Competition in non-linear prices, exclusionary contracts, and market-share discounts

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Abstract

We study the effects of exclusionary contracts and market-share discounts (i.e., discounts conditioned on the share a firm receives of the customer's total purchases) in an adverse selection model where firms supply differentiated products and compete in non-linear prices. We show that exclusionary contracts intensify the competition among the firms, increasing consumer surplus, improving efficiency, and reducing profits. Firms would gain if these contracts were prohibited, but are caught in a prisoner's dilemma if they are permitted. In this case, allowing firms to offer also market-share discounts unambiguously weakens competition, reducing efficiency and harming consumers. However, starting from a situation where exclusionary contracts are prohibited, the effect of market-share discounts (which include exclusionary contracts as a limiting case) is ambiguous.

1 Introduction

The effect of exclusionary contracts on the intensity of competition is a long debated question in economics and antitrust policy.[1] The debate has recently focused also on market-share discounts (i.e., discounts conditioned on the firm's share of the customer's total purchases), which are often viewed as weaker versions of exclusive-dealing arrangements.[2]

One of the main concerns of antitrust authorities is that these strategies may be used by dominant firms to eliminate competition from equally efficient competitors.[3] However, this view is subject to a two-pronged "Chicago critique:" rational buyers must be compensated for accepting exclusionary contracts, and as efficient competitors that are threatened to be foreclosed can in turn respond by offering exclusionary contracts or market-share discounts.

In a model that accounts for these effects, Bernheim and Whinston (1998) show that under complete information exclusionary contracts and market-share discounts are competition neutral. Any equilibrium outcome that can arise when firms use these contracts, that is to say, can also be obtained when firms use standard non-linear prices, i.e., prices that depend only on own sales. However, this result hinges on the fact that under complete information non linear prices suffice to price discriminate perfectly (although buyers may obtain a positive rent because of the competition among the firms). In real life, price discrimination is often imperfect as buyers' willingness to pay is their private information.

In this paper we develop an adverse selection model where two symmetric firms supply horizontally differentiated products and compete in non-linear prices. Firms do not know the consumer's willingness to pay; hence, price discrimination is imperfect, and consumers obtain informational rents. In this framework, we show that exclusionary contracts *intensify* the competition among the firms, increasing consumer surplus, improving efficiency, and reducing profits. Firms would actually gain if these contracts were prohibited; however, if exclusionary contracts are allowed, they are caught in a prisoner's dilemma and hence have an unilateral incentive to propose exclusive-dealing arrangements even if this makes competition fiercer.

The intuitive reason why exclusionary contracts are pro-competitive is very simple. When firms compete in non-linear prices, the intensity of competition is limited by product differentiation. With exclusionary contracts, by contrast, firms compete in utility space, where their products become effectively homogeneous irrespective of the degree of differentiation. Nevertheless, in equilibrium firms' profits need not vanish, for two reasons. First, consumers value product variety and hence

[1] See Motta (2004, ch. 6) for an excellent survey of the literature.

[2] See for instance Inderst and Shaffer (2007), Majumdar and Shaffer (2009), and Mills (2004).

[3] In the *Intel* case, for instance, the European Commission found that "Intel abused its dominant position in the x86 CPU market by implementing a series of conditional rebates." Such rebates ranged from offering discounts for purchasing exclusively Intel CPUs, to conditioning such discounts on the buyer purchasing at least a pre-specified share of its CPU needs from Intel. Among the firms that were required to purchase exclusively from Intel to qualify for the discounts were Dell, Lenovo, and Media Saturn; as for market-share discounts, the critical share was set at 95% for HP and at 80% for NEC.

can be induced to purchase both products at supra-competitive prices even when the exclusionary prices are competitive. As a result, exclusionary contracts are offered but are destined not be accepted in equilibrium. This creates the possibility that firms may tacitly "coordinate" their exclusionary offers, raising the exclusionary prices above marginal costs so as to increase their equilibrium payoffs. This is the second reason why equilibrium profits can be positive. However, while such tacit coordination can occur in a non-cooperative equilibrium, its extent is limited. Thus, exclusionary contracts always provide consumers with an extra option that constrains firms' ability to increase non exclusionary prices as compared to the case in which exclusionary contracts are banned.

As for market-share discounts, their effects depend on the benchmark they are compared to. If exclusionary contracts are allowed, permitting firms to offer also market-share discounts unambiguously *weakens* competition, reducing efficiency and harming consumers. The reason why market-share discounts are anti-competitive in the presence of exclusionary contracts is that they effectively allow firms to "tax" their competitor's output. This results in a double marginalization effect, which is so strong that the equilibrium quantities (at least for consumers with a sufficiently large demand) are the same as if the products were perfect complements, even if they are in fact close substitutes.

This result implies that it may be misleading to view market-share discounts simply as weaker versions of exclusive-dealing arrangements: their strategic effects are in fact subtler and deeper. However, it is clear that firms can actually use market-share discounts to re-produce exclusive-dealing arrangements, simply by charging arbitrarily large prices when their share of a buyer's total purchases is lower than one. As a result, starting from a situation where exclusionary contracts are prohibited, permitting market-share discounts must bring about the same pro-competitive effects as exclusionary contracts, in addition to the anti-competitive double-marginalization effect mentioned above. The total effect is generally ambiguous. A regime where both exclusionary contracts and market-share discounts are prohibited is more likely to be more competitive than one where they are both allowed, the greater is the degree of tacit coordination among the firms and the less differentiated are their products.

From an analytical point of view, the paper contributes to the large and increasing literature on common agency under incomplete information. Differently from most of the literature, however, ours is a model where common agency is *delegated*, not intrinsic. In other words, the buyer has not only the option of purchasing from both firms or not purchasing at all, but also that of purchasing from only one firm. The theory of delegated common agency under incomplete information has been developed by Martimort and Stole (2009), who provide a useful characterization of the equilibrium and arrive at an explicit solution for the case of quadratic utility, constant marginal costs, and uniform distribution of types. However, their analysis is confined to the case of private common agency, where firms can condition their prices only on own sales.

Using the Martimort and Stole (2009) uniform-quadratic model, we extend their analysis to the case of *public* common agency, where firms can condition their prices not only on their own quantity, but also on their competitor's. We also consider the intermediate case where firms can offer exclusionary contracts but cannot engage in market share discounts, so they can condition prices only on whether or not the consumer purchases a positive quantity from their competitor (the *semi-public* common agency case). To the best of our knowledge, this is the first paper that analyzes such a model of delegated, public (or semi-public) common agency under incomplete information.

The rest of the paper is organized as follows. Section 2 sets up the model. In section 3, we re-obtain the private common agency solution. We adapt the Martimort and Stole approach so as to make it easier to apply, later, the case of public and semi-public common agency. Like Martimort and Stole, we start by guessing a specific functional form of the equilibrium price schedules, so that they are fully identified by few parameters. We then consider a restricted game where firms can choose only these parameters, and use the equilibrium of the restricted game as the candidate equilibrium of the original game. Finally, we verify that the candidate equilibrium is, indeed, an equilibrium of the game where firms' strategy space is unrestricted.

In section 4, we apply this approach to the case in which firms can offer exclusionary contracts but not market-share discounts. We show that in this case there is a continuum of equilibria. This multiplicity arises because in equilibrium firms must offer contracts that are destined not be accepted and hence do not affect directly the firms' payoffs. However, these contracts may constrain the payments that firms can request in the contracts that are accepted, and hence may affect their equilibrium payoffs. We characterize the most competitive equilibrium and the most "cooperative" one, showing that even the latter is more efficient and favorable to consumers than the private common agency equilibrium.

Section 5 develops the analysis of the game where firms can condition their prices on their competitor's sales arbitrarily. Again, there is a continuum of equilibria. For any degree of tacit coordination among the firms, we show that the extra-flexibility provided by market-share contracts harms consumers if exclusionary contracts are already feasible. However, starting from a situation where exclusionary contracts are prohibited, the effect of permitting market-share discounts (which include exclusionary contracts as a limiting case) is generally ambiguous.

Section 6 summarizes the paper, discusses the implications of the results for competition policy, and considers several possible directions for future work.

2 The model

Two symmetric risk-neutral firms, denoted by $i = A, B$, supply differentiated products q_A and q_B to a final consumer (a she). Firms' marginal cost is constant and is normalized at zero. The consumer's utility function in monetary terms, $u(q_A, q_B, \theta)$, depends on consumption and a parameter, θ, which is the consumer's private information. To get explicit solutions, we assume that θ is uniformly distributed over the interval $[0, 1]$ and posit a quadratic function

$$u(q_A, q_B, \theta) = \theta(q_A + q_B) - \kappa(q_A^2 + q_B^2) - \gamma q_A q_B.$$

With this formulation, the consumer's reservation utility is independent of her type θ and is normalized to zero. In an alternative interpretation of the model, A and B are manufacturers that sell their products through a common retailer, and u is the retailer's gross profit.

The parameter γ captures the degree of substitutability or complementarity between the goods. Following Shubick and Levitan (1980), to prevent changes in γ to affect the size of the market we set

$$\kappa = \frac{1-\gamma}{2}.$$

This implicitly normalizes the efficient quantities of the consumer with the largest demand (i.e., $\theta = 1$) to one for any value of γ. The parameter γ can then vary from $\frac{1}{2}$ (perfect substitutes) to $-\infty$ (perfect complements); the goods are independent when $\gamma = 0$.

Timing

Firms simultaneously and independently offer a menu of contracts, the form of which will be specified presently. The consumer observes the firms' offers and then decides whom to trade with and the quantities to purchase. If she refuses to trade with a firm, no payment is due to it (in other words, common agency is delegated, not intrinsic). Finally, sales and payments are made, and payoffs are realized.

Strategies

We distinguish between three different games according to the type of contracts firms are allowed to offer.

When exclusionary contracts and market-share discounts are banned, each firm can request a payment that depends only on its own quantity. In the jargon of the common agency literature, this is known as the case of *private* common agency. Since in this case the quantity the consumer purchases from the other firm is not contractible, a strategy for firm i is a set of quantity-payment pairs (q_i, p_i) where $q_i \geq 0$ is the quantity that firm i is willing to supply and $p_i \geq 0$ is the corresponding total payment requested. We shall refer to a quantity-payment pair as a *contract* and to the menu of contracts offered by a firm as a *price schedule*. That is, a price schedule is a function[4] $p_i(q_i) : \mathcal{Q}_i \to \Re_+$ where the domain $\mathcal{Q}_i$ is a compact subset of $\Re_+$. We assume that firm i can choose the domain $\mathcal{Q}_i$, allowing it for certain quantities not to submit any offer. We set almost no restriction on price schedules, assuming only that $p_i(q_i)$ is non decreasing (to allow for free disposal) and that the null contract $(0, 0)$ is always offered (this is just for notational convenience). Let $\mathcal{S}_{pr}$ be the set of feasible strategies under private common agency.

In the second game we consider, firms can offer exclusionary contracts but cannot engage in market share discounts (the *semi-public* common agency case).[5] A contract then is a triple (q_i, I_j, p_i), where $I_j \in \{0, 1\}$ is an indicator function that is zero when $q_j = 0$ and is one when $q_j > 0$. In this case, a strategy for firm i reduces to a menu of two price schedules, $p_i^E(q_i) : \mathcal{Q}_i^E \to \Re_+$ and $p_i^C(q_i) : \mathcal{Q}_i^C \to \Re_+$. The former applies to exclusionary contracts ($q_j = 0$), the latter to non exclusionary ones ($q_j > 0$). (For future reference, in the semi-public common agency game we define

[4]This reformulation implicitly adds the innocuous assumption that firms offer at most one contract for any quantity level. If they offered more than one, only the contract with the lowest requested payment would matter.

[5]One interpretation of this case is that firms can observe whether the buyer purchases from their competitor or not, but cannot observe the exact quantity purchased and thus cannot condition their requested payments on it. Another interpretation is that competition policy permits exclusionary contracts but prohibits market-share discounts – a policy that now is rarely observed, but is in fact optimal in our model.

$\mathcal{Q}_i \equiv \mathcal{Q}_i^E \cup \mathcal{Q}_i^C$.) Each of these schedules must contain the null contract and be non decreasing. The set of feasible strategies under semi-public common agency is denoted by $\mathcal{S}_{sp}$. This formulation is flexible enough to allow firms to unilaterally enforce exclusivity. This can be accomplished by setting $\mathcal{Q}_i^C = \varnothing$, or by requesting exorbitant high payments $p_i^C(q_i)$. In general, however, whether a consumer of type θ ends up purchasing only one product or both may depend on the strategies of both firms, and possibly also on the consumer's type θ.

Finally, in our last game firms' requested payments can depend on their competitor's quantity arbitrarily (the *public* common agency case). This formulation captures the case where both exclusionary contracts and market-share discounts are allowed. Now a contract is a triple (q_i, q_j, p_i), and firm i's strategy is a bivariate price schedule $p_i(q_i, q_j) : \mathcal{Q}_i^i \times \mathcal{Q}_j^i \to \Re_+$.[6] (For future reference, in the public common agency game we define $\mathcal{Q}_i \equiv \mathcal{Q}_i^i \cap \mathcal{Q}_i^j$.) Again, we assume that $p_i(0, q_j) = 0$ and that p_i is non decreasing in q_i. The set of feasible strategies in the public common agency case is denoted by $\mathcal{S}_{pu}$.

In all games, a strategy for the consumer is a function $\mathbf{q}(p_A, p_B) : \mathcal{S}^2 \to \mathcal{Q}_A \times \mathcal{Q}_B$ that expresses her consumption as a function of the firms' offers.

For notational convenience, we shall sometimes use the notation $p_i(q_i, q_j)$ to denote also the price schedules under private and semi-public common agency, with the understanding that p_i cannot depend on q_j at all under private common agency and can depend only on I_j under semi-public common agency.

