Oligopolistic Equilibrium and Financial Constraints*

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Abstract

In this paper we provide a model of dynamic oligopoly in which firms take into account the financial constraints of all firms. The study of the equilibria of our dynamic game leads to the concept of Bankruptcy-Free outputs (BF) in which no firm can drive another firm to bankruptcy without becoming bankrupt itself. For a duopoly with sufficently patient firms all equilibria yield BF outputs. When there are more than two firms, outputs other than BF can be sustained as equilibria but the set of BF outputs is still useful in explaining the shape of the equilibrium set. Cournot one-shot equilibrium and joint profit maximization are more difficult to sustain as equilibria of our game than under the standard repeated games.

Key words: Financial Constraints, Bankruptcy, Firm Behavior, Dynamic Games.

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1. Introduction

There is ample evidence that financial constraints play an important role in the behavior of firms (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). We begin with the observation that the punishment for violation of a financial constraint must be severe or otherwise firms would default all the time. Suppose that the punishment is so severe that firms violating financial constraints lose the capacity to compete and disappear (Sharfstein and Bolton, 1990).[1] Firms might then have incentives to take actions that would make it impossible for competitors to fulfill financial constraints in the hope of getting rid of them.

In this paper we provide a model of dynamic oligopoly in which firms take fully into account the financial constraints of all other firms and not only their own financial constraints. To simplify our task we make two assumptions: profits cannot be transferred from one period to the next and the financial constraint in each period requires that profits must be non-negative in each period. The second assumption entails just a normalization of profits. However, the first assumption is not innocuous and is discussed later on.

The study of the equilibria of our dynamic game leads naturally to the concept of Bankruptcy-Free outputs (BF in the sequel). This is the set of outputs in which profits for all firms are not negative (so no firm goes bankrupt) and no firm can make another firm bankrupt without becoming bankrupt. The concept of BF captures the opportunities for ruining other players that exist in our set up but are not captured by standard concepts such as Cournot equilibrium. Moreover Cournot equilibrium may be not BF. Thus consider a market with two firms in which a firm has a better technology, but both produce a positive output in Cournot equilibrium. These outputs are not BF if the most efficient firm can produce an output for which its profits are zero and profits for the inefficient firm are negative. Why would a firm like to make such a move? Because in a dynamic game this move gets rid of a competitor so if the efficient firm is very patient this move will pay off in the future. This implies that the use of trigger strategies in our framework is limited because reversion to Cournot output does not guarantee that firms have incentives to stay there. However when the Cournot equilibrium is BF it can always be supported as a Subgame Perfect Nash Equilibrium ($SPNE$) of the dynamic game for any discount factor.

[1] Even though firms can be reorganized after bankruptcy and continue business, the survival rate of firms after bankruptcy is typically low, 18% US, 20% in UK and 6% in France, see Couwenberg (2001).

In general, for the duopoly case when firms hardly discount the future, any NE must yield BF outputs. This result highlights the difference between our approach and the standard repeated game in which when the discount factor is sufficiently close to one, any individually rational payoff equals the average payoff of some SPNE. Thus, the consideration of bankruptcy has a bite because, in some cases, it makes it impossible to sustain as equilibria of the dynamic game some well-known solution concepts like Cournot equilibrium.

The interaction among more than two firms is richer and non BF outputs can be supported as NE. This is because there are strategies which prescribe collusion when firms that might have gone bankrupt are active in the market and prescribe, say, Cournot outputs (whenever they are BF) when a firm has gone bankrupt (so, once "war" has been fought, the surviving agents do not trust each other anymore and thus rendering collusion impossible).

Nevertheless, since no firm goes bankrupt in equilibrium, for a given discount factor, any NE outcome of our game can be sustained as an NE outcome of a game without bankruptcy considerations[2]. Thus the consideration of bankruptcy does not bring new outputs that can be supported as equilibrium with respect to those that can be supported in standard repeated games. This is good news in the light of the large multiplicity of outputs that can be sustained as equilibria in the standard repeated game when firms hardly discount the future. An implication of this result is that sustaining collusion does not become easier (and in some cases it becomes more difficult) when we take bankruptcy into consideration.

In order to continue our exploration we make the additional assumption that the technology is characterized by increasing average costs. Our aim is to characterize the average payoffs that can be sustained as SPNE when discount is small.[3] The concept of minimax payoff plays an important role here (as in the folk theorem for repeated games) but it has to be adapted because minimaxing firms may become bankrupt as a consequence of their own action. We define a new concept, the minimax BF, where the min and the max operators are taken over the outputs that are BF. We show that any BF output profile that gives a payoff greater than the minimax BF payoff can be supported as an $SPNE$ and that payoffs less than the minimax BF payoff cannot be sustained in

[2]This result can be strengthened to the consideration of subgame perfection under the additional assumption that only trigger strategies are used.

[3]Recall that under no bankruptcy, any individually rational payoff can be achieved by the average payoff of an SPNE (the folk theorem).

any $SPNE$ for a discount factor close to one. Thus the failure of Cournot to be supported as an equilibrium can be traced to Cournot profits being smaller than minimax BF for some firm. Also, joint profit maximization could yield ouptputs that are BF but with payoff below the minimax BF payoff. Thus, in those cases, joint profit maximization can not be supported as an SPNE.

We end this introduction with a preliminary discussion of the literature (see more on this in the final section). Although a number of papers demonstrate that the financial structure does affect market outcomes in oligopoly, most previous studies adopt either static or two-stage models. Kawakami and Yoshida (1997) and Spagnolo (2000) are two exceptions. Both papers make use of repeated games like ours. The former incorporates a simple exit constraint into the repeated prisoners' dilemma. In their model, each firm must exit from the market no matter how it plays if the rival deviates over a certain period of time. Fixing the length of such an endurable period of time intrinsic to each firm, they show that predation can occur when firms hardly discount the future. The latter study examines the role of stock options in repeated Cournot games. In that model, unlike standard repeated games, firms do not necessarily maximize average discounted profits because stock options affect managers' incentives. Considering this effect, Spagnolo (2000) shows that collusion is easily achieved.

2. The model

There are n firms. They play a dynamic game with infinite horizon and discounting. In each period firms play a constituent game where they simultaneously choose quantities. In order to focus in the strategic decisions regarding outputs we assume that firms cannot accumulate profits. We also assume that firms become bankrupt as long as they have negative profits in a period, and when they do so, they disappear from the market. When making its quantity decision in period t, each firm knows what any firm has produced in all previous periods and which firms became bankrupt.

The constituent game

Each firm, say i, produces a unique output denoted by x_i. Let the aggregate output $\sum_{i=1}^{n} x_i$ be denoted by X. Vectors are denoted in boldface. Let $\mathbf{x} = (x_1, .., x_{i-1}, x_i, x_{i+1}, .., x_n)$ be an output profile. Let $\mathbf{x}_{-i} = (x_1, .., x_{i-1}, x_{i+1}, .., x_n)$. Thus $\mathbf{x} = (x_i, \mathbf{x}_{-i})$. The product is homogeneous. Let $p_i(X)$ be the inverse demand function for firm i assumed to be strictly decreasing in X for any positive price. Let $c_i(x_i)$ be the cost of producing x_i for firm i with $c_i(0) = 0$. The average cost of producing x_i is denoted by $AVC_i(x_i)$. Profits for any given firm i are defined by $\pi_i \equiv p_i(X)x_i - c_i(x_i)$ and written as $\pi_i(\mathbf{x})$ or sometimes $\pi_i(x_i, \mathbf{x}_{-i})$. We assume that profits for any firm are a concave function of its own output.

The infinite horizon dynamic game

In each period, say t, each firm chooses an output denoted by x_i^t. Let $\mathbf{x}^t$ be a profile of outputs in period t. The payoffs obtained by firm i in period t are $\pi_i(\mathbf{x}^t)$ $t = 0, 1, ..., \tau, ...$ Firms cannot accumulate profits, and hence they become bankrupt as long as they have negative profits in a period. Once a firm is bankrupt produces zero output in all subsequent periods. Let $\delta \in (0, 1)$ be the common discount factor. Payoffs for firm i are $P_i = \sum_{t=0}^{\infty} \delta^t \pi_i(\mathbf{x}^t)$. The continuation payoff in period t is given by $P_i^t = \sum_{r=0}^{\infty} \delta^r \pi_i(\mathbf{x}^{t+r})$. At period 0 the game begins with the null history h^0. For $t \geq 1$, let $h^t = (\mathbf{x}^0, \mathbf{x}^1, .., \mathbf{x}^{t-1})$ be the realized profiles of outputs at all periods before t. A strategy for firm i, σ_i, is a sequence of maps, one for each period t, mapping all possible period t histories to an output in period t. Let $\sigma = (\sigma_1, ..., \sigma_n)$ denote a strategy profile. A Nash Equilibrium (NE in the sequel) is a collection of strategies from which no agent finds it profitable to deviate. A Subgame Perfect Nash Equilibrium ($SPNE$ in the sequel) is a collection of strategies which are a NE in every possible subgame. Throughout the paper, we restrict our attention to pure-strategy equilibria.

