

# Hot and Spicy: Ups and Downs on the Price Floor and Ceiling at Japanese Supermarkets\*

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## Abstract

This paper develops a dynamic pricing model of a monopolistic retail store who sells a storable good to ex ante homogeneous customers. We show that frequent price changes between a few focal prices can be the profit maximizing price policy of the store. The key mechanism is that customers' willingness to pay depends on whether they buy the good for immediate consumption or for their inventory. Our empirical findings support the characteristics of this price policy: (1) stores tend to lower the price when the share of customers without inventory is lower, and when the shopping intensity is higher; (2) the demand exhibits negative dependence on price duration at lower sales prices, and positive dependence at a higher regular price. This price policy is consistent with the behavior of household inventories, accumulating during low price periods and decumulating during high price periods – the driving factor of short-run fluctuations in demand.

JEL Classification Numbers: D43, L11, L16, L81

Key Words: Retail price policy, Customer inventories, Sales, Intertemporal price discrimination

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# 1 Introduction

Figure 1 speaks better than any other form of introduction in clarifying the objective of the paper. The figure plots daily price movements of *House Vermont Curry*, a national brand of curry paste sold at a store belonging to one of the nationwide supermarket chains in Japan.<sup>1</sup> The figure traces the price from December 1990 till December 2005, during which this store was open for 5009 days and the price changed 1679 times. Remarkably, about 58% of the recorded prices are concentrated in only two specific levels in this brand, 253 and 198 yen. There are 120 other prices observed for this brand but these observations constitute only 2,088 days, or 41%, of the price observations. See the histogram in Figure 2 where we show the frequency distribution of posted price for House Vermont Curry at store #1.

Among 1652 completed spells, more than 86% of them are less than 6 days, with one day spell comprising 58% of these spells. Not surprisingly, frequent price changes between a few focal prices induce customers to wait for the price mark-down to concentrate their purchases: among the 20 stores which belong to one national chain of supermarket, the price was below 188 yen for 17,857 store-days, or 21% of 84,602 store days of the total observation, during which the stores sold 2,147,036 units, or, 48.1% of the total sales, 4,467,878 units.

What exactly is the underlying logic behind this type of pricing? Although it is clear that the customers do recognize such a pricing pattern and adjust their purchasing time to the lowest prices, it is not at all obvious why these stores employ such a pricing policy. The intertemporal pricing pattern like the one we saw in Figure 1, is commonly observed among retailers, although ours seems to exhibit exceptionally high frequency of price changes.<sup>2</sup>

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<sup>1</sup>Curry paste is sold generally in a package containing half-solid bars which can be cooked with vegetables and meats. Curry served with rice is extremely popular in Japan. The annual sales of the curry paste is roughly 80 billion yen in recent years, which translates into 1.5 billion curry rice dishes cooked and consumed in Japanese household in a typical year. Two brands in our sample are by far the most well known and longtime best sellers among the curry pastes. The market shares of these two brands are relatively small; although no official statistics exist, the share is likely to be less than 10% for both brands. The House with commanding 40 – 45% market shares of the curry pastes has 5 major brands and each brand comes in 3 to 4 different versions (typically distinguished by the spiciness). Our House sample is one version of the five major brands offered by House. Curry paste in Japan is similar to Cambell's soup cans in the United States: you are bound to find at least a few packages of curry pastes in any randomly picked Japanese house.

<sup>2</sup>See similar figures in Slades (1998), Aguirregabiria (1999), and Chevalier, Kashyap, and Rossi (2001). In Slades (1998), for example, the average duration of price is about 5 weeks, compared to 5 to 25 days in our data (see Table 3). In Aguirregabiria (1999), the distribution of average price duration (for 534 brands) ranges between 1 to 2.3 months (in his Table 2).

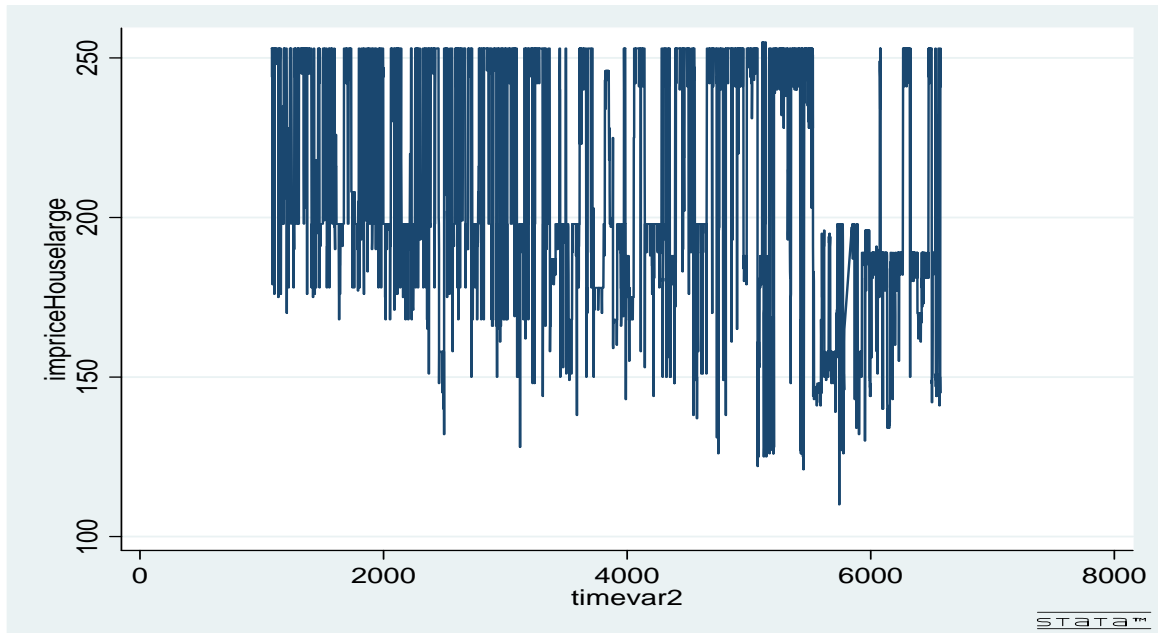


Figure 1: Daily price of curry past sold in Japanese supermarket

Figure 3 plots the average sales quantity at each posted price. At the most frequently posted regular price of 253 yen, the average sales quantity is about 10 units per day. At 198 yen, another focal price, the sales volume increases to 35 units, which translates into the price elasticity of 5. At around 150 yen, the average sales volume is roughly 100 units, 10 times of the average sales at regular price. At around 100 yen, the sales volume jumps to roughly 500 units, 50 times of the average at regular price. If we apply a static simple demand function estimate, our back of the envelope computation of price elasticity is confirmed: using the price data from 61 large scale retailers in Japan, we ran a fixed effect regression of the log of sales quantity on the log of price and the number of visitors. The estimated price elasticity turns out to be 4.57 and the estimated elasticity with respect to the log of visitors (those with purchase records) is around 0.73.

As noted in Hendel and Nevo (2006a,b), there exists strong reason to suspect that such an estimate of the (static) price elasticity of the demand is severely biased upward. This is the case primarily because the large impact of the price reduction on sales reflect a large demand stemming from the purchase for inventory, not for immediate consumption.

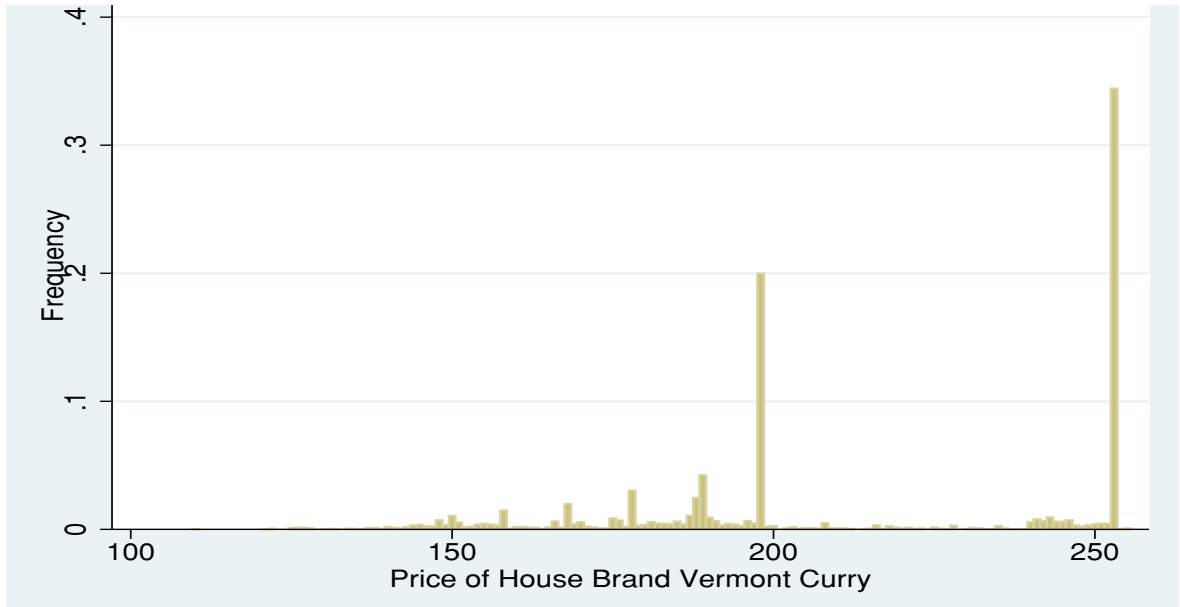


Figure 2: Histogram of House Vermont Curry Price at Store 1

This paper develops a model of sales in the spirit of Varian’s (1980) pioneering work. The key ingredient of our model is inventory holdings of customers. We consider a pricing problem of a monopolistic store who caters to ex ante homogeneous regular customers in a local retail market, and focus on particular brands of a storable good. We assume that the opportunity of consumption and visiting the store arrives randomly and independently to customers. This creates a possible difference in the timing between purchase and consumption, rendering customers with an incentive to hold an inventory at home – forward looking customers anticipate a future event in which they wish to consume but do not have the opportunity to visit the store. The customers’ reservation price therefore depends on whether they purchase the good for current consumption or for future consumption. Facing (ex post) heterogeneous buyers, even though their objective of buying is not observable, the store can use time as a discrimination device: for some periods the store sells at a high price only to those who happen to buy for immediate consumption, while for the other periods the store induces with a lower price the purchase of those who wish to buy for their inventory.

In our model, the intertemporal price patterns, like the one we observed in Figure 1, can emerge as an outcome of the store’s profit maximizing price policy. It takes the form of a price

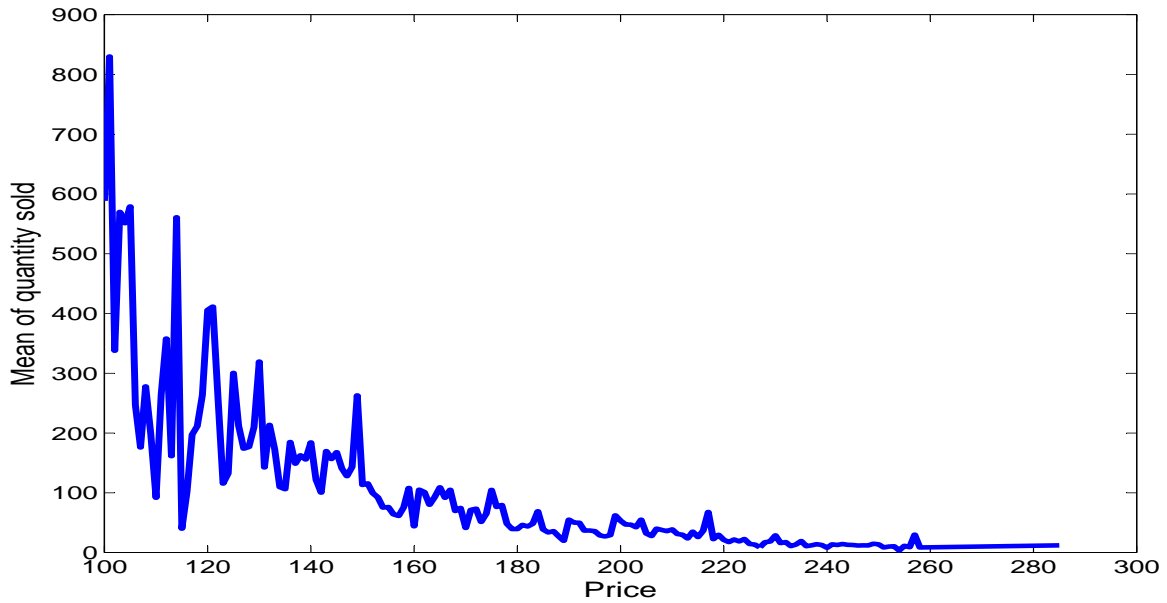


Figure 3: Price and Sales Quantity for House Brand at Store 1

cycle. During the period in which the store posts a high price, no one accumulates inventory and thus the aggregate household inventory decreases over time. For a sufficiently large pool of the customers without inventory, it becomes profitable for the store to capture them with a price markdown; with the price low enough to induce them to buy for their inventory. During the low price period, the aggregate inventory of customers increases. Eventually, the share of customers with un-replenished inventory declines enough that it becomes profitable to stop catering to this type of customers. Then, a high price period resumes.

Given the price cycle described above, customers time their inventory restocking to the occasional low price periods whenever possible. Along the path of the price cycle, one should observe a relatively long period of high regular price with a small quantity sold, punctuated by a large increase in the quantity for a short period of lower sales price. The underlying inventory behaviors of customers imply that the longer the time elapsed since the last sale, the larger (smaller) the size of the pent-up inventory demand waiting for a sale, when the current price is high (low). Our empirical result supports this aspect of the price policy: the demand exhibits positive dependence on price duration at the high regular price, and negative dependence at low sales price.

