Interest Rates, Market Power, and Financial Stability

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Abstract
This paper analyzes the effects of safe rates on financial intermediaries’ risk-taking decisions. We consider an economy where (i) intermediaries have market power in granting loans, (ii) intermediaries monitor borrowers which lowers their probability of default, and (iii) monitoring is not observable which creates a moral hazard problem. We show that lower safe rates lead to lower intermediation margins and higher risk-taking when intermediaries have low market power, but the result reverses for high market power. We examine the robustness of this result to introducing non-monitored market finance, heterogeneity in monitoring costs, and entry and exit of intermediaries. We also consider replacing uninsured by insured deposits, market power in raising deposits, and funding with both deposits and capital.

JEL Classification: G21, L13, E52

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1 Introduction

Lax monetary conditions leading to low levels of real interest rates have been identified as an important driver of financial crises.¹ This paper analyzes, from a theoretical perspective, how interest rates affect the risk-taking decisions of financial intermediaries. Its key contribution is to highlight the relevance of the financial sector’s market structure in shaping such relationship.

We model a one-period risk-neutral economy in which a fixed number of financial intermediaries raise uninsured funding from deep pocket investors and compete à la Cournot in providing loans to penniless entrepreneurs. Intermediaries privately decide the monitoring intensity of their loans, where higher monitoring results in lower probabilities of default. Crucially, we assume that the monitoring decision is unobservable, which creates a standard moral hazard problem between the financial intermediary and its financiers. The expected return that investors require for their funds is assumed to be equal to an exogenous safe rate, which could be taken as a proxy for the stance monetary policy.

We show that the effect of the safe rate on the risk of loan portfolios of financial intermediaries depends on their market power. In competitive loan markets the conventional prediction obtains: lower rates result in higher risk-taking by intermediaries. However, in concentrated loan markets we get the opposite prediction: lower rates result in lower risk-taking. These contrasting results obtain because, although lower interest rates lead to lower funding costs for intermediaries, the intensity of the pass-through of financing rates to loan rates depends on their market power. Hence, lower safe rates can lead to either lower (in competitive markets) or higher (in concentrated markets) intermediation margins, which in turn determine lower or higher monitoring incentives for financial intermediaries. We conclude that underlying market structure of the financial sector is key to assess the effects of the safe rate on the stability of the financial sector. Moreover, in line with the traditional (charter value) literature on competition and financial stability,² we also show that higher

¹See the discussion in Adrian and Liang (2018), as well as the empirical papers by Jimenez et al. (2014) and Iannadou et al. (2015), among many others.

²See, for example, Keeley (1990), Allen and Gale (2000), Hellmann et al. (2000), and Repullo (2004).
competition results in higher risk-taking for any level of the safe rate.

After stating our main results linking interest rates, market structure, and financial stability, we analyze three relevant aspects of competition in the loan market: (i) the possibility of direct market finance by investors that (unlike financial intermediaries) do not monitor entrepreneurs, (ii) monitoring cost asymmetries among intermediaries, and (iii) entry and exit decisions of intermediaries.

We first consider a situation in which entrepreneurs also have the possibility of being directly funded by competitive investors that do not monitor their projects. We show that the equilibrium interest rate that intermediaries can charge is affected by the entrepreneurs’ outside funding option. In particular, direct market finance imposes a constraint on equilibrium loan rates. We show that this constraint is more likely to bind in concentrated loan markets and when the safe rate is low. This implies that, in the presence of direct market finance, concentrated loan markets exhibit a U-shaped relationship between the safe rate and the intermediaries’ risk-taking decisions. For low (high) levels of the safe rate decreasing such rate increases (decreases) the probability of loan default. In contrast, for fairly competitive loan markets the results of the basic setup do not change, since direct market finance is not a competitive threat for financial institutions, and therefore it does not affect the Cournot equilibrium outcome.

We next analyze a situation in which financial intermediaries differ in their monitoring abilities. We assume that there are two observable types of intermediaries: those with high and those with low cost of monitoring entrepreneurs. In equilibrium, intermediaries with high monitoring costs have lower market shares and their loans have higher probabilities of default. We show that lower safe rates decrease (increase) the market share of those intermediaries with lower (higher) cost of monitoring and increase (decrease) the probability of default of their loans. We conclude that, in the presence of heterogenous monitoring costs, lower safe rates can have opposite effects on the risk of different intermediaries. By increasing the market share of those intermediaries with higher cost of monitoring (which grant riskier loans) lower safe rates have an additional impact on the risk of the financial sector.

We conclude our analysis of financial market structure by taking into account entry
and exit decisions of financial intermediaries. We consider these decisions as a longer run phenomenon compared to the decisions to grant and monitor loans, with the aim of shedding light on the widespread view that interest rates that are “too low for too long” are detrimental to financial stability. We model entry decisions by assuming that intermediaries have to pay an ex-ante fixed cost to operate. We show that, when entry is taken into account, lower safe rates induce higher competition in the loan market, adding an “entry effect” to our basic results on low policy rates, which increases the probabilities of loan default.

We next analyze the three alternative funding scenarios for financial intermediaries: (i) replacing uninsured by insured deposits, (ii) introducing competition à la Cournot in the deposit market, and (iii) funding intermediaries with both equity capital and uninsured deposits.

Solving the model with insured deposits is much simpler, since intermediaries are then able to borrow at the safe rate. We show that in this case an increase in the safe rate always leads to an increase in the probability of loan default. The intuition for this result is that, in the perfect competition limit, insured deposits lead to zero intermediation margins and hence zero monitoring, so the relationship between the safe rate and the probability of loan default becomes flat. Away from this limit, i.e. when intermediaries have some market power, lower rates allow them to widen intermediation margins, which translates into higher monitoring and lower probabilities of default. Hence, the results for the model with insured deposits on the effect of safe rates on risk-taking are qualitatively similar to the results for the model with uninsured deposits when banks have significant market power.

We next consider the effects of changes in safe rates when banks also compete à la Cournot in the deposit market. In this case we show that the results are qualitatively identical to those of the basic model: low interest rates have a negative impact on financial stability when market power is low, and a positive impact when market power is high.

Finally, our main setup analyzes a situation in which intermediaries are entirely funded with uninsured deposits. What happens when intermediaries can also be funded with inside capital, that is funds provided by those responsible for the monitoring decisions? As

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3Outside equity capital plays essentially the same role as uninsured deposits.
Dell’Ariccia et al. (2014) point out, a relevant determinant of risk-taking decisions is their capital structure, which can be affected by policy rates. We find that when the leverage of financial intermediaries is endogenously determined, market structure can still be a relevant variable in shaping how safe rates affect their risk-taking.

