Sequencing bilateral negotiations with externalities*

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Preliminary Version

Abstract

We study the optimal sequence of bilateral negotiations between one principal and two agents, whereby the agents have different bargaining power. The principal chooses whether to negotiate first with the agent who is the stronger or the weaker bargainer. We show that the joint surplus of all three players is highest when the principal negotiates with the stronger bargainer first, independent of externalities between agents being positive or negative. The sequence chosen by the principal maximizes the joint surplus if there are negative or no externalities. If, instead, externalities are positive, the principal often prefers to negotiate with the weaker bargainer first.

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1 Introduction

In many situations, a principal needs to negotiate bilaterally with each of several agents, and the outcome of the negotiation between the principal and one agent imposes externalities on the other agents. Examples include the following situations:

1. Vertical relations between a supplier and retailers who compete in the consumer market. Externalities between the retailers are negative, if they sell substitutes, but are positive when they sell complements.

2. A seller of a product contracts with R&D firms (e.g., research labs) to improve the product's quality. Again, externalities between R&D firms can be negative (e.g., because research labs provide similar quality improvements) or positive (e.g., because one improvement makes the other more effective).

We study the optimal sequence of such negotiations in a stylized model where a principal bargains with two agents who differ in their bargaining power. Bargaining is modeled as random proposer take-it-or-leave-it bargaining. The principal chooses with which agent to bargain first. We focus on the case where negotiations are over binding contracts that fix a vector of quantities and a transfer, and do not condition on any actions taken later in the game. While there is, in general, an incentive to renegotiate a contract signed in the first negotiation (or to reopen failed negotiations) after the principal has come to an agreement with the second agent, in practice, requirements of time, or legal costs, can make renegotiations difficult. We focus on the case where no renegotiation is possible.

We study which sequence of negotiations maximizes the payoff of the principal, and which maximizes welfare (defined as the joint surplus of all three players). To trace out the effect of unequal bargaining power, we derive our main results under the assumption that agents are symmetric except for bargaining power. To keep the model as simple as possible, an agent's bargaining power is modeled as the probability of making the offer.

We demonstrate that two effects are at work, which drive the privately and socially optimal negotiation sequence. The first effect arises because the principal (in expectations) only obtains a fraction of the joint surplus in the second-stage negotiation. Knowing this, the decision about quantities in the first stage will be distorted due to externality that these quantities have on the negotiation in the second stage. This distortion is the larger, the smaller is the share the principal obtains in the second stage. We call this effect the *forward effect*. The second effect occurs because negotiated quantities in the second stage affect the payoff of the agent with whom the principal bargained in the first stage. However, this is not taken into account in the negotiation in stage 2, which only maximizes the bilateral surplus of those bargaining in the stage 2. We call this effect the *backward effect*.

We first show that welfare is maximized if the principal bargains first with the agent who has higher bargaining power. This result holds under very general assumptions on the payoff functions and is independent of externalities between agents being positive or negative. The intuition is easiest to grasp in the extreme case in which one agent has no bargaining power. When bargaining with this agent in the second stage, the principal obtains the full surplus. Therefore,she will take the externalities that arise from the negotiation in the first stage fully into account. As a consequence, there is no distortion in the first stage. In other words, with the negotiation sequence of bargaining first with the stronger agent, there is no distortion through the forward effect. The distortion implied by the backward effect is the same in both timings because in the second stage players always maximize their bilateral surplus. As a consequence, joint surplus is higher when the principal negotiates first with the agent who has some bargaining power. We show that this insight carries over to the case in which both agents have positive bargaining power but one of them is the stronger bargainer, as long as both agents are symmetric but for bargaining power.

We then look at the sequence chosen by the principal. We find that the principal chooses the welfare maximizing sequence if externalities are negative. With negative externalities, both the forward effect and the backward effect favor the sequence of negotiating with the stronger bargainer first and the weaker bargainer later. First, the principal obtains a larger surplus in the negotiation in the second stage, which implies that the second-stage surplus is considered to larger amount in the first stage. Therefore, the distortion implies by the forward effect is smaller when bargaining with the stronger bargainer first. Second, because externalities are negative, the surplus in the first-stage negotiation is lower if the bargainers in the second-stage agree on a positive quantities (i.e., the backward effect). The principal suffers less from the backward effect if she negotiates with the stronger bargainer first because she then obtains a relatively low share of the surplus. Due to the fact that she gets a comparably large share in the second negotiation, the principal secures herself a high outside option in the first negotiation allowing her to demand a higher piece of the cake in this negotiation.

With positive externalities, however, the principal may prefer to bargain with the weaker agent first. This results in an inefficient timing. With positive externalities, the backward effect favors the sequence of negotiating with the weaker bargainer first. In particular, the joint surplus of the negotiation in the first stage is now increased through the positive externality. The principal benefits more from this increase if she bargains first with the weaker agent because she obtains a larger share in this negotiation. The principal is therefore willing to sacrifice efficiency to obtain a larger piece of a smaller pie.

We show that this inefficient timing occurs if the positive externality is sufficiently small. By contrast, if the positive externality is large, the efficiency effect becomes more important, inducing the principal to bargain first with the stronger player. Therefore, the sequence chosen by the principal changes non-monotonically in the externalities between agents: when externalities are negative, the principal prefers to bargain with the stronger agent first, if externalities are moderately positive, she prefers to bargain with the weaker agent first, but when externalities are sufficiently positive, she prefers again to bargain with the stronger agent first.

Finally, we obtain that even without externalities the principal prefers to bargain with the stronger agent first if the principal's payoff function is not additive separable in the quantities of the agents. This holds, for example, if agents are retailers but monopolists in their respective product market (i.e., exerting no externalities on each other) and the principal is the supplier with a cost function that is convex in quantities (i.e., not additive separable). This scenario implies that the quantities to the two agents are interdependent in the principal's payoff function. In this case (without externalities) the backward effect is immaterial for the principal. However, the forward effect is still present because the principal cares more about the second-stage surplus in the first stage negotiation, when receiving larger fraction of it. This forward effect ultimately favors negotiating first with the stronger player.

Related literature. Our paper relates to a growing literature on one-to-many negotiations. Stole and Zwiebel (1996), Horn and Wolinsky (1988), and Cai (2000) study one-to-many negotiations with an exogenously given bargaining sequence. The sequencing of such negotiations has been analyzed in several recent papers, including Banerji (2002), Noe and Wang (2004), Raskovich (2007), Marx and Shaffer (2007, 2010), Krasteva and Yildirim (2012a, 2012b), and Guo and Iyer (2013). The papers by Marx and Shaffer (2007, 2010) also focus on the role of bargaining power. They study a buyer who bargains with two sellers, and assume the quantity purchased from one seller has, other things being equal, no impact on the payoff of the second seller. In contrast, we study the case of such direct externalities.[1] The sequencing of negotiations has also been studied in the literature on bargaining over multiple issues (e.g. Winter 1997, Inderst 2000).