Payoffs

Firm $i = A, B$ maximizes its expected profits

$$\pi_i = E[p_i(q_i, q_j)],$$

and the consumer maximizes her net utility

$$U(q_A, q_B, \theta) = u(q_A, q_B, \theta) - p_A(q_A, q_B) - p_B(q_B, q_A).$$

Equilibrium

Given the timing of the game, it is natural to focus on subgame perfect equilibria where the consumer maximizes U for any possible pair of price schedules submitted by the firms, not only the equilibrium one. Thus, an equilibrium is a triple of feasible strategies $\{\tilde{p}_A, \tilde{p}_B, \tilde{\mathbf{q}}(p_A, p_B)\}$ such that

$$\tilde{\mathbf{q}}(p_A, p_B) \in \arg\max_{q_A, q_B} U(q_A, q_B, \theta) \quad \forall \{p_A, p_B\} \in \mathcal{S}^2$$

and

$$E\{\tilde{p}_i(\tilde{q}_i[p_i, \tilde{p}_j], \tilde{q}_j[p_i, \tilde{p}_j])\} \geq E\{p_i(\tilde{q}_i[p_i, \tilde{p}_j], \tilde{q}_j[p_i, \tilde{p}_j])\} \quad \forall p_i \in \mathcal{S} \quad i = A, B$$

[6] Notice that we allow firm i to refuse to deal with the consumer unless she purchases from its rival a quantity $q_j \in \mathcal{Q}_j^i$.

Benchmarks

Before proceeding, we present two useful benchmarks: the efficient solution and the monopoly solution. The first-best quantities maximize the social surplus $u(q_A, q_B, \theta)$ (i.e., the sum of the consumer surplus and firms' profits) under full information, yielding

$$q_A^{fb}(\theta) = q_B^{fb}(\theta) = \theta.$$

In this first-best solution, all types θ trade with both firms.

Next consider a monopolistic firm that supplies both products under asymmetric information. Since high-type consumers will necessarily obtain an informational rent, the monopolist will distort quantities downward so as to reduce the consumer's rent and increase its profits. Using standard techniques one gets:

$$q_A^m(\theta) = q_B^m(\theta) = 2\theta - 1.$$

Clearly, $q_i^m(\theta) \leq q_i^{fb}(\theta)$, with equality only for $\theta = 1$ (no distortion at the top). The marginal consumer is $\theta = \frac{1}{2}$, and types from 0 to $\frac{1}{2}$ do not consume.

3 Private common agency

In this section we derive the private common agency equilibrium. We re-obtain the Martimort and Stole (2009) solution using a new approach, which we later apply also to the case of semi-public and public common agency. Before covering new ground, however, it may be useful to demonstrate our approach in a familiar framework.

The difficulty of finding the Nash equilibria of games like ours lies not so much in deriving the best response functions as in locating their fixed points. When calculating the firms' best responses, one can indeed proceed as if firms were monopolists facing a consumer with a suitably defined indirect utility function, which accounts for any benefit the consumer can obtain by optimally trading with the other firm. As argued by Martimort and Stole, provided that this indirect utility function satisfies appropriate regularity conditions,[7] the best response can be calculated by focusing on direct mechanisms.[8] After finding the optimal solution, one can then recover the price schedule that supports it, which is the firm's best response. However, the problem of characterizing the fixed points is generally more difficult since the best response functions are defined in a broad and relatively unstructured functional space.

To overcome this difficulty, our approach is to guess a specific functional form of the equilibrium price schedules. This allows us to restrict the firms' strategy space to a small subset $\tilde{S}$ of the

[7] In particular, the indirect utility function must satisfy the familiar sorting condition, and the participation constraint must be binding only for the lowest type, $\theta_{\min} = 0$.

[8] In a direct mechanism, the consumer is asked to reveal her type and firms condition quantities and payments on it. Under monopoly, the revelation principle guarantees that there is no loss of generality in focusing on direct mechanisms only. However, the revelation principle may fail with multiple principals (see e.g. Peters, 2001 and Martimort and Stole, 2002), so a game in which firms choose direct mechanisms would generally have different equilibria from the games analyzed in this paper.

original set $\mathcal{S}$, where price schedules are fully identified by few coefficients.[9] The Nash equilibrium of such a restricted game can be calculated much more easily. The equilibrium of the restricted game then becomes the candidate equilibrium of the original game.[10] To verify that the candidate equilibrium is, indeed, an equilibrium, one must then verify that the best response property holds over the unrestricted strategy set $\mathcal{S}$. Luckily, however, this can be done using direct mechanisms, as explained above.

In this procedure, the critical step often is the initial guess. Fortunately, useful hints can be obtained by analogy with the properties of the monopoly solution, which often are well known or can be determined as a matter of routine.

For example, when the surplus function is quadratic and the distribution of types is uniform, the monopoly quantities are linear in θ and hence can be supported by quadratic price schedules. This suggests that in a private common agency equilibrium firms will submit quadratic price schedules, i.e.[11]

$$p_i(q_i) = \alpha_{0,i} + \alpha_{1,i} q_i + \alpha_{2,i} q_i^2 \quad \text{for } q_i \in [0,1].$$

Each of these price schedules is fully identified by the three coefficients $\alpha_{s,i}$ for $s = 0, 1, 2$. But we can further refine our guess by exploiting two other well known properties of the monopoly solution. The first is the no-distortion-at-the-top property, which in our setting means that type $\theta_{\max} = 1$ must purchase the undistorted quantities $q_A^{fb}(1) = q_B^{fb}(1) = 1$. This implies that the equilibrium price schedules must be flat at $q_i = 1$, entailing

$$\alpha_{2,i} = -\frac{\alpha_{1,i}}{2}.$$

The second property is that no fixed fees are charged when the market is uncovered (Wilson, 1995), implying

$$\alpha_{0,i} = 0.$$

Consider, then, a restricted game where firms are constrained to submit price schedules of the type

$$p_i(q_i) = \alpha_{1,i} q_i - \frac{\alpha_{1,i}}{2} q_i^2 \quad \text{for } q_i \in [0,1].$$

A strategy for firm i then becomes simply a value of $\alpha_{1,i}$. This restricted game can be solved easily (see Appendix 3.1). There is a unique equilibrium, which is symmetric and is given by

$$\alpha_1^* = \frac{1}{4}\left[3(1-\gamma) - \sqrt{1 - 2\gamma + 9\gamma^2}\right]. \tag{1}$$

[9] This is similar to the Martimort and Stole approach. However, given such a guess, they immediately proceed to construct the firms' best responses using direct mechanisms. Instead, we focus on the restricted games where firms can choose only the coefficients of the guessed price schedules. Calculating the equilibrium of the restricted games allows us to pin down the coefficients, and we then apply the direct mechanism analysis only to verify that the best response property holds. This procedure is simpler, especially when the number of coefficients to be determined grows large.

[10] Our approach exploits a simple and yet fundamental property of Nash equilibria, namely, that if the Nash equilibrium $\mathbf{s}^*$ of a symmetric n-players game with strategy set S lies in a proper subset $\tilde{S}^n$ of S^n, then $\mathbf{s}^*$ is also a Nash equilibrium of the restricted game where each player's strategy set is $\tilde{S} \subset S$.

[11] These schedules can be extended arbitrarily to $q_i > 1$. Provided that the extension is non decreasing, this will not disrupt the equilibrium.

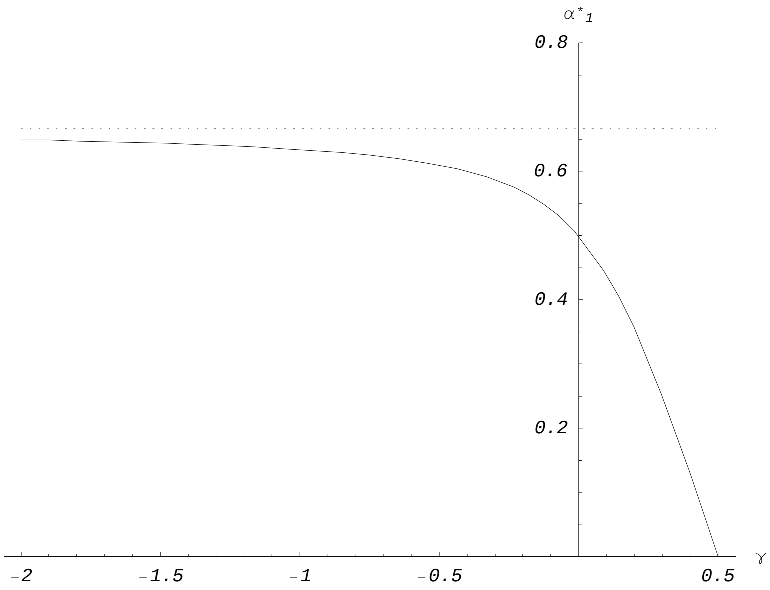

Figure 1: The coefficient α_1^* as a function of the degree of product differentiation γ.

The variable α_1^* plays a key role in all the subsequent analysis. It decreases with γ, vanishes when $\gamma = \frac{1}{2}$ (the perfect substitutes case, where one re-obtains the Bertrand paradox) and converges to $\frac{2}{3}$ as $\gamma \to -\infty$ (the case of perfect complements), as shown in Figure 1.

Our candidate equilibrium then becomes $p_i^* = \alpha_1^* q_i - \frac{\alpha_1^*}{2} q_i^2$. The final step of the procedure must verify that these price schedules are an equilibrium of the original (i.e., unrestricted) game. This is done in Appendix 3.2, where we prove that if firm j submits the price schedule $p_j^* = \alpha_1^* q_j - \frac{\alpha_1^*}{2} q_j^2$, then firm i's optimal strategy in the unrestricted strategy set $\mathcal{S}_{pr}$ is, indeed, $p_i^* = \alpha_1^* q_i - \frac{\alpha_1^*}{2} q_i^2$, and vice versa.

The above discussion can be summarized as follows.

Proposition 1 *The price schedules*

$$p_i^*(q_i) = \alpha_1^* q_i - \frac{\alpha_1^*}{2} q_i^2 \qquad \textit{for } 0 \le q_i \le 1, \tag{2}$$

where α_1^* is given by (1), are an equilibrium of the private common agency game.

Before proceeding, we briefly discuss some important properties of the private common agency equilibrium, which is a fundamental benchmark for our future analysis. The equilibrium quantities are

$$q_i^*(\theta) = \frac{\theta - \alpha_1^*}{1 - \alpha_1^*}, \tag{3}$$

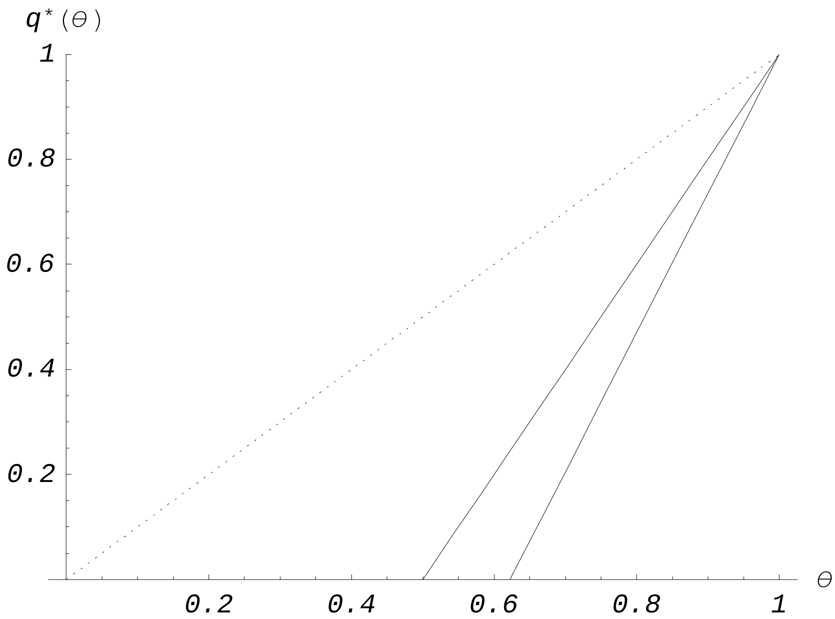

Figure 2: Equilibrium quantities under private common agency for $\gamma = \frac{1}{2}$ (perfect substitutes, the highest line), $\gamma = 0$ (independent good, the intermediate line) and $\gamma = -\infty$ (perfect complements, the lowest line).

and are depicted in Figure 2 for representative values of γ. In particular, one re-obtains the efficient solution for $\gamma = \frac{1}{2}$ (the Bertrand paradox) and the monopoly solution for $\gamma = 0$;[12] the equilibrium quantities are smallest, i.e., $q_i^*(\theta) = 3\theta - 2$, when $\gamma \to -\infty$, since with complementary goods prices are excessively high due to the problem of Cournot complements.[13]

Apart from the special case $\gamma = \frac{1}{2}$, the market is uncovered since types θ from 0 to $\alpha_1^* > 0$ do not consume. The consumer's rent is zero for $\theta \leq \alpha_1^*$ and is

$$U^*(\theta) = \frac{(\theta - \alpha_1^*)^2}{1 - \alpha_1^*} \tag{4}$$

for $\theta \geq \alpha_1^*$. Notice that $\frac{dU^*}{d\theta} = q_A^*(\theta) + q_B^*(\theta)$; this property, which follows by the envelope theorem, reflects the fact that equilibrium quantities are chosen by the consumer optimally.

[12] Thus, under private common agency firms do not effectively interact strategically when the goods are independent. As we shall see below, this property does not carry over to the case of semi-public or public common agency.

[13] As is well known, when several separate firms supply complementary components of the final product and price each component non-cooperatively, the price to the final consumer exceeds the monopoly price, reducing output (and profits) below monopoly output (and profits).

4 Exclusionary contracts

In this section we allow firms to use exclusionary contracts, but continue to rule out market-share discounts. Thus, each firm i can offer a menu of two price schedules, $p_i^E(q_i)$ and $p_i^C(q_i)$: the former applies to exclusionary contracts ($q_j = 0$), the latter to non exclusionary contracts ($q_j > 0$). We shall refer to this environment as the semi-public common agency case.