Our results differ from those of Dell’Ariccia et al. (2014) because, while they assume an infinitely elastic supply of equity capital at a constant spread above the policy rate, we also consider a situation in which equity capital is either fixed or increasingly costly to raise. In this case the results are qualitatively identical to those of the basic model. However, in the case of an infinitely elastic supply of capital we get the same result as theirs: low safe rates are always detrimental to financial stability. The reason is that they increase the cost of equity finance relative to the cost of debt finance, so banks react by increasing their leverage, thereby reducing their monitoring incentives. However, to the extent that inside equity may be in limited supply, we conclude that adding leverage is not likely to change our benchmark results.

Literature review  TBC

Structure of the paper  Section 2 presents the basic model of Cournot competition in the loan market with uninsured deposits and unobservable monitoring by intermediaries, and analyzes how market power affects the relationship between the safe rate and the equilibrium monitoring intensity, which determines the probability of default of the loans. Section 3 examines the robustness of our results when we incorporate three relevant aspects of competition in the loan market, namely the presence of competitive market lenders that do not monitor borrowers, heterogeneity in monitoring costs, and entry and exit decisions of financial intermediaries. Section 4 examines the robustness of our results when we consider three alternative funding scenarios, namely when intermediaries are funded with insured deposits, when they compete à la Cournot in the deposit market, and when they can also be funded with equity capital. Section 5 contains our concluding remarks. Proofs of the analytical results are in the Appendix.
2 Basic Model

Consider an economy with two dates \((t = 0,1)\) populated by three types of risk-neutral agents: a continuum of deep pocket investors, a continuum of penniless entrepreneurs, and \(n\) identical financial intermediaries, which for brevity we refer to as banks.\(^4\) Investors are characterized by an infinitely elastic supply of funds at an expected return equal to \(R_0\) (the safe rate). Entrepreneurs have investment projects that can only be funded by banks. Banks in turn have no capital and are funded by investors.\(^5\)

Entrepreneurs’ projects require a unit investment at \(t = 0\) and yield a stochastic return at \(t = 1\) given by

\[
\hat{R} = \begin{cases} 
R, & \text{with probability } 1 - p + m, \\
0, & \text{with probability } p - m,
\end{cases}
\]

where \(p \in (0,1)\) is the probability of failure in the absence of monitoring, and \(m \in [0, p]\) is the monitoring intensity of the lending bank. While \(p\) is known, \(m\) is not observable, so there is a moral hazard problem.

The success return \(R\) is assumed to be a linearly decreasing function of the aggregate investment of entrepreneurs. Given that entrepreneurs only receive funding from banks, their aggregate investment equals the aggregate supply of loans \(L\). Hence, we can write the success return of a project as

\[
R(L) = a - bL,
\]

where \(a > 0\) and \(b > 0\). Free entry of entrepreneurs ensures that the success return \(R(L)\) equals the rate at which they borrow from banks, which means that \(R(L)\) is also the inverse loan demand function.

We assume that the outcome of entrepreneurs’ projects is driven by a single aggregate risk factor \(z\) that is uniformly distributed in \([0,1]\). A project monitored with intensity \(m\) will fail if and only if \(z < p - m\). This assumption implies that the return of projects monitored with the same intensity will be perfectly correlated.

\(^4\)We analyze the relevance of some features that characterize commercial banks such as deposit insurance and imperfect competition in the deposit market in Section 4.

\(^5\)Section 4 also extends our basic framework to allow for banks raising (inside) equity capital.
Banks compete à la Cournot for loans. Specifically, each bank \( j = 1,\ldots,n \) chooses its supply of loans \( l_j \), which determines the total supply of loans \( L = \sum_{j=1}^{n} l_j \) and the loan rate \( R(L) \). After \( R(L) \) is determined, banks offer an interest rate \( B(L) \) to the (uninsured) investors, and once the lending and the funding rates are set banks choose the monitoring intensity of their loans \( m(L) \). Monitoring is costly, and the cost function is assumed to take the functional form

\[
c(m) = \frac{\gamma}{2} m^2, \tag{3}
\]

where \( \gamma > 0 \).

### 2.1 Equilibrium monitoring decisions

Banks’ profits per unit of loans are given by

\[
\pi(L) = [1 - p + m(L)][R(L) - B(L)] - c(m(L)). \tag{4}
\]

To explain this expression, note that with probability \( 1 - p + m(L) \) the bank gets \( R(L) \) from entrepreneurs and pays \( B(L) \) to investors, and then it has to subtract the monitoring costs \( c(m(L)) \). By limited liability, with probability \( p - m(L) \) the loan defaults and the bank obtains zero returns.

To characterize the equilibrium of the model proceed by backwards induction and first determine the banks’ borrowing rate \( B(L) \) and monitoring intensity \( m(L) \) as a function of the total supply of loans \( L \). Since the monitoring intensity \( m \) is not observed by investors, the banks’ borrowing rate cannot depend on \( m \). This means that the banks’ choice of monitoring, for a given borrowing rate \( B(L) \), is

\[
m(L) = \arg \max_{m} \left\{ (1 - p + m)[R(L) - B(L)] - c(m) \right\}. \tag{5}
\]

The first-order condition that characterizes an interior solution to this problem is

\[
R(L) - B(L) = \gamma m(L). \tag{6}
\]
Thus, the banks’ monitoring intensity \( m(L) \) will be proportional to the intermediation margin \( R(L) - B(L) \).\(^6\) Thus, there will be no monitoring when the intermediation margin is zero.

The investors’ participation constraint is given by
\[
[1 - p + m(L)]B(L) = R_0. \tag{7}
\]
Solving for \( B(L) \) in the participation constraint (7), substituting it into the first-order condition (6), and rearranging gives the key equation that characterizes the banks’ monitoring intensity
\[
\gamma m(L) + \frac{R_0}{1 - p + m(L)} = R(L). \tag{8}
\]
The function in the left-hand side of (8) is convex in \( m \). Let us then define
\[
\bar{R} = \min_{m \in [0,p]} \left( \gamma m + \frac{R_0}{1 - p + m} \right). \tag{9}
\]
The following result shows the condition under which banks will be able to raise the required funds from investors.

**Proposition 1** Banks will be able to fund their lending \( L \) if \( R(L) \geq \bar{R} \), in which case the optimal contract between the bank and the investors is given by
\[
m(L) = \max \left\{ m \in [0, p] \mid \gamma m + \frac{R_0}{1 - p + m} = R(L) \right\} \quad \text{and} \quad B(L) = \frac{R_0}{1 - p + m(L)}. \tag{10}
\]

Proposition 1 implies that of the two possible solutions to equation (8), the one with higher monitoring characterizes the optimal contract, which gives
\[
m(L) = \frac{1}{2\gamma} \left[ R(L) - \gamma(1 - p) + \sqrt{[R(L) + \gamma(1 - p)]^2 - 4\gamma R_0} \right]. \tag{11}
\]
From here it follows that \( R'(L) = -b < 0 \) implies \( m'(L) < 0 \). Thus, higher total lending \( L \) (which translates into a lower loan rate \( R(L) \)) implies less incentives to monitor. Also, (11) implies that an increase in the safe rate \( R_0 \) reduces banks’ monitoring intensity (for a given value of \( L \)). The combination of these two results highlights that in order to understand the effects of changes in the safe rate on bank risk-taking, the pass-through of \( R_0 \) to the loan rate \( R(L) \) is key.

