Certifiable Preplay Communication

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Introduction

Motivation. Before most important decisions or strategic interactions, the concerned parties can communicate with each other.

Question: Are there properties of the strategic interactions about to take place that induce full revelation of private information in the communication phase ?

Framework

- General Bayesian games (simultaneous games with incomplete information)
- Communication extension of the game (interim stage)
  - Hard information (certifiable, verifiable messages): agents cannot lie but may reveal their information partially or not at all.
  - Communication is public, costless and simultaneous.

More Motivation

- General question: what equilibrium outcomes could be implemented with communication.
- Focus here on implementation of a complete information outcome.
- When is it sufficient to study a complete information game?
- Does the introduction of deliberation enable to fully aggregate information?
- Should hard information and deliberation be promoted?

Main Results

- Characterization of the games for which full revelation is a sequential equilibrium under a restriction on beliefs.
- Necessary and sufficient condition for FRE in a “large” class of games.
- List of sufficient conditions for full disclosure.
- Applications: multidimensional types, senders-receiver games, deliberation before voting and supermodular games

The Base Game

Bayesian game: $\Gamma = \langle N, (A_i)_{i \in N}, (T_i)_{i \in N}, p, (u_i)_{i \in N} \rangle$

- Players: $N = \{1, \ldots, n\}$
- Actions: $a = (a_1, \ldots, a_n) \in A = A_1 \times \cdots \times A_n$
- Private information of player i: $t_i \in T_i$ where T_i is finite or a convex subset of $\mathbb{R}^k$.
- $p(.|t_i) \in \Delta(T_{-i})$: interim belief of player i of type t_i (full support)
- Utility function: $u_i : A \times T \to \mathbb{R}$

The Complete Information Games

Let $\tilde{\Gamma}(t) = \langle N, (A_i)_{i\in N}, (u_i(.,t))_{i\in N}\rangle$ denote the complete information game that will be played if everybody learns the true type profile $t = (t_1, \cdots, t_n)$.

Assumptions:

- Each game $\tilde{\Gamma}(t)$ admits a pure strategy Nash equilibrium.
- Let $a^*(t)$ be a Nash equilibrium selection: we will focus on fully revealing equilibria that implement $a^*(.)$
- The following best reply function is well defined:

$$BR_i\big(a^*_{-i}(s_i, t_{-i})|t\big) = \arg\max_{a_i \in A_i} \; u_i\big(a_i, a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}\big)$$

Messages

Before choosing his action a_i but after learning his type t_i, every player i publicly and simultaneously sends a costless message $m_i \in M_i(t_i)$.

Hard information: a message $m_i \in M_i = \cup_{t_i \in T_i} M_i(t_i)$ certifies to the other players that player i's type is in

$$M_i^{-1}(m_i) := \{t_i \in T_i : m_i \in M_i(s_i)\}$$

- $S_i \subseteq T_i$ is certifiable if there exists a message $m_i \in M_i$ such that $M_i^{-1}(m_i) = S_i$.
- Assumption: the certifiable sets are compact.
- Full Certifiability: if for every i, every (compact) $S_i \subseteq T_i$ is certifiable.
- Own Type Certifiability: if the singletons $\{t_i\}$ are certifiable.

The Communication Game

- All players simultaneously send a public message about their type, then they form beliefs and play the base game according to their new beliefs.

- We want to analyze conditions for the existence of a fully revealing sequential equilibrium that implements $a^*(.)$.

- In such an equilibrium:

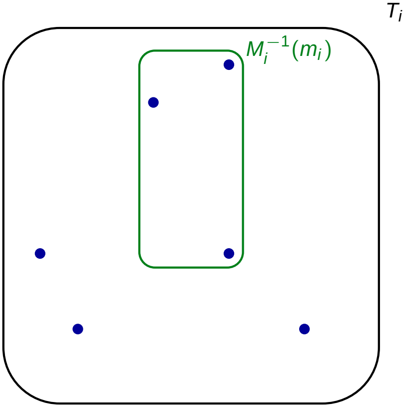
  - Each player i sends a message $m_i \in M_i(t_i)$ that the other players interpret as $\{t_i\}$.

  - On the equilibrium path $a^*(t)$ is played in the second stage.

What happens off the equilibrium path?

Off the Equilibrium Path

For sufficiency arguments, we will construct fully revealing equilibria that satisfy a restriction on beliefs.



