INFORMATION IN TENDER OFFERS WITH A LARGE SHAREHOLDER[1]

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We study takeovers of firms whose ownership structure is a mixture of minority block-holders and small shareholders. We show that the combination of dispersed private information on the side of small shareholders and the presence of a large shareholder can facilitate profitable takeovers. Furthermore, our analysis implies that even if some model of takeovers predicts a profit for the raider, for example due to private benefits, the profit will be underestimated unless the large shareholder and the dispersion of information among the small shareholders are modeled.

KEYWORDS: takeovers, tender offers, lemons problem, large shareholder, block-holders.

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Contents

1. Introduction

The threat of a potential takeover should discipline an incumbent management to serve in the best interest of the shareholders. If the management were to underperform, a more efficient raider should take over the company and manage it better. This reasoning fails, however, when the ownership of the company is widely dispersed. The impact of each small shareholder on the outcome of the takeover attempt is negligible. Therefore, if the price the raider offers is smaller than the post-takeover value of the shares, the small shareholders who anticipate that the takeover should succeed hold onto their shares, which causes the takeover attempt to fail. Hence, the raider has to offer at least the expectation of the post-takeover value of the shares to succeed, rendering successful takeovers unprofitable. In the face of even a small cost the raider will not initiate a value-increasing takeover, making the free-riding problem a fundamental source of inefficiency in the market for corporate control (see Grossman and Hart (1980), Bagnoli and Lipman (1988) and Harrington Jr and Prokop (1993)).

We explore the effects of the interplay between the presence of a minority large shareholder and private information held by the shareholders in takeover contests where the raider is exposed to free-riding.[1] Our main result shows that when a firm is owned by a *large minority shareholder* and a large number of small shareholders, the presence of *dispersed private information* about the post-takeover value on the side of small shareholders can enable the raider to make a profit. The result is striking in light of the following benchmark findings. First, we show that under symmetric information (i.e., no private information) the raider makes no profits. That is, the mere presence of a minority large shareholder is not sufficient to facilitate profitable takeovers and the free-riding continues to be a detrimental friction in the market. Second, Marquez and Yılmaz (2008) show that in a model with only small shareholders and private information on the side of the shareholders the raider cannot make a profit. In that model the raider's inability to make a profit is exacerbated due to the lemons problem arising from the asymmetry of information between the shareholders and the raider. Third, we show that in a model with a large shareholder and asymmetry of information between the shareholders and the raider, but no dispersion (the small shareholders receive perfectly correlated signals), the raider cannot make a profit either. Hence, the existence of a large minority shareholder and the asymmetric information that is dispersed among the small shareholders are central to a raider's ability to make a profit.

In our model, the target firm is owned by a minority large shareholder and many small

[1] Holderness (2009) finds that in a representative sample of U.S. public firms ninety six percent of them have a block holder who owns at least 5% of firm's common stock.

shareholders.[2] With some probability, the state is low and the value added from the takeover is zero. With the complementary probability, the state is high and the value added is positive but only if the takeover is successful. The value of the company is unchanged if the takeover fails. Each shareholder observes a private and imperfectly informative signal about the state. The raider, who has no information about the state of the world beyond the common prior, submits an unconditional offer for the equity shares of the firm by specifying a price per share. Each small shareholder decides whether to accept the offer, and tender his share, or to reject it. The large shareholder, on the other hand, decides how many shares to tender. The takeover succeeds only if the raider acquires at least half of all the shares.

We characterize the unique asymptotic equilibrium outcome of the tender game when the number of shares goes to infinity. If the large shareholder's stake is sufficiently high, the equilibrium price is positive, the takeover succeeds with probability one, and the raider makes a strictly positive profit. The small shareholders use a specific threshold strategy which leaves the large shareholder with only two options. Either he does not sell all his shares and the takeover fails in the high state, or he sells everything and the takeover succeeds in the high state with a positive probability. Opting for the lesser of two evils, the large shareholder sells all his shares, and the raider acquires precisely half of the firm's shares in the high state of the world.

If the large shareholder is sufficiently large and sells all of his shares, the raider needs to acquire only a small fraction of the company from the small shareholders for the takeover to succeed. The key to our result is that due to the *dispersed beliefs* of the small shareholders he can buy the latter from the more pessimistic shareholders, and everything from the large shareholder, by offering a low price. Asymmetric and dispersed information, therefore, diminishes the free-riding incentives of *some* of the small shareholders. The low price in turn results in a profit from the large shareholder's shares, but a loss from the small shareholders' shares due to the lemons problem. The larger is the stake held by the large shareholder the smaller is the lemons problem related to the small shareholders, which results in a higher profit for the raider. Finally, the large shareholder's equilibrium behavior is independent of his information. His information structure has no effect on the equilibrium price offer nor does his information on the probability of a successful takeover.

In addition to the above described results, our paper offers a novel methodology for studying takeovers with a continuum of shareholders and asymmetric information. We introduce an equilibrium concept in the spirit of the rational expectations equilibrium. Our equilibrium concept differs from those commonly used in the literature on takeovers with a continuum

[2] The results are shown to extend to the case of several minority shareholders as long as in total they own a minority stake.

of shareholders (for example, Grossman and Hart (1980) and Shleifer and Vishny (1986)). While the standard models of takeovers with a continuum of shareholders assume that the probability of success is one when exactly half of the shares are sold, we leave this probability to be determined endogenously in equilibrium. Tirole (2010) proposed such an equilibrium concept in a model of takeovers with complete information; see also Dekel and Wolinsky (2012) for a model of takeover competition. We extend this equilibrium concept to environments with incomplete and asymmetric information. In addition, we show that our equilibrium concept captures the behavior present in the model with a large, albeit finite, number of shares. More precisely, we show that the outcomes of perfect Bayes-Nash equilibria of the model with finitely many shares converge to an equilibrium outcome of the continuum-shares model, as the number of shares goes to infinity. In other words, we provide a micro-foundation for the model using standard equilibrium concepts.

1.1. Literature review In what follows we provide the related literature focusing on the work closest to the main ingredients of our model. A more comprehensive review of literature on takeovers can be found in Burkart and Panunzi (2006).

Grossman and Hart (1980) showed that in an environment where it is common knowledge that the takeover would increase the value of the company the raider cannot make a profit, if the takeover is to succeed with certainty, and will therefore not even start it if faced with administrative costs. Bagnoli and Lipman (1988) studied the limit outcomes of the finite models as the shareholders become smaller and smaller. They showed that when there is a large but finite number of small shareholders, the takeover succeeds with a positive probability smaller than one and the raider can make a profit, yet, this profit vanishes as the number of the shareholders grows towards infinity. In our model, instead, positive profits persist even as the number of small shareholders grows towards infinity. Therefore, in the presence of a small cost of initiating a takeover in our model the raider can make a profit even if the majority stake is widely dispersed.

Our incomplete information model builds on Marquez and Yılmaz (2008) who introduced imperfect private information about the post-takeover value of the firm on the side of shareholders. They show that such asymmetric information between the shareholders and the raider causes a lemons problem in the market which exacerbates the raider's inability to make a profit. We introduce a minority large shareholder in the ownership structure of the target company, and show that this allows the raider to make a profit.

One of the important assumptions in both papers is that the shareholders are privately informed, while the raider is not. This can be thought of as a reduced-form of a model in which the raider has some information, yet this information is public. Crucial is that the shareholders have private information, in addition to the public information they share with the raider, and that they are asymmetrically informed. For example, employees of the firm, each of whom owns a very small number of shares, might know about the inner workings of the company, such as its corporate culture, which might affect its post-takeover value. Certainly the case in which the raider has some private information is also relevant. Indeed, our model serves as a building block for the analysis of such a signaling game, since our analysis applies to characterizing the shareholders' behavior after any price offer. For recent work on signaling in corporate takeovers see Marquez and Yılmaz (2012), Burkart and Lee (2010), Stepanov (2012) and Ekmekci and Kos (2014).

Shleifer and Vishny (1986) consider a model with a large shareholder and a continuum of small shareholders. In their model, the large shareholder is the raider, whereas in our model the large shareholder is passive. They show that the raider makes a strictly positive profit, because he already owns a nontrivial share of the company and has strict incentives to facilitate the takeover in order to increase the value of the shares he owned from the start, even at the expense of a loss on the new shares he buys. Holmström and Nalebuff (1992) study a complete-information model in which the firm is owned by several large shareholders. They construct a particular type of equilibrium and show that the raider can extract a significant part of the surplus when the number of shares goes to infinity, while the number of shareholders and their relative position in the firm is held fixed. In their model, unlike in ours, there are only large shareholders. Cornelli and Li (2002) analyze a setting with finitely many risk arbitrageurs participating in a tendering game. The risk arbitrageurs in their paper are similar to the large shareholders in our model. However, the takeover succeeds only when the arbitrageurs collectively hold at least half of the shares. In our model, the large shareholder does not have a controlling stake, yet takeovers can still be successful. Burkart et al. (2006) analyze the effects of a minority shareholder in takeovers. They consider a complete-information environment with a large minority shareholder, in which a successful raider can extract private benefits at the expense of the share value. In their model the large shareholder sells all the shares in equilibrium. Moreover, the large shareholder would like to sell more shares conditional on the takeover succeeding, because of the inefficiency of the post-takeover private-benefit extraction. In contrast, while the large shareholder does sell all of his shares in our model, he would have preferred to keep them if this had no effect on the success of the takeover. In the model of Burkart et al. (2006), the larger is the large shareholder, the higher is the raider's equilibrium price offer, and the lower his profits. We find the opposite in our model, the price is nonincreasing and the profit nondecreasing function of the large shareholder's stake.

While the papers summarized in the preceding paragraph show that the presence of a large minority shareholder can mitigate the free rider problem, the novelty of our paper is that the combination of the large shareholder and dispersed private information among the small shareholders alone enable the raider to achieve profit.

Other mechanisms have been proposed to overcome the free-riding problem. Dilution (Grossman and Hart (1980)), squeeze-outs (Yarrow (1985), Amihud et al. (2004)), and debt financing (Müller and Panunzi (2004)) all reduce the post-takeover value of the shares that a minority shareholder holds, creating pressure on him to tender. However, such mechanisms also create a conflict between minority shareholder protection and efficiency. Since this conflict does not arise in our model, we interpret our result as suggesting that minority shareholder protection can occur with efficient takeovers. Moreover, recently Dalkır and Dalkır (2013) and Dalkır and Dalkır (2014) showed that neither partial tender offers nor freeze-outs enable the raider to make a profit when the shareholders are sufficiently dispersed. Another proposed solution is for the raider to secretly acquire a stake in the company before the takeover attempt (Shleifer and Vishny (1986), Chowdhry and Jegadeesh (1994)). Whether such acquisitions can take place depends on the depth of the market and the regulatory disclosure requirements. In fact, Betton et al. (2009) provide empirical evidence that raiders rarely have a toehold in a company that they are taking over. Finally, private benefits may facilitate takeovers (Grossman and Hart (1980), Bagnoli and Lipman (1988) and Marquez and Yılmaz (2008)). In the latter models, unlike in ours, the raider does not make any profit on the shares he acquires from the shareholders.

Bagnoli and Lipman (1988) show that a mechanism which specifies that the raider pays a certain price per share if and only if all the shares are tendered can extract almost full surplus from the takeover by making every share pivotal. The authors themselves note that such offers are not observed because they are highly impractical. Even if a single shareholder were to fail to respond to the offer for some external reasons, the takeover would fail.

In a representative sample of U.S. public firms Holderness (2009) finds that ninety six percent of them have a block holder who owns at least 5% of firm's common stock. Further empirical support for the importance of large shareholders in takeovers is provided by Gadhoum et al. (2005). In our model, the large shareholder is a passive shareholder, i.e., he does not counter-bid the raider and try to undertake a takeover himself. Possible reasons for such a passive role for the large shareholder are that he may be financially constrained, he may lack managerial skills, or he may be prohibited from such an action; for example, in the case of pension funds. See Burkart et al. (2006) for more details about (i) the empirical evidence that the ownership structure we consider is a widely observed one, and (ii) support of considering a passive minority shareholder.

An important factor contributing to large shareholder formation is hedge fund activism.

Some recent and ever-expanding research explores the consequences of hedge fund activism. Empirical evidence shows that the primary source of positive returns from hedge fund activism is through the takeover premium which such funds get from the acquisition of the target firm (Brav et al. (2008), Brav et al. (2010) and Greenwood and Schor (2009)). Our results suggest that when a hedge fund becomes a minority large shareholder, this can help facilitate an efficient takeover, thereby serving as a powerful incentive to the incumbent management. Moreover, our finding that the large shareholder's information is not reflected in the likelihood of a successful takeover suggests that hedge funds need not have any firm-specific information in order to profit from takeovers.

2. The Model

A firm consists of a continuum of shares of measure one, represented by the interval [0, 1]. A mass of $1-x$ shares is held by a continuum of small shareholders of size $1-x$, each of whom holds a single share, while the mass x of the shares is held by a large shareholder. The large shareholder is not large enough to facilitate the takeover by selling all of his shares, i.e. $x < 1/2$. A raider wishes to take over the company, with the intention to run it. He needs to acquire at least $1/2$ of the shares to successfully take over the company.

Current management's ability to run the company is well understood, therefore the value of the company under the incumbent management is commonly known and normalized to 0. The raider's ability and motives, on the other hand, are uncertain. We model this by assuming that after a successful takeover, the value of the company depends on the state of the world ω, where $\omega \in \{l, h\}$. In the low state of the world ($\omega = l$) the post-take over value of the company is 0 while in the high state ($\omega = h$) the value of the company is 1, if the takeover succeeds. The common prior assigns the probability $\lambda \in (0, 1)$ to $\omega = h$.

Each small shareholder observes a signal drawn from a conditionally i.i.d. $F(s|\omega)$ with support $S := [0, 1]$, and with the density function $f(s|\omega)$, where ω is the true state of the world. Large shareholder observes a signal $s \in S$ drawn from distribution $H(s|\omega)$, with the density $h(s|\omega)$. The large shareholder's signal is conditionally independent of the small shareholders' signals. We assume that the weak monotone likelihood ratio property (MLRP) holds for the shareholders' signal distributions, i.e., $\frac{f(s|h)}{f(s|l)}$ and $\frac{h(s|h)}{h(s|l)}$ are nondecreasing on the interval $[0, 1]$. The general formulation with the weak version of the MLRP allows us to consider both the case of symmetric information, $\frac{f(s|h)}{f(s|l)} = 1$ for all s, as well as the asymmetric information with the strict MLRP within the generally specified model.

The takeover proceeds as follows. The raider offers an unconditional price offer $p \in [0, \infty)$. After the price offer $p \geq 0$, each small shareholder either tenders his share or keeps it, while the large shareholder decides what fraction of his shares to sell. In particular, a mixed strategy for a small shareholder is a measurable mapping that specifies the probability with which he sells his share for each signal:

$$\sigma : S \to [0,1].$$

Large shareholder's strategy is a right-continuous and weakly increasing mapping $\sigma_L : S \times [0,1] \to [0,1]$, which denotes the cumulative distribution function of the fraction of the shares he tenders, and whose marginal on its first coordinate coincides with the distribution of signals. The first argument is the signal, with a generic element s. The second argument is the fraction r. Modeling the strategy as a cumulative distribution function ensures that payoffs are well-defined (see Milgrom and Weber (1985)). The strategy σ_L is weakly increasing in both of its arguments, and for every $s \in [0,1]$ it satisfies

$$\sigma_L(s,1) = \lambda H(s|h) + (1-\lambda)H(s|l). \tag{1}$$

The above condition ensures that the marginal distribution of σ_L on its first coordinate is equal to the signal distribution.[3] The set of strategies of the large shareholder, Σ_L, is the set of all strategies (distributions) satisfying equality (1). In addition, we introduce the conditional distributional strategies, $\sigma_L(s,r|\omega)$, which we derive from $\sigma_L(\cdot,\cdot)$ as follows:

$$\sigma_L(\bar{s},\bar{r}|\omega) := \int_{s=0}^{\bar{s}} \int_{r=0}^{\bar{r}} \frac{h(s|\omega)}{\lambda h(s|h) + (1-\lambda)h(s|l)} d\sigma(s,r).$$

Note that $\sigma_L(s,1|\omega) = H(s|\omega)$, for every $s \in [0,1]$.