The pivotal role of customer inventory in the demand of frequently purchased, storable goods is emphasized by Pesendorfer (2002), Erdem, Imai and Keane (2003) and Hendel and Nevo (2006a,b). In their influential works, Hendel and Nevo (2006a,b) estimate the dynamic demand system and show that the customer purchase is accelerated and the amount is increased by a sale. Their estimates show that duration to the next purchase is longer following the sale, and that the effect of a price discount is larger the longer the duration of high price, confirming the customers' inventory holdings behaviors in anticipation of temporal price reductions. As in most of the existing models of sales, we assume unitary demand of customers, and concentrate our attention on the behavior of customer purchase.<sup>3</sup> Given this concentration, our model incorporates *both*, the inventory purchasing behaviors of forward looking customers in line with Hendel and Nevo (2006a,b), *and* price policies of retail stores that are taken as exogenously given in their study. We confirm the general view, commonly held in this line of works, that the customer inventory can also be the key ingredient to explain the retail stores' price/sales strategies.

A popular assumption made in the literature of sales is to divide customers exogenously into two groups: shoppers and non-shoppers. In Sobel (1984, 1991), shoppers have a lower rate of time discount than non-shoppers. Conlisk, Gerstener, and Sobel (1984) consider the entry of new customers in the context of a durable good monopolist. Our model shares a common feature with these models that a price reduction is related to accumulation of customers with a relatively low reservation price. However, their dynamics differs from ours in the following aspects: in their models price dynamics are driven by demand postponement generating a mass point at the highest price and a smooth distribution of lower prices; in our model price dynamics are driven by inventory demand of forward looking customers, generating a mass point at a few focal prices as we observe in the data. Further, as noted by Hendel and Nevo (2006a), the specification of ex ante heterogeneous time preference seems to

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<sup>3</sup>That we are primarily interested in the interaction of customer purchase and the Hi-Lo price patterns does not mean that we deny the importance of the influence of lower price on the consumption patterns, especially for its long run impact on the average price elasticity. On the other hand, the alternative explanation relying on the impact of price decline on consumption, is simply incredible to account for the observed magnitude of the response shown in the quantity purchased: If the price elasticity is 5, the static profit maximization implies the price cost margin to be around 20% of the price. But then, because almost all the low price is at least 22% or higher discount in comparison to the high price in our sample, their low prices would be all below marginal cost.

be less suitable to describe differential inventory demands of frequently purchased, storable goods. This argument makes sense particularly in our context where we discuss a highly frequent price markdown, which occurs within one week.

In an alternative setup in which non-shoppers do not hold inventory but shoppers may buy for inventory, Hong, McAfee and Nayyar (2002) provide a model of retailers' pricing with customer inventory. A simplification made in their model includes unitary demand and costly inventory holdings up to one unit, which we also employ in our model.<sup>4</sup> In their formulation, while the presence of non-shoppers, who are store-loyal, gives firms an incentive to set a higher price, the firms compete over shoppers who have a lower reservation price to buy for inventory, and sales may occur because of the competition. They show that there exists an equilibrium price dispersion. Hosken and Reiffen (2007) consider multiproduct retailers, who offer two different types of goods, storable and perishable, with the specification of shopper/non-shopper division. This additional ingredient captures the "loss-leader" phenomenon: shoppers are attracted to a retailer by the low price on one good, but end up with paying a high price for the other good given high transaction costs.

In contrast to the previous studies, the differential customer groups arises endogenously, rather than being assumed, in our model with ex ante homogeneous customers. To be perfectly clear, our innovation is to provide a new explanation for the motive of customers' inventory holdings – since consumption and purchase opportunities are not necessarily simultaneous, customers may hold inventory to prepare for future consumption, whenever a chance arises to do so and the price is right. None of the previous works have been explicit about this point. In our framework, such a customer purchase behavior renders the shop with an incentive to have occasional price discounts (i.e., sales). While most of our specifications are standard relative to the existing models of sales, we do not consider explicitly the competition with rival retail stores.<sup>5</sup> Instead, our model offers a novel insight into the role

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<sup>4</sup>In our context, the cost associated with inventory holdings comes from: limited storage space which is an acute concern for the Japanese households; the strong preference for freshness and concerns over the deterioration of quality for foods stored for long time. According to a PR officer of a major manufacturer of the curry paste, most of the curry pastes purchased will be consumed within two months from the purchase and those leftovers beyond two months are more likely to be disposed of, rather than consumed. He also noted, however, that the product stored in room temperature will last at least one year without much deterioration in quality.

<sup>5</sup>From the empirical point of view, we are motivated by our observation that the stores in our sample sell a large variety of grocery items and it is unlikely that customers time their shopping solely on the basis of the

of customer inventory in retail price/sales strategies: (i) the random shopping of customers creates a pent-up demand for inventory that is not disposed in a single sales period, which gives rise to a non-negligible amount of price observations below regular price, just like in Figure 1; (ii) the benefit of holding inventory to customers becomes smaller as they visit the store more often. Therefore, the store tends to lower the price to induce inventory holdings, when customers visits are more frequent. Further, the store can maximize profits by timing the low price period to a high shopping day, since it accelerates the sales to those who buy for inventories, enhancing the effectiveness of intertemporal price discrimination. This result is supported by our evidence and is consistent with Warren and Barsky (1995) who find that sales are more likely to occur on particular days, like weekends and holidays, with relatively high shopping intensity. These price behaviors would be difficult to explain with alternative theories of sales.

The sequel of the paper is organized as follows. Section 2 presents a model of intertemporal price discrimination. In Section 3, we use the scanner data and produce several key facts and empirical evidence which are directly relevant and supportive of the model. Section 4 concludes. All proofs are collected in the Appendix.

## 2 The Model

### 2.1 Basic Setup

Consider a local retail market where a monopolistic store caters to a unit mass of ex ante homogeneous customers. Time is discrete and lasts forever. Each period, the store posts a price of a storable, homogeneous good. Denote by  $\omega \geq 0$  the store's marginal production cost. For the moment, we assume zero cost of changing price. The opportunity of visiting the store arrives randomly to customers each period: a customer is a *buyer* with probability  $s$  in which case he can visit the store. Observing the price, buyers decide whether or not to purchase the good. The customers' preference for the good is random: each period, a customer is a *consumer* with probability  $c$  in which case he wishes to consume one unit of

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price of any particular product. Hence, customers in our sample are unlikely to make their choice of stores based on a price of any single grocery item.



the good. Those who consume obtain per-period utility  $u (> \omega)$ , while those who do not consume obtains zero utility. These probabilities are independent and constant over time.

The randomness described above creates a possible disparity between purchase and consumption, thus providing customers with an incentive to store the good at home. That is, far-sighted customers may wish to hold inventories of the good in preparation for a future event that he wishes to consume but does not have opportunity to visit the store. This implies, two types of buyers exist ex post in our economy, depending on the objective of purchase – those who buy for immediate consumption, referred to as *consumer-buyers*, and those who buy for future consumption, referred to as *inventory-buyers*. Since buyers can buy more than one unit, a buyer can be both types simultaneously, as will become clear shortly below. We assume that holding inventories is costly and that customers can store only up to one unit. Customers face a non-zero constant cost of inventory holding  $\varepsilon \in [\underline{\varepsilon}, u) > 0$  each period. To simplify the analysis, we assume, without loss of generality, that all the agents have zero time discount.

## 2.2 Static Price Policy

We start with a simple benchmark case wherein the store is exogenously constrained to offer time invariant price, i.e., static pricing policy. Since the store cannot recognize the type of individual buyers, the only choice is the constant level of the price for the good.

Suppose first that the store caters only to consumer-buyers who buy for immediate consumption. Then, the optimal price of the store is  $u$ . Under this price the demand is given by  $sc$  each period and the per-period profits, denoted by  $\Pi_H$ , are give by

$$\Pi_H = (u - \omega)sc. \tag{1}$$

Suppose next that the store posts a price low enough to induce inventory-buyers, who buy for inventory, to buy as well. Denote by  $p_L (< u)$  such a price and  $\pi \in [0, 1]$  the share of customers without inventory at home (yet to be determined). Under this price, the per-period demand consists of  $sc$  ( $= sc\pi + sc(1 - \pi)$ ) consumer-buyers and  $s\pi$  ( $= sc\pi + s(1 - c)\pi$ ) inventory-buyers, since the population  $sc\pi$  of buyers buy two units – one for immediate consumption and the other for inventory. Hence, if the store follows this price policy then its profits each

period, denoted by  $\Pi_L$ , are

$$\Pi_L = (p_L - \omega)s(c + \pi). \quad (2)$$

Clearly, the higher the price  $p_L$  or the larger the share of customers with no inventory  $\pi$ , the more profitable the static low-price policy is relative to the static high-price policy.

We now consider the determination of  $\pi$ . If the price is  $p_L$  all the periods, the flow into the total inventories of customers at home is  $s\pi$ , whereas the outflow is  $c(1-s)(1-\pi)$ . Hence, in the steady state where these two quantities are equal, it must hold that

$$\pi = \frac{c(1-s)}{c(1-s) + s}. \quad (3)$$

The price  $p_L$  is determined as follows. Denote by  $V^i$  the expected present value of net utility stream for a customer with  $i$  ( $= 0, 1$ ) unit of inventory. We leave the detailed expression of  $V^i$  to the next subsection. As the reservation price of inventory-buyers is given by  $V^1 - V^0$ , we have  $p_L = V^1 - V^0$  or

$$p_L = u - \frac{\varepsilon}{c(1-s)}, \quad (4)$$

where  $\varepsilon/c(1-s)$  stands for the effective cost of inventory because  $c(1-s)$  represents the average duration of inventory.

Comparison of the profits under two alternative prices, (1) and (2), using (3) and (4) shows that the low price is selected if and only if

$$\left(u - \omega - \frac{\varepsilon}{c(1-s)}\right)s(c + \pi) > (u - \omega)sc,$$

which can be written as

$$\frac{u - p_L}{u - \omega} = \frac{\varepsilon}{c(1-s)(u - \omega)} < \frac{1-s}{1+c(1-s)} = \frac{s\pi}{s(c + \pi)}.$$

**Lemma 1** *Suppose that the store is restricted to post a constant price over time. The profits of selling only to consumer-buyers,  $\Pi_H$ , and the profits of selling to both consumer-buyers and inventory-buyers,  $\Pi_L$ , satisfy  $\Pi_H \stackrel{\leq}{\geq} \Pi_L \iff$*

$$R \equiv \frac{u - p_L}{u - \omega} = \frac{\varepsilon}{c(1-s)(u - \omega)} \stackrel{\leq}{>} \frac{s\pi}{s(c + \pi)} \equiv R_S.$$

Notice that  $R$  is the surplus share of buyers at the low price, or the ratio of effective cost of inventory,  $\varepsilon/c(1-s)$ , relative to the gains from trade,  $u-w$ , and  $R_S$  the proportion of buyers with no inventory to the total demand at the low price. The condition given in Lemma 1 thus simply states that the static low (high) price is chosen if this ratio is smaller (larger) than the threshold,  $R_S$ .

### 2.3 Optimality of Hi-Lo Price Policy: Intuition

With static price policies, the store cannot effectively price discriminate buyers as the store does not observe the objective of buying by individuals. The only choice left for the store is whether or not to price the good low enough to induce buying for inventory. This strategy is chosen if the share of customers without inventory is large enough, or the reservation value of customers to add to inventory is large enough.

Once we allow the store to price discriminate by changing the price from time to time, the store can exploit the property that the lower (higher) price increases profit when the proportion of customers with no inventory at home  $\pi$  is high (low). In what follows, we demonstrate through several steps that a type of Hi-Lo pricing policy emerges as a profit maximizing strategy of intertemporal price discrimination. Notice that conventional devices for price discrimination, such as volume discount, is a sub-optimal policy because two types of buyers with different reservation prices may well purchase the identical unit if the price does not exceed their respective reservation prices.<sup>6</sup> To proceed, we first describe intuitively the idea.

Suppose the store employs a Hi-Lo pricing policy in the following manner: high price is posted for periods  $(0, t_H] \equiv T_H$ , and low price is posted for periods  $(t_H, t_H + t_L] \equiv T_L$ . During the high price period  $T_H$ , customers will not buy the good to store up. Thus, the share of customers without inventory  $\pi_t$  increases gradually as some of the household inventories will be depleted over time. The low price must be such that customers are marginally induced to buy the good to store. The key here is that such a price will not be constant if the

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<sup>6</sup>As virtually all the large scale retailers use scanner at cashers, the volume discount poses an additional problem: to implement volume discounts, multi-unit packages must be given a distinct code, which is also highly cumbersome if the store needs to change prices frequently.

store employs the Hi-Lo policy. Towards the end of the low price period  $T_L$ , forward looking customers are willing to pay more to store up than they are at the beginning of the  $T_L$ . This is because customers get strictly positive net utility from buying and consuming the good at the low price level. The fact that such a valuable opportunity will soon end acts as an inducement to buy during  $T_L$ . Conversely, the customer's willingness to pay for the inventory should be declining during high price period  $T_H$  in anticipation of the  $T_L$  ahead, although such a shadow price is immaterial during  $T_H$ . Hence, the monotone decreasing (increasing) reservation price of forward looking customers during high (low) price period constitutes part of equilibrium under the Hi-Lo price policy.

To summarize, a typical Hi-Lo price cycle starts (arbitrarily) with high price period  $T_H$  during which the inventory will be gradually depleted over time. The demand during the high price period is confined to consumer-buyers who happens to shop and get  $u$  from consumption on the day. At the start of  $T_L$ , the price is reduced to the level at which both consumer-buyers and inventory-buyers are willing to buy. Then, the price rises gradually during  $T_L$ , mirroring the rising reservation price to store up the inventory. The cycle ends with the sudden jump back to the high price. Then, the cycle is repeated.

## 2.4 Hi-Lo Price Policy: Formal Analysis

We now formulate the stationary Hi-Lo price-cycle described above. Denote by  $T \equiv (0, t_H + t_L]$  such a cycle period. Given  $T = T_H \cup T_L$ , we first establish a price cycle in which the store charges a high price  $p_t = u$  for  $t \in T_H$  and a low price  $p_t = W_t$  for  $t \in T_L$ , where  $W_t \equiv V_{t+1}^1 - V_{t+1}^0$  represents the reservation price of inventory buyers and  $V_t^i$  the asset value of a customer with  $i$  ( $= 0, 1$ ) unit of inventory at home in period  $t \in T$ . Given price  $p_t$  in period  $t \in T$ , buyers take a reservation strategy: consumer-buyers buy only if  $p_t \leq u$  and inventory-buyers buy only if  $p_t \leq W_t$ , where the reservation price of inventory buyers  $W_t$  is decreasing in  $t \in T_H$  and increasing in  $t \in T_L$ .