We say that a profile of outputs is *Bankruptcy-Free* if no firm is made bankrupt and no firm can make another firm bankrupt without going bankrupt itself. A motivation to focus on such outputs is that they describe a long run equilibrium in an industry in which all firms have incentives to stay in the market and not to engage in predatory activities. Of course these activities might be important but we look at the industry once the dust has settled and the predatory activities (if any) have been done in the past. Formally,

Definition 1. *An output profile* $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, ..., \hat{x}_n)$ *is bankruptcy-free (BF) if:*
a) $\pi_i(\hat{\mathbf{x}}) \geq 0$, *for all* $i \in \{1, .., n\}$.
b) For all x_j *such that* $\pi_j(x_j, \hat{\mathbf{x}}_{-j}) \geq 0$, $\pi_i(x_j, \hat{\mathbf{x}}_{-j}) \geq 0$ *for all* $i \neq j$.

Note that if firms are required to make $v_i \leq 0$ profits to avoid bankruptcy, we can define a new profit function as $\tilde{\pi}_i(\mathbf{x}) \equiv \pi_i(\mathbf{x}) - v_i$ and redefine BF with respect to this new profit function.

To build intuition on the set of BF outputs, let us consider the following example.

Example 1. *Suppose there are two firms with linear inverse demand $p(X) = \max\{1 - X, 0\}$ and quadratic cost functions $0.1x_1^2$ and $0.04x_2^2$. Profit functions are $\pi_1(x_1, x_2) = (1 - x_1 - x_2)x_1 - 0.1x_1^2$ and $\pi_2(x_1, x_2) = (1 - x_1 - x_2)x_2 - 0.04x_2^2$. Let $\bar{x}_1 \neq 0$ and $\bar{x}_2 \neq 0$ be such that $\pi_1(\bar{x}_1, \bar{x}_2) = \pi_2(\bar{x}_1, \bar{x}_2) = 0$. Writing $\pi_1(x_1, x_2) = (1 - 1.1x_1 - x_2)x_1$ and $\pi_2(x_1, x_2) = (1 - 1.04x_2 - x_1)x_2$, we see that $\bar{x}_1$ and $\bar{x}_2$ are the intersection of the functions $x_2 = 1 - 1.1x_1$ (the solid line in Figure 1) and $x_1 = 1 - 1.04x_2$ (the doted line in Figure 1). These outputs are represented by the thick lines in Figure 1 and amount to $\bar{x}_1 = 0.277$, $\bar{x}_2 = 0.694$.*

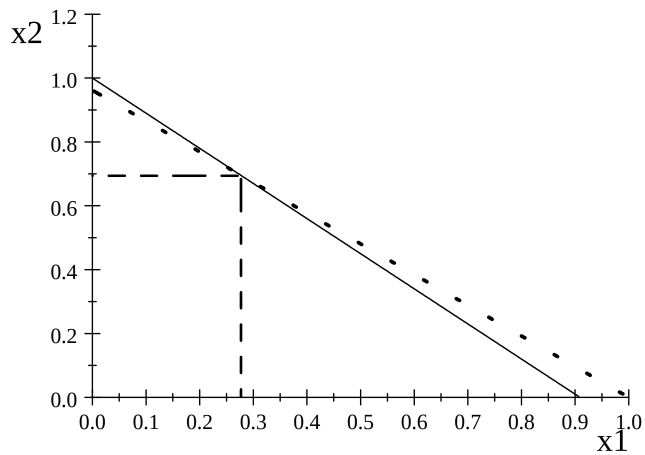

Figure 1

We first show that the set of BF outputs is the rectangle $[0, 0.277] \times [0, 0.694]$. For instance a pair of outputs $(0.5, 0.25)$ are not BF because firm 2 can produce $x_2 = 0.46$ and yield profits of $\pi_1 = -0.005$ and $\pi_2 \simeq 0.001$. The same argument holds for any pair of outputs in the triangle defined by the solid and the dotted line and the x_2 axis in the upper left corner of Figure 1. And what about a pair of outputs in the aforementioned rectangle like $(0.2, 0.2)$? Note that the maximum amount that, say, firm 1 can produce without going bankrupt is .72 and $\pi_2(0.72, 0.2) = 0.014\,4$ we see that it is impossible for firm 1 to make firm 2 bankrupt. This is easily seen in Figure 1 because setting $x_2 = 0.2$ an increase in x_1 hits first the zero profit constraint of firm 1 (solid line).

Note that the (unique) Cournot equilibrium outputs $(0.302, 0.335)$ are not BF because $0.302 > 0.277$.[4] In particular firm 1, by producing 0.302 is exposing itself to bankruptcy if firm 2 produces $0.667 + \epsilon$ for suitably low values of ϵ.

[4]Cournot equilibrium outputs solve the FOC of profit maximization, $1 - x_2 = 2.2x_1$ and $1 - x_1 = 2.08x_2$.

There are several comments to Example 1. Firstly, note that the calculus of BF allocations boils down to finding a solution to the system of equations $\pi_i(\bar{\mathbf{x}}) = \mathbf{0}$. In fact this is a general feature of markets with increasing average costs with two firms (see Lemma 3 in the Appendix).

Secondly, outside the BF set a firm can go bankrupt and this possibility has consequences for the equilibrium of the dynamic game. In this example, the Cournot output profile is not BF. If firms hardly discount the future, a repeated play of the Cournot output in every period is not a NE: For firm 2, it pays to make firm 1 go bankrupt and to get monopoly profits from there on when $\delta > \pi_2^C/\pi_2^M$, where π_2^C denotes firm 2 Cournot profits and π_2^M denotes firm 2 monopoly profits. The same argument can be applied to any output profile which is not BF. Moreover trigger strategies cannot be used in this case because they rely on reversion to the static NE (Cournot output) which is not the part of any NE of the dynamic game.

Thus, the consideration of bankruptcy discards a big chunk of outputs as equilibrium outcomes of the dynamic game when the discount factor is close to one. This contrasts with the folk theorem in standard repeated games where if firms are sufficiently patient, any output yielding non negative profits (the intersection of the areas beneath the solid and the dotted lines in Figure 1) can be sustained as an $SPNE$ of the dynamic game.

The next section formalizes the intuitions above by studying the equilibria of the model and exploring the role of BF in it.

3. Dynamic Competition with Bankruptcy

Let $(x_1^C, x_2^C, ...x_n^C)$ be an output profile corresponding to a Cournot equilibrium of the constituent game, and π_i^C be the profits obtained by i in this Cournot equilibrium. Then we have the following:

Observation. Assume that the output profile corresponding to a Cournot equilibrium of the constituent game is BF. Then:

(i) This output profile can be sustained as an $SPNE$ of the dynamic game for any δ.

(ii) When δ tends to 1, a BF output profile that yield profits larger than π_i^C for all i, can be sustained as an $SPNE$.

Proof. (i) From Fudenberg and Tirole (1991, p. 149) if $(x_1^C, x_2^C, ..., x_n^C)$ is an NE of the constituent game, the repeated play of the static NE is always an SPNE of the repeated game when there are no bankruptcy considerations. Since no player is bankrupt and there is no possibility of making other firm go bankrupt because these outputs are BF, this strategy profile is an $SPNE$ of the dynamic game.

(ii) Let $\mathbf{x}=(x_1,..,x_n)$ be a BF output profile that yield profits larger than π_i^C for all i. Consider the following strategy for a generic player, say i. At period 0 play the output x_i, and continue to play x_i so long as either (i) the output profile in the previous period was $\mathbf{x}$ or (ii) the output profile in the previous period differed from $\mathbf{x}$ in two or more components. In any other case play x_i^C. By the usual reasoning such strategy profile is an $SPNE$ when δ is sufficiently close to one and it yields the desired output profile in each period (as in Friedman, 1971). ■

The above observation requires that the set of BF outputs and the NE of the constituent game have a non-empty intersection. This is not always the case, recall Example 1. [5]

Our first result is an asymptotic result for two firms. The result states that, when δ is sufficiently close to one, any NE of the dynamic game yields BF output profiles in each period. Denoting monopoly profits for firm i as π_i^M, we have the following:

Proposition 1. *Let $n=2$. Let $(x_1^0, x_2^0, ..., x_1^t, x_2^t, ...)$ be a sequence of outputs yielded by a NE for any sufficiently large δ and such that there is an $\epsilon > 0$ with $\pi_i(\mathbf{x}^t) + \epsilon \leq \pi_i^M$ for all $t = 0, 1, ...,$, $i = 1, 2$. Then, when $\delta \to 1$, (x_1^t, x_2^t) is BF for all t.*

Proof. Suppose that in period t, (x_1^t, x_2^t) is not BF. Thus, one firm could cause bankruptcy to the other. Suppose, without loss of generality, it is firm 2. Consider the following strategy for firm 2. In period t choose an output x_2, that drives firm 1 into bankruptcy and choose the output corresponding to monopoly thereafter. In this case, the continuation payoff for firm 2 is

$$\pi_2(x_1^t, x_2) + \delta\pi_2^M + \delta^2\pi_2^M + ... \tag{3.1}$$

The continuation payoff at t for the sequence $(x_1^1, x_2^1, ..., x_1^t, x_2^t, ...)$ is:

$$\pi_2(\mathbf{x}^t) + \delta\pi_2(\mathbf{x}^{t+1}) + \delta^2\pi_2(\mathbf{x}^{t+2}) + ... \tag{3.2}$$

By the definition of an NE,

$$\pi_2(\mathbf{x}^t) + \delta\pi_2(\mathbf{x}^{t+1}) + \delta^2\pi_2(\mathbf{x}^{t+2}) + ... \geq \pi_2(x_1^t, x_2) + \delta\pi_2^M + \delta^2\pi_2^M + ... \tag{3.3}$$

[5] Clearly, if firms are identical the Cournot output profile is BF. More general conditions under which NE and BF outputs have a non-empty intersection are available under request.

or

$$\pi_2(\mathbf{x}^t) - \pi_2(x_1^t, x_2) \geq \delta(\pi_2^M - \pi_2(\mathbf{x}^{t+1})) + \delta^2(\pi_2^M - \pi_2(\mathbf{x}^{t+2})) + .. \geq \delta\epsilon + \delta^2\epsilon + .. = \delta\frac{\epsilon}{1-\delta}. \quad (3.4)$$

Clearly, when δ converges to 1, the inequality 3.4 is impossible, contradicting that we were in an NE. ■

The result is driven by the fact that a firm producing an output outside the BF set will be immediately "killed" because, in this case, crime pays since it means monopoly forever for the killer. Proposition 1 needs two assumptions: $\pi_i(\mathbf{x}^t) + \epsilon \leq \pi_i^M$ and that there are two firms. We discuss both points in turn.

