin only, we have $\phi = \theta$ (under monopoly, it must thus be that $\phi = 1$, because $\theta = 1$). By contrast, $\phi > \theta$ if the share of marginal consumers at the switching margin is strictly positive. The larger the share of marginal consumers at the switching margin, the greater the quality parameter. In the extreme case where all marginal consumers are located at the switching margin and only firms' relative qualities matter (i.e., when symmetric

[26]Our quality parameter is somewhat akin to the "margin weighting" of White and Weyl (2016), a measure of the relative "thickness" of the switching and extensive margins in platform markets.

changes in product qualities do not affect demands), ϕ tends towards infinity.

Moreover, following Katz and Shapiro (2003), we define the aggregate *price* diversion ratio as the percentage of the total sales lost by firm i's product, when its price rises, that are captured by all the other products in the market; that is, $\delta_i^P \equiv \sum_{k \neq i} (\partial q_k / \partial p_i) / (-\partial q_i / \partial p_i) > 0$. Also, we define firm i's aggregate *quality* diversion ratio as $\delta_i^S \equiv \sum_{k \neq i} (-\partial q_k / \partial s_i) / (\partial q_i / \partial s_i) > 0$. This corresponds to the percentage of the total sales lost by firm i's product, when its quality is lowered, that are captured by all the other products in the market, in the spirit of the "innovation diversion ratio" of Farrell and Shapiro (2010). These two diversion ratios underline the relationship between the extensive and the switching margins detailed above. Indeed, given the well-known result that the conduct parameter is equal to the difference between unity and the aggregate price diversion ratio (see Weyl and Fabinger (2013) for a discussion), that is, $\theta = 1 - \delta_i^P$, we find that the quality parameter, ϕ, is equal to the ratio $\left(1 - \delta_i^P\right) / \left(1 - \delta_i^S\right)$, $\forall i$. This notably implies that, in applied work, empirical measures of aggregate diversion ratios could provide an estimate of our quality parameter.[27]

3.3 Existence, Uniqueness, and Stability of the Equilibrium

We now state sufficient conditions ensuring the existence, uniqueness, and stability of the symmetric equilibrium.

First, we assume the following second-order conditions, ensuring strong quasiconcavity of firms' profits in their own strategies.

Assumption 1. *Under price competition, (i)* $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$*, (ii)* $\frac{\partial^2 \pi_i}{\partial s_i^2} < 0$*, and (iii)* $\frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial^2 \pi_i}{\partial s_i^2} > \left(\frac{\partial^2 \pi_i}{\partial s_i \partial p_i}\right)^2$*,* $\forall i$*, over the relevant interval.*

In order to demonstrate the existence of a symmetric, pure-strategy Nash equilibrium, we follow Hefti (2017) in his use of the symmetric opponents form approach. From our model set-up and the assumptions made above,[28] it must be that our symmetric game has at least one symmetric equilibrium.[29]

[27] Conlon and Mortimer (2020) investigate the empirical properties of diversion ratios, and provide a 'treatment effects' framework for interpreting different estimates of diversion.

[28] Symmetry of the (inverse) demand and cost functions implies that firms' profit functions are permutation-invariant, in our model. We thus consider an 'ordinary symmetric' game, according to the taxonomy of Cao and Yang (2018).

[29] It is well known that no pure-strategy Nash equilibrium exist when firms compete *à la* Bertrand

Our focus on the symmetric equilibrium is primarily driven by the fact that it constitutes the most natural equilibrium to consider for many applications. Also, focusing on the symmetric equilibrium allows us to rely on a well-defined stability condition. Indeed, we make the following assumptions which, together with those mentioned above, ensure uniqueness as well as symmetric stability of the symmetric equilibrium:[30]

Assumption 2. *Under price competition, (i)* $\frac{\partial^2 \pi_i}{\partial p_i^2} + (n-1)\frac{\partial^2 \pi_i}{\partial p_k \partial p_i} < 0$*, and (ii)* $\frac{\partial^2 \pi_i}{\partial s_i^2} + (n-1)\frac{\partial^2 \pi_i}{\partial s_k \partial s_i} < 0$*,* $\forall i, k \neq i$*, over the relevant interval.*

Assumption 3. *Under price competition,* $\left[\frac{\partial^2 \pi_i}{\partial p_i^2} + (n-1)\frac{\partial^2 \pi_i}{\partial p_k \partial p_i}\right]\left[\frac{\partial^2 \pi_i}{\partial s_i^2} + (n-1)\frac{\partial^2 \pi_i}{\partial s_k \partial s_i}\right] > \left[\frac{\partial^2 \pi_i}{\partial p_i \partial s_i} + (n-1)\frac{\partial^2 \pi_i}{\partial p_k \partial s_i}\right]\left[\frac{\partial^2 \pi_i}{\partial s_i \partial p_i} + (n-1)\frac{\partial^2 \pi_i}{\partial s_k \partial p_i}\right]$*,* $\forall i, k \neq i$*, over the relevant interval.*

Relying on the variables introduced above, we find that, at the symmetric equilibrium, Assumption 3 is equivalent to $\zeta > 0$ in our baseline model, with:

$$\begin{aligned}\zeta \equiv\ & \left(P_Q + \theta P_Q + Q\theta P_{QQ} + Q\theta_Q P_Q\right)\left(\phi_S P_S + \phi P_{SS} - \frac{\partial^2 c_i}{\partial s_i^2}\right)Q \\ & -\left(\phi_Q P_S + \phi P_{QS}\right)\left(P_S - \phi P_S + Q\theta_S P_Q + Q\theta P_{QS}\right)Q,\end{aligned}$$

where $P_{QQ} \equiv \partial^2 P/\partial Q^2$, $P_{SS} \equiv \partial^2 P/\partial S^2$, $P_{QS} \equiv \partial^2 P/\partial Q \partial S$, $\theta_Q \equiv \partial\theta/\partial Q$, $\theta_S \equiv \partial\theta/\partial S$, $\phi_Q \equiv \partial\phi/\partial Q$, and $\phi_S \equiv \partial\phi/\partial S$. This Assumption will prove particularly useful in order to perform comparative statics (see, e.g., Sections 5 and 6).[31]

3.4 Quantity Competition

In this subsection, we show that the approach developed above naturally extends to the case where firms compete in quantities and qualities.

(i.e., absent any horizontal product differentiation) and set prices and qualities simultaneously. In our setting, we see that the second equality in the equation system (2) cannot be satisfied under Bertrand competition, which gives $\theta = \phi = 0$ in equilibrium, as its left-hand side is null whereas its right-hand side should be positive by assumption. Therefore, we consider that firms sell horizontally-differentiated products, and we only address the limit case of homogeneous products (at similar qualities) under quantity competition, where such existence issue does not arise (Gal-Or, 1983; Moorthy, 1985).

[30]See Hefti (2016) and Hefti (2017) on symmetric stability and uniqueness, respectively.

[31]See Dixit (1986) on the relevance of the stability condition for comparative statics.

Baseline model and equilibrium. When each firm i sets its output, q_i, and product quality, s_i, it faces an inverse demand $P_i(q_i, s_i, \mathbf{q_{-i}}, \mathbf{s_{-i}})$, $\forall i$, where $\mathbf{q_{-i}}$ and $\mathbf{s_{-i}}$ represent the vectors of quantities and qualities set by firm i's competitors, respectively. We assume that the function P_i is twice continuously differentiable, with $\partial P_i/\partial q_i < 0$, $\partial P_i/\partial s_i > 0$, and $\partial P_i/\partial q_k \leq 0$ everywhere over the relevant interval, $\forall i, k \neq i$. This implies that $\sum_k \partial P_i/\partial q_k < 0$. Moreover, we assume that own-effects dominate cross-effects: $\sum_k \partial P_i/\partial s_k > 0$, $\forall i$.

In this case, firm i's profit is $\pi_i \equiv (P_i - c_i)\, q_i$, and its first-order conditions are given by:

$$\begin{cases} \dfrac{\partial \pi_i}{\partial q_i} = 0 \Leftrightarrow P_i + q_i \dfrac{\partial P_i}{\partial q_i} = c_i \\ \\ \dfrac{\partial \pi_i}{\partial s_i} = 0 \Leftrightarrow \dfrac{\partial P_i}{\partial s_i} = \dfrac{\partial c_i}{\partial s_i} \end{cases} \tag{1'}$$

Under quantity competition, the derivatives of the marketwide inverse demand, $P(Q, S)$, with respect to total market quantity and quality are given by $P_Q = (1/n^2) \sum_i \sum_k \partial P_i/\partial q_k$ and $P_S = (1/n) \sum_i \sum_k \partial P_i/\partial s_k$, respectively, at symmetric quantities and qualities. Moreover, the conduct and the quality parameters can be defined as follows, given the equation system (1′). (The properties of both parameters, as well as their economic relevance, remain the same as above.)

