# Money Talks: Information and Seignorage*

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**Abstract**

This paper analyses the consequences for monetary policy in the presence of currencies issued by firms. Such currencies generate seignorage revenues and information on consumers. In a benchmark model of imperfectly competing firms, information breaks the usual portfolio indeterminacy as in Kareken and Wallace (1981): firms do not accept their competitors' currencies. This limits the issuer's seignorage base. Firms then optimally implement the Friedman rule to remove their seignorage income altogether. As a result, public currency is unable to compete unless the central bank follows suit, resulting in deflation. However, private currency market power—modelled as concentration of decision powers in a currency consortium as well as network effects—induces inflationary pressures, breaking the benchmark results.

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# 1 Introduction

The last decade has seen large-scale innovation in the realm of digital currencies. The most prominent example is Bitcoin, a private, decentralised digital currency (PDDC) for which transactions are verified using cryptographic technologies, and which has been issued in large amounts. Not yet in existence in advanced economies but already the subject of research and discussion are central bank digital currencies (CBDC), to be issued by monetary authorities and complementing banknotes, coins and bank reserves. This paper discusses a third type of digital currency: private, centralised digital currencies (PCDC) issued and operationally managed by firms or groups of firms that produce consumption goods. With the announcement of Libra—later renamed Diem and now folded under regulatory pressure—they are perceived as a serious rival to central bank currency in the future. With over a billion users of Alibaba's Alipay and Tencent's WeChat Pay each, they are already much more important in number of transactions than Bitcoin.[1]

Unlike decentralised digital currencies, PCDC do not offer anonymity. The owner of the technology knows the identity of the consumer and verifies transactions centrally. The collected transaction data have great value in understanding consumer tastes, raising the profits of the seller. Therefore, introducing a centralised digital currency brings one benefit: it generates information rents. Unlike CBDC, PCDC generate further income that stays in private hands and is not rebated to the fiscal authorities. Issuing zero-interest currency backed by interest-paying assets, the firms obtain a second benefit: seignorage revenues. This paper studies how these two benefits affect the issuance of PCDC. It also studies their effect on monetary policy, both private and public, once the PCDC is widely used.

To this end, I develop a general equilibrium framework of money as medium of exchange, information frictions and imperfect competition. Consumers value consumption heterogeneously, but their types and purchases are unobservable initially. Firms—best thought of as vertically-integrated platforms, conglomerates or firm consortia which supply the entire range of consumption goods—can

[1] Diem was clearly designed as a currency: tokens held in digital wallets, exchangeable for other currencies at the prevailing exchange rates. However, Alipay and WeChat Pay have features of both currencies and payment technologies which simply access bank accounts. On the one hand, users can hold tokens in their digital wallets. On the other hand, when making payments using the app, users can also debit their bank accounts. In this case, Alipay and WeChat Pay resemble payment technologies such as debit cards, rather than currencies. Furthermore, the value of Alipay and WeChat Pay tokens is pegged to the RMB. The results of the model hold if two conditions are met: tokens are used as media of exchange when purchasing consumption goods; and the issuer of tokens obtains seignorage revenues. Both of these conditions are met for Alipay and WeChat Pay tokens.

only identify consumers after introducing a payment technology that generates transaction data. The model thus contains a notion of information based on past purchasing behaviour. Furthermore, firms have market power and charge prices above marginal costs. Information is useful to attract the most profitable customers.

The assumption of unobservability of types and purchases without a payment technology warrants discussion. Firms already observe a lot of information: Amazon forces users to create accounts before purchasing goods; Apple and Google have indeed introduced payment technologies. Yet the amount of data collected is heterogeneous, with other firms such as Facebook not collecting any such data as of now. Big transaction data greatly help them evaluate the effectiveness of advertising campaigns.[2] Transaction data could also compliment other data collected by digital platforms, allowing for deeper insights into consumer tastes and behavioural patterns. Furthermore, data may be lost to producers when goods are sold through retailers. Last, producing firms may not have the capacities to collect and match consumer data.[3] All of this is captured by assuming that the introduction of a payment technology enhances the firms' understanding about their customers.

Given the Facebook's plans of issuing a currency that shook the policy world, the payment technology is modelled as money: consumers face a cash-in-advance constraint following Lucas and Stokey (1987), forcing them to hold currency corresponding to their consumption expenditure. Since firms issue money, they obtain a second benefit: seignorage. The government also issues public currency, competing with private currency. I endogenise the firms' currency acceptance and the consumers' currency choice. Thus, the money in circulation could be public, private, or both.

In this framework, firms do not accept competitor currencies due to the information rents. This breaks the usual portfolio indeterminacy discovered in Kareken and Wallace (1981) and present in recent models of currency competition.[4] Instead, demand for the private currency of one firm can be pinned down in equilibrium as it corresponds to the consumption purchases with said firm. The model's prediction is consistent with empirical observations from the world of digital platforms and payment technologies. In China, Alipay and WeChat Pay dominate the market for digital payments. However, customers of Alibaba cannot use WeChat Pay to purchase goods. Similarly, Amazon does

[2]Lewis and Rao (2015) show that measuring the returns to advertising is very challenging given the large number of transaction data needed and the noise in individual purchase decisions. Collecting large amounts of transaction data directly at the location of advertising, the mobile phone, can greatly reduce this difficulty.

[3]Shopify, an e-commerce platform and payment provider for small companies and brands, was one of the first members of the Diem consortium.

[4]See i.e. Schilling and Uhlig (2019), Fernández-Villaverde and Sanches (2019) and Benigno et al. (2019).

not accept Google Pay.

Limited acceptance of PCDC due to information collection is key to characterise profit-maximising private monetary policy and its effect on public monetary policy. In models of money as the medium of exchange, seignorage revenues correspond to a tax on consumption. Holding money is costly: it does not pay interest and therefore yields a lower return than bonds. The nominal interest rate of the economy can be interpreted as a tax rate. Since consumers only hold money to purchase consumption, consumption expenditure forms the tax base. By the first result, information limits private currency holdings—and thus the seignorage tax base—to consumption expenditure with the issuing firm. As the recipient of seignorage revenues, this firm perfectly internalises the effect on its product profits and sets interest rates in the private currency to zero: optimally, the firm does not levy a seignorage tax on top of its profit-maximising price.

In the benchmark model, the introduction of a PCDC endangers the central bank's policy autonomy. Private monetary policy restricts public monetary policy, and vice versa. Consumers do not hold a currency if it is associated with higher net price including the seignorage tax. Thus, whenever interest rates in the public currency are larger than zero, competitor firms which only accept the public currency are unable to compete. Consumers do not demand public currency, and the central bank is forced to implement a zero interest rate policy. This limits its policy autonomy but disciplines the central bank: implementing the Friedman rule of zero nominal interest rates improves welfare, and only the firms' market power prevents efficiency. However, while zero interest rates are desirable in a model of money as medium of exchange, they are associated with deflation—an outcome that may be undesirable for reasons outside of this model.[5]

*Breaking the benchmark.* The paper then extends the model in various directions. First, I show that inflationary pressures arise if firms form currency consortia, but decision powers and seignorage claims are concentrated in the hands of one firm. This extension captures an economy in which firms already obtain information from their own transactions, but the consortium overall benefits from observing all transactions of all consortium members. Thus, information is generated with third party firms. In this context, information becomes inflationary if it increases the consortium's seignorage base on which the consortium leader can levy the seignorage tax. I present scenarios in

[5]One example includes economies with nominal wage rigidities. The central bank may want to use inflation as a tool to reduce real wages in response to a negative productivity shock. See i.e. Uhlig and Xie (2021) for a model with nominal rigidities and multiple currencies.

which the public currency disciplines the private currency. Furthermore, I show how the presence of network effects leads to inflationary outcomes.

*Policies to escape the benchmark.* In the final part of the paper, I analyse policies which allow the government to escape the privately-enforced zero interest rate environment. First, I consider interest-bearing CBDC, and find that the central bank indeed regains full autonomy as long as CBDC does not generate any seignorage income.

**Contribution to the literature.** This paper is the first to formally discuss private digital currencies as information-generating devices and the resulting consequences for monetary policy. The paper therefore primarily speaks to the literatures on the digitalisation of money, currency competition, payments and privacy.

Existing papers either analyse the effect of PDDC on public monetary policy, or analyse PCDC but limit the degree to they compete with government currency. In Chiu and Wong (2020), a digital platform faces the choice between introducing private token money and accepting government currency. However, the monopolist platform is fixed in size and their tokens do not fully compete with government currency. I endogenise platform size. Private currency competes with government currency through the prices faced by consumers in each denomination. Brunnermeier et al. (2019) describe, albeit without a model, how the introduction of digital currency areas helps promote platform cohesion and information collection. Central bank policy autonomy may be under pressure if public currencies lose their role as medium of exchange. I develop a formal model, endogenising the issuance and acceptance of PCDC generating information and seignorage, in order to discuss monetary policy consequences. Gans and Halaburda (2015) discuss PCDC as a customer retention device in a partial equilibrium setting. Mayer (2021) analyses the interaction between a platform issuing tokens, speculators, and users. Li and Mann (2018), Catalini and Gans (2018), Garratt and Van Oordt (2019), Prat et al. (2019), Rogoff and You (2020), Cong et al. (2020) and Gryglewicz et al. (2021) analyse PCDC with a focus on optimal financing strategies, rather than competition with government fiat money. Garratt and van Oordt (2021) and Garratt and Lee (2021) discuss how payment data collection leads to welfare losses due to price discrimination and monopoly formation. CBDC preserve privacy in a Kahn et al. (2005) sense. While interest payments on CBDC play a similar role in my analysis, I focus on the consequences for monetary policy.

Schilling and Uhlig (2019) discuss competition among Bitcoin and a public currency, rediscovering the famous portfolio indeterminacy result of Kareken and Wallace (1981). In Fernández-Villaverde and Sanches (2019), entrepreneurs issue PDDC which compete with public currency. The entrepreneurs obtain seignorage revenues by expanding the money supply and may frustrate the government's attempts to implement the Friedman rule. Skeie (2019) discusses digital currency runs when a PDDC competes with public currency experiencing high inflation rates. The findings of Benigno et al. (2019) resonate with the monetary policy consequences in this paper. When a global and a local public currency are perfect substitutes, then the exchange rate must be fixed in equilibrium by a no-arbitrage argument. If the global currency pays interest, the central banks must set interest rates to zero for public currency to be able to compete. When the issuer of currency is a firm, I obtain this result even without interest payments. Cong and Mayer (2021) discuss competition among traditional fiat currencies, cryptocurrencies, and CBDC.[6]

In Lagos and Zhang (2021, 2019), sellers are subject to payment intermediary market power. Monetary policy restricts market power and remains effective through a medium-of-exchange channel even in pure credit economies. Huberman et al. (2021) discuss how the Bitcoin payment system can reduce payment intermediary costs. In this paper, rather than being subject to intermediaries, firms introduce a payment method in the form of currency themselves. Private money competes with public money and restricts the central bank's policy space.

Information and privacy in the digital economy are being tackled from many other directions; see Goldfarb and Tucker (2019) and Bergemann and Bonatti (2019) for extensive surveys. Customer recognition by imperfectly competing firms has been addressed in Villas-Boas (1999) and Fudenberg and Tirole (2000).[7] In more recent work, Bonatti and Cisternas (2020) investigate price discrimination by short-lived monopolists based on a consumer score that aggregates information on past purchases. Abis et al. (2022) find that firms generating data themselves receive better customer ratings and are more profitable.[8] Liu et al. (2021) document a data privacy paradox: evaluating survey and Alipay data, they find no relationship between stated privacy preferences and privacy-preserving actions.Chen et al. (2021) analyse behavioural vulnerabilities, in particular

[6]For further papers discussing pricing of cryptocurrencies, see, among others, Athey et al. (2016), Chiu and Koeppl (2017), Biais et al. (2018), Prat and Walter (2018), Budish (2018), Sockin and Xiong (2020) and Choi and Rocheteau (2020). See John et al. (2021) for a survey.

[7]See also Acquisti and Varian (2005), and Fudenberg and Villas-Boas (2012) for a survey of this literature.