\(^6\)We implicitly assume that the marginal cost of monitoring \( \gamma \) is sufficiently high, so we do not reach the corner solution \( m(L) = p \) in which bank loans are safe.
2.2 Equilibrium lending decisions

To compute the Cournot equilibrium of the loan market, we note that the objective function of an individual bank is given by the product of its lending $l$ by the profits per unit of loans $\pi(L)$, which depend on the lending by the other $n - 1$ banks.

A symmetric Cournot equilibrium $l^*$ is defined by

$$ l^* = \arg \max_l [l \pi(l + (n - 1)l^*)], $$

and is characterized by the first-order condition

$$ L^* \pi'(L^*) + n \pi(L^*) = 0, $$

where $L^* = nl^*$. The function $\pi(L)$ in (4) satisfies

$$ \pi'(L) = -b[1 - p + m(L)] + B(L)m'(L) < 0, $$

where we have used (2), (6), (7), and the result $m'(L) < 0$. Although the sign of $\pi''(L)$ is in principle ambiguous, in what follows we assume that parameter values are such that $L\pi''(L) + (n + 1)\pi'(L) < 0$, so the second-order condition $L^* \pi''(L^*) + 2n \pi'(L^*) < 0$ is satisfied.

The equilibrium loan rate is $R^* = R(L^*)$, and the rate at which banks borrow from investors is $B^* = B(L^*)$. The probability of loan default is given by $PD = p - m^*$, where $m^* = m(L^*)$ is the banks’ equilibrium monitoring intensity. The assumption of a single aggregate risk factor implies that probability of loan default equals the probability of bank failure, which is therefore the key driver of financial stability.

We are interested in analyzing the effect on the probability of default $PD$ of changes in two parameter values, namely the expected return $R_0$ required by investors, and the number $n$ of banks in the market, which measures (the inverse of) banks’ market power.

The effect of changes in the number of banks $n$ is straightforward. Differentiating the first-order condition (13) and using the assumption $L\pi''(L) + (n + 1)\pi'(L) < 0$ gives

$$ \frac{dL^*}{dn} = -\frac{\pi(L^*)}{L^* \pi''(L^*) + (n + 1)\pi'(L^*)} > 0. $$

This condition is satisfied in all of our numerical results.
Thus, increasing the number of banks \( n \) increases equilibrium total lending \( L^* \). But since \( m'(L) < 0 \), this lowers the equilibrium monitoring intensity \( m^* \) and consequently increases the probability of default \( PD \).

Interestingly, the effect of changes in the safe rate \( R_0 \) on the probability of default \( PD \) depends on the number of banks \( n \). To see this, let us consider two limit cases, the monopoly case \( (n = 1) \) and the perfect competition case \( (n \to \infty) \).

Starting with the monopoly case, we first note that by the participation constraint \( (7) \) the bank’s profits per unit of loans may be written as

\[
\pi(L) = [1 - p + m(L)]R(L) - R_0 - c(m(L)).
\]  

(16)

Now using \( (3) \) and \( (6) \) we have

\[
\pi(L) = [1 - p + m(L)]\gamma m(L) - \frac{\gamma}{2} m(L)^2 = (1 - p)\gamma m(L) + \frac{\gamma}{2} m(L)^2.
\]  

(17)

Hence, \( \pi(L) \) is monotonic in \( m(L) \). Now let \( R_0 \) and \( R_1 \) denote two safe rates with \( R_0 < R_1 \), and let \( \pi_0^* \) and \( \pi_1^* \) denote the corresponding equilibrium profits per unit of loans for the monopoly bank. Assuming that the monopolist’s profits per unit of loans are decreasing in the safe rate \( R_0 \), that is \( \pi_0^* > \pi_1^* \), we conclude that \( m_0^* > m_1^* \). In other words, higher safe rates reduce the monitoring intensity of the monopoly bank and consequently increase the probability of default of its loans.

The perfect competition case is essentially identical to the model analyzed in Martinez-Miera and Repullo (2017). As shown in \( (15) \), increasing the number of banks \( n \) increases the equilibrium aggregate lending \( L^* \) and reduces the equilibrium loan rate \( R^* \). By Proposition 1, there will be a point in which the constraint \( R(L) \geq R \) will be binding, in which case the equilibrium monitoring intensity satisfies the condition

\[
\frac{d}{dm} \left( \frac{R_0}{1 - p + m(L)} \right) = \gamma - \frac{R_0}{(1 - p + m)^2} = 0,
\]  

(18)

which implies

\[
m^* = \sqrt{\frac{R_0}{\gamma} - (1 - p)}.
\]  

(19)

\( ^8 \)This condition is satisfied in all of our numerical results.
From here it follows that an increase in the safe rate $R_0$ increases the monitoring intensity $m^*$ of the competitive banks and consequently reduces the probability of default of their loans.

Summing up, we have shown that under monopoly increases in the safe rate $R_0$ increase the probability of default of bank loans, while under perfect competition increases in the safe rate $R_0$ reduce the probability of default of bank loans. These results suggest that the slope of the relationship between $R_0$ and $m^*$ will change from positive to negative as we increase the number of banks $n$, so that $\partial^2 m^*/\partial R_0 \partial n < 0$.

Indeed, as Figure 1 illustrates, an increase in the number of banks $n$ leads to a flattening of the relationship between the safe rate $R_0$ (in the horizontal axis) and the equilibrium probability of loan default $P_D$ (in the vertical axis). For sufficiently high $n$ the slope changes sign from positive to negative. The conclusion is that market power matters for assessing the effect of interest rates on financial stability. In particular, low interest rates are detrimental to financial stability when banks’ market power is low, but not at all when their market power is high.

The intuition for these results is as follows. A reduction in the safe rate reduces banks’ funding cost which translates into lower loan rates. In monopolistic markets this pass-through
from financing costs to loan rates is not very intense and results in higher intermediation margins, and hence higher monitoring incentives; see equation (6). In competitive markets the pass-through is more intense and results in lower intermediation margins and lower monitoring incentives. This is illustrated in Figure 2, where we show the effect of changes in the safe rate $R_0$ on equilibrium loan rates $R^*$ (Panel A) and intermediation margins $R^* - B^*$ (Panel B) for different values of the number of banks $n$. The slopes of the lines in Panel A become steeper (a higher pass-through) with increases in $n$, which leads to the change in the slope of the lines in Panel B from positive (for high $n$) to negative (for low $n$).