Restriction:

- Extremal Beliefs on Single Deviators: $\beta_j^i(m_i)$ assigns m_i to a single type in $M_i^{-1}(m_i)$

Consequences:

- Homogeneity: all players share the same belief about i.
- No Conditioning on other messages: this belief is independent of m_{-i}.

$\rightarrow$ Another Restriction: If off the equilibrium path, a player believes $s \in T$ she plays according to $a^*(.)$.

Sketch of Proof

Assume own type certifiability, and consider a FRE in which every player certifies $\{t_i\}$.

Suppose j plays m_j, off path. By sequentiality, there exists a sequence of fully mixed trembles σ_j^k such that for every $t_j \in M_j^{-1}(t_j)$:

$$\beta_i^k(t_j|m_j, t_{-j}) = \left(\sum_{t'_j \in T_j} \frac{\sigma_j^k(m_j|t'_j)p(t'_j|t_{-j})}{\sigma_j^k(m_j|t_j)p(t_j|t_{-j})} \right)^{-1} \xrightarrow[k\to\infty]{} \mathbb{1}_{t_j=s_j}$$

Then for every $t'_j \neq s_j$

$$\frac{\sigma_j^k(m_j|t'_j)p(t'_j|t_{-j})}{\sigma_j^k(m_j|s_j)p(s_j|t_{-j})} \xrightarrow[k\to\infty]{} 0 \Leftrightarrow \frac{\sigma_j^k(m_j|t'_j)}{\sigma_j^k(m_j|s_j)} \xrightarrow[k\to\infty]{} 0$$

So s_j cannot depend on t_{-j} (No Conditioning on other messages).

Remarks

- With this formulation, beliefs are the only disciplining device.
- In order to dissuade a player from sending off-path messages, other players must be skeptical.
- With extremal beliefs, they must assign any off-path message to a type that no potential sender would want to masquerade as.
- Our restriction on beliefs restrains the ability to punish deviators.

The Masquerade

Understanding Incentives: If i is of type t_i, her payoff of masquerading as s_i is:

$$v_i(s_i|t_i) = E_i\left(\underbrace{u_i\left(BR_i\left(a^*_{-i}(s_i, t_{-i})\right), a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}\right)}_{v_i(s_i|t_i;t_{-i})}|t_i\right)$$

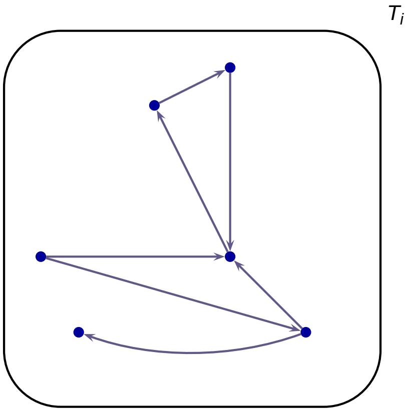
Assumption: $v_i(s_i|t_i)$ is lower semicontinuous in s_i.

Masquerade Relation:

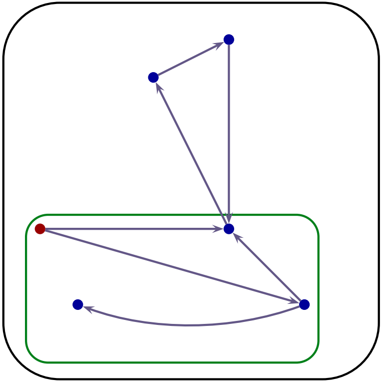
- t_i wants to masquerade as s_i if $v_i(s_i|t_i) > v_i(t_i|t_i)$.
- This defines the masquerade relation denoted by $t_i \xrightarrow{\mathcal{M}} s_i$
- An oriented graph on T_i.

The Masquerade Relation

Example:



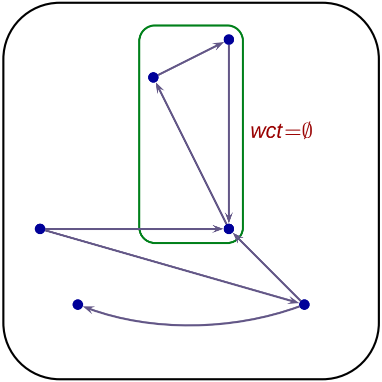
The Worst Case Type



- A worst case type in $S_i \subseteq T_i$ is a type s_i that no $t_i \in S_i$ wants to masquerade as.

$$wct(S_i) = \left\{ s_i \in S_i \mid \nexists t_i \in S_i, t_i \xrightarrow{\mathcal{M}} s_i \right\}.$$

The Worst Case Type



- A worst case type in $S_i \subseteq T_i$ is a type s_i that no $t_i \in S_i$ wants to masquerade as.

$$wct(S_i) = \left\{ s_i \in S_i \mid \nexists t_i \in S_i, t_i \xrightarrow{\mathcal{M}} s_i \right\}.$$

Evidence Base

The existence of worst case types will allow players to punish deviations to off path messages.