The literature our paper connects to most is the one on contracting with externalities. Segal (1999, 2003) studies the *offer game* where the principal has all the bargaining power. Möller (2007) studies the endogenous timing of contracting. He assumes that the principal

[1]Another difference is that Marx and Shaffer (2007, 2010) focus on the case where the contracting space is so rich that the whole surplus is extracted from the second seller, while in our setup a contract between the principal and an agent unconditionally fixes a quantity and a transfer.

has all the bargaining power (as in the offer game), and focuses on the impact of early negotiations on the outside option of the agents who bargain later. In contrast, we study the impact of differences in bargaining power, assuming that there are no externalities on the nontraders, such that the outside option of the agent who bargains late is not affected by the earlier negotiations. Bernheim and Whinston (1986) study the *bidding game* where the agents make the offers. Galasso (2008) shows that the outcome of sequential bargaining may differ remarkably from the outcomes of the offer game and the bidding game. Not much is known, however, in the intermediate cases where both the principal and the agents have some bargaining power.

2 Model

Assumptions. There are three players: a principal (A, "she") and two agents (B and C). A and B negotiate over a decision $b \in \mathcal{B} \subset \mathbb{R}_+^{n_b}$, with $0 \in \mathcal{B}$, and a monetary transfer $t_B \in \mathbb{R}$ from B to A. Similarly, A and C negotiate over a decision $c \in \mathcal{C} \subset \mathbb{R}_+^{n_c}$, $0 \in \mathcal{C}$, and a transfer $t_C \in \mathbb{R}$. The payoff of the principal is $u_A(b,c) + t_B + t_C$, the payoffs of the agents are $u_B(b,c) - t_B$ and $u_C(b,c) - t_C$, respectively.

Negotiations are bilateral, and the order is chosen by A. Within each stage, there is random proposer take-it-or-leave-it bargaining.[2] Bargaining power is modelled as the probability of making the offer: B proposes with probability $\beta \in [0,1]$, C proposes with $\gamma \in [0,1]$. Without loss of generality, assume that $\beta \geq \gamma$; that is, B is the stronger bargainer among the agents. There is no renegotiation of the outcome of stage 1. Moreover, we assume that the contract negotiated in stage 1 cannot condition on any actions chosen later in the game, because of exogenous legal constraints, or other reasons for incomplete contracting. For example, if A is an upstream firm serving two retailers B and C, a contract between A and B that conditions on c might be in conflict with competition law. As noted by Möller (2007), in practice, contingent contracts are rare, and hard to enforce.

The timing of the game is as follows. In stage 0, A chooses timing BC or timing CB. In *timing* BC, in stage 1, A bargains with B. With probability β, B proposes a contract $(b, t_B) \in \mathcal{B} \times \mathbb{R}$, and A either accepts or rejects. With probability $1-\beta$, A proposes, and B then accepts or rejects. If A and B reach an agreement on a contract (b, t_B), the decision b is implemented and the transfer t_B is made. In case of rejection, $b = t_B = 0$. In $t = 2$, C observers the outcome of stage 1. Then A and C bargain. With probability γ, C proposes a contract $(c, t_C) \in \mathcal{C} \times \mathbb{R}$; with probability $1-\gamma$, A proposes. If they reach an agreement

[2] Alternatively, one can think of the outcome of each negotiation as given by an asymmetric Nash bargaining solution (see, for example, Muthoo 1999).

on a contract (c, t_C), the decision c is implemented and the transfer t_C is paid. Otherwise, $c = t_C = 0$.[3] *Timing CB* is similar, except that A bargains with C in stage 1 and with B in stage 2.

We assume that there are no externalities on the nontraders: $u_B(0, c)$ is constant in c, and $u_C(b, 0)$ is constant in b. Moreover, we normalize the utility functions such that $u_A(0, 0) = u_B(0, c) = u_C(b, 0) = 0$.

We say that (i) b has negative (no, positive) externalities on C if $u_C(b, c)$ is decreasing (constant, increasing) in b; (ii) c has negative (no, positive) externalities on B if $u_B(b, c)$ is decreasing (constant, increasing) in c; (iii) there are negative (no, positive) externalities if b has negative (no, positive) externalities on C, and c has negative (no, positive) externalities on B.

Moreover, we say that b has strictly negative (strictly positive) externalities on C if $u_C(b, c)$ is strictly decreasing (strictly increasing) in b whenever $c \neq 0$, and (ii) c has strictly negative (strictly positive) externalities on B if $u_B(b, c)$ is strictly decreasing (strictly increasing) in c whenever $b \neq 0$.

To isolate the impact of differences in bargaining power, our main results assume some degree of symmetry between players B and C. We say that *agents are symmetric except for bargaining power* if $\mathcal{B} = \mathcal{C}$ and for all $(b, c) \in \mathcal{B}^2$, (i) u_A is a symmetric function, i.e. $u_A(b, c) = u_A(c, b)$, and (ii) $u_C(c, b) = u_B(b, c)$. Note that under symmetry, b (c) has negative externalities on C (B) if and only if there are negative externalities, and similarly for positive externalities.

Define welfare as the joint surplus of all three players, $W(b, c) := \sum_{i \in \{A,B,C\}} u_i(b, c)$. We impose the tie-breaking rule that, if A is indifferent, but welfare is strictly higher in one of the timings, A selects the welfare maximizing timing.

Preliminaries. Since within each stage there is take-it-or-leave-it-bargaining, the decisions reached in the stage maximize the joint expected surplus of the two bargaining players. Moreover, whoever proposes chooses the transfer such that the other player is just willing to accept.

Consider timing BC (timing CB can be analyzed similarly). In stage 2, the decision b and transfer t_B are already fixed. The decision reached in stage 2 maximizes the joint surplus of A and C, given b. We assume that, for any b, there exists a unique solution

$$c^*(b) := \arg\max_{c \in \mathcal{C}} \{u_A(b, c) + u_C(b, c)\}.$$

[3] After stage 2, a game between $i = A, B, C$ might ensue, provided it has unique expected equilibrium payoffs $u_i(b, c)$ for all $(b, c) \in \mathcal{B} \times \mathcal{C}$, and the contracts cannot condition on any actions taken in the game.

Existence is ensured when (i) the sets $\mathcal{B}$ and $\mathcal{C}$ are finite, or (ii) the payoff functions u_i $(i = A, B, C)$ are continuous on $\mathcal{B} \times \mathcal{C}$ and the sets $\mathcal{B}$ and $\mathcal{C}$ are compact. A sufficient condition for uniqueness of decisions in case (ii) is that $u_A(b, c) + u_B(b, c)$ is strictly quasiconcave in b, and $u_A(b, c) + u_C(b, c)$ is strictly quasiconcave in c.

The expected payoff of A in stage 2 of timing BC is

$$(1 - \gamma)(u_A(b, c^*(b)) + u_C(b, c^*(b))) + \gamma u_A(b, 0) + t_B.$$

When $b = t_B = 0$, the expected payoff of A in stage 2 is

$$O_A^{BC} = (1 - \gamma) \max_{c \in \mathcal{C}} \{u_A(0, c) + u_C(0, c)\}.$$

This is the expected utility of A when the first stage negotiation with B fails; it therefore is the outside option of A in the first stage.

In the first stage of timing BC, the joint surplus of A and B consists of player B 's payoff, and the expected payoff of A in stage 2:

$$S_{AB}^{BC}(b) := u_B(b, c^*(b)) + (1 - \gamma)(u_A(b, c^*(b)) + u_C(b, c^*(b))) + \gamma u_A(b, 0). \qquad (1)$$

In any equilibrium of timing BC, A and B reach a decision $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$,[4] and the expected payoff of A is

$$U_A^{BC} = (1 - \beta) S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC}.$$

In case that there exists several $b \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$, note that they all lead to the same payoffs for A and B. In case they lead to different welfare, we asssume that a decision that maximizes $W(b, b^*(c))$ is selected. Therefore, the welfare in any equilibrium of timing BC is unique, even if the first stage decisions are not unique. We impose the corresponding assumptions on timing CB.