Definition 1′ (Conduct parameter under quantity competition)**.** *Under quantity competition, at the symmetric equilibrium, for any $i \in \{1, ..., n\}$, the conduct parameter is given by:*

$$\theta(Q, S) \equiv \frac{\partial P_i/\partial q_i}{\sum_k (\partial P_k/\partial q_i)}.$$

Definition 2′ (Quality parameter under quantity competition)**.** *Under quantity competition, at the symmetric equilibrium, for any $i \in \{1, ..., n\}$, the quality parameter is given by:*

$$\phi(Q, S) \equiv \frac{\partial P_i/\partial s_i}{\sum_k (\partial P_k/\partial s_i)}.$$

As a result, the system of first-order conditions given in (1′) is equivalent to the equation system (2), at the symmetric equilibrium. The equation system (2) thus represents the symmetric equilibrium of a game where firms compete in qualities and either in prices or quantities.

Existence, uniqueness, and stability of the equilibrium. Under quantity competition, the assumptions which ensure existence, uniqueness and (symmetric) stability of the symmetric equilibrium are as follows.

Assumption 1′. *Under quantity competition, (i)* $\frac{\partial^2 \pi_i}{\partial q_i^2} < 0$*, (ii)* $\frac{\partial^2 \pi_i}{\partial s_i^2} < 0$*, and (iii)* $\frac{\partial^2 \pi_i}{\partial q_i^2}\frac{\partial^2 \pi_i}{\partial s_i^2} > \left(\frac{\partial^2 \pi_i}{\partial s_i \partial q_i}\right)^2$*,* $\forall i$*, over the relevant interval.*

Assumption 2′. *Under quantity competition, (i)* $\frac{\partial^2 \pi_i}{\partial q_i^2} + (n-1)\frac{\partial^2 \pi_i}{\partial q_k \partial q_i} < 0$*, and (ii)* $\frac{\partial^2 \pi_i}{\partial s_i^2} + (n-1)\frac{\partial^2 \pi_i}{\partial s_k \partial s_i} < 0$*,* $\forall i, k \neq i$*, over the relevant interval.*

Assumption 3′. *Under quantity competition,* $\left[\frac{\partial^2 \pi_i}{\partial q_i^2} + (n-1)\frac{\partial^2 \pi_i}{\partial q_k \partial q_i}\right]\left[\frac{\partial^2 \pi_i}{\partial s_i^2} + (n-1)\frac{\partial^2 \pi_i}{\partial s_k \partial s_i}\right] > \left[\frac{\partial^2 \pi_i}{\partial q_i \partial s_i} + (n-1)\frac{\partial^2 \pi_i}{\partial q_k \partial s_i}\right]\left[\frac{\partial^2 \pi_i}{\partial s_i \partial q_i} + (n-1)\frac{\partial^2 \pi_i}{\partial s_k \partial q_i}\right]$*,* $\forall i, k \neq i$*, over the relevant interval.*

These three assumptions play, under quantity competition, a role similar to Assumptions 1, 2, and 3, respectively. Importantly, Assumption 3′ is equivalent to $\zeta > 0$ at the symmetric equilibrium, as in the case of price competition.

4 Quality Distortions under Imperfect Competition

As a first application of our approach, we analyze the distortions in the supply of quality induced by the profit-maximizing behavior of the firms. This also allows us to capture the role played by the quality parameter introduced above, ϕ, in particular in combination with the marketwide change in marginal value of quality as willingness-to-pay varies, P_{QS}.

Spence (1975) and Sheshinski (1976) have shown that a monopolist setting its product quality would distort the allocation because of its inability to provide inframarginal consumers with their optimal (average) quality. Such market failure comes in addition to the classical distortion arising from the ability to raise prices above marginal costs.[32] They demonstrated that, for a given quantity, quality is under-supplied (respectively, over-supplied) by the monopolist if consumers with higher willingness-to-pay value quality improvements more (resp., less) than consumers with a lower willingness-to-pay; that is, $P_{QS} < 0$ over the relevant range

[32]This classical distortion due to market power also exists in our model. Indeed, at a given quality, the first-order condition $P + Q\theta P_Q = c_i$ implies that $P > c_i$, whereas total welfare would be maximized at $P = c_i$, $\forall i$.

(resp., $P_{QS} > 0$). In this section, we extend their analysis to the case of imperfect competition. We start by introducing the following definition.

Definition 3 (Marginal value of quality (MVQ))**.** *Consumers' marginal value of quality (MVQ) is given by* $P_S = \partial P(Q,S)/\partial S$. *It is evaluated at symmetric prices and qualities across firms. We say that the MVQ decreases when* $P_{QS} < 0$, *that it is constant when* $P_{QS} = 0$, *and that it increases when* $P_{QS} > 0$, *with* $P_{QS} \equiv \partial^2 P(Q,S)/\partial Q \partial S$.

We follow Spence (1975) and compare the equilibrium outcome to the (symmetric) outcome which maximizes total welfare, given by $W \equiv \int_0^Q P(X,S)\,dX - (Q/n)\sum_k c_k$, for a given market quantity, where S denotes the symmetric quality set by all firms in the market (i.e., $S = s_i$, $\forall i$). At a given market quantity Q, firms' profits are respectively maximized with a choice of quality given by $\phi P_S = \partial c_i/\partial s_i$, $\forall i$, as shown by equation system (1′), using Definition 2′. Instead, total welfare reaches a maximum when $\partial W/\partial S = 0$, $\forall i$. When, at a given market quantity, firms set qualities to maximize their respective profits, we thus obtain:

$$\frac{\partial W}{\partial S} = \int_0^Q P_S(X,S)\,dX - Q\phi P_S.$$

Therefore, assuming that W admits a single, interior maximum, quality is under-supplied by the market (that is, $\partial W/\partial S > 0$) when $(1/Q)\int_0^Q P_S\,dX > \phi P_S$, and over-supplied (i.e., $\partial W/\partial S < 0$) when $(1/Q)\int_0^Q P_S\,dX < \phi P_S$.

When $P_{QS} = 0$ over the relevant interval, we obtain $\int_0^Q P_S\,dX = QP_S$ and, following the analysis above, quality is under-supplied by the market if and only if $\phi < 1$. Conversely, when $\phi = 1$, quality is under-supplied if and only if $P_{QS} < 0$, as in the monopoly case of Spence (1975) and Sheshinski (1976). Instead, if $P_{QS} \neq 0$, firms under-supply quality relative to the optimum when $P_{QS} < 0$ and $\phi \leq 1$, and they over-supply quality when $P_{QS} > 0$ and $\phi \geq 1$. This also implies that, when $\phi < 1$ and $P_{QS} > 0$ (or when $\phi > 1$ and $P_{QS} < 0$), quality can be optimally supplied by the market (at a given quantity).[33]

These results are summarized in the following proposition, which extends Spence's Proposition 1 to imperfect competition.

[33] In the monopoly setting of Spence (1975) and Sheshinski (1976), $P_{QS} = 0$ implies that the monopolist supplies the optimal quality because marginal and inframarginal consumers all have the same marginal value of quality. Note that, in our setting, quality is also optimally supplied by the market when $P_{QS} = 0$ and $\phi = 1$.

Proposition 1. *For a given Q, the market under-supplies quality relative to the optimum when* $(1/Q) \int_0^Q P_S \, dX > \phi P_S$*, and conversely. Moreover, assuming that the sign of* P_{QS} *remains unchanged over the relevant interval:*

(a) it is sufficient for $(1/Q) \int_0^Q P_S \, dX > \phi P_S$ *that (i)* $P_{QS} < 0$ *and* $\phi \leq 1$*, or that (ii)* $P_{QS} = 0$ *and* $\phi < 1$*;*

(b) conversely, it is sufficient for $(1/Q) \int_0^Q P_S \, dX < \phi P_S$ *that (i)* $P_{QS} > 0$ *and* $\phi \geq 1$*, or that (ii)* $P_{QS} = 0$ *and* $\phi > 1$*.*

Proof. Omitted. □

The results above indicate that, under imperfect competition, whether quality is optimally supplied by the market or not depends not only on the shape of the marketwide (inverse) demand but also on the strategic interaction between competing firms, as captured by the quality parameter.[34] More precisely, we observe that the signs of P_{QS} and of $1 - \phi$ are informative about the direction of quality distortions. As shown in the following sections, these two components also play an important role in other applications. We now discuss these two components, as well as the associated economic effects, separately.

In his analysis, Spence (1975) emphasized the role of consumers' marginal value of quality on a monopolist's distortions. For instance, when the MVQ decreases (i.e., $P_{QS} < 0$), a monopolist would under-supply quality compared to the social optimum, for a given quantity. Indeed, the monopolist would set its product quality based on the MVQ of the marginal consumer; hence, if the MVQ decreases (i.e., if it is lower for consumers with smaller absolute willingness-to-pay), the quality selected by the monopolist must be lower than that set by the social planner who cares, instead, about the 'average MVQ' across consumers. This '*MVQ-effect*' is still present under imperfect competition at the market level, as our approach allows us to see clearly.[35]

[34] We can apply the results from Proposition 1 to the various examples detailed in Appendix A. For instance, in our Example A.1, which corresponds to the linear demand system *à la* Häckner (2000), we have $P_{QS} = 0$ and $\phi = 1$, and, hence, quality is optimally supplied by the market. A similar outcome arises with the logit model corresponding to our Example A.4. By contrast, the quality-adjusted Cournot model which corresponds to our Example A.3, for which $\phi < 1$, can lead to either $P_{QS} < 0$, $P_{QS} = 0$ or $P_{QS} > 0$ depending on its specification.