A strategy for a small shareholder, σ, is a threshold strategy if there exists a signal $\gamma \in S$ such that $\sigma(s) = 1$ for every $s < \gamma$ and $\sigma(s) = 0$ for every $s > \gamma$. A strategy profile in a tender subgame is a collection $\{\sigma_i\}_{i \in [0,1-x] \cup \{L\}}$. A strategy profile is *symmetric* if $\sigma_i = \sigma_j$ for every $i, j \in [0, 1-x]$; (σ, σ_L) denotes a typical symmetric strategy profile.

2.1. Payoffs A small shareholder's payoff from tendering his share is the price p. The expected payoff from keeping his share depends on his belief $q \in [0,1]$ that the takeover is

[3] We could also model the strategy of the small shareholder as a distributional strategy. However, as we will see later, in all equilibria the small shareholders' strategies have a threshold structure. Hence, we do not have to assume that the strategy of the small shareholder be representable by a distribution function.

successful in state h and his belief that the state is h.[4] Let

$$\beta(s) := \frac{\lambda f(s|h)}{\lambda f(s|h) + (1-\lambda) f(s|l)},$$

be a small shareholder's posterior belief that the state is h, given his signal s. Then, a small shareholder's payoff function is:

$$U(p, s, q, keep) = \beta(s) q,$$

and

$$U(p, s, q, sell) = p.$$

Similarly, let the large shareholder's posterior belief that the state is h when he observes signal s be $\beta_L(s) := \frac{\lambda h(s|h)}{\lambda h(s|h)+(1-\lambda)h(s|l)}$. For a collection of beliefs $q_L := q(r)_{r\in[0,1]}$, the expected payoff from tendering a fraction $r \in [0,1]$ of his shares is:

$$U_L(p, s, q(r), r) = x\left(rp + (1-r)q(r)\beta_L(s)\right),$$

with the interpretation that $q(r)$ is the belief the large shareholder attaches to the takeover succeeding in the high state when he tenders a fraction r of his shares.

Finally, the raider's payoff (i) when he offers the price p, (ii) the shareholders use the symmetric strategy profile (σ, σ_L), and (iii) he believes the probability of takeover success as a function of the large shareholder's behavior is determined by the collection q_L, is given by:

$$\begin{aligned} U_R(p, \sigma, \sigma_L, q_L) &= \lambda \int_{r,s} q(r) \left(xr + (1-x) \int_{s\in[0,1]} \sigma(s) f(s|h) ds \right) d\sigma_L(s, r|h) \\ &- p \left[(1-x) \int_{s\in[0,1]} \sigma(s)[\lambda f(s|h) + (1-\lambda) f(s|l)] ds + x \int_{s,r} r d\sigma_L(s,r) \right]. \end{aligned}$$

The first term represents the raider's benefits when the takeover is successful and the state is high. The second term represents the total payment the raider makes to acquire the shares. As a reminder, small shareholders' strategy σ is a standard behavior strategy which prescribes for each signal s the probability with which a small shareholder tenders his share. On the other hand, σ_L is a distributional strategy.

[4] The role and meaning of the concepts, such as the belief q introduced here, will be clearer when we define the equilibrium concept below.

2.2. Equilibrium A tuple $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$ is an equilibrium of a tender subgame with a price offer p if the following conditions hold:

$$U(p, s, q, \sigma(s)) \geq U(p, s, q, a),\ \forall a \in \{keep, sell\},\ \forall s \in [0, 1]. \tag{2}$$

$$\int_{s\in[0,1],r\in[0,1]} U_L(p, s, q(r), r)d\sigma_L(s, r) \geq \int_{s\in[0,1],r\in[0,1]} U_L(p, s, q(r), r)d\bar{\sigma}_L(s, r),\ \forall \bar{\sigma}_L \in \Sigma_L. \tag{3}$$

$$q(r) = \begin{cases} 0, & \text{if } (1-x)\int_0^1 \sigma(s)dF(s|h) + xr < 1/2 \\ 1, & \text{if } (1-x)\int_0^1 \sigma(s)dF(s|h) + xr > 1/2 \\ \in [0,1], & \text{if } (1-x)\int_0^1 \sigma(s)dF(s|h) + xr = 1/2. \end{cases} \tag{4}$$

$$q = \int_{s\in[0,1],r\in[0,1]} q(r)d\sigma_L(s, r|h). \tag{5}$$

The first two conditions are the standard conditions requiring that the shareholders' behavior is optimal given their beliefs. Condition (4) describes the large shareholder's beliefs about the probability of the successful takeover in the high state when he tenders a fraction r of his shares, given the fixed behavior of the small shareholders. The fraction of shares tendered by the small shareholders in the high state, given strategy σ, is $(1-x)\int_0^1 \sigma(s)dF(s|h)$. Therefore, if the large shareholder tenders fraction r of his shares and if $(1-x)\int_0^1 \sigma(s)dF(s|h) + xr$ is larger (smaller) than $1/2$, then the takeover succeeds (fails) with certainty. The indeterminate case is when $(1-x)\int_0^1 \sigma(s)dF(s|h) + xr = 1/2$. We leave the large shareholder's beliefs, in such a knife-edge case, to be determined in equilibrium. Finally, condition (5) requires that the small shareholders' belief q about the success of the takeover in the high state be derived from q_L, using the large shareholder's strategy σ_L.

REMARK 1 *Our equilibrium concept differs from Nash equilibrium or refinements thereof in that it contains variables such as the probability of a successful takeover. This is in the spirit of the rational-expectations equilibrium concept, and allows for the probability of a successful takeover when the fraction of shares acquired is one-half to be determined endogenously. We show in Section 7 that the limit of the equilibrium outcomes of the takeover model with finitely many shares is an equilibrium outcome of the model with a continuum of shares. We will comment further on the equilibrium concept and the interpretation of the limits in the subsequent sections.*

The raider's continuation payoff from offering p when the tuple $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$ is played in the tender subgame is denoted by:

$$\Pi(p, T) := U_R(p, \sigma, \sigma_L, q_L).$$

Note that a price $p > 1$ can never result in a positive payoff for the raider; we therefore restrict his price offers to the interval $[0, 1]$. We say that the collection $(p, T(p')_{p' \in [0,1]})$ is an equilibrium of the takeover game if each $T(p')$ is an equilibrium of the tender subgame with price offer p', and if $p \in argmax_{p' \in [0,1]} \Pi(p', T(p'))$.

When there is a unique equilibrium of a tender subgame for a given price p, we write $\Pi(p)$ for the raider's profit when all other players play the unique equilibrium of the tender subgame.

3. Symmetric Information

We first consider the symmetric information benchmark. The benchmark case can be represented by completely uninformative signals for the small and the large shareholders, i.e., by an information structure such that $f(s|h) = f(s|l) = f(s)$ and $h(s|h) = h(s|l) = h(s)$ for all s. Signal s here represents the role of a private randomization device. That is, $\int_0^1 \sigma(s) dF(s)$ can be interpreted as the probability with which a small shareholder sells his share. To simplify the notation we will assume that f and h are uniform distributions on the interval $[0, 1]$ and that σ is a threshold strategy which prescribes the small shareholder to sell his share if $s \leq \gamma$ and not sell otherwise. Then, γ becomes the probability with which a small shareholder sells his share.

Given that the information is symmetric, each shareholder attaches an expected value of λ to a share after the takeover is successful, and value of 0 if the takeover does not succeed. Our analysis here corresponds to the complete information case in Bagnoli and Lipman (1988) and the symmetric information analysis in Marquez and Yılmaz (2008).

If the raider offers price $p \geq \lambda$, then either all the shareholders tender their shares and the raider makes a loss (if $p > \lambda$) or the raider just breaks even (if $p = \lambda$). On the other hand, if the raider offers price 0, the probability of success needs to be 0 in equilibrium. If it were larger than zero, it would be optimal for the shareholders to keep their shares, but then the probability of success could not be positive. Therefore the raider's profit after offering the price zero is zero. In what follows we explore what happens when the raider offers a price $p \in (0, \lambda)$.

Since the shareholders are uninformed about the state of the world, the probability of the takeover succeeding is identical in both states. Nevertheless, since the value of the company is 0 in the low state we will focus on the probability of success in the high state, which we denote by q, with an understanding that the latter is equal to the expected probability of success.

THEOREM 1 *For any price offer $p \in (0, \lambda)$ there is a unique equilibrium of the tender subgame. In this equilibrium the large shareholder sells all of his shares, each small shareholder sells his share with probability $\frac{0.5-x}{1-x}$, the takeover succeeds with probability $q = p/\lambda$ and the raider's profit is* 0.

PROOF: See Appendix A. □

While the literature so far has emphasized the raider's inability to make a profitable tender offer for a widely dispersed company (see, for example, Grossman and Hart (1980) and Bagnoli and Lipman (1988)), Theorem 1 shows that profitable tender offers are impossible even in the presence of a large shareholder as long as he does not hold a majority stake.[5] In light of Theorem 1, the inability of the raider to make a profitable tender offer is not a consequence of the dispersed ownership of the whole firm, but is rather a consequence of the fact that the majority stake is widely dispersed.

In what follows we provide some intuition underlying the above theorem. For any $p \in (0, \lambda)$, the probability of the takeover succeeding in the high state, q, has to be strictly between 0 and 1; which is established similarly as in Grossman and Hart (1980) and Bagnoli and Lipman (1988). If q was 0, then keeping a share would be valueless and all the shareholders would be selling their shares, contradicting the assumption that the probability of success is 0. On the other hand, if the probability of success in the high state was to be 1, then the shareholders would keep their shares, which would lead the takeover to fail with certainty. Therefore, $q \in (0, 1)$.

Furthermore, in any equilibrium after a price offer $p \in (0, \lambda)$ it has to be the case that the small shareholders are selling their shares with a probability γ such that $(1 - x)\gamma + x \geq 1/2$. If that was not the case, then less than half of the shares in total would be sold even if the large shareholder was to sell all of his shares. But then the takeover would fail with certainty, contradicting the above observation that $q \in (0, 1)$. That is to say, the small shareholders sell their shares with a sufficiently high probability to make the large shareholder pivotal. Also, $(1 - x)\gamma \leq 1/2$, since otherwise the takeover would be successful with probability 1, contradicting the above observation that $q \in (0, 1)$. Given that in equilibrium the small shareholders are mixing between selling and not selling their share, they must be indifferent between these two choices. Selling the share yields the payoff p while keeping it yields λq. Indifference then requires $p = q\lambda$, or $q = p/\lambda$.

[5]In our model the large shareholder can not initiate a takeover. Shleifer and Vishny (1986) have shown that a raider with an initial stake in the company can make a profit.

Now we turn attention to the large shareholder. The large shareholder never sells with positive probability a fraction of shares such that the takeover fails with certainty. Selling all of his shares is a more profitable option. Large shareholder is also the only shareholder who could potentially tinker with the probability of the takeover. In particular, if the probability of success is $q \in (0,1)$ when he is selling a fraction $r < 1$ of his shares, then the value of the shares he is keeping is $(1-r)xq\lambda$. In this case he can increase the success rate from q to 1 by selling slightly more shares, which gets the total of the sold shares over 1/2. The only case in which such a profitable deviation does not exist is if the small shareholders sell with a probability γ such that exactly half of the shares are sold in total only if the large shareholder sells everything, i.e., $(1-x)\gamma + x = 0.5$. Thereby the large shareholder is not only made pivotal, but is made pivotal with every share he has.

The above reasoning shows that the only potential candidate for an equilibrium of a continuation game after a price offer $p \in (0, \lambda)$ has the small shareholders sell their share with probability $\frac{0.5-x}{1-x}$ and the large shareholder sell everything, with the probability of takeover succeeding being equal to p/λ. It is easy to verify that this profile of strategies indeed constitutes an equilibrium. The small shareholders are willing to mix between selling and not selling because the payoff from selling p is equal to the payoff from not selling $q\lambda$. The large shareholder strictly prefers to sell all of his shares. If he withheld some, less than half of the shares would be sold in total and the probability of success would plunge to 0, rendering the withheld shares worthless.

Finally, after a price offer $p \in (0, \lambda)$ the total value created is $q\lambda$. Each small shareholder receives payoff $q\lambda$, therefore the small shareholders in total receive $(1-x)q\lambda$. Large shareholder's payoff is $px = q\lambda x$, thus the shareholders' welfare in total $q\lambda$. This, in turn, implies that the raider's profit must be 0.

Large shareholder with a controlling stake. Our result depends on the assumption that the large shareholder owns a minority stake of the firm. If the large shareholder were to own a majority stake, then for every positive price offer, there would be an equilibrium of the subgame in which the large shareholder would sell exactly half of the firm's shares, and the small shareholders would sell none. Therefore, in equilibrium, the takeover would succeed and the raider would obtain half of the surplus of the takeover activity. If, contrary to what we assume, the large shareholder holds a fraction $x > 1/2$ of the company, then the raider can facilitate a successful takeover by offering any positive price. Namely, it is in the large shareholder's interest to ensure a successful takeover. Small shareholders' holdings, on the other hand, are not necessary for a successful takeover.

The equilibrium concept. The equilibrium concept used in our paper suffers from a slight drawback of not being able to pinpoint the raider's limiting equilibrium price. Namely, in the continuum model the raider's profit is 0 irrespective of which price in the interval $[0, \lambda]$ the raider offers, and hence any price offer in that interval can be sustained as part of an equilibrium. Despite this, the equilibrium concept provides an intuitive characterization of the raider's equilibrium payoff in a model with a finite and large number of shares and shareholders. In particular, in Appendix C we show that any sequence of symmetric subgame perfect equilibrium outcomes of finite games converges, as the number of shares and shareholders go to infinity, to one of the tender equilibrium outcomes of the continuum shares model. Because there may be multiple tender equilibria, we can not say which one of these tender equilibria will be the limit point of the equilibrium outcomes of the finite takeover games. But since all of the tender equilibria yield zero profit for the raider, we know that this will also be the case in the limit of the equilibria of those finite games. One can further show that, along any sequence $(p^n, \sigma^n, \sigma_L^n)_n$ in which $p^n \in [0, \lambda]$ for all n, and (σ^n, σ_L^n) are symmetric equilibria of continuation games after p^n, the raider's profits converge to 0; as is shown in an earlier version of this paper Ekmekci and Kos (2012b). This is why our equilibrium concept yields profit 0 for the raider for every $p \in [0, \lambda]$.

4. Asymmetric Information

Here we assume that the strong MLRP holds for the shareholders' signal distributions, i.e., $\frac{f(s|h)}{f(s|l)}$ and $\frac{h(s|h)}{h(s|l)}$ are strictly increasing on the interval $[0, 1]$, and that the densities $f(\cdot|h)$ and $f(\cdot|l)$ are continuous. Notice that the strong MLRP implies that all the densities are larger than zero and finite for all $s \in (0, 1)$.

Below we characterize the equilibrium behavior of the shareholders for both on and off the equilibrium price offers. In particular, in Theorem 2 we show that there is a unique equilibrium of each tender subgame after a positive price offer, and we calculate the raider's profits in each of these equilibria. In the following development, we show that under certain parameters the raider makes a positive price offer, the takeover is successful in the high state, and the raider makes a strictly positive profit. We start with a preliminary observation that the small shareholders use threshold strategies in any tender subgame.

Lemma 1 *In any equilibrium of the tender subgame where $p > 0$, the small shareholders use a threshold strategy, i.e., there is a $\gamma \in [0, 1]$ such that each small shareholder tenders his share if $s < \gamma$ and keeps it if $s > \gamma$.*

Proof: Strict MLRP condition implies that a small shareholder's belief $\beta(s)$ is a strictly increasing function. Fix an equilibrium of the tender subgame, $T = (\sigma, \sigma_L, q, q(r)_{r \in [0,1]})$, for some $p > 0$. A small shareholder's payoff from tendering a share is p, while keeping the share yields $q\beta(s)$. Therefore, if $\sigma(s) > 0$ for some s, then for every $s' < s$, it follows that $q\beta(s') < p$ and hence $\sigma(s') = 1$. Similarly, if $\sigma(s) < 1$ for some s, then $q\beta(s') > p$ for all $s' > s$, and therefore $\sigma(s') = 0$ for every $s' > s$.[6] $\square$

We call a signal $s^* \in [0, 1]$ *pivotal* if it has the property that when the small shareholders use the threshold s^*, and the large shareholder tenders all his shares, then the fraction of tendered shares in the high state is $1/2$. Since $x < 1/2$, the pivotal type $s^* \in (0, 1)$ is uniquely defined by

$$F(s^*|h)(1 - x) + x = 1/2.$$

The *critical price* $\bar{p}$ is the price that would keep the pivotal type indifferent between tendering his share and keeping it, if he believed that the takeover would be successful with probability one in state h. In particular,

$$\bar{p} := \beta(s^*). \tag{6}$$

REMARK 2 *Note that both s^* and $\bar{p}$ are decreasing in x.*

We now present the unique equilibrium of each tender subgame with a price offer $p > 0$. The structure of equilibria depends on whether the price offer is below or above the critical price.