We start with the buyers' problem. For any  $p_t$ , the asset values of a customer with  $i$

(= 0, 1) unit of inventory at home in period  $t \in T$  are given by

$$\begin{aligned} V_t^1 &= -\varepsilon + c(1-s)(u + V_{t+1}^0) + cs \max(u - p_t + V_{t+1}^1, u + V_{t+1}^0) + (1-c)V_{t+1}^1 \\ V_t^0 &= s(1-c)(-p_t + V_{t+1}^1, V_{t+1}^0) + cs \{ \max(u - p_t, 0) + \max(-p_t + V_{t+1}^1, V_{t+1}^0) \} + (1-s)V_{t+1}^0. \end{aligned}$$

These equations are interpreted as follows. A customer with inventory already at hand has to pay  $\varepsilon$  per period for the inventory holdings. Consumption opportunity arises with probability  $c$ . If the customer turns out not to be a buyer, which happens with probability  $1-s$ , she consumes the inventory and the continuation value is given by  $V_{t+1}^0$ . If she happens to be a buyer, then she can either consume and buy another unit, in which case she gets  $u - p_t$ , or only consume by depleting the inventory, in which case she gets  $u$ . The continuation value is  $V_{t+1}^1$  ( $V_{t+1}^0$ ) in the former (latter) case. If no consumption opportunity arises, then the state does not change and the customer goes to the next period with the inventory. An inventory-buyer with no inventory and no consumption opportunity can buy one unit, in which case she pays  $p_t$  at that period and goes to the next period with inventory – see the first term in the second equation (for  $V_t^0$ ). If she chooses not to buy then her continuation value is  $V_{t+1}^0$ . If a consumption opportunity arises to the customer with no inventory, then she can consume the good only if she happens to be a buyer with probability  $s$ , upon which she gets  $u - p_t$ . At the same time, she can purchase one more unit for inventory, as represented by the second term. If there is no chance of visiting the store, then the customer goes to the next period without inventory. Along the Hi-Lo price path in which  $p_t = u$  for  $t \in T_H$  and  $p_t = W_t$  for  $t \in T_L$ , it holds that

$$V_t^1 = -\varepsilon + c(u - W_t) + V_{t+1}^1 \quad \text{for } t \in T \quad (5)$$

$$V_t^0 = \begin{cases} V_{t+1}^0 & \text{for } t \in T_H \\ cs(u - W_t) + V_{t+1}^0 & \text{for } t \in T_L. \end{cases} \quad (6)$$

The solution  $W_t(t_H, t_L) = V_{t+1}^1 - V_{t+1}^0$  to equations (5) and (6), i.e., the equilibrium reservation price of inventory buyers, can be obtained by using the terminal condition,  $W_{t_H+t_L} = W_0$ , where  $W_0$  represents the initial value at  $t = 0$ .

Given the optimal response of buyers, the share of customers without inventory  $\pi_t$  along

the Hi-Lo price path should satisfy

$$\pi_t = \begin{cases} \pi_{t-1} + c(1 - \pi_{t-1}) & \text{for } t \in T_H \\ (1 - s)\pi_{t-1} + c(1 - s)(1 - \pi_{t-1}) & \text{for } t \in T_L, \end{cases} \quad (7)$$

since for  $t \in T_H$ , no one accumulates inventory and those who hold inventory at home consume it with probability  $c$ . For  $t \in T_L$ , the inventory will be accumulated if and only if an opportunity of visiting the store arises, and the inventory at home will be depleted with probability  $c$ . Using the terminal condition  $\pi_{t_H+t_L} = \pi_0$ , one can obtain the solution to (5), which we denote by  $\pi_t = \pi_t(t_H, t_L)$ .

**Lemma 2** *Consider a price cycle which consists of high price period  $T_H \equiv (0, t_H]$ , where the store posts  $p_t = u$ , and low price period  $T_L \equiv (t_H, t_H + t_L]$ , where the store posts  $p_t = W_t(t_H, t_L)$  ( $< u$ ). Given the cycle period  $T = T_H \cup T_L$ ,*

1. *the reservation price to buy for inventory  $W_t(\cdot)$  is strictly decreasing in  $t \in T_H$  and strictly increasing in  $t \in T_L$ ;*
2. *the share of customers with no inventory  $\pi_t(\cdot)$  is strictly increasing in  $t \in T_H$  and strictly decreasing in  $t \in T_L$ .*

The lemma confirms the intuition provided before. For  $t \in T_H$ , the store posts the high price  $p_t = u$  and the reservation price of buying for inventory  $W_t$  decreases over time, reflecting the customers' expectation that the high price period will soon end and then there will emerge an opportunity to buy at a lower price. As consumer-buyers buy at this high price and no one accumulate inventory, the share of customers without inventory at home  $\pi_t$  increases over time during the high price period. For  $t \in T_L$ , the store posts the low price  $p_t = W_t(\cdot)$  and the reservation price increases over time. As both consumer-buyers and inventory buyers buy at this price, the share  $\pi$  decreases during the low price period.

We now consider the store's problem. Given  $W_t(\cdot)$  and  $\pi_t(\cdot)$  described above, we consider the store' problem selects  $t_H, t_L$  to maximize the average per-period profit per-cycle given by  $\Pi_C(t_H, t_L) =$

$$\left[ (u - \omega)sc \sum_{t \in T_H} \pi_{t-1}(t_H, t_L) + s \sum_{t \in T_L} (W_t(t_H, t_L) - \omega)(c + \pi_{t-1}(t_H, t_L)) \right] / (t_H + t_L),$$

which allows us to compare the profitability of the profit maximizing price cycle with the static price policies described before (see below). The demand during high price period is by those consumer-buyers who do not have inventory at home, and the demand during the low price period is by both consumer-buyers and inventory buyers. Notice that  $\pi_t$  represents the number realized at the end of period  $t$ , hence the store faces the  $\pi_{t-1}$  customers with no inventory at period  $t$ . In Appendix, we prove that any policy starting from arbitrary level of  $\pi_0$  indeed converges to the stationary policy analyzed here. As it turns out, with costless price changes, it is profitable to change price as frequently as possible, so as to make the total cycle-length as short as possible. This minimizes the deviation of  $\pi_t$  around its steady state value and maximizes the sustainable size of  $\pi_t$  consistent with the low price. Thus, the problem can be converted to the choice of the high price ratio  $\theta \equiv t_H/(t_H + t_L)$  to maximize an average profit denoted by  $\Pi_0(\theta)$  (see the proof of Proposition 1), which satisfies  $\Pi_0(1) = \Pi_H$  as given by (1) when posting the high price for ever, and  $\Pi_0(0) = \Pi_L$  as given by (2) when posting the low price for ever. In what follows, we shall adopt the following terminology.

**Definition 1** *We say that the price policy of the store with costless price changes is: a static high price policy, if  $\theta = 1$ ; a static low price policy, if  $\theta = 0$ ; a Hi-Lo policy, if  $\theta \in (0, 1)$ .*

We are now in a position to describe the profit maximizing price policy of the store by comparing the profitability of the above price policies. Notice that whenever a Hi-Lo price policy is profit maximizing, the price cycle established in Lemma 2 is an equilibrium cycle – given the reservation strategy of buyers, the store has no incentive to deviate from  $p_t = u$  for  $t \in T_H$  and  $p_t = W_t$  for  $t \in T_L$ , while given the price cycle the buyers’ response to the store’s price is optimal. The following proposition summarizes the main result.

**Proposition 1** *Suppose there is no cost of changing prices. There exists a subset of parameters  $(c, s) \in (0, 1)^2$  and critical values  $R_L, R_H \in (0, 1)$ , with  $R_L < R_S < R_H$ , such that the profit maximizing price policy of the store is: the static high price policy if and only if  $R \equiv \varepsilon/(u - \omega)c(1 - s) \geq R_H$ ; the static low price policy if and only if  $R \leq R_L$ ; the Hi-Lo price policy if and only if  $R \in (R_L, R_H)$ .*

A subset of parameters exists in which a Hi-Lo price policy is the profit maximizing price policy of the store. The store implements price discrimination, selling only to consumer-buyers who buy for immediate consumption at a high price, and to both consumer-buyers and inventory-buyers who buy to store for future consumption at a low price. In order for the store to employ this Hi-Lo price policy, the effective storage costs relative to the size of surplus,  $R$ , must lie within some intermediate range,  $(R_L, R_H)$ . If this ratio exceeds the upper bound  $R_H$  the store sells only to consumer-buyers at the high price, whereas if the ratio is below the lower bound  $R_L$  the store sells to both types of buyers simultaneously at a low price. The threshold for the static low and high price policies  $R_S$  introduced in Lemma 1 lies in between these lower and upper bounds.

**Corollary 1** *The profit maximizing Hi-Lo policy described in Proposition 1 satisfies that:*

1. *the low price  $p_t = W_t$  is greater than the price under the static low price policy  $p_L$ ;*
2. *the share of customers with no inventory at home  $\pi_t$  is greater than the stationary level under the static low price policy  $\pi$  for all the periods.*

Using the Hi-Lo price policy, the store can induce inventory-buyers to buy at a higher price than the reservation level under the static low price policy, precisely because the price discount is known to be temporary. As a result, the total level of customers' inventories given by  $1 - \pi_t$  is always lower than what is implied by the static low price policy.

## 2.5 Extensions

Hi-Lo price policy with menu cost: The profit maximizing Hi-Lo price policy with costless price changes, described in Proposition 1, accompanies infinite alternations of high and low price. We now introduce the cost of changing prices, i.e. menu cost, and consider the possibility of non-negligible cycle length to emerge as a profit maximizing Hi-Lo price policy.

To formulate the Hi-Lo price policy with costly price changes, suppose that the store changes price  $k \geq 1$  times during the low price period.  $k \in Z_{++}$  is a choice variable of



the store and we will describe below how it is determined. Each Hi-Lo cycle is represented by a cycle period  $T \equiv T_H \cup T_L$ , where low price period is now modified to  $T_L \equiv \bigcup_{j=1}^k T_L^j$  and  $T_L^j \equiv (t_H + \sum_{i=1}^{j-1} t_L^i, t_H + \sum_{i=1}^j t_L^i]$ ,  $j = 1, 2, \dots, k$ , represents the  $j$ -th low price period during which price is kept constant. The price to be posted is modified accordingly: For  $t \in T_H \equiv (0, t_H]$ , the store posts a constant price  $p_t = u$  as before; At the beginning of  $T_L$ , i.e. at  $t = t_H + 1$ , the price is reduced to  $p_t = W_{t_H+1}$ , and it lasts  $t_L^1$  periods; Then the price is changed to  $p_t = W_{t_H+t_L^1+1}$ , which lasts  $t_L^2$  periods, and so on, until the price is finally increased back to  $u$  after  $t_L \equiv \sum_{j=1}^k t_L^j$  periods. Notice that during the cycle the store changes price  $k + 1 \geq 2$  times and incurs  $(k + 1)\sigma$  of menu costs, where  $\sigma > 0$  is a fix cost per price change. Figure 4 illustrates the Hi-Lo price policy in the case  $k = 2$ , exactly as described above – the cycle starts (arbitrarily) with the high price  $p_t = u$  for  $T_H$  periods, after which the price is reduced to  $p_t = W_{t_H+1}$  at  $t = t_H + 1$ . This low price lasts  $T_L^1$  periods, and the price is increased to  $p_t = W_{t_H+t_L^1+1}$  ( $> W_{t_H+1}$ ) at  $t = t_H + t_L^1 + 1$ , reflecting the increasing reservation price  $W_t$  over time (as formally shown below). This price lasts  $T_L^2$  periods, and then the cycle ends.

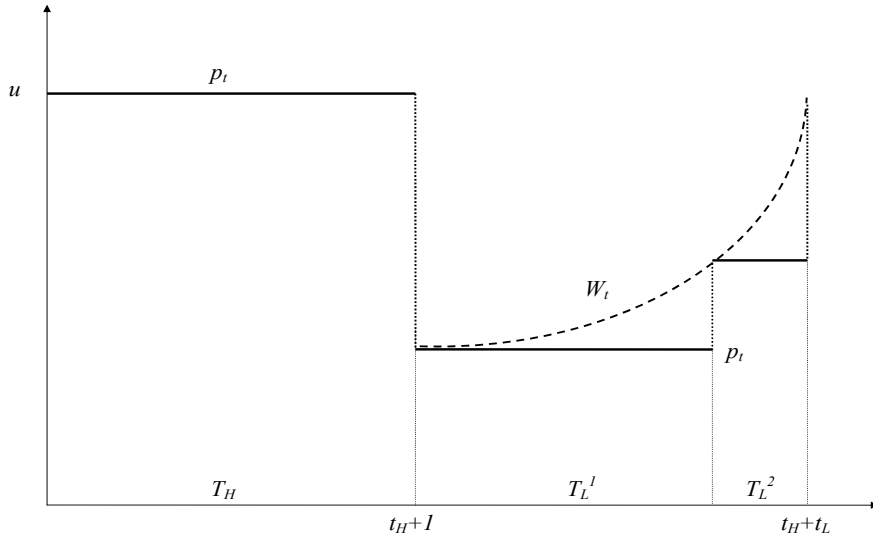


Figure 4: Hi-Lo price policy ( $k = 2$ )

The average per-period profit per cycle with menu cost, denoted by  $\Pi_M(t_H, t_L, k)$ , is given

by  $(t_H + t_L)\Pi_M(t_H, t_L, k) =$

$$-(k+1)\sigma + (u - \omega)sc \sum_{t \in t_H} \pi_{t-1}(t_H, t_L, k) + s \sum_{j=1}^k \sum_{t \in T_L^j} (p_t(t_H, t_L, k) - \omega)(c + \pi_{t-1}(t_H, t_L, k)),$$

where  $\pi_t(t_H, t_L, k)$  is the solution to (7), and

$$p_t(t_H, t_L, k) = W_{t_H + \sum_{i=1}^{j-1} t_L^i + 1}(t_H, t_L, k)$$

for  $t \in T_L^j$ ,  $j = 1, \dots, k$ , is the solution to (5) and (6), defined over the modified price cycle.