[35] Beyond making the comparison to the monopoly case simpler, this also sheds light on the properties of numerous oligopoly models, for which the sign of P_{QS} can easily be derived.

The second relevant component controlling the direction of quality distortions is the sign of $1-\phi$. Let us focus on the case of quantity competition in order to grasp the intuition behind the corresponding effect, played by imperfect competition. There, the quality parameter is strictly lower than unity (respectively, greater than unity) when $\partial P_k/\partial s_i > 0$ (resp., $\partial P_k/\partial s_i < 0$) at the symmetric equilibrium, $\forall i, k \neq i$. In other words, in our symmetric setting, $\phi < 1$ if and only if an increase in firm i's quality, s_i, (locally) shifts the inverse demands for other firms' products outwards. Such effect of firm i's quality on the inverse demands for other products is not internalized by firm i, and it pushes in favor of quality under-supply by market forces. Instead, a social planner internalizing this '*competition effect*' on competitors' sales would raise quality above the level set by the market. The case of price competition, for which $\phi < 1 \Leftrightarrow (\partial q_k/\partial p_i)\,/\,(\partial q_i/\partial p_i) < (\partial q_k/\partial s_i)\,/\,(\partial q_i/\partial s_i)$ at the symmetric equilibrium, $\forall i, k \neq i$, follows the same intuition.

When both effects mentioned above are aligned, or when one of them is null, we can conclude on whether the market under- or over-supply quality compared to a welfare-maximizing social planner, as stated in Proposition 1.

Moreover, the case of price competition allows us to relate our findings to diversion ratios. Relying on the aggregate price diversion ratio, δ_i^P, and the aggregate quality diversion ratio, δ_i^S, defined above in Section 3, we find that, $\forall i$:

$$\delta_i^P - \delta_i^S = \frac{\theta}{\phi}\left(1-\phi\right).$$

The sign of $1-\phi$ is thus equivalent to the sign of the difference between the aggregate price diversion ratio and the aggregate quality diversion ratio.[36]

Building further on the approach developed in this section, future work could compare the equilibrium under imperfect competition to the optimum set by a benevolent social planner, or the effects of price, quantity or quality regulation on the equilibrium outcome, as in the monopoly analysis of Sheshinski (1976). Finally, our approach could also be used in order to compare equilibrium outcomes under

[36] In a recent working paper, Bourreau, Jullien, and Lefouili (2021) have demonstrated that the impact of duopoly-to-monopoly mergers on quality supply depends on the difference between those two diversion ratios. In their work, they distinguish between environments where (i) the quality diversion ratio is greater than the price diversion ratio, where (ii) the reverse holds, and where (iii) both ratios are equal. Our analysis shows that the difference between both diversion ratios plays a key role in various applications, beyond its effects in relation to mergers and innovation, and that it can be expressed as a function of the conduct and the quality parameters.

either quantity or price competition, as in Motta (1993) and Symeonidis (2003).

5 Technology Shocks and Tax Incidence

In this section, we rely on the approach developed above in order to study the effects of technology shocks and commodity taxation on the supply of quality and quantity in an oligopolistic industry.

5.1 Technology Shocks

We now assume that firm i's marginal cost of production is given by $c_i(s_i; \xi)$, where ξ is a parameter which indexes the technology available to firms. We consider a symmetric setting where all firms have access to the same production technology and where technology shocks affect the entire market. Note that a marketwide shock affects each firm directly through its impact on its own technology (i.e., how that firm's cost function is affected by the shock), but also through the strategic effects related to the changes in competitors' technologies. In order to capture the effects of marketwide shocks on quantity and quality, respectively, we define $dQ/d\xi \equiv \sum_i dq_i/d\xi$, and $dS/d\xi \equiv (1/n) \sum_i ds_i/d\xi$.

In this section, for clarity of exposition, we assume that θ and ϕ are independent of Q and S (e.g., $\partial\theta/\partial S = 0$). In Appendix B, we relax this assumption which does not qualitatively affect our results. Total differentiating the equation system (2) with respect to ξ, we obtain:

$$\begin{cases} \dfrac{dQ}{d\xi}\left[P_Q(1+\theta)+Q\theta P_{QQ}\right]+\dfrac{dS}{d\xi}\left(P_S+Q\theta P_{QS}\right)=\dfrac{\partial c_i}{\partial \xi}+\dfrac{dS}{d\xi}\dfrac{\partial c_i}{\partial s_i} \\ \\ \dfrac{dQ}{d\xi}\phi P_{QS}+\dfrac{dS}{d\xi}\phi P_{SS}=\dfrac{\partial^2 c_i}{\partial\xi\partial s_i}+\dfrac{dS}{d\xi}\dfrac{\partial^2 c_i}{\partial s_i^2} \end{cases} \tag{3}$$

Solving this system of equations, we obtain the following result.

Lemma 1. *The marginal effects of a marketwide technology shock on total market quantity and on market quality are respectively given by:*

$$\frac{dQ}{d\xi}=\frac{Q}{\zeta}\left\{\frac{\partial c_i}{\partial\xi}\underbrace{\left(\phi P_{SS}-\frac{\partial^2 c_i}{\partial s_i^2}\right)}_{<0}-\frac{\partial^2 c_i}{\partial\xi\partial s_i}\left[P_S\left(1-\phi\right)+Q\theta P_{QS}\right]\right\}, \tag{4}$$

and:

$$\frac{dS}{d\xi} = \frac{Q}{\zeta}\left\{-\frac{\partial c_i}{\partial \xi}\,\phi P_{QS} + \frac{\partial^2 c_i}{\partial \xi \partial s_i}\underbrace{\left[P_Q\,(1+\theta) + Q\theta P_{QQ}\right]}_{<0}\right\}. \tag{5}$$

Proof. Directly obtained by solving the equation system (3). Parts (*i*) and (*ii*) of Assumptions 2 and 2′ imply that $P_Q\,(1+\theta) + Q\theta P_{QQ} < 0$ and $\phi P_{SS} - \partial^2 c_i/\partial s_i^2 < 0$, respectively, whereas Assumptions 3 and 3′ imply that $\zeta > 0$. □

The effects of a technology shock on the equilibrium quality and quantity are captured by our main variables of interests; in particular by P_{QS}. The quality parameter, ϕ, is also relevant to evaluate the impact of a technology shock under imperfect competition.[37] The effects on the equilibrium price follow easily, given that $dP/d\xi = P_Q\,(dQ/d\xi) + P_S\,(dS/d\xi)$ in our model. We obtain the following results.

Proposition 2. *Marketwide technology shocks have the following marginal effects:*

(a) a shock which reduces the cost level of quality without affecting the cost of quality increments (i.e., $\partial c_i/\partial \xi < 0$ *and* $\partial^2 c_i/\partial \xi \partial S = 0$*, respectively) always raises total market quantity, and it raises quality if and only if* $P_{QS} > 0$*;*

(b) instead, a shock which makes quality increments less costly without affecting the cost level of quality (i.e., $\partial^2 c_i/\partial \xi \partial S < 0$ *and* $\partial c_i/\partial \xi = 0$*, respectively) always increases quality, and it raises total market quantity if and only if* $P_S\left(1-\phi\right) + Q\theta P_{QS} > 0$*.*

Proof. When $\partial c_i/\partial \xi < 0$ and $\partial^2 c_i/\partial \xi \partial S = 0$, we have $dQ/d\xi > 0$ from equation (4), and equation (5) indicates that $dS/d\xi > 0 \Leftrightarrow P_{QS} > 0$. This proves part (*a*). Moreover, if $\partial c_i/\partial \xi = 0$ and $\partial^2 c_i/\partial \xi \partial S < 0$, equation (5) implies that $dS/d\xi > 0$, and $dQ/d\xi > 0 \Leftrightarrow P_S\left(1-\phi\right) + Q\theta P_{QS} > 0$ from equation (4). This proves part (*b*). □

A technology shock ξ can in theory alter both the level and the slope of $c_i(s_i;\xi)$. Assume, first, that $\partial^2 c_i/\partial \xi \partial S = 0$ so that only the level of the cost function is affected by the change in technology. The first part of Proposition 2 states that a technology enhancement which reduces the cost level of quality (i.e., $\partial c_i/\partial \xi < 0$) always raises the equilibrium quantity. Moreover, such shock raises quality if and

[37] Setting $n = 1$, $\theta = 1$ and $\phi = 1$, equations (4) and (5) indicate the effects of a technology shock on quantity and quality, respectively, in the monopoly case.

only if $P_{QS} > 0$. Intuitively, if a technology shock raises equilibrium quantities and the MVQ increases, firms have the incentives to raise product quality.