To get FRE will also need the message space to allow players to reveal their type.

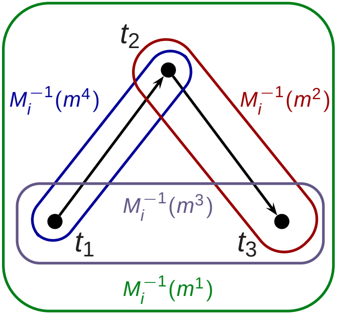
Evidence Base: an evidence base for i is a set $\mathcal{E}_i \subseteq M_i$ such that there exists a one-to-one function $e_i : T_i \to \mathcal{E}_i$ that satisfies:

(i) $e_t(t_i) \in M_i(t_i)$.

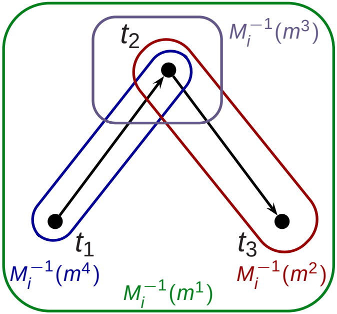
(ii) t_i is a worst case type of $M_i^{-1}\big(e_i(t_i)\big)$.

Remark: under Own Type Certifiability, there exists an evidence base. Also Milgrom's piano player example.

Evidence Base: Example



$$\mathcal{E}_i = \{m^1, m^2, m^3\}$$
$$\mathcal{E}_i = \{m^4, m^2, m^3\}$$



No evidence base

Necessity of Evidence Bases

Proposition

If there exists a FRE, then every player i has an evidence base

Argument:

If player i uses a message m_i on the equilibrium path, then:

- m_i only uses this message when of type t_i.
- t_i is a worst case type of $M_i^{-1}(m_i)$.

A Characterization

Proposition

There exists a FRE that satisfies our restrictions (extremal beliefs, a^*) iff.

(i) *Every player has an evidence base.*

(ii) *Every certifiable subset has a worst case type.*

In sender-receiver models, the relationship between worst-case types and the existence of a full disclosure equilibrium was first pointed out by Seidmann and Winter (1997) and Mathis (2008) has an “evidence base” condition.

When (i) is satisfied, adding messages makes it harder to get a FRE.

Acyclic Masquerade

Acyclic Masquerade Property: if for every player i, the masquerade relation $\xrightarrow{\mathcal{M}_i}$ has no finite cycle.

Proposition

There is equivalence between:

(i) $\xrightarrow{\mathcal{M}_i}$ *is acyclic.*

(ii) *Every finite subset $S_i \subseteq T_i$ admits a worst case type.*

(iii) *Weak Representation: there exists a lsc function $w_i : T_i \to \mathbb{R}$ such that:*

$$t_i \xrightarrow{\mathcal{M}_i} s_i \;\;\Rightarrow\;\; w_i(s_i) > w_i(t_i) \tag{WR}$$

(iv) *Directional Masquerade: There exists a complete and transitive lsc order $\succeq$ on T_i such that:*

$$t_i \xrightarrow{\mathcal{M}_i} s_i \;\;\Rightarrow\;\; s_i \succ t_i \tag{DM}$$

FRE and Acyclic Masquerade

Proposition

Suppose that (Γ, a^*) satisfies the acyclic masquerade property. Then there exists a Fully Revealing Equilibrium iff. every player has an evidence base.

Corollary:

- Under Full Certifiability: there exists a FRE with extremal beliefs iff. (Γ, a^*) satisfies the acyclic masquerade property.

The Multidimensional Case

An Example: Conformity

- Multidimensional Sender/Receiver with conformity (à la Bernheim, 1994).
- $T \subseteq \mathbb{R}^k$ = preferred action of the receiver.
- $C \subseteq T$ a finite set: central positions associated with social norms or lobbies.
- Preferences of the sender:

$$v(s|t) = f(s,t) - \underbrace{\lambda(t)}_{>0} d(s,C)$$

- Where $t \in \arg\max_x \; f(x,t)$.