![Figure 2. Effect of the safe rate on loan rates and intermediation margins](image)

This figure shows the relationship between the safe rate and the equilibrium loan rates (Panel A) and intermediation margins (Panel B) for loan markets with 1 (bold line), 2, 5, 7, and 10 (light line) banks.

### 3 Alternative Competition Scenarios

This section reviews our previous results on the relationship between the safe rate and banks’ risk-taking decisions when we incorporate three relevant aspects of competition in the loan market. First, we consider the effect of introducing competitive market lenders that do not monitor borrowers, but can limit the monopoly rents that banks are able to capture. Second, we analyze at the effect of introducing heterogeneity in banks’ monitoring costs. Finally, we
discuss the long run effects that obtain when we allow for entry (or exit) of banks in the loan market.

### 3.1 Direct market finance

Consider a variation of our model in which entrepreneurs can obtain funding for their projects from banks and also directly from investors. We assume that investors are not able to monitor entrepreneurs’s projects (because they may be dispersed and subject to a free rider problem). They are also assumed to be competitive in the sense that they are willing to lend at a rate \( \overline{R} \) that satisfies the participation constraint

\[
(1 - p)\overline{R} = R_0. \tag{20}
\]

The presence of market lenders imposes a constraint on banks’ lending, since the loan rate \( R(L) \) cannot exceed the market rate \( \overline{R} \). This means that the inverse loan demand function (2) now becomes

\[
R(L) = \min\{a - bL, \overline{R}\}. \tag{21}
\]

The upper bound \( \overline{R} \) will be binding whenever the original equilibrium (in the absence of the bound) is such that \( R(L^*) > \overline{R} \). In such case the candidate equilibrium lending will be \( \overline{L} > L^* \) such \( R(\overline{L}) = a - b\overline{L} = \overline{R} \). By our previous results, the banks’ borrowing rate and monitoring intensity will be given by \( B(\overline{L}) \) and \( m(\overline{L}) \), respectively. The question is: will a bank \( j \) want to deviate when the other \( n - 1 \) banks choose \( \overline{l} = \overline{L}/n \) ?

There are two cases to consider. First, note that setting \( l_j < \overline{l} \) is not profitable, since given the upper bound in loan rates the profits per unit of loans would not change from \( \pi = \pi(\overline{L}) \). Second, setting \( l_j > \overline{l} \) is not profitable either since the assumption \( L\pi''(L) + 2n\pi'(L) < 0 \) together \( \overline{L} > L^* \) implies

\[
\left. \frac{d}{dl} \left[ l\pi(l + (n - 1)\overline{L}) \right] \right|_{l=l_j} = \overline{l}\pi''(\overline{L}) + \pi(\overline{L}) < l^*\pi'(L^*) + \pi(L^*) = 0, \tag{22}
\]

where the last equality is just the equilibrium condition in the absence of direct market finance.
Hence, we conclude that whenever the upper bound $\bar{R}$ is binding, the equilibrium bank lending will be $\bar{L}$. Interestingly, although direct market finance is zero, it has a significant effect on equilibrium lending and interest rates by limiting banks’ market power. It also has an effect on the relationship between the safe rate $R_0$ and the probability of loan default $PD$. In particular, substituting the loan rate $\bar{R} = R_0/(1 - p)$ into (11) yields an equilibrium level of monitoring

$$m^* = \frac{R_0}{\gamma(1 - p)} - (1 - p),$$

which is increasing in $R_0$. Thus, when the presence of market lenders binds the loan rate, increases in the safe rate $R_0$ increase the monitoring intensity $m^*$ of the banks, and consequently reduce the probability of default of their loans.

Figure 3 illustrates the effect of changes in the safe rate $R_0$ on equilibrium loan rates $R^*$ (Panel A) and intermediation margins $R^* - B^*$ (Panel B) in the presence of direct market finance. The solid lines in Panel A show the relationship between $R^*$ and $R_0$ for different values of $n$. The dashed line shows the upper bound $\bar{R} = R_0/(1 - p)$, which is binding for fairly monopolistic markets (low $n$) and for low values of the safe rate $R_0$. The lines in Panel B show the relationship between $R^* - B^*$ and $R_0$ for different values of $n$.

Figure 4 shows the effect of introducing market finance on the equilibrium probability of loan default $PD$ for different values of the safe rate $R_0$ and the number of banks $n$. The horizontal axis represents the safe rate $R_0$, and the vertical axis represents the probability of loan default $PD$. The different lines show the relationship between $PD$ and $R_0$ for different values of $n$. For competitive markets (high $n$), the relationship is still negative, that is higher safe rates translate into lower bank risk-taking. However, in contrast with the result in Section 2, in monopolistic markets (low $n$) the effect is U-shaped: lower safe rates initially decrease banks’ risk-taking, but below certain point they increase risk-taking. This result follows from the fact that, as shown in Figure 3, when the safe rate is low the equilibrium loan rate $R^*$ in monopolistic markets equals the market rate $\bar{R}$, so by (23) lower rates reduce monitoring intensities, thereby increasing the probability of default of bank loans.
Figure 3. Effect of the safe rate on loan rates and intermediation margins in the presence of market finance

This figure shows the relationship between the safe rate and the equilibrium loan rates (Panel A) and intermediation margins (Panel B) in the presence of market finance for loan markets with 1 (bold line), 2, 5, 7, and 10 (light line) banks. The dashed line in Panel A represents the loan rate under direct market finance.

Figure 4. Effect of the safe rate on the probability of loan default in the presence of market finance

This figure shows the relationship between the safe rate and the probability of default for loan markets with 1 (bold line), 2, 5, 7, and 10 (light line) banks in the presence of direct market finance.
3.2 Heterogenous monitoring costs

We next consider the effect of changes in the safe rate $R_0$ in a loan market in which banks have different monitoring costs. Specifically, suppose that there are two types of banks that differ in the parameter $\gamma$ of their monitoring cost function (3): $n_1$ banks have high monitoring costs, characterized by parameter $\gamma_1$, while $n_0 = n - n_1$ banks have low monitoring costs, characterized by parameter $\gamma_0 < \gamma_1$. It is assumed that a bank’s type is observable to investors, so they can adjust the rates at which they are willing to fund them accordingly.

To characterize the equilibrium of the model with heterogeneous banks, note first that the critical values $R_0$ and $R_1$ defined in (9) by setting $\gamma$ equal to $\gamma_0$ and $\gamma_1$, respectively, satisfy $R_0 < R_1$, except in the corner case where $R_0 = R_1 = R_0/(1-p)$.\footnote{This case obtains when the derivative of the function in the left-hand side of (8) evaluated at $m = 0$ is positive for the low (and hence for the high) monitoring cost banks, that is when $R_0 \leq \gamma L(1-p)^2$.} From here it follows that whenever the total supply of loans $L$ is such $R_0 < R(L) < R_1$, only the low monitoring cost banks will operate.