If we assume instead that $\partial c_i / \partial \xi = 0$, so that only the slope of the function c_i is affected by the technological change, the second part of Proposition 2 states that a technology enhancement which makes quality increments less costly always increases quality; an intuitive result. Also, it increases (respectively, decreases) quantity when $P_{QS} > 0$ and $\phi \leq 1$ (resp., $P_{QS} < 0$ and $\phi \geq 1$).

When a technology shock affects both the level and the slope of the marginal cost curves at the same time, the two types of effects mentioned above occur simultaneously. Finally, note that the results stated in parts (a) and (b) of Proposition 2 extend to discrete (i.e., non-infinitesimal) technology shocks, as long as the signs of P_{QS} and $P_S\left(1-\phi\right) + Q\theta P_{QS}$ remain unchanged over the relevant interval.

5.2 Tax Incidence

The results developed above can also guide our understanding of the incidence of per-unit commodity taxes in a symmetric environment under a flexible demand system, as in the work of Weyl and Fabinger (2013) or Miklós-Thal and Shaffer (2021). In contrast to these papers, however, our model also allows us to also discuss the effects of taxes on quality supply.

In our model, a per-unit tax borne by all firms simply corresponds to an upwards shift of firms' marginal cost curves. In other words, when the government levies a per-unit tax t in the market, a firm's marginal cost can be expressed as $c_i(s_i; t) = \tilde{c}_i(s_i) + t$. As the marginal cost function is additively separable in the tax rate, only the *level* of the firms' cost function is affected by the tax. Therefore, building on the results above for the case of technology shocks, we can state the following proposition.

Corollary 1. *The marginal effects of an increase in the per-unit tax, t, on total market quantity and market quality are respectively given by:*

$$\frac{dQ}{dt} = \frac{Q}{\zeta}\left(\phi P_{SS} - \frac{\partial^2 c_i}{\partial s_i^2}\right) < 0, \tag{6}$$

and:

$$\frac{dS}{dt} = \underbrace{\frac{-Q}{\zeta}}_{<0}\phi\, P_{QS}. \tag{7}$$

Proof. Immediate from Lemma 1, with $\xi = t$, $c_i(s_i;t) = \tilde{c}_i(s_i) + t$, and, hence, $\partial c_i(s_i;t)/\partial t = 1$ and $\partial^2 c_i(s_i;t)/\partial t \partial S = 0$. □

This confirms that when the government raises a per-unit tax by an infinitesimal amount, total market quantity always declines in equilibrium. The marginal effect of the per-unit tax on market price can also be derived from the equations above, as $dP/dt = P_Q\,(dQ/dt) + P_S\,(dS/dt)$. Corollary 1 also shows that the direction of the effects of such a tax on quality solely depends on whether the MVQ decreases or increases, as stated clearly below in the case of a discrete tax.

Proposition 3. *When the government levies a discrete per-unit tax, total market quantity decreases. Moreover, when the sign of P_{QS} does not change over the relevant interval:*

(a) if $P_{QS} \leq 0$, the per-unit tax weakly raises quality;

(b) if $P_{QS} \geq 0$, the per-unit tax weakly reduces quality.

Proof. Consider that the discrete tax raised by the government is $t > 0$. The effect on total market quantity is given by $\int_0^t (Q/\zeta)\left(\phi P_{SS} - \partial^2 c_i/\partial s_i^2\right) d\tau$ and is negative, given equation (6). Moreover, the effect of the tax on quality is given by $\int_0^t (-Q/\zeta)\,\phi P_{QS}\, d\tau$. Hence, it must be negative if $P_{QS} \geq 0$ and positive if $P_{QS} \leq 0$, given equation (7). □

Intuitively, when $P_{QS} \leq 0$, raising the per-unit tax increases the equilibrium quality because consumers' MVQ is greater at reduced quantity levels; the opposite result obtains when $P_{QS} \geq 0$ instead.

Building on these simple results, further work could compare the effects of per-unit vs. *ad valorem* commodity taxation on quality supply, as in the papers by Kay and Keen (1983, 1991), Cremer and Thisse (1994), or Delipalla and Keen (2006).[38] It could also address the effects of 'sin taxes' under imperfect competition on prices and qualities, such as sugar-sweetened beverage taxes (Allcott, Lockwood, and Taubinsky, 2019; Cremer, Goulão, and Lozachmeur, 2020). Another interesting avenue for future research would be to consider specific types of technology or

[38]See also Auerbach and Hines (2002), Section 6.4, for a discussion of the various effects at play.

cost shocks, such as tariffs, and their effects on prices, quantities and qualities, in the context of tax policy (Zhou, Spencer, and Vertinsky, 2002) or international trade (see, e.g., Eckel et al., 2015; Ludema and Yu, 2016). Our approach could also be used to analyze the effects of cost shocks on firms' profits, as in Seade (1985).

6 Market Concentration

We now rely on the model developed in Section 3 to study the effects of changes in market concentration on market quantity and quality.

In order to study the impact of market concentration on the equilibrium allocation, we vary the number of firms in the market, n.[39] That is, we investigate the effects of exogenous variations in the number of firms in the market on total market quantity, Q, and quality, S. In order to do so, we consider the direct effects of marginal changes in n on both the conduct and the quality parameters: $\partial\theta/\partial n$ and $\partial\phi/\partial n$. We make the natural assumption that the conduct parameter decreases in the number of firms competing in the market (i.e., $\partial\theta/\partial n < 0$).[40]

For clarity of exposition, in this section we limit ourselves to the case where θ and ϕ do not directly vary with Q nor S (e.g., $\partial\theta/\partial S = 0$), and we also assume that the total size of the market is invariant in the number of varieties offered. These assumptions are satisfied for our Examples A.1 to A.3, for instance, and can easily be relaxed without qualitatively affecting our results (see Appendix B).

We consider the total derivative of each of the two equations of the equilibrium system (2) with respect to n. We obtain the following:

$$\begin{cases} \dfrac{dQ}{dn}\left[P_Q(1+\theta) + Q\theta P_{QQ}\right] + \dfrac{dS}{dn}\left(P_S + Q\theta P_{QS}\right) + QP_Q\dfrac{\partial\theta}{\partial n} = \dfrac{dS}{dn}\dfrac{\partial c_i}{\partial s_i} \\[2ex] \dfrac{dQ}{dn}\phi P_{QS} + \dfrac{dS}{dn}\phi P_{SS} + P_S\dfrac{\partial\phi}{\partial n} = \dfrac{dS}{dn}\dfrac{\partial^2 c_i}{\partial s_i^2} \end{cases} \tag{8}$$

[39] Dasgupta and Stiglitz (1980), Gal-Or (1983), Banker, Khosla, and Sinha (1998), Vives (2008), Gaynor and Town (2011), López and Vives (2019), or Marshall and Parra (2019), for instance, adopted a similar approach. Although the number of retailers active in the market can in practice only be an integer, we treat this variable as a continuous one. As we are primarily interested in the direction of the relevant effects, this approach is not problematic as long as marginal effects keep the same sign over the range corresponding to a discrete change in n.

[40] This assumption is satisfied for all of our examples as well as for the CES model of price competition, for instance. See Gaudin (2018) for a discussion.

Solving this system of equations, we obtain the following results.

Lemma 2. *The marginal effects of a change in market concentration on total market quantity and on market quality are respectively given by:*

$$\frac{dQ}{dn} = \frac{Q}{\zeta}\left\{\frac{\partial\phi}{\partial n}P_S\left[P_S\left(1-\phi\right)+Q\theta P_{QS}\right] - \underbrace{\frac{\partial\theta}{\partial n}QP_Q\left(\phi P_{SS} - \frac{\partial^2 c_i}{\partial s_i^2}\right)}_{<0}\right\}, \tag{9}$$

and:

$$\frac{dS}{dn} = \frac{Q}{\zeta}\left\{-\frac{\partial\phi}{\partial n}\underbrace{P_S\left[P_Q\left(1+\theta\right)+Q\theta P_{QQ}\right]}_{<0} + \underbrace{\frac{\partial\theta}{\partial n}Q\phi P_Q}_{>0}P_{QS}\right\}. \tag{10}$$

Proof. Directly obtained by solving the equation system (8). Parts (*i*) and (*ii*) of Assumptions 2 and 2′ imply that $P_Q\left(1+\theta\right)+Q\theta P_{QQ} < 0$ and $\phi P_{SS} - \partial^2 c_i/\partial s_i^2 < 0$, respectively, whereas Assumptions 3 and 3′ imply that $\zeta > 0$. □

Lemma 2 indicates that the effects of market concentration on Q and S are driven by the signs of P_{QS} and $\partial\phi/\partial n$. Before we review the various possible cases below, it is worth noticing that the sign of $\partial\phi/\partial n$ is strongly related to whether the quality parameter, ϕ, is larger than, equal to, or smaller than unity. First, when $\phi = 1, \forall n$, as with linear demand systems *à la* Häckner (2000), we obtain $\partial\phi/\partial n = 0$. Also, given that $\phi = 1$ when $n = 1$ (regardless of the demand system) we find that $\partial\phi/\partial n < 0 \Leftrightarrow \phi < 1$ whenever ϕ is monotonic in n. This monotonicity condition is commonly satisfied; it holds for all the examples detailed in Appendix A or the CES model, for instance.[41] We can now state our results. Below, we study the effects of changes in market concentration when either total market quantity or quality remains unaffected (Proposition 4), or when both vary (Proposition 5).