An Example: Conformity

Then:

$$t \xrightarrow{\mathcal{M}} s \;\Leftrightarrow\; \lambda(t)\big(d(t,C) - d(s,C)\big) > f(t,t) - f(s,t) \geq 0$$

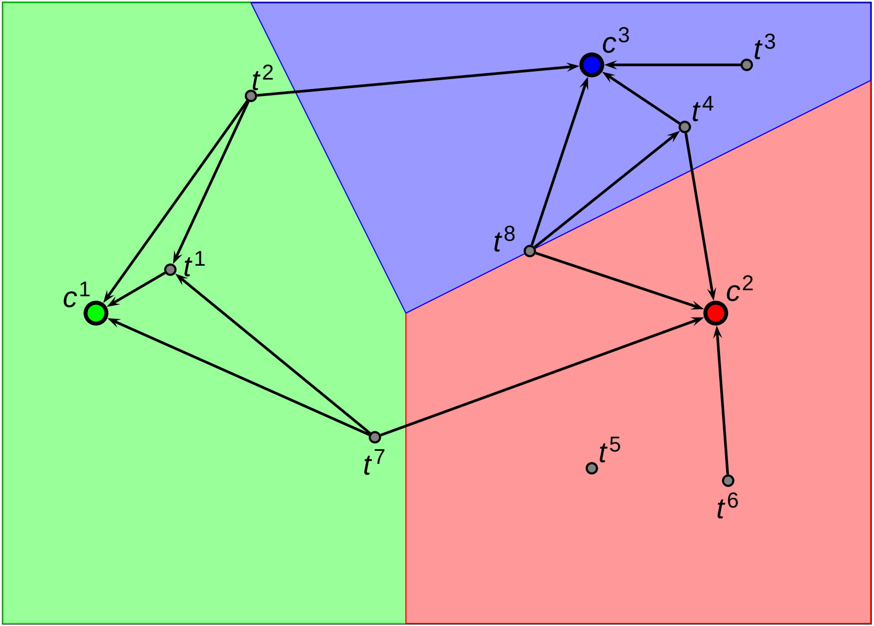
$$\Rightarrow\; -d(s,C) > -d(t,C)$$

Hence the acyclic masquerade property is satisfied (Weak Representation).

More generally, the AMP is satisfied if:

$$v(s,t) = \underbrace{f(s,t)}_{\text{max at } s=t} + \underbrace{\lambda(t)}_{>0} g(s) + h(t)$$

Illustration



Biases

Suppose :

$$v(s,t) = -||s - t - b(t)||^2$$

Where $b : T \to \mathbb{R}^k$

What conditions on $b(.)$ ensure existence of a FRE?

Seidmann and Winter (1997): $T \subseteq \mathbb{R}$, and $b(t)$ is single crossing from below $\Rightarrow$ FRE.

Otherwise they need an additional no reciprocal masquerade condition.

Biases

Proposition

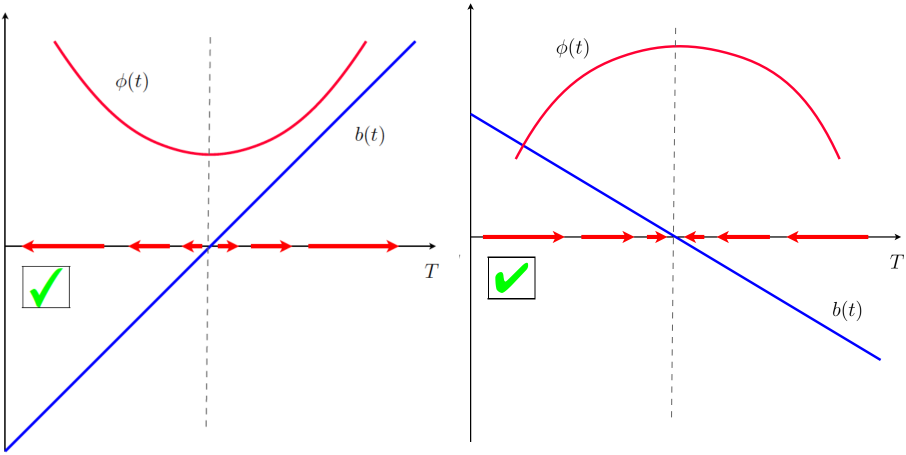
Suppose that there exists a function $\phi : \mathbb{R}^k \to \mathbb{R}$ *such that*