4.1 A non neutrality result

In the complete information model of Bernheim and Whinston (1998), the same equilibrium outcome is obtained with and without exclusionary contracts: in both cases, the allocation is efficient and each firm obtains a profit exactly equal to the incremental value of its product. Thus, exclusionary contracts are irrelevant, or competition neutral.

Things are different with incomplete information. The next Proposition (proved in Appendix 4.1) shows that if exclusionary contracts are permitted, they disrupt the private common agency equilibrium, at least when the market is uncovered.

Proposition 2 *When $\gamma < \frac{1}{2}$, exclusionary contracts are offered in equilibrium, and the private common agency equilibrium outcome cannot be supported by any semi-public common agency equilibrium.*

Assumption $\gamma < \frac{1}{2}$ guarantees that in the private common agency equilibrium the market is uncovered. Some low-type consumers then are not served even if their demand for the goods is positive. The intuitive reason is that in order to serve these consumers, firms should offer contracts that would attract also some high type consumers, increasing by too much their informational rents. However, starting from the private common agency equilibrium, exclusionary contracts allow firms to profitably serve low-type consumers without attracting high-type ones. This is possible because high-type consumers value the opportunity to purchase both products more than low types, and hence have more to lose from accepting exclusionary contracts.

Since they impose on consumers an unnecessary cost, exclusionary contracts resemble the strategy of damaging one's goods analyzed by Deneckere and McAfee (1996). Focusing on monopoly, they show that the conditions under which damaging goods can be a profitable discriminating strategy are rather restrictive.[14] But the scope for profitable discrimination expands considerably when firms damage goods contractually through exclusionary clauses, since most of the cost of this strategy then falls as a negative externality on their rival. As Proposition 2 shows, contractually damaging one's good through exclusionary clauses is unprofitable only if the goods are perfect substitutes (in which case the private common agency equilibrium already entails zero prices).

Having shown that permitting exclusionary contracts disrupts the private common agency equilibrium, we now turn to the analysis of the new equilibrium.

[14] Anderson and Dana (2005) have shown that a monopolist price discriminates if and only if the condition of *increasing percentage differences* holds. This condition requires that the ratio of the marginal social value from an increase in product quality to the total social value of the good increases with consumers' willingness to pay and is very restrictive when the unit production cost is constant or decreasing in quality, as in the case of damaged goods.

4.2 The most competitive equilibrium

We start by noting that when firms offer exclusionary contracts, they compete in utility space where their products become effectively homogeneous. As a result, if exclusionary contracts involving strictly positive payments were accepted by some consumers, the standard Bertrand logic would imply that firms would start undercutting each other, driving exclusionary prices to cost. This is, indeed, what happens in the models of one-stop shopping of Armstrong and Vickers (2001), Rochet and Stole (2002) and others.

This suggests that we look for an equilibrium in which firms offer competitive exclusionary price schedule involving zero payments:

$$p_i^E(q_i) = 0 \text{ for all } q_i \geq 0,\ i = A, B. \tag{5}$$

To show that these contracts can, indeed, be part of an equilibrium, observe that if firm j offers $p_j^E(q_j) = 0$, firm i will make zero profits with any non negative exclusionary price schedule. Moreover, acting unilaterally firm i cannot affect the reservation utility that is provided to the consumers by $p_j^E(q_j) = 0$. That means that given $p_j^E(q_j) = 0$, firm i is actually indifferent between any exclusionary price schedule, and hence its best reply includes the schedule $p_i^E(q_i) = 0$.

With these exclusionary contracts, consumer θ's optimal choice is to purchase a quantity

$$q^E(\theta) = \frac{\theta}{1-\gamma}$$

of either good,[15] obtaining a strictly positive type-dependent utility

$$U^E(\theta) = \frac{\theta^2}{2(1-\gamma)}. \tag{6}$$

Now consider the restricted game in which firms choose only their non exclusionary price schedules $p_i^C(q_i)$, given (5). This restricted game is one of private common agency. Differently from the previous section, however, now there is a strictly positive and type dependent reservation utility, $U^E(\theta)$. This is an important difference, which radically changes the non exclusionary price schedules.[16]

To guess the possible shape of the new equilibrium schedules, we use an important result in the mechanism design literature due to Armstrong, Cowan and Vickers (1994) and Jullien (2000). They show that if the reservation utility is "weakly" convex (in a sense that will be made precise below), then in the monopoly solution there exists a threshold $\hat{\theta}$ such that the participation constraint binds

[15] At first, it might seem surprising that the demand under exclusive dealing depends on the product differentiation parameter γ. However, recall that we set $\kappa = (1-\gamma)/2$, so when the consumer purchases only one good, her total demand does depend on γ.

[16] There is another, subtler difference. In the restricted game the consumer cannot choose a contract from the common agency schedule $p_i^C(q_i)$ without trading also with firm j: either she purchases from both firms, or she obtains $U^E(\theta)$. Thus, the restricted game can be viewed as a game of *intrinsic* common agency, where both firms must take care that the participation constraint is satisfied.

for a set of types $[0, \hat{\theta}]$ and does not bind over the interval $(\hat{\theta}, 1]$. If a similar pattern is to hold under duopoly, then the non exclusionary price schedules must comprise two parts, one intended for consumers who will obtain exactly their reservation utility $U^E(\theta)$, and one for those who will obtain strictly more.

Therefore, we posit non exclusionary price schedules that again depend only on own output but now are piecewise quadratic :

$$p_i^C(q_i) = \begin{cases} \underline{\alpha}_{0,i} + \underline{\alpha}_{1,i} q_i + \underline{\alpha}_{2,i} q_i^2 (\equiv \underline{p}_i^C(q_i)) & \text{for } 0 \le q_i \le \hat{q}_i \\ \bar{\alpha}_{0,i} + \bar{\alpha}_{1,i} q_i + \bar{\alpha}_{2,i} q_i^2 (\equiv \bar{p}_i^C(q_i)) & \text{for } \hat{q}_i < q_i \le 1 \end{cases} \tag{7}$$

We denote by $\underline{q}_i(\theta)$ the optimal quantities for consumers who choose $q_i \le \hat{q}_i$ and by $\bar{q}_i(\theta)$ the optimal quantities for those who choose $q_i > \hat{q}_i$; $\underline{U}(\theta)$ and $\bar{U}(\theta)$ denote the corresponding net utilities. In the restricted game where firms can choose only schedules of type (7), for each firm there are now seven coefficients to be determined: the $\underline{\alpha}_{s,i}$'s, the $\bar{\alpha}_{s,i}$'s, and $\hat{q}_i$.

Consider the lower part of the non exclusionary price schedules first. Under monopoly, this would be pinned down by the type dependent participation constraint. This turns out to be true also under duopoly, but the reasoning is subtler. Notice first of all that if consumers opted for the exclusionary contracts (5), firms would make zero profits. However, since consumers value the opportunity to purchase both products, firms can obtain positive profits by inducing "common participation".

From this perspective, common participation can be viewed as an indivisible public good that is jointly provided by the two firms, since both firms must take care that the participation constraint is satisfied. To do so, they must lower their non exclusionary prices. But since "common participation" is an indivisible good, any firm will provide it as long as the residual contribution that is needed is lower than the benefit. In other words, for any given $\underline{p}_j^C(q_j)$ set by firm j, firm i will induce the consumer to accept the non exclusionary contracts if only it can do so and still charge positive prices $\underline{p}_i^C(q_i) > 0$. Anticipating this behavior, each firm can free ride on its rival by charging high prices and letting the cost of the provision of "common participation" fall on it. This creates the possibility of multiple equilibria, as in many other models of private provision of an indivisible public good. For simplicity, in what follows we shall focus on the unique symmetric equilibrium, where the cost of inducing common participation is divided evenly among the two firms.

The above discussion implies that the coefficients $\underline{\alpha}_{s,i}$ must indeed guarantee to low type consumers a rent equal to their reservation utility $U^E(\theta)$. If the equality is to hold for a non degenerate set of types $[0, \hat{\theta}]$, we must have

$$\underline{q}_A(\theta) + \underline{q}_B(\theta) = q^E(\theta) \tag{8}$$

for all $\theta \in [0, \hat{\theta}]$. This condition follows from the envelope theorem, which implies that $\frac{dU^E}{d\theta} = q^E$ and $\frac{d\underline{U}}{d\theta} = \underline{q}_A(\theta) + \underline{q}_B(\theta)$. Condition (8) immediately implies

$$\underline{\alpha}_0^{**} = \underline{\alpha}_1^{**} = 0.$$

It also implies

$$\alpha_{2,A} \times \alpha_{2,B} = \left(\frac{1}{2} - \gamma\right)^2,$$

an equation that implicitly determines the entire set of possible asymmetric candidate equilibria (i.e., the set of its non negative solutions). Focusing on the unique symmetric candidate equilibrium where $\underline{q}_A(\theta) = \underline{q}_B(\theta) = \frac{q^E(\theta)}{2}$, we therefore set

$$\underline{\alpha}_2^{**} = \frac{1}{2} - \gamma.$$

Now we turn to the upper part of the non exclusionary price schedules, which is intended for high-type consumers. Since the participation constraint is not binding for them, it is natural to conjecture that the equilibrium quantities are the same as under private common agency. This conjecture can be justified as follows. At $\theta = 1$, the equilibrium quantities are fully determined by the no-distortion-at-the-top property. For lower values of θ, the equilibrium quantities then can be determined proceeding "backwards". Consider any interval $(\hat{\theta}, 1]$ where the participation constraint does not bind. Given its rival's price schedule $\bar{p}_j^C(q_j)$, firm i's best response will be determined by pointwise maximization of an indirect virtual surplus function that depends only on the consumer's indirect utility and the hazard rate $\frac{F(\theta)-1}{f(\theta)}$ over the interval $(\hat{\theta}, 1]$. For any given $\bar{p}_j^C(q_j)$, the indirect virtual surplus, and hence the best response function, is the same as in the private common agency game. This implies that the equilibrium quantities are the same as under private common agency, i.e., $\bar{q}_i(\theta) = q_i^*(\theta)$.[17] This requires that

$$\bar{\alpha}_1^{**} = \alpha_1^* \text{ and } \bar{\alpha}_2^{**} = -\frac{\alpha_1^*}{2};$$

now, however, the fixed fees $\bar{\alpha}_{0,i}$ can be different from zero.

Before proceeding to calculate the $\bar{\alpha}_{0,i}$'s, we pause here to note that the conjecture $\bar{q}_i(\theta) = q_i^*(\theta)$ implies also that $\bar{U}(\theta) = U^*(\theta) - \bar{\alpha}_{0,A} - \bar{\alpha}_{0,B}$. This means that the reservation utility $U^E(\theta)$ is less strongly convex than $\bar{U}(\theta)$,[18] so it is, indeed, "weakly" convex in the sense of Jullien (2000).

To determine the $\bar{\alpha}_{0,i}$'s and the $\hat{q}_i$'s, we impose the condition that the non exclusionary price schedules be continuous and continuously differentiable at $\hat{q}_i$. This guarantees that the equilibrium quantities are continuous in θ. Since this property must be satisfied by any best response (see e.g. Laffont and Martimort, 2002), it is natural to conjecture that it must hold also in equilibrium. (Appendix 4.2 develops an explicit analysis of the restricted game where firms choose only the $\bar{\alpha}_{0,i}$'s

[17] Another difference between the restricted game considered here and the private common agency game is that here common agency is intrinsic, not delegated, as we remarked in the preceding footnote. However, Calzolari and Scarpa (2008) and Martimort and Stole (2009) have shown that if the participation constraint does not bind, equilibrium quantities under intrinsic and delegated common agency coincide.

[18] That is,

$$\frac{d^2\bar{U}(\theta)}{d\theta^2} = \frac{2}{1-\alpha_1^*} > \frac{1}{1-\gamma} = \frac{d^2U^E(\theta)}{d\theta^2}.$$

and the $\hat{q}_i$'s, given all the other postulated properties of the exclusionary and non exclusionary price schedules, and proves that in equilibrium the non exclusionary price schedules must, indeed, be continuous and continuously differentiable at $\hat{q}_i$.)

Continuity requires (with a slight abuse of notation):

$$\underline{p}_i^C(\hat{q}_i) = \bar{p}_i^C(\hat{q}_i). \tag{9}$$

At $\hat{q}_i$ the left- and right-hand side derivatives of the non exclusionary price schedules coincide if

$$2\left(\frac{1}{2} - \gamma\right)\hat{q}_i = \alpha_1^*(1 - \hat{q}_i).$$

The latter equation directly yields

$$\hat{q}^{**} = \frac{\alpha_1^*}{1 - 2\gamma + \alpha_1^*}.$$

Substituting this value into (9) and using symmetry one finally obtains

$$\bar{\alpha}_0^{**} = -\frac{\alpha_1^{*2}}{2\left(1 - 2\gamma + \alpha_1^*\right)}.$$

Notice that the fixed fee is negative. The intuitive explanation is that in the absence of the fixed fee, firms would earn more on the upper part of the non exclusionary price schedules than on the lower part. Thus, firms must bribe consumers into the upper part of the schedule by setting negative fixed fees.