3 Welfare maximizing sequence

There are two reasons why, in general, the equilibrium decisions are not welfare maximizing. The first is that, in the negotiation in the first stage, A cannot commit to the decision that will be taken in the second stage. The negotiation in the second stage maximizes the surplus

[4] Existence of a maximum of $S_{AB}^{BC}(b)$ is ensured under the conditions discussed above (in case (ii), $b^*(c)$ is continuous by the Maximum Theorem, thus $S_{AB}^{BC}(b)$ is continuous, and a solution to $\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$ exists by the Weierstrass Theorem).

of the two players involved, and, therefore does not take into account its effect on the agent with whom A has already signed a contract. We call this the *backward effect*. The backward effect works through the externality of c on B in timing BC, and through the externality of b on C in timing CB. As an example, suppose A is a supplier and B and C are competitors in a downstream market. Then, agreeing on a larger quantity in the second-stage negotiation has a negative effect on the agent with whom A bargained first.

The second reason why equilibrium decisions are not welfare maximizing is because A only receives a fraction of the surplus in the second-stage negotiation. This implies that, in the first stage, A does only partially consider the second-stage surplus. Therefore, first stage decisions may be distorted away from the welfare maximizing outcome. We call this the *forward effect*; it works through the externality of b on C in timing BC, and through the externality of c on B in timing CB. In the example above, if A signs a contract with a large quantity in the first stage, the surplus A and his negotiation partner can achieve in the second stage is lower due to the negative externalities of the decisions.

Remark 1 illustrates that the forward effect and the backward effect are indeed the *only* reasons for inefficiencies. It shows that the equilibrium decisions maximize welfare in timing BC if $\gamma = 0$ (which shuts down the forward effect because A receives the full surplus in the negotiation with C) and c has no externality on B (which shuts down the backward effect). Denote the welfare in timing BC by W^{BC}, and welfare in timing CB by W^{CB}.

Remark 1 *Suppose that $1 \geq \beta > \gamma = 0$, and c has no externalities on B. Then $W^{BC} = W^{FB} \geq W^{CB}$.*

Proof. Consider timing BC. In the second stage, the decision reached is

$$\begin{aligned} c^*(b) &= \arg\max_{c \in \mathcal{C}} \{u_A(b,c) + u_C(b,c)\} \\ &= \arg\max_{c \in \mathcal{C}} \{u_A(b,c) + u_B(b,c) + u_C(b,c)\} \\ &= \arg\max_{c \in \mathcal{C}} W(b,c). \end{aligned}$$

Since $u_B(b,c)$ is independent of c, b is predetermined from the first stage, and adding a constant does not change the location of the maximum. In the first stage, the decision maximizes the joint surplus $S_{AB}^{BC}(b)$ of A and B. Since $\gamma = 0$, $S_{AB}^{BC}(b) = W(b, c^*(b))$. Therefore, $W^{BC} = \max_{b \in \mathcal{B}} W(b, c^*(b)) = W^{FB} \geq W^{CB}$. ■

The next proposition shows that the insight derived in the remark also applies if C has some bargaining power (i.e., $\gamma > 0$) and agents are symmetric but for bargaining power.

Proposition 1 *(i) W^{BC} is decreasing in γ and constant in β. Similarly, W^{CB} is decreasing in β and constant in γ. (ii) Suppose that agents are symmetric except for bargaining power, and $1 \geq \beta > \gamma \geq 0$. Then $W^{BC} \geq W^{CB}$. The inequality is strict (i.e., $W^{BC} > W^{CB}$) if the set of maximizers is different in the different timings.*

The proof of part (i) of Proposition 1 uses the following Lemma:

Lemma 1 *Suppose that $w : \mathcal{B} \to \mathbb{R}$ and $v : \mathcal{B} \to \mathbb{R}$ are functions and suppose that*

$$b^*(\gamma) := \arg\max_{b \in \mathcal{B}} (1-\gamma)\, w(b) + \gamma v(b)$$

exists for all $\gamma \in [0,1]$. Then for all $\gamma_1 \in [0,1]$ and $\gamma_0 \in [0,1]$, $\gamma_1 > \gamma_0$ implies $w(b^*(\gamma_1)) \leq w(b^*(\gamma_0))$.

Proof. See Appendix 6.1. ■

Proof of Proposition 1. Part (i). Consider timing BC (the result concerning timing CB can be established similarly). It is evident from (1) that the equilibrium decisions $\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ do not depend on β. Therefore, $W^{BC} = W\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is constant in β. Moreover, $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$, where

$$\begin{aligned} S_{AB}^{BC}(b) &= u_B(b, c^*(b)) + (1-\gamma)\left(u_A(b, c^*(b)) + u_C(b, c^*(b))\right) + \gamma u_A(b, 0) \\ &= (1-\gamma)\, W(b, c^*(b)) + \gamma\left[u_A(b, 0) + u_B(b, c^*(b))\right] \end{aligned}$$

Applying Lemma 1 with $w(b) = W(b, c^*(b))$ and $v(b) = u_A(b, 0) + u_B(b, c^*(b))$ shows that $W\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is decreasing in γ.

Part (ii). Suppose agents are symmetric. If $\beta = \gamma$, timings BC and CB differ only in the names of the agents. Since equilibrium welfare is unique, $W^{BC} = W^{CB}$. Part (i) therefore implies that, if $\beta > \gamma$, $W^{BC} \geq W^{CB}$.

If b^* and c^* (i.e., the set of maximizers) are different in timings BC and CB, welfare in the two timings must be strictly different. This is because agents are symmetric except for bargaining powers, and bargaining power does matter for welfare only insofar as it influences the optimal decisions b^* and c^*. It follows that $W^{BC} > W^{CB}$ then. ■

The Proposition shows that, under symmetry, welfare is higher when the principal bargains with the stronger agent first, irrespective of whether externalities are negative or positive.[5] The intuition is rooted in the forward effect: with symmetry, the backward effect plays

[5]Interestingly, it also does not matter whether the principal has more or less bargaining power than the agents, or one of them. Whenever $\beta \geq \gamma$, $W^{BC} \geq W^{CB}$, no matter whether the principal's bargaining power is low compared with the agents' bargaining power.

out similarly in the two timings. This is because the players in the second-stage negotiation always maximize their joint profits.

However, the forward effect is different in both timings. If the principal negotiates with the weaker agent in the second stage, she receives a larger share of the surplus in this stage. Therefore, the utility of the agent with whom the principal bargains in the second stage is taken into account to a larger extent in the first stage negotiation. As a consequence, first-stage decisions are closer to the welfare optimal ones than in case the principal bargains with the stronger player in the second stage. Therefore, the forward effect leads to a higher distortion when the bargaining power of the agent with whom the principal negotiates in stage 2 increases. This explains our main insight that the welfare optimal bargaining sequence is BC independent of the externalities. As we proceed to show, the sequence preferred by the principal depends on the nature of externalities.