$$b(t) = \nabla\phi(t)$$

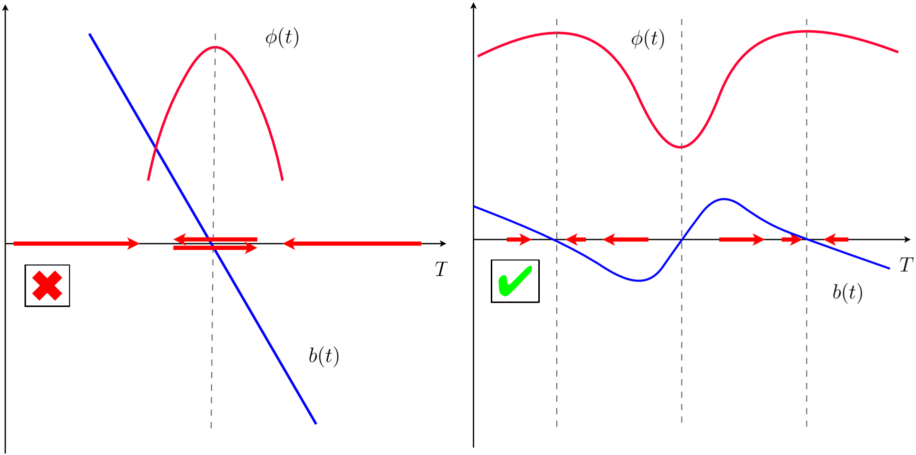
and $\psi(t) = \phi(t) + \frac{1}{2}||t||^2$ *is convex. Then the Acyclic Masquerade Property is satisfied.*

- In particular it is sufficient that $b(t)$ be the gradient of a convex function (centrifugal biases).
- But the function can also have a mild concavity (mildly centripetal biases).

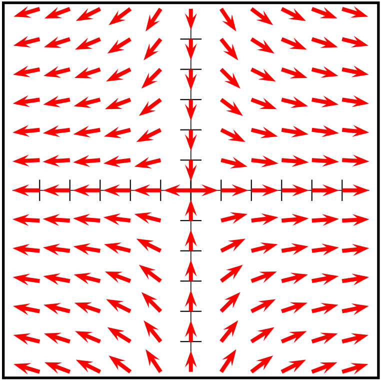
One-Dimensional Examples



One-Dimensional Examples



Two Dimensional Example



- Bias: $b(t) = \begin{pmatrix} \alpha_1 t_1 \\ -\alpha_2 t_2 \end{pmatrix}, \quad \alpha_1, \alpha_2 > 0.$
- Gradient of: $\phi(t) = \frac{1}{2}\left(\alpha_1 t_1^2 - \alpha_2 t_2^2\right).$
- $b(t)$ is centrifugal on the first dimension, and "mildly" centripetal on the second dimension.
- Condition:

$$Db(t) + I = \begin{pmatrix} 1+\alpha_1 & 0 \\ 0 & 1-\alpha_2 \end{pmatrix} \geq 0$$

$$\Leftrightarrow$$

$$\alpha_2 \leq 1$$

The Unidimensional Case

Some Obvious Sufficient Conditions

In all these cases it is easy to find a worst-case type:

Monotonicity (MON): $(T_i, \succeq)$ is linearly ordered and $v_i(.|t_i)$ is monotonic.

Directional Masquerade (DM): $(T_i, \succeq)$ is linearly ordered and

$$t_i \xrightarrow{\mathcal{M}} s_i \ \Rightarrow \ s_i \succeq t_i$$

Remark: almost all the literature on hard information lies here.

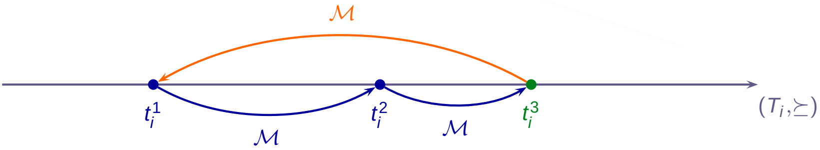
Sufficient Condition: (SP-NRM)

- $(T_i, \succeq)$ is linearly ordered.
- $v_i(.|t_i)$ is Single-Peaked.
- It satisfies the No Reciprocal Masquerade ($\xrightarrow{\mathcal{M}}$ has no 2-cycles):