We have thus completed the calculation of the candidate equilibrium of the semi-public common agency game. Summarizing, firm $i = A, B$ offers an exclusionary contract

$$p^E(q) = 0 \text{ for all } q \geq 0$$

and a non-exclusionary price schedule

$$p^C(q) = \begin{cases} \left(\frac{1}{2} - \gamma\right) q^2 & \text{for } 0 \leq q \leq \frac{\alpha_1^*}{1-2\gamma+\alpha_1^*} \\ -\frac{\alpha_1^{*2}}{2\left(1-2\gamma+\alpha_1^*\right)} + \alpha_1^* q - \frac{\alpha_1^*}{2} q^2 & \text{for } \frac{\alpha_1^*}{1-2\gamma+\alpha_1^*} \leq q \leq 1. \end{cases}$$

Proposition 3 below shows that this is, indeed, a semi-public common agency equilibrium. However, this equilibrium is not unique (even after imposing symmetry). To see why, recall that our initial guess $p_i^E(q_i) = 0$ was justified by the argument that if some exclusionary contracts are accepted, the familiar Bertrand undercutting process must drive exclusionary prices to zero. However, in the semi-public common agency equilibrium we have just derived, no exclusionary contract is actually accepted. This means that firms need not start undercutting each other's exclusionary prices. Although there is always an equilibrium in which both firms set $p_i^E(q_i) = 0$, this observation opens the possibility that there may exist other semi-public common agency equilibria, in which firms may offer strictly positive exclusionary price schedules.

4.3 The set of equilibria with exclusionary contracts

To confirm that multiple equilibria can indeed exist, observe that even if exclusionary price schedules are supra-competitive, they cannot directly generate any positive profit for the firms, since exclusionary contracts are destined not be accepted in equilibrium. However, these contracts may constrain the payments that firms can request in non exclusionary contracts, and hence their equilibrium payoffs. In particular, the less aggressively firms bid for exclusionary contracts, the lower the reservation utility left to consumers, and hence the greater the payments firms can obtain for non exclusionary contracts. This suggests that firms can tacitly "coordinate" their exclusionary offers in order to increase their equilibrium payoffs, as in Chiesa and Denicolò (2009).[19] The equilibrium with $p_i^E(q_i) = 0$ derived above is the most competitive equilibrium, in which no such coordination takes place.

To characterize the set of all possible equilibria, we must therefore determine the maximum degree of coordination among firms that is consistent with playing the game non-cooperatively. In other words, supposing that firms tacitly agree to request very large payments in their exclusionary contracts, what are the maximum payments that make the tacit agreement stable?

To answer this question, suppose that both firms offer the same strictly positive exclusionary price schedules $p^E(q)$ and also the same non exclusionary schedules $\underline{p}^C(q)$.[20] In equilibrium, the participation constraint binds for a non-degenerate interval of types, so certain low-type consumers must be exactly indifferent between exclusionary and non exclusionary contracts. This implies that one must have $q^E(\theta) = 2\underline{q}(\theta)$, and also that the following condition must hold:[21]

$$2\underline{p}^C(\underline{q}(\theta)) - p^E(2\underline{q}(\theta)) = (1 - 2\gamma)\underline{q}^2(\theta). \tag{10}$$

The economic interpretation of (10) is simple. The left-hand side is the difference between the cost of purchasing quantity $\underline{q}(\theta)$ from both firms and the cost of purchasing the same total quantity,

[19] Under private common agency, by contrast, all contracts offered in equilibrium are accepted by some consumer θ, so there is no scope for varying the requested payments arbitrarily without directly affecting the payoffs.

[20] As we proceed, it will become clear that the upper bound on the exclusionary payments is largest when firms behave symmetrically. Thus, given that our goal is to characterize the most "cooperative" equilibrium, there is no loss of generality in assuming symmetry.

[21] To prove equality (10), we begin by noting that in any equilibrium the reservation utility guaranteed by the exclusionary price schedules $p^E(q)$ must be matched by the non exclusionary schedules $\underline{p}^C(q)$; otherwise, some consumers would accept the exclusionary contracts and Bertrand competition in utility space would then drive the exclusionary prices to zero. The maximum rent consumer θ could obtain by choosing an exclusionary contract is

$$\theta q^E(\theta) - \frac{1-\gamma}{2}\left[q^E(\theta)\right]^2 - p^E(q^E(\theta)),$$

where

$$q^E(\theta) = \arg\max_{q_i}\left[\theta q_i - \frac{1-\gamma}{2}q_i^2 - p_i^E(q_i)\right]$$

On the other hand, the net utility obtained by consumer θ if she purchases $\underline{q}(\theta)$ from both firms is

$$2\theta\underline{q}(\theta) - (1-\gamma)\left[\underline{q}(\theta)\right]^2 - \gamma\left[\underline{q}(\theta)\right]^2 - 2\underline{p}(\underline{q}(\theta)).$$

Equating the net utility consumers obtain with exclusionary and non exclusionary contracts, equation (10) follows.

i.e. $q^E(\theta) = 2\underline{q}(\theta)$, exclusively from one firm. The right-hand side is the value of variety, i.e., the difference in the utility obtained by consumer θ with those two strategies. For the consumer to be indifferent between exclusionary and non exclusionary contracts, the extra cost of common agency must equal the extra benefit of product variety.

Now consider the largest exclusionary price schedules that can be part of an equilibrium. Since consumer $\theta \in [0, \hat{\theta}]$ must be indifferent between exclusionary and non exclusionary contracts, any arbitrary small discount would suffice to induce her to switch to exclusionary contracts. But no such discount can be profitable in equilibrium. Since the deviating firm in equilibrium earns $\underline{p}(\underline{q}(\theta))$ on consumer θ while it would earn $p^E(2\underline{q}(\theta)) - \varepsilon$ by inducing her to switch, where $\varepsilon > 0$ is arbitrary small, the no deviation condition requires $\underline{p}^C(\underline{q}(\theta)) \geq p^E(2\underline{q}(\theta))$. Using (10), this condition can be rewritten as

$$p^E(q) \leq \frac{1-2\gamma}{4} q^2. \tag{11}$$

This inequality places an upper bound on the maximum payment that can be requested for exclusionary contracts.

This upper bound is tight, meaning that for any $0 \leq \mu \leq \frac{1-2\gamma}{4}$ there exists a semi-public common agency equilibrium with

$$p_i^E(q_i) = \mu q_i^2 \qquad i = A, B. \tag{12}$$

The equilibrium where $p_i^E(q_i) = 0$ corresponds to $\mu = 0$ and is, clearly, the most competitive equilibrium. For $\mu = \frac{1-2\gamma}{4}$, by contrast, we obtain the least competitive equilibrium, which minimizes the consumer's rent.

To determine the structure of these equilibria, we proceed as for the case $p_i^E(q_i) = 0$. First of all, note that facing exclusionary schedules (12) consumer θ would purchase

$$q_i^E(\theta, \mu) = \frac{\theta}{1 - \gamma + 2\mu},$$

obtaining a reservation utility of

$$U^E(\theta, \mu) = \frac{\theta^2}{2\,(1 - \gamma + 2\mu)}.$$

With this new and lower reservation utility, the analysis then proceeds as before. The coefficients of the lower part of the non exclusionary price schedules must guarantee to low type consumers a rent equal to $U^E(\theta, \mu)$. This requires $\underline{\alpha}_0^{**} = \underline{\alpha}_1^{**} = 0$, as in the most competitive equilibrium, but now we must have

$$\underline{\alpha}_{2,A} \times \underline{\alpha}_{2,B} = \left(\frac{1}{2} - \gamma + 2\mu\right)^2.$$

Focusing again on a symmetric candidate equilibrium,[22] this implies

$$\underline{\alpha}_2^{**} = \frac{1}{2} - \gamma + 2\mu.$$

[22] Since now firms would obtain positive profits from their exclusionary contracts, not all the non-negative solutions

As for the upper part of the common agency price schedule, we have again $\bar{\alpha}_1^{**} = \alpha_1^*$ and $\bar{\alpha}_2^{**} = -\frac{\alpha_1^*}{2}$. The $\bar{\alpha}_{0,i}$'s and the $\hat{q}_i$'s must guarantee that the non exclusionary price schedules are continuous and continuously differentiable at $\hat{q}_i$. This requires

$$\bar{\alpha}_0^{**} = -\frac{\alpha_1^{*2}}{2\,(1-2\gamma+\alpha_1^*+4\mu)}.$$

and

$$\hat{q}^{***} \equiv \frac{\alpha_1^*}{1-2\gamma+\alpha_1^*+4\mu}.$$

This completes the calculation of the candidate equilibrium for any $\mu \in [0, \frac{1-2\gamma}{4}]$.

Appendix 4.3 verifies that the candidate equilibria satisfy the best response property in the unrestricted strategy space $\mathcal{S}_{sp}$. Hence, we have:

Proposition 3 *The following are equilibria of the semi-public common agency game. For any*

$$0 \leq \mu \leq \frac{1-2\gamma}{4},$$

firm i offers an exclusionary contract

$$p_i^{E**}(q_i) = \mu q_i^2 \text{ for all } q_i \geq 0$$

and a non-exclusionary price schedule

$$p_i^{C**}(q_i) = \begin{cases} \left(\frac{1}{2}-\gamma+2\mu\right) q_i^2 & \text{for } 0 \leq q \leq \hat{q}^{**} \\ -\dfrac{\alpha_1^{*2}}{2\,(1-2\gamma+\alpha_1^*+4\mu)} + \alpha_1^* q - \frac{\alpha_1^*}{2} q^2 & \text{for } \hat{q}^{**} \leq q \leq 1. \end{cases}$$

The equilibrium price schedules are depicted in Figure 3. They are at first convex (meaning that consumers pay quantity premia) and then concave (so firms offer quantity discounts eventually, as under private common agency). For any admissible value of μ, the equilibrium non exclusionary price schedules – the dotted curves in Figure 3 – lie below the private common agency schedules. This fact suggests that exclusionary contracts are pro-competitive. To confirm this intuition, now we turn to the welfare analysis.

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to the equation

$$\underline{\alpha}_{2,A} \times \underline{\alpha}_{2,B} = \left(\frac{1}{2} - \gamma + 2\mu\right)^2$$

are an equilibrium of the game of private provision of "common participation." If, for instance, consumers randomize when both firms offer the same exclusionary price schedule $p_i^E(q_i) = \mu q_i^2$, patronizing either firm with a probability of a half, only the roots satisfying the condition $\underline{\alpha}_{2,i} \geq \mu$ would correspond to an equilibrium.

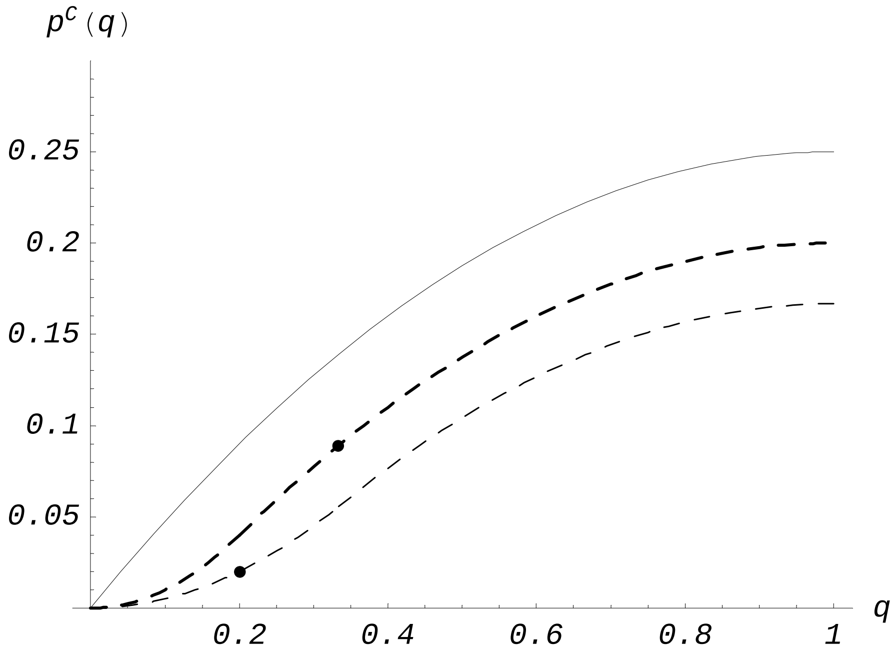

Figure 3: Equilibrium price schedules under private common agency (the continuous curve) and with exclusionary contracts (the thick dashed line is the most cooperative equilibrium, the thin dashed line the most competitive).

4.4 Welfare comparison

In this subsection we compare the equilibria with exclusionary contracts to the private common agency equilibrium in terms of consumer surplus, profits, and social welfare. Although multiple equilibria exist with exclusionary contracts, we obtain unambiguous predictions irrespective of which equilibrium is selected.

To begin with, notice that since the marginal prices are zero at $q_i = 0$, now in equilibrium the market is covered and all consumers purchase both products. The equilibrium quantities are

$$q_i^{**}(\theta) = \begin{cases} \dfrac{\theta}{2\,(1-\gamma+2\mu)} & \text{for } 0 \le \theta \le \hat{\theta}^{**} \\ q_i^*(\theta) \quad \text{for } \hat{\theta}^{**} \le \theta \le 1, \end{cases} \tag{13}$$

where $\hat{\theta}^{**}$ is the threshold where consumers switch from the lower to the upper part of the non exclusionary schedules, and is

$$\hat{\theta}^{**} \equiv \frac{2\alpha_1^*(1-\gamma+2\mu)}{1-2\gamma+\alpha_1^*+4\mu}.$$

Figure 4 depicts the special case $\gamma = 0$, but the qualitative pattern is more general. The equilibrium quantities with exclusionary contracts decrease with μ, but for any value of μ they are at least as large as under private common agency. In particular, consumers of type $\theta \in [0, \alpha_1^*]$ now purchase positive quantities whereas under private common agency they did not purchase at all, consumers of type $\theta \in (\alpha_1^*, \hat{\theta}^{**}]$ increase their consumption, which was already positive, and only for $\theta \in [\hat{\theta}^{**}, 1]$ do equilibrium quantities stay unchanged.