We finally note that while part (i) of Proposition 1 does not need symmetry, part (ii) does. In fact, if agents were asymmetric, welfare can be higher in timing BC than in timing CB.[6]

4 The sequence preferred by the principal

We start this section by considering the special case in which $\beta = 1$, that is, B has all bargaining power. This case shows in a particularly transparent way how the externalities affect the principal's preference over the bargaining sequences.

Let U_A^{BC} $\left(U_A^{CB}\right)$ denote the expected payoff of A in timing BC (CB).

Remark 2 *Suppose that $\beta = 1$, $\gamma \in [0,1)$. If b has negative (no, positive) externalities on C, then $U_A^{BC} \geq U_A^{CB}$ $\left(U_A^{BC} = U_A^{CB},\ U_A^{BC} \leq U_A^{CB}\right)$. Moreover, when externalities are strictly negative (strictly positive) and equilibrium decisions in timing CB are not zero, then $U_A^{BC} > U_A^{CB}$ $\left(U_A^{BC} < U_A^{CB}\right)$.*

Proof. Since $\beta = 1$, $U_A^{BC} = O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A(0,c) + u_C(0,c)\right\}$. In contrast, in timing CB, $U_A^{CB} = (1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A(0,c) + u_C(b^*(c),c)\right\}$ where

$$b^*(c) = \arg\max_{b\in\mathcal{B}}\left\{u_A(b,c) + u_B(b,c)\right\}.$$

[6]The following example illustrates the point. Assume that $\gamma = 0 < \beta < 1$. Suppose that $b \in [0,1/2]$ and $c \in \{0,1\}$. Let $u_A(b,c) = 0$; $u_B(b,1) = -b$ and $u_B(b,0) = k\left(b - b^2\right)$ with $k > 4/(1-\beta)$; $u_C(b,0) = 0$ and $u_C(b,1) = 1 - b$. In timing BC, in the second stage $c^*(b) = 1$; therefore, $b^{BC} = 0$ and $U_A^{BC} = u_C(0,1) = 1$. Welfare is $W^{BC} = 1$. In timing CB, if $c = 0$, then in the second stage $b^*(c) = 1/2$ and the principal achieves a payoff of $(1-\beta)k/4$; if $c = 1$, the principal achieves 1 as before. Since by assumption $k > 4/(1-\beta)$, $c^{CB} = 0$, and $U_A^{CB} = (1-\beta)k/4$. Welfare is $W^{CB} = k/4 > W^{BC}$.

Therefore,

$$U_A^{BC} - U_A^{CB} = (1-\gamma)\left(\max_{c\in\mathcal{C}}\{(u_A(0,c)+u_C(0,c))\} - \max_{c\in\mathcal{C}}\{u_A(0,c)+u_C(b^*(c),c)\}\right)$$

When there are negative externalities of b on C, then $u_C(0,c) \geq u_C(b,c)$ for all b,c. Hence $U_A^{BC} \geq U_A^{CB}$. Moreover, when externalities are strictly negative and $c \neq 0 \neq b^*(c)$, then $U_A^{BC} > U_A^{CB}$. The results on positive and no externalities can be established similarly. ■

The remark shows that for $\beta = 1$, the principal's preference is solely driven by the externality of b on C. The principal prefers timing BC if externalities are negative and CB if externalities are positive. The intuition behind the result is again rooted in the interplay between the backward and the forward effect. If B has full bargaining power, the principal only receives a profit from the negotiation with C. When bargaining with B first, the backward effect plays no role for the principal because she receives no surplus in the negotiation with B. Only the forward effect is important. The principal's threat in the first stage is to reject B's offer, which implies $b = 0$. Hence, A can always assure herself a payoff that gives her $1-\gamma$ of the joint surplus of A and C, where the decision c maximizes this surplus, given $b = 0$.

By contrast, when bargaining with C first, the forward effect is immaterial for A because A receives no surplus in the second stage. However, the backward effect is important because the decision A and B agree upon in the second stage affects the surplus made in the first stage. In fact, C will foresee the decision that A and B will make in the second stage. Therefore, A and C will maximize the joint surplus, taking into account that b is decided upon in the second stage. As a consequence, A can assure herself a payoff that gives her $1-\gamma$ of the joint surplus of A and C, given that b will be positive.

The optimal sequence for the principal follows from this consideration. If externalities are negative, C's profit is higher if b equals zero than if b is positive. Since the principal obtains a share of C's profit, she prefers the sequence BC, where $b = 0$. By contrast, if externalities are positive, the joint surplus of those who bargain in stage 1 is increased through the externality. The principal then prefers the sequence CB where b is positive.

Finally if there are no externalities, the principal is indifferent. As we will show later, this last result only holds for $\beta = 1$. If the principal has strictly positive bargaining power against both agents, perhaps surprisingly, she prefers the sequence BC even without externalities.

Note that Remark 2 does not assume any symmetry. In particular, the externality of c on B does not influence the principal's choice of the bargaining sequence. To understand why, note that in timing BC, the backward effect on the joint stage-one-surplus of A and B is fully borne by B when $\beta = 1$. Likewise, in timing CB, the forward effect in the second-stage surplus is fully borne by B.

Remarks 1 and 2 have a straightforward implication for the efficiency of equilibrium timing in the case where B has all the bargaining power and C has no bargaining power.

Remark 3 *Suppose that $\beta = 1$, $\gamma = 0$, and c has no externalities on B. The equilibrium timing is efficient if b has negative externalities or no externalities on C. If b has positive externalities on C, the equilibrium timing is inefficient, unless the principal is indifferent between the two timings.*

Proof. By Remark 1, $W^{BC} \geq W^{CB}$. Suppose that b has negative externalities, or no externalities, on C. By Remark 2, $U_A^{BC} \geq U_A^{CB}$. Moreover, we assumed that if $U_A^{BC} = U_A^{CB}$ but $W^{BC} > W^{CB}$, A selects the timing BC. It follows that the equilibrium timing is welfare maximizing. Now suppose that b has positive externalities on C. By Remark 2, $U_A^{BC} \leq U_A^{CB}$. Thus, if the principal is not indifferent between the timings, $U_A^{BC} < U_A^{CB}$. ■

We now turn to the analysis of the case in which the bargaining power of both agents is strictly below 1. In particular, we are interested how the conclusions of Remark 2 need to be modified if $\beta < 1$. To isolate the effect of differing bargaining power, we focus our analysis on the symmetric case, that is, agents are symmetric but for bargaining power. We start with the case of negative externalities.