By our results in Section 2, if $R(L) \geq R_j$ the monitoring intensity chosen by bank $j = 0,1$ is

$$m_j(L) = \frac{1}{2\gamma_j} \left[ R(L) - \gamma_j(1-p) + \sqrt{(R(L) + \gamma_j(1-p))^2 - 4\gamma_j R_0} \right],$$

and the corresponding borrowing rate is

$$B_j(L) = \frac{R_0}{1-p + m_j(L)}.$$  \hspace{1cm} (24)

It is immediate to show that $m_0(L) > m_1(L)$,\footnote{This follows from the fact that the function in the left-hand side of (8) is increasing in $\gamma$, so the highest intersection with $R(L)$ must be decreasing in $\gamma$.} which implies $B_0(L) < B_1(L)$. That is, low monitoring cost banks will choose a higher monitoring intensity, and consequently will be able to borrow from investors at lower rates. Finally, by (16) profits per unit of loans for banks of type $j = 0,1$ may be written as

$$\pi_j(L) = [1-p + m_j(L)]R(L) - R_0 - c_j(m_j(L)).$$ \hspace{1cm} (26)

Since

$$\frac{\partial \pi(L)}{\partial \gamma} = B(L)\frac{\partial m(L)}{\partial \gamma} - \frac{1}{2} m(L)^2 < 0,$$ \hspace{1cm} (27)
it follows that $\pi_0(L) > \pi_1(L)$.

A Cournot equilibrium is defined by a pair of strategies $(l_0^*, l_1^*)$ that satisfy

$$l_0^* = \arg \max_l [l \pi_0(l + (n_0 - 1)l_0^* + n_1 l_1^*)],$$
$$l_1^* = \arg \max_l [l \pi_1(l + (n_1 - 1)l_1^* + n_0 l_0^*)].$$

From here it follows that the Cournot equilibrium will be characterized by the first-order conditions

$$L_0^* \pi_0'(L^*) + n_0 \pi_0(L^*) = 0,$$
$$L_1^* \pi_1'(L^*) + n_1 \pi_1(L^*) = 0,$$

where $L_0^* = n_0 l_0^*$, $L_1^* = n_1 l_1^*$, and $L^* = L_0^* + L_1^*$.

Figure 5 shows the effect of changes in the safe rate $R_0$ on equilibrium lending by low and high monitoring cost banks, $L_0^*$ and $L_1^*$, and equilibrium total lending $L^*$. Increases in the safe rate $R_0$ reduce lending by both types of banks, but the effect is more significant for high monitoring cost banks. In particular, the market share of low monitoring cost banks, denoted $\lambda = L_0^*/L^*$, increases with the safe rate, reaching 100% for high values of $R_0$.

Figure 5. Effect of the safe rate on loan supply with heterogeneous monitoring costs

This figure shows the relationship between the safe rate and the aggregate supply of loans (solid line), and the supply of loans by banks with low (dashed line) and high monitoring costs (dotted line).
Since low monitoring cost banks choose a higher monitoring intensity, their loans have a lower probability of default. Given that the market share of these banks increases with the safe rate, it follows that the average probability of loan default will get closer to that of the low monitoring cost banks. Figure 6 illustrates the effect of changes in the safe rate $R_0$ on the probability of loan default of low and high monitoring cost banks, $PD_0 = p - m_0^*$ and $PD_1 = p - m_1^*$, as well as on the average probability of default defined by

$$PD = \lambda PD_0 + (1 - \lambda)PD_1.$$  \hspace{1cm} (32)

Increases in the safe rate $R_0$ translate into increases in the probability of default of the loans granted by high monitoring cost banks, and decreases in the probability of default of the loans granted by low monitoring cost banks. But due to the effect of increases in $R_0$ on the market share of the latter, the average probability of loan default $PD$ goes down, approaching $PD_0$ for large values of $R_0$.

![Figure 6. Effect of the safe rate on the probability of loan default with heterogeneous monitoring costs](image)

This figure shows the relationship between the safe rate and the average probability of default (solid line), and the probability of default of loans by banks with low (dashed line) and high monitoring costs (dotted line).

A conclusion that can be drawn from this analysis is that, when banks have different monitoring costs, the composition effect of increases in the safe rate, which leads to a greater
market share of low monitoring cost banks, makes the results closer to those of the basic model with low market power (high $n$).

3.3 Bank Entry

We next consider the effects of changes in the safe rate when we allow for entry (and exit) of banks into (or out of) the loan market. In this manner, we intend to shed light on the widespread view that interest rates that are “too low for too long” are detrimental to financial stability.

In order to endogenize the number of banks, we assume that each bank incurs a fixed cost to operate. Banks may have different fixed costs. In particular, let $f_j$ denote the fixed cost of bank $j = 1, 2, 3, \ldots$, and assume that $f_{j+1} = f_j + z$, for all $j$, with $z \geq 0$. We consider two possible cases: one in which all banks have the same fixed cost ($z = 0$), and another one in which the fixed cost is increasing in the number of banks ($z > 0$).

Let $\pi_n^*$ denote the equilibrium level of profits (before subtracting the fixed costs) in a market in which $n$ otherwise identical banks operate. Ignoring integer constraints, the free entry equilibrium is characterized by a number $n$ of banks that satisfy a zero net profit condition for the marginal bank, namely $\pi_n^* - f_n = 0$.

In what follows we analyze the effect of introducing either constant or increasing fixed costs on the relationship between the safe rate $R_0$ and the probability of loan default $PD$. The benchmark for this analysis will be the monopoly case ($n = 1$), in which as shown in Section 2 lower rates translate into lower probabilities of default.

Figure 7 shows the effect of introducing fixed costs on the equilibrium number of banks $n$ for different values of the safe rate $R_0$. The horizontal axis represents the safe rate $R_0$, and the vertical axis represents the equilibrium number of banks $n$. The horizontal solid line corresponds to the benchmark monopoly case, the dotted line is the increasing fixed cost case, and the dashed line is the constant fixed cost case. As expected, with lower rates there will be entry which will be more pronounced for constant fixed costs.

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11This implies that the fixed cost for any $n > 1$ is $f_n = f_1 + (n - 1)z$. 
Figure 7. Effect of the safe rate on the number of banks

This figure shows the relationship between the safe rate and the equilibrium number of banks for a constant fixed cost (dashed line) and an increasing fixed cost of entry (dotted line). The solid line represents the fixed number of banks benchmark.