Firstly we show by means of an example that without the condition $\pi_i(\mathbf{x}^t)+\epsilon \leq \pi_i^M$, Proposition 1 might fail because it might not pay to kill a firm whose output shrinks quickly over time.

Example 2. *Suppose a linear inverse demand $p(X) = \max\{20-X, 0\}$ and two firms with constant average cost $AVC_1(x_1) = 0$ and $AVC_2(x_2) = 4$. Consider the following pair of strategies: firm 2 , if it is in the market in period t, produces $x_2^t = (0.5)^t$ regardles of what happened in the past so the output of firm 2 shrinks over time. Firm 1 produces $x_1^0 = 9.5$, and for all $t \geq 1$, $x_1^t = (20-(0.5)^t)/2$ (the best reply to $(0.5)^t$) if $x_2^{t-r} = (0.5)^{t-r}$ for all $r \in \{1, ..., t\}$, otherwise it produces $x_1^t = 16$ (the minimax strategy against firm 1). We first notice that the output profile in every period is not BF because in period t firm 1 may produce for example $19-(0.5)^t$ and ruin firm 2 without ruining itself (at those outputs the price will be 1 which is below the average cost of firm 2). We show that for a sufficiently large δ this strategy constitutes a NE of the dynamic game.*
Consider first firm 2. The best reply to $x_1^1 = 9.75$, is 3.125 so if firm 2 deviates in period 1 receives, at most, $(20-9.75-3.125-4)3.125 = 9.7656$ in that period and the minimax payoff from there on (in this model the minimax payoff is zero). If firm 2 does not deviate, total output in period t is $(20+(0.5)^t)/2$ and market price is $(20-(0.5)^t)/2$. Thus firm 2 discounted profits are

$$\sum_{t=0}^{\infty}\delta^t(\frac{20-(0.5)^t}{2} - 4)(0.5)^t = \frac{10}{(1-\delta(0.5))} - \frac{1}{2(1-\delta(0.5)^2)} - \frac{4}{1-\delta(0.5)}, \quad (3.5)$$

which is increasing in δ. For $\delta \simeq 0.85$ the expression in (3.5) equals 9.7656. Thus, for any $\delta > 0.85$, firm 2 does not have incentives to deviate in period 1. And deviations in subsequent periods will be even less profitable because the output of firm 1 in these periods is larger than its output in period 1.

Consider now firm 1. If firm 1 does not deviate, total output in period t is $(20+(0.5)^t)/2$ and market price is $(20-(0.5)^t)/2$. Thus firm 1 discounted profits are

$$\sum_{t=0}^{\infty}\delta^t(\frac{20-(0.5)^t}{2})^2=\frac{20^2}{4(1-\delta)}+\frac{1}{4(1-\delta(0.5)^2)}-\frac{40}{4(1-\delta(0.5))}. \tag{3.6}$$

The only possible profitable deviation for firm 1 would be to make firm 2 bankrupt and to produce the monopoly output (which in this example is 10) from then on. If this deviation happens in period 1, given that firm 2 is producing $x_2^t=0.5$, firm 1 needs to produce slightly above the quantity that makes price equal to the average cost of firm 2 (which in this example amounts to 15.5).Thus, profits for firm 1 will be, approximately, $(15.5)4=62$ in the first period and monopoly profits ($\pi_1^M=100$) in any subsequent period. Discounted profits associated with this strategy are

$$62+\delta\frac{100}{1-\delta}. \tag{3.7}$$

Let us see that

$$\frac{20^2}{4(1-\delta)}+\frac{1}{4(1-\delta(0.5)^2)}-\frac{40}{4(1-\delta(0.5))}>62+\delta\frac{100}{1-\delta}, \tag{3.8}$$

or equivalently,

$$100+\delta\frac{100}{1-\delta}+\frac{1}{4(1-\delta(0.5)^2)}-\frac{10}{(1-\delta(0.5))} > 62+\delta\frac{100}{1-\delta}\Leftrightarrow \tag{3.9}$$

$$38+\frac{1}{4(1-\delta(0.5)^2)}-\frac{10}{(1-\delta(0.5))} > 0 \tag{3.10}$$

Given that $1/(4(1-\delta(0.5)^2))>0$ and $10/(1-\delta(0.5))<20$, the left hand side of the last inequality is greater than 18 and therefore greater than 0. Thus, firm 1 does not have incentives to make firm 2 bankrupt in period 1. For subsequent periods, the output that ruins firm 2 is larger and larger with an upper bound of 16 and profits for firm 1 of, at most, 64. A similar argument to the one used above shows that this deviation is not profitable.

In Proposition 1 the assumption of two firms is also essential. With more than two firms, after one firm becomes bankrupt, the strategies of the remaining firms may prescribe outputs which yield less profits than before, something that cannot happen in the duopoly case. Thus ruining a firm may have unfortunate payoff consequences despite the fact that it makes the market (apparently) less competitive because there are less competing firms. Therefore firms may abstain from ruining other firms because this might not pay off. We now present an example in which after any deviation from a non BF collusive output, firms revert to the Cournot equilibrium, which in the example is BF, and thus the aforementioned output can be sustained as an SPNE.

Example 3. *Let us consider a market with three firms and an inverse demand function* $p(X) = \max\{10 - X, 0\}$. *Firms have constant average cost such that* $ACVC_1(x_1) = AVC_2(x_2) = 0$ *and* $AVC_3(x_3) = 4$. *The best reply functions are:*

$$x_i = \max\{0, \frac{10 - c_i - \sum_{i \neq j} x_j}{2}\}, \ i = 1, 2, 3. \tag{3.11}$$

The unique Cournot equilibrium output vector is $(3.33, 3.33, 0)$ *yielding profits* $(10, 10, 0)$. *These outputs are* BF *because since firm* 3 *does not produce we have a duopoly with two identical firms and outputs so the bankruptcy of one implies the bankruptcy of the other.*

Now consider the vector of outputs $(2.4, 2.4, 0.2)$ *yielding profits* $(12, 12, 0.2)$. *These outputs are not* BF *because, firm* 1 *or firm* 2, *can produce* 6 *units of output and make firm* 3 *bankrupt. But we will see that these outputs are yielded by an SPNE.*

Now consider the following trigger strategies: $x_i^0 = 2.4$, $i = 1, 2$, $x_3^0 = 0.2$. *If* x_i^{t-r} *equals the previous outputs for all* $r \in \{1, 2, .., t\}$ *and* $i \in \{1, 2, 3\}$, *then each firm produces the same output than in perido* 0. *Otherwise* $x_i^t = 3.33$, $i = 1, 2$, $x_3^t = 0$.

For δ *sufficiently close to* 1, *the previous strategies constitute an* $SPNE$ *that generates outputs* $x_i^t = x_i^J$ *for all* $t = 0, 2, ..., T, ...$ *and* $i = 1, 2, 3$. *The proof is virtually identical to that of the observation at the beginning of the Section.*

Thus, when $n > 2$ the outputs yielded by $SPNE$ are not a subset of BF allocations. But the concept of BF allocations plays an important role in the results in the next section.

To close this section, we show that the introduction of bankruptcy constraints reduces the outputs which can be sustained as NE with respect to those sustained as an NE of a standard repeated game for a given δ.

Let $\Gamma(\delta)$ be the infinitely repeated constituent game without bankruptcy considerations when the discount factor is δ. Let $\Gamma^{BC}(\delta)$ be the game with bankruptcy constraints when the discount factor is δ.

We first prove that no bankruptcy occurs in NE. This is due to the fact that firms can escape bankruptcy by producing zero.

Lemma 1. *No firm goes bankrupt in any NE outcomes of* $\Gamma^{BC}(\delta)$.

Proof. Suppose that firm i goes bankrupt in some period t, which happens only if its profit in t is negative. Since the profits after bankruptcy are always 0, the i's continuation payoff on t is also negative. However, producing nothing at t assures 0 profits, so firm i can profitably deviate by choosing $x_i^t = 0$ on t. Thus we derive contradiction. ■

Note that the statement in Lemma 1 refers to Nash equilibrium (NE) outcomes. This implies that bankruptcy never happens both on and off the equilibrium path of any subgame perfect Nash equilibrium (SPNE).