Proposition 2 *Assume that the agents are symmetric except for bargaining power, and $1 > \beta > \gamma$. If externalities are negative, then $U_A^{BC} \geq U_A^{CB}$, with strict inequality if externalities are strictly negative and equilibrium decisions are not zero.*

Proof. The symmetry of the agents has two implications that will be used in the proof. First,

$$\arg\max_{c\in\mathcal{C}} \{u_A(x,c) + u_C(x,c)\} = \arg\max_{b\in\mathcal{B}} \{u_A(b,x) + u_C(b,x)\} =: f(x) \tag{2}$$

for all $x \in \mathcal{B} = \mathcal{C}$. The function f defined in (2) gives the second stage decision that ensues after a first stage decision x; under symmetry, it is the same function in both timings. Second, symmetry implies that

$$\max_{c\in\mathcal{C}} \{u_A(0,c) + u_C(0,c)\} = \max_{b\in\mathcal{B}} \{u_A(b,0) + u_B(b,0)\}.$$

Since the outside options of A in stage one are

$$\begin{aligned} O_A^{BC} &= (1-\gamma) \max_{c\in\mathcal{C}} \{u_A(0,c) + u_C(0,c)\}, \\ O_A^{CB} &= (1-\beta) \max_{b\in\mathcal{B}} \{u_A(b,0) + u_B(b,0)\}, \end{aligned}$$

it follows that symmetry implies that

$$\beta O_A^{BC} - \gamma O_A^{CB} = (\beta - \gamma) \max_{c \in \mathcal{C}} \{u_A(0,c) + u_C(0,c)\}. \tag{3}$$

The surplus of A and B in timing BC as a function of b is

$$S_{AB}^{BC}(b) = (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b))) + \gamma u_A(b, 0) + u_B(b, f(b)).$$

In equilibrium of timing BC, $b = b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$. Similarly, the surplus of A and C in timing CB as a function of c is

$$S_{AC}^{CB}(c) = (1-\beta)(u_A(f(c), c) + u_B(f(c), c)) + \beta u_A(0,c) + u_C(f(c), c).$$

In equilibrium of timing CB, $c = c^{CB} \in \arg\max_{c \in \mathcal{C}} S_{AC}^{CB}(c)$. The expected payoffs of A in timing BC and CB are, respectively,

$$\begin{aligned} U_A^{BC} &= (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC} \\ U_A^{CB} &= (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) + \gamma O_A^{CB}. \end{aligned}$$

Since $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$,

$$S_{AB}^{BC}\left(b^{BC}\right) \geq S_{AB}^{BC}\left(c^{CB}\right). \tag{4}$$

Moreover, by symmetry,

$$S_{AB}^{BC}\left(c^{CB}\right) = \left((1-\gamma)\left(u_A\left(f\left(c^{CB}\right), c^{CB}\right) + u_B\left(f\left(c^{CB}\right), c^{CB}\right)\right) + \gamma u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right)$$

and therefore

$$(1-\beta) S_{AB}^{BC}\left(c^{CB}\right) - (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) = (\gamma - \beta)\left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right). \tag{5}$$

From (3), (4), and (5),

$$\begin{aligned} U_A^{BC} - U_A^{CB} &\geq (\beta - \gamma)\left(\max_{c \in \mathcal{C}} \{u_A(0,c) + u_C(0,c)\} - \left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right)\right) \\ &\geq (\beta - \gamma)\left(\max_{c \in \mathcal{C}} \{u_A(0,c) + u_C(0,c)\} - \max_{c \in \mathcal{C}} \{(u_A(0,c) + u_C(f(c), c))\}\right). \end{aligned}$$

Negative externalities imply $u_C(0,c) \geq u_C(b,c)$ for all $b \geq 0$, and therefore $U_A^{BC} \geq U_A^{CB}$.

Moreover, whenever externalities are strictly negative and $b > 0$, $u_C(0, c) > u_C(b, c)$ for all $c > 0$, therefore $U_A^{BC} > U_A^{CB}$. ■

The proposition shows that the main insight obtained in Remark 2 carry through if externalities are negative. If $\beta < 1$, in both timings the forward and the backward effect are present. When externalities are negative, both effects point in the same direction of favoring the timing BC over CB. First, in the timing BC the distortion implied by the forward effect is smaller than in timing CB because the utility of the agent A bargains with in the second stage is taken into account in the first stage to larger extent. Because A obtains a fraction of the full surplus, she favors BC over CB. Second, because externalities are negative, the agent in the first stage negotiation knows that she will receive a lower utility if the decision in the second stage is large. This distorts first-stage decisions (i.e., the backward effect). The principal suffers less from this effect if she negotiates with B first because she then obtains a lower share of the surplus in the first stage than when bargaining with C first.

It is interesting that the principal strictly prefers timing BC even if the decisions are the same in both timings, as long as externalities are strict. This in in contrast to the timing that maximizes welfare: if decisions are the same in both timings, both timings are welfare-equivalent. The intuition for why the principal strictly prefers BC is that she obtains a different share of the surplus in the two timings, even if decisions are the same. His outside option in timing BC is strictly larger because receives a larger share in the negotiation with C. The is the decisive effect for the principal if b^{ast} and c^{ast} are the same, leading to a strict timing preference.

We now turn to the case in which there are no externalities between agents. As demonstrated in Remark 2, if $\beta = 1$, then the principal is indifferent between the two timings. However, this is no longer true if $\beta < 1$. The reason is that even without externalities, the two bargaining problems are not independent of each other because the decisions b and c interact through the principal's payoff function. The timing of negotiations then still plays a role. As the next proposition shows the principal still prefers timing BC over CB in this case.

Proposition 3 *Assume agents are symmetric except for bargaining power, there are no externalities, and $1 > \beta > \gamma$. Then $U_A^{BC} \geq U_A^{CB}$. Moreover, the inequality is strict if first-stage decisions in the two timing differ from each other; a sufficient condition is that (i) equilibrium first-stage and second-stage decisions are interior, (ii) u_A, u_B, u_C and $c^*(b)$ are differentiable, and (iii) whenever $c \neq c'$, then for any $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$ there exists* *some* $i = 1, ..., n_B$, *such that*

$$\frac{\partial}{\partial b_i} u_A\left(b^{BC}, c\right) \neq \frac{\partial}{\partial b_i} u_A\left(b^{BC}, c'\right). \tag{6}$$

Proof. See Appendix 6.2 ■

Conditions (i)-(iii) are used to ensure that the first-stage decisions in the two timings problems differ from each other.[7]

We point out that (iii) will be satisfied in many economic applications. A sufficient condition for (iii) is that the marginal returns to some b_i are strictly monotone (increasing or decreasing) in c.[8] It is satisfied, for example, when A is a supplier, sells a single homogeneous good to B and C, and has strictly increasing marginal costs. Assumption (iii) rules out the case of an additively separable u_A where there is no interaction between the bargaining problems. Assumption (iii) alone is not sufficient to rule out the possibility that first-stage decisions might be identical in the two timings, be it because they occur at a boundary of the feasible set, or because the payoff functions are not differentiable; assumptions (i) and (ii) serve to rule these possibilities out.[9]

The intuition behind result of the last Proposition lies in the forward effect, which favors timing BC. Without externalities, the backward effect is immaterial because the second-stage decision has no effect on the utility of the agent negotiating in the first stage. However, the forward effect is still important. Since b and c interact only through A's payoff function, A would like to decide about both variables at one stage. In the timing BC, she takes the second-stage maximization into account to a greater extent than in the timing CB. Therefore, the distortion in the two decisions is smaller in the timing BC. It is worth mentioning that the result holds independent of the concrete way b and c interact in u_A.

We now turn to the case of positive externalities. As shown above, with $\beta = 1$, the principal unambiguously prefers timing CB. However, in what follows we demonstrate that this is no longer true if $\beta < 1$. In fact, both timings BC and timing CB can emerge in equilibrium, even with additive separability of b and c in the principal's utility function. In order to focus on the pure effect of positive externalities, we give more structure to the utility

[7] More generally, the proof of Proposition 2 shows that, if there are no externalities, and $b^{BC} = c^{CB}$, and c^{CB} maximizes $(u_A(0, c) + u_C(0, c))$, then $u_A^{BC} = u_A^{CB}$. This is the case in Krasteva and Yildirim (2012a) in the benchmark case with commonly known valuations.