[8]See also Bian et al. (2022).

limited resistance to purchases of consumption goods that do not increase utility, in the context of data privacy regulation. Parlour et al. (2020) analyse the welfare effects if banks lose payment information due to FinTech innovations. Modelling information based on past purchase behaviour in a general equilibrium framework, I endogenise the acceptance of firms' digital currencies in order to discuss consequences for monetary policy.[9]

**Organisation of this paper.** Section 2 presents the benchmark model. Optimal private monetary policy and the resulting consequences for public monetary public are characterised in Section 3. Section 4 extends the framework to break the benchmark model's results. Section 5 discusses interest-bearing CBDC and policies that force firms to hold public currency. Section 6 concludes.

# 2 Model framework

## 2.1 Environment

### 2.1.1 Households

The model features overlapping generations (OLG) of consumers who live for three periods and discount the future at rate $\beta$. At each point in time, three cohorts of consumers—the young, the adolescent, and the old—co-exist. Each generation consists of a continuum of consumers on the unit interval. They consume a credit good $X$ and a money good $C$, and supply labour $N$. Their age is denoted by $A \in \{y, a, o\}$. Period utility for consumer $j$ is quasi-linear and given by[10]

$$U_{A,j} \ = \ U(X) \ + \ \theta_{A,j}^{1-\alpha} \left(C\right)^{\alpha} \ - \ N \tag{1}$$

I assume that the Inada conditions hold for the credit good utility function, $U(X)$. This implies the existence of a consumption level $X^*$ satisfying $U'(X^*) = 1$. The market for the credit good is a useful modelling device. First, the credit good serves as the numeraire. Second, the market for

[9]Keister and Monnet (2020) discuss how CBDC can generate information on the quality of banks' balance sheets. In this paper, PCDC generates information for firms over consumer tastes.

[10]The consumer's period utility corresponds to a buyer's period utility in Lagos and Wright (2005). As in their paper, this specification generates a degenerate distribution of money holdings which makes the money tractable. This paper however does not focus on the hold-up problem of the buyer in a decentralised exchange, but instead focuses on monetary policy conducted by firms which sell goods in exchange for money.

the credit good pins down the real wage $w_t$, described in detail in the following subsection. Third, the separability and quasi-linearity of the utility function ensure that credit good consumption is independent of monetary policy in equilibrium. Intuitively, consumers can always supply an additional unit of labour at constant marginal disutility in order to purchase $w_t$ more units of the credit good. This pins down the real interest rate. Effectively, the credit side of the economy is super-neutral with respect to monetary policy which allows me to focus on its direct effect on the levels of money good consumption—and thus on the role of money as medium of exchange.

Turning to the money good, consumers value consumption heterogeneously according to their type $\theta_j$. I assume that this type is private information. At the beginning of the game, each consumer draws their type from a publicly known, common binary distribution: $\theta_j \in \{\theta_L, \theta_H\}$, $P[\theta_j = \theta_H] = q$, with $\theta_H > \theta_L \geq 0$.[11] Consumers do not value consumption of the money good in the final period of their life:

$$\left(\theta_{y,j},\, \theta_{a,j},\, \theta_{o,j}\right) \;=\; \left(\theta_j,\, \theta_j,\, 0\right) \tag{2}$$

Two firms $i \in \{f, g\}$ supply the money good and charge a price $p_t^i$. Each period, consumers must choose a firm from which they purchases the money good without knowledge of the price. Firms cannot transmit any information about their price to consumers. Consumers thus engage in directed search during which they learn the chosen firm's price. The OLG structure is useful to avoid Folk theorem-type results: it cuts off the purchase history and thus limits the degree of learning to the first period of consumer lives; it also allows the game among firms and consumers to be solved backwards from the final period in which consumers derive utility from money good consumption. Yet the model requires an infinite horizon for money to achieve its equilibrium value.[12]

Consumers form portfolios consisting of money and nominal bonds which are denoted in the public currency or—if private currencies have been introduced—in private currencies. Let the public currency be denoted by $M^{\$}$. Firm $f$ issues private currency $M^{\approxeq}$, firm $g$ issues currency $M^{\mathbb{G}}$. Going forward, I refer to the public currency as the *Dollar* and to firm $f's$ private currency as *Diem*. Let $\phi_t^z$ denote the value of currency $z \in \{\$, \approxeq, \mathbb{G}\}$ in terms of the credit good, and let $\boldsymbol{\phi}_t = (\phi_t^{\$}, \phi_t^{\approxeq}, \phi_t^{\mathbb{G}})$ denote the vector of real prices. Omitting all $(A, j)$-subscripts to indicate individual decision and

[11] The exponent on $\theta_j$ is included for exposition purposes and without loss of generality. As a result, consumer demand schedules and equilibrium firm profits are linear in consumer types.

[12] The OLG structure does not make money essential as a store of value, as in Wallace (1980).

state variables of consumer $j$ aged $A$, the real money balances of the adolescent consumer $j$ are given by $\boldsymbol{\phi}_t \boldsymbol{M}_t$. Consumers are subject to short-sale constraints for each currency.

Let $Q_t^z$ denote the price of a nominal bond denoted in currency $z$, paying one nominal unit in the following period. Let $\boldsymbol{Q_t} = (Q_t^{\$}, Q_t^{\approx}, Q_t^{\mathbb{G}})$ denote the vector of bond prices. The real value of the bond portfolio formed at time-$t$ is then given by $\boldsymbol{\phi}_t \boldsymbol{Q}_t \boldsymbol{B}_t$. As usual, bond prices are inversely related to the interest rate prevailing in the respective currencies: $Q_t^z = \frac{1}{1+i_t^z}$. In sum, the budget constraint is given by:

$$X_{j,t} + p_t^i C_{j,t} + \boldsymbol{\phi}_t \boldsymbol{M}_{j,t} + \boldsymbol{\phi}_t \boldsymbol{Q}_t \boldsymbol{B}_{j,t} \leq w_t N_{j,t} + \boldsymbol{\phi}_t \boldsymbol{M}_{j,t-1} + \boldsymbol{\phi}_t \boldsymbol{B}_{j,t-1} + T_{j,t} \tag{3}$$

where $T_t$ denotes the total real lump-sum transfer from firms and government to consumer $j$. I assume that firms and government transfer all proceeds to the young in equal proportions.[13]

### 2.1.2 Firms

There are two sectors, the credit good and the money good sector. All firms maximise lifetime profits, discounting future profits with discount factor $\beta$.[14] The firms two firms $i \in \{f, g\}$ in the money good sector supply a homogeneous non-durable good and choose a price $p^i$. Each firm in each sector produces given production functions that are linear in the only input factor labour:

$$Y_t^X = N_t^X \qquad Y_t^C = N_t^C \tag{4}$$

I assume perfect competition in the credit good market. Given the linearity of the production function, the real wage and thus real marginal costs for all firms in this economy are given by $w_t = 1$ for all $t$.

### 2.1.3 Money

Consumers need to hold currency in order to facilitate transactions of the money consumption good. In particular, they face a cash-in-advance (CIA) constraint. The timing assumption is that of Lucas

[13] As long as $X^*$ is sufficiently high, this assumption ensures that the labour supply is always positive, without affecting any other equilibrium outcomes.

[14] In equilibrium, given the quasi-linearity of the period utility function, consumer discount factors are equalised at $\beta$. Firms thus use the representative discount factor.

and Stokey (1987): the "cash market" opens after the "credit market". Firms set their menu of prices in their accepted currencies at the beginning of the period. Consumers choose a firm when the credit market is open, learn about firm prices and then choose their cash balances. Consumption takes place at the end of the period.

At the beginning of the game, firms may pay a fixed cost to introduce a private currency which reveals otherwise unobservable purchases of the young.[15] Firms then decide whether to accept the competitor's and the public currency for the remainder of the game. Denote firm $i$'s decision whether to accept currency $z$ by $\gamma^{i,z}$; the vector $\boldsymbol{\gamma}^i$ summarises $i$'s decision for all currencies in circulation. The CIA constraint faced by consumer $j$ at firm $i$ is therefore given by

$$p_t^i C_{j,t} \ \leq \ \boldsymbol{\gamma}^i \boldsymbol{\phi}_t \boldsymbol{M}_t \tag{5}$$

Equation 5 implies that consumers require unit of real money that is accepted by firm $i$ for each unit of real expenditure, all measured in units of the credit good.

**Assumption.** *Private currency introduction and acceptance decisions are fully observable to all agents in the economy. Consumers thus perfectly anticipate which currencies each firm accepts.*

This assumption ensures that firms do not forgo any revenues by not accepting a particular currency. Since firm currencies are newly being introduced, this assumption initially boils down to the public currency being widely used as form of payment. I relax this assumption in Section 4.2.

Given this setup, the OLG structure helps address a feature of the cash-in-advance model: consumers need to hold real money balances proportional to their consumption expenditure but there is no direct exchange of money and goods. Consumers enter the following period with exactly the same amount of nominal money even if they have consumed. Including a third period in which money holdings unravel is useful since money has no value to consumers when they have died, and they would otherwise strategically reduce their money good consumption when adolescent.

[15]Firms already match a lot of transactions to individuals, i.e. because consumers need to open online accounts with them first. In principal, I could allow for heterogeneity in the degree to which firms obtain information without a private currency. Section 4.1 discusses a set-up in which information are generated by third-party firms.

### 2.1.4 Seignorage

Define $\tau_t^z = 1 - Q_t^z$ as the opportunity cost of holding money: both money and bonds pay one nominal unit in the following time period but bonds cost $Q_t^z \leq 1$. I assume that firms back their currencies with bonds issued by the household and denominated in said currency.[16,17] As an example, firm $f$ purchases a unit of bonds at price $Q_t^{\approx}$ for every unit of Diem issued at price one. Hence firm $f$'s real seignorage revenues, $s_t^{\approx}$, are given by $s_t^{\approx} = \tau_t^{\approx}\phi_t^{\approx}M_t^{\approx}$.

### 2.1.5 Information and choice of firm

Correctly anticipating the firms' currency acceptance decisions, consumers need to choose a firm before learning their price.[18] I assume that consumers choose randomly whenever they expect both firms to charge the same price. Young consumers choose each firm with equal probability of 1/2. Adolescent consumers which have not been identified as high types do the same. Crucially, information help firms increase their profits. In particular, I assume that adolescent consumers, which expect both firms to charge the same price but have been identified as high types by one firm, choose this firm with probability $\xi > 1/2$.[19] This assumption is the simplest way of introducing information based on past purchasing behaviour in this monetary framework. In equilibrium, all firms charge the same price to all consumers, regardless of type and time period. Unless firms can direct the consumers' choice towards their firm, they have no means of making use of information.

# 3 Equilibrium

## 3.1 Money balances and demand schedules of the adolescent

Given the problem's set-up, this section provides a summary and intuitive discussion of the consumer optimality conditions. A formal, step-by-step solution to the consumer problem, including the full

[16] Households are perfectly happy supplying bonds in exchange for money as long as the real bond return does not exceed $1 + r_t = \beta^{-1}$. Since $w_t = 1$, every unit of interest payments will have to be made up by supplying one unit of labour in the future, but the disutility from supplying labour is discounted by the rate of time preference $\beta$.

[17] Section 5.2 discusses an economy in with Dollar-denominated bonds only. The results are unchanged.

[18] Alternatively, one could assume that consumers have to search sequentially for price quotes with a given period. This gives rise to a dynamic Diamond paradox, as shown in a previous version of the paper (Guennewig, 2021).