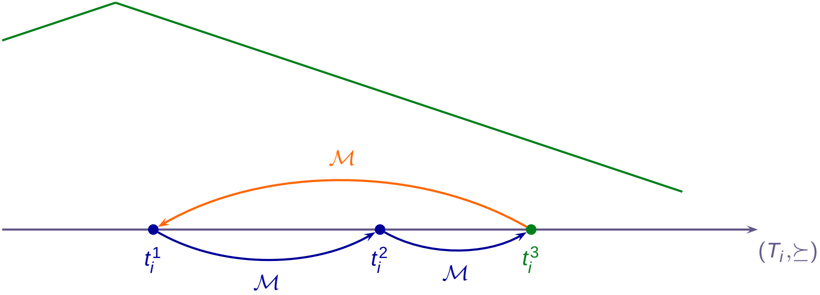
$$v_i(s_i|t_i) > v_i(t_i|t_i) \;\Rightarrow\; v_i(s_i|s_i) \geq v_i(t_i|s_i)$$

- Remark: This idea underlies the results of Giovanonni and Seidmann (2004) on sender-receiver models, and also Seidmann and Winter (1997).

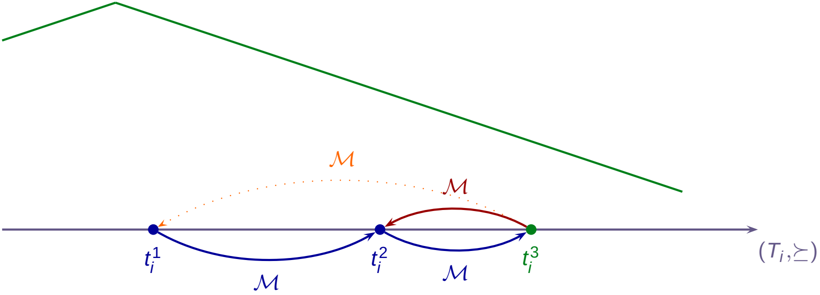
Illustration



Illustration



Illustration



Sufficient Condition: (ID)

- $(T_i, \succeq)$ is linearly ordered.
- $v_i(s_i|t_i)$ has Increasing Differences in (s_i, t_i):

$$s_i' \succeq s_i \quad \text{and} \quad t_i' \succeq t_i$$
$$\Downarrow$$
$$v_i(s_i'|t_i') - v_i(s_i|t_i') \geq v_i(s_i'|t_i) - v_i(s_i|t_i)$$

- Intuition: the return of masquerading as a higher type increases with one's true type.

Sufficient Condition: (SCD)

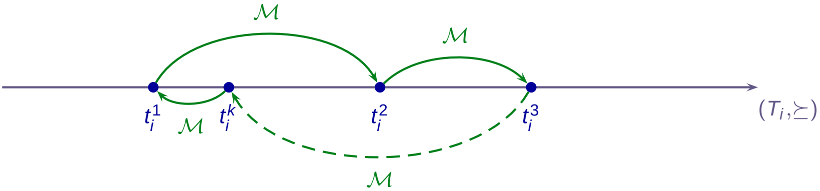
- $(T_i, \succeq)$ is linearly ordered.

- $v_i(s_i|t_i)$ has Single Crossing Differences in (s_i, t_i):

$$s_i' \succeq s_i \quad \text{and} \quad t_i' \succeq t_i$$
$$\Downarrow$$
$$v_i(s_i'|t_i) \geq (>) v_i(s_i|t_i) \Rightarrow v_i(s_i'|t_i') \geq (>) v_i(s_i|t_i')$$

- Intuition: if the return of masquerading as a higher type is positive for t_i, then it is positive for $t_i' \succeq t_i$.

Sketch of the Proof (for ID):

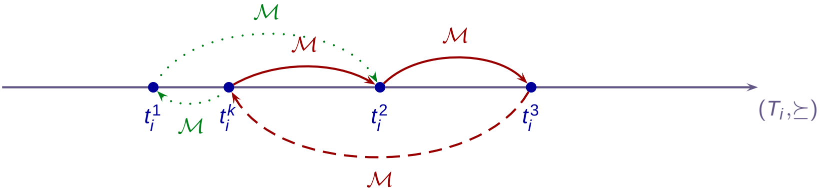


$$
\begin{aligned}
v_i(t_i^2|t_i^k) - v_i(t_i^k|t_i^k) &= v_i(t_i^2|t_i^k) - v_i(t_i^1|t_i^k) + v_i(t_i^1|t_i^k) - v_i(t_i^k|t_i^k) \\
&\geq \underbrace{v_i(t_i^2|t_i^1) - v_i(t_i^1|t_i^1)}_{>0} + \underbrace{v_i(t_i^1|t_i^k) - v_i(t_i^k|t_i^k)}_{>0} \\
&> 0
\end{aligned}
$$