THEOREM 2 *(Characterization) For any $p > 0$, there is a unique equilibrium of the tender subgame, $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$. σ is a threshold strategy with a threshold $\gamma \in [0, 1]$.*

(i) If $p \le \bar{p}$, then

a) $\gamma = s^*$.

b) $\sigma_L(s, r) = 0$ for every $s \in [0, 1]$ and every $r < 1$.

c) $q = \frac{p}{\beta(s^*)}$, $q(1) = \frac{p}{\beta(s^*)}$ and $q(r) = 0$ for all $r < 1$.

Moreover, the raider's profit is

$$\Pi(p) = \lambda\frac{q}{2} - p\left(\lambda\frac{1}{2} + (1 - \lambda)[(1 - x)F(s^*|l) + x]\right). \tag{7}$$

(ii) If $p > \bar{p}$, then

a) $\gamma = 1$ if $p \ge \beta(1)$, and otherwise is the unique solution to the equality $\beta(\gamma) = p$.

b) There is a signal $s_L \in [0, 1]$ and a fraction $a < 1$ such that, if the large shareholder's

[6] Here we use a convention that if q and p are such that $q\beta(s) < p$ for all s, then the small shareholders use the threshold $\gamma = 1$; in other words, they tender irrespective of their signal. Similarly, if $q\beta(s) > p$ for all s, the threshold $\gamma = 0$, which means that they never tender.

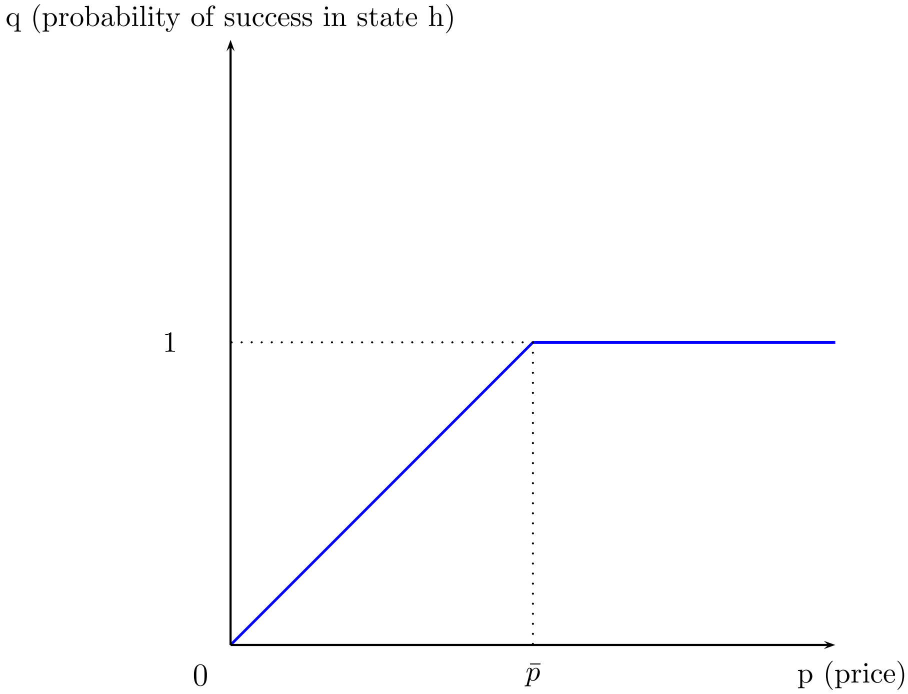

Figure 1: This figure shows how the equilibrium probability of a successful takeover in state h, denoted by q, varies with the raider's price offer. In particular, $q = \frac{p}{\bar{p}}$ for $p \leq \bar{p}$ and $q = 1$ for $p > \bar{p}$.

signal $s > s_L$, then he tenders fraction a of his shares; and if $s < s_L$, then he tenders all of his shares.

c) $q = 1$, $q(r) = 0$ for $r < a$, and $q(r) = 1$ for $r \geq a$.

Moreover, the raider's profit is

$$
\begin{aligned}
\Pi(p) = {} & \lambda \left[(1-x)F(\gamma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\
& -p\lambda \left[(1-x)F(\gamma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\
& -p(1-\lambda)[(1-x)F(\gamma|l) + x(a(1-H(s_L|l)) + H(s_L|l))].
\end{aligned} \tag{8}
$$

Proof: See Appendix B. □

The theorem characterizes the unique equilibrium of tender subgames under two cases. The first one is when the price p is smaller than or equal to the critical price $\bar{p}$. In this case, the small shareholders' equilibrium threshold is s^*, and is independent of the exact value of p. The probability of a successful takeover in the high state, q, is determined endogenously so that a shareholder receiving the signal s^* is indifferent between tendering and not tendering his share. If he tenders his share, he receives the price p, whereas if he keeps it, it is worth zero in the low state, and one in the high state but only if the takeover succeeds. Such a shareholder believes that the state is high and the takeover succeeds with probability $\beta(s^*)q$.

Therefore, the probability of a successful takeover is linear in the price offer (see Figure 1 for a depiction).

The large shareholder, on the other hand, tenders all of his shares regardless of his signal. If he were to tender anything less, he would cause the takeover to fail in the high state with certainty, by the definition of the pivotal signal s^*. Such a takeover failure would render the shares he held back worthless. His behavior is therefore independent of his signal.

The shareholders' behavior, as described for $p \leq \bar{p}$, and the definition of s^* imply that exactly half of the shares are sold in the high state. The raider's payoff can now be decomposed into two parts. He benefits only from the shares that he holds in the high state conditional on the takeover succeeding, as captured in the first term in equation (7). Since (*i*) the probability of the high state is λ, (*ii*) the probability of the success of the takeover in the high state is q, and (*iii*) exactly half of the shares are being sold in the high state, this yields $\lambda q/2$. The second term in equation (7) represents the expected amount the raider pays for shares in the equilibrium. It is the price p times the expected quantity of shares he has bought. Notice that more shares are sold in the low state than in the high. This is because the large shareholder's tendering decision is independent of his signal, and the small shareholders tender their shares when they observe the low signals, which are in turn more likely in the low state. This is a so-called lemons problem. Only the more pessimistic small shareholders are willing to tender their shares. Therefore, the raider who wants to induce a successful takeover in the high state must accept losses on the shares he buys from the small shareholders.

The second case is when the price offer p is larger than $\bar{p}$. In this case, the small shareholders' equilibrium threshold γ is greater than the pivotal signal, s^*. Namely, the small shareholder with a signal s^* is indifferent between tendering and keeping his share even when the probability of success is one and the price is $\bar{p}$. He, therefore, strictly prefers to tender when the higher price is offered and the probability of success is smaller than or equal to one. Consequently the equilibrium threshold must be above s^*. An argument similar to the one above implies that the large shareholder either tenders a fraction a of his shares, which is barely sufficient to ensure a successful takeover in the high state, or he tenders all of his shares. He certainly tenders all the shares when he deems the high state unlikely, in which case even a successful takeover does not generate a high post-takeover share value. On the other hand, he tenders only fraction a of his shares, the smallest fraction which renders success in the high state certain, when his signal favors the high state. More importantly, when $p > \bar{p}$, the takeover succeeds with probability one in both states.

Before we proceed to the characterization of the raider's equilibrium price offers, we show that the raider's profit from offering price zero is zero.

Lemma 2 $\Pi(0, T) = 0$ *for any equilibrium of the tender subgame where* $p = 0$.

Proof: When $p = 0$, $q = 0$. If on the contrary $q > 0$, it would be optimal for the small shareholders not to tender their shares and to obtain a positive payoff, which contradicts $q > 0$. Since the probability of success in the high state is zero, the raider expects a payoff of zero regardless of how many shares are tendered.[7] □

In the following result we show that the raider, depending on the parameters of the environment, either offers the price zero and makes zero profit, or the price $\bar{p}$, in which case the profit is positive.

Lemma 3 *The raider's profit is maximized at either* $p = 0$ *or* $p = \bar{p}$. *If*

$$\Pi(\bar{p}) := \lambda\frac{1}{2} - \bar{p}\left(\lambda\frac{1}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]\right) > 0,$$

then the raider offers the price $\bar{p}$, *and the takeover is successful with probability one in both states. If instead* $\Pi(\bar{p}) < 0$, *then the raider offers price zero, and the takeover fails with certainty in the high state.*

Proof: See Appendix B. □

Only one of the two prices may arise in equilibrium. Either the raider is not willing to offer a positive price for the shares, or he pays the lowest price that ensures a successful takeover in the high state.

To see that no price strictly between zero and $\bar{p}$ is offered, notice that for any such price the threshold type of the small shareholders is s^*. Moreover, the indifference condition for the type s^* implies that $p = q\beta(s^*)$. Hence, the price and the probability of a successful takeover in the high state are linearly related to each other. The fraction of shares that the raider acquires does not depend on the price, as long as $p \leq \bar{p}$, because the large shareholder tenders all of his shares at any such price. Therefore, if offering a positive price is a better strategy than offering a zero price, then the marginal cost of increasing the probability of success in the high state is strictly smaller than the marginal benefit from increasing such a probability. Consequently, offering $\bar{p}$ dominates any offer below $\bar{p}$.

The raider does not want to make an offer larger than $\bar{p}$. Note that the total surplus is equal to the probability of a successful takeover in the high state. This probability is 1 as long as the price is at least $\bar{p}$. Therefore, the total surplus is independent of the price offer

[7] Shareholders' equilibrium behavior for $p = 0$ is restricted only by $q = 0$. In the low state the takeover could succeed with any probability.

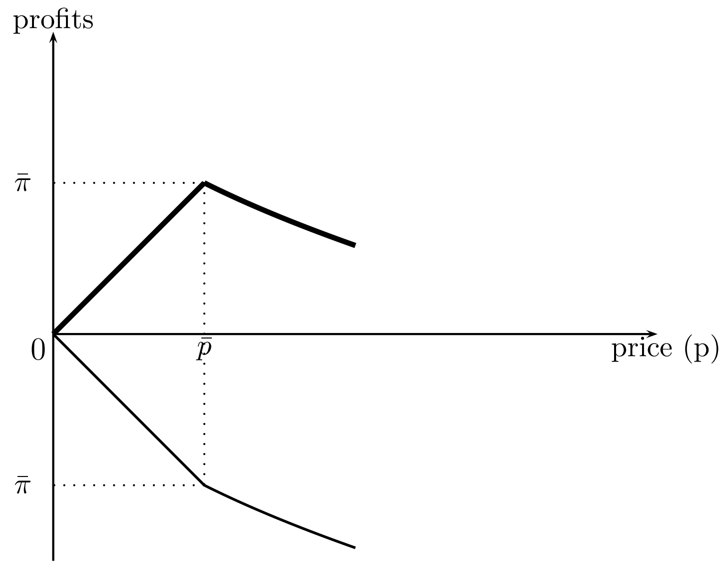

Figure 2: This figure shows the relationship between the raider's profits and the price offer he makes. If $\lambda\frac{1}{2} - \beta(s^*)\left(\lambda\frac{1}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]\right) > 0$, then the profit function has the shape of the thick curve (higher curve); otherwise, it has the shape of the thin curve (lower curve). In either case, the profit function is strictly decreasing in the price offer in the range $p \geq \bar{p}$.

in that range. However, the surplus that goes to the shareholders strictly increases with the price offer. Hence, the raider's profit is lower with higher price offers. (See Figure 2, which shows that the raider's profits are linear in price until the price reaches $\bar{p}$ and are decreasing when the price is above $\bar{p}$.)

The large shareholder's information is irrelevant for the outcome of the takeover and the raider's profit. Indeed, Lemma 3 shows that the raider's profit is maximized at either 0 or $\bar{p}$. The raider's profit is 0 when he offers price 0, thus independent of large shareholder's information. When the raider offers price $\bar{p}$ the large shareholder sells all of his shares regardless of his private information; as shown in Theorem 2. Since $\bar{p}$ is independent of the large shareholder's information, so are the raider's strategy and the profit.

So far we have argued that the raider will only ever offer either price zero or the lowest price at which the takeover succeeds with probability one in both states, $\bar{p}$. To put it differently, we have ruled out the possibility that any other price is offered in equilibrium. We have yet to establish, however, that the raider can make a profit from a positive price offer. We will show that the raider's ability to make a profit depends on the size of the large shareholder x. In general, the term $\bar{p}$ depends on x. However, we often suppress this dependence in notation to keep the exposition simple.

The next lemma shows that the raider's payoff from offering the lowest price that renders the takeover successful is strictly increasing in the large shareholder's stake.

Lemma 4 *Let* $\Pi(x) := \Pi(\bar{p}(x))$ *be the raider's profit when he offers the price* $\bar{p}$ *and the large shareholder holds a fraction* $x < 1/2$ *of the shares. Then* $\Pi(x)$ *is a strictly increasing function.*

Proof: See Appendix B. $\square$

Theorem 2 implies that at price $\bar{p}$ the large shareholder sells everything and the small shareholders use such a threshold that precisely half of the shares are sold in the high state. Moreover, for a given size of the large shareholder x, price $\bar{p}$ is set by the indifference condition of the threshold type, i.e., $\bar{p} = \beta(s^*)$. Therefore, the larger the large shareholder, less shares the raider needs to acquire from the small shareholders to reach the controlling stake. In other words, larger x implies smaller s^*, and therefore smaller $\beta(s^*)$, which in turn implies a smaller $\bar{p}$. To sum up, larger x enables the raider to guarantee the success of the takeover at a lower price. Consequently the raider's profit from offering the smallest price at which the takeover succeeds with probability one is increasing in the size of the large shareholder's stake.

We now argue that $\Pi(x)$ is positive if x is close to but smaller than $1/2$, and negative if x is close to zero.

Lemma 5 $\lim_{x \searrow 0} \Pi(x) < 0$ *and* $\lim_{x \nearrow 1/2} \Pi(x) = \frac{1}{2}[\lambda - \beta(0)] > 0$.

Proof: See Appendix B. $\square$

The term $\Pi(x)$ is the raider's profit when the large shareholder owns a fraction x of the shares and the raider offers the price $\bar{p}$. The negative limit when x tends towards zero means that by offering $\bar{p}$, the raider would incur a loss. Therefore, when x is close to zero, the raider prefers to offer price 0. This is in line with the results from Marquez and Yılmaz (2008), who have shown that when the firm is owned only by small shareholders, the raider cannot make a profit.

In the limit as x approaches 0.5, given that the large shareholder at $\bar{p}$ sells everything, the raider, loosely speaking, only needs to obtain a share from the shareholder with the signal 0. This can be done by offering the price equal to his belief $\beta(0)$. Since the prior probability of the high state is λ and we assume MLRP, it follows that the lowest possible posterior $\beta(0)$ is strictly smaller than the prior λ. However, $\beta(0)$ can be anywhere in $[0, \lambda)$. If it is closer to zero, the agent who is observing the lowest signal is almost certain that he is in the low state. In such an environment the low signal is very informative about the state. If $\beta(0)$ is closer to λ, then the shareholder who observes signal 0 holds almost the same beliefs as he did prior to observing the signal. Such a signal is rather uninformative. Our result shows that as x tends to one-half, the raider's profit is higher when the bottom signal is more informative. This is not only because of the conveyed information per se, but because the greater informativeness of the bottom signal helps the raider to differentiate the shareholders. This in turn enables him to buy the shares from the most pessimistic shareholders relatively cheaply. It is worthwhile to remark that this observation relies on looking at the limit as x goes to one-half. For a fixed x, the impact of increasing the informativeness of the signals of the small shareholders is ambiguous. A more precise information structure may increase the pivotal type for some parameters.[8]

Lemma 3 shows that only two prices can be offered (generically) in the equilibrium of the takeover game: 0 or $\bar{p}$. Lemma 4 implies that the raider's profit from offering $\bar{p}$, $\Pi(x)$, is strictly increasing in x. Moreover, Lemma 5 shows that $\Pi(x) < 0$ for x close to 0 and $\Pi(x) > 0$ for x close to 0.5. The three results together then imply the existence of $x^* < 1/2$ such that $\Pi(x) \leq 0$ for $x < x^*$, and, $\Pi(x) > 0$ for $x > x^*$, thus proving the following theorem.