However, even consumers whose consumption does not change now enjoy negative fixed fees. As a result, the net surplus obtained by consumer θ, which is

$$U^{**}(\theta) = \begin{cases} \dfrac{\theta^2}{2\,(1-\gamma+2\mu)} & \text{for } 0 \le \theta \le \hat{\theta}^{**} \\ U^*(\theta) + \dfrac{\alpha_1^{*2}}{(1-2\gamma+\alpha_1^*+4\mu)} & \text{for } \hat{\theta}^{**} \le \theta \le 1, \end{cases} \tag{14}$$

is everywhere strictly greater than under private common agency (see Figure 5).[23]

Since the new equilibrium quantities are everywhere closer to the first best equilibrium quantities and u is concave, social welfare u is greater with exclusionary contracts. However, not even in the most competitive equilibrium (i.e., $\mu = 0$) is the efficient solution attained. The intuition is that while in the most competitive equilibrium exclusionary prices vanish, non exclusionary prices are still strictly positive, and thus inefficient.

Finally, one can show that for any value of μ and γ, firms' profits are always lower in the semi-public common agency equilibrium than under private common agency (see Figure 6).[24] This

[23] By the envelope theorem, this follows immediately from the fact that equilibrium quantities are larger.

[24] Although this results follows from simple algebraic calculations, from an economic point of view it is not obvious when the goods are complements. For permitting exclusionary contracts lowers prices, but when the goods are complements the equilibrium prices under private common agency are too high not only from the social viewpoint, but also from the point of view of joint profit maximization, due to the problem of Cournot complements.

Figure 4: The equilibrium quantities with exclusionary contracts when the goods are independent ($\gamma = 0$) in the most competitive ($\mu = 0$, the dashed line) and the most cooperative ($\mu = \frac{1-2\gamma}{4}$, the continuous line) equilibrium.

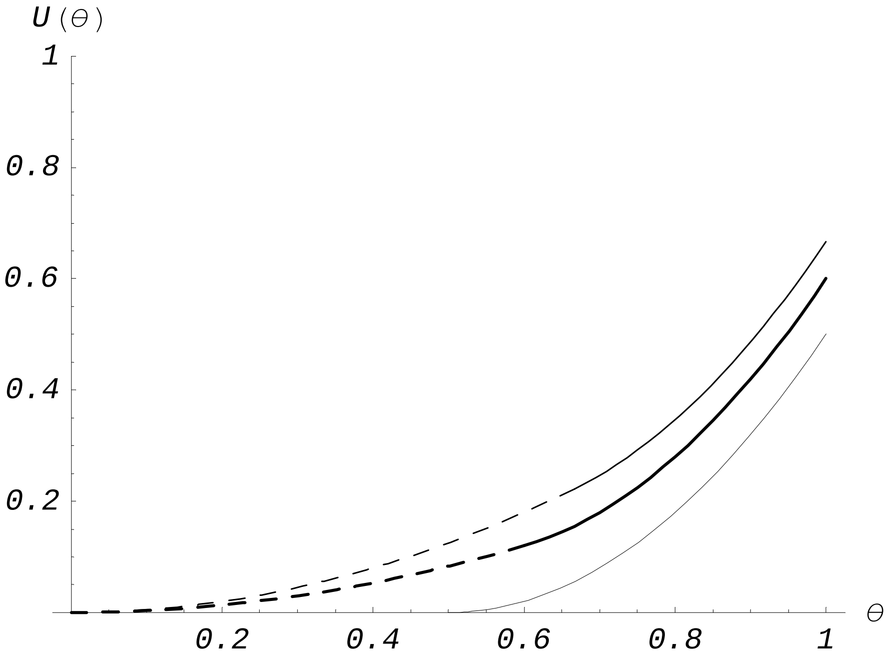

Figure 5: The consumer surplus under private common agence (the lowest curve) and with exclusionary contracts in the most competitive equilibrium (the highest curve) and the most cooperative one (the intermediate curve).

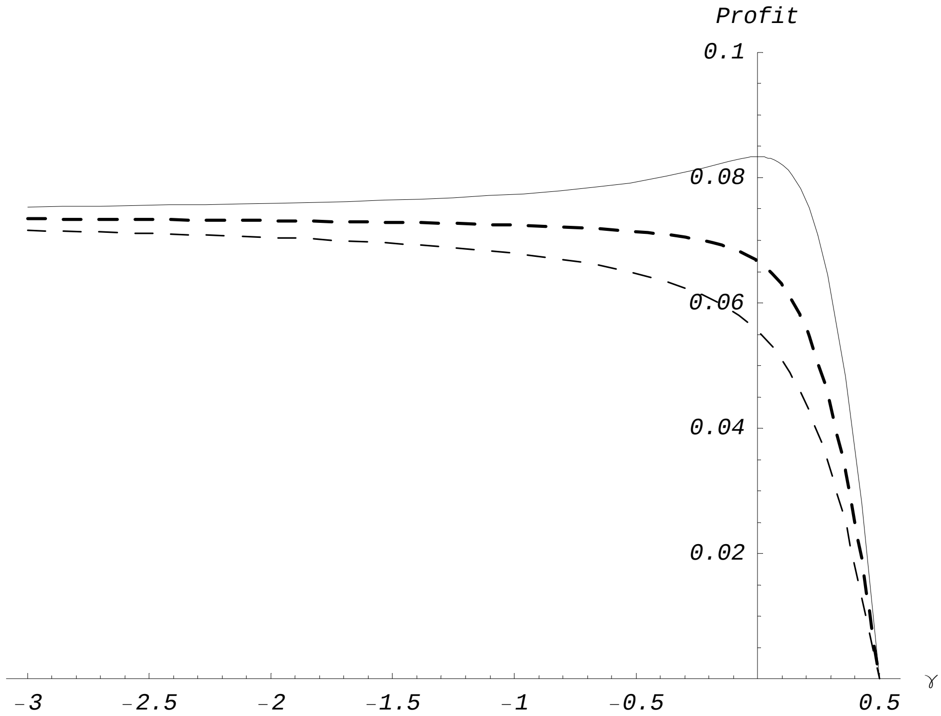

Figure 6: Firms' profits under private common agency (continuous line) and with exclusionary contracts in the most competitive equilibrium (the thin dashed line) and in the most cooperative one (the thick dashed line).

means that when exclusionary contracts are permitted, firms are trapped in a prisoners' dilemma. Both would gain by entering a binding agreement not to offer exclusive dealing arrangements, but if such an agreement is not binding, each has a unilateral incentive to offer exclusionary contracts (Proposition 2).

We can summarize the above discussion as follows.

Proposition 4 *Exclusionary contracts increase consumer surplus and social welfare and decrease profits.*

In other words, the analysis confirms that exclusionary contracts are pro-competitive for any possible degree of tacit coordination among the firms. Obviously, the more limited is such coordination (i.e., the lower is μ), the more pro-competitive are exclusionary contracts.

5 Market-share discounts

Now we turn to the case where not only exclusionary contracts but also market-share discounts are allowed. In this case, each firm i can request a payment $p_i(q_i, q_j)$ that depends not only on its own output, q_i, but also on its rival's, q_j. In the jargon of the common agency literature, this is known as the public common agency case.

As usual, we start by guessing a specific functional form of the price schedules. At first, it would seem natural to posit quadratic price schedules of the type

$$p_i(q_i, q_j) = \alpha_{0,i} + \alpha_{1,i}q_i + \alpha_{2,i}q_i^2 + \alpha_{3,i}q_j + \alpha_{4,i}q_j^2, \tag{15}$$

and it might seem redundant to allow firms to submit a separate exclusionary price schedule, since the schedules (15) already include exclusionary contracts for $q_j = 0$. Notice, however, that the specification (15) does not satisfy the condition that no payment can be due to a firm if nothing is purchased from it, i.e., $p_i(0, q_j) = 0$. Thus, the true schedules $p_i(q_i, q_j)$ must be discontinuous at $q_i = 0$. But this discontinuity may entail another discontinuity, this time at $q_j = 0$. For if $p_j(q_j, q_i)$ is discontinuous at $q_j = 0$, one must allow firm i to respond discontinuously at $q_j = 0$. Thus, after all, we must posit that each firm offers a separate exclusionary price schedule $p_i^E(q_i)$, just as in the previous section.

As we know, given any pair of exclusionary price schedules $p_i^E(q_i)$, the restricted game in which firms must choose only non exclusionary contracts can be viewed as an intrinsic common agency game with type dependent reservation utility. Again, this suggests that equilibrium non exclusionary price schedules may have two parts, one intended for consumers who will obtain exactly their reservation utility $U^E(\theta)$, and one for those who will obtain strictly more.

Another property of the semi-public common agency game that carries over to the public common agency case is the multiplicity of equilibria, which is due to the fact that firms must make offers that are destined not to be accepted by any consumer. Since now the set of permitted contracts is even broader than under semi-public common agency, the scope for multiple equilibria is even wider. However, it turns out that varying the exclusionary price schedules $p_i^E(q_i)$ suffices to generate the entire set of equilibria. Hence, the upper bound on the maximum payment that can be requested for exclusionary contracts is again given by (11).

Thus, suppose that both firms offer exclusionary schedules

$$p_i^E(q_i) = \mu q_i^2 \text{ for all } q_i \geq 0$$

with $\mu \in \left[0, \frac{1-2\gamma}{4}\right]$. The non exclusionary price schedules now can depend both on own output and the rival's output. Assuming a piecewise quadratic specification, now we conjecture:

$$p_i(q_i, q_j) = \begin{cases} \underline{\alpha}_{0,i} + \underline{\alpha}_{1,i}q_i + \underline{\alpha}_{2,i}q_i^2 + \underline{\alpha}_{3,i}q_j + \underline{\alpha}_{4,i}q_j^2 . (\equiv \underline{p}_i^C(q_i, q_j)) & \text{for } 0 \leq q_i \leq \hat{q}_i \\ \bar{\alpha}_{0,i} + \bar{\alpha}_{1,i}q_i + \bar{\alpha}_{2,i}q_i^2 + \bar{\alpha}_{3,i}q_j + \bar{\alpha}_{4,i}q_j^2 . (\equiv \bar{p}_i^C(q_i, q_j)) & \text{for } \hat{q}_i < q_i \leq 1 \end{cases} \tag{16}$$

In the restricted game where firms choose only schedules of type (16), for each firm there are now eleven coefficients to be determined: the $\underline{\alpha}_{s,i}$'s, the $\bar{\alpha}_{s,i}$'s, and $\hat{q}_i$.[25]

As in the semi-public common agency case, the $\underline{\alpha}_{s,i}$'s are determined by the requirement that the lower part of the schedules must re-produce the reservation utility $U^E(\theta, \mu)$, since firms can gain by inducing the consumer to purchase both products. This requires

$$\underline{\alpha}_{0,A}^{***} = \underline{\alpha}_{0,B}^{***} = \underline{\alpha}_{1,A}^{***} = \underline{\alpha}_{1,B}^{***} = \underline{\alpha}_{3,A}^{***} = \underline{\alpha}_{3,B}^{***} = 0$$

and

$$\left(\underline{\alpha}_{2,A} + \underline{\alpha}_{4,B}\right) \times \left(\underline{\alpha}_{2,B} + \underline{\alpha}_{4,A}\right) = \left(\frac{1}{2} - \gamma + 2\mu\right)^2. \tag{17}$$

Now we have several degrees of freedom in the choice of the coefficients that appear in equation (17). This is partly due to the fact that a game of private provision of an indivisible public good (i.e., common participation) may have multiple equilibria in which firms contribute asymmetrically, as discussed above.

Imposing symmetry (i.e. $\underline{\alpha}_{2,A} = \underline{\alpha}_{2,B}$ and $\underline{\alpha}_{4,A} = \underline{\alpha}_{4,B}$) no longer suffices to select a unique equilibrium, but makes the remaining indeterminacy payoff irrelevant. To see why, notice that under symmetry the cost of inducing common participation is divided evenly among the firms; but now, within the limits of the total payment it can request, each firm can choose to charge a positive price for its own output or for its rival's. To fix ideas, we assume that firms charge equally for both goods, and thus we set $\underline{\alpha}_{2,i} = \underline{\alpha}_{4,i}$. With this convention, we obtain a unique solution

$$\underline{\alpha}_{2,A}^{***} = \underline{\alpha}_{2,B}^{***} = \underline{\alpha}_{4,A}^{***} = \underline{\alpha}_{4,B}^{***} = \left(\frac{1}{4} - \frac{\gamma}{2} + \mu\right).$$

Thus, in our candidate equilibria the lower part of the non exclusionary price schedules is

$$\underline{p}_i(q_i, q_j) = \left(\frac{1}{4} - \frac{\gamma}{2} + \mu\right) q_i^2 + \left(\frac{1}{4} - \frac{\gamma}{2} + \mu\right) q_j^2.$$

Next, consider the upper part of the schedules, $\bar{p}_i^C(q_i, q_j)$. These are intended for consumers whose participation constraint does not bind. It is therefore natural to conjecture that these parts of the non exclusionary schedules must coincide (with the possible exception of the fixed fees) with the equilibrium schedules in an hypothetical game where the consumer's reservation utility is zero. Intuitively, in both the true and the hypothetical game the equilibrium quantities can be determined starting from $\theta = 1$ and then proceeding backwards, so the equilibrium quantities must coincide as long as the participation constraint does not bind.

Consider then the hypothetical game in which each firm i chooses the coefficients of a quadratic price schedule

$$p_i(q_i, q_j) = \alpha_{0,i} + \alpha_{1,i} q_i + \alpha_{2,i} q_i^2 + \alpha_{3,i} q_j + \alpha_{4,i} q_j^2 \qquad \text{for all } 0 \leq q_i, q_j \leq 1$$

[25] We have also analyzed the consequences of adding a further term $\alpha_{5,i} q_i q_j$ in both parts of the schedules, but this term turns out to be irrelevant, so the coefficients $\alpha_{5,i}$ can be set equal to zero with no loss of generality.

so as to maximize its expected profits, and the consumer's reservation utility is zero. In searching for the equilibrium of this game, we can exploit the no-distortion-at-the-top property to set

$$\alpha_{2,i} = -\frac{\alpha_{1,i}}{2} \text{ and } \alpha_{4,i} = -\frac{\alpha_{3,i}}{2}.$$

In addition, we know that no fixed fee is charged when the market is uncovered, so we set $\alpha_{0,i} = 0$. As a consequence, now each firm must choose only two coefficients, namely $\alpha_{1,i}$ and $\alpha_{3,i}$.