Hence:

$$t_i^k \xrightarrow{\mathcal{M}} t_i^2.$$

Sketch of the Proof (for ID):



$$\begin{aligned}
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&> 0
\end{aligned}$$

Hence:

$$t_i^k \xrightarrow{\mathcal{M}} t_i^2.$$

Ex post sufficient conditions

- $v_i(s_i|t_i) = \sum_{t_{-i} \in T_{-i}} v_i(s_i|t_i; t_{-i}) p(t_{-i}|t_i)$.
- We want joint conditions on $v_i(s_i|t_i; t_{-i})$ and beliefs $p(t_{-i}|t_i)$.

Proposition

The Acyclic Masquerade Property is satisfied whenever for every i there exists a linear order $\succeq$ on T_i such that either of the following conditions is satisfied:

1. $v_i(s_i|t_i; t_{-i})$ *is nondecreasing in* s_i
2. $v_i(s_i|t_i; t_{-i}) > v_i(t_i|t_i; t_{-i}) \Rightarrow s_i \succ t_i$.
3. $v_i(s_i|t_i; t_{-i})$ *has ID in* (s_i, t_i)*; types are affiliated.*

Point 3 implies (SCD) for interim masquerading payoffs: a consequence of Quah and Strulovici (2012) who provide necessary and sufficient conditions to aggregate the SC property.

Application 1: Voting

Setup

- Two alternatives: a proposal and a status quo.
- The committee: $\mathcal{C} \subseteq N$ of size C.
- Voting Rule: proposal accepted if supported by at least q members of $\mathcal{C}$.
- Preferences: normalize the value of the status quo to 0 and let u_i be the uncertain gain derived from the proposal.
- Information: each agent has a private signal t_i about the proposal. T_i is linearly ordered.
- Assumption: $U_i(t) = E(u_i \mid t_1, \ldots, t_n)$ nondecreasing in t.
- Equilibrium: sincere equilibrium of the perfect information voting game: vote in favor of the proposal if $U_i(t) > 0$

Unanimity

Proposition (Unanimity)

Under unanimity, if all players are members of the committee the voting game satisfies the Acyclic Masquerade Property. If some players are not members of the committee and types are affiliated, then the AMP is also satisfied.

- Tally in favor of the proposal:

$$S_i(s_i; t_{-i}) = \sum_{j \in \mathcal{C} \setminus \{i\}} \mathbb{1}_{U_j(s_i; t_{-i}) > 0} \uparrow (s_i; t_{-i})$$

- Unanimity Rule: $q = C$.

- Ex post masquerading payoff if $i \in \mathcal{C}$:

$$v_i(s_i | t_i; t_{-i}) = U_i(t) \mathbb{1}_{U_i(t) > 0} \mathbb{1}_{S_i(s_i; t_{-i}) = C-1} \uparrow s_i$$

- Ex post masquerading payoff if $i \notin \mathcal{C}$: for $t'_i \succ t_i$

$$v_i(s_i | t'_i; t_{-i}) - v_i(s_i | t_i; t_{-i}) = \big(U_i(t'_i, t_{-i}) - U_i(t_i, t_{-i})\big) \mathbb{1}_{S_i(s_i; t_{-i}) = C} \uparrow s_i$$

Other Voting Rules

- Masquerading payoff if $i \notin \mathcal{C}$: for $t_i' \succ t_i$

$$v_i(s_i|t_i'; t_{-i}) - v_i(s_i|t_i; t_{-i}) = \big(U_i(t_i', t_{-i}) - U_i(t_i, t_{-i})\big)\mathbb{1}_{S_i(s_i;t_{-i})\geq q} \uparrow s_i$$

- Masquerading payoff if $i \in \mathcal{C}$:

$$v_i(s_i|t_i; t_{-i}) = U_i(t)\mathbb{1}_{U_i(t)>0}\mathbb{1}_{S_i(s_i;t_{-i})\geq q-1} + U_i(t)\mathbb{1}_{U_i(t)<0}\mathbb{1}_{S_i(s_i;t_{-i})\geq q}$$