[19] This assumption can be microfounded in a model of advertising in which firms face convex cost to advertising or have to allocate scarce advertising capacity. Having identified high types, they advertise to direct consumers' search.

set of optimality conditions, is presented in Appendix C.[20]

First, the optimality conditions for credit good consumption establish the equilibrium relationship between nominal and real interest rates as well as the inflation rates. Given the separability and quasi-linearity of the utility function, combined with constant wages $w_t = 1$, renders the credit side of the economy super-neutral with respect to monetary policy. It follows that credit good consumption is equal for all time periods $t$ and consumer ages $A$:

$$X_{A,t} \;=\; X^* \tag{6}$$

Intuitively, given the unit real wage, consumers can always purchase one more unit of the credit good by supplying an additional unit of labour. The real interest rate of the economy is then pinned down by the time rate of preference: $1 + r_t = \beta^{-1}$. For all currencies $z \in \{\$, \approx, \mathbb{G}\}$, define the gross inflation rate as $(1 + \pi^z_{t+1}) = \phi^z_t / \phi^z_{t+1}$. The first order conditions for bonds for all consumers, regardless of their type, simplify to the Fisher equation (here expressed in bond prices):

$$Q^z_t \;=\; \beta(1 + \pi^z_{t+1})^{-1} \tag{7}$$

The price of bonds needs to compensate for the fact that consumers discount the future and that nominal bonds lose real value over time, captured by the inflation rate.

Second, adolescent consumers only hold money in order to enable consumption purchases. Money is dominated by bonds in terms of returns whenever $\tau^z_t > 0$. If this is true for all currencies $z$, then consumers do not hold real money balances in excess of their real money good expenditure. Furthermore, consumers do not hold currencies that are not accepted by firm $i$; if multiple currencies are accepted, they hold the currency with the lowest opportunity cost. Let $\tau^i_t$ denote the lowest opportunity cost among all currencies accepted by firm $i$. Whenever $\tau^i_t > 0$, consumer $j$'s CIA constraint is binding :

$$\boldsymbol{\phi}_t \boldsymbol{M}_{j,t} \;=\; p^i_t \, C_{j,t} \tag{8}$$

If $\tau^i_t = 0$, there is no opportunity cost of holding money and the CIA constraint is slack.[21]

[20] The equilibrium of this economy is defined in Appendix B.

[21] The solution also implies a zero lower bound on nominal interest rates for all currencies $z \in \{\$, \approx, \mathbb{G}\}$: $Q^z_t \leq 1 \;\Leftrightarrow$

Third, the adolescent consumer's demand schedule for the money good is given by

$$C_{j,t} \;=\; C(\theta_j, p_t^i, \tau_t^i) \;=\; \theta_j \left[ \frac{\alpha}{p_t^i(1+\tau_t^i)} \right]^{\frac{1}{1-\alpha}} \tag{9}$$

The demand schedule is a function of the *seignorage-adjusted price*: firms charge a price $p^i$ which is scaled up by the opportunity cost of having to hold a currency that is accepted in exchange. If $\tau_t^i = 0$, bonds and money have the same return. There is no opportunity cost of money and consumers pay the real price only once. If $\tau_t^i > 0$, consumers pay the full price once to firms, and another $(\tau_t^i)$-times to the issuer of currency.

Finally, adolescent consumers choose the firm which they expect to charge the lowest seignorage-adjusted price. When consumers expect both firms to charge the same seignorage-adjusted price, they randomise.

## 3.2 *Characterising monopoly prices*

Before characterising the equilibrium firm pricing strategies, I characterise the prices a monopoly firm would charge in a one-firm one-period economy. First, suppose money is not necessary in order to purchase consumption goods. This is equivalent to a world without an opportunity cost of holding money, and thus the consumption demand schedule of Equation 9 simplifies to

$$C(\theta_j, p) \;=\; \theta_j \left[ \frac{\alpha}{p} \right]^{\frac{1}{1-\alpha}} \tag{10}$$

Given the demand schedule and unit marginal costs, the profits from selling to consumer $j$ are given by $\Pi(p, \theta_j) = (p-1)C(\theta_j, p)$. The profit function is continuous and concave in $p$ for all $p > 0$, and there exists a unique profit-maximising price given by:

$$p(\theta_j) \;=\; arg \max_p \Pi(p, \theta_j) \;=\; \frac{1}{\alpha} \;\equiv\; \tilde{p} \tag{11}$$

The profit-maximising monopoly price is a constant mark-up over marginal costs independent of the consumer's type $\theta$. It follows that monopoly profits are linear in the consumer type, $\Pi(\theta_j) = \kappa\theta_j$,

$i_t^z \geq 0$. For negative interest rates, markets for bonds and money do not clear: consumers want to borrow infinite amounts at negative rates to purchase money which pays zero interest.

where $\kappa$ is a constant.

Two assumptions yield this result: type $\theta_j$ does not affect the price elasticity of consumption; and marginal costs are constant. In the monetary framework, firms obtain full market power but do not price discriminate. Thus, high valuation consumers do not have any incentives to mimic the low valuation consumers' actions in equilibrium. The advantage is of these assumptions is tractability: the framework includes notions of imperfect competition and information based on based purchase behaviour in an otherwise standard monetary framework. The disadvantage is that the model is silent on strategic consumer behaviour.[22] Information here is not useful to price discriminate. Instead, it affects the consumers' choice of firms which can be microfounded as improving the allocation of scarce advertising capacity.

Consider next a one-firm one-period economy in which money is required to purchase consumption. The demand schedule of Equation (9) reveals that consumers consider two factors: the real price $p_t^i$ in terms of the numeraire, and the opportunity cost of money captured by $\tau_t^i$. For a firm without private currency, the firm sets the price and the central bank implements a particular seignorage tax rate. A producer that issues private currency chooses both: it charges a price and controls monetary policy for the currency used in the transaction. The following Lemma characterises the monopoly prices depending on the type of money in circulation:

**Lemma 1** (**One-firm one-period monopoly prices**). *Consider an economy in which the only money in circulation is public currency. In this economy, the monopolist charges the product price as characterised in Equation 11. It follows that their seignorage-adjusted price is given by $\tilde{p}(1+\tau_t^{\$}) > \tilde{p}$ whenever $\tau_t^{\$} > 0$.*

*Consider next an economy in which the only money in circulation is issued by the monopoly firm. This firm charges a seignorage-adjusted price given by $\tilde{p}$. In other words, the firm fully removes the seignorage tax either by pursuing a private monetary policy of $\tau^{\approx} = 0$ or by providing compensating product discounts.*

The Lemma is derived as follows. Consider a monopolist that transacts in the public currency and does not obtain seignorage revenues on a given transaction with consumer $j$. The firm's corresponding profits are given by $\Pi_t = (p_t - 1)\ C(\theta_j, p_t, \tau_t^{\$})$. Importantly, firms optimally do not internalise

[22] See Bonatti and Cisternas (2020) for a recent example of price discrimination based on past purchase behaviour.

the Dollar opportunity cost: $\tilde{p} = arg \underset{p_t}{max} \, \Pi_t$. Profits are directly reduced by the seignorage tax:

$$\Pi\big(\theta_j, \tau_t^{\$}\big) \; = \; \kappa\theta_j\big(1+\tau_t^{\$}\big)^{\frac{1}{\alpha-1}} \tag{12}$$

Next consider a monopolist which also issues private currency. Given Equation 5, the CIA constraint binds and private currency holdings are exactly equal to consumption good purchases whenever seignorage revenues are positive: $s_t \; = \; \tau_t p_t \, C(\theta_j, p_t, \tau_t)$. The profit-maximisation problem is then given by

$$\underset{p_t, \tau_t}{max} \quad \Big(p_t(1+\tau_t) - 1\Big)\, \theta_j \, \left[\frac{\alpha}{p_t(1+\tau_t)}\right]^{\frac{1}{1-\alpha}} \tag{13}$$

The seignorage-adjusted price of Lemma 1 is the solution to this problem.

Importantly, if a transaction takes place in private currency, the monopolist also sets the seignorage tax rate. Maximising the sum of producer profits and seignorage tax income, the monopolist perfectly internalise any equilibrium effects. Optimally, they do not set a tax on top of their profit-maximising price. Thus, the firm optimally implements a private currency variant of the Friedman rule, removing the tax income altogether. Firms obtain a degree of freedom: they can implement any private monetary policy and then set prices with compensating discounts accordingly.

## *3.3 Firms issuing private currency fully remove the seignorage tax*

Suppose that firm $f$ has introduced its currency Diem, the only private currency in circulation. The model then contains two types of money good producers: one that issues private currency, and one that doesn't. For the remainder of this subsection, I postulate that firm $g$ does not accept Diem in order not to generate information for their competitor. I verify this postulate having jointly characterised optimal product pricing and private monetary policy for firm $f$, and having discussed the consequences for public monetary policy. It follows that—whenever seignorage revenues are positive and the CIA constraint binds—Diem holdings are exactly equal to purchases from firm $f$ using Diem.

**Proposition 1.** *Both firms charge the monopoly prices as characterised by Lemma 1 to all consumers in all time periods. It follows that, in equilibrium, firm $g$'s transaction are subject to the government's seignorage tax rate. Firm $f$, who issues private currency, optimally removes the seignorage tax rate.*

*Proof:* Adolescent consumers choose a money good producer without knowledge of their price. Once made, their choice cannot be reverted. Hence, firms charge the monopoly prices to adolescent consumers which are independent of consumer types. Optimally, adolescent consumers choose the firm which they expect to charge the lowest seignorage-adjusted price; when they expect both firms to charge the same seignorage-adjusted price, they randomise. Anticipating these future monopoly prices independent of their type, young high valuation consumers have no incentive to hide their type. Hence the young consumers' optimality conditions perfectly mirror those of the adolescent consumers: the demand schedule is given by Equation (9) and they visit the firm which they expect to charge the lowest seignorage-adjusted price, randomising for equal expected seignorage-adjusted prices. Firms again charge their monopoly prices. □

**Corollary 1 (Currency design equivalence).** *If firms were allowed to pay interest on private currency in the model, Proposition 1 would extend to interest payments. Firms are indifferent between implementing $\tau_t^{\approx} = 0$, or fully compensating for inflation using either price discounts or interest payments on currency.*

Given the private currencies' digital nature, it is technologically feasible to pay interest on money holdings. Considering the regulators' worries about the consequences arising from the introduction of PCDC, this result is again surprising. One suggested policy to mitigate the consequences is to prevent issuers of PCDC from paying interest. The model suggests that such a policy does not have any bite. It prevents issuers of PCDC from adding a feature to their currency which they do not want or need to add. It also cannot avoid the consequences for monetary policy outlined below.

Crucially, the results obtained in this section rely on the fact that the issuer of PCDC obtains seignorage revenues corresponding exactly to its product sales. Consumers only hold Diem in order to transact with firm $f$. This section should therefore be considered a benchmark. Section 4.1 discusses currency consortia more generally in which seignorage revenues may not be distributed according to sales shares. Section 5 introduces policies that affect seignorage revenues, i.e. through macroprudential policies which limit how firms can invest the proceeds from issuing money.

## *3.4 The central bank loses its policy autonomy*

Having characterised the seignorage-adjusted prices that firms charge in different currencies, I am now ready to derive the consequences for private and public monetary policy.

**Lemma 2 (Choice of firm and currency).** *Suppose $\tau_t^{\$} > 0$. Consumers only choose firms which have introduced their own private currencies. Whenever these firms accept the Dollar and their private currency, consumers prefer to transact using the private currency.*

**Proposition 2 (Central bank loses policy autonomy).** *Suppose the central bank supplies a strictly positive amount of money, $M_t^{\$,S} > 0$. Government fiat money is only valued, $\phi_t^{\$} > 0$, if the central bank removes all seignorage income and sets the nominal Dollar interest rate to zero: $\tau_t^{\$} = i_t^{\$} = 0$. This policy is associated with deflation: $\pi_{t+1}^{\$} = \beta - 1 < 0$.*

Consumers rationally form beliefs about firms' prices. Given optimal product pricing combined with private monetary policy, and unless $\tau_t^{\$} = 0$, purchasing at firm $f$ using Diem is less costly than a) using the Dollar at firm $f$, and b) purchasing at firm $g$. Suppose $\tau_t^{\$} > 0$. No consumer visits firm $g$'s store, and thus aggregate consumption of the money good provided by firm $g$ is zero. Since no consumer uses the Dollar at firm $f$, the non-negativity constraint binds, and $\phi_t^{\$} M_t^{\$} = 0$. Therefore government fiat money is not valued if the central bank supplies a positive Dollar supply, $M_t^{\$,S} > 0$, unless $\tau_t^{\$} = 0$. Seignorage revenues accruing to the currency-supplying firm lower their seignorage-adjusted prices; these prices are only matched by the competing firm in the absence of any government seignorage revenues. That is, the central bank's interest rate needs to satisfy $i_t^{\$} = 0$.