Proposition 4. *Assume that ϕ is monotonic in n.*

(a) Marginal increases in market concentration that leave quality unchanged reduce total market quantity.

(b) Marginal increases in market concentration that leave total market quantity unchanged reduce (respectively, raise) quality if $\phi > 1$ (resp., $\phi < 1$).

[41]We can also relate our analysis to the recent work of Bourreau, Jullien, and Lefouili (2021), who show that a key determinant of the effects of mergers on quality supply corresponds to the sign of the difference between the price diversion and quality diversion ratios. In our setting, this difference between the diversion ratios has the same sign as $-\partial\phi/\partial n$ as long as ϕ is monotonic in n.

Proof. When $dS/dn = 0$, solving for the first equation in system (8) gives $dQ/dn = -(\partial\theta/\partial n)QP_Q/[P_Q(1+\theta)+Q\theta P_{QQ}]$, which is positive given that $P_Q(1+\theta)+Q\theta P_{QQ} < 0$ from Assumptions 2 and 2′, part (*i*), and $\partial\theta/\partial n < 0$. This proves part (*a*). When $dQ/dn = 0$, solving system (8) gives $dS/dn = -(\partial\phi/\partial n)P_S/\left(\phi P_{SS} - \partial^2 c_i/\partial s_i^2\right)$. If ϕ is monotonic in n, and given that $\phi P_{SS} - \partial^2 c_i/\partial s_i^2 < 0$ from Assumptions 2 and 2′, part (*ii*), we obtain $dS/dn < 0 \Leftrightarrow \phi < 1$. This proves part (*b*). □

This proposition shows the effects of changes in market concentration in the 'extreme cases' where either quantity or quality remains unaffected, either endogenously or because of some external forces such as market regulation. First, Proposition 4, part (*a*), confirms the standard result that total market quantity must decrease with market concentration when other strategic variables do not adjust. In that case, prices must increase with market concentration. The second part of the proposition shows that greater concentration may either raise or reduce quality if 'quantity-adjusting' does not occur, and that, in this case, the effect on quality is driven by the sign of $1 - \phi$, which we discussed in Sections 3 and 4 above.

When both total market quantity and quality adjust following changes in market concentration, the effects become more intricate. In particular, they rest on whether the MVQ increases or decreases, as demonstrated in the following proposition.

Proposition 5. *Assume that the sign of P_{QS} does not change over the relevant interval as n varies. Assume further that ϕ is monotonic in n.*

(a) When $\phi < 1$, increasing market concentration raises quality if $P_{QS} \leq 0$, and it decreases total market quantity if $P_S\left(1-\phi\right) + Q\theta P_{QS} \leq 0$.

(b) When $\phi = 1$, increasing market concentration reduces (respectively, raises) quality if and only if $P_{QS} > 0$ (resp., $P_{QS} < 0$), and it always decreases total market quantity. Moreover, if $P_{QS} = 0$, quality does not vary with market concentration.

(c) When $\phi > 1$, increasing market concentration reduces quality if $P_{QS} \geq 0$, and it decreases total market quantity if $P_S\left(1-\phi\right) + Q\theta P_{QS} \geq 0$.

Proof. When ϕ is monotonic in n, we have $\partial\phi/\partial n < 0 \Leftrightarrow \phi < 1$. Assume first that $\partial\phi/\partial n < 0$. From equation (10), $P_{QS} \leq 0 \Rightarrow dS/dn < 0$ and increasing market concentration raises quality. Moreover, from equation (9) we find that $P_S\left(1-\phi\right) + Q\theta P_{QS} \leq 0 \Rightarrow dQ/dn > 0$. This proves part (*a*). Second, assume that $\partial\phi/\partial n = 0$. Equation (9) implies $dQ/dn > 0$, while equation (10) implies that dS/dn has the same sign as P_{QS}. This proves part (*b*). Finally, assume that $\partial\phi/\partial n > 0$. From equation (10), $P_{QS} \geq 0 \Rightarrow dS/dn > 0$. Moreover, from equation (9) we obtain $P_S\left(1-\phi\right) + Q\theta P_{QS} \geq 0 \Rightarrow dQ/dn > 0$. This proves part (*c*). □

In order to interpret these results, let us consider, first, that market concentration does not affect the quality parameter ϕ (i.e., $\partial\phi/\partial n = 0$) and, hence, that a change in a firm's quality has no direct effect on its competitors' inverse demands (i.e., $\phi = 1$). This corresponds to part (*b*) of Proposition 5. This holds, for instance, with models of either quantity or price competition building on a linear demand system *à la* Häckner (2000), as in Example A.1. In this case, the total equilibrium quantity in the market always increases with the number of firms (i.e., $dQ/dn > 0$). Moreover, the direction of the impact of competition on quality is dictated by the sign of P_{QS}. Intuitively, as output goes down with greater concentration firms have the incentives to reduce quality if and only if the MVQ increases (i.e., $P_{QS} > 0$).

Consider now that $\partial\phi/\partial n < 0$ (that is, $\phi < 1$), as in part (*a*) of Proposition 5. This is satisfied, for instance, for models of either quantity or price competition building on a linear demand system *à la* Sutton (1997), as in Example A.2, and for the quality-adjusted Cournot models of Example A.3. The 'competition effect' dictated by $1-\phi > 0$ shifts the threshold on P_{QS} at which $dS/dn = 0$ to a strictly positive value and, hence, we must have $dS/dn < 0$ when the MVQ is constant. Also, it must be the case that quality decreases in n whenever $P_{QS} \leq 0$ (or, stated differently, that greater concentration raises quality). Turning to the impact of competition on total market quantity, we observe from equation (9) that dQ/dn must be positive when $(\partial\phi/\partial n)P_S[P_S(1-\phi) + Q\theta P_{QS}]$ is positive, null or not too negative.

Finally, a similar reasoning applies to the case where $\partial\phi/\partial n > 0$ (i.e., $\phi > 1$), as in part (*c*) of Proposition 5. The CES model of price competition satisfies this condition, for instance. The inequality $1-\phi < 0$ implies that the threshold on P_{QS} at which $dS/dn = 0$ must be strictly negative and that we must have $dS/dn > 0$ when the MVQ is constant. Therefore, quality increases with the number of firms in the market when the MVQ increases. As above, dQ/dn must be positive when $(\partial\phi/\partial n)P_S[P_S(1-\phi) + Q\theta P_{QS}]$ is positive, null or not too negative.

Although we focused on changes in market concentration, whereby variations in the number of firms in the market correspond to variations in the number of product varieties available to consumers, our results also apply to more complex environments. In particular, as detailed in Subsection 7.3 below, our approach can be adjusted in order to consider the case of multi-product firms, thus allowing us to disconnect the number of firms from the number of varieties in the market, and, hence, to rely on Lemma 2 in order to investigate the effects of horizontal mergers on quality supply and market quantity.

Future work could also examine the impact of mergers in regulated markets, as Brekke, Siciliani, and Straume (2017b), and various research questions relevant to antitrust policy beyond merger control could also be analyzed within our setting, such as the interplay between common ownership and quality supply, in the spirit of the work of López and Vives (2019) on cost-reducing innovation. One could also perform comparative statics in order to study the effects of heightened competition under various regulatory regimes.[42]

7 Extensions

We now extend our model of analysis to more general cost functions, a two-stage 'quality-then-price' (or 'quality-then-quantity') timing, multi-product firms, and gross complements, respectively.

7.1 Costs

Our approach can be adjusted to account for non-linear marginal cost of production.

General cost functions. We can assume that firm i faces a total cost function $C_i(q_i, s_i)$, and that its profit is given by $\pi_i = p_i\, q_i(p_i, s_i, \mathbf{p_{-i}}, \mathbf{s_{-i}}) - C_i(q_i(p_i, s_i, \mathbf{p_{-i}}, \mathbf{s_{-i}}), s_i)$ under price competition, and by $\pi_i = P_i(q_i, s_i, \mathbf{q_{-i}}, \mathbf{s_{-i}})\, q_i - C_i(q_i, s_i)$ under quantity competition. All firms face the same cost function. Firm i's marginal cost of supplying products of quality s_i at quantity q_i is denoted by $\partial C_i(q_i, s_i)/\partial q_i$. We assume that the function C_i is twice continuously differentiable, and that marginal costs increase in product quality, for any q_i.