- This function has ID in (s_i, t_i): $v_i(s_i'|t_i, t_{-i}) - v_i(s_i|t_i, t_{-i}) =$

$$\underbrace{U_i(t)\mathbb{1}_{U_i(t)>0}}_{\uparrow t_i}\underbrace{\left\{\mathbb{1}_{S_i(s_i';t_{-i})\geq q-1} - \mathbb{1}_{S_i(s_i;t_{-i})\geq q-1}\right\}}_{\geq 0}$$

$$+\underbrace{U_i(t)\mathbb{1}_{U_i(t)<0}}_{\uparrow t_i}\underbrace{\left\{\mathbb{1}_{S_i(s_i';t_{-i})\geq q} - \mathbb{1}_{S_i(s_i;t_{-i})\geq q}\right\}}_{\geq 0}$$

Other Voting Rules

Proposition

If types are affiliated, the voting game satisfies the Acyclic Masquerade Property for any voting rule.

Application 2: Supermodular Games

Setup

- We assume that the perfect information games are supermodular.
  - Each $(A_i, \succeq_i)$ is a lattice.
  - Own Action Complementarities: $u_i(a_i, a_{-i}|t_i; t_{-i})$ is supermodular in a_i.
  - Strategic Complementarities: $u_i(a_i, a_{-i}|t_i; t_{-i})$ has increasing differences in (a_i, a_{-i}).
  - Own Action/Type Complementarities: $u_i(a_i, a_{-i}|t_i; t_{-i})$ has increasing differences in (a_i, t)
- Then $NE(t)$ is a complete lattice and its extremal elements are nondecreasing in t.
- We focus on one of these extremal equilibria $a^*(.)$

Result

- Suppose: $A_i \subseteq \mathbb{R}^m$, $T_i \subseteq \mathbb{R}$.
- Assumption: smooth preferences and interior best response so that the best response always satisfies a FOC.

Proposition (Supermodular Games)

The supermodular game satisfies the Acyclic Masquerade Property whenever:

1. *Types are affiliated.*
2. $u_i(a_i, a_{-i}|t)$ *has increasing differences in* (a_{-i}, t_i).
   (Complementarities in own type and the actions of other players)

Proof

$$\frac{\partial v_i}{\partial s_i} = \sum_{j\neq i} \frac{\partial u_i}{\partial a_j}\left(BR_i(a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}), a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}\right) \frac{\partial a^*_j(s_i, t_{-i})}{\partial s_i}$$
$$+ \underbrace{\frac{\partial u_i}{\partial a_i}\left(BR_i(a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}), a^*_{-i}(s_i, t_{-i}), t\right)}_{=0 \; by \; (FOC)} \times (sthg)$$

$$\frac{\partial^2 v_i}{\partial s_i \partial t_i}$$
$$= \sum_{j\neq i} \underbrace{\frac{\partial^2 u_i}{\partial a_j \partial t_i}\left(BR_i(a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}), a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}\right)}_{\geq 0} \underbrace{\frac{\partial a^*_j(s_i, t_{-i})}{\partial s_i}}_{\geq 0}$$
$$+ \sum_{j\neq i} \frac{\partial}{\partial a_j}\Big\{\underbrace{\frac{\partial u_i}{\partial a_i}\left(BR_i(a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}), a^*_{-i}(s_i, t_{-i})|t_i, t_{-i}\right)}_{=0 \; by \; (FOC)}\Big\} \frac{\partial a^*_j(s_i, t_{-i})}{\partial s_i} \times (sthg)$$

Examples

An Influence Game (Galeotti, Ghiglino and Squintani, 2011):

$$u_i(a_i, a_{-i}|t) = -\sum_j \alpha_{ij}\big(a_j - \theta_i(t)\big)^2$$

with $\theta_i(.)\ \uparrow$.

A Coordination Game (Hagenbach and Koessler, 2011):

$$u_i(a_i, a_{-i}|t) = -\big(a_i - \theta_i(t)\big)^2 - \sum_{j \neq i} \alpha_{ij}\big(a_i - a_j\big)^2$$

with $\theta_i(.)\ \uparrow$.

Final Remarks

- A better understanding of communication with hard information.
- We focused on the existence of full disclosure equilibria.
- Some remaining questions:
  - When is every equilibrium fully revealing?
  - Refinements?
  - Partially revealing equilibria?
  - Mechanism design with evidence?

Thank you!