## *3.5 Firms do not accept competitor currencies*

The following proposition verifies the initial postulate:

**Proposition 3 (No interoperability).** *Firms do not accept their competitor's currencies:*

$$\gamma^{g,\approxeq} = \gamma^{f,\mathbb{G}} = 0 \tag{14}$$

*Proof:* If firm $g$ accepts both Diem and Dollar, consumers use the currency with the lowest opportunity cost. In equilibrium, both currencies must have the same opportunity cost for both to be valued: $\tau_t^{\$} = \tau_t^{\approx} = \tau_t$. By Equation (12), profits are then in linear in the type-weighted measure of firm $g$'s customers, $\bar{\theta}^g$: $\Pi^g = \bar{\theta}^g \kappa (1+\tau_t)^{\frac{\alpha}{\alpha-1}}$. Profits are increasing in both the number as well as the average type of firm $g$'s customers. Since firm $f$ identifies any of firm $g$'s high valuation customers when using Diem, they are able to improve their customer base at the expense of firm $g$. Conditional on being subject to a given level of seignorage taxes, firm $g$'s profits decrease linearly in the measure of high valuation consumers using Diem with firm $g$. Hence, firm $g$ prefers not to accept Diem if this measure is strictly positive. Since not accepting Diem leads to the lowest feasible level of seignorage taxes $\tau_t = 0$ for all $t$, doing so is indeed optimal.

The model therefore predicts that digital giants including Amazon, Apple and Google would not have accepted Diem as means of payment. This prediction is consistent with observations from the real world. In China, the market for digital payments is dominated by Alipay and WeChat Pay. The social media platform WeChat resembles Facebook, and its payment technology is not accepted by Alibaba, the owner of Alipay. Similarly, Amazon does not accept Google Pay.

*Exchange rate determination.* The firms' optimal currency acceptance decisions ensure that the Dollar-Diem exchange rate is determined:

**Corollary 2** (**Acceptance of public currency).** *Issuers of PCDC do not accept the public currency in order to maximise information rents:* $\gamma^{f,\$} = 0$. *Alternatively, if forced to accept the Dollar given its status as legal tender, firms can charge higher prices in Dollars in order to encourage Diem rather than Dollar transactions.*

**Corollary 3** (**Exchange rate determination).** *The Dollar-Diem exchange rate—defined by the ratio of relative real price of both currencies,* $\phi_t^{\$}/\phi_t^{\approx}$*—is determined in equilibrium.*

In models of currency competition, such as Schilling and Uhlig (2019) and Fernández-Villaverde and Sanches (2019), total real money balances are determined in equilibrium. This is does not apply to the portfolio breakdown among the competing currencies. Consumers are only willing to hold perfectly substitutable currencies with equal returns. Since consumers are indifferent between all currencies, there are not enough equilibrium conditions to pin down individual currency balances and thus the exchange rates between currencies. This result was initially obtained by Kareken and Wallace (1981). In this framework, firm $f$ only accepts Diem and firm $g$ only accepts the Dollar.

Given equilibrium consumption levels for each firm, the real Diem and Dollar balances are pinned down which allows me to write down the profit-maximisation problem of the Diem-issuing firm $f$.

### *3.6 Welfare and efficiency*

Upon the introduction of a PCDC, the obtained consumption levels are those that would be achieved in a pure Dollar economy in which the central bank follows the Friedman rule. Here, in equilibrium, this Friedman rule is privately enforced. The PCDC disciplines the public currency, and the central is forced to remove the opportunity cost of holding money by setting nominal interest rates to zero. Forcing the government to implement the Friedman rule is welfare-improving. Money serves the vital role of facilitating transactions, and levying a tax on money balances reduces consumption. However, the allocations for the Friedman rule are not efficient due to monopoly pricing.

### *3.7 Both firms introduce a private currency if information rents are large*

The equilibrium private currency introduction decisions are discussed in detail in Appendix A. In short, the model features a first mover advantage. Both firms reap the equilibrium seignorage rents equally from a one-sided introduction of PCDC. Firm $f$ forces the government to implement the Friedman rule and remove their seignorage tax, which by Equation (12) leads to an increase in firm $g$'s profits. However, once information on consumers is being generated, firm $f$ exploits it to improve their customer base at the cost of firm $g$. The first mover therefore trades off the sum of seignorage and information gains from introducing a private currency against a fixed cost of doing so. The second mover then only trades off the information gains against the fixed cost. If the information gains are small relative to the fixed cost, only one firm introduces a PCDC and gains an information advantage on the competitor firm. Importantly, given the privately-enforced Friedman rule, the firm $g$'s counter-innovation need not take the form of a currency, but may resemble a payment technology instead.

## 4 Breaking the benchmark

In the benchmark of Section 3, firms internalised the opportunity cost of holding money due to seignorage revenues. Monetary policy was forced to follow suit and implement a zero interest rate policy. In this section, I consider three extension to the benchmark model which break its results: firms form currency consortia, network effects prevent competitor firms from not accepting private currencies, and consumers' dislike for providing information.

## 4.1 Industrial organisation of currency consortia

Inspired by the previously proposed institutional set-up of Diem as a currency consortium consisting of multiple firms and initiated by Facebook, I now consider information generation with third party firms—the consortium members.

### 4.1.1 Environment

In this section, consumption utility is derived from *two* money goods:

$$U_{A,j,t} \;=\; U(X_t) \;+\; \theta_{A,j}^{1-\alpha}\Big[\big(C_t\big)^{\alpha} \;+\; \omega\big(\hat{C}_t\big)^{\alpha}\Big] \;-\; N_t \tag{15}$$

As before, firms $f$ and $g$ supply the first money good $C$. Two firms $(\hat{f}, \hat{g})$ produce the second money good. Its market mirrors the market of the first money good, and so do the consumers' optimality conditions. Importantly, the parameter $\omega$ captures the relative size of the second market. I restrict the parameter space such that $\omega \geq 1$. In this context, information gains in the second money good market relative to the information gains in the first money good market are scaled by $\omega$. Hence this parameter captures not only the size of the different markets, but it can also be interpreted as the relative importance of information in these markets.

Consider the scenario in which firms $\Big(f, \hat{f}\Big)$ have formed a currency consortium that issues Diem, the only private money in the economy. I assume that firm $f$ is the *consortium leader*, deciding on Diem monetary policy and thus on the corresponding seignorage tax rate.[23] I refer to firm $\hat{f}$ as the *consortium member.*

As by Proposition 3, competitor firms $g$ and $\hat{g}$ do not accept Diem given the consortium's information rents. Thus, the consortium only obtains seignorage revenues that correspond to the Diem transactions with consortium firms. I assume that the consortium leader and member share

[23] One interpretation is that the leading firm determines the initial private currency set-up, including private monetary policy, and the second firm joins the currency consortium afterwards taking this set-up as given. The Diem consortium's proposal to issue many public currency-denominated stablecoins before all of the proposed 100 members have joined is one such scenario.

the total Diem seignorage revenues equally. The leader's total profits are then given by:

$$\Pi_t^f \;=\; \left[p^f\left(1+\frac{\tau^{\approx}}{2}\right)-1\right]\,C\big(p^f(1+\tau^{\approx})\big) \;+\; \frac{\tau^{\approx}}{2}\,p^{\hat{f}}\,\hat{C}\big(p^{\hat{f}}(1+\tau^{\approx}),\omega\big) \tag{16}$$

The first term captures firm $f$'s product profits and the half of the corresponding seignorage dividends that firm $f$ receives. The second term captures the seignorage dividends corresponding to firm $\hat{f}$'s transactions that firm $f$ receives. Appendix D provides the corresponding profit function for firm $\hat{f}$ and derives equilibrium pricing strategies for both firms, yielding the leader's profits as a function purely of the Diem seignorage tax rate and parameter $\omega$. The consortium leader maximises profits with respect to its price $p^f$ and the Diem seignorage tax rate $\tau^{\approx}$, subject to the consortium member's optimal pricing strategy and an upper bound on $\tau^{\approx}$:

$$\tau^{\approx} \;\leq\; \bar{\tau}\big(\tau^{\$}\big) \tag{17}$$

Consumers only demand the consortium firms' goods if they charge a weakly lower seignorage-adjusted price than their competitors. If the desired Diem seignorage tax rate is too high, no consumers purchase consumption goods using Diem. It follows that the Diem seignorage tax rate is bounded from above.[24] Figure 1 illustrates Equations (16) and (17) for $\omega = 2$ and $\tau^{\$} = 0.06$.[25]

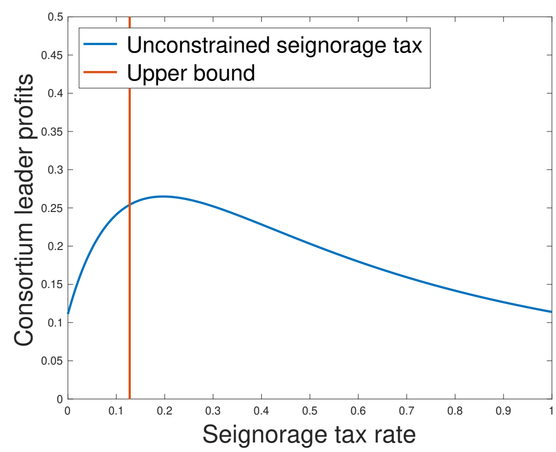


Figure 1: Consortium leader profit function

[24] Appendix D.2 provides the upper bound on $\tau^{\approx}$.

[25] The curvature of the utility function is governed by parameter $\alpha = 0.9$.

To complete the set-up, I define a notion of private currency market power:

**Definition (Ownership concentration).** *Only if $\omega = 1$, both markets are equally large and both firms obtain seignorage dividend shares corresponding to their transaction shares. Whenever $\omega > 1$, the consortium leader obtains a seignorage dividend share larger than its transaction share, and ownership of the currency consortium is concentrated.*

#### *4.1.2 Inflationary pressures if ownership is concentrated*

Given this set-up, I am now ready to characterise sufficient conditions such that the consortium leader levies seignorage taxes on its consortium member firms.

**Proposition 4.** *If ownership is concentrated, the consortium leader implements a strictly positive seignorage tax, inducing inflationary pressures. If $\omega$ is sufficiently large, then the seignorage tax rate is bound from above by the government's desired seignorage tax rate.*

Proof: see Appendix D.3. Intuitively, the consortium leader trades off maximising its own product profits and corresponding seignorage revenues against collecting seignorage income generated from the consortium member's transactions. The former is maximised as $\tau_t^{\approx} = 0$ but this sets the latter to zero. As ownership becomes concentrated, the trade-off tips in the favour of the latter: the tax base available to the consortium leader grows relative to its own product profits, and so does the temptation to levy a seignorage tax. One interpretation of Proposition 4 is that inflationary pressures arise as the private currency becomes more commonly used in the economy. Another interpretation is that information becomes inflationary:

**Corollary 4 (Inflationary information).** *When information collection leads to a relative increase in the consortium member's transactions relative to the consortium leader, increasing the seignorage tax base, then information becomes inflationary.*

The optimal Diem seignorage tax rate—if the consortium leader were unconstrained by central bank policy—is the solution to a non-linear equation. Figure 2 plots the numerical solution as a function of the consortium leader's transaction share given by $\frac{1}{1+\omega} \leq \frac{1}{2}$. Figure 2 demonstrates that the profit-maximising, unconstrained seignorage tax rate is decreasing in the leader's transaction share (or, equivalently, increasing in the size of the consortium member market $\omega$). Thus, both growing the consortium as well as relatively larger information gains in the consortium member goods market

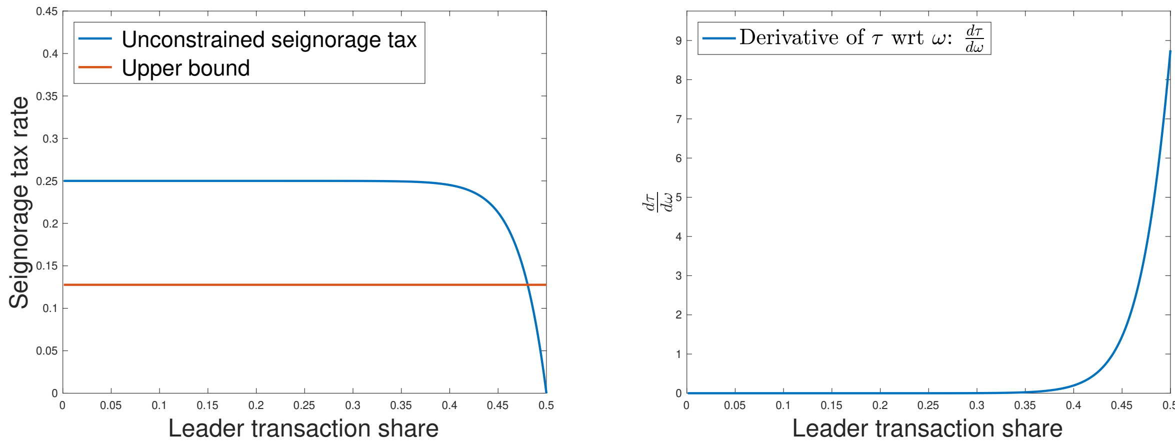


Figure 2: Unconstrained profit-maximising Diem seignorage tax rate and its upper bound

are (weakly) inflationary. Whenever the unconstrained Diem seignorage tax rate lies above the upper bound, then public monetary policy disciplines private monetary policy. The central bank enjoys full policy autonomy.