[8] This sufficient condition, however, rules out some economically interesting cases covered by (iii). For example, (iii) is also satisfied when $u_A(b, c) = -\sum_{i=1}^{n}(b_i + c_i)^2$. Here, there is no single good i such that the marginal returns to b_i are strictly monotone in c. Moreover, (iii) assumes that marginal returns are unequal, not that they are monotone.

[9] Similarly, Edlin and Shannon (1998) rely on interiority and differentiability assumptions for strictly monotone comparative statics.

function by considering the case of "parametric externalities". This allows us to show that given u_A is additive separable and some differentiability assumptions, A strictly prefers CB when externalities are small.

Case of parametric externalities. *The utility functions of B and C are parametrized by $k \in \mathbb{R}$ and written $u_B(b,c,k)$ and $u_C(b,c,k)$. k parametrizes the importance of externalities in the following sense: (1) u_A is constant in k; (2) if $k=0$ there are no externalities, thus $u_B(b,c;0)$ is constant in c; (3) k has no effect on u_B when $c=0$;*[10] *(4) for all $b>0$,*[11] *all c and $c'>c$, $u_B(b,c';k)-u_B(b,c;k)$ is strictly increasing in k. Since $u_B(b,c';0)=u_B(b,c;0)$, it follows that, for all $k>0$, $u_B(b,c';k)>u_B(b,c';0)$.* We *employ the slightly stronger assumption*[12] *that u_B is differentiable in k and*

$$\frac{\partial u_B(b,c;k)}{\partial k}>0. \tag{7}$$

whenever $b>0$ and $c>0$.

In the following Proposition, let $c^*(b,k) := \arg\max_{c\in\mathcal{C}}(u_A(b,c)+u_C(b,c,k))$ denote the second stage decision in timing BC, and define $b^*(c,k)$ similarly.

Proposition 4 *Consider the case of parametric externalities. Suppose agents are symmetric except for bargaining power, $1>\beta>\gamma$, u_A is additively separable, and (i) u_i $(i=A,B,C)$ is C^1 in (b,c,k), (ii) $c^*(b,k)$ is interior and C^1 in (b,k), and (iii) $\mathcal{B}=\mathcal{C}$ is compact. Then, there exists exists a $\hat{k}>0$ such that $U_A^{BC}<U_A^{CB}$ for all $k\in\left(0,\hat{k}\right)$.*

Proof. See Appendix 6.3. ■

If externalities are small, the principal prefers the inefficient timing CB. For the intuition behind the result, first note that the principal is more interested in the joint surplus of her and agent C because she receives a larger share in this negotiation than in the negotiation with B. With positive externalities, the surplus realized by A and C is larger if A and B decide on a positive b in the second stage. Therefore, the backward effect favors timing CB with positive externalities. In particular, as outlined after Remark 2, when $\beta=1$, in the timing BC the principal receives a surplus in the negotiation with C given $b=0$, whereas b is positive in the timing CB. This is less extreme for $\beta<1$. However, by continuity, b is

[10] This assumption is motivated from the idea that k should parametrize externalities *and nothing else.*

[11] We use the vector inequality notation where $b>0$ means that $b_i\geq 0$ for all $i=1,...,n$ and $b_i>0$ for at least one $i=1,...,n$.

[12] The issue is that $u_B(b,c',k)-u(b,c,k)$ could have a zero derivative with respect to k on sets of measure zero.

still higher in timing CB than in BC. Hence, with positive externalities the surplus in the negotiation with C is higher in the timing CB. If u_A is additive-separable and externalities are small, this effect is decisive in the preferred timing of the principal. With positive externalities, the principal is therefore willing to sacrifice overall surplus to obtain a larger piece of a smaller cake.

Proposition 4 focused on the small externalities. The question remains if the principal also prefers the timing CB if externalities are positive but large. Remark 2 showed that this is true for $\beta = 1$. However, as we will demonstrate in the next example, the result is not true in general if $\beta < 1$.[13]

Example 1 *Let* $u_A(b, c) = -b-c$, $u_B(b, c) = 2\left(\sqrt{b} + k\sqrt{c}\right)$ *if* $b > 0$, $u_C(b, c) = 2\left(\sqrt{c} + k\sqrt{b}\right)$ *if* $c > 0$, *and* $u_B(0, c) = u_C(b, 0) = 0$. *Moreover, let* $\beta > \gamma$. *Then* $U_A^{CB} > U_A^{BC}$ *whenever* $0 < k < \hat{k} := 2/\left((1-\beta)(1-\gamma)\right)$, *and* $U_A^{CB} < U_A^{BC}$ *whenever* $k > \hat{k}$.

Example 1 shows that the principal's chosen sequence changes non-monotonically in the externalities. If externalities are negative, the principal prefers BC, if externalities are positive but small she prefers CB whereas if externalities are positive and large, she prefers BC again. The intuition is that if externalities become large, efficiency considerations become more important. In particular, the difference in efficiency between the sequences BC and CB increases with k; this leads to $U_A^{BC} > U_A^{CB}$ when k becomes sufficiently large.[14]

We also note that the equilibrium sequence need not be non-monotonic in the externalities. For instance, replace k in Example 1 by $1 - 1/(1+k)$. The example then still fulfills all requirements for parametric externalities. However, it is then easy to show that $U_A^{CB} > U_A^{BC}$ for all $k > 0$ (i.e., even if $k \to \infty$). The reason why the equilibrium sequence does not change with k here is that when k grows large, the effect of b on u_C (and by symmetry the effect of c on u_B) stays bounded. Therefore, although the importance of externalities increases with k, they will not become dominant. From the principal's perspective, efficiency considerations are then dominated by the effect that she obtains a higher surplus in the negotiation with C.

Finally, our results allow us to compare the equilibrium sequence with the efficient one. The last proposition summarizes these insights.

[13] See Appendix 6.4 for details on the derivation of the equilibrium timing in Example 1.

[14] The threshold value $\hat{k}$ at which the principal's preferred sequence changes from CB to BC is in fact larger than 1 in Example 1. This implies that the decision variable b (c) must have a stronger effect on C (B) to render timing BC optimal for the principal.

Proposition 5 *Suppose agents are symmetric except for bargaining power. The equilibrium timing is efficient when there are negative or no externalities. The equilibrium timing is inefficient if externalities are positive and small and can either be inefficient or efficient if externalities are positive and large.*

5 Conclusion

This paper has studied optimal sequence of negotiations between two agents and one principal. If the agents are symmetric except for bargaining power, welfare is higher when the principal bargains with the stronger agent first. The principal chooses the welfare maximizing sequence if externalities are negative, or there are no externalities. With positive externalities, however, the equilibrium timing can be inefficient.

One limitation of our study is the symmetric setup considered in our main results. Without the assumption of symmetry, we have derived two results concerning limiting cases of bargaining power. If one agent has all the bargaining power, the principal will negotiate with him first if externalities are negative, she will negotiate with the weaker agent first if externalities are negative. If one agent has no bargaining power at all and there are no externalities, the welfare maximizing sequence is to talk to the stronger agents first.