THEOREM 3 *There exists $x^* < 1/2$ such that, for $x > x^*$ in the equilibrium of the takeover game the raider offers price $\bar{p}$ and makes a strictly positive profit. Moreover, for $x > x^*$ the raider's profit is strictly increasing in the size of the large shareholder, x. For $x < x^*$ the raider offers price* 0.

The raider can make a profit if the large shareholder is large enough. Moreover, the raider's profit is non-decreasing in the size of the large shareholder. It is constant, and equal to zero, for $x < x^*$, and strictly increasing for $x > x^*$. In the latter case the raider offers price $\bar{p}$. The monotonicity of profits follows from Lemma 4.

In Section 7 we establish that a limit of symmetric PBE of finite games must be an equilibrium of the game with a continuum of shares. Since in our model the equilibrium outcome is generically unique (as shown in Lemma 3), it follows that such a unique outcome is the limit (as the number of shares goes to infinity) of the sequence of symmetric PBE equilibria of the finite games.

Two assumptions play the critical roles in enabling the raider to make a profit. The first, of course, is the existence of the large shareholder. The second is the dispersion of the information among the small shareholders, captured by the assumption that signals are conditionally i.i.d. distributed and satisfying strict MLRP. As argued after Lemma 4, a successful takeover requires fewer shares from the small shareholders when the large shareholder's stake is larger. When the small shareholders' beliefs are dispersed, the raider can acquire these shares from the most pessimistic shareholders, and the shares of the large shareholder, at a lower price.

[8] We thank an anonymous referee for this observation.

We demonstrate the need of dispersed beliefs for the raider's profits in the following subsection. The following analysis shows that when the small shareholders' signals are perfectly correlated, and therefore their beliefs about the value of the shares are common, the raider cannot make a profit.

4.1. Asymmetric but Perfectly Correlated Information. We consider an environment as presented in Section 2 with the difference that all the small shareholders observe the same signal. The small shareholders' signal is imperfectly informative, and the strong MLRP holds. To keep the analysis tractable, we also assume that the large shareholder is uninformed (i.e., his signal is uninformative). There is asymmetry of information between the small shareholders and the raider, but no asymmetry among the small shareholders.

A caveat of this setup is that the equilibrium concept has to be augmented to capture the fact that the probability of success of the takeover in the high state might also depend on the signal s the small shareholders observe. In particular, in equilibrium there is a mapping $\{q(r,s)\}$ that indicates the success rate in the high state, if the large shareholder sells fraction r of his stake and the small shareholders observe signal s. The mapping $q(r,s)$ is similar to the mapping $q(r)$ introduced in Section 2 in that it is determined endogenously in equilibrium when exactly half of the shares are sold in the high state. The additional dependence on the signal s is needed, because the small shareholders can jointly vary their strategies according to s. We will now argue that the raider cannot make a profit after any price offer.

For any $p > 0$ the following equilibrium delivers the unique equilibrium outcome of the tender subgame. Define a signal s^* through $\beta(s^*) = p$. In equilibrium all small shareholders sell their shares if $s < s^*$. If however $s > s^*$, each small shareholder sells his share with probability $\gamma = \frac{0.5-x}{1-x}$. The large shareholder sells all his shares. To complete the description of the equilibrium we need to specify the beliefs that the takeover succeeds in the high state for every combination of (r,s):

$$(9) \qquad q(r,s) = \begin{cases} \frac{p}{\beta(s)}, & \text{if } r = 1, \text{ and } s > s^*, \\ 0, & \text{if } r < 1 \text{ and } s > s^*, \\ 1, & \text{if } r \leq 1 \text{ and } s < s^*. \end{cases}$$

In words, the first equality represents the probability of success in the high state when the large shareholder is selling all of his shares, while the small shareholders are observing a signal $s > s^*$ and therefore selling with probability γ as defined above. The second equality states that, if the small shareholders observe a signal $s > s^*$ and the large shareholder is not selling all of his shares the takeover will surely fail. Meanwhile, the last equality asserts that the takeover succeeds with certainty upon the small shareholders observing $s < s^*$, irrespective of the large shareholder's behavior.[9]

First we verify the optimality of the small shareholders' behavior. Indeed, for any $s < s^*$, by the strong MLRP, $\beta(s) < \beta(s^*) = p$. Therefore, even if the takeover was to succeed with probability 1 the small shareholders would prefer to sell. Since all of the small shareholders are selling, at least a fraction $1 - x > 0.5$ is sold for signals $s < s^*$, therefore $q(s, r) = 1$ for any r and $s < s^*$. That is, if the small shareholders observe a signal $s < s^*$ they sell everything and the takeover succeeds with probability one, regardless of what fraction of his shares the large shareholder sells. For signals $s > s^*$ the small shareholders expect the takeover to succeed with probability $q(s, 1) = \frac{p}{\beta(s)}$, since the large shareholder is supposed to sell everything. But then they are indifferent between selling and not selling.

Before we verify that the large shareholder wants to sell everything, let's remember that he is uninformed. Therefore, if he were to observe the signal s that the small shareholders observe, his posterior would also be $\beta(s)$. In what follows we show that the large shareholder would prefer to sell everything conditional on each of the realizations of the signal s that the small shareholders observe. He then clearly wants to sell everything when he has no information. If he was informed that the small shareholders observed a signal s such that $s < s^*$ he would have a posterior $\beta(s) < p$ and would have preferred to sell everything. On the other hand, if he was to condition on a signal $s > s^*$, he would infer that the small shareholders are selling with such a probability that the takeover can succeed with positive probability in the high state only if he is selling everything. Therefore again, it would be optimal for him to sell everything.

Finally, we argue that the raider cannot make a profit after any price offer $p > 0$. Even more, the raider cannot make a profit conditional on any s. For $s < s^*$ the shares are in expectation worth less than the price p even if the takeover is successful. For $s > s^*$ the probability of success in the high state is such that the shareholders are indifferent between selling and not selling. The raider is therefore just breaking even. But then after aggregating over all the signals s the raider must be loosing money in expectation. The only remaining case is when $p = 0$. In this case, the probability of a successful takeover after any signal has to be 0. Otherwise, if after a signal s, the takeover were to succeed with a strictly positive probability, then none of the small shareholder would be willing to sell their shares, contradicting the initial hypothesis that the takeover succeeds with positive probability. The uniqueness of the above equilibrium in the tender subgame after price offer $p > 0$ follows

[9] Note that what happens if $s = s^*$ is not uniquely pinned down in equilibrium, and hence we did not specify it in our description, however, in all tender subgame equilibria, when $s = s^*$, takeover succeeds with probability one. The multiplicity arises because the fraction of shares that are sold may be any fraction above $1/2$, however, this indeterminacy does not affect the raider's profit.

from a similar argument as the case of dispersed information.

The above reasoning provides a proof for the following result.

Proposition 1 *If the small shareholders' signals are perfectly correlated and the large minority shareholder is uninformed, the raider offers price* 0 *in the unique equilibrium of the takeover game. The equilibrium results in a zero profit for the raider.*

When all the small shareholders observe the same signal, their posterior beliefs about the state are identical. Therefore, no shareholder is more pessimistic than the others, which prevents the raider from catering only to more pessimistic small shareholders by offering a low price.

5. Multiple large shareholders

Our results extend to a setting with multiple large shareholders. Suppose there are K large shareholders, $j \in \{1, 2..., K\}$. Large shareholder j holds a fraction x_j of the total number of shares. We assume that $x := \sum_{j=1}^{K} x_j < 1/2$.

First we consider the symmetric-information setup, where the value of the company in the case the takeover succeeds is λ. In a tender subgame after a price offer p, the large shareholder j's strategy $\sigma_{L,j}$ is a probability distribution over the fraction of his shares he tenders to the raider. In a symmetric equilibrium the small shareholders use mixed strategies,[10] i.e., they are indifferent between tendering and not tendering their shares. This means that the probability they assign to the takeover succeeding must be equal to p/λ. In turn, the ex ante expected surplus is equal to p, and the small shareholders' surplus is $(1-x)p$. On the other hand, each of the large shareholders can guarantee himself the payoff px_j by selling all of his shares; thus the large shareholders together get at least xp. But then there is no surplus left for the raider.

Now suppose that there is incomplete information about the state of the world, and the large shareholder j receives a private signal, which is distributed according to the probability distribution function $H_j(s|\omega)$. In this case, there might a multiplicity of equilibria in the continuum model due to the possibility of coordination failures among large shareholders. However, there also exists an equilibrium in which, after the price offer $\bar{p} = \beta(s^*)$, the large shareholders tender all their shares, and the takeover is successful with certainty. In this equilibrium, the raider's profit is the same as in the unique equilibrium of the tender subgame with price offer $\bar{p}$ in the model with a single large shareholder who holds fraction x

[10]Symmetry here means that all the small shareholders use the same strategy.

of all the shares. Hence, if $\Pi(\bar{p}) > 0$, the raider offers price $\bar{p}$ and the takeover succeeds with certainty.

6. Private benefits

Suppose that the raider obtains some private benefit, $B > 0$, if the takeover is successful, irrespective of the state of the world.

In the symmetric-information scenario, the raider's equilibrium payoff is B, and the probability that the takeover succeeds is one. This is very similar to the result obtained by Marquez and Yılmaz (2008) in an environment without a large shareholder. The reason is that, given that the raider can not make a profit on the shares he acquires, he makes certain that he receives the private benefit B. He can always achieve this by offering the shareholders the full post-takeover value of the company.

In the incomplete-information game with a continuum of shares, the shareholders' behavior is the same regardless of the size of B; therefore, the characterization of the shareholders' behavior carries over from the environment with $B = 0$. For any strictly positive price offer, the raider receives at least half of all the shares in the high state of the world. As in the case in which $B = 0$, prices above $\bar{p}$ are dominated for the raider by $\bar{p}$.[11] In the remainder of the analysis we focus on the case in which $p \in [0, \bar{p}]$. For the prices in $(0, \bar{p}]$, exactly half the shares are sold in the high state. This in turn means that the raider receives a fraction of shares that strictly exceeds one-half, and therefore the takeover succeeds with probability one, in the low state of the world.

There is a multiplicity of equilibria in the tender subgame after $p = 0$, with the common feature that the takeover succeeds with probability zero in the high state, and that all the shareholders have payoff zero. Nevertheless, the equilibrium outcome is unique, since when the raider offers any positive price, the takeover succeeds in the low state with probability one.

In equilibrium the raider offers either price $\bar{p}$ or zero. Prices above zero increase the probability of a successful takeover in the high state, but do not affect the probability of a successful takeover in the low state or, for that matter, the fraction of shares that the raider acquires. In particular, for any price $p \leq \bar{p}$, the probability that the takeover succeeds in the high state is $p/\bar{p}$. Hence, if the raider prefers to offer a positive price to offering a zero price, then he also prefers to offer $\bar{p}$ to offering any price strictly smaller than $\bar{p}$. The equilibrium price is $\bar{p}$ if $\Pi(\bar{p}) + \lambda B > 0$. In other words, there is a threshold $\bar{B} \geq 0$ such that if $B \geq \bar{B}$, then

[11] For all those prices, the takeover in the high state occurs with probability 1; thus, the expected surplus is $\lambda + B$. Increasing the price from $\bar{p}$ can only increase shareholders' payoffs, and thus, decrease the raider's payoff.

the takeover succeeds with probability one in both states. Otherwise, it is only successful in the low state. Moreover, this threshold is strictly smaller than the threshold identified by Marquez and Yılmaz (2008) if $x > 0$, and coincides with their threshold if $x = 0$.

Some care is required when $\Pi(\bar{p}) + \lambda B < 0$; equivalently, $\Pi(\bar{p}) + B < (1-\lambda)B$. In this case, the raider offers price zero, the takeover succeeds with probability one in the low state and with probability zero in the high state. While we pointed out that there are equilibria of the tender subgame after price offer zero in which the takeover succeeds in the low state with any probability, the tender equilibrium requires that after the price offer zero, an equilibrium of the tender subgame is played in which the takeover succeeds with probability one in the low state. Otherwise, the raider would have a profitable deviation to a price just slightly above zero, which would ensure that he obtains the private benefit B at least in the low state.

To sum up, we state the results argued above in the form of a Theorem, without providing a formal proof.

Theorem 4 *In an incomplete-information model in which the raider has some commonly known private benefits $B \geq 0$, the raider offers either price $\bar{p}$ or zero. He offers price $\bar{p}$ if $\Pi(\bar{p}) + \lambda B > 0$. Moreover, given the size of the large shareholder's stake $x < 1/2$, there exists a $\bar{B}_x$ such that if $B > \bar{B}_x$, the takeover succeeds with probability one in both states and the raider makes a profit.*

It is easy to see that B_x in the above theorem is non-increasing. It is strictly decreasing for the values of x so low that without any private benefits the raider could not make a profit. On the other hand, when x is large enough so that the raider can make a profit even without the private benefits, $B_x = 0$.

Our result has an important implication for the estimation of the raider's profits in takeovers. Even if the raider can make a profit in a certain model – as for example in a model with only small shareholders, symmetric information, and private benefits – such a profit will underestimated if the large shareholder and the dispersion of information are not modeled.

7. Finite Model and Convergence

In this section we introduce the tender game with finitely many shares. There are n shares in total.[12] Each one of the $(1 - x)n$ small shareholders holds a single share, while the large shareholder holds xn shares. We allow for the two information structures as specified in

[12] In the following development, if any number that corresponds to a number of shares is not an integer, it should read as the smallest integer not larger than that number.

Sections 3 and 4, i.e., when the signals are uninformative or when the strict version of MLRP is satisfied. While the result could be proven beyond the two information structures, such generality would unnecessarily complicate the exposition.

7.1. Strategies and Payoffs A mixed strategy for the raider is a distribution over a set of prices, $[0, 1]$. In a tender subgame after the raider's price offer, a strategy for a small shareholder is a mapping from his signal to the probability with which he tenders his share, denoted by $\sigma^n(s)$. A strategy for the large shareholder describes the number of shares he tenders for every realization of his private signal. It will be convenient to describe his strategy as a joint probability distribution function over his signals and the fraction of shares that he is tendering. Remembering that Σ_L is the set of all strategies for the large shareholder in the model with continuum shares, let the set $\Sigma_L^n \subset \Sigma_L$ be the set of strategies such that for any $\sigma_L^n \in \Sigma_L^n$, for every $s \in [0, 1]$, and for every $i \in \{0, 1, ..., nx - 1\}$, the strategy $\sigma_L^n(s, r)$ is constant in the interval $r \in \left[\frac{i}{nx}, \frac{i+1}{nx}\right)$. A strategy for the large shareholder in the game with n shares is an element in Σ_L^n, and a typical strategy is σ_L^n. The large shareholder's strategy induces a probability distribution on the fraction of tendered shares conditional on state h, denoted by $g^n : \{0, 1, ..., nx\} \to [0, 1]$, and defined as:

$$g^n(i) := \int_{s\in[0,1]} (\sigma_L^n(s, i/nx) - \sigma_L^n(s, (i-1)/nx))dH(s|h) \text{ , for } i > 0;$$
$$g^n(0) := \int_{s\in[0,1]} \sigma_L^n(s, 0)dH(s|h).$$

We specify the payoffs of the small shareholders and the large shareholder in the same way that we did for the continuum shares case in Section 4. In particular, the small shareholders' payoffs are

$$U(p, s, q, keep) = \beta(s)q,$$

$$U(p, s, q, sell) = p,$$

where p denotes the price offered for the firm, s denotes a shareholder's signal, and q represents the probability of the success of the takeover in the high state, conditional on the shareholder keeping his share.

Remark 3 *We interpret the raider's price offer p as the price he offers for the whole company, which converts into offering a price p/n per share. The small shareholders' payoffs are then p/n if they tender and $\beta(s)q/n$ if they do not tender. The latter payoff is due to the fact that in the high state the company is worth 1, with a per share value of $1/n$. But now notice that the small shareholders' behavior is determined by the ratio of p and $\beta(s)q$. Therefore, the payoffs defined above capture the relevant behavior.*

The large shareholder's payoff when tendering a fraction r of his shares is

$$U_L(p, s, q(r), r) = x\left[rp + (1-r)q(r)\beta_L(s)\right],$$

where $q(r)$ is the probability he attaches to the takeover succeeding in the high state, when he is selling a fraction r of his shares.