We relegate its lengthy expression to Appendix. The Hi-Lo price policy with costly price changes is then described by

$$\{t_H^*, t_L^1, \dots, t_L^k, k^*\} = \operatorname{argmax}_{\{t_H, t_L^1, \dots, t_L^k, k\}} \Pi_M(t_H, t_L, k)$$

and  $\Pi_M^* \equiv \Pi_M(t_H^*, t_L^*, k^*)$ . The definition of the profit maximizing price policy is modified as follows.

**Definition 2** *The profit maximizing price-policy of the store with costly price changes is: a static high price policy if  $\Pi_H > \max\{\Pi_M^*, \Pi_L\}$ ; a static low price policy if  $\Pi_L > \max\{\Pi_M^*, \Pi_H\}$ ; a Hi-Lo policy if  $\Pi_M^* \geq \max\{\Pi_H, \Pi_L\}$ .*

Unfortunately, it is practically impossible to derive the solution analytically. While we will verify under what parameter space it can be profit maximizing later using numerical approach, the Hi-Lo price policy has the same characteristics as before.

**Proposition 2** *Suppose that there exist positive costs of changing prices and consider a Hi-Lo price policy under which the store changes price  $k \geq 1$  times during the low price period  $T_L \equiv \bigcup_{j=1}^k T_L^j$ . Given the cycle period  $T = T_H \cup T_L$ ,*

1. *the reservation price to buy for inventory  $W_t(\cdot)$  is strictly decreasing in  $t \in T_H$  and strictly increasing in  $t \in T_L$ ;*
2. *the share of customers with no inventory  $\pi_t(\cdot)$  is strictly increasing in  $t \in T_H$  and strictly decreasing in  $t \in T_L$ .*

The intuition behind the above results remains the same as before and we will not repeat it here. It is worth noting that the solution, if it exists, should strike the balance between the menu cost and the loss from setting the length of the price cycle too long – the profit maximizing cycle length is zero if price change is costless. This implies, it should hold that for any  $k \geq 1$ ,

$$\Pi_0(\theta^*) > \Pi_M^* + \frac{(k+1)\sigma}{t_H^* + t_L^*},$$

where  $\theta^* \in (0, 1)$  represents the profit maximizing Hi-Lo policy without menu costs described in Proposition 1. Therefore, when the Hi-Lo price policy is profit maximizing with menu cost, the following must be true:

$$\Pi_0(\theta^*) > \max\{\Pi_H, \Pi_L\} + \frac{(k^*+1)\sigma}{t_H^* + t_L^*}.$$

This necessary condition is more stringent than the one without menu cost, because by construction the necessary (and sufficient) condition without menu cost is given by setting  $\sigma = 0$ , in which case we recover Proposition 1.

Numerical examples of optimal price policy with menu cost: The baseline parameters we select are:  $u = 10$ ,  $\omega = 5.11$ ,  $s = 0.35$ ,  $c = 0.19$ ,  $\varepsilon = 0.35$ , and  $\sigma = 0.05$ .

In the example benchmark case, we used House brand Vermont Curry sold at shop #1 as our benchmark. Its regular price is 253 yen, and the sales weighted average selling price is 174 yen, or 0.687 of the regular price. Assuming 30% gross margin and 5% marginal operating cost, we arrive at 0.511 as the marginal cost of the good. Based on the mean average frequency of shopping (data below), we use 0.35 as the daily probability of visiting store, which translates into twice a week visit to the store. Then  $c$  is chosen so that the duration of price conforms closely to the one we observe, roughly 2.5 days. Finally, setting  $\varepsilon = 0.35$  translates into 2.845 as the effective cost of inventory, hence, consumers are induced to buy the good to store only if they are given at least 28% discount under the static low pricing policy. This amount of discount is close to the ratio of the focal price, 198 yen, to the regular price, 253 yen. At the benchmark value of  $\sigma = 0.05$ , or, a half percentage of the regular price, we obtain the benchmark results with only three prices appear in each price

cycle: i.e., the numerical solution exhibits clear pattern of a few focal prices. Under this set of parameter values, the profit maximizing price policy is a price cycle of 6.76 days (ignoring the integer problem), roughly one week, in which the high price is set for the first 0.67 of the cycle, or, 4.53 days, then the price is cut to 7.3 from the high price, 10. This first low price period lasts 1.15 days, then the price is increased slightly to 7.47, which last for 1.08 days, and the cycle ends by returning to the high price level, i.e.,  $k^* = 2$ . The average discount during the low price period is 26%, i.e., the average price during the low price period is about 7.4. The maximand per unit of time in this case is 1.03977. Compared to this value, the maximand is 0.33%, 0.52%, 1.23%, 2.07%, and 10.64% lower if  $k = 1, 3, 4, 5$  and 0 (static pricing policy), respectively. On the other hand, the maximand is 8.52% smaller than the maximum value which we would obtain if the menu cost,  $\sigma$ , is zero.

Table 1 summarizes the comparative statics results. The impacts of changes in  $s$  are shown in the second row of the Table 1 and they are quite intuitive. Both cycle length and average price duration are increasing in  $s$ . As visiting the store becomes more frequent, the benefit of holding inventory declines. Hence, to induce customers to store inventory, the shop needs to discount more during the low price period and to shorten the low price period relative to the total cycle length. Eventually, beyond the threshold value of  $s$ , it becomes profitable to sell always at the high price. Conversely, with  $s$  below the threshold, it is profitable to set the price always at the low threshold level.

	$T$	$T/(k+1)$	$TH/T$	$M$	$E(p)$	$E(pL)$
	cycle length	price duration	share of high price periods	maximand	mean price (time average)	mean price during sales
$s$	2.59	3.87	1.05	0.65	0.18	0.09
$c$	-0.6	-0.8	-0.36	1.29	-0.22	0.08
$\omega$	3.16	4.9	1.49	-1.44	0.24	0.1
$\varepsilon$	4.59	6.31	1.32	-0.45	0.18	-0.16
$\sigma$	1.13	1.39	0.12	-0.05	0.04	-0.04

Notes: The figure in each cell shows the average % change of an endogenous variable (column) when one of the parameters (row) is increased by 1%, with the rest of the parameters set at the benchmark values. In computing the figures, only the results from interior equilibria are used (we exclude equilibria wherein static price policies are optimal).

Table 1. Effects of parameter change

Similar logic applies to the impact of a change in  $c$ . A higher  $c$  implies a greater likelihood of stock-out and thus a higher reservation price of inventory. Hence, as  $c$  increases, low

price period increases but the average discount during the low price period becomes smaller. Eventually, beyond the threshold value, the profit maximizing policy calls for setting low price for ever.

An increase in the marginal cost  $\omega$  obviously raises the average price not only by shortening the relative share of the low price period but also by raising the average price during the period. The impact of  $\varepsilon$  is quantitatively large: the impact is through its direct effect on the cost of holding inventory at household. As a result, the store must set lower price during the low price period, whereas the consequent decline in the profit from low price sales reduces the share of the low price period. The impacts on the price level and discounts are relatively small and it is unclear if the results shown in the Table 1 are robust.

In Table 2 we show the impact of a change in the menu cost on the choice of  $k$ . To begin with, the profit maximizing policy calls for longer cycle length and larger value of  $k^*$  as we decrease  $\sigma$  (bottom row). Starting with the benchmark value of 0.05, the profit maximizing policy changes from  $k^* = 2$  to  $k^* = 1$  at  $\sigma = 0.088$  or larger. Reducing  $\sigma$  further, it changes to  $k^* = 3$  at or around  $\sigma = 0.0073$ , and then to  $k^* = 4$  at  $\sigma = 0.00063$ . Correspondingly, the cycle length (not shown in Table 3) gets shorter and shorter as we decrease  $\sigma$ . For example, at  $\sigma = 0.24$ , which turns out to be the maximum value in our numerical example, the cycle length is 16.6 days, whereas it is less than 2.5 days when  $k^*$  first becomes 3. Notice  $\sigma = 0.05$  corresponds to 0.5% of the high price. The simulation results suggest that thrice or more frequent price changes during the low price period can be found only in extreme configuration of parameter values and policies, e.g., extremely small menu costs (less than 0.1% of the price tag) and extremely short cycle length (less than 3 days). Aside from the changes in menu cost, we did not find any parameter configurations in which the profit maximizing policy calls for more than three ( $k^* > 3$ ) price changes. Frequent price changes during a cycle is typically dominated by either a static low or static high price policy.

Hi-Lo price cycle and high shopping intensity: In the base model, we assumed that the flow of buyers is constant. We now examine the effect of deterministic variations of  $s$  that occurs within each Hi-Lo price cycle. Consider a sequence of shopping intensity  $\{s_t\}$  defined over

	parameter range	static low	$k=2$	$k=1$	$k>2$	static high
$s$	[.122 .999]	<.296	[.296 .423]	[.423 .521]	n.a.	>.521
$c$	[.036 .999]	<.141	[.166 .237]	[.141 .166]	n.a.	>.237
$\omega$	[2.26 9.65]	<3.74	[3.74 5.63]	[5.63 6.11]	n.a.	>5.63
$\varepsilon$	[.122 .999]	<.284	[.284 .396]	[.396 .460]	n.a.	>.460
	parameter range	static policy	$k=1$	$k=2$	$k=3$	$k \geq 4^\dagger$
$\sigma$	[.00013	high price if $\sigma > .24$	[.088 .24]	[.0073 .088]	[.00063 .0073	<.00063

$\dagger$  We programmed simulation model up to  $k=5$ . Obviously, as we reduce  $\sigma$  further, the optimal number of price changes increases indefinitely. We find that  $k=5$  overtakes  $k=4$  at .00013.

Table 2. Optimal number of price changes during a cycle

each cycle  $T$  that takes the following form:

$$\{s_t\} = \begin{cases} s & \text{if } t \neq t_S \\ (1 + \gamma)s & \text{if } t = t_S \end{cases}$$

with  $\gamma > 0$ . The high shopping date  $t_S \in T$  is known to all the agents. We are interested in the timing that the store wishes to have this high shopping date within the cycle. Denote by  $\Theta_O \equiv \{\omega, \varepsilon, u, c, s, \sigma\}$  a configuration of parameters with constant  $s$ , and by  $\Theta_S \equiv \{\omega, \varepsilon, u, c, \{s_t\}, \sigma\}$  the one with the time variation. With a slight abuse of notation, let  $\Pi_M(t_H, t_L, k \mid \Theta_i)$  be the average per-period profit per cycle for a parameter configuration  $\Theta_i$ ,  $i = O, S$ . We then define a temporal increase in profits,

$$\Delta(t_S) \equiv \Pi_M(t_H^*, t_L^*, k^* \mid \Theta_S) - \Pi_M(t_H^*, t_L^*, k^* \mid \Theta_O),$$

taking as given the profit maximizing policy  $t_H^*, t_L^*, k^*$  under  $\Theta_O$ . The profit increase  $\Delta(t_S)$  depends on the timing of the shock within the cycle  $t_S \in T$ , and is given by

$$\Delta(t_S) = \begin{cases} \gamma(u - \omega)sc\pi_{t_S-1} & \text{if } t_S \in T_H \\ \gamma(W_{t_S} - \omega)s(c + \pi_{t_S-1}) & \text{if } t_S \in T_L. \end{cases}$$

Given  $t_H^*, t_L^* > 0$ , Proposition 2 implies that  $\pi_{t_S-1}$  is strictly increasing in  $t_S \in T_H$  and is strictly decreasing in  $t_S \in T_L$ . This property leads to the followings.

**Proposition 3** *Taking the profit maximizing cycle  $t_H^*, t_L^* > 0$  as given, consider a one-time increase in shopping intensity  $s$  at  $t_S \in T$ . Then, the resulting increase in profits  $\Delta(t_S)$  is maximized when  $t_S \in T_L$ .*

Timing the low price period to the high shopping day helps contribute to increase the share of purchase by inventory-buyers for  $t \in T_L$ , during which there are a relatively large fraction of customers with no inventory  $\pi_t$ . Then, the store can accelerate the sales to the inventory-buyers, thus enhancing the effectiveness of the dynamic price discrimination. Setting price high in the high shopping day is less profitable because it reduces the total amount of sales.

## 2.6 Testable Implications

We now summarize testable implications of the model presented above.

- *Implication 1.* The store changes the price frequently between a few focal prices.
- *Implication 2.* The quantity sold depends positively on the past prices.
- *Implication 3.* The quantity sold depends negatively on price duration at lower sales prices, and positively at a higher regular price.
- *Implication 4.* The quantity sold depends positively (negatively) on the duration of the previous price, if the previous price was at the high regular (low sales) price.
- *Implication 5.* The sales promotion and price reduction tend to be concentrated in heavy shopping days.

Whenever the Hi-Lo price policy is profit maximizing, one should observe a few focal points between which the store alternates its price, hence Implication 1. To the best of our knowledge, establishing the existence of a few focal prices within a single firm for homogeneous good is new relative to the literature.<sup>7</sup> The next three implications are related to the dependence of quantity sold on the customers' inventory accumulation and decumulation, which are consistent with Hendel and Nevo (2006a,b). Given the price cycle, because inventory buyers buy only during low price periods, the share of customers without inventory at home

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<sup>7</sup>In Salop and Stiglitz (1982), there exist two focal prices in market equilibrium where some stores charge a high price and the others a low price – not the ones within single firm. Lal (1990), Agrawal (1996), and Lal and Villas-Boas (1998) establish a similar outcome in the context of national/private brands firms.

increases over time during high price period and decreases over time during low price period. This behavior of customer inventories implies that current demand tends to be higher (lower) when the store has posted a higher (lower) price in the previous periods, leading to Implication 2. It also implies that the longer the time elapsed since the last sale, the larger (smaller) the size of the pent-up inventory demand waiting for a sale, when the current price is high (low). Thus, Implication 3 follows. The latter implication can be further extended to the effects of the previous price duration on current demand, as stated in Implication 4. Finally, the expected benefit of customers to hold inventory declines if they have more opportunities of visiting the store, thereby the store has to lower the price to induce inventory holdings. As we have seen, the store can maximize profits by shortening the sales periods in response to larger customers visits, or by timing its low price period to the period of high shopping intensity. One implication of this result is that if the store is able to pinpoint the sales promotion and advertising activities to increase the shopping intensity of a particular day, then the number of buyers during the low price periods can be increased by such an activity, and the store can time its sale to that particular day (Implication 5). This implication is new relative to the literature, and would be difficult to explain with alternative theories of sales. As we will see in the following section, all the above implications are supported by the empirical evidence in our sample.