The symmetric equilibrium is then determined by the following system under

[42] For the case of price regulation, a highly relevant topic in health care markets, Gaynor and Town (2011) explained that equilibrium quality increases with the number of firms in the market (see their Section 4.2); see also the analysis of White (1972) on the comparison between quality supplied under monopoly or under perfect competition.

either quantity or price competition, $\forall i$:[43]

$$\begin{cases} P + Q\theta P_Q = \dfrac{\partial C_i}{\partial q_i} \\ \\ \phi P_S = \dfrac{n}{Q}\dfrac{\partial C_i}{\partial s_i} \end{cases} \tag{11}$$

The definitions of θ and ϕ given in the baseline model remain unchanged. The various applications provided in Sections 4, 5, and 6 can also be easily extended to this setting with more general cost functions. As an example, following the logic of Section 4, Proposition 1 directly applies to settings with non-constant marginal costs, as in the monopoly analysis of Spence (1975).

Fixed costs and cost-reducing investments. Furthermore, we can consider the special case with both constant marginal costs, $c_i(s_i)$, and fixed costs of production, $F_i(s_i)$, $\forall i$; that is, $C_i(q_i, s_i) = q_i\, c_i(s_i) + F_i(s_i)$. In such a case, the equilibrium system (11) becomes:

$$\begin{cases} P + Q\theta P_Q = c_i \\ \\ \phi P_S = \dfrac{\partial c_i}{\partial s_i} + \dfrac{n}{Q}\dfrac{\partial F_i}{\partial s_i} \end{cases} \tag{12}$$

Our approach can thus easily incorporate fixed costs of investment, in addition to (or in lieu of) marginal costs, as in the work of Ma and Burgess (1993), for instance. An interesting aspect of this extension is that it allows us to explore the effects of process innovation; that is, investments which reduce marginal costs. Indeed, assuming that $\partial F_i/\partial s_i > 0$ and that, in contrast to our baseline model, $\partial c_i/\partial s_i < 0$, s_i could represent the costly effort produced by firm i in order to reduce its marginal cost of production.[44] When quality does not play any role in the analysis and only process innovation is to be considered, the equilibrium is given by the equation system (12) above by setting $P_S = \phi = 0$.

[43]Assumptions 1 to 3 (or 1′ to 3′ under quantity competition) ensure the existence, uniqueness and symmetric stability of the symmetric Nash equilibrium in pure strategies.

[44]See, e.g., the analyses of Vives (2008), Motta and Tarantino (2016), or López and Vives (2019).

7.2 Two-stage Game

Instead of considering that firms set all of their strategic variables at once, another standard approach in the literature is to consider that they play a multi-stage game (see Shaked and Sutton, 1982). In this extension, we consider that, in a first stage, firms choose their qualities simultaneously and that, in a second stage, after having observed all firms' quality choices, they simultaneously set their second strategic variables (either quantities or prices).

In this two-stage game, which can be solved by backward induction, the choice of qualities will directly affect rivals' quantities and prices. In a symmetric equilibrium, the second-stage, first-order conditions lead to the same expression as in the main analysis above. However, the first-order conditions corresponding to the first stage of the game, i.e., relating to the choice of quality, differ from that in our baseline model. This is so, because firm i's choice of product quality, s_i, directly affects its competitors' second-stage strategic variables in equilibrium. We denote firm k's second-stage equilibrium variable expressed as a function of the first-stage strategic variables by $p_k^*(s_k, \mathbf{s_{-k}})$ under price competition, and $q_k^*(s_k, \mathbf{s_{-k}})$ under quantity competition, $\forall k$. The symmetric equilibrium of the two-stage game is thus determined by, $\forall i$:[45]

$$\begin{cases} P + Q\theta P_Q = c_i \\ \\ \phi P_S + (1-\theta)\,\lambda = \dfrac{\partial c_i}{\partial s_i} \end{cases}$$

where λ captures how a firm's quality in the first stage affects its competitors' strategic variables in the second stage. Under price competition, $\lambda \equiv dp_k^*/ds_i$, whereas under quantity competition we have $\lambda \equiv nP_Q(dq_k^*/ds_i)$, $\forall k \neq i$. The derivatives dp_k^*/ds_i and dq_k^*/ds_i are obtained by total-differentiating the second-stage, first-order conditions with respect to s_j, $\forall j$, under price and quantity competition, respectively.

Interestingly, this change in the timing of game directly affects the first-order conditions only through the additive component $(1-\theta)\lambda$, when compared to our baseline equilibrium given by the equation system (2). This implies, first, that the

[45]Considering that any marginal change in quality affects (inverse) demands both directly and through the second-stage equilibrium variables, Assumptions 1 to 3 (or 1′ to 3′ under quantity competition in the second stage) ensure the existence, uniqueness and symmetric stability of the symmetric, subgame perfect Nash equilibrium in pure strategies.

conduct and quality parameters defined above remain relevant for the analysis. Second, this means that λ captures the strategic effects resulting from firms' ability to observe competitors' choice of quality, prior to setting quantities or prices.[46]

Our approach can thus relatively easily be modified to consider a two-stage timing. The various applications provided in Sections 4, 5, and 6 could be replicated with this two-stage game, and our variables of interest would remain key determinants of the overall effects. However, the presence of λ would also affect the comparative statics, notably through the third derivatives of demand.[47]

Note that, in this extension, we still assumed that all firms set their strategic variables simultaneously: qualities in the first stage, and either price or quantity in the second stage. In an alternative timing, one could consider that some firms move before their competitors, *à la* Stackelberg, as in the paper by Moorthy (1988).

7.3 Multi-product Firms

In this extension, we assume that each of the n firms in the market sells m horizontally-differentiated product varieties. Demands for different varieties are symmetric, and we denote by $M \equiv n\,m$ the total number of varieties in the market.

When firms compete in prices, demand for firm i's j-th product is denoted by the function q_{ij}, which depends on all prices and qualities, and firm i sets the prices p_{ij}, $\forall j = \{1, ..., m\}$. Instead, when firms compete in quantities, the inverse demand for firm i's j-th product is denoted by P_{ij}, which is a function of all quantities and qualities, while firm i sets the quantities q_{ij}, $\forall j = \{1, ..., m\}$. The quality of firm i's j-th product is denoted by s_{ij}. Moreover, the function c_{ij}, which takes s_{ij} as an argument, represents the marginal cost of firm i's j-th product. We assume that marginal costs are constant in quantities, and that cost functions are symmetric across product varieties and firms. We also assume existence, uniqueness and symmetric stability of the equilibrium.

[46] The effect captured by λ is thus similar to the "price-reaction effect" of Barigozzi and Ma (2018), who decomposed, for a different model, the effects due to competition in qualities and prices.

[47] For instance, in relation to our results from Section 6, Banker, Khosla, and Sinha (1998) considered a model with a two-stage game as well as fixed costs of investment and a variant of the linear demand system *à la* Häckner (2000), and found a negative relationship between quality supply and the number of firms in the market.

Conduct and quality parameters in the multi-product case. Under price competition, firm i's profit is equal to $\sum_{r=1}^{m} (p_{ir} - c_{ir}) q_{ir}$. The first-order conditions associated with firm i's problem are given by, $\forall j \leq m$: $q_{ij} + \sum_{r=1}^{m} p_{ir} \left(\partial q_{ir}/\partial p_{ij}\right) = \sum_{r=1}^{m} c_{ir} \left(\partial q_{ir}/\partial p_{ij}\right)$ and $\sum_{r=1}^{m} (p_{ir} - c_{ir}) \left(\partial q_{ir}/\partial s_{ij}\right) = q_{ij} \left(\partial c_{ij}/\partial s_{ij}\right)$. In order to keep the exposition simple, we assume that demands for each products are not only symmetric across firms but also across product varieties.[48] Observing that $P_Q = \left[\sum_{t=1}^{n} \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial q_{kr}/\partial p_{tj}\right)\right]^{-1}$ and $P_S = -\left[\sum_{t=1}^{n} \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial q_{kr}/\partial s_{tj}\right)\right] P_Q$, we can redefine our key parameters. In this symmetric, multi-product setting, the conduct and the quality parameters are respectively given by:

$$\theta(Q,S) \equiv \frac{\sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial q_{kr}/\partial p_{ij}\right)}{\sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial q_{ir}/\partial p_{ij}\right)} \quad \text{and} \quad \phi(Q,S) \equiv \theta \, \frac{\sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial q_{ir}/\partial s_{ij}\right)}{\sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial q_{kr}/\partial s_{ij}\right)}$$

Summing firm i's first-order conditions over its m products, we find that the symmetric equilibrium is given by the equation system (2), with the newly defined θ and ϕ, as well as with $P = p_{ir}$, $c_i = c_{ir}$, and $\partial c_i/\partial s_i = \partial c_{ir}/\partial s_{ir}$, $\forall i, r$.