7.2. Equilibrium We say that a couple $T^n = (\sigma_L^n, \sigma^n)$ is a symmetric Nash equilibrium of the tender subgame, when there are n shares, and when the price offer for the firm's total shares is p, if the following two conditions hold:

$$U(p, s, q_{-1}^n, \sigma^n(s)) \geq U(p, s, q_{-1}^n, a),\ \forall a \in \{keep, sell\}\ , \forall s \in [0,1],$$

and

$$\sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p, s, q^n(i/nx), i/nx)d\sigma_L^n(s, i/nx) \geq$$
$$\sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p, s, q^n(i/nx), i/nx)d\bar{\sigma}_L^n(s, i/nx),\ \forall \bar{\sigma}_L \in \Sigma_L^n,$$

where

$$q^n(i) := \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k},$$

$$q_{-1}^n := \sum_{i=0}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n-1} \binom{(1-x)n-1}{k} \phi_n^k (1-\phi_n)^{(1-x)n-1-k}.$$

The term $q^n(i/nx)$ is the probability of a successful takeover in the high state when the large shareholder tenders exactly i shares, while q_{-1}^n is the probability of a successful takeover in the high state, conditional on a particular small shareholder keeping his share. The term ϕ_n refers to the probability that a small shareholder tenders in the high state, and is defined as follows:

$$\phi_n := \int_{s\in[0,1]} f(s|h)d\sigma^n(s).$$

The probability of the takeover succeeding in the high state is:

$$q^n := \sum_{i=0}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k}.$$

For any $p \geq 0$ and $T^n = (\sigma^n, \sigma_L^n)$, the raider's payoff function, $\Pi^n(p, T^n)$, is defined similarly as the payoff function in the model with a continuum of shares:

$$\begin{aligned}\Pi^n(p,T^n) &= \frac{1}{n}\lambda\left(\sum_{i=0}^{nx} g^n(i)\left(\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}(k+i)\right)\right)\\ &- p\left[(1-x)\int_{s\in[0,1]}\sigma(s)[\lambda f(s|h)+(1-\lambda)f(s|l)]ds + x\int_{s,r} r d\sigma_L(s,r)\right].\end{aligned}$$

We say that a tuple $\Gamma^n := (p^n, T^n(p')_{p'\in[0,1]})$ is an equilibrium of the tender game if each $T^n(p')$ is a symmetric Nash equilibrium of the tender subgame with price offer p', and $\Pi^n(p^n, T^n(p)) = \max_{p'\in[0,1]}\Pi^n(p', T^n(p'))$.

REMARK 4 *Note that our equilibrium concept is equivalent to a perfect Bayesian equilibrium when the strategies are defined as measurable with respect to the prices. In fact, even after off-equilibrium price offers by the raider, the shareholders' beliefs about the state of the world are unchanged and equal to their posterior belief obtained by Bayesian updating using the common prior λ and their signal.*[13]

7.3. Convergence Let an outcome $\theta := (p,\pi,q)\in[0,1]\times[0,1]\times[0,1]$ be a tuple, where p is a price, π is the raider's profit, and q is the probability of success in the high state. Every equilibrium of a finite game induces an equilibrium outcome.

THEOREM 5 *If $\{\theta^n = (p^n,\pi^n,q^n)\}_{n=1,2,\ldots}$ is a sequence of equilibrium outcomes in the finite shares model with n shares, then any limit point $\theta = (p,\pi,q)$ of the sequence is a tender equilibrium outcome of the continuum shares model.*

PROOF: See Appendix C. □

The proof proceeds by showing that the equilibrium strategies of the small shareholders, σ^n, and that of the large shareholder, σ_L^n, as well as prices, p^n, and probabilities of success in the high state, q^n, converge to their counterparts, (σ,σ_L,p,q), in the game with a continuum of shares. From these limiting objects we derive a mapping $q(r)$, representing the large shareholder's beliefs about the success of the takeover in the high state when selling a fraction r of his shares. We conclude by showing that in the model with a continuum of shares, the raider's equilibrium price offer is p and $(\sigma,\sigma_L,q,q(r)_{r\in[0,1]})$ is a tender equilibrium of the tender subgame after price offer p.

[13] For a precise statement of PBE, see Fudenberg and Tirole (1991, Definition 8.2).

8. Discussion

We have analyzed the impact of a large shareholder (or shareholders) on takeovers. The main finding of the paper is that, without any informational asymmetries, the raider cannot carry out profitable takeovers even with a minority large shareholder. However, when the shareholders are privately and asymmetrically informed, while the raider is not, the presence of a large shareholder can facilitate profitable takeovers. More precisely, the raider makes a positive profit if the large shareholder is large enough. In addition to the large shareholder, the crucial ingredient of the model is the dispersion of information among the small shareholders. The dispersion provides the existence of the shareholders who due to unfavorable information sell their shares even at low prices. Our model allows for several comparative statics that offer testable predictions about the relationship between ownership structure, takeover success, and the distribution of takeover gains.

In this paper we focus on a simple mechanism in which the raider offers a price per share. In a related paper, Ekmekci and Kos (2012a), we show that conditional offers in which the raider pays a price p per share if at least half of the shares are tendered, do not increase the raider's expected profit in the continuum shares model. In fact, the raider's profit is the same across the two mechanisms.[14]

A. Proof of Theorem 1

We break down the proof of the theorem into a sequence of lemmata. In particular, we start by deriving the properties that any equilibrium of a continuation game must satisfy. These properties yield a unique candidate for an equilibrium. We then show that the candidate is indeed an equilibrium.

First we offer some clarifications. Signals the shareholders observe here are completely uninformative. Therefore, since the signals across the shareholders were conditionally independent to start with, they are independent here. In essence, they can be thought of as independent randomization devices. Any small shareholder's strategy specifies the probability γ with which the small shareholder sells his share. Alternatively, and with a slight abuse of notation, γ can be thought of as a threshold strategy from the set of signals $[0, 1]$ into sell/not sell which prescribes the shareholder to sell his share if the signal he observes is below γ. Also, since the shareholders are uninformed of the state, the probability of success in the low state is the same as the probability of success in the high state q, which is therefore equal to the expected probability of success.

[14] A similar result was established in an environment without a large shareholder in Marquez and Yılmaz (2007).

As in the supposition of the theorem in everything that follows we will assume $p \in (0, \lambda)$.

LEMMA 6 *After a price offer $p \in (0, \lambda)$, the takeover can neither succeed nor fail with certainty, i.e., $q \in (0, 1)$.*

PROOF: If the takeover was to succeed with certainty, $q = 1$, then the small shareholders would be better off keeping their shares and obtaining the payoff λ rather then selling them for a price $p < \lambda$. But then less than half of the shares would be sold which contradicts the assumption that $q = 1$.

On the other hand, if the takeover was certain to fail, $q = 0$, the small shareholders would be strictly better of by selling, rather than keeping their worthless shares. Thereby more than half of the shares would be sold, which would render $q = 0$ an impossibility. □

The next lemma shows that the small shareholders sell their shares with a probability high enough to make the large shareholder pivotal. More precisely, they sell their shares with a probability high enough so that if the large shareholder sells everything at least half of the shares are sold.

LEMMA 7 *In any equilibrium of the continuation game after a price offer $p \in (0, \lambda)$ the small shareholders use a strategy σ with a threshold γ such that $(1-x)\gamma + x \geq 0.5$ and $\gamma < 1$.*

PROOF: Suppose to the contrary that there was an equilibrium of the continuation game such that $(1-x)\gamma + x < 0.5$. Then $q = 0$, because even if the large shareholder sells all of his shares less than half of the shares are sold in total. But then the contradiction is obtained with Lemma 6. □

The large shareholder is the only shareholder who can affect the probability of success. The only way to prevent him from trying to increase the probability of success up to 1, which would be contrary to Lemma 6, is to have him sell all of his shares. Indeed this makes him not just pivotal, but pivotal with every share he owns.

LEMMA 8 *In any equilibrium of the continuation game after a price offer $p \in (0, \lambda)$ the large shareholder sells all of his shares and the small shareholders sell their shares with probability $\frac{0.5-x}{1-x}$.*

PROOF: First we establish that the large shareholder never sells with a positive probability a fraction of shares r such that less than half of the shares are sold in total, $(1-x)\gamma + xr < 0.5$, and the takeover fails with certainty. Notice that in such a case the shares that the large shareholder would keep, $x(1-r)$ in total, would be worthless. By selling them he would receive $px(1-r)$ instead. Therefore selling all the shares would present a profitable deviation.

Lemma 6 states that $q \in (0,1)$ which together with the above paragraph implies that the raider must with positive probability sell a fraction of shares r such that $q(r) \in (0,1)$.

Now, suppose that for some $r < 1$ we have $0 < q(r) < 1$, thus $(1-x)\gamma + xr = 0.5$. The large shareholder cannot sell such a fraction r with positive probability. If he did his payoff from doing so would be $prx + q(r)rx\lambda$. By selling an $\epsilon > 0$ more, for an ϵ small enough, the probability of success would increase to 1 and the value of the shares he keeps to λ, yielding him a payoff of $p(r+\epsilon)x + (r-\epsilon)x\lambda$ and thus constituting a profitable deviation.

Since the raider must be selling with positive probability a fraction r such that $q(r) \in (0,1)$ and by the previous paragraph this r cannot be smaller than one, it must be the case that $q(1) \in (0,1)$. Which, in turn, is possible only if $(1-x)\gamma + x = 0.5$. Consequently, the large shareholder sells all of his shares and the small shareholders sell with probability $\gamma = (0.5-x)/(1-x)$. □

The probability of success of the takeover is pinned down by the price.

LEMMA 9 *In any equilibrium of the continuation game after a price offer $p \in (0,\lambda)$, the takeover must succeed with the probability $q = p/\lambda$.*

PROOF: Lemma 7 shows that the small shareholders sell their shares with positive probability. Clearly they do not sell with probability one, in which case more than half of the shares would be sold in total and the takeover would succeed with probability 1, contradicting Lemma 6.

The above paragraph establishes that a small shareholder is mixing between selling his share or not. He will only do that if he is indifferent. Selling a share gives him payoff p while keeping it delivers $q\lambda$. Therefore $p = q\lambda$, which proves our claim. □

Finally, we show that the above derived candidate indeed is an equilibrium.

LEMMA 10 *The strategy for the small shareholders to sell their shares with probability $\frac{0.5-x}{1-x}$, for the large shareholder to sell all of his shares, together with $q(r) = 0$ for $r < 1$, $q(r) = p/\lambda$ for $r = 1$, and $q = p/\lambda$ is an equilibrium of the continuation game with the price $p \in (0,\lambda)$.*

PROOF: Clearly $(1-x)\frac{0.5-x}{1-x} + x = 0.5$, thus exactly half of the shares are sold if the shareholders use the strategies prescribed by the candidate equilibrium. A small shareholder's payoff from selling is p and from keeping a share $q\lambda$. These two are by the definition of q equal rendering the small shareholder indifferent between selling and not selling. Thus his strategy is optimal.

Since $q(r) = 0$ for $r < 1$ the large shareholder causes the takeover to fail if he sells anything less than all of his shares. Thus making the withheld shares worthless. I.e., the large shareholder's payoff from selling a fraction r of his shares is prx. This is clearly maximized at $r = 1$.

As the last step we need to show that q is derived from the function $q(r)$ according to the large shareholder's equilibrium strategy. Since the large shareholder sells all of his shares with probability 1, $q = q(1)$ which concludes our proof. □

Lastly, it is easy to see that the raider's payoff after any price offer $p \in (0, \lambda)$ is 0. Namely, the total created value is $q\lambda$, $(1-x)q\lambda$ accrues to the small shareholder and $xp = xq\lambda$ to the large shareholder, leaving nothing on the table for the raider.

B. Proofs for the Asymmetric Information Model

Proof of Theorem 2: Suppose that there exists an equilibrium of the tender subgame after some price offer $p > 0$, and denote it by $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$. Lemma 1 implies σ is a threshold strategy. We denote its threshold by γ. In the following development, we will fix this candidate equilibrium, and characterize its properties. Then we will verify that such an equilibrium exists.

Before we characterize the equilibrium, we remind the reader of two definitions from the main text. The pivotal type, $s^* \in [0,1]$ is the type such that if the small shareholders use the threshold strategy s^*, and if the large shareholder tenders all his shares, the fraction of tendered shares is 1/2. Since $x < 1/2$, there is indeed an interior threshold signal $s^* \in (0,1)$, which satisfies the equality, $F(s^*|h)(1-x) + x = 1/2$. Price $\bar{p}$ is the price that makes the small shareholder observing the threshold signal $s^* \in (0,1)$ indifferent between tendering her share or keeping it, if she believes that the takeover is successful in state h with probability one. In particular, $\bar{p} := \beta(s^*)$.

The first claim shows that the probability of the success of the takeover is larger than zero.

Claim 1 $q > 0$.

Proof: Suppose, on the way to a contradiction that $q = 0$. Then, not tendering yields an expected payoff of zero, while tendering yields $p > 0$, therefore $\gamma = 1$. Consequently, at least $1-x$ of shares are sold regardless of what the large shareholder does, resulting in $q(r) = 1$ for every $r \in [0,1]$. In turn $q = 1$, which contradicts the hypothesis that $q = 0$. □

Next we establish a lower bound on the threshold used in an equilibrium by small shareholders.

Claim 2 $(1-x)F(\gamma|h)+x \geq 1/2$.

Proof: If the claim was not true, less than half the shares in total would be sold in the high state, even if the large shareholder were to tender all of his shares. Implying $q(r)=0$ for every $r \in [0,1]$. Therefore, $q=0$ which would contradict Claim 1. □

The above claim establishes that in any equilibrium the strategy of the small shareholders is such that the large shareholder could guarantee that at least half of the shares are sold in the high state, if he wanted to.

In what follows we define the notation for the share of the large shareholder that needs to be tendered so that exactly half of the shares are tendered in the high state, given the equilibrium strategy of the small shareholders. Notice that such a share of the large shareholder exists in an equilibrium due to the previous claim.

Definition 1 *Let*

$$a := \max\{0, \frac{1/2-(1-x)F(\gamma|h)}{x}\}.$$

In the next claim we show, roughly speaking, that the large shareholder never sells less than fraction a of his shares. Moreover, if he is selling precisely a and if $q(a)<1$ then it must be that $a=1$. Indeed, if the large shareholder were to sell $a<1$ with positive probability and $q(a)<1$, then he would be better off by selling just slightly more than a which would ensure the success of the takeover in the high state and yield a significantly larger payoff on the shares the large shareholder keeps.

Claim 3 *If $\sigma_L(s,r)>0$ for some $r<1$ and $s \in [0,1]$, then $q(r)=1$.*

Proof: The proof is in two steps. First, if $\sigma_L(s,r)>0$ for some $r<1$, then $r \geq a$. To see this, note that $q(r)=0$ for $r<a$, in which case the large shareholder gets p for the shares he tendered and zero for the ones he keeps. Therefore, tendering all shares does strictly better than tendering $r<a$. Hence, $\sigma_L(s,r)=0$ for every $r<a$.

For the second step, we show that the claim is true under the two cases: i) $r>a$ and ii) $r=a$.

i) When $r>a$, $q(r)=1$ by the definition of a and by the equilibrium requirement on $q(r)$.

ii) Now suppose that $r=a$, $\sigma_L(s,a)>0$, $r<1$, and suppose contrary to the assertion of the claim, that $q(a)<1$. Then the large shareholder has a profitable deviation by tendering a fraction arbitrarily close to but above a, by which he pushes the probability of the success in the high state to 1. This contradicts the equilibrium condition that σ_L maximizes the large shareholder's payoff. □

If the small shareholders expect the success of the takeover in the high state to be less than certain, then it must be the case that the large shareholder is selling all of his shares. Otherwise he could increase the probability to one by selling just slightly more shares.

Claim 4 *If $q < 1$, then $\sigma_L(s, r) = 0$ for every $r < 1$ and $s \in [0, 1]$.*

Proof: Let $q < 1$. If $a = 1$, then the claim is true because $\sigma_L(s, r) = 0$ for every $r < a = 1$ as shown in Claim 3.