Appendix 5.1 shows that this restricted game has a unique equilibrium, which is symmetric and is given by

$$\alpha_1^+ = \alpha_3^+ = \frac{1}{3}.$$

The corresponding quantities are

$$q^+(\theta) = 3\theta - 2.$$

This solution has two remarkable properties. First, for any value of γ the equilibrium quantities are the same as in a private common agency game where the goods are perfect complements (i.e., the limiting case $\gamma = -\infty$). Second, the direct coefficients (resp., α_1^+ and α_2^+) coincide with the cross coefficients (resp., α_3^+ and α_4^+).

These two properties are closely related. The fact that each firm can charge the consumer both for consuming its own output and its rival's creates an externality, which is exactly similar to that arising when the goods are perfect complements. In both cases, the total price faced by the final consumer is the sum of the prices charged by two separate firms, each of which acts non-cooperatively and thus internalizes only partially the negative consequences of an increase in its own prices.

In particular, starting from the private common agency equilibrium where it cannot charge for good j, firm i always has an incentive to charge a positive price for good j, since this affects negatively only the revenue accruing to firm j. In fact, firm i has an incentive to increase the marginal price it charges on good j precisely as long as firm j has an incentive to increase its own price, so in equilibrium both firms must charge the same marginal price for each good. But this implies that in equilibrium each firm charges equally for both own output and its rival's output.[26]

To proceed with our guess, the upper part of the non exclusionary price schedules must result in the same equilibrium quantities as the hypothetical game, i.e., $q^+(\theta) = 3\theta - 2$. This requires

$$\begin{aligned} \bar{\alpha}_1^{***} &= \bar{\alpha}_3^{***} = \frac{1}{3} \\ \bar{\alpha}_2^{***} &= \bar{\alpha}_4^{***} = -\frac{1}{6}. \end{aligned}$$

To complete the calculation of the candidate equilibrium, we must determine the $\bar{\alpha}_{0,i}$'s and the $\hat{q}_i$'s. As in section 4, we require that the non exclusionary price schedules $p_i(q_i, q_j)$ be continuous

[26] With positive marginal production costs, firms would equalize the price-cost margins. Thus, each firm would charge a greater marginal price on own output (which alone entails a positive marginal cost) than on its rival's.

and continuously differentiable in q_i.[27] Continuity requires

$$\underline{p}_i(\hat{q}_i, \hat{q}_j) = \bar{p}_i(\hat{q}_i, \hat{q}_j). \tag{18}$$

On the other hand, equating the right and left derivative of $p_i(q_i, q_j)$ at $q_i = \hat{q}_i$ we get

$$\left(\frac{1}{2} - \gamma + 2\mu\right)\hat{q}_i = \frac{1}{3} - \frac{1}{3}\hat{q}_i. \tag{19}$$

The solution to the system (18)-(19) is

$$\bar{\alpha}_0^{***} = -\frac{2}{3(5 - 6\gamma + 12\mu)}$$

and

$$\hat{q}^{***} = \frac{2}{(5 - 6\gamma + 12\mu)}.$$

This completes the calculation of the candidate equilibrium. As usual, the proof that the candidate equilibrium satisfies the best response property is relegated to the Appendix.

Proposition 5 *The following are equilibria of the semi-public common agency game. For any*

$$0 \le \mu \le \frac{1 - 2\gamma}{4},$$

firm i offers an exclusionary contract

$$p_i^{E***}(q_i) = \mu q_i^2 \text{ for all } q_i \ge 0$$

and a non-exclusionary price schedule

$$p_i^{C***}(q_i, q_j) = \begin{cases} \left(\frac{1}{4} - \frac{\gamma}{2} + \mu\right) q_i^2 + \left(\frac{1}{4} - \frac{\gamma}{2} + \mu\right) q_j^2 & \text{for } 0 \le q_i \le \hat{q}^{***} \\ -\dfrac{2}{3(5 - 6\gamma + 12\mu)} + \frac{1}{3}q_i - \frac{1}{6}q_i^2 + \frac{1}{3}q_j - \frac{1}{6}q_j^2 & \text{for } \hat{q}^{***} \le q_i \le 1. \end{cases}$$

Figure 7 compares the public common agency equilibrium price schedules with those obtained under private and semi-public common agency. To make the comparison possible, Figure 7 depicts the projection of the schedules $p_i^C(q_i, q_j)$ onto the plane $q_j = q_i$; thus, the curves in Figure 7 represent the total payment due in equilibrium to firm i for purchasing the same amount of both products.

[27] Again, these properties can be derived from explicit analysis of a restricted game in which firms choose only the $\bar{\alpha}_{0,i}$'s and the $\hat{q}_i$'s, as in Appendix 4.2.

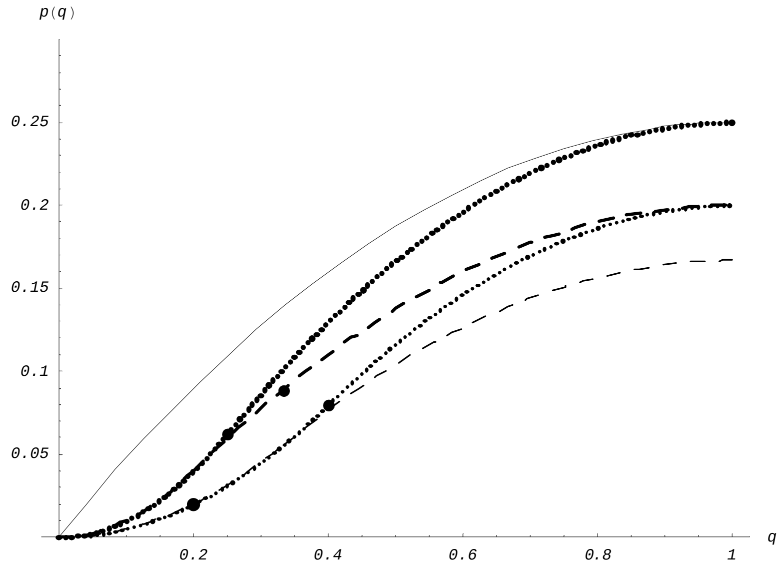

Figure 7: Equilibrium price schedules under private common agency (the continuos line), with exclusionary contracts (the dashed lines) and with market share discounts (the dotted lines). In the last two cases, thin and thik lines represent the most competitive and the most cooperative equilibria, respectively.

In equilibrium, the market is covered and all consumers purchase in common agency. The equilibrium quantities are

$$q_i^{***}(\theta) = \begin{cases} \dfrac{\theta}{2(1-\gamma+2\mu)} & \text{for } 0 \leq \theta \leq \hat{\theta}^{***} \\ 3\theta - 2 & \text{for } \hat{\theta}^{***} \leq \theta \leq 1, \end{cases} \tag{20}$$

where the critical threshold $\hat{\theta}$ now is

$$\hat{\theta}^{***} = \frac{4(1-\gamma+2\mu)}{(5-6\gamma+12\mu)}$$

They are depicted in Figure 8 for the special case $\gamma = 0$. Equilibrium quantities are lower than in the semi-public common agency case (strictly so when $\theta > \hat{\theta}^{**}$), but the comparison with the private common agency case in ambiguous: equilibrium quantities increase for low-type consumers (i.e., consumers θ such that $\theta < \hat{\theta}^{***}$), but decrease for high-type consumers.

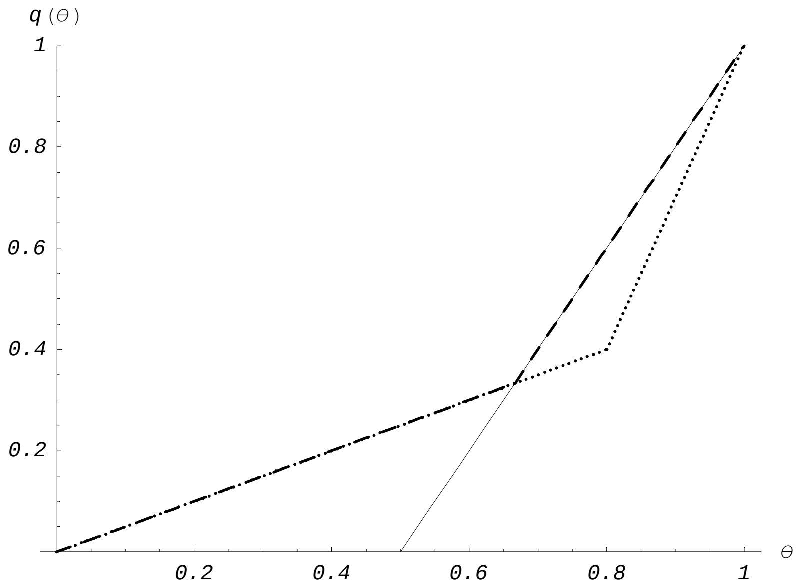

Figure 8: Equilibrium quantity under private common agency (continuous line), and those of the most competitive equilibrium with exclusionary contracts (dashed lines) and market share discounts (dotted lines).

The net surplus obtained by consumer θ is

$$U^{***}(\theta) = \begin{cases} \dfrac{\theta^2}{2\,(1-\gamma+2\mu)} & \text{for } 0 \le \theta \le \hat{\theta}^{***} \\ \dfrac{(3\theta-2)^2}{3} + \dfrac{4}{3(5-6\gamma+12\mu)} & \text{for } \hat{\theta}^{***} \le \theta \le 1. \end{cases} \tag{21}$$

Now we can compare the public common agency equilibrium to the equilibrium with exclusionary contracts found in the previous section. Contrasting (13) to (20) and (14) to (21), and noting that for any given μ, $\hat{\theta}^{***} > \hat{\theta}^{**}$, one immediately obtains:

Proposition 6 *For any given degree of "cooperation" μ, if exclusionary contracts are permitted, allowing firms to offer also market-share discounts reduces consumption, consumer surplus, and social welfare.*

The comparison of the public and private common agency equilibria is more ambiguous. Under public common agency, the consumption of low-type consumers is less distorted than under private common agency, but the opposite is true for high-type consumers. However, under public common agency high-type consumers now obtain a negative fixed fee, which can at least partly offset the negative effect of reduced consumption. Nevertheless, when the goods are substitutes, in the most cooperative public common agency equilibrium some high-type consumers are worse off than in

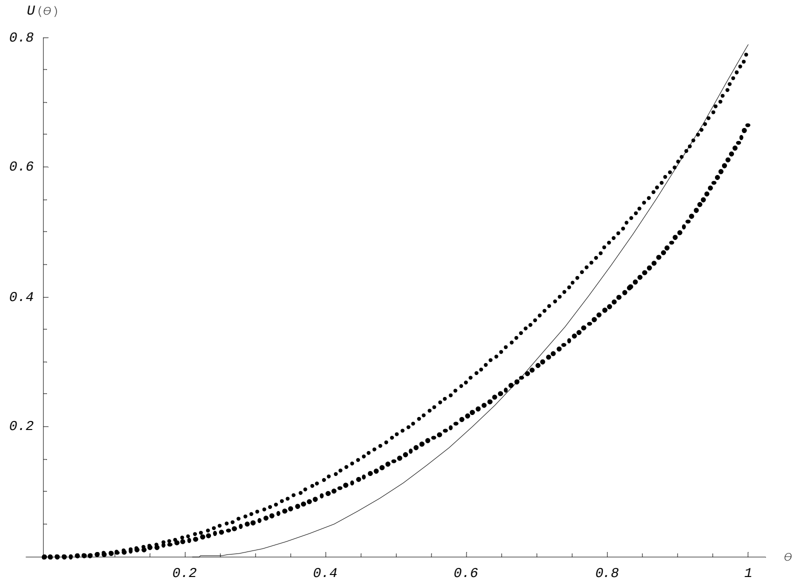

Figure 9: Equilibrium utilities with $\gamma = \frac{1}{3}$ under private common agency (continuous line) and market share discounts (thin dotted line for the most competitive equilibrium and the think dotted line for the most cooperative one).

the private common agency equilibrium. Focusing on the most competitive public common agency equilibrium makes the comparison more favorable for market-share discounts, but still some high-type consumers necessarily lose when the goods are sufficiently close substitutes (to be precise, the condition is $\gamma > \frac{1}{5}$), as shown in the following figure.

Continuing to focus on the most competitive equilibrium, Appendix 5.3 shows that on average, allowing for market-share discounts (starting from a situation where exclusionary contracts are prohibited) increases consumer surplus and welfare and reduces profits.

Proposition 7 *When $\mu = 0$, expected consumer surplus and social welfare are greater under public common agency than under private common agency, but expected profits are lower.*

When $\mu > 0$, however, even the *ex ante* comparison becomes ambiguous: a move from private to public common agency is pro-competitive if the goods are not too close substitutes, but now can be anti-competitive when γ is sufficiently close to $\frac{1}{2}$. The intuition is that when the degree of product substitutability increases, $\hat{\theta}^{***}$ goes to zero, so a move from private to public common agency decreases the equilibrium quantities for almost all consumers.

6 Conclusions

We have studied competition in non-linear prices for horizontally differentiated products when firms may offer exclusionary contracts and market-share discounts to consumers who are privately informed on their demand. If exclusionary contracts are allowed, firms have an incentive to offer them although they intensify competition, reducing profits (a prisoner's dilemma) and increasing consumers' surplus and welfare. The reason why exclusionary contracts are pro-competitive is that they force firms to compete in utility space where their products are effectively homogeneous, irrespective of the degree of product differentiation. Firms can still induce consumers to purchase both products, but to do so they must reduce their non exclusionary prices, since now exclusionary contracts provide an attractive outside option. If firms are allowed to offer also market-share discounts, however, competition is weakened, since market-share discounts create a double-marginalization effect that makes the products perfectly complementary, again irrespective of the actual degree of product differentiation. However, a move from a situation where both exclusionary contracts and market-share discounts are prohibited to one where they are both allowed has ambiguous effects on consumer surplus and social welfare. Typically, low-type consumers benefit from the exclusionary contracts, but high-type consumers are harmed by the double-marginalization effect created by market-share discounts.