The next three propositions establish the connections between equilibria of $\Gamma(\delta)$ and $\Gamma^{BC}(\delta)$. Note that bankrupt firms must choose zero output in $\Gamma^{BC}(\delta)$ while no such constraint is imposed in $\Gamma(\delta)$. This implies that equilibrium strategies after a history such that some firm goes bankrupt in $\Gamma^{BC}(\delta)$ must be different from those in $\Gamma(\delta)$ unless producing zero output (by that firm) happens to be an equilibrium in $\Gamma(\delta)$. That is, equilibrium strategies in two models usually fail to exactly coincide. However, as the following propositions show, equilibrium outcomes, i.e., action profiles induced by equilibrium strategies, have strong connections.

Let us first show a benchmark result which connects SPNE outcomes of the repeated Cournot game and those of our financially constrained model.

Proposition 2. *Any SPNE outcome of $\Gamma(\delta)$ is sustained by an SPNE of $\Gamma^{BC}(\delta)$ if every action profile induced by the former equilibrium strategy (after any history) is BF.*

Proof. Let σ be an SPNE strategy of $\Gamma(\delta)$ such that action profiles induced by σ after any history are BF. Then, no firm goes bankrupt if the firms follow (equilibrium path of) σ in $\Gamma^{BC}(\delta)$, and each firm cannot make its rival bankrupt by unilaterally deviating from that. This property, combined with Lemma 1, implies that if some strategy σ_i' is a profitable deviation for firm i in $\Gamma^{BC}(\delta)$, the same σ_i' must also be profitable in $\Gamma(\delta)$. However, by assumption, σ is an SPNE in $\Gamma(\delta)$, so there is no such profitable deviation in $\Gamma(\delta)$. Therefore, as long as equilibrium path of σ is played in $\Gamma^{BC}(\delta)$, no firm can profitably deviate, which completes the proof. ■

Roughly speaking, Proposition 2 states that an SPNE of $\Gamma(\delta)$ also becomes an SPNE of $\Gamma^{BC}(\delta)$ under the condition that the output profiles on and off the equilibrium path are all BF. This proposition implies that introducing the possibility of bankruptcy does not change the SPNE if the output profiles induced by the equilibrium are BF.

If we focus on NE rather than SPNE, direct connection can be obtained. Note that, since our equilibrium concept is NE, we no longer need to check whether equilibrium off-path strategies are a Nash equilibrium of the relevant subgame.

Proposition 3. *Any NE outcome of* $\Gamma^{BC}(\delta)$ *is sustained by an NE of* $\Gamma(\delta)$.

Proof. Note that a feasible output after any history in $\Gamma^{BC}(\delta)$ (i.e. any nonnegative output if the firm has not gone bankrupt until the previous period and zero otherwise) is always feasible in $\Gamma(\delta)$. This implies that, for any strategy σ in $\Gamma^{BC}(\delta)$, its continuation payoffs (both on and off the equilibrium paths) can be completely replicated in $\Gamma(\delta)$. Therefore, if σ is NE in $\Gamma^{BC}(\delta)$, the corresponding strategy in $\Gamma(\delta)$ must also be NE. ■

Proposition 3 shows that any NE outcome in the model with bankruptcy can be sustained as an NE in the model without bankruptcy. This means that the introduction of financial constraints does not create new outcomes, which is good news given the multiplicity of outcomes that can be sustained as NE in the Cournot repeated game.

Since SPNE outcomes are subset of NE outcomes, the following holds immediately.

Corollary 1. *Any SPNE outcome of* $\Gamma^{BC}(\delta)$ *is sustained by an NE of* $\Gamma(\delta)$.

Let us now consider the situation in which repeated play of static NE, i.e., Cournot output, constitutes SPNE in our financially constrained model no matter which subset of the firms remains active. Further, we assume that active firms always employ trigger strategies as a punishment device. Finally, we assume that profits associated to Cournot outputs of any subset of firms are decreasing with the entry of a new firm. This property holds under reasonable conditions which include concave inverse demand and convex costs.[6] Then, the following proposition holds.

Proposition 4. *Any SPNE outcome of* $\Gamma^{BC}(\delta)$ *sustained by the trigger strategy is also sustained as an SPNE by the trigger strategy in* $\Gamma(\delta)$.

Proof. Since the individual payoff (of any active firm) associated with Cournot output weakly increases as the set of active firm shrinks, the static equilibrium payoff is weakly larger when the market becomes less competitive. This implies that punishment payoff achieved by a trigger

[6] See, for example, Proposition 2 in Corchón (1994) and the references therein about previous work.

strategy in $\Gamma^{BC}(\delta)$ (depending on which firms are active) must be weakly higher than that in $\Gamma(\delta)$, since all firms are trivially active in $\Gamma(\delta)$. Therefore, if no deviation is profitable (against the trigger strategy) in $\Gamma^{BC}(\delta)$, it must also be the case when no financial constraint is imposed, because the punishment induced by the trigger strategy becomes severer in $\Gamma(\delta)$. ■

This proposition implies the following.

Corollary 2. *For any output profile x, the set of discount factors under which trigger strategies sustain x is (weakly) larger in a game without financial constraints than in the same game with financial constraints.*

That is, given that punishing strategy is uniquely fixed to the repeated play of static equilibrium, financial constraints make collusive behaviors more difficult to sustain.

4. Equilibrium with Increasing Average Cost and Patient Firms

In the previous section, we have seen that when firms are patient, not all the average payoffs larger than the minimax payoff can be supported as an SPNE when $n = 2$, only those yielded by a sequence of BF outputs (Proposition 1). This contrasts with the folk theorem of repeated games which says that when firms are sufficiently patient, arbitrary feasible payoffs larger than the minimax can be obtained as the average payoff of an SPNE of the repeated game. Thus a natural question is to ask what kind of payoffs can be supported as SPNE in our model. This section is devoted to this task under the following additional assumption:

Assumption 1. All firms have an increasing average cost, and for any subset $S \subseteq N$, there is a unique $\mathbf{x}^S = (x_i^S)_{i \in S}$ with $x_i^S \neq 0$ for all $i \in S$ such that $\pi_i(\mathbf{x}^S) = 0$ for all $i \in S$.

It is easy to find sufficient conditions on demand and cost functions such that Assumption 1 holds (for example Assumption 1 holds in Example 1). What this assumption requires is that firms do not be very different from each other. In what follows, whenever we use the notation $\mathbf{x}^S$ for any $S \subseteq N$ we refer to the vector described in the Assumption 1.

Assumption 1 guarantees that the set of BF output profiles is not empty. In particular, the output profile $\mathbf{x}^N$ is BF because for a firm to make another one bankrupt, it has to increase output, but since at $\mathbf{x}^N$ its profit is zero and average cost is increasing, when this firm increases output it gets negative profits.

We now adapt the standard definition of a minimax payoff to the case in which outputs are constrained to be BF. Let B_{-i} be the set of outputs $\mathbf{x}_{-i}$ such that there exists an output for firm i such that $(x_i, \mathbf{x}_{-i})$ is BF (since the set of BF output profiles is not empty, this set is well defined). For each $\mathbf{x}_{-i} \in B_{-i}$, let $B_i(\mathbf{x}_{-i}) = \{x_i \mid (x_i, \mathbf{x}_{-i}) \text{ is } BF\}$. **The minimax BF payoff for firm i** is defined as:

$$\pi_{im} = \min_{\mathbf{x}_{-i} \in B_{-i}} \max_{x_i \in B_i(\mathbf{x}_{-i})} \pi_i(x_i, \mathbf{x}_{-i}). \tag{4.1}$$

The following lemma gives us a handier expression for the minimax BF payoff under Assumption 1. The proof is in the Appendix.

Lemma 2. *Under Assumption 1, the minimax BF payoff is*

$$\pi_{im} = \max_{x_i \in [0, x_i^N]} \pi_i(x_i, \mathbf{x}_{-i}^N). \tag{4.2}$$

In the following example we show the isoprofits corresponding to the minimax BF payoff for two firms.

Example 4. *Let us consider two firms with payoffs* $\pi_1(x_1, x_2) = (10 - x_1 - x_2)x_1 - x_1^2$, *and* $\pi_2(x_1, x_2) = (10 - x_1 - x_2)x_2 - \frac{1}{5}x_2^2$. *In Figure 3, the intersection of the linear solid lines provides* (x_1^N, x_2^N). *The dash lines correspond to the best replies of the firms and the curve lines corresponds to the minimax BF isoprofits.*

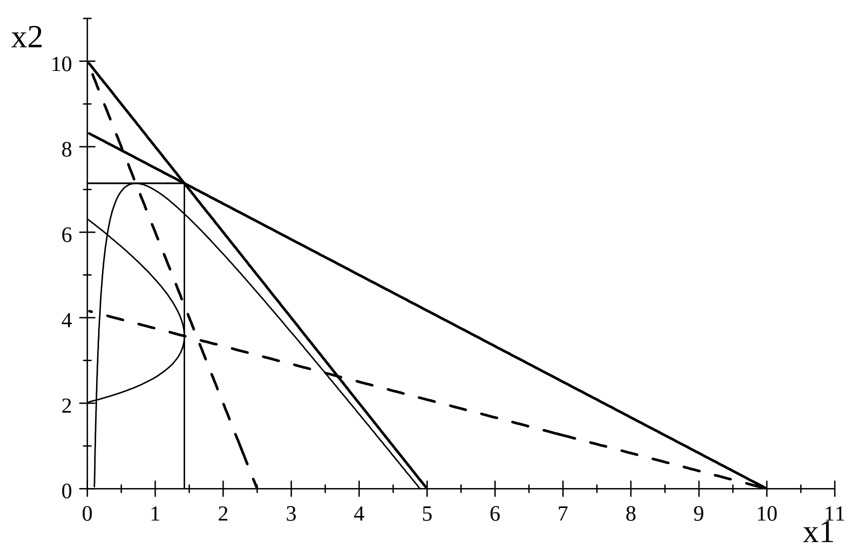

Figure 3

Note that the standard minimax, when applied to our model, yields a minimax payoff of zero because firms other than i could produce an output, call it $\bar{\mathbf{x}}_{-i}$, such that the best reply of i is to produce zero (in Figure 3, $\bar{\mathbf{x}}_{-1} = 10$ and $\bar{\mathbf{x}}_{-2} = 10$). But $\bar{\mathbf{x}}_{-i}$ might not fulfill the definition of minimax BF payoffs because it might drive all firms but i to bankruptcy . To highlight the difference between the standard minimax and our minimax BF we note the following:

Remark 1. *Let i be a firm such that $AVC_i(0) \leq AVC_j(0)$ all j. Let $\bar{\mathbf{x}}$ be an output profile which yields the standard minimax payoffs to i. Then, all firms other than i producing a positive output at $\bar{\mathbf{x}}$ have negative profits.*

The Remark follows from the fact if $\bar{x}_j > 0$, $AVC_j(\bar{x}_j) > AVC_j(0) \geq AVC_i(0) \geq p(\bar{X})$ where $\bar{X}$ is the aggregate output associated to $\bar{\mathbf{x}}$. The implication is that standard minimax punishes severely the minimaxing firms. In the minimax BF this extreme punishment is avoided. This implies that the minimax BF payoff are strictly positive because under Assumption 1, $x_i^N > 0$ and $\pi_i(x_i^N, \mathbf{x}_{-i}^N) = 0$ imply that reducing x_i^N the payoff for firm i becomes positive.

We are now ready to begin our study. We first consider the possibility of sustaining payoffs less than minimax BF payoffs. The next proposition shows that, for a sufficiently large δ, no $SPNE$ of the dynamic game can give any firm a payoff lower than its minimax BF payoff. The proof is in the Appendix.

Proposition 5. *Under Assumption 1, $\delta' \in (0, 1)$ exists such that for all $\delta \in (\delta', 1)$, $\pi_i < \pi_{im}$ cannot be sustained in any $SPNE$.*

It might appear surprising that Proposition 5 needs patient firms. But when discount is heavy, firms may have little incentives to engage in predatory activities and allocations which are not BF might be sustained. For instance if $\delta = 0$ only the payoffs corresponding to the Cournot equilibrium can be sustained as NE, but Cournot equilibrium outputs may be not BF, see Figure 3.

In the next example we show an implication of Proposition 5. For firms that hardly discount the future, a BF collusive outcome can not be sustained as $SPNE$.

Example 5. *There are two firms with linear inverse demand $p(X) = \max\{3 - X, 0\}$ and cost functions $c_1(x_1) = x_1$ and $c_2(x_2) = x_2$. To avoid bankruptcy firm 1 is required to make at least -0.25 profits and firm 2 is required to make at least -1 profits. As we noted below the definition of a*

BF profile, we can define a new profit function as $\tilde{\pi}_i(\mathbf{x}) \equiv \pi_i(\mathbf{x}) - v_i$ *and redefine* BF *with respect to this new profit function. Disregarding the non negativity constraint in the inverse demand function we have that* $\tilde{\pi}_1(x_1, x_2) = (3 - x_1 - x_2)x_1 - x_1 + 0.25$ *and* $\tilde{\pi}_2(x_1, x_2) = (3 - x_1 - x_2)x_2 - x_2 + 1$*. Thus it is as if firms had cost functions* $\tilde{c}_1(x_1) = x_1 - 0.25$ *and* $\tilde{c}_2(x_2) = x_2 - 1$ *so average costs are increasing.*

Let $x_1^N \neq 0$ *and* $x_2^N \neq 0$ *be such that* $\tilde{\pi}_1(x_1^N, x_2^N) = \tilde{\pi}_2(x_1^N, x_2^N) = 0$*. These outputs amount to* $x_1^N = 0.5$, $x_2^N = 2$*. Recall from Example 1 (see Lemma 3 in the Appendix for a formal proof), that the set of* BF *output profiles is*

$$BF = \{(x_1, x_2)/0 \leq x_1 \leq 0.5,\ 0 \leq x_2 \leq 2\}. \tag{4.3}$$

The minimax BF *payoffs for each firm are:*

$$\pi_{1m} = \max_{x_1 \in [0, x_1^N]} \tilde{\pi}_1(x_i, \mathbf{x}_2^N) = \tilde{\pi}_1(0, 2) = 0.25, \tag{4.4}$$

$$\pi_{2m} = \max_{x_2 \in [0, x_2^N]} \tilde{\pi}_2(\mathbf{x}_1^N, x_2) = \tilde{\pi}_2(0.5, 0.75) = 1.562. \tag{4.5}$$

The Cournot output profile is $(x_1^C, x_2^C) = (2/3, 2/3)$*. Note that the Cournot output profile is not* BF *because* $x_1^C > 0.5$*. Firm 2 can make firm 1 bankrupt. The collusive outcome implies that* $x_1 + x_2 = 1$*. If the output is equally shared among firms the resulting output profile is* BF *with associated payoffs* $\tilde{\pi}_1(0.5, 0.5) = 0.75$ *and* $\tilde{\pi}_2(0.5, 0.5) = 1.5$*. Under this collusive outcome firm 2 has a payoff below its minimax* BF *payoff. If firms discount the future very little, this collusive outcome can not be supported as an SPNE of the dynamic game as we have shown in Proposition 5. For this particular example that will happen for a discount factor* $\delta \geq 0.8$*.*

We are now ready to prove a folk theorem regarding BF allocations. We say that π_i is an **individually rational** BF **payoff** if $\pi_i > \pi_{im}$. An individually rational BF vector payoff $(\pi_i)_{i \in N}$ is **feasible** if a BF output profile $(x_1, ..., x_n)$ exists such that $\pi_i = \pi_i(x_1, x_2, .., x_n)$ for all $i \in N$. In Figure 3 the BF output profiles that give an individually rational BF payoff are the ones in the area limited by the minimax BF isoprofits.

Proposition 6. *Suppose Assumption 1 holds. Let* $\pi = (\pi_i)_{i \in N}$ *be a feasible and individually rational* BF *payoff vector. Then,* $\delta' \in (0, 1)$ *exists such that for all* $\delta \in (\delta', 1)$, π *is the average payoffs in some* $SPNE$*.*

The proof is given by constructing an equilibrium adapting the one proposed by Fudenberg and Maskin (1986) to this framework with BF considerations. Deviations of a firm from the BF output profile that gives payoffs π are punished by firms minimaxing each other in the BF set during finite periods. Details are given in the Appendix.[7]

When $n > 2$, we saw in Example 3 that payoffs corresponding to outputs outside the BF set can be sustained as SPNE. Our next result extends this example to the case where payoffs are above minimax BF payoffs and deviations are punished not with Cournot output forever but with firms mutually minimaxing each other in the BF set for a certain period. For simplicity, we work out the case of $n = 3$, even though our results can be extended to any $n > 3$ at the cost of introducing some additional notation. We denote by π_{im}^{ij} the minimax BF payoff of firm i when only firms i and j are in the market.

Proposition 7. *Suppose Assumption 1 holds. Let* (x_1, x_2, x_3) *be a non* BF *output profile such that* $\pi_i(x_1, x_2, x_3) > \pi_{im}$ *for all* $i \in \{1, 2, 3\}$*, and* $\pi_i(x_1, x_2, x_3) > \pi_{im}^{ij}$ *for all* $i, j \in \{1, 2, 3\}$*. Then,* $\delta' \in (0, 1)$ *exists such that for all* $\delta \in (\delta', 1)$*,* $\pi = (\pi_i)_{i \in \{1,2,3\}}$ *is the average payoffs in some* $SPNE$*.*

5. Final Remarks

In this paper we have developed a theory of dynamic competition in which firms may make each other bankrupt. We have shown that this theory is tractable and provides new insights into the theory of dynamic games.

Our results, though, are obtained at the cost of making several simplifications to make the model tractable. For instance, we did not consider coalitions of firms in the definition of BF allocations or refinements of $SPNE$ (such as renegotiation-proof) to get rid of some equilibria. It is likely that these extensions would not qualitatively alter the nature of our results. However, other issues neglected here might affect our conclusions significantly. Among these the following might be of particular importance.

[7] The proofs of Propositions 6 and 7 rely on the idea of "mutual minimaxing" as a punishment, which is originally proposed by Fudenberg and Maskin (1986) for only two players cases. In general, this method does not extend to three or more players. For example, suppose n = 3. Then, there may exist no combination of action profiles (a_1, a_2, a_3) such that (a_2, a_3) minimax player 1, (a_1, a_3) minimax player 2, and (a_1, a_3) minimax player 3. However, thanks to Lemma 2 the existence of such an action profile (within BF profiles) is always guaranteed in our model, which enables players to minimax each other even when n is larger than 2. This dramatically simplifies our proofs.