Let $a < 1$, and on the way to a contradiction assume that $\sigma_L(s, r) > 0$ for some $r < 1$. Then $q(r) = 1$ for all such r, by Claim 3. Moreover, if $a < 1$, then $q(1) = 1$. Therefore $q = 1$, contradicting the supposition that $q < 1$. □

So far we have established several properties of the equilibrium. In what follows, we will complete the characterization under two cases: i) when $p < \bar{p}$ and ii) when $p \geq \bar{p}$.

CASE 1: Suppose that $p < \bar{p}$.

When the price p is below the threshold $\bar{p}$ the takeover cannot succeed with certainty in the high state.

Claim 5 *If $p < \bar{p}$, then $q < 1$.*

Proof: Suppose on the way to a contradiction that $q = 1$. Then, either $\gamma = 0$ or $\gamma > 0$. If $\gamma > 0$, then $p = q\beta(\gamma)$, $p < \bar{p} = \beta(s^*)$, and $\beta(\cdot)$ is strictly increasing imply $\gamma < s^*$. Therefore,

$$(1 - x)F(\gamma|h) + x < (1 - x)F(s^*|h) + x = 1/2,$$

hence $q(r) = 0$ for every $r \in [0, 1]$. Consequently $q = 0$, which contradicts the assumption we started with. A similar analysis yields a contradiction when $\gamma = 0$. □

Claim 1 and 5 put together yield that in any equilibrium, it has to be the case that $q \in (0, 1)$. From the previous analysis we also know that in such an equilibrium, the large shareholder would try to tip the scale in the favor of the certain success, if he could. The only way he can be prevented from doing so is if he is already selling all of his shares.

Claim 6 *$\sigma_L(s, r) = 0$ for every $s \in [0, 1]$ and $r < 1$.*

Proof: From Claim 5, we know that $q < 1$. The result is then implied by Claim 4. □

In words, the large shareholder tenders all his shares regardless of his information.

Claim 7 *$a = 1$, $\gamma = s^*$ and $q = \frac{p}{\beta(s^*)}$.*

Proof: If $a < 1$, then $q(1) = 1$, and since the large shareholder would be tendering all his shares by claim 6, $q = 1$. This contradicts the statement of Claim 5 that $q < 1$. Therefore $a = 1$, and using the definition of a, $\gamma = s^*$. Finally, since γ is the threshold type, $q = \frac{p}{\beta(\gamma)} = \frac{p}{\beta(s^*)}$, completing the proof. □

CASE 2: Suppose that $p \geq \bar{p}$.

First we establish that if the price offer is high, then in equilibrium, the takeover succeeds with probability one in the high state.

Claim 8 *If $p \geq \bar{p}$ then $q = 1$.*

Proof: On the way to a contradiction, assume that $q < 1$. Then by Claim 4, $\sigma_L(1, r) = 0$ for every $r < 1$, i.e., the large shareholder tenders all his shares. Moreover, $p \geq \bar{p}$ and $q < 1$ imply $\beta(\gamma) = \frac{p}{q} > \bar{p} = \beta(s^*)$ and therefore $\gamma > s^*$. But then $q = 1$, because the large shareholder tenders all his shares and $(1 - x)F(\gamma|h) + x > 1/2$. This contradicts the initial hypothesis that $q < 1$. □

Remark 5 *Since $q = 1$, γ is found by the identity $\beta(\gamma) = p$. Since $p \geq \bar{p}$, $\gamma \geq s^*$.*[15]

At low prices, $p < \bar{p}$, the takeover succeeds in the high state with a probability q smaller than one. The only way to prevent the large shareholder from increasing this probability to one was if he was already tendering all of his shares in the equilibrium. For a high price $p \geq \bar{p}$, however, the probability of the takeover succeeding in the high state must be one, as shown in Claim 8. This leaves scope for the large shareholder to keep some of the shares if he deems them more valuable then the price the raider is offering.

Definition 2 *Let s_L be the signal that satisfies $\beta_L(s_L) = p$, if such a signal exists. Let $s_L := 0$ if $\beta_L(0) > p$ and let $s_L := 1$ if $\beta_L(1) < p$.*

s_L is the signal at which the large shareholder's expected value for each of his shares is equal to the price, when he is expecting the takeover to succeed with probability one.

The following claim shows that the large shareholder tenders fraction a of his shares when his signal is high and all of his shares when the signal is low.

[15]If $p \geq \beta(1)$, we set $\gamma = 1$, in which case the small agents tender their shares for all the signals.

CLAIM 9 *i)* $\sigma_L(s,r) = 0$ *for* $r < a$ *and any* $s \in [0,1]$. *In words, the large shareholder does not tender a fraction less than* a.

ii) For $s < s_L$ *and* $r < 1$*:* $\sigma_L(s,r) = 0$. *For* $s > s_L$ *and* $r \in [a,1)$*:*

$$\sigma_L(s,r) = \lambda[F(s|h) - F(s_L|h)] + (1-\lambda)[F(s|l) - F(s_L|l)].$$

Notice that $\sigma_L(s,1) = \lambda F(s|h) + (1-\lambda)F(s|l)$ *for all* $s \in [0,1]$, *by the definition of distributional strategies.*

In words, the large shareholder tenders exactly fraction a *if his signal is above the threshold signal* s_L. *He tenders all his shares if his signal is below* s_L.

iii) If $s_L < 1$, *then* $q(a) = 1$.

PROOF: Part i) follows directly from Claim 3.

We will argue part ii) by considering two cases: $\gamma = s^*$ and $\gamma > s^*$. If $\gamma = s^*$, then $a = 1$ by the definition of a and s^*. Since the large shareholder never tenders a fraction smaller than a, as in part i), $\sigma_L(s,r) = 0$ for every $r < 1$ and every $s \in [0,1]$. $q = 1$ then implies that $q(a) = 1$.

If $\gamma > s^*$, then $a < 1$. When $s < s_L$, the price p is greater than $\beta_L(s)$, by the definition of s_L. Therefore, it is optimal for the large shareholder to tender all his shares. Hence, if $s < s_L$, then $\sigma_L(s,r) = 0$ for every $r < 1$.

If $s_L = 1$, then the proof is complete. So, let $s_L < 1$. A direct calculation shows that tendering any fraction $r > a$ is dominated by tendering fraction $\frac{a+r}{2}$ of shares. Moreover, if $q(a) < 1$, then tendering a fraction arbitrarily close to a from above is a profitable deviation. Therefore, the optimality condition for the large shareholder's strategy delivers that $q(a) = 1$. Since $\sigma_L(s,r) = 0$ for $r < a$, in any equilibrium from part i), $\sigma_L(s,a) = \lambda[F(s|h) - F(s_L|h)] + (1-\lambda)[F(s|l) - F(s_L|l)]$ for $s > s_L$, which concludes the proof. □

We have proven that all equilibria of the tender subgame have the following structure:

i) If $0 < p < \bar{p}$, then $\gamma = s^*$, $q = \frac{p}{\beta(s^*)}$, $\sigma_L(s,r) = 0$ for every $s \in [0,1]$ and $r < 1$. Moreover, the raider's profit is:

$$\Pi(p) = \lambda(q/2 - p/2) + (1-\lambda)(-p[(1-x)F(s^*|l) + x]).$$

ii) If $p = \bar{p}$, then $\gamma = s^*$, $q = 1$, $\sigma_L(s,r) = 0$ for every $s \in [0,1]$ and $r < 1$; the raider's profit is:

$$\Pi(p) = \lambda(1/2 - p/2) + (1-\lambda)(-p[(1-x)F(s^*|l) + x]).$$

iii) If $p > \bar{p}$, then γ satisfies $\beta(\gamma) = p$, $q = 1$, $\sigma_L(s,r)$ satisfies the findings above; the raider's profit is:

$$\begin{aligned}\Pi(p) = \lambda\left[(1-x)F(\gamma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ -p\lambda\left[(1-x)F(\gamma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ -p(1-\lambda)[(1-x)F(\gamma|l) + x(a(1-H(s_L|l)) + H(s_L|l))].\end{aligned}$$

□

Proof of Lemma 3: In what follows we use the characterization of profits obtained in Theorem 2. We start by rewriting $\Pi(p)$ for $p \leq \bar{p}$. Inserting $q = p/\beta(s^*)$ into (7), we obtain

$$\Pi(p) = p\left[\frac{\lambda}{2}\left(\frac{1}{\beta(s^*)} - 1\right) - (1-\lambda)[(1-x)F(s^*|l) + x]\right].$$

Since the profit function is linear in p, for $p \in [0, \bar{p}]$, if $\lambda/2(1/\beta(s^*) - 1) - (1-\lambda)[(1-x)F(s^*|l) + x]$ is non-negative, then any price less that $\bar{p}$ is weakly dominated by $\bar{p}$. If on the other hand, the expression is negative, then any price $p \in (0, \bar{p}]$ is dominated by zero.

Next we will provide four inequalities to argue that any price $p > \bar{p}$ is dominated by $\bar{p}$. Before we proceed, notice that $p > \bar{p}$ implies $\gamma > s^*$.

For the first inequality, $\gamma > s^*$ and $p = \frac{\lambda f(\gamma|h)}{\lambda f(\gamma|h) + (1-\lambda) f(\gamma|l)}$ imply:

$$\lambda(1-p)F(\gamma|h) + (1-\lambda)(-p)F(\gamma|l) < \lambda(1-p)F(s^*|h) + (1-\lambda)(-p)F(s^*|l).$$

To see this, we rewrite the inequality by plugging in the value of p and rearranging to obtain:

$$\frac{F(\gamma|h)}{f(\gamma|h)} - \frac{F(\gamma|l)}{f(\gamma|l)} < \frac{F(s^*|h)}{f(\gamma|h)} - \frac{F(s^*|l)}{f(\gamma|l)}.$$

The above inequality holds because the derivative of $K(s) := \frac{F(s|h)}{f(\gamma|h)} - \frac{F(s|l)}{f(\gamma|l)}$ with respect to s is negative when $s < \gamma$, due to MLRP.

The second inequality follows directly from $p > \bar{p}$:

$$\lambda(1-p)F(s^*|h) + (1-\lambda)(-p)F(s^*|l) < \lambda(1-\bar{p})F(s^*|h) + (1-\lambda)(-\bar{p})F(s^*|l).$$

The first two inequalities together yield

(10) $$\lambda(1-\bar{p})F(s^*|h) + (1-\lambda)(-\bar{p})F(s^*|l) > \lambda(1-p)F(\gamma|h) + (1-\lambda)(-p)F(\gamma|l).$$

The third inequality follows from the MLRP condition in a similar fashion as the first inequality, but by using the condition $p = \beta_L(s_L)$ (The case of $p > \beta_L(1)$ leads to $s_L = 1$, and the inequality follows similarly.):

$$\lambda(1-p)[a(1-H(s_L|h)) + H(s_L|h)] + (1-\lambda)(-p)[a(1-H(s_L|l)) + H(s_L|l)] \\ \leq \lambda(1-p) + (1-\lambda)(-p).$$

Indeed, rearranging shows that the above inequality is true if and only if:

$$\frac{1-H(s_L|h)}{1-H(s_L|l)} \geq \frac{h(s_L|h)}{h(s_L|l)}.$$

This last inequality follows directly from MLRP.

The fourth inequality again follows directly from $p > \bar{p}$

$$\lambda(1-p) + (1-\lambda)(-p) < \lambda(1-\bar{p}) + (1-\lambda)(-\bar{p}).$$

The last two inequalities yield

(11)
$$\lambda(1-\bar{p})+(1-\lambda)(-\bar{p}) > \lambda(1-p)[a(1-H(s_L|h))+H(s_L|h)]+(1-\lambda)(-p)[a(1-H(s_L|l))+H(s_L|l)].$$

Now, for $p > \bar{p}$:

$$\begin{aligned}
&\Pi(\bar{p}) - \Pi(p) = \\
&\lambda(1-\bar{p})F(s^*|h) + (-\bar{p})(1-\lambda)(1-x)F(s^*|l) - \lambda(1-p)(1-x)F(\gamma|h) + p(1-\lambda)(1-x)F(\gamma|l) \\
&+ x\left[\lambda(1-\bar{p}) - p(1-\lambda) - \lambda(1-p)[a(1-H(s_L|h)) + H(s_L|h)] + p(1-\lambda)[a(1-H(s_L|l)) + H(s_L|l)]\right] \\
&> 0,
\end{aligned}$$

where the first equality uses the identity $1/2 - x = F(s^*|h)$, the term in the second line is larger than zero due to (10) and the term in the third line due to (11). □

Proof of Lemma 4: $s^*(x)$ is strictly increasing in x, which is readily seen from

$$(1-x)F(s^*|h) + x = 1/2. \tag{12}$$

Therefore $\beta(s^*(x))$ is strictly decreasing in x. $\Pi(x)$ can be rewritten as follows:

$$\Pi(x) = \frac{\lambda}{2} - \beta(s^*)[\frac{\lambda}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]].$$

We show that the second term is decreasing in x. First, $\beta(s^*)$ is decreasing in x. Second, as we will show below, $(1-x)F(s^*|l)+x$ is decreasing in x, completing the proof. Differentiating (12) with respect to x yields

$$0 = 1 - F(s^*|h) + (1-x)f(s^*|h)\frac{ds^*}{dx}. \tag{13}$$

Now:

$$\begin{aligned}
\frac{d((1-x)F(s^*|l)+x)}{dx} &= 1 - F(s^*|l) + (1-x)f(s^*|l)\frac{ds^*}{dx} \\
&= 1 - F(s^*|l) - f(s^*|l)\frac{1-F(s^*|h)}{f(s^*|h)} \\
&< 0,
\end{aligned}$$

where the second line follows from (13) and the third from MLRP. This concludes the proof. □

Proof of Lemma 5: Let $s^*(x)$ be defined by

$$(1-x)F(s^*(x)|h) + x = \frac{1}{2},$$

for all $x \in [0, 1/2]$. Then

$$\begin{aligned}
\lim_{x \searrow 0} \Pi(x) &= \lim_{x \searrow 0} \left(\frac{\lambda}{2} - \beta(s^*(x)) \left[\frac{\lambda}{2} + (1-\lambda)[(1-x)F(s^*(x)|l) + x] \right] \right) \\
&= \frac{\lambda}{2} - \beta(s^*(0)) \left[\frac{\lambda}{2} + (1-\lambda)F(s^*(0)|l) \right] \\
&= (1-\lambda)f(s^*(0)|l)\beta(s^*(0)) \left[\frac{F(s^*(0)|h)}{f(s^*(0)|h)} - \frac{F(s^*(0)|l)}{f(s^*(0)|l)} \right] \\
&< 0,
\end{aligned}$$

where the second equality follows from continuity of $\beta(\cdot)$ and $s^*(\cdot)$, and the third line follows by observing $1/2 = F(s^*(0)|h)$ and rearranging. The inequality is due to MLRP.

On the other hand,

$$\begin{aligned}
\lim_{x \nearrow 1/2} \Pi(x) &= \lim_{x \nearrow 1/2} \left(\frac{\lambda}{2} - \beta(s^*(x)) \left[\frac{\lambda}{2} + (1-\lambda)[(1-x)F(s^*(x)|l) + x] \right] \right) \\
&= \frac{1}{2} [\lambda - \beta(0)] \\
&> 0,
\end{aligned}$$

where the second line follows after observing $s^*(1/2) = 0$.

$\square$

C. Finite Model and Convergence: Proof of Theorem 5

We prove Theorem 5 via a sequence of lemmata. In particular, we show that symmetric equilibrium outcomes of the finite shares model converge to an equilibrium outcome of the model with a continuum of shares. In the following development, we denote the strategies in the finite shares model with n shares using the superscript n.

C.1. Threshold Strategies The next lemma is a preliminary observation showing that the equilibrium strategies of small shareholders in finite shares model either have a threshold structure, or there is an outcome equivalent equilibrium in which small shareholders use a threshold strategy.