To the best of our knowledge, this is the first paper that analyzes the effects of exclusionary contracts and market share discounts in a model of oligopoly where firms cannot price discriminate perfectly without making *ad hoc* assumptions on the type of contracts firms can offer. Thus, our results have an obvious policy relevance. Although some are clear-cut and others are more ambiguous, the collection taken together suggests that a *per se* illegality rule, such as that adopted by competition authorities and the courts in Europe, is unwarranted. Within the limits of our model, a *per se* legality rule would seem more appropriate.

However, our analysis has focused only on the case of symmetric firms. The current policy debate suggests that exclusionary contracts and market-share discounts may become more dangerous when firms are asymmetric. For a dominant firm might use these contracts to foreclose a rival that is not equally efficient, and yet should stay in the market since it supplies a differentiated product for which there is consumers' demand. We plan to extend our analysis to the case of asymmetric firms in a follow-on paper.

Other extensions are also awaiting for future work. One is to the case where the parameter θ is distributed over a support $[\theta_{\min}, 1]$ with $\theta_{\min} > 0$, so that the market can be covered also under private common agency. In the limiting case $\theta_{\min} \to 1$, one then re-obtains a complete information model, where we know from Bernheim and Whinston (1998) that exclusionary contracts and market-share discounts are competition neutral. Preliminary investigation of this case suggest that with exclusionary contracts, the neutrality results may still hold provided that $\theta_{\min}$ is sufficiently close to 1, whereas with market-share discounts the neutrality result is re-obtained only in the limiting case of complete information.

Another obvious extension is to consider more general utility and distribution functions. The uniform-quadratic specification we have used is analogous, for instance, to positing a linear demand in a standard oligopoly model. Thus, our qualitative results shoukd hold under much more general specifications.

Finally, it would be interesting to extend the analysis to the case of three or more firms. The analysis in Chiesa and Denicolò (2009) suggests that in this case the scope for tacit coordination among firms may increase considerably, so the effects of exclusionary contracts and market-share discounts may become be less pro-competitive.

Appendices

Appendix 3.1

The candidate equilibrium under private common agency

Let us consider a restricted game where firms are constrained to submit price schedules of the type

$$p_i(q_i) = \alpha_{1,i} q_i - \frac{\alpha_{1,i}}{2} q_i^2 \quad \text{for } q_i \in [0,1].$$

A strategy for firm i then becomes simply a value of $\alpha_{1,i}$. Facing these price schedules, consumer θ maximizes

$$u(q_A, q_B, \theta) - \alpha_{1,A} q_A - \frac{\alpha_{1,A}}{2} q_A^2 - \alpha_{1,B} q_B - \frac{\alpha_{1,B}}{2} q_B^2.$$

The consumer's optimal choice depends on whether $\alpha_{1,A}$ is greater or lower than $\alpha_{1,B}$. Hence, firm A's best response function has two branches, according to whether $\alpha_{1,A}$ is greater or lower than $\alpha_{1,B}$. Since the branching point is $\alpha_{1,A} = \alpha_{1,B}$ and the candidate equilibrium is symmetric, either branch can be used to calculate the candidate equilibrium, but the calculations are simpler if one assumes $\alpha_{1,A} \geq \alpha_{1,B}$. Focusing on this case we get:

$$\tilde{q}_A(\theta) = \max\left[0, \frac{(1-2\gamma-\alpha_{1,B})\theta - (1-\gamma-\alpha_{1,B})\alpha_{1,A} + \gamma\alpha_{1,B}}{(1-\gamma-\alpha_{1,B})(1-\gamma-\alpha_{1,A}) - \gamma^2}\right]$$

Firm A's profit then is

$$\begin{aligned}\pi_A &= \int_0^1 \left[\alpha_{1,A}\tilde{q}_A(\theta) - \frac{\alpha_{1,A}}{2}\tilde{q}_A^2(\theta)\right] d\theta \\ &= \frac{(1-2\gamma) - (1-\gamma)(\alpha_{1,A}+\alpha_{1,B}) + \alpha_{1,A}\alpha_{1,B}}{3(1-2\gamma-\alpha_{1,B})}\alpha_{1,A},\end{aligned}$$

and its best response function

$$\alpha_{1,A} = \frac{(1-2\gamma)-(1-\gamma)\alpha_{1,B}}{2(1-\gamma-\alpha_{1,B})} \qquad \text{for } \alpha_{1,A} \geq \alpha_{1,B}.$$

Imposing symmetry, $\alpha_{1,A} = \alpha_{1,B}$, we can solve to get

$$\alpha_1^* = \frac{1}{4}\left[3(1-\gamma) - \sqrt{1-2\gamma+9\gamma^2}\right].$$

It is easy to check that there are no asymmetric equilibria.

Appendix 3.2

Proof of Proposition 1

We must verify that the candidate equilibrium is, indeed, an equilibrium of the original game. That is, we must prove that if firm j offers a price schedule $p_j = \alpha_1^* q_j - \frac{\alpha_1^*}{2} q_j^2$, then firm i's best response in the set of all possible price schedules $\mathcal{S}_{pr}$ is indeed $p_i = \alpha_1^* q_i - \frac{\alpha_1^*}{2} q_i^2$.

Given $p_j = \alpha_1^* q_j - \frac{\alpha_1^*}{2} q_j^2$, firm i faces a standard monopolistic screening problem where the consumer's utility is given by the following indirect utility function

$$v_i^*(q_i, \theta) = \max_{q_j \geq 0} \left[u(q_i, q_j, \theta) - \left(\alpha_1^* q_j - \frac{\alpha_1^*}{2} q_j^2 \right) \right],$$

which is the maximum utility that a consumer of type θ can obtain by purchasing q_i and trading optimally with firm j. Assuming an interior solution,[28] this indirect utility function can be easily calculated as

$$v_i^*(q_i, \theta) = A_0 + A_1 q_i - A_2 q_i^2$$

where

$$\begin{aligned} A_0 &= \frac{(\theta - \alpha_1^*)^2}{2(1 - \gamma - \alpha_1^*)} \\ A_1 &= \frac{(1 - 2\gamma - \alpha_1^*)\theta + \gamma \alpha_1^*}{(1 - \gamma - \alpha_1^*)} \\ A_2 &= \frac{1 - \alpha_1^* - 2\gamma + \gamma \alpha_1^*}{2(1 - \gamma - \alpha_1^*)} \end{aligned}$$

Now, however, the consumer's reservation utility

$$v_i^*(0, \theta) = A_0$$

is type dependent. Thus, in order to apply the standard approach of pointwise maximization of the virtual surplus function (see e.g. Laffont and Martimort, 2002), we must check not only that the sorting condition is satisfied, but also that the equilibrium rent increases with θ more rapidly that the reservation utility, so that the consumer's participation constraint $v_i^*(q_i, \theta) \geq v_i^*(0, \theta)$ binds only at $\theta = \alpha_1^*$. The sorting condition is

$$\frac{\partial^2 v_i^*}{\partial \theta \partial q_i} = \frac{1 - 2\gamma - \alpha_1^*}{1 - \gamma - \alpha_1^*} > 0;$$

it can be checked that it always holds. The second condition (which in fact is always entailed by the first one, as shown by Calzolari and Scarpa, 2008) requires that

$$2 q_i^*(\theta) > \frac{\theta - \alpha_1^*}{1 - \gamma + \alpha_1^*},$$

[28] When $q_j = 0$, the indirect utility function reduces to $\theta q_i - \frac{1-\gamma}{2} q_i^2$.

since by the envelope theorem the derivative of the equilibrium rent equals the sum of the quantities purchased. This condition also reduces to $1 - 2\gamma - \alpha_1^* > 0$, and so it is always satisfied, too.

Thus, firm i's problem reduces to finding a function $q_i(\theta)$ that pointwise maximizes the "indirect virtual surplus"

$$\begin{aligned} s_i(q_i, \theta) &= v_i^*(q_i, \theta) - (1-\theta)\frac{dv_i^*}{d\theta} \\ &= v_i^*(q_i, \theta) - (1-\theta)\left[\frac{(\theta - \alpha_1^*)}{(1-\gamma-\alpha_1^*)} + \frac{(1-2\gamma-\alpha_1^*)}{(1-\gamma-\alpha_1^*)}q_i\right]. \end{aligned}$$

The first order condition for a maximum is

$$A_1 - 2A_2 q_i - (1-\theta)\frac{(1-2\gamma-\alpha_1^*)}{(1-\gamma-\alpha_1^*)} = 0$$

which implies

$$q_i(\theta) = \frac{2\theta(1-2\gamma-\alpha_1^*) - (1-2\gamma-\alpha_1^*-\gamma\alpha_1^*)}{1-\alpha_1^*-2\gamma+\gamma\alpha_1^*}.$$

Using the definition of α_1^* in (1), tedious algebra shows that $q_i(\theta) = q_i^*(\theta)$, so the optimal mechanism for firm i must induce the consumer to choose the equilibrium quantities. Clearly, the only price schedule that supports the equilibrium quantities is $p_i = \alpha_1^* q_i - \frac{\alpha_1^*}{2} q_i^2$. ■

Appendix 4.1

Proof of Proposition 2

To proof the proposition, it suffices to show that starting from the private common agency equilibrium, each firm can unilaterally increase its profits by offering exclusionary contracts. In other words, when exclusionary contracts are permitted, if firms offered non exclusionary price schedules that coincide with the private common agency equilibrium schedules, there would exist a profitable deviation.

To construct such a profitable deviation, we focus on a quantity forcing exclusionary contract that is targeted to a particular type $\tilde{\theta}$. For consumer $\tilde{\theta}$ alone to be induced to purchase exclusively from firm i, firm i must offer a quantity-forcing exclusionary schedule that consists of a single contract (p^E, q^E), where

$$q^E = 2q^*(\tilde{\theta}).$$

When firm i offers the contract (p^E, q^E), the net utility consumers may obtain by choosing to purchase exclusively from firm i is

$$2\theta q^*(\tilde{\theta}) - 2(1-\gamma)\left[q^*(\tilde{\theta}))\right]^2 - p^E.$$

Hence, net utility is linear in θ and has the same slope as $U^*(\theta)$ at $\theta = \tilde{\theta}$. It follows that by appropriate choice of the requested payment p^E, the two curves can be made tangent, implying that only type $\tilde{\theta}$ is induced to accept the exclusionary contract. To achieve this outcome, the requested payment p^E must be

$$p^E = 2\theta q^*(\tilde{\theta}) - 2(1-\gamma)\left[q^*(\tilde{\theta}))\right]^2 - U^*(\tilde{\theta}) - \varepsilon$$

where is arbitrarily small. Since the exclusionary contract (p^E, q^E) will be chosen only by type $\tilde{\theta}$, a necessary and sufficient condition for the deviation to be profitable is that the payment p^E exceeds the revenue firm i obtains from consumer $\tilde{\theta}$ in equilibrium, i.e.,

$$p^E > \alpha_1^* q^*(\tilde{\theta}) - \frac{\alpha_1^*}{2}\left[q^*(\tilde{\theta}))\right]^2$$

Now we show that this condition can always be met by suitable choice of $\tilde{\theta}$. The above inequality rewrites as

$$2\tilde{\theta} q^*(\tilde{\theta}) - 2(1-\gamma)\left[q^*(\tilde{\theta})\right]^2 - U^*(\tilde{\theta}) > \alpha_1^* q^*(\tilde{\theta}) - \frac{\alpha_1^*}{2}\left[q^*(\tilde{\theta})\right]^2$$

or, taking into account that $U^*(\tilde{\theta}) = (1-\alpha_1^*)\left[q^*(\tilde{\theta}))\right]^2$,

$$\left(2\tilde{\theta} - \alpha_1^*\right) - \left(1 - 2\gamma + \frac{\alpha_1^*}{2}\right) q^*(\tilde{\theta}) > 0.$$

Since $q^*(\tilde{\theta})$ converges to zero as $\tilde{\theta}$ goes to α_1^*, it is clear that as long as $\gamma < \frac{1}{2}$ and hence $\alpha_1^* > 0$, one can always find $\tilde{\theta} > 0$ close enough to α_1^* that the above inequality holds. ■

Appendix 4.2

Alternative calculation of the candidate equilibrium under semi-public common agency

Consider the restricted game in which firms can choose only the $\bar{\alpha}_{0,i}$'s and the $\hat{q}_i$'s, given all the other postulated properties of the price schedules. The candidate equilibrium must be an equilibrium of this restricted game. In such a game, firm i's profit is

$$\pi_i = \int_0^{\hat{\theta}} (\frac{1}{2} - \gamma)\left[\frac{q^E(\theta)}{2}\right]^2 d\theta + \int_{\hat{\theta}}^1 \left[\bar{\alpha}_{0,i} + \alpha_1^* q_i^*(\theta) - \frac{\alpha_1^*}{2}\left[q_i^*(\theta)\right]^2\right] d\theta,$$

where the first integral is the profit made by selling to consumers who choose a contract along the lower part of the price schedule (and hence purchase $\underline{q}_A(\theta) = \underline{q}_B(\theta) = \frac{q^E(\theta)}{2}$) and the second integral captures the profit on the upper part of the price schedule (where consumers purchase $\bar{q}_A(\theta) = \bar{q}_B(\theta) = q_i^*(\theta)$). The cutoff $\hat{\theta}$ is implicitly determined by the condition

$$U^E(\hat{\theta}) = U^*(\hat{\theta}) - (\bar{\alpha}_{0,A} + \bar{\alpha}_{0,B}), \tag{A4.1}$$

which states that for the critical type $\hat{\theta}$ it is indifferent to choose the optimal contract on the lower or the upper part of the common agency price schedule.