### 3 Empirical Evidence

#### 3.1 Data

The scanner data covers millions of products, ranging from fresh vegetables, fish and meats, to processed foods, utensils, and toiletries. Our choice is two popular brands of curry pastes, commonly found in virtually all the supermarkets in Japan.<sup>8</sup> Our sample consists of daily

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<sup>8</sup>Our choice is somewhat arbitrary but not without reasons. First of all, average life span of consumer products is extremely short: in some categories such as snack foods, more than 80% of newly introduced products disappear within a year. Fresh foods are unsuitable for our purpose. Importantly, in order for the sales discount to be effective, the product needs to be well known, high quality national brand item. Otherwise, lower prices may well be mistaken for lower quality. We should also avoid products which average consumers purchase only occasionally, or long lasting such as consumer durables. Given all these considerations, our choice fits well to the requirements. The curry paste is an item that almost every households buy as the rice covered by the curry source is one of the most long lasting popular meals in Japan. The selected two brands



observations on transacted prices and quantities sold at 60 different, large scale retail stores out of which all but one belong to one of the six national and regional supermarket chains in Japan. The maximum sample coverage is from December 1st 1988 to December 31, 2005. During the entire period, retail prices in Japan remained stable and there was virtually no discernible effect of ongoing or future inflation.

### 3.2 Price Changes

Table 3 provides several key indicators which we use to characterize the pricing policy of the stores, confirming Implication 1 of our model. The sample data exhibit extremely high frequency of price changes. The durations of prices are shorter for lower prices, characteristics shared with other scanner data. The price durations in our data are far shorter, however, compared to similar data sets. In our sample, average price durations are less than 10 days in the first chains and none of the sample stores have durations longer than 60 days even at the regular high price level. In Slade (1998, 1999), she reports that 80% of weekly price observations are zero price changes, which implies unconditional mean duration of price is larger than one month. In Pesendorfer (2002), who uses similar scanner sales data on two brands of ketchup sold at US supermarket stores, the number of price changes ranges 11 to 34 in the data spanning more than 3 years. Nakamura (2008) and Nakamura and Steinsson (2008) analyze original micro data used for the compilation of US CPI series and find that the median probability of monthly price change is 0.439, which implies the average price duration of 36 days if we assume that the observed frequency reflects a constant daily probability of price change. Even allowing the under-estimation of the probabilities due to monthly observations, it seems clear that the price changes are far more frequent in our data than in the comparable US data at retail levels.<sup>9</sup>

Although the visual inspection of the price data such as Figure 1 is often sufficient to determine the regular price, it is not always the case. We define the *regular price* as the highest price during a given length of the past price observations. We used 30, 60, 90, and

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are by far the oldest and most well known.

<sup>9</sup>A non-exhaustive list of related recent reference includes Berck et al. (2008), Bills and Klenow (2004), Dhyne et al. (2006), Hosken and Reiffen (2004), and Klenow and Kryvtsov(2008).

180 days as the candidate lengths. Since the qualitative features of the results do not depend on the choice, we use 90 days as our choice.

Chain ##	Number of Stores	Price (yen)			Price observation shares (%)			Average price duration (days)			
		Mean	Min.	Max.	Regular	Low	Sales<.75*regular	All	Regular	Low	Sales
1	20	191	97	272	37.6	62.4	19.4	6.91	8.05	5.91	4.5
2	8	182	76	268	57.4	42.6	5.3	11.29	13.34	8.13	7.69
3	9	195	128	268	45.3	54.7	20.3	8.53	11.29	4.9	4.49
4	7	196	76	268	40.3	59.7	7.3	15.27	15.53	15.08	12.47
5	5	182	100	278	57.1	42.9	7.5	13.87	15.39	11.43	5.2
6	9	194	76	278	52.4	47.6	14.7	13.12	17.19	6.64	5.39
Others	1	195	128	278	54.9	45.1	8.7	11.79	12.53	10.65	5.68

Table 3 (a). Summary statistics of price data: *House* Brand

Chain ##	Number of Stores	Price (yen)			Price observation shares (%)			Average price duration (days)			
		Mean	Min.	Max.	Regular	Low	Sales<.75*regular	All	Regular	Low	Sales
1	20	191	47	272	43.1	56.9	35.8	10.1	11.52	7.2	7.69
2	8	182	76	268	53.2	46.8	17.2	11.76	14.06	7.61	7.02
3	9	195	128	268	41.5	58.5	48.9	8.98	9.69	5.85	4.78
4	7	197	76	268	37.4	62.6	41.9	15.86	16.46	14.79	16.6
5	5	182	100	278	42.4	57.6	42.8	13.84	13.76	14.04	12.07
6	9	194	76	268	47.3	52.7	39.4	22.2	23.47	17.67	11.42
Others	1	195	128	278	33.7	66.3	50.6	14.1	14.69	12.82	11.95

Table 3 (b). Summary statistics of price data: *S&B* Brand

Conditioned by		S&B Brand		Conditioned by		House Brand	
		Regular	Low			Regular	Low
House Brand	Regular	0.480	0.520	S&B Brand	Regular	0.495	0.505
	Low	0.420	0.580		Low	0.434	0.566

Table 3 (c). Within Store Price Correlations between Two Brands

Patterns of price changes: We investigated if there exists any simple time pattern of price changes in the data. We thoroughly checked the relative frequency of price changes in both directions by the day of a week, day of a month, by month, on holidays, all of them for each store. We found a pair of statistically significant regularities. Among the seven stores that belong to a same national chain of supermarket (chain 1), we found that the prices of both brands are reduced on the day 20th of a month, only to be increased again on 21st. As it turns out, the day 20th of each month is a regular monthly store sales day, common to all the stores of this chain. Except for this pattern, we failed to detect any simple regularity in price changes, although we do find mild seasonality in the overall frequency of price changes

over a year.<sup>10</sup>

Although we found no simple rule for price changes, they are far from random pricing either. Using conventional probit model and survival analysis (results not shown), we find robust and strong negative duration dependence of price changes, even after controlling for a variety of seasonality. These findings are consistent with our model specification in several dimensions. First of all, they suggest that the retailers set these prices under their initiative and with information they have. Table 3(c) displays correlations between the prices of the two rival brands within store. Chi square tests statistics indicate that the prices of the two brands are significantly correlated at 0.1% confidence level: when *House* brand is priced below (at) regular price, the probability is significantly higher that the rival brand, *S&B*, is also priced below (at) regular price. The converse is also true.<sup>11</sup> Given the strong positive correlation of the prices of two competing brands *within* each store, it is unlikely that upstream suppliers coincidentally adjust respective prices. Rather, prices of the two brands reflect common factors relevant to stores' price policy. The time pattern we found on the day 20th is a prime example. The store sales day of a month is typically accompanied by sales promotion and advertising activities, and increases of the size of the customers visiting the store. This evidence is supportive of Implication 5. Finally, the extremely high frequency of price changes suggest that the wholesale price is unlikely to be the driving force behind these pricing policies.

Focal prices: Table 4 shows the price observations of 30 stores for the House Vermont Curry Brand. We show top 3 price observations, and their summed shares in total observations. The top 3 share ranges 36% to 81% of total observations, and it typically exceeds 50%. Majority of stores have more than 100 different prices in the record. The unconditional probability of price change is around 14% in each direction, thus the probability of a price change in either

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<sup>10</sup>For example, take weekly fluctuations in the relative frequency of price changes: for House brand, Tuesdays have the highest frequency in both directions (11.2% and 9.7%, respectively), and at 6.3%, Friday is the lowest for the price increase, whereas Thursday figure is the lowest at 6.8% for the decrease. S&B brand exhibits similar pattern: Tuesdays register the highest frequency in both directions (8.4% and 6.2%), with Fridays being lowest for the increase (4.4%) and Thursdays lowest (4.6%) for the decrease.

<sup>11</sup>If we compute within store correlation of two prices using regular or low price dummies, they are positively correlated with significance level at 1% for 38 stores, negatively significant at 8 stores, and not significant at 14 stores. Among the 8 stores with negatively significant stores, 5 of them belong to the same supermarket chain. Across stores, price correlations are virtually absent when we compare stores belonging to different chains. On the other hand, in some chains (most notably among stores in chain #2), pricing policy are highly correlated and in most of pair wise correlations, Spearman rank test easily reject the null that they are independent.

direction is 0.28. Putting these observations together, it is clear that prices are changed highly frequently, whereas most of the observations are concentrated in a few focal prices. This is the Implication 1 of our model.

shopid	Top share		Second share		Third share		topr 3 share	total number of distinct prices
	price	share	price	share	price	share		
1	253	34.32	198	19.97	189	4.23	58.52	87
2	198	19.87	253	18.61	188	7.39	45.87	131
3	252	17.96	198	12.96	188	12.36	43.28	115
4	228	30.61	253	20.96	198	10.7	62.26	81
5	198	22.18	253	15.64	178	6.49	44.31	127
6	198	28.45	253	17.33	228	6.27	52.05	120
7	253	22.34	198	18.56	188	8.43	49.33	125
8	253	22.37	198	16.7	188	8.66	47.73	125
9	198	27.3	253	22.18	188	10.43	59.91	121
10	253	22.67	228	16.82	198	10.66	50.15	29
11	253	15.53	198	10.7	188	10.06	36.29	126
12	198	24.6	253	22.87	188	4.69	52.16	123
13	253	24.37	198	22.19	228	4.61	51.16	126
16	253	29.69	198	18.49	228	6.67	54.85	128
17	253	30.32	198	21.56	228	6.09	57.97	114
18	198	36.14	253	17.78	218	7.02	60.94	112
20	198	48.21	188	11.4	253	5.77	65.38	114
31	198	41.99	258	13.44	218	8.57	64	89
33	198	47.44	218	9.72	228	6.42	63.58	90
34	198	59.4	158	7.87	218	6.06	73.33	75
35	198	50.88	218	8.57	178	6.9	66.35	57
36	198	54.14	218	6.26	188	6.07	66.47	80
37	198	31.63	188	26.61	216	10.21	68.45	71
39	198	40.68	188	29.65	158	5.35	75.68	54
40	198	66.88	158	8.47	188	5.76	81.11	55
42	253	38.4	198	15.62	158	4.04	58.06	105
43	253	50.19	198	9.25	188	3.76	63.19	106
44	253	40.33	198	11.51	208	5.65	57.49	106
45	253	30.96	198	17.27	188	3.37	51.6	113
46	253	26.64	198	12.6	188	4.22	43.46	116

Table 4. Frequency Distribution of Posted Price

Shopping frequency: We use the accompanying data on the customers of one of the national supermarket chains, which is chain 1 in Table 3 above. The data includes information on individual customers' records of the visits and purchase at one of the stores of this chain (store #2 and #8). It covers from January 1, 1998 until December 31, 2001, and in total 3,144 individual customers.

The intervals between the visits for individual customers have simple mean 4.49 days, median 2 days, and mode 1 day. The mean is rather misleading: among 0.8 million spells in the data, only about 1.1% of them are more than 30 days. If we limit our sample to durations less than or equal to 30 days, the mean duration is 3.63 days, while both the median and the mode remain the same as in the full sample. Figure 5 shows the frequency distribution of completed spells, i.e., the number of days between their store visits. Roughly speaking, the average customer visits the store 2 to 3 times a week in our sample. The average amount

of purchase is consistent with this customers shopping pattern: the median of the average purchase per customer visit is 1,433 yen (roughly 15 US dollars). Since the average food expense per household is 73,000 yen per month, according to household survey 2010, the average purchase per visit counts only 2% of the monthly food expenditure.

As expected, regular customers to the supermarkets in Japan visit stores far more often than the comparable customers of the US supermarkets. Higher frequency of sales discount and higher shopping frequency are consistent with each other in our theory. Our view is that the shopping behavior of Japanese customers to visit stores relatively often is responsible for the observed prevalence of sales price discount and, in general, the highly frequent price changes in the Japanese supermarkets.