We have shown that increasing the number of banks increases equilibrium total lending, lowers the monitoring intensity of the banks, and hence increases the probability of loan default. Since there will be more entry with lower rates, we have

$$\frac{\partial PD}{\partial R_0} + \frac{\partial PD}{\partial n} \frac{dn}{\partial R_0} < \frac{\partial PD}{\partial R_0},$$  \hspace{1cm} (33)

where the first term in the left-hand side shows the direct effect for a fixed number of banks, and the second term the indirect effect through bank entry. It follows that entry will tend to strengthen our previous results on the negative relationship between safe rates and bank risk-taking in competitive markets, and possibly reverse our previous results on the positive relationship between safe rates and bank risk-taking in monopolistic markets.

Figure 8 illustrates these results. The horizontal axis represents the safe rate $R_0$, and the vertical axis represents the probability of loan default $PD$. The solid line corresponds to the benchmark monopoly case, the dotted line is the increasing fixed cost case, and the dashed line is the constant fixed cost case. The effect of entry (the second term in the left-hand side of (33)) is clearly more pronounced for the constant than for the increasing fixed cost of entry.
Figure 8. Effect of the safe rate on the probability of loan default with endogenous entry

This figure shows the relationship between the safe rate and the probability of default for a constant fixed cost (dashed line) and an increasing fixed cost of entry (dotted line). The solid line represents the fixed number of banks benchmark.

4 Alternative Funding Scenarios

This section analyzes the robustness of our previous results to incorporating three relevant aspects of banks’ funding sources. First, we consider a variation of the basic model in which deposits are insured. Second, we analyze the effect of assuming that banks also compete à la Cournot in the deposit market. Finally, we introduce bank capital, and analyze whether endogenizing banks’ leverage decision changes the relationship between the safe rate and banks’ risk-taking decisions.

4.1 Insured deposits

When deposits are insured banks can borrow from investors at the safe rate $R_0$, since when they fail the insurer pays them the promised return.\footnote{We assume that such insurance is provided at a flat rate equal to zero.} Hence, we have $B(L) = R_0$, so the banks’ choice of monitoring is given by

$$m(L) = \arg \max_m \{(1 - p + m)[R(L) - R_0] - c(m)\}.$$  \hspace{1cm} (34)
The first-order condition that characterizes an interior solution to this problem is

$$R(L) - R_0 = \gamma m(L). \tag{35}$$

From here it follows that banks’ profits per unit of loans simplify to

$$\pi(L) = [1 - p + m(L)](R(L) - R_0) - c(m(L))$$

$$= (1 - p)[R(L) - R_0] + \frac{1}{2\gamma}[R(L) - R_0]^2. \tag{36}$$

As before, $R'(L) = -b < 0$ implies $\pi'(L) < 0$.

Following the same steps as in Section 2, the first-order condition that characterizes a symmetric Cournot equilibrium is

$$L^*\pi'(L^*) + n\pi(L^*) = 0. \tag{37}$$

As before, we are interested in analyzing the effect on the probability of loan default $PD = p - m(L^*)$ of changes in two parameter values, namely the number $n$ of banks in the market and the expected return $R_0$ required by investors. Differentiating the first-order condition (37), and assuming that parameter values are such that $L^0(L) + (n+1)\pi'(L) < 0$, we get

$$\frac{dL^*}{dn} = -\frac{\pi(L^*)}{L^*\pi''(L^*) + (n+1)\pi'(L^*)} > 0, \tag{38}$$

which is the same result as in the basic model.

Similarly, differentiating the first-order condition (37) gives

$$\frac{dL^*}{dR_0} = -\frac{1}{L^*\pi''(L^*) + (n+1)\pi'(L^*)} \frac{\partial}{\partial R_0} [L^*\pi'(L^*) + n\pi(L^*)]. \tag{39}$$

Now using the expression for $\pi(L)$ in (36) we have

$$\frac{\partial}{\partial R_0} [L^*\pi'(L^*) + n\pi(L^*)] = \frac{bL^*}{\gamma} - n \left[(1 - p) + \frac{1}{\gamma}[R(L^*) - R_0]\right]$$

$$= \frac{1}{b} [L^*\pi''(L^*) + n\pi'(L^*)], \tag{40}$$

which we have assumed to be negative. Hence, we have

$$\frac{dL^*}{dR_0} = -\frac{L^*\pi''(L^*) + n\pi'(L^*)}{b[L^*\pi''(L^*) + (n+1)\pi'(L^*)]} < 0, \tag{41}$$
that is an increase in the safe rate $R_0$ reduces equilibrium lending $L^*$. From here it follows that the effect on the intermediation margin is

$$
\frac{d}{dR_0} \left[ R(L^*) - R_0 \right] = -b \frac{dL^*}{dR_0} - 1 = -\frac{\pi'(L^*)}{L^* \pi''(L^*)} + (n + 1) \pi'(L^*) < 0.
$$

(42)

But then by (35) we know that a decrease in the intermediation margin leads to a decrease in monitoring, so $\partial m^*/\partial R_0 < 0$.

We conclude that when deposits are insured an increase in the safe rate always leads to an increase in the probability of loan default $PD = p - m^*$. Hence, the results for the model with insured deposits on the effect of the safe rate on banks’ risk-taking decisions are qualitatively similar to the results for the model with uninsured deposits when banks have significant market power.

### 4.2 Endogenous deposit rates

We now consider the effects of changes in safe rates when banks also have market power in raising deposits. In particular, we assume that banks compete à la Cournot in a deposit market characterized by a linear inverse supply function of the form

$$
R_D(D) = R_0 - c + dD,
$$

(43)

where $D$ is the aggregate supply of deposits, $R_D$ is the expected return of bank deposits, and $c > 0$ and $d > 0$. In this setup, the safe rate $R_0$ may be interpreted as the rate that investors could obtain by investing in a safe asset such as government bonds.

The function in (43) can be derived from a model in which investors differ in a liquidity premium associated with bank deposits. Specifically, suppose that there is a measure $c$ of atomistic risk-neutral investors with wealth $1/d$ characterized by a liquidity premium $s$ uniformly distributed in $[0, c]$. An investor of type $s$ will deposit her wealth in a bank offering a return $R_D$ if

$$
R_D + s \geq R_0.
$$

(44)

The liquidity premium could also be interpreted as an individual-specific cost of accessing the government bond market.
From here it follows that if the equilibrium return is $R_D$, the aggregate supply of deposits $D$ will be the wealth of investors with a liquidity premium higher than $R_0 - R_D$, that is

$$ D = \frac{c - (R_0 - R_D)}{d}. \quad (45) $$

Solving for $R_D$ in this equation gives the inverse supply function (43).

Banks compete à la Cournot for loans and deposits. Specifically, each bank $j = 1, \ldots, n$ chooses its supply of loans $l_j$ and its demand for deposits $d_j$ subject to the balance sheet constraint $l_j = d_j$. Given this constraint, in what follows we will simply denote by $l_j$ the size of the balance sheet of bank $j$.