**Corollary 5 (Information generated with consortium member firms).** *Since the consortium not only generates information for transactions with the consortium leader but also with the consortium member, information rents are higher than in the benchmark model.*

Relative to the benchmark model, the incentives to counter-innovate increase given the inflationary pressures and the increased consortium information rents.

### *4.1.3 Discussion of the previously proposed set-up of Diem*

The fact that the consortium leader may benefit from private currency inflation has implications for the optimal design of private currencies. In particular, the model offers an explanation why the Diem consortium were planning on issuing many stablecoins denominated in public currencies, effectively adopting the public currency as unit of account. Through the lens of the model, this corresponds to a scenario in which the public currency disciplines the PCDC, and the consortium leader maximises their profits by implementing the government's preferred inflation policy. In the same vein, the model helps explain why the Diem association designed their currency as non-interest-bearing.

## 4.2 Network effects

I now consider network effects: a measure $\eta^{error}$ of consumers may erroneously want to use Diem in transactions with firm $g$ even though this firm does not accept Diem or charges a strictly higher price in the competitor's currency. I assume that, conditional on choosing firm $g$, consumers of all types and ages are equally likely to err.

**Proposition 5.** *If consumers make errors, firm $g$ accepts Diem but sets a strictly higher product price $p^{error} = \tilde{p} + \varepsilon$, with $\varepsilon > 0$ but very small. Firm $f$ sets product price $\tilde{p}$. The optimal Diem inflation rate is increasing in the measure of consumers making errors and bound from above by the government's desired seignorage tax rate.*

*Proof:* Firm $g$ chooses from three strategies when consumers make errors: a) never accept Diem, b) accept Diem but charge a strictly higher price $p^{error} > \tilde{p}$, or c) fully accept Diem. Fully accepting Diem is only optimal, taking desired private and public seignorage tax rates as given, when information rents are zero (Proposition 3). If information rents are strictly positive, it is always more profitable to accept Diem but charge a strictly higher price $p^{error} = \tilde{p} + \varepsilon$, with $\varepsilon > 0$ but sufficiently small. Conditional on being chosen by consumer $j$ who erroneously uses Diem rather than the Dollar even though firm $g$ charges a strictly higher price, the profit function is continuous in $p^g$. Thus, firm $g$ can set a price sufficiently close to $\tilde{p}$ such that profits are only reduced on the margin and but information generation jumps: it is limited to the erroneous consumers rather than the whole measure of consumers using Diem when it is fully accepted at equal prices. Accepting Diem and charging higher prices is more profitable than never accepting Diem which would lead to a loss of profits on all erroneous consumers: young and adolescent, high valuation and low valuation. The downside is that information is generated on the erroneous high valuation consumers, leading to a worsening in tomorrow's customer base for firm $g$. Yet, firm $g$ only loses a fraction of the erroneous high valuation consumers, and only does so tomorrow for which profits are discounted with factor $\beta$. Hence firm $g$ prefers to generate some information on erroneous consumers over turning them all away. $\square$

Let $\bar{\theta}$ denote the average customer base of firm $f$ relative to firm $g$. The unconstrained profit-maximising Diem seignorage tax rate is given by:

$$\tau^{\approx} = \frac{1-\alpha}{\alpha}\frac{\eta^{error}}{\bar{\theta}+\eta^{error}} \tag{18}$$

Hence $\tau^{\approx}$ is increasing in $\eta^{error}$ for similar reasons as in the previous subsection: The more Diem transactions take place with firm $g$, the larger is the seignorage tax base that firm $f$ can access when setting $\tau^{\approx} > 0$ without giving compensating product discounts, and so the larger is the temptation to access this seignorage tax base. At the same time, information becomes deflationary:

**Corollary 6 (Disinflationary information).** *The optimal Diem seignorage tax rate, and thus also inflation, is decreasing in the value of information.*

Collecting information on consumers erroneously using Diem with firm $g$ allows firm $f$ to improve its customer base at the expense of firm $g$. Thus, the higher the value of information in this economy as captured by $\bar{\theta}$, the larger are firm $f$'s Diem transactions due to information gains following consumer errors, and hence the lower is the temptation to impose a seignorage tax on Diem transactions with firm $g$.

# 5 Policies to escape the benchmark

## 5.1 *Interest-bearing CBDC*

Suppose the central bank introduces central bank digital currency (CBDC). Given its digital nature, it is technologically feasible to pay interest on CBDC.

**Proposition 6.** *The central bank can escape the benchmark equilibrium outcome by issuing interest-bearing digital currency as long as the interest rate on digital currency matches the interest rate paid on bonds.*

An extension of the benchmark model proving Proposition 6 is presented in Appendix E.1. Intuitively, the firm issuing private currency forces the central bank to implement the Friedman Rule, removing the seignorage tax and thus the opportunity cost of holding money. Instead of setting interest rates on bond to zero, the central bank can also remove the seignorage tax by paying equivalent interest on money.

## 5.2 Forcing firms to hold Dollar-denominated assets

Suppose the regulator requires the issuer of PCDC to back its money using assets denominated in the public currency.

**Proposition 7.** *If the firm is required to hold bonds, either issued by the household or the government, then the results of Propositions 1 and 2 are unchanged. If the firm is required to hold government currency, then both firm $f$'s profits and consumer welfare are increasing in Diem inflation, and are maximised as*

$$\tau^{\approx} \;=\; 1 \qquad \Leftrightarrow \qquad \pi^{\approx} \;\to\; \infty \tag{19}$$

Proof: see Appendix E.2 which presents an extension of the baseline model. Intuitively, and beginning with bonds, total profits are broken down into three components: product profits, direct seignorage revenues from bond holdings, and indirect capital gains.

$$\Pi_t^{f,total} \;=\; \underbrace{\Pi_t^f\left(p_t^f, \tau_t^{\approx}\right)}_{\text{product profits}} \;+\; \underbrace{\tau_t^{\$}\phi_t^{\approx}M_t^{\approx}}_{\text{seignorage}} \;+\; \underbrace{\left(\tau_t^{\approx} - \tau_t^{\$}\right)\phi_t^{\approx}M_t^{\approx}}_{\text{capital gains}} \tag{20}$$

Seignorage revenues are governed by the Dollar seignorage tax rate $\tau_t^{\$}$. Capital gains however are governed by the difference in the Dollar and Diem seignorage tax rates: for every unit of real Diem balances the firm holds real Dollar bonds, and their real value relative to Diem currency increases proportionally in the Diem inflation rate and therefore its seignorage tax rate. On the consumer's side, money holdings are determined by the opportunity cost of holding Diem. The breakdown between direct seignorage revenues and indirect capital gains, captured by the Dollar seignorage tax rate, is irrelevant for consumers. Thus, profits are only affected by the Diem seignorage tax rate. The firm's profit function outlined in Section 3.2 is unchanged and the firm implements the private monetary policy as characterised in Proposition 1.

Turning to money, suppose that firm $f$ faces a macroprudential constraint:

$$\phi_t^{\approx}M_t^{S,\approx} \;\leq\; \phi_t^{\$}M_t^{f,\$} \tag{21}$$

where $M_t^{S,\approx}$ denotes the Diem supply and $M_t^{f,\$}$ denotes the Dollar holdings of firm $f$. It is immediately clear that such a policy leads to positive demand for Dollars whenever firm $f$ sells any goods to consumers, even if they only accept Diem. Total firm profits are given by:

$$\Pi_t^{f,total} = \underbrace{\Pi_t^f\left(p_t^f, \tau_t^{\approx}\right)}_{\text{product profits}} + \underbrace{\left(\tau_t^{\approx} - \tau_t^{\$}\right)\phi_t^{\approx} M_t^{\approx}}_{\text{capital gains}} \tag{22}$$

Forcing firms to hold public currency removes the direct seignorage revenues, breaking the previous one-for-one relationship with capital gains. The firm does not collect any interest but only benefits from the capital gains on money. The balance of internalising the opportunity cost of money either through discounts or through deflation tips in favour of high inflation and corresponding product discounts, in order to achieve capital gains.[26] Macroprudential policies—aimed at delivering price stability—actually induce high levels of inflation, which, surprisingly, are welfare-enhancing.

# 6 Conclusion

This paper is the first work to formally analyse the relationship between information and seignorage for private digital currencies issued by firms. The model highlights two important interactions between information and seignorage. First, seignorage revenues accrue to the issuer of private currency, but information limits the degree to which firms can collect such seignorage proceeds. The resulting private and public inflation outcomes depend on whether large firms issue currencies on their own, or whether many smaller firms form currency consortia which are dominated by one firm. The central bank loses its policy autonomy in the former case but retains it in the latter. In the second case, information becomes inflationary when it increases the seignorage tax base which the consortium leader can exploit using higher inflation rates. The model helps explain the Diem consortium's previous plans to issue public currency-denominated stablecoins, and offers policy prescriptions to escape any undesired consequences for the central bank.

[26] Infinite inflation arises because there is no direct exchange between goods and money in the CIA framework. In other frameworks of money as medium of exchange, optimal private currency inflation may be bound away from infinity. Firms should still find it optimal to generate capital gains in order to circumvent the government's policy and compensate consumers with product discounts.

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# Appendices

# A General Equilibrium PCDC introduction decisions

I now allow firms to counter-innovate by also introducing a private currency. This section highlights a first mover advantage: while information rents reduce the competing firm's profit, seignorage revenues accrue to both firms in equilibrium. The second mover therefore may not find it profitable to also introduce a PCDC.

The first mover gains can be neatly decomposed into two components: information rents in a zero interest rate environment, and seignorage revenues. Prior to the introduction of a private currency, firms split the market equally. The opportunity cost of holding Dollars acts as a tax on consumption, lowering profits for all firms in every time period. Proposition 1 showed that this opportunity cost is fully removed for all currencies upon the introduction of one private currency. Both firms benefit equally and achieve the lifetime seignorage gains, denoted by $\Delta^{\$}\left(\left\{\tau^{\$}_{t+s}\right\}_{s\geq 0}\right)$.

Turning to information, let $\Delta^I$ denote the lifetime information rents for an economy with zero interest rates.[27] The first mover trades off the total gains from introducing a private currency against a fixed cost of doing so. The second mover then trades off only the information gains $\Delta^I$ against the fixed cost.