Similarly, under quantity competition, firm i's profit is given by $\sum_{r=1}^{m} (P_{ir} - c_{ir}) q_{ir}$, and its first-order conditions by $P_{ij} + \sum_{r=1}^{m} q_{ir} \left(\partial P_{ir}/\partial q_{ij}\right) = c_{ij}$ and $\sum_{r=1}^{m} q_{ir} \left(\partial P_{ir}/\partial s_{ij}\right) = q_{ij} \left(\partial c_{ij}/\partial s_{ij}\right)$, $\forall j \leq m$. In this setting with quantity competition between multi-product firms, assuming that inverse demands are symmetric across product varieties and firms, we have $P_Q = (n^2 m^2)^{-1} \sum_{t=1}^{n} \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial P_{kr}/\partial q_{tj}\right)$ and $P_S = (n\, m)^{-1} \sum_{t=1}^{n} \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial P_{kr}/\partial s_{tj}\right)$. We can respectively express the conduct and the quality parameters by:

$$\theta(Q,S) \equiv \frac{\sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial P_{ir}/\partial q_{ij}\right)}{\sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial P_{kr}/\partial q_{ij}\right)} \quad \text{and} \quad \phi(Q,S) \equiv \frac{\sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial P_{ir}/\partial s_{ij}\right)}{\sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{r=1}^{m} \left(\partial P_{kr}/\partial s_{ij}\right)}$$

As in the case of price competition, we find that the symmetric equilibrium can be expressed by the equation system (2), as in our baseline model, with the newly defined θ and ϕ, and with $P = P_{ir}$, $c_i = c_{ir}$, and $\partial c_i/\partial s_i = \partial c_{ir}/\partial s_{ir}$, $\forall i, r$.

[48]Our approach can also accommodate the case where demands are symmetric across firms but not across product varieties.

Our results from Sections 4 and 5 thus apply to the case of multi-product firms, using the adjusted definitions of the conduct and quality parameters given above. Moreover, the case of multi-product firms allows us to extend our analysis of Section 6 beyond simple changes in market concentration and also to analyze the effects of horizontal mergers on quantity and quality.

Horizontal mergers. In the analysis of Section 6, when the number of firms varies from n to $n-1$, the number of available product varieties also decreases from n to $n-1$. Instead, horizontal mergers should allow for the coordination of decisions over several varieties, without implying that the assets and product of one of the merging firms simply disappear (Federico, Scott Morton, and Shapiro, 2020).

By considering multi-product firms, we can determine the effects of mergers on quality when the total number of varieties supplied in the market, $M \equiv n\,m$, remains unaffected by the mergers. For instance, our approach can be used to analyze the effects of mergers to (multi-product) monopoly, of pairwise mergers reducing the number of participants in the market from four single-product firms to two two-product competitors, or a switch from four three-product firms to three competitors selling four products each. The only requirement of our approach is that symmetry in the market must be preserved, both before and after the mergers.

We obtain the following results. Firstly, for demand systems for which neither θ nor ϕ (as defined in this subsection) depend on m, the results presented in Proposition 2 and the discussion thereafter apply directly to the case of horizontal mergers.[49] Secondly, for demand systems for which either or both parameters vary with the number of varieties controlled by each firm, $m = M/n$, one can consider that any marginal change in the number of firms, n, also triggers a change in m in order to keep the total number of varieties in the market, M, constant. Hence, it suffices to replace, in equations (9) and (10), the partial derivatives of the conduct and quality parameters with respect to n by $\partial\theta/\partial n - (M/n^2)\,\partial\theta/\partial m$ and $\partial\phi/\partial n - (M/n^2)\,\partial\phi/\partial m$, respectively. This demonstrates the robustness of our analysis to horizontal mergers.

We can also restate some results from the literature with our approach. For instance, combining together the extensions of multi-product firms and fixed costs of investment in quality, our approach can be used to express the findings of

[49] The demand systems corresponding to the multi-product versions of our Examples A.1 or A.2 display the properties that $\partial\theta/\partial m = \partial\phi/\partial m = 0$, for instance.

Bourreau, Jullien, and Lefouili (2021) as functions of ϕ and P_{QS} as well as to extend such findings to the cases of quantity competition and of pairwise mergers. Similarly, our approach could prove useful to analyze the effects of horizontal mergers on quality when firms' investments, in the form of fixed costs, reduce marginal costs (as in the baseline model of Motta and Tarantino (2016)) while simultaneously raising quality.

7.4 Gross Complements

Our approach also naturally extends to the case where firms sell complement goods, instead of substitutes. Indeed, if we consider that $\partial q_i/\partial p_k \leq 0$ and $\partial q_i/\partial s_k \geq 0$ under price competition, or that $\partial P_i/\partial q_k \geq 0$ under quantity competition, $\forall i, k \neq i$, the symmetric equilibrium can still be expressed by the equation system (2).

The main difference with the case of gross substitutes is that, by construction, the conduct parameter θ is now larger than unity. Moreover, under price competition, complement goods imply that our quality parameter ϕ lies in the interval $(0, \theta]$.[50]

Importantly, several results of Section 6 change if firms sell complement goods, because gross complementarity implies that $\partial\theta/\partial n > 0$. This reverses the sign of the last terms in both equations (9) and (10), in Lemma 2. Therefore, this also reverses some of the conclusions made in the case where firms sell substitute goods and implies, for instance, the classic result that an increase in market concentration that leaves quality unchanged will raise market output, due to the internalization of pricing externalities.

8 Conclusion

In this paper, we developed a flexible model of imperfect competition between firms selecting both the quality and either the price or quantity of their products. We found that the equilibrium can be expressed in a concise manner when we introduce our quality parameter in complement to the so-called conduct parameter. Whereas the conduct parameter relies on firms' aggregate diversion ratios obtained when they raise their price or quantity, the quality parameter captures relative changes in demand when firms alter the quality of their products instead.

[50] Examples A.1 and A.2 can be used to illustrate the case where firms sell complements, for quantity or price competition, with the parameter $\mu \in (-1, 0)$.

We relied on our approach for various applications. First, we showed that quality distortions in an oligopolistic industry depend on (i) whether or not the marginal value of quality decreases as willingness-to-pay falls, as in the monopoly case, but also on (ii) whether our quality parameter lies above or below unity. Second, we investigated the impact of cost shocks (including commodity taxes) on equilibrium outcomes, and we found that both determinants of quality distortions listed above govern the marginal effects of a marketwide technology shock on market quantity and quality. Third, the same two determinants are also key elements in understanding the effects of a change in market concentration on market quantity and quality. Note that, although we mostly discussed the determinants of the *direction* of those economic effects, we also provided the relevant formulas for the analysis of their *magnitude*.

Finally, we provided various examples in order to discuss the application and robustness of our results. We demonstrated that our approach can easily be adapted to alternative modelling choices, including fixed costs of investment, a two-stage timing, multi-product firms, and selling complement goods.

This paper thus highlights the key determinants that need to be uncovered in a given market in order to make robust predictions about the outcomes of potential changes in such market. As mentioned above, whether the marginal value of quality increases or decreases governs various effects in our comparative statics. Given that a decreasing marginal value of quality implies that consumers with higher willingness-to-pay value quality improvements more than consumers with a lower willingness-to-pay – a relatively natural assumption made, for instance, by Mussa and Rosen (1978) – this case could be potentially seen as the most relevant in practice. Moreover, whether the quality parameter is smaller or greater than unity depends on the demand system and, in particular, on whether consumers acquired after a firm's increase of its product quality were at the extensive margin or poached from competitors.

Our approach could be tailored further to other modelling choices. For instance, one could introduce spillovers in our setting. Ultimately, our baseline model can be adapted to many specific situations, by introducing the relevant features or modifications on a case-by-case basis. The scope for follow-up applications remains vast, as already discussed at the end of Sections 4, 5, and 6.

Appendices

A Examples

In this appendix, we provide some details about the various examples we rely on in the main text.

A.1 Linear Demand System *à la* Häckner (2000)

Example A.1 is a linear demand system *à la* Häckner (2000), which is altered in order to maintain an invariant market size in n.[51] The utility of the representative consumer is given by:

$$U = \sum_i s_i q_i - \frac{\mu + n}{2(1+\mu)} \sum_i q_i^2 - \frac{\mu}{1+\mu} \sum_i \sum_{k>i} q_i q_k + \kappa,$$

with $\mu > 0$, and where κ denotes the consumption of the outside good. This gives the following inverse demand for firm i:

$$P_i = s_i - \left(\frac{\mu + n}{1+\mu}\right) q_i - \frac{\mu}{1+\mu} \sum_{k \neq i} q_k.$$

Note that firms' products become independent when $\mu \to 0$, and homogeneous when $\mu \to +\infty$. The (direct) demand for firm i is given by:

$$q_i = \frac{1}{n}\left[\left(1 + \mu - \frac{\mu}{n}\right)(s_i - p_i) - \frac{\mu}{n} \sum_{k \neq i} (s_k - p_k)\right].$$

At symmetric outcomes, this system gives $P = S - Q$, $P_Q = -1$, $P_S = 1$ and $P_{QS} = 0$. Moreover, under quantity competition, we obtain $\theta = (1 + \mu/n)/(1+\mu)$ and $\phi = 1$. Instead, under price competition: $\theta = n/(n + n\mu - \mu)$ and $\phi = 1$.

[51] See Choné and Linnemer (2020) for an extensive overview of linear demand systems.