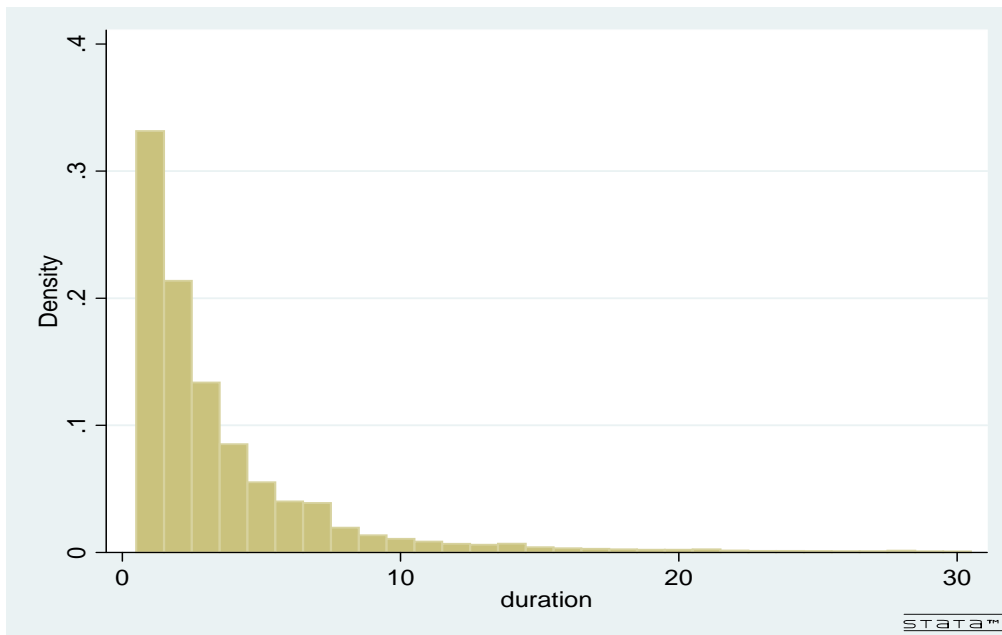


Figure 5: Distribution of durations between store visits

‘Stylized’ facts and alternative models: Frequent price changes without any apparent time trend or seasonality are observed regularly in many grocery items sold typically in supermarket stores. Recurrent price increases and decreases without accompanying changes in costs can be profit maximizing only if some type of the inter-temporal linkage in demand

is important. As pointed out correctly in Slade (1998), many candidate explanations imply gradually decaying *negative* impact of the past price on current demand. Think, for example, a customer capital model. Lower than a ‘fair’ price gradually accumulates goodwill or customer capital so that the (current) demand size also increases. Similar patterns can be obtained if consumers over time grow tastes or habits in consumption of a good: lower price attracts first time customers and the stores can build up the customer base. Information imperfection is yet another possibility consistent with such a pattern. Suppose customers collect price information only occasionally so that, at any point of time, customers differ in the time elapsed since they collected price information for the last time. In this case, lower prices set in the past induce some of the customers to shop, resulting again in gradually decaying negative effect of the past price on current demand.

In contrast, current demand depends *positively* on the past prices in the case of models of sales – demand is higher (lower) when stores have posted a higher (lower) price in the previous periods. Table 5 displays the result of fixed effect regressions of the quantity sold on the average past prices. It leaves little doubt on the positive impact of the past prices on current demand, and confirms Implication 2.

	House Brand						SB Brand					
	Quantity sold						Quantity sold					
Price	-1.680 (0.007)***	-1.680 (0.134)***	-1.457 (0.006)***	-1.457 (0.117)***	-1.297 (0.006)***	-1.297 (0.104)***	-0.682 (0.004)***	-0.682 (0.049)***	-0.572 (0.003)***	-0.572 (0.041)***	-0.488 (0.003)***	-0.488 (0.036)***
Average past price (1week)	1.126 (0.008)***	1.126 (0.103)***					0.471 (0.004)***	0.471 (0.040)***				
Average past price (2 weeks)			0.987 (0.008)***	0.987 (0.094)***					0.384 (0.004)***	0.384 (0.033)***		
Average past price (4 weeks)					0.885 (0.009)***	0.884 (0.089)***					0.311 (0.004)***	0.310 (0.029)***
Constant	146.468 (2.509)***	149.817 (7.749)***	129.206 (2.586)***	132.534 (6.834)***	117.434 (2.697)***	120.764 (7.042)***	61.154 (1.184)***	62.036 (2.938)***	56.380 (1.197)***	57.286 (2.619)***	54.011 (1.238)***	54.932 (2.691)***
Observations	223892	223892	223465	223465	222611	222611	205977	205977	205550	205550	204696	204696
R2		0.203		0.190		0.176		0.157		0.144		0.129
Number of shop*brands	61	61	61	61	61	61	61	61	61	61	61	61

p<0.05, \* p<0.1.

Table 5. Effects of past prices on demand

### 3.3 Dependence of Demand on Past Pricing Patterns

Unfortunately, we have no data on customer inventories. Nevertheless, as Hendel and Nevo (2006a,b) point out, we can exploit the scanner data to test some implications of customers

inventory holdings on demand. Below, we summarize the evidence for the remaining testable implications listed in the previous section.

Dependence of demand on price duration: The quantity sold should depend on the price duration, if customers' inventory holdings affect their purchase decisions. In our model, the inventory accumulation (decumulation) during high (low) price period implies a positive (negative) dependence of demand on the price duration. Table 6 and 7 show the results of panel regression on the quantity sold, incorporating the differential impacts of price durations: At the regular high (low sales) price, the impact is significant and positive (negative). The results are supportive of Implication 3, suggesting that the longer the time elapsed since the last sale, the larger (smaller) the size of the pent-up inventory demand waiting for sales, when the price is high (low).

	House Brand				SB Brand			
	Quantity Sold		Quantity Sold/Visitors		Quantity Sold		Quantity Sold/Visitors	
Price/Regular price	-235.023 (1.076)***	-235.023 (1.076)***	-0.054 (0.000)***	-0.054 (0.000)***	-211.731 (0.388)***	-211.662 (0.388)***	-0.055 (0.000)***	-0.055 (0.000)***
Price Duration* Low Price Dummy	-1.757 (0.025)***	-1.757 (0.025)***	-0.000 (0.000)***	-0.000 (0.000)***	-1.525 (0.010)***	-1.525 (0.010)***	-0.000 (0.000)***	-0.000 (0.000)***
Price Duration	0.270 (0.013)***	0.270 (0.013)***	0.000 (0.000)***	0.000 (0.000)***	-0.044 (0.003)***	-0.044 (0.003)***	-0.000 (0.000)***	-0.000 (0.000)***
Constant	10.308 (0.223)***	10.308 (0.223)***	0.003 (0.000)***	0.003 (0.000)***	6.867 (0.056)***	-11.008 (0.856)***	0.002 (0.000)***	-0.003 (0.000)***
Observations	214535	214535	214535	214535	1007294	1007294	1007294	1007294
R-squared	0.190	0.190	0.227		0.248		0.248	
Number of shopbrandid	60	60	60	60	61	61	61	61

Standard errors in parentheses. \*\*\* indicates the significance level,  $p < 0.01$ , \*\* $p < 0.05$ , \*  $p < 0.1$ .

Table 6. Demand Regressions on relative price and price duration

Dependence of demand on previous price duration: Given that customers' purchase decision depends on their inventories, the quantity sold in the current period should also be influenced by the duration of the preceding price period. As summarized in Implication 4, the inventory behaviors of our model imply that the impact of the previous price duration depends positively on the previous price level: If the price was at discount during the previous price periods, the inventories must have been increasing, whereas the inventories must have been decreasing over the previous price periods if the price was at the regular high level. Hence, the longer the duration of the previous price, the larger (smaller) the current demand, if the previous price was at the regular high (low sales) level. The results in Table 7 are consistent with this

implication: the effect of the previous price duration on the current quantity is significant and positive, if the previous price is at the high regular level.

	House Brand				SB Brand			
	Quantity Sold	Quantity Sold	Quantity Sold/Visitors	Quantity Sold/Visitors	Quantity Sold	Quantity Sold	Quantity Sold/Visitors	Quantity Sold/Visitors
Number of Customers	0.014 (0.000)***	0.015 (0.002)***	0.000 (0.000)***	0.000 (0.000)**	0.003 (0.000)***	0.003 (0.001)***	-0.000 (0.000)***	-0.000 (0.000)*
Price/Regular Price	-230.463 (1.052)***	-230.534 (16.700)***	-0.054 (0.000)***	-0.054 (0.004)***	-84.374 (0.483)***	-84.435 (8.543)***	-0.022 (0.000)***	-0.022 (0.002)***
Price Duration*Low Price Dummy	-1.581 (0.024)***	-1.578 (0.186)***	-0.000 (0.000)***	-0.000 (0.000)***	-0.574 (0.011)***	-0.573 (0.080)***	-0.000 (0.000)***	-0.000 (0.000)***
Price Duration*Regular PriceDummy	0.239 (0.013)***	0.238 (0.044)***	0.000 (0.000)***	0.000 (0.000)***	0.080 (0.007)***	0.081 (0.034)**	0.000 (0.000)***	0.000 (0.000)**
Previous Price Duration*Previous Regular Price Dummy	0.426 (0.025)***	0.432 (0.100)***	0.000 (0.000)***	0.000 (0.000)***	0.108 (0.009)***	0.111 (0.032)***	0.000 (0.000)***	0.000 (0.000)***
Constant	-47.058 (1.741)***	-47.333 (10.079)***	0.001 (0.000)**	0.001 (0.001)	-8.203 (0.739)***	-7.640 (3.395)**	0.002 (0.000)***	0.002 (0.001)***
Observations	214508	214508	214508	214508	200264	200264	200264	200264
R-squared		0.233		0.229		0.151		0.140
Estimation method	RE	FE	RE	FE	RE	FE	RE	FE
Number of shopbrandid	60	60	60	60	60	60	60	60

Standard errors in parentheses. \*\*\* indicates the significance level, p<0.01, \*\* p<0.05, \* p<0.1.

Table 7. Demand regressions including the previous price duration

### 3.4 Sales, Shopping Intensity and Menu Cost

Shopping intensity and sales: Implication 5 suggests that sales are more likely to occur in heavy shopping days. To test this implication, we run a random effect probit regression on the dummy variable for low price period, i.e., the period in which price is set below regular price. The results summarized in Table 8 not only confirm our prediction on the correlation between shopping intensity and occurrence of sales, but also indicate that the effect is even more pronounced in exceptionally heavy shopping days: the result shows the squared term has a significant and positive impact; if we replace the squared term with the dummy variable for exceptionally heavy shopping days, which equals 1 if the deviation is more than two standard deviations away from the mean, then the effect is significant for one brand but not for the other brand.

Cross sectional evidence: One clear implication of the menu cost model is the negative impact of menu cost on frequency of price change. Although we have no data that can be used to represent variations in menu cost, we know that the cost involved in price change does not depend on the sales volume. As the menu cost per unit of sales is decreasing with the sales



Estimation Method : Random Effect Probit Model. Dependent variable: dummy variable for low price						
	House Brand			SB Brand		
Number of Customers	0.072 (0.004)***	0.060 (0.005)***	0.053 (0.006)***	0.018 (0.007)***	0.010 (0.007)	0.010 (0.008)
Number of Customers^2		0.009 (0.002)***			0.008 (0.003)***	
Heavy Shopping Dummy†			0.119 (0.022)***			0.057 (0.035)
Constant	-1.195 (0.091)***	-1.204 (0.091)***	-1.202 (0.091)***	-0.753 (0.114)***	-0.762 (0.114)***	-0.757 (0.114)***
Log( $\sigma^2$ )††	-0.689 (0.204)***	-0.690 (0.204)***	-0.691 (0.204)***	-0.474 (0.233)**	-0.471 (0.233)**	-0.475 (0.233)**
Observations	165089	165089	165089	65519	65519	65519
Number of shopbrandid	61	61	61	49	49	49

Standard errors in parentheses. \*\*\* indicates the significance level,  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

† A dummy variable=1 if the number of customer exceeds at each shop exceed the shop average by more than 2 standard deviations.

†† Log of the variance due to the panel level component

Table 8. Shopping intensity and price discount

volume, we expect that the frequency of price change must be positively correlated with the average sales volume. The results summarized in Table 9 does support such prediction. Indeed, even after controlling for possible idiosyncratic elements of menu cost (by brand or by store), the average sales volume enters significantly in the regression of mean price change probability for each brand-store combination. Table 10 shows a similar cross sectional regression on the share of low price periods. Our analysis shows that the share of low-price periods is decreasing in menu cost. Given the negative relation between the menu cost per unit of sales and the average sales, we expect a significant dependence of the low price share on the mean quantity sold. The results confirm it.

Dependent Variable	Mean Probability of Price Change		
Mean of Quantity Sold	0.00405	0.01150	0.01142
Stn Dev. of Quantity Sold	0.00076	0.00076	0.00077
Chain Dummy	yes	yes	no
Brand Dummy	yes	no	no
Adjusted R2	0.829	0.524	0.496
Number of Observaitons	232	232	232

All the estimated coefficients are significant at 1% level

Table 9. Cross section regression of price change on average sales quantity

Dependent Variable	Share of Low Price Periods		
Mean of Quantity Sold	0.0214	0.0237	0.0072
Stn Dev. of Quantity Sold	0.0020	0.0020	0.0014
Chain Dummy	no	yes	yes
Brand Dummy	no	no	yes
Adjusted R2	0.32400	0.38990	0.89390
Number of observations	232	232	232

All the estimated coefficients are significant at 1% level

Table 10. Cross section regression on the share of low price periods

### 3.5 Model Implications and Applicability

Our model thus captures important facets of the price changes applicable to many grocery items typically found in large scale retailers. At the same time, we should also point out the limitations.

First of all, our model does not explicitly incorporate multi product nature of the retailers. Multi product retailers may face a variety of issues not properly analyzed in our model. Among others, our model is not suitable to explain a certain type of deep discounting in a selected product: milk, eggs, sugar, etc. These items are often sold below marginal cost. Their main objective is to entice customers to visit the store, and hopefully they will purchase other items as well, i.e., they are *loss leaders*. Our model obviously fails to account for this type of pricing. We refer our readers to Hoskin and Reiffen (2007) in which they develop a model of retailers with multiple products facing (ex ante) heterogenous customers.

The second major short coming is that our model is deterministic. Thus, at equilibrium, barring any change in underlying parameters, our model predict repetition of the identical price cycles, which obviously contradicts the data as shown, for example, in our Figure 1. In principle, our model can be extended to incorporate uncertainty, and stochastic changes in parameters to account for the variable price cycles. On the other hand, it is not very clear what we can gain in terms of additional insights by introducing randomness to the model. As a matter of fact, virtually all the past modeling of the retail pricing was done in the deterministic environment. One interesting venue for extension of the model into explicitly stochastic modeling is the stochastic fluctuations in the shopping intensity.

Last, but not the least, our model, as well as the data, do not cover price adjustments due to price changes upstream. As we indicated above, however, our data indicates rather limited, if at all, role of the manufacturer or wholesaler in the observed retail pricing patterns. Rival brand prices within stores are positively correlated, whereas across store correlation of the same brand is virtually nil. Needless to say, this is not to deny the possibility that in other countries, or in other types of goods, the pricing dynamics is dictated by the manufacturer's pricing strategy, although we doubt that the type of price changes we observed in our data can be explicable as a pricing under a manufacturer's initiative: its high frequency, dominant

role of a few focal prices, and coincidence of heavy discount at store wide sales day of the month, etc., all indicate strongly that it is the retailer's, not up stream firms' decision.