Mixed strategies

Throughout the paper we have assumed that firms only use pure strategies, but a good way of avoiding bankruptcy might be to use mixed strategies as boxers use random movements to avoid easy hits. We argue that when $n = 2$, if the outputs played in equilibrium involve a randomization and one of these outputs is not BF, for δ close enough to 1, the best strategy of the other firm consists of choosing an output that will make this firm bankrupt. This is because sooner or later the probability that the output which is not BF is played is close to 1 so this firm will be ruined and the predating firm will enjoy monopoly profits forever. Thus, in this case the BF set gives us a indication of which type of outputs will arise in equilibria, regardless of what kind of strategy is played by the agents. However, in other cases the introduction of mixed strategies might substantially enlarge the set of allocations that might be supported as equilibria of the dynamic game.

No accumulation

In this paper we focused on outputs that make other firms bankrupt, but we did not consider the other side of bankruptcy, namely the funds that might support or deter aggressive strategies (the "deep pocket" argument). Our result when $n = 2$ might survive when accumulation is considered. Indeed, suppose as a first approximation that in each period the firms transfer an exogenous quantity of their wealth to the next period. Then in each period we can define a BF set that depends on the wealth accumulated by each firm. If in a period the output chosen by, say, firm 1, is not BF, firm 2 may get rid of firm 1 and enjoy monopoly profits forever. When δ is sufficiently close to 1, this is optimal for firm 2. In other cases, accumulation of profits might play an important role in shaping the NE set as in the model of Rosenthal and Rubinstein (1984).[8]

Credit

If credit is given on the basis of past performance, the redefinition of the BF set sketched in the previous paragraph can be applied here and credits can be incorporated into the model. However, if credit is given on the basis of future performance, we have a problem because future performance also depends on credit (via the BF constraints), which makes this problem extremely complex. This points to a deep conceptual problem about credit in oligopolistic markets where firms might

[8] They characterize a subset of the Nash equilibria in the repeated game with no discounting (i.e., $\delta = 1$) where each player regards ruin of the other player as the best possible outcome and his own ruin as the worst possible outcome.

be made bankrupt. This topic should be the subject of future research.

Entry

In this paper we assumed a given number of competitors. This implies that the disappearance of a firm does not bring a new one into the market. Of course this should not be taken literally. What we mean is that if entry does not quickly follow, it makes sense, as a first approximation, to analyze the model with a given number of firms. For instance when $n = 2$ and demand and costs are linear, ruining a firm is a good investment even if monopoly lasts for one period (this example is available under request). In other cases, though, the nature of equilibria will be altered if, for instance, entry immediately follows the ruin of a competitor as in the model of Rosenthal and Spady (1989).[9]

Buying Competitors

In our model, there is no option to buy a firm. Sometimes it is argued that buying an opponent may be a cheaper and safer strategy than ruining it. We do not deny that buying competitors plays an important role in business practices. However, we contend that under the option of buying, ruining a competitor is irrational. First, buying competitors may be forbidden by a regulatory body because of anticompetitive effects. Second, when the owner of a firm sells it to competitors, this does not stop her from creating a new firm and financing it with the money received from selling the old one. In other words, selling a firm is not equivalent to a contract in which the owner commits not to enter into a market again. Thus, bankruptcy may be the only credible way of getting rid of a competitor. Finally, buying and ruining competitors may complement each other because the acquisition value may depend on the aggressiveness of the buyer in the past; see Burns (1986) for some evidence in the American tobacco industry. Thus, it seems that a better understanding of the mechanism of ruin might help to further enhancement of our understanding of how the buying mechanism works in this case.

Summing up, the model presented in this paper sheds some light on certain aspects of the equilibrium in oligopolistic markets in which firms may make each other bankrupt . We hope that the insights obtained here can be used in further research in this area.

[9] They consider a prisoner's dilemma in continuous time in a market with room for two firms only. When a firm goes bankrupt, this firm is immediately replaced by a new entrant. They show that some kind of predatory behavior can arise in equilibrium.

6. APPENDIX

Proof of Lemma 2. Since the payoff of firm i is affected by the aggregate output of the other firms but not by which firm is producing it, the worst situation for firm i in the BF set is the one with the maximal aggregate output in the set B_{-i}. In order to find this maximal aggregate output we consider a superset of B_{-i} (denoted by $\bar{B}_{-i}$). We show that this superset is compact and therefore the maximal aggregate output in the superset exists and we show that it is attained at $\mathbf{x}_{-i}^N$. Since $\mathbf{x}^N = (x_i^N, \mathbf{x}_{-i}^N)$ is BF, $\mathbf{x}_{-i}^N \in B_{-i}$. Thus, the maximal aggregate output in the set B_{-i} is attained at $\mathbf{x}_{-i}^N$.

We denote by $\bar{B}_{-i}$ the set of all ouputs, $\mathbf{x}_{-i}$, such that firm i cannot make any other firm bankrupt. Note that the set $\bar{B}_{-i}$ might be different from the set B_{-i}. This is because given $\mathbf{x}_{-i}$ firm i cannot make any other firm bankrupt, but a firm different from i, say j, can make firm k bankrupt. But, $\bar{B}_{-i}$ is a superset of B_{-i} because for all $\mathbf{x}_{-i} \in B_{-i}$ there is x_i such that $(\mathbf{x}_{-i}, x_i)$ is BF and therefore firm i cannot make any other firm bankrupt.

Finally, note that the set $\bar{B}_{-i}$ is characterized by the following inequalities:

$$\pi_j(\bar{x}_i, \mathbf{x}_{-i}) \geq 0, \text{ for all } j \neq i, \tag{6.1}$$

where $\bar{x}_i = \bar{x}_i(\mathbf{x}_{-i}) > 0$ is such that

$$\pi_i(\bar{x}_i, \mathbf{x}_{-i}) = 0. \tag{6.2}$$

Thus, the set $\bar{B}_{-i}$ is compact and therefore the maximal aggregate output in $\bar{B}_{-i}$ exists. We show that the maximum is attained at $\mathbf{x}_{-i}^N$. Let $\mathbf{x}_{-i} \in \arg\max_{\mathbf{x}_{-i} \in \bar{B}_{-i}} X_{-i}$ where $X_{-i} = X - x_i$. We show first in Step 1 that $X_{-i} = X_{-i}^N$. Using this information we show in Step 2 that $\mathbf{x}_{-i} = \mathbf{x}_{-i}^N$.

Step 1. Suppose $X_{-i} > X_{-i}^N$. Thus, there is at least one agent $k \neq i$ such that $x_k > x_k^N$. Note first that $X_{-i} \leq X^N$, because otherwise $p(X_{-i}) - AVC_k(x_k) < p(X^N) - AVC_k(x_k^N) = 0$. In this case, at $\mathbf{x}_{-i}$, agent i can make agent k bankrupt by producing zero, which contradicts that $\mathbf{x}_{-i} \in \bar{B}_{-i}$. Thus, $X_{-i}^N < X_{-i} \leq X^N$. Let $\hat{x}_i$ be such that $\hat{x}_i + X_{-i} = X^N$. Since $X_{-i}^N < X_{-i}$, then $\hat{x}_i < x_i^N$. By producing $\hat{x}_i$ firm i can make firm k bankrupt and keep positive profits because $p(\hat{x}_i + X_{-i}) - AVC_k(x_k) < p(X^N) - AVC_k(x_k^N) = 0$. Which again contradicts that $\mathbf{x}_{-i} \in \bar{B}_{-i}$. Thus, $X_{-i} = X_{-i}^N$.

Step 2. Finally, we show that $x_j = x_j^N$ for all $j \neq i$. If this is not the case, there is at least one firm k such that $x_k > x_k^N$. But then, firm i by producing x_i^N can make firm k bankrupt keeping

non negative profits for itself in contradiction with $\mathbf{x}_{-i} \in \bar{B}_{-i}$.
Thus, the maximum is reached at $\mathbf{x}^N_{-i}$. Since $(x^N_i, \mathbf{x}^N_{-i})$ is BF, $\mathbf{x}^N_{-i} \in B_{-i}$. Therefore, $\mathbf{x}^N_{-i} = \arg\max_{x_{-i} \in B_{-i}} X_{-i}$. Since $B_i(\mathbf{x}^N_{-i}) = [0, x^N_i]$, the minimax BF payoff is reduced to:

$$\pi_{im} = \max_{x_i \in [0, x_i^N]} \pi_i(x_i, \mathbf{x}^N_{-i}), \tag{6.3}$$

and the proof is completed. ■

To formally prove Proposition 5, we need the following lemmas.