Lemma 11 *Let $T^n = (\sigma_L^n, \sigma^n)$ be a symmetric Nash Equilibrium of the tender subgame with price offer p when there are n shares. Then there is unique symmetric equilibrium strategy profile $\tilde{T}^n := (\sigma_L^n, \tilde{\sigma}^n)$ that induces the same distribution over outcomes as T^n, and the small shareholders' strategy $\tilde{\sigma}^n$ is a threshold strategy.*

Proof: The MLRP property on the signal distribution implies that $\beta(s)$ is weakly increasing in s. A small shareholder's payoff from tendering a share is p, whereas the payoff from keeping it is $\beta(s)q_{-1}^n$. Therefore, if $U(p, s, q_{-1}^n, keep) > U(p, s, q_{-1}^n, sell)$ for a signal $s \in [0, 1]$, then $U(p, s', q_{-1}^n, keep) > U(p, s', q_{-1}^n, sell)$ for every $s' < s$. Similarly, if $U(p, s, q_{-1}^n, sell) > U(p, s, q_{-1}^n, keep)$ for a signal $s \in [0, 1]$, then $U(p, s', q_{-1}^n, sell) > U(p, s', q_{-1}^n, keep)$ for every $s' > s$. Therefore, there are signals $s_1 \le s_2$ such that $\sigma^n(s) = sell$ for all $s < s_1$ and $\sigma^n(s) = keep$ for all $s > s_2$, and $\beta(s_1) = \beta(s_2)$. Let $s^{**} \in [s_1, s_2]$ be the unique signal such that $F(s^{**}|h) - F(s_1|h) = \int_{s_1}^{s_2} \mathbf{1}_{\{\sigma^n(s)=sell\}} dF(s|h)$; where the term $\mathbf{1}$ is the indicator function. It is then clear that the strategy $\tilde{\sigma}^n$ defined as $\tilde{\sigma}^n(s) = sell$ for all $s \le s^{**}$ and $\tilde{\sigma}^n(s) = keep$ for all $s > s^{**}$ yields identical outcomes as σ^n when the price offer is p and the large shareholder strategy is σ_L^n. Also it is clear that $\tilde{T}^n$ is an equilibrium strategy profile, since the distribution of action profiles remains unchanged. $\square$

Henceforth, without loss of generality, we will confine attention to equilibria in which small shareholders are using a threshold strategy. The term γ^n denotes the threshold type of the threshold strategy σ^n. Hence, $\phi_n = F(\gamma^n|h)$.

C.2. Convergence Let the collection $\{p^n, T^n, q^n\}_{n=1,2...}$ be a sequence where each $p^n \in [0,1]$ is a price, T^n is a symmetric Nash equilibrium of the tender subgame with price offer p^n and n shares, and q^n is derived from T^n as described in the main body of the paper. In the following development, we fix this sequence.

C.2.1. *Method of proof* We first prove that the equilibrium outcomes of the sequence $\{p^n, T^n, q^n\}_n$ converge to an equilibrium outcome of the continuum game we identified in the main text. On the way to the result, we first argue that there is a strategy $\sigma_L \in \Sigma_L$ to which the sequence σ_L^n converges. Moreover, γ^n, the threshold type of σ^n, converges to a threshold γ, prices, p^n, converge to a price p, and q^n converges to q. From these limit objects, we *derive* a new mapping $q(r)_{r\in[0,1]}$ and show that $(\sigma_L, \sigma, q, q(r)_{r\in[0,1]})$ where σ is a threshold strategy with the threshold γ, is an equilibrium of the tender subgame with a price offer p in the continuum game. Finally we show that the equilibrium prices of the finite games, p^n, converge to an equilibrium price, p, of the continuum game.

The lemma below shows that every collection $\{p^n, T^n, q^n\}_n$ has a convergent subsequence.

Lemma 12 *There exists a subsequence of $\{p^n, \gamma^n, \sigma_L^n, q^n\}_n$ and an increasing and right-continous function $\sigma_L \in \Sigma_L$ such that $\sigma_L^n(s,r) \to \sigma_L(s,r)$ at every continuity point of $\sigma_L(s,r)$, $p^n \to p$, $q^n \to p$, $\gamma^n \to \gamma$.*

Proof: Since $p^n, \gamma^n, q^n \in [0,1]$ for all n, the sequence $\{p^n, \gamma^n, \sigma_L^n, q^n\}_n$ has a convergent subsequence $\{p^{nk}, \gamma^{nk}, \sigma_L^{nk}, q^{nk}\}_{nk}$. Sequence $\{\sigma_L^{nk}\}_{nk}$ has a subsequence $\{\sigma_L^{nkj}\}_{nkj}$ which converges to a distribution due to Helly's theorem; distributions here have a bounded support (see Billingsley (1986), Thm 25.10). Moreover, since all the distributional strategies along the sequence $\{\sigma_L^{nkj}\}_{nkj}$ satisfy equation (1), and so does the limit. Therefore, the limit distribution of $\{\sigma_L^{nkj}\}_{nkj}$ is in Σ_L and $\{p^{nkj}, \gamma^{nkj}, \sigma_L^{nkj}, q^{nkj}\}_{nkj}$ is a convergent subsequence of $\{p^n, \gamma^n, \sigma_L^n, q^n\}_n$. □

From this point on, we use the term $lim_{n\to\infty}$ to take the limit over the convergent subsequence identified in the previous lemma. We denote the limit point to which the subsequence converges with the collection of the price p, threshold signal γ, large shareholder's strategy σ_L and the probability of success in the high state, q.

So far we identified a limit of a sequence of equilibria of finite games. In the following development, we establish that the limiting strategies constitute, a part of, an equilibrium of the continuum game. Note, however, that an equilibrium in the continuum game is identified by two strategies, one for small shareholders and one for the large shareholder, a probability q and a mapping $\{q(r)\}_{r\in[0,1]}$. In our description of the equilibrium of the continuum game we will use the limit distributions for the strategies, the limit of q^n for q, while in the next definition we describe how to specify the mapping $\{q(r)\}_{r\in[0,1]}$. Let $a := min\{a^*, 1\} \in [0, 1]$ where a^* is the solution to $(1-x)\gamma + xa^* = 1/2$.

DEFINITION 3 *Let $q(r) = 0$ for all $r < a$, $q(r) = 1$ for all $r > a$. If $\sigma_L(1, a|h) - \lim_{y\to a^-} \sigma_L(1, y|h) > 0$, then let*

$$q(a) = \frac{q - (1 - \sigma_L(1, a|h))}{\sigma_L(1, a|h) - \lim_{y\to a^-} \sigma_L(1, y|h)},$$

otherwise let $q(a)$ be an arbitrary number between 0 and 1.

First we establish that $q(a)$ as defined above can actually be interpreted as a probability.

LEMMA 13 $q(a) \in [0, 1]$.

PROOF: We start by showing $q(a) \geq 0$. On the way to the result, note that for every $\epsilon > 0$,

$$1 - \sigma_L^n(1, a + \epsilon|h) = \sum_{i=(a+\epsilon)nx}^{nx} g^n(i).$$

Definition of a and the fact that ϕ_n converges to ϕ imply

$$\sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1 - \phi_n)^{(1-x)n-k} \to_{n\to\infty} 1,$$

uniformly over all $i \geq (a+\epsilon)nx$. Indeed, a was defined so that whenever the large shareholder tenders more than fraction a of his shares in the continuum game, given the fixed behavior of the small shareholders, he expects the takeover in the high state to succeed with probability one. Now

$$\sum_{i=(a+\epsilon)nx}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1 - \phi_n)^{(1-x)n-k} - \sum_{i=(a+\epsilon)nx}^{nx} g^n(i) \to 0.$$

The above observations can be put together to show that there exists an N such that for

$n > N$,

$$
\begin{aligned}
(1-\epsilon)(1-\sigma_L^n(1,a+\epsilon|h)) &= (1-\epsilon)\sum_{i=(a+\epsilon)nx}^{nx} g^n(i) \\
&\leq \sum_{i=0}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} \\
&= q^n.
\end{aligned}
$$

Since the inequality is true for every large n, it has to be true in the limit:

$$q \geq (1-\epsilon)(1-\sigma_L(1,a+\epsilon|h)).$$

Moreover, the last inequality holds for every $\epsilon > 0$, and σ_L is right continuous, therefore

$$q \geq 1-\sigma_L(1,a|h).$$

Next we argue that $q(a) \leq 1$. For this, it suffices to show that

$$1-q \geq \lim_{y\to a^-} \sigma_L(1,y|h).$$

Suppose, to the contrary that $1-q < \lim_{y\to a^-}\sigma_L(1,y|h)$. Then there exists an $\epsilon_1 > 0$ such that

$$1-q < \sigma_L(1,a-\epsilon_1|h). \tag{14}$$

$1-q$ is the probability the small agents attach in the limit to the failure of the takeover in the high state. It is easy to verify that

$$1-q = \lim_{n\to\infty}\sum_{i=0}^{nx} g^n(i) \sum_{k=0}^{\frac{n}{2}-i-1} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k}.$$

Fix an ϵ_2 such that $0 < \epsilon_2 \leq \epsilon_1$ and notice that definition of a and the fact that ϕ_n converges to ϕ imply

$$\sum_{k=0}^{\frac{n}{2}-i-1} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} \to 1,$$

uniformly for all $i \leq (a-\epsilon_2)nx$. The idea is, in the limit game a is the fraction of shares that the large shareholder needs to sell, so that exactly half of the shares are sold, given the small shareholders' strategy. Thus, if he is selling a fraction smaller than a, and the small shareholders' strategies are converging to the limit strategy, it has to be the case that for large n the takeover is failing with probability 1 in the high state.

But then there exist an $\epsilon_3 > 0$ and N such that for $n > N$

$$\begin{aligned}
1 - q &= \lim_{n\to\infty} \sum_{i=0}^{nx} g^n(i) \sum_{k=0}^{\frac{n}{2}-i-1} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} \\
&\geq \sum_{i=0}^{(1-\epsilon_2)nx} g^n(i) + \sum_{i=(1-\epsilon_2)nx}^{nx} g^n(i) \sum_{k=0}^{\frac{n}{2}-i-1} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} - \epsilon_3 \\
&\geq \sum_{i=0}^{(1-\epsilon_2)nx} g^n(i) - \epsilon_3.
\end{aligned}$$

Since the above inequality holds for every ϵ_3 and $\sigma_L(1, a-\epsilon_2|h) = \lim_n \sum_{i=0}^{(1-\epsilon_2)nx} g^n(i)$,

$$1 - q \geq \sigma_L(1, a - \epsilon_2|h),$$

which contradicts (14). Hence, $1 - q \geq \lim_{y\to a^-} \sigma_L(1, y|h)$. □

Now that we have established the limiting structure of the game we need to show that the limiting strategies, together with the beliefs, form an equilibrium of the continuum game. First we show that the limit threshold signal of the small shareholders represents the optimal strategy for them in the continuum game.

Lemma 14 *The limit threshold type γ is such that $U(p, s, q, keep) > (<) p$ if $s > (<) \gamma$.*

Proof: We start by calculating the term $q^n - q^n_{-1}$:

$$q^n := \sum_{i=0}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k}.$$

$$q^n_{-1} := \sum_{i=0}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n-1} \binom{(1-x)n-1}{k} \phi_n^k (1-\phi_n)^{(1-x)n-1-k}.$$

We use the following identity:

$$\sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} - \sum_{k=n/2-i}^{(1-x)n-1} \binom{(1-x)n-1}{k} \phi_n^k (1-\phi_n)^{(1-x)n-1-k}$$
$$= \binom{(1-x)n-1}{\frac{n}{2}-i-1} \phi_n^{\frac{n}{2}-i} (1-\phi_n)^{(1-x)n-\frac{n}{2}+i}$$

to obtain:

$$q^n - q_{-1}^n = \sum_{i=0}^{nx} g^n(i) \binom{(1-x)n-1}{\frac{n}{2}-i-1} \phi_n^{\frac{n}{2}-i} (1-\phi_n)^{(1-x)n-\frac{n}{2}+i},$$

which is easily seen to converge to 0 as n goes to infinity. Now, $q_{-1}^n \to q_n$ and $q_n \to q$ imply $q_{-1}^n \to q$.

Pick any arbitrary $s < \gamma$. Since $\gamma^n \to \gamma$, there is a N such that for every $n > N$, $s < \gamma^n$. Observing that small shareholders with signals less than the threshold signal γ^n weakly prefer to tender in the game with n shares delivers that $U(p^n, s, q_{-1}^n, keep) \leq p^n$. Moreover, because U is continuous in q_{-1}^n, because $q_{-1}^n \to q$, and because $p^n \to p$, we have that $U(p, s, q, keep) \leq p$. Because s is arbitrarily chosen, the previous inequality holds for any $s < \gamma$. A similar argument shows that $U(p, s, q, keep) \geq p$ for any $s > \gamma$. □

The last piece of the puzzle in the limit result for the equilibria of the tender subgames is to establish that the large shareholder's limiting strategy σ_L is a best response to the small shareholders' limiting strategies in the continuum game, given the limiting price and beliefs.

Lemma 15

$$\int_{s,r} U_L(p, s, q(r), r) d\sigma_L(s, r) \geq \int_{s,r} U_L(p, s, q(r), r) d\tilde{\sigma}_L(s, r),$$

for every $\tilde{\sigma}_L \in \Sigma_L$.

Proof: Suppose, contrary to the assertion of the lemma, that there exists a $\tilde{\sigma}_L \in \Sigma_L$ such that

$$\int_{s,r} U_L(p, s, q(r), r) d\sigma_L(s, r) < \int_{s,r} U_L(p, s, q(r), r) d\tilde{\sigma}_L(s, r).$$

We will consider two cases: $a < 1$ or $a = 1$.

Case 1: $a < 1$. There exist an $\epsilon > 0$, and a $\bar{\sigma}_L \in \Sigma_L$ such that:

$$(15) \qquad \int_{s,r} U_L(p, s, q(r), r) d\sigma_L(s, r) < \int_{s,r} U_L(p, s, q(r), r) d\bar{\sigma}_L(s, r),$$
$$\bar{\sigma}_L(1, a + \epsilon) = \bar{\sigma}_L(1, a - \epsilon),$$

and $\bar{\sigma}_L(1, y) = \tilde{\sigma}_L$, for $y \in [0, a - \epsilon)$. The idea is to take the strategy $\tilde{\sigma}_L$ and construct a new strategy $\bar{\sigma}_L$ by shifting the probability mass that $\tilde{\sigma}_L(1, \cdot)$ assigns to the interval $[a - \epsilon, a + \epsilon]$ toward the endpoint of the interval, $a+\epsilon$. The existence of a $\bar{\sigma}_L$ satisfying the above inequality is guaranteed because, given $a < 1$, shifting the shares slightly above a cannot decrease the payoff discontinuously.

Let for every $r \in [0, 1]$,

$$\bar{\sigma}^n_L(s, r) := \bar{\sigma}_L(s, i/nx),$$

for the unique i that satisfies $i - 1 < rnx \leq i$.

In what follows, we will show that the large shareholder's equilibrium payoffs in the finite games converge to his payoffs in the continuum game with price p and tuple T. We will use this finding together with the hypothesis that σ_L is not a best response to find a profitable deviation from σ^n_L when n is large, obtaining a contradiction.