Summary: We found that neither simple time-dependent rule or explanations based on information imperfection, consumer habit formation, or customer capital model can account for the statistical regularities documented above. The estimation results support the implications of our model of sales in terms of the impacts of past prices on current sales and the dependence of current demand on the past and current price durations. These are consistent with the implied behavior of household inventory holdings. We have established equilibrium price cycles that involve only a few prices, as observed in data. We also confirmed that the effects of the shopping intensity on the occurrence of sales are consistent with our theory. Finally, cross sectional regressions are supportive of the role played by the menu cost in our model.

## 4 Conclusion

In this paper, we developed a dynamic model of sales for a storable good. The key idea is that the inventory holdings of customers can be the driving force of short-run fluctuations in demand. We showed that frequent price changes between a few focal prices can be profit maximizing for the store who implements intertemporal price discrimination. We applied this theory to the pricing policy of Japanese supermarkets, and provided empirical evidence that supports the implications of our model. Overall, the model fits quite well with our empirical findings.

One of the major unexplored issues is the interactions between brands within a store – the issue studied in Hendel and Nevo (2006b). As we noted in Section 3, statistical evidence indicates significant correlations between the pricing of the two brands within a store. It seems likely that customers may switch between brands in response to occasional sales of one of the brands. Another issue of importance is inventory holdings on the side of retail stores, the avenue taken by Aguirregabiria (1999). Finally, it would be interesting to explore for further implications of the linkage between pricing and sales promotion activities.

## 5 Appendix

### 5.1 Proof of Lemma 1

In text. ■

### 5.2 Proof of Lemma 2

We shall start with the solution of  $\pi_t$ . Solving the system of difference equations in (5),

$$\pi_t = \begin{cases} 1 - (1-c)^t(1-\pi_0) & \text{for } t \in T_H \\ \pi + \{(1-c)(1-s)\}^{t-t_H} \{1-\pi - A(1-\pi_0)\} & \text{for } t \in T_L, \end{cases}$$

where  $A = A(t_H) \equiv (1-c)^{t_H} < 1$  and  $\pi = c(1-s)/(c(1-s) + s)$  is the stationary level given in (3). Using the terminal condition,  $\pi_{t_H+t_L} = \pi_0$ , we obtain the initial value

$$\pi_0 = 1 - (1-\pi)(1-B)(1-AB)^{-1}$$

where  $B = B(t_L) \equiv \{(1-c)(1-s)\}^{t_L} < 1$ , and the solution

$$\pi_t(t_H, t_L) = \begin{cases} 1 - (1-c)^t(1-\pi)(1-B)(1-AB)^{-1} & \text{for } t \in T_H \\ \pi + \{(1-c)(1-s)\}^{t-t_H} (1-\pi)(1-A)(1-AB)^{-1} & \text{for } t \in T_L, \end{cases} \quad (8)$$

which shows that  $\pi_t(\cdot)$  is strictly increasing in  $t \in T_H$  and strictly decreasing in  $t \in T_L$ .

Consider next  $W_t$ . Solving (5) and (6) using  $W_t \equiv V_{t+1}^1 - V_{t+1}^0$ ,

$$W_t = \begin{cases} W_H - \left(\frac{1}{1-c}\right)^t (W_H - W_0) & \text{for } t \in T_H \\ W_L + \left(\frac{1}{1-c(1-s)}\right)^{t-t_H} \{W_H - W_L - D(W_H - W_0)\} & \text{for } t \in T_L, \end{cases}$$

where  $W_H \equiv u - \frac{\varepsilon}{c}$ ,  $W_L \equiv u - \frac{\varepsilon}{c(1-s)}$  ( $= p_L$ ) and  $D = D(t_H) \equiv A(t_H)^{-1} > 1$ . Using the terminal condition  $W_{t_H+t_L} = W_0$ , we obtain the initial value

$$W_0 = (DE - 1)^{-1} \{E(D - 1)W_H + (E - 1)W_L\}$$

where  $E = E(t_L) \equiv \{1 - c(1-s)\}^{-t_L} > 1$ , and the solution

$$W_t(t_H, t_L) = \begin{cases} W_H - \left(\frac{1}{1-c}\right)^t W_\Delta (DE - 1)^{-1} (E - 1) & \text{for } t \in T_H \\ W_L + \left(\frac{1}{1-c(1-s)}\right)^{t-t_H} W_\Delta (DE - 1)^{-1} (D - 1) & \text{for } t \in T_L, \end{cases} \quad (9)$$

where  $W_\Delta \equiv W_H - W_L = s\varepsilon/c(1-s) > 0$ , implying that  $W_t(\cdot)$  is strictly decreasing in  $t \in T_H$  and strictly increasing in  $t \in T_L$ . ■

### 5.3 Proof of Proposition 1

To prove the claim, we first establish the following property.

**Lemma 3** *The store's problem can be converted to the choice of the ratio,  $\theta \equiv t_H/(t_H+t_L) \in [0, 1]$ , to maximize an average profit  $\Pi_0(\theta)$ , which satisfies  $\Pi_0(1) = \Pi_H$  and  $\Pi_0(0) = \Pi_L$ .*

**Proof of Lemma 3.** Substituting (8) and (9) for  $\pi_t$  and  $W_t$ , in the objective function  $\Pi_C(t_H, t_L)$ , with tedious but straight-forward computations and re-arranging terms, we obtain:  $\Pi_C(t_H, t_L) =$

$$\begin{aligned} & \theta\Pi_H + (1-\theta)\Pi_L + [(W_L - \omega) - \{c(1-s) + s\}(u - \omega)](1-\pi)^2 \frac{(1-A)(1-B)}{(1-AB)(t_H+t_L)} \\ & + \frac{s(c+\pi)W_\Delta}{c(1-s)} \frac{(D-1)(E-1)}{(DE-1)(t_H+t_L)} + W_\Delta(1-\pi) \frac{(1-A)(D-1)(1-BE)}{(1-AB)(DE-1)(t_H+t_L)} \end{aligned} \quad (10)$$

where we used  $\theta \equiv t_H/(t_H+t_L)$ . Note  $BE = \{(1-c)(1-s)/(1-c(1-s))\}^{t_L} < 1$ . To investigate the behavior of the last three terms in the above expression, we use the following properties.

Property 1. A function

$$\psi(x; a, b, \alpha, \beta) \equiv \frac{(1-a^{\alpha x})(1-b^{\beta x})}{(1-a^{\alpha x}b^{\beta x})x}, \quad 0 < a, b < 1, \quad \alpha, \beta > 0,$$

is monotonically decreasing in  $x \in (0, \infty]$  and satisfies

$$\psi_0(a, b, \alpha, \beta) \equiv \lim_{x \rightarrow +0} \psi(x; \cdot) = \frac{\alpha\beta \ln(1/a) \ln(1/b)}{\alpha \ln(1/a) + \beta \ln(1/b)}.$$

Property 2. A function

$$\phi(x; d, e, \alpha, \beta) \equiv \frac{(d^{\alpha x} - 1)(e^{\beta x} - 1)}{(d^{\alpha x}e^{\beta x} - 1)x}, \quad d, e > 1, \quad \alpha, \beta > 0,$$

is monotonically decreasing in  $x \in (0, \infty]$  and satisfies

$$\phi_0(d, e, \alpha, \beta) \equiv \lim_{x \rightarrow +0} \phi(x; \cdot) = \frac{\alpha\beta \ln(d) \ln(e)}{\alpha \ln(d) + \beta \ln(e)}.$$

Property 3. A function

$$\xi(x; a, b, d, e, \alpha, \beta) \equiv \frac{(1-a^{\alpha x})(d^{\alpha x} - 1)(1-(be)^{\beta x})}{(1-a^{\alpha x}b^{\beta x})(d^{\alpha x}e^{\beta x} - 1)x}, \quad 0 < a, b < 1 < d, e, \quad \alpha, \beta > 0, \quad be < 1,$$

is monotonically decreasing in  $x \in (0, \infty]$  and satisfies

$$\xi_0(a, b, d, e, \alpha, \beta) \equiv \lim_{x \rightarrow +0} \xi(x; \cdot) = \frac{\alpha^2 \beta \ln(1/a) \ln(1/be) \ln(d)}{(\alpha \ln(1/a) + \beta \ln(1/b)) (\alpha \ln(d) + \beta \ln(e))}.$$

Property 1 and 2 can be shown by using the L'Hopital's rule twice, and Property 3 by using the L'Hopital's rule thrice. Applying Property 1-3, with  $x = t_H + t_L$ ,  $\alpha = \theta$  and  $\beta = 1 - \theta$ , one can see that the sum of the last three terms in the objective function given in (10) is continuous and monotonically decreasing in  $x \in [0, \infty]$  given values of  $\alpha, \beta > 0$ . On the other hand, the first two terms in (10) are constant once we fix values of  $\theta \in [0, 1]$ . Therefore, it follows that for any given  $\theta \in [0, 1]$  and for all  $t_H, t_L > 0$ ,

$$\Pi_0(\theta) > \Pi_C(t_H, t_L)$$

where

$$\begin{aligned} \Pi_0(\theta) \equiv & \theta \Pi_H + (1 - \theta) \Pi_L + [(W_L - \omega) - \{c(1 - s) + s\}(u - \omega)](1 - \pi)^2 \psi_0(a, b, \theta, 1 - \theta) \\ & + \frac{s(c + \pi) W_\Delta}{c(1 - s)} \phi_0(d, e, \theta, 1 - \theta) + W_\Delta (1 - \pi) \xi_0(a, b, d, e, \theta, 1 - \theta). \end{aligned} \quad (11)$$

In this expression, the parameters are  $a \equiv 1 - c$ ,  $b \equiv (1 - c)(1 - s)$ ,  $d \equiv (1 - c)^{-1}$ ,  $e \equiv \{1 - c(1 - s)\}^{-1}$ , and  $\psi_0(\cdot)$ ,  $\phi_0(\cdot)$ ,  $\xi_0(\cdot) > 0$  are derived in Property 1, 2, 3, respectively. Finally, observe that when  $\theta = 0$  or  $\theta = 1$ , we have  $\psi_0(\cdot) = \phi_0(\cdot) = \xi_0(\cdot) = 0$ , hence  $\Pi_0(1) = \Pi_H$  and  $\Pi_0(0) = \Pi_L$ . This completes the proof of Lemma 3. ■

We now prove the claims in the proposition. Differentiation yields

$$\frac{d\Pi_0(\theta)}{d\theta} = -C_u(\theta)(u - \omega) + C_\varepsilon(\theta) \frac{\varepsilon}{c(1 - s)}$$

where

$$\begin{aligned} C_u(\theta) & \equiv s\pi - (1 - c)(1 - s)(1 - \pi)^2 \frac{d\psi_0(\cdot)}{d\theta}, \\ C_\varepsilon(\theta) & \equiv s(c + \pi) - (1 - \pi)^2 \frac{d\psi_0(\cdot)}{d\theta} + \frac{s^2(c + \pi)}{c(1 - s)} \frac{d\phi_0(\cdot)}{d\theta} + s(1 - \pi) \frac{d\xi_0(\cdot)}{d\theta}. \end{aligned}$$

The derivatives of the functions given in Property 1-3 have the following characteristics:

- $d\psi_0(\cdot)/d\theta$  and  $d\phi_0(\cdot)/d\theta$  are both monotonically decreasing in  $\theta$ , strictly positive at  $\theta = 0$  and negative at  $\theta = 1$ ;
- $d\xi_0(\cdot)/d\theta$  is strictly convex in  $\theta$ , zero at  $\theta = 0$ , positive at some  $\theta \in (0, 1)$  and negative at  $\theta = 1$ .

It then follows that:

- $C_u(\theta) > 0$  is monotonically increasing in  $\theta \in [0, 1]$ ;

- $C_\varepsilon(\theta) > 0$  is monotonically decreasing in  $\theta \in [0, 1]$ .

The interior solution to the maximization problem, if it exists, is therefore characterized by the first order condition,  $d\Pi_0(\cdot)/d\theta = 0$  or

$$R \equiv \frac{\varepsilon}{(u - \omega)c(1 - s)} = \frac{C_u(\theta)}{C_\varepsilon(\theta)}.$$

Using the approximation  $z \doteq \ln(1/(1 - z))$  for small positive  $z$  ( $= \{c, s\} < 1$ ), we have

$$R_L \equiv \frac{C_u(0)}{C_\varepsilon(0)} = \frac{c(1 - s)^2}{\{c(1 - s) + s\}^2 + c(1 - s)^2} < \frac{1}{1 + c} = \frac{C_u(1)}{C_\varepsilon(1)} \equiv R_H.$$

As  $C_u(\cdot)/C_\varepsilon(\cdot)$  is monotone increasing in  $\theta \in [0, 1]$ , the optimal solution should satisfy that:  $\theta^* \in (0, 1)$  if and only if  $R \in (R_L, R_H)$ ;  $\theta^* = 1$  if and only if  $R \geq R_H$ ;  $\theta^* = 0$  if and only if  $R \leq R_L$ . Finally,  $R_S \equiv (1 - s)/(1 + c(1 - s)) \in (R_L, R_H)$  follows from the above expression. ■

#### 5.4 Proof of Corollary 1

The claim is immediate from (8) and (9). ■

#### 5.5 Omitted Derivation of Solutions in the Model with Menu Cost

⊙ **Solution of  $\pi_t(t_H, t_L, k)$ :** For the time path of  $\pi_t$ , the equations derived for the case of no-menu cost are still applicable for all  $t \in T$ , hence the solution  $\pi_t = \pi_t(t_H, t_L, k)$  is given by (8), where it should be noted that  $t_L \equiv \sum_{j=1}^k t_L^j$  and thus  $B = \prod_{j=1}^k B_j$  where  $B^j \equiv \{(1 - c)(1 - s)\}^{t_L^j}$ . ■

⊙ **Solution of  $W_t(t_H, t_L, k)$  and  $p_t(t_H, t_L, k)$ :** The time path of  $W_t$  for  $t \in T_H$  is the same as before and is described by (9). During low price period, the difference equation of  $W_t(\cdot)$  is modified to

$$W_{t-1} = -\varepsilon + c(1 - s)(u - W_t) - s(W_t - p_t) + W_t$$

for  $t \in T_L^j$  and for all  $j \in \{1, \dots, k\}$ . Let  $p^j$  ( $= p_t$ ) represent a constant price posted during the  $j$ -th low price period  $t \in T_L^j$  (yet to be determined). In what follows, we shall adopt the following notation:

$$\tau^j \equiv t_H + \sum_{\iota=1}^{j-1} t_L^\iota,$$

which represents the end of each  $(j - 1)$ -th low price period  $T_L^j$ . Using this notation, the above difference equation can be solved to obtain

$$W_t = W_u + W_s p^j + \lambda^{t-\tau^j} \{W_{\tau^j} - (W_u + W_s p^j)\} \quad (12)$$

where  $\lambda \equiv \{(1-c)(1-s)\}^{-1} > 1$ ,  $W_u \equiv (c(1-s)u - \epsilon)/(c(1-s) + s)$  and  $W_s \equiv s/(c(1-s) + s)$ .