**Proposition 8** (**First mover advantage**). *If the fixed cost of introducing the currency is smaller than the lifetime information gain, $k \leq \Delta^I$, then both firms introduce private digital currencies. Neither accept the Dollar nor their competitor's currency. The Dollar loses its role as medium of exchange, and thus money market clearing for a positive supply $M^{\$,S}_t > 0$ again requires $\tau^{\$}_t = 0$.*

*For an intermediate cost $k$, with $\Delta^I < k \leq \Delta^I + \Delta^{\$}\left(\left\{\tau^{\$}_{t+s}\right\}_{s\geq 0}\right)$, only one firm introduces a private currency. The competitor firm does not accept it. The first mover achieves both information and seignorage rents. While information rents impose a negative externality on the competitor firm, forcing the central bank to implement the Friedman rule imposes a positive externality.*

Whether the public currency loses its role as medium of exchange depends on the size of the economy's information rents. For sufficiently large information rents, all firms form digital currency

[27] Let $\Delta$ denote the period information gain for an economy with zero interest rates. Firms discount future profits using the household's real discount factor, given by $\beta$ in equilibrium. Since the firm benefits from information for the first time in the period after introducing the currency, the total lifetime information gain is given by $\Delta^I = \frac{\beta}{1-\beta}\,\Delta$.

areas as introduced by Brunnermeier et al. (2019): although transactions take place within one economy, they are settled using different currencies in different marketplaces. Information generated by purchases is valuable, and thus firms aim to maximise their own information set while minimising that of the competitor. This is achieved if each firm only accepts their private currency. It follows that firms prefer not to transact in the Dollar even for a central bank monetary policy of zero interest rates. However, given the privately-enforced Friedman rule result, payment technologies introduced by second movers need not take the form of private currencies. One currency already disciplines the government, and firms can rely on other technologies to generate transaction data.

Proposition 8 also shows that it is more profitable to introduce a PCDC in countries with high inflation. Incentives are higher if the central bank is ill-disciplined and monetary policy effectively taxes transactions between firms and consumers. Even in the absence of any PCDC, public currency interest rates are capped. As $\tau_t^{\$}$ increases—or equivalently as $\pi_{t+1}^{\$}$ increases—profits decrease. Unless the cost of introducing a PCDC approaches infinity, there exists a threshold level of public currency ill-discipline at which one firm introduces a PCDC, disciplining the government forever. This finding resonates with the observed dollarisation in economies which experienced high inflation rates.

Finally, the model's first mover advantage also resonates with frequently made arguments for the clear dominance of one currency in the majority of economies.[28] In all likelihood, handling multiple currencies is mentally costly, and the marginal cost of holding another currency is increasing. General adoption also depends on network effects: consumers only want to hold a particular currency if they expect others to accept it. Interestingly, the People's Bank of China is planning on issuing CBDC very shortly. One doubt about its future success is whether the CBDC can compete with Alipay and WeChat Pay which have successfully established themselves economy-wide. The example of China shows that two digital payment technologies can co-exist, but the number of currencies that can circulate in an economy may well be limited. Such considerations strengthen this paper's result. Together, they help explain why Facebook is pushing ahead with their currency project, although they have been unable to convince the desired amount of 100 firms to join their consortium so far.

[28] One argument for the success of public currencies is that the government can force agents to pay taxes in the public currency. I evaluate the equilibrium effects of such a policy in Section **??**.

# B Equilibrium definition

The competitive equilibrium of this economy is given by the

1. Set of firm strategies that solve the profit maximisation problems, given beliefs $\mu_t^i$:

$$\left(p_t^i, M_t^{z,S}\right)_{i\in\{f,g\},\, z\in\{\approxeq,\mathbb{G}\},\, t\geq 0}$$

2. Set of initial currency introduction decisions

3. Set of currency acceptance decisions, $\left(\gamma^i\right)_{i\in\{f,g\}}$

4. Set of consumer strategies that solve the utility maximisation problem, given beliefs $\mu_t^j$:

$$\left(\psi_{j,y,t}, \psi_{j,a,t}, C_{j,A,t}, X_{j,A,t}, \boldsymbol{M}_{j,A,t}, \boldsymbol{B}_{j,A,t}, N_{j,A,t}\right)_{j\in[0,1],\, A\in\{y,a,o\},\, z\; t\geq 0}$$

5. Set of prices $\left(w_t, \phi_t^z, Q_t^z\right)_{z\in\{\$,\approxeq,\mathbb{G}\},\, t\geq 0}$

such that the markets for labour, the credit good, the money good, bonds and money clear. Beliefs are formed rationally, and updated according to Bayes' Law.

# C Appendix to Section 2—consumer decisions

## *C.1 The adolescent consumer's maximisation problem*

Consider the second period of consumer $j$'s life, born at time $t-1$, having visited firm $f$'s shop and learnt their price $p_t^f$. The consumer maximises remaining lifetime utility subject to budget constraints when adolescent and old (Equation 3) and the CIA constraint when adolescent (Equation 5). As in the main body of text, drop all $j$-subscripts for readability. Formally, the non-negativity constraints on their real money holdings reads:

$$\phi_t^{\$} M_{a,t}^{\$},\ \phi_t^{\approxeq} M_{a,t}^{\approxeq},\ \phi_t^{\mathbb{G}} M_{a,t}^{\mathbb{G}} \ \geq\ 0 \tag{IC.1}$$

The full utility maximisation problem is given by

$$\max_{\substack{\{X_{a,t},X_{o,t+1},C_{a,t},N_{a,t},\\ N_{o,t+1},B_{a,t},M_{a,t}\}}} U(X_{a,t}) \;+\; \theta_j^{1-\alpha}(C_{a,t})^{\alpha} \;-\; N_{a,t} \;+\; \beta\Big[U(X_{o,t+1}) - N_{o,t+1}\Big]$$

$$\begin{aligned}
\text{s.t.}\quad X_{a,t} + p_t^f C_{a,t} + \boldsymbol{\phi_t(M_{a,t} + Q_t B_{a,t})} \;&\leq\; N_{a,t} + \boldsymbol{\phi_t(M_{y,t-1} + B_{y,t-1})} \\
X_{o,t+1} \;&\leq\; N_{o,t+1} + \boldsymbol{\phi_{t+1}(M_{a,t} + B_{m,t})} \\
p_t^f C_{a,t} \;&\leq\; \boldsymbol{\gamma^f \phi_t M_{m,t}} \\
\phi M_{a,t}^{\$},\; \phi_t^{\approx} M_{a,t}^{\approx},\; \phi_t^{\mathbb{G}} M_{a,t}^{\mathbb{G}} \;&\geq\; 0
\end{aligned} \tag{IC.2}$$

The first order conditions (FOCs) are then given by

$$X_{a,t}:\quad U'(X_{a,t}) \;=\; \lambda_{a,t} \tag{IC.3}$$

$$X_{o,t+1}:\quad U'(X_{o,t+1}) \;=\; \lambda_{a,t+1} \tag{IC.4}$$

$$C_{a,t}:\quad \alpha\theta_j^{1-\alpha}(C_{a,t})^{\alpha-1} \;=\; p_t^f(\lambda_{a,t} + \nu_{a,t}) \tag{IC.5}$$

$$N_{a,t}:\quad 1 \;=\; \lambda_{a,t} \tag{IC.6}$$

$$N_{o,t+1}:\quad 1 \;=\; \lambda_{o,t+1} \tag{IC.7}$$

$$B_{a,t}^z:\quad Q_t^z \;=\; \beta\,\frac{\lambda_{o,t+1}}{\lambda_{a,t}}\,\frac{\phi_{t+1}^z}{\phi_t^z} \tag{IC.8}$$

$$M_t^z:\quad 1 \;=\; \beta\frac{\lambda_{o,t+1}}{\lambda_{a,t}}\frac{\phi_{t+1}^z}{\phi_t^z} \;+\; \gamma^{f,z}\nu_{a,t} \;+\; \rho_{a,t}^z \tag{IC.9}$$

for all currencies $z \in \{\$, \approx, \mathbb{G}\}$. The Lagrange and Kuhn-Tucker multipliers of the budget and CIA constraints are denoted by $\lambda_{A,t}$ and $\nu_{a,t}$. The Kuhn-Tucker conditions for the CIA and the non-negativity constraints are given by

$$\nu_{a,t}\left(\boldsymbol{\gamma^f \phi_t M_{a,t}} - p_t^f C_{a,t}\right) = 0 \qquad \text{and} \qquad \nu_{a,t} \geq 0 \tag{IC.10}$$

$$\rho_{a,t}^z\,\phi_t^z M_{a,t}^z = 0 \qquad \text{and} \qquad \rho_{a,t}^z \geq 0 \qquad \forall\, z \tag{IC.11}$$

Combining FOCs for consumption of the credit good and labour supply immediately yields that

$X_{a,t} = X_{o,t+1} = X^*$. Because consumption of the credit good is equal across time, the real interest rate of the economy is pinned down by the discount factor $\beta$. The FOCs for bonds in all currencies simplify to

$$Q_t^z = \beta(1 + \pi_{t+1}^z)^{-1} \tag{IC.12}$$

Using this expression for bond prices, money FOCs become

$$\gamma^{i,z} \nu_{m,t} = 1 - Q_t^z - \rho_{m,t}^z \tag{IC.13}$$

Consumers only hold currencies accepted by firms, and only hold the one with the lower inflation rate (higher bond price) if multiple currencies are accepted. Consider first the case where firm $f$ only accepts one currency: $\gamma^f = (\gamma^{f,\$}, \gamma^{f,\approxeq}, \gamma^{f,\mathbb{G}}) = (0, 1, 0)$. The LHS of the above is zero, requiring $(\rho_t^\$, \rho_t^{\mathbb{G}}) > 0$ whenever $(Q_t^\$, Q_t^{\mathbb{G}}) < 1$; it follows that $M_{a,t}^\$ = M_{a,t}^{\mathbb{G}} = 0$. Consider next a firm that accepts multiple currencies, i.e. the Dollar and Diem. Combining the two FOCs for $M^\$$ and $M^{\approxeq}$ shows that whenever $Q_t^\$ < Q_t^{\approxeq}$, it must be that $\rho_{a,t}^\$ > \rho_{a,t}^{\approxeq}$; since $\rho_{a,t}^{\approxeq} \geq 0$, this requires $\rho_{a,t}^\$ > 0$, yielding $M_{a,t}^\$ = M_{a,t}^{\mathbb{G}} = 0$.

The FOC for money also implies a zero lower bound on the nominal interest rate. All of $\gamma^{f,z}$, $\nu_{a,t}$ and $\rho_{a,t}^z$ are non-negative, and hence $Q_t^z$ can take a maximum value of one ($i_t^z$ can take a minimum value of zero).