In particular, let $U_L(p^n, \sigma^n, \sigma^n_L)$ be the large shareholder's payoff from following the strategy σ^n_L when the small shareholders follow the symmetric threshold strategy σ^n, in the finite game with n shares and a price offer p^n. We argue that:

$$(16) \qquad \lim_{n\to\infty} U_L(p^n, \sigma^n, \sigma^n_L) = \int U_L(p, s, q(r), r) d\sigma_L(s, r),$$
$$(17) \qquad \lim_{n\to\infty} U_L(p^n, \sigma^n, \bar{\sigma}^n_L) = \int U_L(p, s, q(r), r) d\bar{\sigma}_L(s, r).$$

We start with the first equality. Note that for every $\epsilon > 0$, $q^n(r)$ converges uniformly to 1 in the domain $r > a + \epsilon$ and converges uniformly to zero in the domain $r < a - \epsilon$. The large shareholder's payoff can now be rewritten as:

$$\begin{aligned}
U_L(p^n,\sigma^n,\sigma^n_L) &= \sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma^n_L(s,i/nx)\\
&= \sum_{i\in\{(a+\epsilon)nx,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma^n_L(s,i/nx)\\
&+ \sum_{i\in\{0,1,\ldots,(a-\epsilon)nx\}} \int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma^n_L(s,i/nx)\\
&+ \sum_{i\in\{(a-\epsilon)nx+1,\ldots,(a+\epsilon)nx-1\}} \int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma^n_L(s,i/nx).
\end{aligned}$$

We use the facts that $q^n(\cdot)\to q(\cdot)$ uniformly for $r\in[0,a-\epsilon]\cup[a+\epsilon,1]$, σ^n_L converges to σ_L, and that $U_L(p,s,q(r),r)$ is continuous in $r\in[0,a-\epsilon]\cup[a+\epsilon,1]$ and in its first argument, to argue that there is an N such that for $n>N$:

$$\begin{aligned}
(18)\qquad &\sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma^n_L(s,i/nx) \geq\\
&\geq \int_{r>a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_{r<a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)+\\
&x(a-\epsilon)p\int_{a-\epsilon}^{a+\epsilon} d\sigma_L(1,r) + x(1-a-\epsilon)\lambda\left(\sum_{i=(a-\epsilon)xn}^{(a+\epsilon)xn} g^n(i)\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}\right)-\epsilon.
\end{aligned}$$

In the above expression, the first two terms on the right-hand side are the limit payoffs in the specified regions. We obtain the third term by explicitly rewriting the U_L term in the integral, and bounding it generously. The second inequality below bounds the large shareholder's payoff from above, in a similar fashion as the first inequality did from below:

$$\begin{aligned}
(19)\qquad &\sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma^n_L(s,i/nx) \leq\\
&\int_{r>a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_{r<a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)+\\
&x(a+\epsilon)p\int_{a-\epsilon}^{a+\epsilon} d\sigma_L(1,r) + x(1-a+\epsilon)\lambda\left(\sum_{i=(a-\epsilon)xn}^{(a+\epsilon)xn} g^n(i)\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}\right)+\epsilon.
\end{aligned}$$

Remember that:

$$q=\lim_{n\to\infty}\sum_{i=0}^{xn} g^n(i)\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}.$$

The above sum can be split into three parts where the first sum is from 0 to $(a-\epsilon)xn-1$, the second from $(a-\epsilon)xn$ to $(a+\epsilon)xn$ and the third from $(a+\epsilon)xn+1$ to xn. The first sum converges, due to the definition of a, to 0, and the third to $\lim_{n\to\infty}\sum_{i=(a+\epsilon)xn+1}^{xn} g^n(i)$, which is in turn equal to $1-\sigma_L(1,a+\epsilon|h)$. Therefore:

$$q = 1-\sigma_L(1,a+\epsilon|h) + \lim_{n\to\infty}\sum_{i=(a-\epsilon)xn}^{(a+\epsilon)xn} g^n(i)\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}.$$

(18) can now be rewritten as

$$\begin{aligned}
&\sum_{i\in\{0,1,\ldots,nx\}}\int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma_L^n(s,i/nx) \leq \\
&\leq \int_{r>a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_{r<a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) \\
&+x(a+\epsilon)p\int_{a-\epsilon}^{a+\epsilon} d\sigma_L(1,r) + x(1-a+\epsilon)\lambda(q-(1-\sigma_L(1,a+\epsilon|h))) + 2\epsilon.
\end{aligned}$$

and (19) as

$$\begin{aligned}
&\sum_{i\in\{0,1,\ldots,nx\}}\int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma_L^n(s,i/nx) \geq \\
&\geq \int_{r>a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_{r<a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) \\
&+x(a-\epsilon)p\int_{a-\epsilon}^{a+\epsilon} d\sigma_L(1,r) + x(1-a-\epsilon)\lambda(q-(1-\sigma_L(1,a+\epsilon|h))) - 2\epsilon.
\end{aligned}$$

Since the above inequalities hold for every $\epsilon > 0$, and since the cumulative distributions are right-continous functions:

$$\begin{aligned}
&\lim_{n\to\infty}\sum_{i\in\{0,1,\ldots,nx\}}\int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma_L^n(s,i/nx) = \\
&= \int_{r>a}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_{r<a}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) \\
&+xap[\sigma_L(1,a) - \lim_{y\to a^-}\sigma_L(1,y)] + x(1-a)\lambda[q-(1-\sigma_L(1,a|h))].
\end{aligned}$$

Replacing the definition of $q(a)$, and rewriting the definition of $U_L(p,s,q(a),a)$, we get:

$$\begin{aligned}
&\lim_{n\to\infty}\sum_{i\in\{0,1,\ldots,nx\}}\int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma_L^n(s,i/nx) = \\
&= \int_{r>a}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_{r<a}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_s U_L(p,s,q(a),a)d\sigma_L(s,a) \\
&= \int_{r,s} U_L(p,s,q(r),r)d\sigma_L(s,r).
\end{aligned}$$

This completes the proof of (16). (17) can be shown using the same method, after first observing that $\bar{\sigma}_L^n$ is constructed so that it converges to $\bar{\sigma}_L$ at every continuity point of $\bar{\sigma}_L$, and that:

$$\begin{aligned} U_L(p^n, \sigma^n, \bar{\sigma}_L^n) &= \sum_{i \in \{0,1,\dots,nx\}} \int_{s \in [0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\bar{\sigma}_L^n(s, i/nx) \\ &= \sum_{i \in \{(a+\epsilon)nx,\dots,nx\}} \int_{s \in [0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\bar{\sigma}_L^n(s, i/nx) \\ &+ \sum_{i \in \{0,1,\dots,(a-\epsilon)nx\}} \int_{s \in [0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\bar{\sigma}_L^n(s, i/nx), \end{aligned}$$

because $\bar{\sigma}_L(1, a+\epsilon) = \bar{\sigma}_L(1, a-\epsilon)$, by construction. Now the steps used in the proof of (16) can be used to show $\lim_{n\to\infty} U_L(p^n, \sigma^n, \bar{\sigma}_L^n) = \int U_L(p, s, q(r), r) d\bar{\sigma}_L(s, r)$.

(15), (16) and (17) imply

$$\begin{aligned} \lim_{n\to\infty} U_L(p^n, \sigma^n, \bar{\sigma}_L^n) &= \int U_L(p, s, q(r), r) d\bar{\sigma}_L(s, r) \\ &> \int U_L(p, s, q(r), r) d\sigma_L(s, r) \\ &= \lim_{n\to\infty} U_L(p^n, \sigma^n, \sigma_L^n). \end{aligned}$$

Therefore there exists an n such that

$$U_L(p^n, \sigma^n, \bar{\sigma}_L^n) > U_L(p^n, \sigma^n, \sigma_L^n),$$

contradicting the fact that σ_L^n is a best response.

Case 2: $a = 1$. We first argue that, if $a = 1$, and if $p > 0$, then the strategy $\bar{\sigma}_L$, according to which the large shareholder tenders all shares at every signal gives the large shareholder a strictly higher payoff then any other strategy in the continuum game. In particular, let $\bar{\sigma}_L(1,1) = 1$ and $\bar{\sigma}_L(1, r) = 0$ for every $r < 1$. Then, $\int U_L(p, s, q(r), r) d\bar{\sigma}_L(s, r) > \int U_L(p, s, q(r), r) d\sigma'_L(s, r)$ for every $\sigma'_L \neq \bar{\sigma}_L$. Indeed, when the large shareholder tenders less than fraction a in the continuum game, the probability of a successful takeover, by the definition of a, is zero. Therefore it is profitable for the large shareholder to deviate towards tendering fraction a of his shares. Moreover, the large shareholder's payoff from tendering all his shares is px.

If σ_L is suboptimal, then $\bar{\sigma}_L$ yields the large shareholder a strictly higher payoff than σ_L: $\int U_L(p, s, q(r), r) d\sigma_L(s, r) < xp$. Moreover, $\int U_L(p, s, q(r), r) d\sigma_L(s, r) = \lim_{n\to\infty} U_L(p^n, \sigma^n, \sigma_L^n)$.

Since $p^n \to p$, we conclude that, for some n, $U_L(p^n, \sigma^n, \sigma_L^n) < xp^n$. But then tendering all the shares in the game with n shares is a profitable deviation from σ_L^n for the large shareholder, contradicting the assumption that σ_L^n is a best response.

The only remaining case is $p = 0$. In this case, all strategies give the large shareholder zero payoff in the continuum game, and hence all strategies are best responses. $\square$

The above lemmata can be combined to establish that the limit object $(\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$ that we have derived from the sequence $\{\sigma^n, \sigma_L^n, q^n\}_n$ is an equilibrium of the continuum game after the price offer p.

LEMMA 16 *The tuple $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$ is an equilibrium of the tender subgame in the continuum game with price p.*

PROOF: The above lemmata show how to construct the belief function $q(r)$, and establish that σ and σ_L are best responses for the small shareholders and the large shareholder, respectively. In particular, Lemma 15 shows that σ_L is a best response to $q(r)$ and p, Lemma 14 shows that small shareholder's strategy is a best response to q and p. In Definition 3, we construct $q(r)$ in a way that it satisfies the equilibrium conditions for $q(r)$. Moreover, $q(r)$ integrates to q using σ_L, by construction. The only caveat that the definition does not deliver is if the large shareholder's strategy does not have a mass on selling a fraction a. In this case, $q = 1 - \sigma_L(1, a)$, which follows from the two inequalities above inequality 14. $\square$

The last thing to argue is that the limit of equilibrium prices coincides with an equilibrium price of the continuum game.

LEMMA 17 *Let $\{p^n, \sigma^n, \sigma_L^n\}_n$ be a sequence of prices and equilibrium strategies of the tender subgames with price offer p^n in the finite shares model with n shares. If $\lim_n p^n = p$, then, $\lim_n \Pi^n(p^n, \sigma^n, \sigma_L^n) = \Pi(p)$, where $\Pi(p)$ is raider's equilibrium profit of the tender subgame with a price offer p in the model with continuum shares.*

PROOF: We omit the formal proof to this result, as it is very similar to showing how the large shareholder's payoff in the finite shares model converges to his payoff in the model with a continuum of shares; as in equality (16). The only caveat is the possible multiplicity of equilibria for a price p in the game with a continuum of shares. However, the raider's equilibrium profits after the price offer p are uniquely pinned down both in the symmetric information as well as the asymmetric information set up, as analyzed in sections 3 and 4. $\square$

In what follows we complete the proof of Theorem 5. We show that if $\{p^n, \sigma^n, \sigma_L^n\}_n$ is a convergent sequence of equilibrium price offers and equilibrium strategies of the tender subgames with price offer p^n in the finite shares model with n shares, then $p := \lim p^n$ is an equilibrium price offer for the raider in the model with a continuum of shares. This is sufficient for the proof of Theorem 6, because Lemmata 16 and 17 posit that the probability of a successful takeover, q^n, and the profits of the raider, Π^n converge to their equilibrium counterparts in the continuum shares model, after price offer p.

Let $\hat{p}$ be a price at which the raider achieves the maximum profit in a model with a continuum of shares. Such a price exists both in the models we analyzed in sections 3 and 4. Also, in both models, although there may be multiple tender subgame equilibria after some price offers, the raider's payoff is identical across all equilibria. As stated in Lemma 17, $\Pi(p')$ denotes the raider's equilibrium payoff in the tender subgame after price offer p'. Any price p' for which $\Pi(p') = \Pi(\hat{p})$ can be sustained as a tender equilibrium price offer by the raider. Therefore proving that $\Pi(p) = \Pi(\hat{p})$ will complete the proof. Clearly $\Pi(p) \leq \Pi(\hat{p})$, because $\Pi(\hat{p})$ is the maximum of $\Pi(.)$.

On the way to a contradiction, suppose that $\Pi(p) < \Pi(\hat{p})$. Then it has to be the case that there is an $\epsilon > 0$ and an integer N such that, whenever $n > N$, $\Pi^n(p^n, \sigma^n, \sigma_L^n) < \Pi(\hat{p}) - \epsilon$. On the other hand, if the raider instead offered the price $\hat{p}$ in the finite games, then for sufficiently large n, his equilibrium payoffs from following this strategy gives him a payoff arbitrarily close to $\Pi(\hat{p})$, due to Lemma 17. But this is a contradiction to p^n being an equilibrium price offer by the raider, since offering $\hat{p}$ is a profitable deviation when n is sufficiently large.

References

Amihud, Y., M. Kahan, and R. Sundaram (2004): "The foundations of freezeout laws in takeovers," *The Journal of Finance*, 59, 1325–1344.

Bagnoli, M. and B. Lipman (1988): "Successful takeovers without exclusion," *Review of Financial Studies*, 1, 89–110.

Betton, S., B. E. Eckbo, and K. S. Thorburn (2009): "Merger negotiations and the toehold puzzle," *Journal of Financial Economics*, 91, 158–178.

Billingsley, P. (1986): *Probability and Measure*, John Wiley and Sons, Inc., second ed.

Brav, A., W. Jiang, and H. Kim (2010): "Hedge fund activism: a review," *Foundations and Trends in Finance, Vol. 4, No. 3, 2009*.

Brav, A., W. Jiang, F. Partnoy, and R. Thomas (2008): "Hedge fund activism, corporate governance, and firm performance," *The Journal of Finance*, 63, 1729–1775.

Burkart, M., D. Gromb, and F. Panunzi (2006): "Minority blocks and takeover premia," *Journal of Institutional and Theoretical Economics*, 162, 32–49.

Burkart, M. and S. Lee (2010): *Signaling in Tender Offer Games*, Centre for Economic Policy Research.

Burkart, M. and F. Panunzi (2006): "Takeovers," *CEPR Discussion Paper No. 5572.*

Chowdhry, B. and N. Jegadeesh (1994): "Pre-tender offer share acquisition strategy in takeovers," *Journal of Financial and Quantitative Analysis*, 29.

Cornelli, F. and D. Li (2002): "Risk arbitrage in takeovers," *Review of Financial Studies*, 15, 837–868.

Dalkir, E. and M. Dalkir (2013): "Freeze-out Mergers," *Working Paper, University of New Brunswick.*

——— (2014): "On the optimality of partial tender offers," *Journal of Economic Theory*, 151, 561–570.

Dekel, E. and A. Wolinsky (2012): "Buying shares and/or votes for corporate control," *The Review of Economic Studies*, 79, 196–226.

Ekmekci, M. and N. Kos (2012a): "Conditional Offers in Takeovers with a Large Shareholder," *Working paper, MEDS/KSM, Northwestern University.*

——— (2012b): "Information in Tender Offers with a Large Shareholder," *Working paper, MEDS/KSM, Northwestern University.*

——— (2014): "Value of Information and Fairness Opinions in Takeovers," Tech. rep.

Fudenberg, D. and J. Tirole (1991): *Game Theory.*

Gadhoum, Y., L. Lang, and L. Young (2005): "Who controls US?" *European Financial Management*, 11, 339–363.

Greenwood, R. and M. Schor (2009): "Investor activism and takeovers," *Journal of Financial Economics*, 92, 362–375.

Grossman, S. and O. Hart (1980): "Takeover bids, the free-rider problem, and the theory of the corporation," *The Bell Journal of Economics*, 42–64.

Harrington Jr, J. and J. Prokop (1993): "The dynamics of the free-rider problem in takeovers," *Review of Financial Studies*, 6, 851–882.

Holderness, C. G. (2009): "The myth of diffuse ownership in the United States," *Review of Financial Studies*, 22, 1377–1408.

Holmström, B. and B. Nalebuff (1992): "To The Raider Goes The Surplus? A Reexaminationof the Free-Rider Problem," *Journal of Economics & Management Strategy*, 1, 37–62.

Marquez, R. and B. Yilmaz (2007): "Conditional versus Unconditional Offers in Takeovers," *Working Paper, Wharton.*

——— (2008): "Information and efficiency in tender offers," *Econometrica*, 76, 1075–1101.

——— (2012): "Takeover bidding and shareholder information," *Review of Corporate Finance Studies*, 1, 1–27.

Milgrom, P. and R. Weber (1985): "Distributional Strategies for Games with Incomplete Information," *Mathematics of Operations Research*, 10.

Müller, H. and F. Panunzi (2004): "Tender offers and leverage," *The Quarterly Journal of Economics*, 119, 1217–1248.

Shleifer, A. and R. Vishny (1986): "Large shareholders and corporate control," *The Journal of Political Economy*, 461–488.

Stepanov, S. (2012): "Takeovers under Asymmetric Information: Block Trades and Tender Offers in Equilibrium," .

Tirole, J. (2010): *The theory of corporate finance*, Princeton University Press.

Yarrow, G. (1985): "Shareholder protection, compulsory acquisition and the efficiency of the takeover process," *The Journal of Industrial Economics*, 3–16.