The individual bank decisions determine the total supply of loans $L = \sum_{j=1}^{n} l_j$ and the loan rate $R(L)$, as well as the total demand for deposits $D = \sum_{j=1}^{n} d_j$ and the required return on deposits $R_D(L)$. After $R(L)$ and $R_D(L)$ are determined, banks offer a deposit rate $B(L)$ to the investors, and once the lending and the funding rates are set they choose the monitoring intensity of their loans $m(L)$.

As in the basic model, to characterize the equilibrium of this model we first determine the banks’ deposit rate $B(L)$ and monitoring intensity $m(L)$ as a function of the total supply of loans $L$ (and demand for deposits $D = L$). The banks’ choice of monitoring is given by

$$ m(L) = \arg \max_m \{(1 - p + m)[R(L) - B(L)] - c(m)\}. \quad (46) $$

and the investors’ participation constraint is now

$$ [1 - p + m(L)]B(L) = R_D(L). \quad (47) $$

Following the same steps as in the basic model of Section 2, one can show that if $L$ is such that

$$ R(L) \geq \frac{R_D(L)}{1 - p + m}, \quad (48) $$

then we have

$$ m(L) = \frac{1}{2\gamma} \left[ R(L) - \gamma(1 - p) + \sqrt{[R(L) + \gamma(1 - p)]^2 - 4\gamma R_D(L)} \right]. \quad (49) $$
and

\[ B(L) = \frac{R_D(L)}{1 - p + m(L)} \tag{50} \]

From here it follows that

\[ \frac{dm(L)}{dL} = -b \frac{\partial m(L)}{\partial R(L)} + d \frac{\partial m(L)}{\partial R_D(L)} < 0. \tag{51} \]

The second term in this expression is new, relative to the model with an infinitely elastic supply of funds at the safe rate \( R_0 \). This term amplifies the negative impact of total lending on bank monitoring, via the additional reduction in the intermediation margin \( R(L) - B(L) \), due to the increase in \( R_D(L) \), and hence in \( B(L) \).

A Cournot equilibrium is defined as in the basic model, with \( m(L) \) and \( B(L) \) in (49) and (50) replacing the previous expressions in (4). Solving the first-order condition (13) gives the equilibrium amount of lending \( L^* \) (and deposit taking \( D^* = L^* \)). As before, the equilibrium loan rate is \( R^* = R(L^*) \), the deposit rate is \( B^* = B(L^*) \), and the probability of loan default is given by \( PD = p - m(L^*) \).

Figure 9 shows that the qualitative effects of changes in the safe rate \( R_0 \) on the probability of default \( PD \) for different values of \( n \) is similar to the ones in Figure 1. Increasing the number of banks \( n \) leads to a flattening of the relationship between the safe rate \( R_0 \) (in the horizontal axis) and the equilibrium probability of loan default \( PD \) (in the vertical axis). For sufficiently high \( n \) the the slope changes sign from positive to negative. The conclusion is that adding Cournot competition in the deposit market does not change our initial results on the effect of safe rates on banks’ risk-taking: low interest rates have a negative impact on financial stability when banks’ market power is low, and a positive impact when market power is high.
Figure 9. Effect of the safe rate on the probability of loan default with Cournot competition for deposits and loans

This figure shows the relationship between the safe rate and the probability of default for markets with 1 (bold line), 2, 5, 7, and 10 (light line) banks that compete à la Cournot for both deposits and loans.

4.3 Endogenous leverage

Finally, we analyze the effect of changes if the safe rate when banks can adjust their leverage. As highlighted by Dell’Ariccia et al. (2014), leverage decisions are an important driver of the risk-taking effects of monetary policy.\footnote{It is important to note that in our model bank equity is taken to be inside equity, that is funds provided by agents that either make the unobservable risk-taking decisions or have no conflict of interest with those that take them.}

In what follows we consider two models with equity capital: one in which the aggregate supply of bank capital is fixed at $K$ (in which case each bank will have $K/n$ capital), and one in which, as in Dell’Ariccia et al. (2014), there is an infinitely elastic supply of capital at the rate $R_0 + \delta$, where $\delta > 0$ is an exogenous equity premium.

In the former case, the sequence of moves is as in the basic model, except for the fact that the supply of loans $l_j$ by each bank $j = 1, \ldots, n$ determines not only the total supply of loans $L = \sum_{j=1}^{n} l_j$ and the loan rate $R(L)$, but also its capital per unit of loans $k_j = K/nl_j$. In the latter case, each bank $j = 1, \ldots, n$ first chooses its supply of loans $l_j$, which determines
the total supply of loans $L = \sum_{j=1}^{n} l_j$ and the loan rate $R(L)$, and then chooses its capital per unit of loans $k_j$.

In both cases, after $R(L)$ is determined, banks offer an interest rate $B(L)$ to the (uninsured) investors, and once the lending and the funding rates are set they choose the monitoring intensity of their loans $m(L)$. Notice that each bank $j$ only has to raise $(1 - k_j)l_j$ funds from investors, since the rest is funded with equity.

Given a loan rate $R = R(L)$, a safe rate $R_0$, and a capital per unit of loans $k$, a bank’s choice of borrowing rate $B^*$ and monitoring intensity $m^*$ is a solution to the problem

$$m^* = \arg \max_m [(1 - p + m)[R - (1 - k)B^*] - c(m)],$$

subject to the investors’ participation constraint

$$(1 - p + m^*)B^* = R_0. \tag{53}$$

By the convexity of the monitoring cost function (3), the solution to (52) is characterized by the first-order condition

$$\gamma m^* + (1 - k)B^* = R. \tag{54}$$

Solving for $B^*$ in the participation constraint (53) and substituting it into the first-order condition (54) gives the key equation that characterizes the banks’ monitoring intensity

$$\gamma m^* + \frac{(1 - k)R_0}{1 - p + m^*} = R. \tag{55}$$

The right-hand side of (55) is convex in $m^*$, so in general there will be two solutions for $m^*$. By the same arguments as in Proposition 1, we can show that the banks prefer the highest one, which is

$$m(R, k) = \frac{1}{2\gamma} \left[ R - \gamma(1 - p) + \sqrt{[R + \gamma(1 - p)]^2 - 4\gamma(1 - k)R_0} \right]. \tag{56}$$

It follows that a higher loan rate $R$ and a higher a capital per unit of loans $k$ increase the bank’s monitoring intensity $m^*$. Let us then write $m^* = m(R, k)$, with $\partial m^*/\partial R > 0$ and $\partial m^*/\partial k > 0$. 