Denote the lowest seignorage tax rate with firm $f$ by $\tau_t^f$. The FOC for this money then reads

$$\nu_{a,t} = \tau_t^f$$

which implies that $\nu_{a,t} > 0$ whenever $\tau_t^f > 0$. Then the CIA constraint holds with equality:

$$m_{a,t} = p_t^f C_{a,t}(p_t^f, \tau_t^f)$$

where $m_{a,t}$ denote real money balances of currencies held by the consumer. Turning to the FOC for money good consumption, the demand schedule is given by

$$C_{a,t}(p_t^f, \tau_t^f) = \theta_j \left[\frac{\alpha}{p_t^f(1+\tau_t^f)}\right]^{\frac{1}{1-\alpha}} \tag{IC.14}$$

The value function of the middle aged consumer $j$ at time-$t$ having chosen firm $i$ is given by

$$V_{j,a,t} = U(X^*) \ + \ \theta_j^{1-\alpha}(C_{a,t})^\alpha \ - \ N_{a,t} \ + \ \beta\big[U(X^*) - N_{o,t+1}\big] \tag{IC.15}$$

where

$$N_{a,t} = X^* + p_t^i C_{a,t} + \boldsymbol{\phi}_t(\boldsymbol{M}_{a,t} + \boldsymbol{Q}_t\boldsymbol{B}_{a,t}) - \boldsymbol{\phi}_t(\boldsymbol{M}_{y,t-1} + \boldsymbol{B}_{y,t-1}) \tag{IC.16}$$

$$N_{o,t+1} = X^* - \boldsymbol{\phi}_{t+1}(\boldsymbol{M}_{a,t} + \boldsymbol{B}_{a,t}) \tag{IC.17}$$

$$C_{a,t} = \theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)}\right]^{\frac{1}{1-\alpha}} \tag{IC.18}$$

Plugging in and rearranging, the expression becomes

$$V_{j,a,t} \ = \bar{V}_{j,a} \ + \ \theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)}\right]^{\frac{\alpha}{1-\alpha}} \ - \ p_t^i\theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)}\right]^{\frac{1}{1-\alpha}} \tag{IC.19}$$

$$- \ \boldsymbol{\phi}_t(\boldsymbol{M}_{a,t} + \boldsymbol{Q}_t\boldsymbol{B}_{a,t}) \ + \ \boldsymbol{\phi}_t(\boldsymbol{M}_{y,t-1} + \boldsymbol{B}_{y,t-1}) \ + \ \beta\boldsymbol{\phi}_{t+1}(\boldsymbol{M}_{a,t} + \boldsymbol{B}_{a,t}) \tag{IC.20}$$

Using the nominal stochastic discount factors, $Q_t^z = \beta(1+\pi_{t+1}^z)^{-1}$ for all $z \in \{\$, \approxeq, \mathbb{G}\}$, the expression for $\boldsymbol{M}_{a,t}$ simplifies substantially:

$$V_{j,a,t} \ = \bar{V}_{j,a} \ + \ \theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)}\right]^{\frac{\alpha}{1-\alpha}} \ - \ p_t^i\theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)}\right]^{\frac{1}{1-\alpha}} \tag{IC.21}$$

$$- \ (\boldsymbol{1} - \boldsymbol{Q}_t)\boldsymbol{\phi}_t\boldsymbol{M}_{a,t} \ + \ \boldsymbol{\phi}_t(\boldsymbol{M}_{y,t-1} + \boldsymbol{B}_{y,t-1}) \tag{IC.22}$$

Optimally consumers only hold the currency with the lowest inflation rate among those accepted by firm $i$. The cash-in-advance constraint holds with equality. It follows that

$$p_t^i C_{a,t} = \boldsymbol{\phi}_t\boldsymbol{M}_{a,t} \tag{IC.23}$$

and the expression for the value function becomes

$$V_{j,a,t} \ = \ \bar{V}_{j,a} \ + \ \theta_j \left[ \frac{\alpha}{p_t^i(1+\tau_t^i)} \right]^{\frac{\alpha}{1-\alpha}} \ - \ p_t^i(1+\tau_t^i)\theta_j \left[ \frac{\alpha}{p_t^i(1+\tau_t^i)} \right]^{\frac{1}{1-\alpha}} \ + \ \boldsymbol{\phi}_t(\boldsymbol{M}_{y,t-1} \ + \ \boldsymbol{B}_{y,t-1}) \tag{IC.24}$$

Combining the two middle terms, I can write down the adolescent household's value function at time-$t$ as

$$V_{j,a,t}(\boldsymbol{M}_{y,t-1}, \boldsymbol{B}_{y,t-1}) = \bar{V}_{j,a} \ + \ \boldsymbol{\phi}_t(\boldsymbol{M}_{y,t-1} + \boldsymbol{B}_{y,t-1}) \ + \ \theta_j \tilde{\kappa} \Big[ p_t^i(1+\tau_t^i) \Big]^{\frac{\alpha}{\alpha-1}} \tag{IC.25}$$

where $\tilde{\kappa} > 0$ is a constant. Utility derived from the money good consumption at time-$t$ is only affected by the price that consumers face at the firm they visit, and by the inflation rate on the money that they need to hold. Clearly, consumers visit the firm which they expect to charge the lowest seignorage-adjusted real price.

## *C.2 The young consumer's maximisation problem*

The adolescent consumer's value function is independent of any consumer's decisions taken when young, apart from asset holdings. Consider a consumer born at time $t$. Since consumers face monopoly prices when adolescent, they can only affect future utility through asset holdings. Hence I proceed using the following adolescent value function:

$$V_{j,a,t+1} = \bar{V}_{j,a} + \boldsymbol{\phi}_{t+1}(\boldsymbol{M}_{y,t} + \boldsymbol{B}_{y,t}) \tag{IC.26}$$

Having visited firm $i$'s shop and learnt their price $p_t^i$, the young consumer's utility maximisation problem is given by

$$\max_{\{X_{y,t},C_{y,t},N_{y,t},\boldsymbol{B}_{y,t},\boldsymbol{M}_{y,t}\}} U(X_{y,t}) \;+\; \theta_j^{1-\alpha}(C_{y,t})^{\alpha} \;-\; N_{y,t} \;+\; \beta\, V_{j,a,t+1}(\boldsymbol{M}_{y,t},\boldsymbol{B}_{y,t})$$

$$\begin{aligned} \text{s.t.} \quad X_{y,t} + p_t^i C_{y,t} + \boldsymbol{\phi}_t(\boldsymbol{M}_{y,t} + \boldsymbol{Q}_t\boldsymbol{B}_{y,t}) &\leq N_{y,t} \;+\; T_{y,t} \\ p_t^i C_{y,t} &\leq \boldsymbol{\gamma}^i \boldsymbol{\phi}_t \boldsymbol{M}_{y,t} \\ \boldsymbol{\phi}\boldsymbol{M}_{y,t} &\geq 0 \end{aligned} \tag{IC.27}$$

The resulting equilibrium conditions below, together with the budget and CIA constraint holding with equality, mirror those for the adolescent consumer:

$$X_{y,t} = X^* \tag{IC.28}$$

$$C(\theta_j, p_t^i, \tau_t^i) = \theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)}\right]^{\frac{1}{1-\alpha}} \tag{IC.29}$$

The value function of the young consumer $j$ at time-$t$ is found in analogy to the middle age value function above. Starting point is the middle-age value function:

$$V_{j,a,t+1} = \bar{V}_{j,a} + \boldsymbol{\phi}_{t+1}(\boldsymbol{M}_{y,t} + \boldsymbol{B}_{y,t}) \tag{IC.30}$$

Given the time-$(t+1)$ equilibrium outcomes for consumption of the credit and money goods, time-$t$ decisions only affect future utility through asset holdings: for every unit of real assets in the next period, consumers need to supply one unit less labour.

$$V_{j,y,t} = U(X^*) + \theta_j^{1-\alpha} C_{y,t} - N_{y,t} + \beta\left[\bar{V}_{j,a} + \boldsymbol{\phi}_{t+1}(\boldsymbol{M}_{y,t} + \boldsymbol{B}_{y,t})\right] \tag{IC.31}$$

where

$$N_{y,t} = X^* + p_t^i C_{y,t} + \boldsymbol{\phi}_t(\boldsymbol{M}_{y,t} + \boldsymbol{Q}_t\boldsymbol{B}_{y,t}) - T_{y,t} \tag{IC.32}$$

Plugging in the expression for $N_{y,t}$ and $C_{y,t}$, and again making use of the expression for the nominal stochastic discount factor and equilibrium money holdings gives

$$V_{j,y,t} = \bar{V}_y + \theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)}\right]^{\frac{\alpha}{1-\alpha}} - p_t^i(1+\tau_t^i)\theta_j \left[\frac{\alpha}{p_t^i(1+\tau_t^i)}\right]^{\frac{1}{1-\alpha}} + T_{y,t} + \beta\,\bar{V}_{j,a} \tag{IC.33}$$

As for the middle age value function, combining the middle terms yields the value function of consumer $j$ is given by

$$V_{j,y,t} = \bar{V}_y \ + \ T_{y,t} \ + \ \theta_j \tilde{\kappa}\Big[p_t^i(1+\tau_t^i)\Big]^{\frac{\alpha}{\alpha-1}} \ + \ \beta\,\bar{V}_{j,a} \tag{IC.34}$$

Consumers simply visit firms seeking to minimize the seignorage-adjusted cost of purchasing the money good when young.

# D Appendix to Section 4.1

## D.1 *Optimal pricing strategies*

Consider firm $\hat{f}$'s profit function:

$$\Pi_t^{\hat{f}} \ = \ (p_t^{\hat{f}} - 1)\, C_t^{\hat{f}}(p_t^f, \tau_t^{\approx}) \ + \ \frac{\tau_t^{\approx}}{2}\Big[p_t^f C_t^f(p_t^f, \tau_t^{\approx}) \ + \ p_t^{\hat{f}} C_t^{\hat{f}}(p_t^{\hat{f}}, \tau_t^{\approx})\Big] \tag{ID.1}$$

Since both firms obtain customers with the same expected type and only the scaling parameter $\omega$ matters, I ignore all $\theta$ terms. The demand schedules, conditional on purchase using Diem, resemble the demand schedules of the previous sections:

$$C_t^f(p_t^f, \tau_t^{\approx}) \ = \ \left[\frac{\alpha}{p_t^f(1+\tau_t^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{ID.2}$$

$$C_t^{\hat{f}}(p_t^{\hat{f}}, \tau_t^{\approx}) \ = \ \left[\frac{\alpha\omega}{p_t^{\hat{f}}(1+\tau_t^{\approx})}\right]^{\frac{1}{1-\alpha}} \tag{ID.3}$$

The firms' first order conditions then reveal that firms charge the following prices in Diem taking $\tau_t^{\approx}$ as given:

$$p_t^f(\tau_t^{\approx}) = \frac{\tilde{p}}{1+\frac{\tau_t^{\approx}}{2}} \qquad p_t^{\hat{f}}(\tau_t^{\approx}) = \frac{\tilde{p}}{1+\frac{\tau_t^{\approx}}{2}} \tag{ID.4}$$

The equilibrium profit function of the consortium-leading firm $f$ is then given by:

$$\Pi_t^f\left(\tau_t^{\approx},\omega\right) = (\tilde{p}-1)\left[\frac{1+\frac{\tau_t^{\approx}}{2}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} + \frac{\tau_t^{\approx}}{2}\,\omega^{\frac{1}{1-\alpha}}\,\tilde{p}\left[\frac{\left[1+\frac{\tau_t^{\approx}}{2}\right]^{\alpha}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{ID.5}$$

where the first term captures firm $f$'s product profits and seignorage revenues due to its own transactions; the second term captures the seignorage revenues generated by firm $\hat{f}$ which accrue to firm $f$.

## *D.2 Upper bound on the Diem seignorage tax rate*

To derive the lower bound on the Diem seignorage tax rate, begin by noting that firm $\hat{f}$ only wants to remain part of the consortium if they pay weakly lower seignorage-adjusted prices than their competitor—otherwise they do not sell any goods and prefer to only accept the Dollar.

$$p_t^f(1+\tau_t^{\approx}) \;\leq\; \tilde{p}(1+\tau_t^{\$}) \tag{ID.6}$$

where the above expression imposed that the non-consortium firms charge a real product price of $\tilde{p}$ as by Proposition 1. Given the consortium firms' pricing strategies as above, rearrange to find

$$\tau_t^{\approx} \;\leq\; \frac{\tau_t^{\$}}{1-\frac{1+\tau_t^{\$}}{2}} \tag{ID.7}$$

whenever this expression's numerator is strictly greater than zero. If the numerator is weakly less than zero, Diem inflation is unconstrained by central bank policy.

### D.3 Proof of Proposition 4

Given the firms' optimal pricing strategies, the consortium leader's profits are given by

$$\Pi^f \;=\; (\tilde{p}-1)\left(\frac{1+\frac{\tau^{\approx}}{2}}{1+\tau^{\approx}}\right)^{\frac{1}{1-\alpha}} \;+\; \frac{\tau^{\approx}}{2}\,\tilde{p}\,\omega^{\frac{1}{1-\alpha}}\left(\frac{\left[1+\frac{\tau^{\approx}}{2}\right]^{\alpha}}{1+\tau^{\approx}}\right)^{\frac{1}{1-\alpha}} \tag{ID.8}$$

Taking the first order condition with respect to $\tau^{\approx}$, and evaluating it at zero reveals that $\tau^{\approx}=0$ is a critical point whenever the consortium leader's dividend share corresponds to their transaction share: $(\omega=1)$. Evaluating the second order conditions at zero, imposing $\omega=1$, reveals that $\tau^{\approx}=0$ is indeed a maximum.