Lemma 3. *Suppose Assumption 1 holds and $N = \{1, 2\}$. Then the set of BF output profiles is:*

$$BF = \{(x_1, x_2)/0 \leq x_1 \leq x^N_1, 0 \leq x_2 \leq x^N_2\}. \tag{6.4}$$

Proof. Let us see first that an output profile (x_1, x_2) such that $x_1 \leq x^N_1$, and $x_2 \leq x^N_2$ is BF. Note that at (x_1, x_2) profits for both firms are non negative. Trivially if one firm is producing zero output this firm can no be made bankrupt by the other firm. Suppose, without lost of generality, that $x_2 \neq 0$. Let us see that firm 1 can not make firm 2 bankrupt (the same argument will apply for firm 2 against firm 1). Let $\hat{x}_1$ be such that $p(\hat{x}_1 + x_2) - AV(x_2) = 0$. Since $x_2 \leq x^N_2$, $p(x^N_1 + x^N_2) - AV(x^N_2) = 0$, and average cost is increasing, $\hat{x}_1 + x_2 \geq x^N_1 + x^N_2$, which implies that $\hat{x}_1 \geq x^N_1$ thus profits for firm 1 at $(\hat{x}_1 + x_2)$ are non positive, which implies that firm 1, by increasing its output, can not make firm 2 bankrtupt without making itself bankrupt also.
Finally, let (x_1, x_2) be an output profile such that both firms have non negative profits, and suppose that $x_2 > x^N_2$. Let us see that firm 1 by increasing its output can make firm 2 bankrupt keeping positive profits for itself. Let $\hat{x}_1$ be such that $\hat{x}_1 + x_2 = x^N_1 + x^N_2$, since $x_2 > x^N_2$, $\hat{x}_1 < x^N_1$, thus, at $(\hat{x}_1, x_2)$ firm 1 has positive profits. But since average cost is increasing and $x_2 > x^N_2$, firm 2 at $(\hat{x}_1, x_2)$ is bankrupt. ■

Lemma 4. *Let $\mathbf{x} = (x_1, .., x_n)$ be such that all firms have non negative profits. Under Assumption 1, if $X_{-i} > X^N_{-i}$, firm i can make some of the other firms bankrupt.*

Proof. Given that $X_{-i} > X^N_{-i}$, there is at least one agent $k \neq i$ such that $x_k > x^N_k$. If $X_{-i} > X^N$, then $X > X^N$. But in this case, $p(X) - AVC_k(x_k) < p(X^N) - AVC_k(x^N_k) = 0$. Which contradicts that at $\mathbf{x}$ firms have non negative profits. Thus, $X_{-i} \leq X^N$. Let $\hat{x}_i$ be such that $\hat{x}_i + X_{-i} = X^N$. Since $X^N_{-i} < X_{-i}$, then $\hat{x}_i < x^N_i$. By producing $\hat{x}_i$ firm i can bankrupt firm k

and keep non negative profits because $p(\hat{x}_i + X_{-i}) - AVC_k(x_k) < p(X^N) - AVC_k(x_k^N) = 0$, and $AVC_i(\hat{x}_i) < AVC_i(x_i^N)$. ■

Lemma 5. *Let S and T be two groups of firms such that $S \subset T$, and let $k \in S$. Under Assumption 1, the minimax BF payoff for firm k when the set of firms is S (denoted by π_{km}^S) is larger than the minimax BF payoff for firm k when the set of firms is T (denoted by π_{km}^T).*

Proof. Let $\mathbf{x}^S$ and $\mathbf{x}^T$ as described in Assumption 1. Note first that $X^S < X^T$ and since $AVC_k(x_k^T) = p(X^T) < p(X^S) = AVC_k(x_k^S)$ and average cost is increasing, then $x_k^T < x_k^S$. Thus, $X_{-k}^S < X_{-k}^T$. Therefore,

$$\pi_{km}^T = \max_{x_k \in [0, x_k^T]} \pi_k(x_k, X_{-k}^T) < \max_{x_k \in [0, x_k^T]} \pi_k(x_k, X_{-k}^S) \tag{6.5}$$

because profits are decreasing in the sum of the outputs of the other firms. And finally, since $x_k^T < x_k^S$,

$$\pi_{km}^T = \max_{x_k \in [0, x_k^T]} \pi_k(x_k, X_{-k}^T) < \max_{x_k \in [0, x_k^S]} \pi_k(x_k, X_{-k}^S) = \pi_{km}^S. \tag{6.6}$$

Thus, $\pi_{km}^T < \pi_{km}^S$. ■

Proof of Proposition 5. We prove the proposition by induction on the number of firms. We start by showing that the statement is true when there are only two firms in the market, $N = \{1, 2\}$. Let $\delta^i \in (0, 1)$ be such $\delta^i \pi_i^M = \pi_{im}$ where π_i^M is the monopoly profit and π_{im} is the minimax BF payoff. Since $\pi_i^M > \pi_{im}$, δ^i exists. Let $\delta' = \max_{i \in N} \delta^i$ and let $\delta \in (\delta', 1)$.
Note first that by Lemma 2, when $N = \{1, 2\}$ the minimax BF payoff can be expressed as:

$$\pi_{im} = \min_{x_j \in [0, x_j^N]} \max_{x_i \in [0, x_i^N]} \pi_i(x_i, x_j). \tag{6.7}$$

If $x_j^t \in [0, x_j^N]$ for all t on and off the equilibrium path, firm i could have achieved at least π_{im} irrespective of δ by choosing an output $x_i^t \in [0, x_i^N]$ (the standard argument in repeated games can be applied here because in this case, as Lemma 3 shows, the output profile at each t is in the BF set). Therefore, if $\pi_i < \pi_{im}$ happens in equilibrium, $x_j^t > x_j^N$ must hold for some t either on or off the equilibrium path. We show that if this is the case, the continuation payoff for i at t in equilibrium, P_i^t, must be such that $P_i^t \geq \delta \pi_i^M$, where π_i^M is the monopoly profit. Suppose that $P_i^t < \delta \pi_i^M$; since $x_j^t > x_j^N$, by the characterization of the BF set given in Lemma 3, firm i can make firm j bankrupt retaining non-negative profits, and can achieve a monopoly profit in every period from $t+1$. Although the bankruptcy of firm j has a cost at period t, the continuation payoff for firm i if it deviates from equilibrium will be at least $\delta\pi_i^M$. However, if $\delta\pi_i^M > P_i^t$ such a deviation would be profitable for firm i and would contradict the notion that we are in equilibrium. Thus, $P_i^t \geq \delta\pi_i^M$. Since $\delta \in (\delta', 1)$, $P_i^t > \pi_{im}$. Thus, π_i must exceed π_{im} which concludes the proof for $n = 2$.

Suppose that the proposition is true for $n-1$ firms. We show that it is true for n firms.

By 4.2, firm i could have achieved at least π_{im} if $X_{-i}^t \leq X_{-i}^N$ for all t on and off the equilibrium path irrespective of δ. Therefore, if $\pi_i < \pi_{im}$ occurs in equilibrium, $X_{-i}^t > X_{-i}^N$ for some t, and if this is the case, at t firm i could make some other firm bankrupt. Suppose, without lost of generality, that firm i can make firm k bankrupt. The equilibrium strategies constitute a NE in any subgame, in particular in the subgame in which all firms but k survive. Let denote by π_i^{N-k} a possible payoff that firm i can obtain in the equilibrium of the subgame with all firms but firm k. Let Π_i^{N-k} be the set of all those possible payoffs. Notice first that, by the induction hypothesis, for each firm k, there is δ^{N-k} such that for all $\delta \in (\delta^{N-k}, 1)$, $\pi_i^{N-k} \geq \pi_{im}^{N-k}$, where π_{im}^{N-k} is the minimax BF payoff of firm i when the firms in the market are $N \setminus \{k\}$. By Lemma 5, $\pi_{im}^{N-k} > \pi_{im}^N$. Let $\delta^i \in (0,1)$ be such that $\delta^i \pi_{im}^{N-k} = \pi_{im}^N$, and let $\delta' = \max(\max_k \delta^{N-k}, \max_i \delta^i)$. Let $\delta \in (\delta', 1)$. Let us see that the continuation payoff for i at t in equilibrium, P_i^t, must be such that $P_i^t \geq \delta\pi_i^{N-k}$ for some $\pi_i^{N-k} \in \Pi_i^{N-k}$. Suppose that $P_i^t < \delta\pi_i^{N-k}$ for all $\pi_i^{N-k} \in \Pi_i^{N-k}$. If this is the case, firm i can deviate in period t making firm k bankrupt and retaining non-negative profits and conforming with the initial strategy thereafter. Thus, firm i can achieve π_i^{N-k} profits in every period from $t+1$. Under this situation, the continuation payoff for firm i will be greater than $\delta\pi_i^{N-k}$. However, $\delta\pi_i^{N-k} > P_i^t$, which contradicts the notion that we are in equilibrium. Thus, $P_i^t \geq \delta\pi_i^{N-k}$ for some $\pi_i^{N-k} \in \Pi_i^{N-k}$. By the induction hypothesis, for $\delta \in (\delta', 1)$, $\pi_i^{N-k} \geq \pi_{im}^{N-k}$. Thus, $P_i^t \geq \delta\pi_i^{N-k} \geq \delta\pi_{im}^{N-k} > \delta^i\pi_{im}^{N-k} = \pi_{im}^N$, which implies that π_i must exceed π_{im}^N at some point which concludes the proof. ■

Proof of Proposition 6. The proof is given by constructing an equilibrium which is originally proposed by Fudenberg and Maskin (1986). Let $(\pi_i)_{i \in N}$ be feasible and individually rational BF payoff vector. By the definition of feasibility, there is a BF output profile $(x_1, .., x_n)$ such that

$$\pi_i = \pi_i(x_1, .., x_n) \text{ for } i \in N. \tag{6.8}$$

Suppose each firm $i \in N$ produces this x_i in each period if no deviation has occurred, but all $i \in N$

produces x_i^N, for T periods once one of them unilaterally deviates from the equilibrium path. If no one deviates during these T periods, then firms go back to the original path. Otherwise, if one of them deviates, then firms restart this phase for T more periods. We prove that this strategy actually constitutes an $SPNE$.