6 Appendix

6.1 Proof of Lemma 1

Proof. Suppose to the contrary that $w(b^*(\gamma_1)) > w(b^*(\gamma_0))$. From the definition of $b^*(\gamma)$,

$$(1-\gamma_0)\,w(b^*(\gamma_0)) + \gamma_0 v(b^*(\gamma_0)) \geq (1-\gamma_0)\,w(b^*(\gamma_1)) + \gamma_0 v(b^*(\gamma_1)),$$

or equivalently,

$$(1-\gamma_0)\,(w(b^*(\gamma_0)) - w(b^*(\gamma_1))) \geq \gamma_0\,(v(b^*(\gamma_1)) - v(b^*(\gamma_0))) \tag{8}$$

Since $w(b^*(\gamma_1)) > w(b^*(\gamma_0))$ and $1 \geq \gamma_1 > \gamma_0$, the left side of inequality (8) is strictly negative. Therefore, $v(b^*(\gamma_1)) < v(b^*(\gamma_0))$.

Similarly,

$$-(1-\gamma_1)\,(w(b^*(\gamma_0)) - w(b^*(\gamma_1))) \geq -\gamma_1\,(v(b^*(\gamma_1)) - v(b^*(\gamma_0))) \tag{9}$$

Adding (9) to (8) shows that

$$(\gamma_1-\gamma_0)\,w(b^*(\gamma_0)) - w(b^*(\gamma_1)) \geq (\gamma_0-\gamma_1)\,(v(b^*(\gamma_1)) - v(b^*(\gamma_0)))$$

This is a contradiction because the left hand side is strictly smaller than zero, and the right hand is strictly greater than zero. ■

6.2 Proof of Proposition 3

Proof. The proof of Proposition (2) also establishes that with no externalities, $U_A^{BC} \geq U_A^{CB}$. Moreover, when $b^{BC} \neq c^{CB}$, then inequality (4) is strict. Since $\beta < 1$, it follows that $U_A^{BC} > U_A^{CB}$ when $b^{BC} \neq c^{CB}$ for any $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ and $c^{CB} \in \arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$. We show that (i)-(iii) imply this is the case.

By (ii), $S_{AB}^{BC}(b)$ and $S_{AC}^{CB}(c)$ are differentiable. Since any $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ is interior by (i), it satisfies the first order condition

$$\begin{aligned}\frac{\partial S_{AB}^{BC}\left(b^{BC}\right)}{\partial b_i} &= \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i} + (1-\gamma)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \gamma\frac{\partial}{\partial b_i}u_A\left(b^{BC}, 0\right) \\ &\quad + (1-\gamma)\left(\sum_k \frac{\partial}{\partial c_k}\left(u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + u_C\left(f\left(b^{BC}\right)\right)\right)\frac{df_k\left(b^{BC}\right)}{db_i}\right) = 0.\end{aligned}$$

Since $f\left(b^{BC}\right)$ is interior by (i), and $u_A(b,c)+u_C(c)$ is differentiable by (ii), the first order condition

$$\frac{\partial}{\partial c_k}\left(u_A\left(b^{BC}, f\left(b^{BC}\right)\right)+u_C\left(f\left(b^{BC}\right)\right)\right)=0$$

holds, thus

$$\frac{\partial u_B\left(b^{BC}\right)}{\partial b_i}+(1-\gamma)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right)+\gamma\frac{\partial}{\partial b_i}u_A\left(b^{BC},0\right)=0.$$

Since $f\left(b^{BC}\right)$ is interior by (i), $f\left(b^{BC}\right)>0$. Thus (6) implies

$$\begin{aligned}
& \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i}+(1-\gamma)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right)+\gamma\frac{\partial}{\partial b_i}u_A\left(b^{BC},0\right)\\
\neq\; & \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i}+(1-\beta)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right)+\beta\frac{\partial}{\partial b_i}u_A\left(b^{BC},0\right)\\
=\; & \frac{\partial u_C\left(b^{BC}\right)}{\partial c_i}+(1-\beta)\frac{\partial}{\partial c_i}u_A\left(f\left(b^{BC}\right), b^{BC}\right)+\beta\frac{\partial}{\partial c_i}u_A\left(0,b^{BC}\right)\\
=\; & \frac{\partial S_{AC}^{CB}\left(b^{BC}\right)}{\partial c_i}
\end{aligned}$$

where the first equality is from symmetry. We have shown that

$$\frac{\partial S_{AC}^{CB}\left(b^{BC}\right)}{\partial c_i}\neq 0.$$

Since any $c^{CB}\in\arg\max_{c\in\mathcal{C}}S_{AC}^{CB}(c)$ is interior by (i), it satisfies the first order condition

$$\frac{\partial S_{AC}^{CB}\left(c^{CB}\right)}{\partial c_i}=0,$$

thus $b^{BC}\neq c^{CB}$. ■

6.3 Proof of Proposition 4

Proof. In what follows, we denote the first stage decision in timing BC, which depends on k, by $b^{BC}(k)$.

To show the result in the most concise way, we first determine in the next lemma how the social surpluses in the two timings change with k. We note that the proof of the lemma uses a version of the envelope theorem applied to the joint first-stage surplus. We cannot directly apply to standard versions of the envelope theorem (e.g., Simon and Blume 1994, Theorem 19.4) for two reasons. First, we do not assume $b^{BC}(k)$ to be differentiable in k.

We solve this issue by using an envelope theorem from Milgrom and Segal (2002) that does not presuppose differentiability of the maximizer. Second, the choices in the second-stage do in general not maximize the joint surplus of those who bargain in the first stage. As in the envelope theorem for Stackelberg games (Caputo 1998), we need to take into account the effect of k on the second-stage reaction function. Under the assumptions of Proposition 4, however, at $k=0$ the second-stage decision also maximizes the surplus of the negotiation in the first stage, therefore the corresponding terms disappear.

Lemma 2 *Under the assumptions of Proposition 4,* $S_{AB}^{BC}(k) = \max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)$ *and* $S_{AC}^{CB}(k) = \max_{c\in\mathcal{C}} S_{AC}^{CB}(b^*(c,k), c, k)$ *are differentiable in* k *at* $k=0$*, and*

$$\frac{d}{dk}\left((1-\gamma) S_{AC}^{CB}(k) - (1-\beta) S_{AB}^{BC}(k)\right)|_{k=0} = (\beta-\gamma)\frac{\partial}{\partial k} u_B(b,c;k)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} > 0.$$

Proof. If $k=0$, there is no interaction between the two bargaining problems, and

$$\arg\max_b S_{AB}^{BC}(b,0) = \arg\max_b u_A(b,0) + u_B(b,0,0)$$

Our assumption that second stage decision are unique ensures that $\arg\max_b u_A(b,0) + u_B(b,0,0)$ is unique. Therefore, when $k=0$, the first stage decision in timing BC is unique. Since $c^*(b,0)$ is interior by assumption (ii), symmetry implies that if $k=0$, $c^*(b,0) = b^{BC}(0)$. Thus $b^{BC}(0)$ is interior. Moreover, the function $S_{AB}^{BC}(b, c^*(b,k), k)$ is continous in b and continuously differentiable in k.