Next, I show that the optimal seignorage tax rate for concentrated ownership is strictly positive. Take the derivative of profits with respect to the Diem seignorage tax rate and evaluate it at zero. This derivative is strictly greater than zero whenever ownership is concentrated:

$$\omega \;>\; 1 \quad\Rightarrow\quad \left.\frac{d\Pi^f}{d\tau^{\approx}}\right|_{\tau^{\approx}=0} \;>\; 0 \tag{ID.9}$$

This implies that there is a $\tau^{\approx}>0$ in the neighbourhood of zero associated with larger profits. Hence it must be that the maximum of the profit function over the permissible domain of $\tau^{\approx}\in[0,1]$ is strictly positive.

# E Appendix to Section 5

### E.1 Formal model demonstrating Proposition 6: CBDC

Consider the following variant of the model, demonstrating how the central bank can escape the zero interest rate environment if the public currency has lost its role as medium of exchange. A representative consumer derives period utility according to

$$U(X_t)+\big(C_t\big)^{\alpha}-N_t \tag{IE.1}$$

The model features two currencies: the Diem issued by firm $f$, and the Dollar issued by the central bank. The Dollar here is a digital currency that pays interest at rate $i_t^{M\$}$. The budget constraint is thus given by

$$X_t + p_t C_t + \boldsymbol{\phi}_t \boldsymbol{Q}_t \boldsymbol{B}_t + \boldsymbol{\phi}_t \boldsymbol{M}_t \leq w_t N_t + \boldsymbol{\phi}_t \boldsymbol{B}_{t-1} + \phi_t^{\$}(1 + i_{t-1}^{M\$}) M_{t-1}^{\$} + \phi_t^{≋} M_{t-1}^{≋} \tag{IE.2}$$

Firms produce according to the linear production functions as in the main body of the text. The credit market is perfectly competitive, and the real wage is again pinned down as $w_t = 1$. Firm $f$ supplies the money good as a monopolist at price $p_t$ and does not accept the Dollar in exchange. The consumer thus faces a CIA constraint as below:

$$p_t C_t \leq \phi_t^{≋} M_t^{≋} \tag{IE.3}$$

Real money balances in both currencies need to be weakly positive at all points in time:

$$\phi_t^{≋} M_t^{≋}, \phi_t^{\$} M_t^{\$} \geq 0 \tag{IE.4}$$

The resulting optimality conditions are largely unchanged. The first order condition for Dollar bonds simplifies to

$$Q_t^{\$} = \beta(1 + \pi_{t+1}^{\$})^{-1} \tag{IE.5}$$

The simplified first order condition for Dollar balances is given by

$$1 \;=\; \beta \frac{1 + i_t^{M\$}}{1 + \pi_{t+1}^{\$}} \;+\; \rho_t^{\$} \tag{IE.6}$$

where $\rho_t^{\$}$ is the multiplier of the non-negativity constraint on real Dollar balances. Combining the two equations yields

$$1 \;=\; Q_t^{\$}\,(1 + i_t^{M\$}) \;+\; \rho_t^{\$} \tag{IE.7}$$

The Kuhn-Tucker conditions for Dollar balances imply that

$$\phi_t^{\$} M_t^{\$}\, \rho_t^{\$} \;=\; 0 \tag{IE.8}$$

And hence positive real money balances, for which $\rho_t^{\$} = 0$, require that

$$Q_t^{\$}\,(1 + i_t^{M^{\$}}) = 1 \qquad \Rightarrow \qquad i_t = i_t^{M^{\$}} \tag{IE.9}$$

For a one-sided introduction of private currencies, the main body of text established that the issuer of PCDC charges a seignorage-adjusted price satisfying

$$p^f (1 + \tau_t^{\approx}) \;=\; \tilde{p} \tag{IE.10}$$

While still achieving monopoly rents, the firm fully removes the opportunity cost of holding money. A firm that transacts in the public currency does not internalise the opportunity cost of holding Dollars and charges a real price $p^g = \tilde{p}$. Thus, no consumer visits firm $g$ if the opportunity cost of holding Dollars is positive. Consider now the problem for the consumer as before but with two currencies in circulation, the Dollar and Diem. The Dollar pays interest, and the period budget constraint is given by Equation (IE.2). Firm $g$ only accepts the Dollar, and so $(\gamma^{g,\$}, \gamma^{g,\approx}) = (1, 0)$. Having chosen firm $g$, the first order condition for Dollar holdings for a adolescent consumer becomes

$$1 \;=\; \beta \frac{(1 + i_t^{M^{\$}})}{1 + \pi_{t+1}^{\$}} \;+\; \nu_{m,t} \;+\; \rho_{m,t}^{\$} \tag{IE.11}$$

where $\nu$ and $\rho$ denote the Kuhn-Tucker multipliers of the CIA and non-negativity constraints. Having visited firm $g$, the consumer can only purchase goods from this firm and thus holds positive real Dollar balances. Combining with the first order condition for Dollar-denominated bonds, the above becomes

$$1 - Q_t^{\$}(1 + i_t^{M^{\$}}) \;=\; \nu_{m,t} \tag{IE.12}$$

First of all, $Q_t^{\$}(1 + i_t^{M^{\$}}) = 1$ already implies that $\nu_{m,t} = 0$. The CIA constraint does not bind: there is no opportunity cost of holding Dollars that pay sufficient interest and the constraint forcing money holdings is slack. Combining Equation (IE.12) with the first order conditions, the consumer's money good demand becomes

$$C_{m,t}(p_t^g, Q_t^\$, i_t^{M^\$}) = \theta_j \left[ \frac{\alpha}{p_t^g \Big(2 - Q_t^\$ (1 + i_t^{M^\$})\Big)} \right]^{\frac{1}{1-\alpha}} \tag{IE.13}$$

The seignorage-adjusted price face by consumer $j$ at firm $g$ is thus given by $p_t^g \Big(2 - Q_t^\$ (1 + i_t^{M^\$})\Big)$. Consumers visit firm $g$'s store, leading to positive demand for Dollar goods in equilibrium, if

$$p_t^g \Big(2 - Q_t^\$ (1 + i_t^{M^\$})\Big) \;=\; \tilde{p}\Big(2 - Q_t^\$ (1 + i_t^{M^\$})\Big) \;\leq\; p^f (1 + \tau_t^{\approx}) \;=\; \tilde{p} \tag{IE.14}$$

which requires $Q_t^\$ (1 + i_t^{M^\$}) = 1$, or equivalently, $i_t^\$ = i_t^{M^\$}$.

## *E.2 Forcing firms to hold Dollar-denominated assets*

### *E.2.1 Bonds*

Consider the flow budget constraint of a firm that issues Diem currency, $M_t^{\approx}$, and holds household bonds denominated in the Dollar:

$$\Pi_t^f \;+\; \phi_t^{\approx}(M_t^{\approx,S} \;-\; M_{t-1}^{\approx,S}) \;=\; T_t^f \;+\; \phi_t^\$(Q_t^\$ B_t^f \;-\; B_{t-1}^f) \tag{IE.15}$$

where $\Pi_t^f$ denote firm $f$'s product profits. As before, there is full backing of the currency: $\phi_t^{\approx} M_t^{\approx,S} = \phi_t^\$ B_t^f$. Plugging in the market clearing condition for Diem, $M_t^{\approx,S} = M_t^{\approx}$, defining $m_t^{\approx} = \phi_t^{\approx} M_t^{\approx}$, and using the expression for the inflation rates, the above becomes

$$\Pi_t^f \;+\; \tau_t^\$ m_t^{\approx} \;+\; \left[1 - \frac{1 + \pi_t^\$}{1 + \pi_t^{\approx}}\right] \frac{m_{t-1}^{\approx}}{1 + \pi_t^\$} \;=\; T_t^f \tag{IE.16}$$

From consumers first order conditions, I obtain $Q_t^\$ = \beta(1 + \pi_t^\$)^{-1}$. Since this economy does not feature Diem-denominated bonds, define $Q_t^{\approx} = \beta(1 + \pi_{t+1}^{\approx})^{-1}$. Effectively, if a Diem bond would exist, its price would account for the time rate of preferences and the change in the value of Diem relative to the numeraire. Using the definition of seignorage tax rates $\tau$, I obtain

$$\Pi_t^f \; + \; \tau_t^{\$} m_t^{\approxeq} \; + \; \frac{\tau_{t-1}^{\approxeq} - \tau_{t-1}^{\$}}{\beta} m_{t-1}^{\approxeq} \; = \; T_t^f \tag{IE.17}$$

where the $\frac{\tau_{t-1}^{\approxeq} - \tau_{t-1}^{\$}}{\beta}$ captures the capital gains due to inflation rate differentials on the two currencies. Note how higher inflation on Diem, or equivalently $\tau_{t-1}^{\$} < \tau_{t-1}^{\approxeq}$, yields capital gains in the following period. The firm's profit maximisation problem is now dynamic: a choice of $(Q_t^{\approxeq}, p_t^f)$ affects both profits today and tomorrow. Since there is no meaningful economic connection between time periods other than the fact that some profits only accrue tomorrow, I the firm's total profit maximisation problem becomes:

$$\begin{aligned} \max_{p_t^f, \tau_t^{\approxeq}} \Pi_t^{f,total} \; &= \; \max_{p_t^f, \tau_t^{\approxeq}} \Pi_t^f \; + \; \tau_t^{\$} m_t^{\approxeq} \; + \; \beta \frac{\tau_t^{\approxeq} - \tau_t^{\$}}{\beta} m_t^{\approxeq} \\ &= \; \max_{p_t^f, \tau_t^{\approxeq}} \Pi_t^f \; + \; \tau_t^{\approxeq} m_t^{\approxeq} \\ &= \; \max_{p_t^f, \tau_t^{\approxeq}} \left( p_t^f (1 + \tau_t^{\approxeq}) - 1 \right) \left[ \frac{\alpha \theta^{1-\alpha}}{p_t^f (1 + \tau_t^{\approxeq})} \right]^{\frac{1}{1-\alpha}} \end{aligned} \tag{IE.18}$$

where tomorrow's profits are discounted at rate $\beta$. The problem and also its solution are exactly as before.

### *E.2.2 Money*

Consider firm $f$'s flow budget constraint:

$$\Pi_t^f \; + \; \phi_t^{\approxeq} (M_t^{S,\approxeq} - M_{t-1}^{S,\approxeq}) \; = \; T_t^f \; + \; \phi_t^{\$} (M_t^{f,\$} - M_{t-1}^{f,\$}) \tag{IE.19}$$

Diem is backed using government currency: $\phi_t^{\approxeq} M_t^{S,\approxeq} = \phi_t^{\$} M_t^{f,\$}$. The firm's profit function is derived in analogy to Appendix E.2.1, and the firm's flow budget constraint becomes

$$\Pi_t^f \; + \; \frac{\tau_{t-1}^{\approxeq} - \tau_{t-1}^{\$}}{\beta} \, m_{t-1}^{\approxeq} \; = \; T_t^f \tag{IE.20}$$

Again, the firm's profits are only dynamic in the sense that some profits accrue with a one-period delay. The period profit function is thus given by

$$\Pi_t^{f,total} \;=\; \big[p_t^f - 1\big] C(\theta_{A,j}, p_t^f, \tau_t^{\approx}) \;+\; \big(\tau_t^{\approx} - \tau_t^{\$}\big)\, p_t^f C(\theta_{A,j}, p_t^f, \tau_t^{\approx}) \tag{IE.21}$$

Firm $f$ optimally charges a real price given by

$$p_t^f = \frac{1}{\alpha\big[1 + \tau_t^{\approx} - \tau_t^{\$}\big]} \tag{IE.22}$$

Plugging this expression into the profit function and the consumer's money good consumption function, reveals both are increasing in $\tau_t^{\approx}$:

$$\Pi^f \;=\; \kappa\theta_{A,j}\left[1 \;-\; \frac{1+\tau_t^{\$}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \qquad C_t \;=\; \theta_{A,j}\left[1 \;-\; \frac{1+\tau_t^{\$}}{1+\tau_t^{\approx}}\right]^{\frac{1}{1-\alpha}} \tag{IE.23}$$

where $\kappa$ is constant as in the main body of text. It follows that Diem monetary policy is characterised by the corner solution $\tau_t^{\approx} = 1$, corresponding to infinite inflation.