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For the model with a fixed aggregate supply of bank capital, bank’s profits per unit of loans are
\[
\pi(R, k) = [1 - p + m(R, k)]R - (1 - k)R_0 - c(m(R, k)).
\] (57)

Given that \( R = R(L) \) and \( k = K/nl \), with a slight abuse of notation we can write
\[
\pi(L, l) = \pi(R(L), K/nl) = [1 - p + m(L, l)]R(L) - (1 - K/nl)R_0 - c(m(L, l)).
\] (58)

A symmetric Cournot equilibrium is then defined by
\[
l^* = \arg \max_l [l\pi(l + (n - 1)l^*, l)].
\] (59)

For the model with an infinitely elastic supply of bank capital, bank’s profits per unit of loans are
\[
\pi(R, k) = [1 - p + m(R, k)]R - (1 - k)R_0 - k(R_0 + \delta) - c(m(R, k)).
\] (60)

Given that \( R = R(L) \), let us define
\[
\pi(L) = \max_k \pi(R(L), k).
\] (61)

A symmetric Cournot equilibrium is then defined by
\[
l^* = \arg \max_l [l\pi(l + (n - 1)l^*)].
\] (62)

Figure 10 illustrates the effects of changes in the safe rate \( R_0 \) for the model with a fixed aggregate supply of capital on capital per unit of loans \( k \) (Panel A) and the probability of default \( PD \) (Panel B) for different values of \( n \). Panel A shows that an increase in the number of banks \( n \) leads to a reduction in \( k \), due to the higher equilibrium supply of loans (recall that \( k = K/L \)). It also shows that an increase in the safe rate \( R_0 \) leads to an increase in \( k \), due to the lower equilibrium supply of loans. Panel B shows that the results for this model of endogenous leverage are similar to those of the basic model. For sufficiently high \( n \) the slope of the relationship between the safe rate \( R_0 \) and the equilibrium probability of default \( PD \) changes sign from positive to negative. Thus, low interest rates are detrimental.
to financial stability when banks’ market power is low, but not when their market power is high. A comparison between Panels A and B shows that while lower rates lead to an increase in leverage regardless of \( n \), this does not always increase the probability of default.

![Figure 10. Effect of the safe rate on the equity ratio and the probability of loan default with a fixed aggregate supply of capital](image)

This figure shows the relationship between the safe rate and the capital per unit of loans (Panel A) and the probability of default (Panel B) for loan markets with 1 (bold line), 2, 5, 7, and 10 (light line) banks with a fixed aggregate supply of equity capital.

Figure 11 illustrates the effects of changes in the safe rate \( R_0 \) for the model with an infinitely elastic supply of capital on capital per unit of loans \( k \) (Panel A) and the probability of default \( PD \) (Panel B) for different values of \( n \). Panel A shows that the effects of an increase in the number of banks \( n \) and in the safe rate \( R_0 \) on banks’ capital per unit of loans are qualitatively the same as those for the model with a fixed aggregate supply of bank capital. However, the results in Panel B are different: although an increase in the number of banks \( n \) also leads to an increase in the probability of default \( PD \), now the relationship between the safe rate \( R_0 \) and the probability of default \( PD \) is always decreasing. Thus, as previously shown by Dell’Ariccia et al. (2014), when banks can raise capital at a fixed equity premium \( \delta \) low interest rates are always detrimental to financial stability. The intuition for this result is as follows: low safe rates increase the cost of equity finance, \( R_0 + \delta \), relative to the cost of debt finance, \( R_0 \), so banks react by increasing their leverage, as shown in Panel A. This, in turn, leads to higher risk-taking, as shown in Panel B. In the case of high market power,
this means that the effect of a higher intermediation margin is more than compensated by the increase in leverage.

![Figure 11. Effect of the safe rate on the equity ratio and the probability probability of loan default with an infinitely elastic supply of capital](image)

This figure shows the relationship between the safe rate and the capital per unit of loans (Panel A) and the probability of default (Panel B) for loan markets with 1 (bold line) and 2 (light line) banks with an infinitely elastic supply of equity capital.

More generally, we can consider intermediate cases between the fixed and the infinitely elastic aggregate supply of bank capital. For example, we could assume that the differential cost of equity finance is an increasing and convex function $\delta(K)$ of the aggregate supply of bank capital. When $\delta(K) = \delta$ we have the case of an infinitely elastic supply, while when $\delta(K) = 0$ for $K \leq \overline{K}$ and $\delta(K) = \infty$ for $K > \overline{K}$ we have the case of a fixed supply of bank capital. By changing the shape of the function $\delta(K)$ we can obtain results that are close to one of the two limit cases examined above. However, in a model in which bank equity is inside equity, it may be reasonable to assume that it is in limited supply. For this reason, we may conclude that adding leverage does not essentially change our initial results on the effect of safe rates on banks’ risk-taking: low interest rates are expected to have a negative impact on financial stability when banks’ market power is low, and a positive impact when market power is high.
5 Conclusion

TBC
Appendix

Proof of Proposition 1\textsuperscript{15} To simplify the notation, let $R$ denote $R(L)$. If $R < R_0$, for any $m \in (0, p]$ we have

$$R - \frac{R_0}{1 - p + m} - \gamma m < 0,$$

which implies that the bank has an incentive to reduce $m$. But for $m = 0$ we have

$$R - \frac{R_0}{1 - p} < 0,$$

which violates the banks’ participation constraint $R \geq B$.

If $R \geq R_0$, by the convexity of the function in the right-hand side of (9) there exist an interval $[m^-, m^*] \subset [0, p]$ such that

$$R - \frac{R_0}{1 - p + m} - \gamma m \geq 0 \quad \text{if and only if} \quad m \in [m^-, m^*].$$

By our previous argument, for any $m \in (0, p]$ for which

$$R - \frac{R_0}{1 - p + m} - \gamma m < 0,$$

the bank has an incentive to reduce $m$. Similarly, for any $m \in [0, p)$ for which

$$R - \frac{R_0}{1 - p + m} - \gamma m > 0,$$

the bank has an incentive to increase $m$. Hence, there are three possible values of monitoring in the optimal contract: $m = m^*$, $m = m^-$, and $m = 0$ (when $m^- > 0$).

To prove that the bank prefers $m = m^*$, notice that our assumptions on the monitoring cost function together with the definition of $m^*$ imply

$$\frac{d}{dm} [(1 - p + m)R - c(m)] = R - \gamma m > R - \gamma m^* = \frac{R_0}{1 - p + m^*} > 0,$$

for $m < m^*$. Hence, we have

$$(1 - p + m^*)R - R_0 - c(m^*) > (1 - p + m)R - R_0 - c(m),$$

for either $m = m^-$ (when $m^* > m^-$) or $m = 0$ (when $m^- > 0$), which proves the result. \(\square\)

\textsuperscript{15}The proof is almost identical to the proof of Proposition 1 in Martinez-Miera and Repullo (2017)
References