Therefore, Corollary 4 from Milgrom and Segal (2002) applies (here we use assumption (iii)), and $\max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)$ is differentiable in k at $k=0$, with

$$\begin{aligned}\frac{d}{dk}\max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)|_{k=0} &= \frac{\partial}{\partial k}\left(u_B(b, c^*(b;k);k) + (1-\gamma) u_C(b, c^*(b;k);k)\right)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} \\ &+ \sum_{i=1}^{n} \frac{\partial S_{AB}^{BC}(b,c,k)}{\partial c_i}\frac{\partial c_i^*(b;k)}{\partial k}\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}}.\end{aligned}$$

The first term of the right-hand side is the direct effect of k, keeping b and c constant, whereas the second term captures that the second-stage reaction function depends on k.

We next show that

$$\left.\frac{\partial S_{AB}^{BC}\left(b,c,k\right)}{\partial c_i}\right|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^{*}\left(b^{BC},0\right)}} = 0$$

for all $i = 1, ..., n$. We have

$$\frac{\partial S_{AB}^{BC}\left(b,k\right)}{\partial c_i} = \frac{\partial}{\partial c_i} u_B\left(b, c^*\left(b\right); k\right) + \left(1-\gamma\right)\frac{\partial}{\partial c_i}\left(u_A\left(b, c^*\left(b;k\right)\right) + u_C\left(b, c^*\left(b;k\right); k\right)\right).$$

Since $c^*\left(b,0\right)$ maximizes $u_A\left(b,c\right) + u_C\left(b,c;0\right)$ and is interior,

$$\left.\frac{\partial}{\partial c_i}\left(u_A\left(b,c\right) + u_C\left(b,c,k\right)\right)\right|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^{*}\left(b^{BC},0\right)}} = 0.$$

Moreover, at $k = 0$ there are no externalities, thus

$$\left.\frac{\partial}{\partial c_i} u_B\left(b,c,k\right)\right|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^{*}\left(b^{BC},0\right)}} = 0.$$

It follows that

$$\frac{d}{dk} S_{AB}^{BC}\left(k\right) = \left.\frac{\partial}{\partial k}\left(u_B\left(b,c;k\right) + \left(1-\gamma\right) u_C\left(b,c;k\right)\right)\right|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^{*}\left(b^{BC}(0),0\right)}}.$$

Similarly,

$$\frac{d}{dk} S_{AC}^{CB}\left(k\right) = \left.\frac{\partial}{\partial k}\left(u_C\left(b,c;k\right) + \left(1-\beta\right) u_B\left(b,c;k\right)\right)\right|_{\substack{k=0\\ b=b^{*}\left(c^{CB}(0),0\right)\\ c=c^{CB}(0)}}.$$

By symmetry, for all x, y, and k, $u_B\left(x,y;k\right) = u_C\left(y,x;k\right)$ and thus

$$\frac{\partial}{\partial k} u_B\left(x,y;k\right) = \frac{\partial}{\partial k} u_C\left(y,x;k\right) \tag{10}$$

Moreover, symmetry implies $b^{BC}\left(0\right) = c^{CB}\left(0\right)$ and $b^*\left(c^{CB}\left(0\right);0\right) = c^*\left(b^{BC}\left(0\right),0\right)$. Evaluating (10) at $k = 0$, $x = b^{BC}\left(0\right) = c^{CB}\left(0\right)$, and $y = b^*\left(c^{CB}\left(0\right);0\right) = c^*\left(b^{BC}\left(0\right),0\right)$

gives

$$\frac{\partial}{\partial k} u_B(b,c;k)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC}(0),0\right)}} = \frac{\partial}{\partial k} u_C(b,c;k)\Bigg|_{\substack{k=0\\ b=b^*\left(c^{CB}(0),0\right)\\ c=c^{CB}(0)}}.$$

Similarly, evaluating (10) at $k=0$, $x=b^*\left(c^{CB}(0);0\right)=c^*\left(b^{BC}(0),0\right)$, and $y=b^{BC}(0)=c^{CB}(0)$ gives

$$\frac{\partial}{\partial k} u_B(b,c;0)\Bigg|_{\substack{k=0\\ b=b^*\left(c^{CB}(0),0\right)\\ c=c^{CB}(0)}} = \frac{\partial}{\partial k} u_C\left(b^{BC},c^*\left(b^{BC},0\right);0\right)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC}(0),0\right)}}.$$

Therefore,

$$\frac{d}{dk}\left((1-\gamma)S_{AC}^{CB}(k)-(1-\beta)S_{AB}^{BC}(k)\right)|_{k=0} = (\beta-\gamma)\frac{\partial}{\partial k} u_B(b,c;k)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC}(0),0\right)}},$$

which is strictly positive since by assumption $c^*(b,k)>0$ and, as shown above, $b^{BC}(0)>0$. ■

We can now show the result of Proposition 4. Since u_A does not depend on k, and $u_C(0,c;k)$ is independent of k,

$$O_A^{BC}=(1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A(0,c)+u_C(0,c;k)\right\}$$

does not depend on k. Similarly, O_A^{CB} is independent of k. They payoff of A in timings BC and CB is

$$\begin{aligned} U_A^{BC}(k) &:= (1-\beta)S_{AB}^{BC}(k)+\beta O_A^{BC}, \\ U_A^{CB}(k) &:= (1-\gamma)S_{AC}^{CB}(k)+\gamma O_A^{CB}. \end{aligned}$$

Therefore, Lemma 2 implies that

$$\frac{\partial}{\partial k}\left(U_A^{CB}(k)-U_A^{BC}(k)\right)|_{k=0}>0.$$

If $k=0$, the bargaining problems do not interact, and $U_A^{BC}(0)=U_A^{CB}(0)$. By continuity, it follows that for sufficiently small $k>0$, $U_A^{CB}(k)>U_A^{BC}(k)$. ■

6.4 Details on Example 1

In timing BC, the second stage decision is $c^* (b) = 1$, the joint surplus of A and B on stage 1 is

$$S_{AB}^{BC} (b) = 2 \left(\sqrt{b} + k\right) + (1-\gamma) \left(-b - 1 + 2 \left(1 + k\sqrt{b}\right)\right) - \gamma b,$$

which is maximized by

$$b^{BC} = (k (1-\gamma) + 1)^2 .$$

Thus

$$S_{AB}^{BC} \left(b^{BC}\right) = (1 + (1-\gamma) k)^2 + 1 - \gamma + 2k.$$

The outside option of A in the first stage of timing BC is $O_A^{BC} = (1-\gamma)$. Therefore,

$$U_A^{BC} = (1-\beta) \left((1 + (1-\gamma) k)^2 + 1 - \gamma + 2k\right) + \beta (1-\gamma)$$

A similar argument shows that in timing CB, $b^* (c) = 1$,

$$\begin{aligned} c^{CB} &= (k (1-\beta) + 1)^2 , \\ U_A^{CB} &= (1-\gamma) \left((1 + (1-\beta) k)^2 + 1 - \beta + 2k\right) + \gamma (1-\beta) . \end{aligned}$$

Moreover,

$$U_A^{BC} - U_A^{CB} = k (\beta - \gamma) (k (1-\beta) (1-\gamma) - 2) .$$

Thus $U_A^{BC} > U_A^{CB}$ if, and only if,

$$k > \hat{k} := \frac{2}{(1-\beta)(1-\gamma)}.$$

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