As mentioned in the text, the price should satisfy  $p_t = W_{\tau^{j+1}}$  for each  $t \in T_L^j$ ,  $j = 1, \dots, k$ , along the Hi-Lo price cycle. Applying  $p^j = W_{\tau^{j+1}}$  and evaluating (12) at  $t = \tau^j + 1$ ,

$$p^j = \frac{-(\lambda - 1)W_u + \lambda W_{\tau^j}}{1 + (\lambda - 1)W_s}. \quad (13)$$

We now derive the solution of  $W_{\tau^j}$ . Evaluating (12) at  $t = \tau^{j+1}$  and applying (13),

$$W_{\tau^{j+1}} = -\frac{(\lambda^{t_L^j} - 1)W_u}{1 + (\lambda - 1)W_s} + \frac{\lambda W_s + \lambda^{t_L^j}(1 - W_s)}{1 + (\lambda - 1)W_s} W_{\tau^j},$$

which has a general solution in the form given by

$$W_{\tau^{j+1}} = -\Theta_u^j + \Theta_s^j W_{t_H}$$

where

$$\begin{aligned} \Theta_u^j &\equiv \sum_{\iota=1}^j \frac{(\lambda^{t_L^\iota} - 1)W_u}{1 + (\lambda - 1)W_s} \prod_{\iota'=\iota+1}^j \frac{\lambda W_s + \lambda^{t_L^{\iota'}}(1 - W_s)}{1 + (\lambda - 1)W_s} \\ \Theta_s^j &\equiv \prod_{\iota=1}^j \frac{\lambda W_s + \lambda^{t_L^\iota}(1 - W_s)}{1 + (\lambda - 1)W_s} > 1. \end{aligned}$$

Finally, we know from (9) that  $W_{t_H} = W_H - D(W_H - W_0)$ . The terminal condition  $W_{\tau^{k+1}} = W_0$  is then used to obtain

$$W_0 = (\Theta_s^k D - 1)^{-1} \{ \Theta_s^k (D - 1) W_H + \Theta_u^k \};$$

which can be substituted back into  $W_H$  and  $W_{\tau^{j+1}}$  to obtain

$$W_{\tau^j} = (\Theta_s^k D - 1)^{-1} [ \Theta_s^{j-1} (D - 1) W_H - \Theta_u^{j-1} (\Theta_s^k D - 1) + \Theta_s^{j-1} D \Theta_u^k ]. \quad (14)$$

Hence, the solution  $p^j = p_t(t_H, t_L, k)$  for  $t \in T_L^j$  is given by (13) where  $W_{\tau^j}$  satisfies (14), and the solution  $W_t = W_t(t_H, t_L, k)$  for  $t \in T_L^j$  is given by (12) where  $p^j$  satisfies (13) and  $W_{\tau^j}$  satisfies (14). ■

⊙ **Closed Form Expression of  $\Pi_M(t_H, t_L, k)$ :** Using the notations introduced above, we have  $(t_H + t_L)\Pi_M(t_H, t_L, k) =$

$$-(k+1)\sigma + (u - \omega)sc \sum_{t=1}^{t_H} \pi_{t-1} + s \sum_{j=1}^k \sum_{t=\tau^j+1}^{\tau^{j+1}} (p^j - \omega)(c + \pi_{t-1}),$$



where  $\pi_t = \pi_t(t_H, t_L, k)$  and  $p^j = p_t(t_H, t_L, k)$  are the solutions derived above. From

$$s \sum_{t=\tau^j+1}^{\tau^{j+1}} \pi_{m-1} = s\pi t_L^j + (1-\pi)^2 \frac{(1-A) \prod_{l=1}^{j-1} B_l (1-B_j)}{1-AB}$$

and  $\theta \equiv t_H/(t_H + t_L)$ , it follows that  $\Pi_M(t_H, t_L, k) =$

$$\begin{aligned} & -\frac{(k+1)\sigma}{t_H + t_L} + \theta \Pi_H + (1-\theta)s \sum_{j=1}^k (p^j - \omega)(c + \pi) - (u - \omega)s(1-\pi) \frac{(1-A)(1-B)}{(1-AB)(t_H + t_L)} \\ & + s(1-\pi)^2 \sum_{j=1}^k (p^j - \omega) \frac{(1-A) \prod_{l=1}^{j-1} B_l (1-B_j)}{(1-AB)(t_H + t_L)}. \end{aligned} \quad (15)$$

■

## 5.6 Proof of Proposition 2

(8) shows  $\pi_t$  is strictly increasing in  $t \in T_H$  while strictly decreasing in  $t \in T_L$ . (9) shows  $W_t$  is strictly decreasing in  $t \in T_H$ . Observe in (12) that  $W_t$  is strictly increasing in  $t \in T_L^j$  if and only if  $W_{\tau^j} > W_u - W_s p^j$ . Using (13), the latter condition can be written as

$$p^j > \frac{W_u}{1 - W_s} = u - \frac{\varepsilon}{c(1-s)} \equiv W_L.$$

Consider first the case  $j = 1$ ; (13) implies

$$p^1 = \frac{-(\lambda-1)W_u + \lambda W_{t_H}}{1 + (\lambda+1)W_s} = W_L + \frac{W_H - W_L + D(W_0 - W_H)}{(1-c)(1-s) + s}.$$

Applying the initial value  $W_0$  derived above, it turns out that the second term is positive if and only if  $D(W_H + \Theta_u^k - \Theta_s^k W_L) > W_H - W_L$ , which holds true if  $\Theta_s^k - 1 \leq \Theta_u^k / W_L$  (since  $D \equiv (1-c)^{-t_H} > 1$ ). In what follows, we show

$$\Theta_s^k - 1 = \Theta_u^k / W_L.$$

For convenience, define:

$$\theta_u^\iota \equiv \frac{(\lambda^{t_L} - 1)W_u}{1 + (\lambda-1)W_s}; \quad \theta_s^\iota \equiv \frac{s + \lambda^{t_L} c(1-c)(1-s)^2}{(c(1-s) + s)\{(1-c)(1-s) + s\}},$$

so that  $\Theta_u^j = \sum_{\iota=1}^j \theta_u^\iota \prod_{\iota'=\iota+1}^j \theta_s^{\iota'}$  and  $\Theta_s^j = \prod_{\iota=1}^j \theta_s^\iota$ . Observe that for any  $\iota = 1, \dots, j$ , it holds that

$$\frac{\theta_u^\iota}{W_L} = \frac{(\lambda^{t_L} - 1)W_u}{(1 + (\lambda-1)W_s)W_L} = \frac{(\lambda^{t_L} - 1)c(1-c)(1-s)^2}{(c(1-s) + s)\{(1-c)(1-s) + s\}} = \theta_s^\iota - 1.$$

This implies  $\Theta_s^j - 1 = \Theta_u^j/W_L$  for  $j = 1$ . For  $j = 2, \dots, k$ , we have

$$\frac{\Theta_u^j}{W_L} = \sum_{\iota=2}^j \frac{\theta_u^\iota}{W_L} \prod_{\iota'=\iota+1}^j \theta_s^{\iota'} + \frac{\theta_u^1}{W_L} \prod_{\iota=2}^j \theta_s^\iota = \sum_{\iota=2}^j \frac{\theta_u^\iota}{W_L} \prod_{\iota'=\iota+1}^j \theta_s^{\iota'} + (\theta_s^1 - 1) \prod_{\iota=2}^j \theta_s^\iota.$$

Repeating a recursive substitution of  $\theta_u^\iota/W_L = \theta_s^\iota - 1$  to the last expression above  $j - 1$  times,

$$\begin{aligned} \frac{\Theta_u^j}{W_L} &= \sum_{\iota=3}^j \frac{\theta_u^\iota}{W_L} \prod_{\iota'=\iota+1}^j \theta_s^{\iota'} + \left( \frac{\theta_u^2}{W_L} - \theta_s^2 \right) \prod_{\iota=3}^j \theta_s^\iota + \Theta_s^j \\ &= \sum_{\iota=4}^j \frac{\theta_u^\iota}{W_L} \prod_{\iota'=\iota+1}^j \theta_s^{\iota'} + \left( \frac{\theta_u^3}{W_L} - \theta_s^3 \right) \prod_{\iota=4}^j \theta_s^\iota + \Theta_s^j \\ &\quad \vdots \\ &= \frac{\theta_u^j}{W_L} + \left( \frac{\theta_u^{j-1}}{W_L} - \theta_s^{j-1} \right) \theta_s^j + \Theta_s^j = -1 + \Theta_s^j. \end{aligned}$$

This proves  $\Theta_s^j - 1 = \Theta_u^j/W_L$  for  $j = 2, \dots, k$ . Setting  $j = k$  implies the result. Hence, we have shown that  $p^1 > W_L$  and so  $W_t$  is strictly increasing in  $t \in T_L^1$ . Applying  $W_{\tau^2} > W_{\tau^1}$  ( $= W_{t_H}$ ) to (13), we have  $p^2 > p^1 (> W_L)$ , implying  $W_t$  is strictly increasing in  $t \in T_L^2$ . Applying  $W_{\tau^3} > W_{\tau^2}$  to (13), we have  $p^3 > p^2 (> W_L)$ , implying  $W_t$  is strictly increasing in  $t \in T_L^3$ . Repeating this process for  $j = 4, \dots, k$ , we establish that  $W_t$  is strictly increasing in  $t \in T_L$ . ■

## 5.7 Proof of Proposition 3

Notice first that  $\pi_{t_S-1}$  and  $\Delta(t_S)$  are strictly increasing in  $t_S \in T_H$ . Hence to prove the claim, it is sufficient to show that  $\Delta(t_H) < \Delta(t_H + 1)$ . We know from (9) (without menu costs) and from the proof of Proposition 2 (with menu costs) that

$$W_{t_H+1} - \omega > W_L - \omega = (1 - R)(u - \omega).$$

This implies

$$\Delta(t_H + 1) = \gamma(W_{t_H+1} - \omega)s(c + \pi_{t_H}) > \gamma(1 - R)(u - \omega)s(c + \pi_{t_1}) \equiv \Delta_1.$$

On the other hand, using (7) we can write

$$\Delta(t_1) = \gamma(u - \omega)sc \frac{\pi_{t_H} - c}{1 - c}.$$

Then it is sufficient to show  $\Delta_1 > \Delta(t_1)$  or equivalently,

$$R < \frac{\pi_{t_H}(1 - 2c) + c}{(c + \pi_{t_H})(1 - c)}.$$

Observe that the right hand side of this inequality is strictly decreasing in  $\pi_{t_H} \in [0, 1]$ , achieving the minimum  $1/(1+c) \equiv R_H$  at  $\pi_{t_H} = 1$ . Therefore, the above inequality holds true whenever the Hi-Lo price policy is optimal because in such a case we must have  $R_H > R$ . ■

## 5.8 Proof of Convergence to Stationary Hi-Lo Price Policy

Consider a sequence of Hi-Lo pricing cycles, as considered in the main text, each of which consists of high price periods followed by low price periods. Index by  $n$  ( $= 1, 2, 3, \dots$ ) each of these cycles. Denote by  $\pi_0 = \bar{\pi}$  the initial value at  $t = 0$ . The profit maximizing price policy denoted by  $k^*, t_H^*, t_L^j$ ,  $j = 1, \dots, k^*$ , can be then represented by a mapping from the set of parameters,  $\omega, \varepsilon, u, c, s, \delta, \bar{\pi}$  to the policy  $k, t_H, t_L^j$ ,  $j = 1, \dots, k$ , to maximize the average per-period profit per cycle.

Now, using (7), we can write

$$\pi_{t_H^*(\bar{\pi})+t_L^*(\bar{\pi})} = [1 - \pi - A(t_H^*(\bar{\pi}))]B(t_L^*(\bar{\pi})) + \pi + A(t_H^*(\bar{\pi}))B(t_H^*(\bar{\pi}))\bar{\pi},$$

given the other parameters. Since  $\pi_{t_H^*+t_L^*}$  is the initial value of the next Hi-Lo price cycle, we must have

$$\bar{\pi}_{n+1} = [1 - \pi - A(t_H^*(\bar{\pi}_n))]B(t_L^*(\bar{\pi}_n)) + \pi + A(t_H^*(\bar{\pi}_n))B(t_L^*(\bar{\pi}_n))\bar{\pi}_n$$

where  $\bar{\pi}_n$  stands for the initial value of the  $n$ -th cycle. Rewriting the above,

$$\bar{\pi}_{n+1} - \pi^* = \Phi(\bar{\pi}_n - \pi^*)(\bar{\pi}_n - \pi^*)$$

where  $\pi^* \equiv 1 - (1 - AB)^{-1}(1 - B)(1 - \pi)$  and

$$\Phi(\bar{\pi}_n) \equiv A(t_H^*(\bar{\pi}_n))B(t_L^*(\bar{\pi}_n)) \in (0, 1).$$

The mapping  $\Phi(\cdot)$  given above is a contraction and the conventional argument is applied to confirm that the sequence  $\bar{\pi}_n$  converges monotonically towards the fixed point,  $\pi^*$ . ■

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