First consider a deviation from the equilibrium path. Suppose firm i produces $x_i' \neq x_i$ in some period, say period t. By the one-stage-deviation principle (e.g. Fudenberg and Tirole, 1991, p.110), a deviation is profitable if and only if firm i could profit by deviating from the original strategy in period t only and conforming thereafter. Therefore, firm i can benefit by deviation if and only if x_i' exists such that

$$(1-\delta)\pi_i(x_i', \mathbf{x}_{-i}) + (1-\delta)(\delta + .. + \delta^T)\pi_i(x_i^N, \mathbf{x}_{-i}^N) + \delta^{T+1}\pi_i > \pi_i, \tag{6.9}$$

or equivalently,

$$(1-\delta)\pi_i(x_i', \mathbf{x}_{-i}) + \delta^{T+1}\pi_i > (1-\delta)(1 + \delta + ... + \delta^T)\pi_i + \delta^{T+1}\pi_i, \tag{6.10}$$

which it holds whenever:

$$(1-\delta)\{(\pi_i(x_i', \mathbf{x}_{-i}) - \pi_i) - (\delta + ... + \delta^T)\pi_i\} > 0. \tag{6.11}$$

Let $\Delta_i = \max_{x_i'} \pi_i(x_i', \mathbf{x}_{-i}) - \pi_i$ and choose T such that

$$\Delta_i < T\pi_i. \tag{6.12}$$

Note that the left hand side of (6.11) is weakly less than

$$(1-\delta)\{\Delta_i - (\delta + ... + \delta^T)\pi_i\}. \tag{6.13}$$

This term is non-positive when δ is close to 1. Therefore, (6.11) cannot be satisfied for such T.

By the same argument as above, firm i can benefit by deviating from the mutual minmax phase if and only if x_i'' exists such that

$$\begin{aligned}&(1-\delta)\pi_i(x_i'', \mathbf{x}_{-i}^N) + (1-\delta)(\delta + ... + \delta^T)\pi_i(x_i^N, \mathbf{x}_{-i}^N) + \delta^{T+1}\pi_i \\ &> (1-\delta)(1 + \delta + ... + \delta^{T-1})\pi_i(x_i^N, \mathbf{x}_{-i}^N) + \delta^T\pi_i,\end{aligned} \tag{6.14}$$

which can be written as:

$$\pi_i(x_i'', \mathbf{x}_{-i}^N) > \delta^T\pi_i. \tag{6.15}$$

Note that $\pi_i(x_i'', \mathbf{x}_{-i}^N) \leq \max_{x_i \in [0,\hat{x}_i]} \pi_i(x_i, \mathbf{x}_{-i}^N) = \pi_{im}$. Since $\pi_i > \pi_{im}$ by assumption. This implies that (6.15) never holds when δ is close to 1.

Thus, both on and off the equilibrium paths, there is no profitable deviation when δ is sufficiently close to 1. Since π is an arbitrary feasible and individually rational BF payoff vector, the proof is complete. ■

Proof of Proposition 7. Suppose, without loss of generality, that only firm 3 can be made bankrupt. Suppose each firm $i \in \{1, 2, 3\}$ produces x_i in each period but if one of then deviates such that no firm is bankrupt, then firms start to produce x_i^N for T periods . If no one deviates during these T periods, then firms go back to the original path. Otherwise, if one of them deviates in one of these T periods, then firms restart this phase for T more periods. If one firm deviates by making firm 3 bankrupt, then firm 1 and 2 chose x_i^S, $S = \{1, 2\}$ for T periods. If no one deviates during this phase, then firms chose $(\bar{x}_1, \bar{x}_2)$ a BF output profile in the market with those two firms such $\pi_i(x_1, x_2, x_3) > \pi_i(\bar{x}_1, \bar{x}_2) > \pi_{im}^{ij}$. If one of them deviates from this phase, then firms restart this phase for T more periods.

We show that this strategy actually constitutes an $SPNE$.

First consider the deviation on the equilibrium path when no firm is made bankrupt. Suppose firm i produces $x_i' \neq x_i$ in some period, say period t such that this firm does not make any other firm bankrupt with this production level. By the one-stage-deviation principle, deviation is profitable if and only if firm i could profit by deviating from the original strategy in period t only and conforming thereafter. Therefore, firm i can benefit by deviation if and only if x_i' exists such that

$$(1-\delta)\pi_i(x_i', x_j, x_k) + (1-\delta)(\delta + .. + \delta^T)\pi_i(x_i^N, x_j^N, x_k^N) + \delta^{T+1}\pi_i > \pi_i, \tag{6.16}$$

or equivalently

$$(1-\delta)\pi_i(x_i', x_j, x_k) + \delta^{T+1}\pi_i > (1-\delta)(1 + \delta + ... + \delta^T)\pi_i + \delta^{T+1}\pi_i, \tag{6.17}$$

which it holds whenever

$$(1-\delta)\{(\pi_i(x_i', x_j, x_k) - \pi_i) - (\delta + ... + \delta^T)\pi_i\} > 0. \tag{6.18}$$

Let $\Delta_i = \max_{x_i'} \pi_i(x_i', x_j, x_k) - \pi_i$ and choose T such that

$$\Delta_i < T\pi_i. \tag{6.19}$$

Note that the left hand side of (6.18) is weakly less than

$$(1-\delta)\{\Delta_i - (\delta + ... + \delta^T)\pi_i\}. \tag{6.20}$$

This term is non-positive when δ is close to 1. Therefore, (6.18) cannot be satisfied for such T. Deviations from the mutual minmax phase cannot make any firm bankrupt because (x_1^N, x_2^N, x_3^N) is BF. Thus, by the same argument as above, firm i can benefit by deviating from the mutual minmax phase if and only x_i'' exits such that

$$\begin{aligned}&(1-\delta)\pi_i(x_i'', x_j^N, x_k^N) + (1-\delta)(\delta + ... + \delta^T)\pi_i(x_i^N, x_j^N, x_k^N) + \delta^{T+1}\pi_i \\ &> (1-\delta)(1+\delta+...+\delta^{T-1})\pi_i(x_i^N, x_j^N, x_k^N) + \delta^T\pi_i,\end{aligned} \tag{6.21}$$

which can be written as:

$$\pi_i(x_i'', x_j^N, x_k^N) > \delta^T\pi_i. \tag{6.22}$$

Note that $\pi_i(x_i'', x_j^N, x_k^N) \leq \max_{x_i \in [0,\hat{x}_i]} \pi_i(x_i, x_j^N, x_k^N) = \pi_{im}$. Since $\pi_i > \pi_{im}$ by assumption. This implies that (6.22) never holds when δ is close to 1.

Now, consider deviations whereby one firm can make firm 3 bankrupt. Suppose this firm is firm 1. Firm 1 can benefit by deviating if and only if x_1' exits that make firm 3 bankrupt and such that

$$\begin{aligned}&(1-\delta)\pi_1(x_1', x_2, x_3) + (1-\delta)(\delta + .. + \delta^T)\pi_1(x_1^S, x_2^S) + \delta^{T+1}\pi_1(\bar{x}_1, \bar{x}_2) \\ > \quad &\pi_i = (1-\delta)(1+\delta+...+\delta^T)\pi_i + \delta^{T+1}\pi_i.\end{aligned} \tag{6.23}$$

Since $\pi_i > \pi_1(\bar{x}_1, \bar{x}_2)$. The above inequality is true if and only if

$$\begin{aligned}&(1-\delta)\pi_1(x_1', x_2, x_3) + (1-\delta)(\delta + .. + \delta^T)\pi_1(x_1^S, x_2^S) \\ > \quad &(1-\delta)(1+\delta+...+\delta^T)\pi_i,\end{aligned} \tag{6.24}$$

which it holds whenever

$$(1-\delta)\{(\pi_i(x_i', x_j, x_k) - \pi_i) - (\delta + ... + \delta^T)\pi_i\} > 0. \tag{6.25}$$

However, causing bankruptcy to a firm implies an increase in output and since average cost is increasing, $\pi_i(x_i', x_j, x_k) - \pi_i < 0$. Thus, the above inequality can never hold.

Deviations from the mutual minmax phase with two firms cannot make any firm bankrupt because (x_1^S, x_2^S) is BF. Thus, by the same argument as above, firm 1 (the same argument applies to firm 2) can benefit by deviating from the mutual minmax phase if and only if x_i'' exists such that

$$\begin{aligned} & (1-\delta)\pi_1(x_1'', x_2^S) + (1-\delta)(\delta + ... + \delta^T)\pi_1(x_1^S, x_2^S) + \delta^{T+1}\pi_1(\bar{x}_1, \bar{x}_2) \\ > \ & (1-\delta)(1 + \delta + ... + \delta^{T-1})\pi_1(x_1^S, x_2^S) + \delta^T \pi_1(\bar{x}_1, \bar{x}_2). \end{aligned} \tag{6.26}$$

The previous inequality can be written as:

$$\pi_1(x_1'', x_2^S) > \delta^T \pi_1(\bar{x}_1, \bar{x}_2). \tag{6.27}$$

Note that $\pi_1(x_i'', x_2^S) \leq \max_{x_1 \in [0, x_1^S]} \pi_1(x_1, x_2^S) = \pi_{1m}^S$. Since $\pi_1(\bar{x}_1, \bar{x}_2) > \pi_{1m}^S$, (6.27) never holds when δ is closed to one.

Thus, both on and off the equilibrium paths, there is no profitable deviation when δ is sufficiently close to 1. ■

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