A.2 Linear Demand System *à la* Sutton (1997)

Example A.2 is a linear demand system *à la* Sutton (1997), modified in order to maintain a market size constant in n. The utility of the representative consumer is given by:

$$U = \sum_i q_i - \frac{\mu + n}{2(1+\mu)} \sum_i \frac{q_i^2}{s_i^2} - \frac{\mu}{1+\mu} \sum_i \sum_{k>i} \frac{q_i q_k}{s_i s_k} + \kappa,$$

with $\mu > 0$, and where κ denotes the consumption of the outside good. The inverse demand faced by firm i is given by:

$$P_i = 1 - \left(\frac{\mu + n}{1+\mu}\right)\left(\frac{q_i}{s_i^2}\right) - \frac{\mu}{1+\mu} \sum_{k \neq i} \frac{q_k}{s_i s_k}.$$

Firms' products become independent when $\mu \to 0$, and homogeneous when $\mu \to +\infty$. Firm i's (direct) demand is given by:

$$q_i = \frac{s_i}{n}\left[\left(1 + \mu - \frac{\mu}{n}\right)(1 - p_i)\, s_i - \frac{\mu}{n} \sum_{k \neq i} (1 - p_k)\, s_k\right].$$

At symmetric outcomes, this (inverse) demand system gives $P = 1 - Q/S^2$, $P_Q = -1/S^2$, $P_S = 2Q/S^3$, and $P_{QS} = 2/S^3 > 0$. Moreover, under quantity competition, $\theta = (1 + \mu/n) / (1 + \mu)$ and $\phi = (2 + \mu + \mu/n) / (2 + 2\mu) \leq 1$. Under price competition, we have $\theta = n / (n + n\mu - \mu)$ and $\phi = (2n + n\mu - \mu) / (2n + 2n\mu - 2\mu) \leq 1$.

A.3 Quality-adjusted Cournot

Example A.3 is a model of quantity competition where firms sell homogeneous products at symmetric qualities. The inverse demand faced by firm i is equal to:

$$P_i = f(\Psi_i),$$

where $\Psi_i \equiv (1/s_i) \sum_k (q_k/s_k)$, and $f(\cdot)$ is twice differentiable and decreasing in its argument. For instance, when $f(\Psi_i) = 1 - \Psi_i$, the inverse demand P_i is equivalent to that from Example A.2 with $\mu \to +\infty$.

In symmetry, this gives $P = f(Q/S^2)$, $P_Q = (1/S^2)\, f'(Q/S^2)$, $P_S = (-2Q/S^3)\, f'(Q/S^2)$, $P_{QS} = (-2/S^3)\, [f'(Q/S^2) + (Q/S^2) f''(Q/S^2)]$. Defining the curvature of (inverse) demand as $\sigma \equiv -QP_{QQ}/P_Q$, we find that $\sigma = (-Q/S^2)f''(Q/S^2)/f'(Q/S^2)$ and that $P_{QS} < 0 \Leftrightarrow \sigma > 1$. Moreover, we have $\theta = 1/n$ and $\phi = (n+1)/(2n) \leq 1$.

A.4 Logit

Example A.4 is the logit model of price competition, when consumer utility is a linear function of price and of quality (see Anderson, de Palma, and Thisse, 1992). The probability of a consumer choosing product i is given by the following, with $\mu > 0$:

$$q_i \equiv \mathbb{P}_i = \frac{\text{Exp}[(s_i - p_i)/\mu]}{\text{Exp}[V_0/\mu] + \sum_k \text{Exp}[(s_k - p_k)/\mu]}.$$

At symmetric outcomes, we obtain $P_Q = -\mu/(Q - Q^2)$, $P_S = 1$, $P_{QS} = 0$, $\theta = n(1-Q)/(n-Q)$, and $\phi = 1$.

B Additional Properties of Demand Systems

In this appendix, we extend the results provided in Sections 5 and 6 to the case where our parameters θ and ϕ may vary with Q and S. Indeed, in some models, the conduct and the quality parameters are functions of the quantity and/or the quality supplied in the market, at the symmetric equilibrium. For instance, in the logit model considered in Example A.4, we have $\theta = n(1-Q)/(n-Q)$; thus implying that $\theta_Q \neq 0$. In the CES model of price competition, both the conduct and quality parameters vary with quantity and quality.

In addition, we also extend the results of Section 6 to the case where the size of the market varies with the number of varieties supplied. Indeed, several demand systems commonly used in the literature display the property that, at given (symmetric) prices and qualities, total market quantity increases in n.

B.1 Technology Shocks and Tax Incidence

In order to demonstrate the robustness of our main results to the case where θ and ϕ vary with Q and S, we follow the same procedure as in the main text. Total-differentiating the equilibrium system (2) by ξ in the symmetric equilibrium, we obtain:

$$\frac{dQ}{d\xi} = \frac{Q}{\zeta}\left\{\frac{\partial c_i}{\partial \xi}\underbrace{\left(\phi_S P_S + \phi P_{SS} - \frac{\partial^2 c_i}{\partial s_i^2}\right)}_{<0} - \frac{\partial^2 c_i}{\partial \xi \partial s_i}\left[P_S\left(1-\phi\right) + Q\left(\theta_S P_Q + \theta P_{QS}\right)\right]\right\},$$

and:

$$\frac{dS}{d\xi} = \frac{Q}{\zeta}\left\{-\frac{\partial c_i}{\partial \xi}\left(\phi_Q P_S + \phi P_{QS}\right) + \frac{\partial^2 c_i}{\partial \xi \partial s_i}\underbrace{\left[P_Q\left(1+\theta+Q\theta_Q\right) + Q\theta P_{QQ}\right]}_{<0}\right\}.$$

The interpretation of our results is thus analogous to that in the main text, except that the derivatives θ_Q, θ_S, ϕ_Q and ϕ_S also play a role in determining the effects of technology shocks on the equilibrium quality and total market quantity. In particular, the products $\theta_S P_Q$ and $\phi_Q P_S$ affect quantity and quality in a comparable manner as P_{QS}. A similar analysis also shows the robustness of our results on commodity taxation and tax incidence.

B.2 Market Concentration

We now analyze the effects of changes in market concentration on equilibrium quantity and quality, in order to show the robustness of our results from Section 6.

Robustness of our main results when θ and ϕ vary with Q and S. In order to demonstrate the robustness of our main results to the case where θ and ϕ vary with Q and S, we follow the same procedure as in the main text. Total-differentiating the equilibrium system (2) by n, we obtain a system of two equations with two variables: dQ/dn and dS/dn. Solving for these, we have:

$$\frac{dQ}{dn} = \frac{Q}{\zeta}\left\{\frac{\partial \phi}{\partial n}P_S\left[P_S\left(1-\phi\right) + Q\left(\theta_S P_Q + \theta P_{QS}\right)\right] - \underbrace{\frac{\partial \theta}{\partial n}Q P_Q\left(\phi_S P_S + \phi P_{SS} - \frac{\partial^2 c_i}{\partial s_i^2}\right)}_{<0}\right\},$$

and:

$$\frac{dS}{dn} = \frac{Q}{\zeta}\left\{-\frac{\partial \phi}{\partial n}\underbrace{P_S\left[P_Q\left(1+\theta+Q\theta_Q\right) + Q\theta P_{QQ}\right]}_{<0} + \frac{\partial \theta}{\partial n}Q P_Q\left(\phi_Q P_S + \phi P_{QS}\right)\right\}.$$

Therefore, our results are comparable to that in the main text. The main difference comes from the fact that the products $\theta_S P_Q$ and $\phi_Q P_S$ affect total market quantity and quality in a similar manner as θP_{QS} and ϕP_{QS}, respectively.

Robustness of our main results when the market size varies with n. In order to show that our main results remain robust when the market size varies in n, we follow the same procedure as in the main text.[52] Total-differentiating the equilibrium system (2) by n, we obtain a system of two equations with the variables dQ/dn and dS/dn. Solving for these, and defining $\Theta_n \equiv (\partial\theta/\partial n)\, QP_Q + (\partial P/\partial n) + Q\theta \left(\partial P_Q/\partial n\right)$ and $\Phi_n \equiv \left(\partial\phi/\partial n\right) P_S + \phi\,(\partial P_S/\partial n)$, we obtain:

$$\frac{dQ}{dn} = \frac{Q}{\zeta}\left\{\Phi_n\left[P_S\left(1-\phi\right) + Q\theta P_{QS}\right] - \Theta_n \underbrace{\left(\phi P_{SS} - \frac{\partial^2 c_i}{\partial s_i^2}\right)}_{<0}\right\},$$

and:

$$\frac{dS}{dn} = \frac{Q}{\zeta}\left\{-\Phi_n \underbrace{\left[P_Q\,(1+\theta) + Q\theta P_{QQ}\right]}_{<0} + \Theta_n \phi P_{QS}\right\}.$$

This shows the robustness of the results provided in the main text, as changes in market size inherent to variations in the number of competitors are captured through redefining some our variables.

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[52]If the market size varies with market concentration, it could be useful to adjust the demand system such that the underlying consumer preferences are not affected by whether a given variety is available or not; see the short note by Höffler (2008) for an example. Our main results are qualitatively robust to performing such adjustments to our Examples A.1 and A.2, for instance.

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