Now we argue that in any equilibrium of the restricted game, the $\hat{q}_i$'s must guarantee that the equilibrium payment is a continuous function of θ, i.e.,

$$\underline{p}_i^C(\hat{q}_i) = \bar{p}_i^C(\hat{q}_i).$$

Suppose to the contrary that this equality does not hold; to fix ideas, let $\underline{p}_i^C(\hat{q}_i) > \bar{p}_i^C(\hat{q}_i)$. Then firm i could slightly increase $\hat{q}_i$, inducing some consumers (i.e., those with θ just above $\hat{\theta}$) to switch to the lower part of the price schedule, where the per capita profit is larger.

The condition of continuity implicitly determines $\hat{q}_i$'s for any given fixed fee $\bar{\alpha}_{0,i}$. Next consider the first-order condition for a maximum with respect to the fixed fee $\bar{\alpha}_{0,i}$. This is implicitly given by

$$\frac{d\pi_i}{d\bar{\alpha}_{0,i}} = \left(1 - \hat{\theta}\right) + \frac{d\hat{\theta}}{d\bar{\alpha}_{0,i}} \left[\underline{p}_i^C(\hat{q}_i) - \bar{p}_i^C(\hat{q}_i)\right] = 0,$$

where by implicit differentiation of (A4.1)

$$\frac{d\hat{\theta}}{d\bar{\alpha}_{0,i}} = -\frac{1}{\left(\frac{dU^E}{d\theta} - \frac{dU^*}{d\theta}\right)|_{\theta=\hat{\theta}}}$$

Since the term inside square brackets vanishes, at equilibrium $\left|\frac{d\hat{\theta}}{d\bar{\alpha}_{0,i}}\right|$ must be infinitely large. This requires that $\frac{dU^E}{d\theta} = \frac{dU^*}{d\theta}$ at $\hat{\theta}$, so that $\underline{q}_i(\hat{\theta}) = \bar{q}_i(\hat{\theta})$. This in turn requires that

$$\bar{\alpha}_{0,A} + \bar{\alpha}_{0,B} = -\frac{\alpha_1^{*2}}{(1 - 2\gamma + \alpha_1^*)},$$

that is, the aggregate fixed fee must be negative and large enough to make the consumer's net utility under intrinsic common agency tangent to the reservation utility. Focusing again on a symmetric equilibrium, we finally get

$$\bar{\alpha}_0^{**} = -\frac{\alpha_1^{*2}}{2\,(1 - 2\gamma + \alpha_1^*)}.$$

Appendix 4.3

Proof of Proposition 3

Now we must verify that the candidate equilibria satisfy the best response property in the unrestricted strategy space $\mathcal{S}_{sp}$: provided firm j offers such schedules (p_j^{E**}, p_j^{C**}), it is indeed optimal for firm i to offer the same menu of schedules. The proof is in two steps. First, we show that given p_j^{E**}, p_j^{C**}, there is not a profitable deviation for firm i to a exclusionary price schedule different from $p_i^{E**}(q_i)$. Second, we show that p_j^{E**}, p_j^{C**}, there is not a profitable deviation for firm i to a common agency price schedule different from p_j^{C**}.

Step 1. The argument is simplest for the most competitive equilibrium where $\mu = 0$. If firm i offers $p_i^E(q_i) = 0$, firm j will make zero profits with any non negative exclusionary price schedule. Moreover, firm j's cannot affect the reservation utility that is implicitly provided to the consumers by $p_i^E(q_i) = 0$. That means that firm j is indifferent between any $p_j^E(q_j)$, and hence its best reply includes the schedule $p_j^E(q_j) = 0$.

When $\mu > 0$, the argument is more complex. Clearly, any deviation to a schedule $p_i^{E\text{dev}}(q_i) \geq p_i^{E**}(q_i)$ with strict inequality for some q_i is not profitable since the consumer can then purchase in exclusivity at price $p_j^{E**}(q_j) \leq p_i^{E**}(q_i)$. Profitable deviations may exist then only for exclusionary prices that are smaller than p_i^{E**}. However, we now illustrate that such deviations are not profitable either. If a profitable deviation $p_i^{E\text{dev}}(q_i)$ existed, then some consumers $\tilde{\theta}$ must prefer to purchase exclusively from firm i under $p_i^{E\text{dev}}(q_i)$ over purchasing from both firms under common agency or over purchasing exclusively from firm j under $p^{E\max}(q)$ (which by construction must give the same net utility), and firm i must make a profit out of this exclusive sale greater than $\underline{p}^C(q(\tilde{\theta}))$. Conversely, this means that for no profitable deviation to exist it must be not possible to convince any type $\tilde{\theta}$ to purchase exclusively from firm i and obtain a profit from type $\tilde{\theta}$ greater than $\underline{p}^C(q(\tilde{\theta}))$. Next observe that for type $\tilde{\theta}$ alone to be induced to purchase exclusively from firm i, firm i must offer a quantity-forcing exclusionary contract where the quantity is $q^{E\text{dev}} = 2\underline{q}(\tilde{\theta})$ and the corresponding requested payment will be determined shortly. With such a quantity forcing exclusionary contract, the function that gives the net utility which consumers may obtain by choosing to purchase exclusively from firm i, is linear in θ and has a slope equal to the slope of $U^{CA}(\theta)$ at $\theta = \tilde{\theta}$. It follows that by appropriate choice of the requested payment $\tilde{p}^E$, the two curves can be made tangent, meaning that only type $\tilde{\theta}$ accepts the exclusionary contract and that type $\tilde{\theta}$ is indifferent between accepting the deviation or the equilibrium payoff. For no profitable deviation to exist, no such quantity forcing profitable deviation must exist, with quantity $2\underline{q}(\tilde{\theta})$ and price $\tilde{p}^E < \mu(2\underline{q}(\tilde{\theta}))^2$ (recall we are considering deviations to lower exclusionary prices). However, constructing the upper-bound for exclusionary contracts $\frac{1-2\gamma}{4}q^2$, we noticed that the condition $p^E(q) \leq \frac{1-2\gamma}{4}q^2$ is equivalent to $\underline{p}^C(\underline{q}(\theta)) \geq p^E(2\underline{q}(\theta))$ and it is then $\underline{p}^C(\underline{q}(\theta)) \geq p^E(2\underline{q}(\theta)) > \tilde{p}^E$ which proves that the deviation is not profitable.

Step 2. The exclusionary schedule p_j^{E**} guarantees the agent a strictly positive reservation utility $U^E(\theta, \mu) = \frac{\theta^2}{2(1-\gamma+2\mu)}$ with associated quantity $q^E(\theta, \mu)$. We derive the indirect utility function $v_i(q_i, \theta)$ which is continuous since p_j^{C**} is continuous and continuously differentiable. Now, firm i faces a well-behaved monopolistic screening problem $\mathcal{P}_i$ with a consumer characterized by utility $v_i(q_i, \theta)$ and a type-dependent reservation utility $U^E(\theta, \mu)$. This problem can be addressed with the techniques in Jullien (2000) and we next show that $\mathcal{P}_i$ determines an optimal quantity $q_i(\theta)$ for firm i supported in $\mathcal{P}_i$ by a schedule $p_i(q_i) = p_i^{C**}(q_i)$, which is then a best response to

(p_j^{E**}, p_j^{C**}).

Let $q_j(q_i, \theta) = \max\{0, \frac{\theta - \alpha_1^* - q_i \gamma}{1-\gamma-\alpha_1^*}\}$ be the maximizer of the program defining $v_i(q_i, \theta)$, i.e.

$$v_i(q_i, \theta) \equiv \max\left\{u(q_i, 0, \theta), \max_{q_j} u(q_i, q_j, \theta) - p_j^{C**}(q_j)\right\}$$

We can equivalently write

$$v_i(q_i, \theta) = \begin{cases} u(q_i, q_j(q_i,\theta), \theta) - \overline{p}_j^{C**}(q_j(q_i,\theta)) = A_0 + A_1 q_i + A_2 q_i^2 & \text{if } \theta - q_i\gamma \in [\frac{\alpha_1^*(2-3\gamma+4\mu)}{(1-2\gamma+\alpha_1^*+4\mu)}, (1-\gamma)] \\ u(q_i, q_j(q_i,\theta), \theta) - \underline{p}_j^{C**}(q_j(q_i,\theta)) = B_0 + B_1 q_i + B_2 q_i^2 & \text{if } \theta - q_i\gamma \in [\alpha_1^*, \frac{\alpha_1^*(2-3\gamma+4\mu)}{(1-2\gamma+\alpha_1^*+4\mu)}] \\ u(q_i, 0, \theta) = \theta q_i - (1-\gamma)/2 \;\; q_i^2 = C_1 q_i + C_2 q_i^2 & \text{if } \theta - q_i\gamma \leq \alpha_1^* \end{cases}$$

where $B_0 \equiv \frac{\theta^2}{6-10\gamma}$, $B_1 \equiv \frac{\theta 6(1-2\gamma)}{6-10\gamma}$, $B_2 \equiv \frac{4\,(2-\gamma)\gamma-3}{6-10\gamma}$ and $C_1 = \theta$, $C_2 = -(1-\gamma)/2$ (A_0, A_1, A_2 are defined in the proof of Proposition 1). Notice that $v_i(q_i, \theta)$ is continuous and differentiable. In the following we will write $v_i(q_i, \theta)$ with the following compact notation $v_i(q_i, \theta) = K_0 + K_1 q_i + K_2 q_i^2$, for $K_i \in \{A_i, B_i, C_i\}$.

We can now define the (single-principal-agent) problem $\mathcal{P}_i$ between firm i and a consumer with an utility $v_i(q_i, \theta)$ and reservation utility $U^E(\theta, \mu)$. Let $U(\theta)$ be the payoff of the consumer in the solution for $\mathcal{P}_i$ and $q_i(\theta)$ the associated quantity. As in Jullien (2000) we can express the virtual surplus of program $\mathcal{P}_i$ using the hazard rate $\frac{F(\theta)-g}{f(\theta)}$ that contemplates the weight $g \in [0,1]$ to account for the binding participation constraint:

$$v_i(q_i, \theta) + \frac{F(\theta) - g}{f(\theta)} \frac{dv_i(q_i, \theta)}{d\theta} = K_0 + K_1 q_i + K_2 q_i^2 - (\theta - g) \frac{\partial K_1}{\partial \theta} q_i.$$

The maximizer of the virtual surplus is

$$q_i(g, \theta) = \frac{(g - \theta) \frac{\partial K_1}{\partial \theta} - K_1}{2K_2}$$

with $\frac{dq_i(g,\theta)}{d\theta} = -\frac{\partial K_1}{\partial \theta} \frac{1}{K_2}$ for some $K_i \in \{A_i, B_i, C_i\}$. We can now apply Theorem 1 and Proposition 3 in Jullien (2001). To this end we need to show that program $\mathcal{P}_i$ satisfies properties Potential Separation (PS), Homogeneity (H) and Full Participation (FP). Property PS is verified if (i)

$$\frac{\partial}{\partial \theta} \left(\frac{\partial v_i(q_i, \theta)/\partial q_i}{\partial^2 v_i(q_i, \theta)/\partial q_i, \partial \theta} \right) \geq 0$$

and (ii) the hazard rate and the inverse hazard rate are respectively non-increasing and non-decreasing. These conditions are clearly verified in our environment. Property H simply requires that it exists a quantity $\hat{q}(\theta)$ such that the pair $(\hat{q}(\theta), U^E(\theta, \mu))$ is implementable, i.e. in our environment $U^E(\theta, \mu) = \partial u(\hat{q}(\theta), 0, \theta)/\partial \theta$ and $\hat{q}(\theta)$ is non-decreasing. Notice that since $U^E(\theta, \mu)$ is here determined by (out-of-equilibrium) contracting between the consumer and any of the firms in exclusivity for any p_j^E, the property is certainly verified (i.e., Property H is verified for any p_j^E

and not only for p_j^{E**}). Finally, Property FP requires that all types are induced to participate in the program $\mathcal{P}_i$. This is clearly the case since, as explained in the text, it is not possible that in equilibrium some types consume in exclusivity.

Hence, Proposition 3 in Jullien (2001) allows to state that if the program $\mathcal{P}_i$ is weakly convex, i.e. formally if

$$\frac{d^2U^E(\theta,\mu)}{d\theta^2}=\frac{dq^E(\theta,\mu)}{d\theta}=\frac{1}{1-\gamma+2\mu}\leq\frac{d^2U(\theta)}{d\theta^2}=\frac{dq_i(g,\theta)}{d\theta}=-\frac{\partial K_1}{\partial\theta}\frac{1}{K_2}$$

then the solution to $\mathcal{P}_i$ is such that for types in some set $[\theta_1,\theta_2]$ it must be $U(\theta)=U^E(\theta,\mu)$, whereas for all other types $U(\theta)>U^E(\theta,\mu)$, where θ_1 is implicitly defined by $q^E(\theta_1,\mu)=q_i(0,\theta_1)$ and θ_2 is implicitly defined by $q^E(\theta_2,\mu)=q_i(0,\theta_2)$, i.e.

$$\begin{aligned}\frac{\theta_1}{1-\gamma+2\mu} &= -\frac{\theta_1\frac{\partial K_1}{\partial\theta}+K_1}{2K_2},\\ \frac{\theta_2}{1-\gamma+2\mu} &= \frac{(1-\theta_2)\frac{\partial K_1}{\partial\theta}-K_1}{2K_2}\end{aligned}$$

for some $K_1\in\{A_1,B_1,C_1\}$.

Simple calculations show that (i) for any $K_i\in\{A_i,B_i,C_i\}$, then $\frac{d^2U^E(\theta,\mu)}{d\theta^2}\leq\frac{d^2U(\theta)}{d\theta^2}$ (equivalent to $\frac{1}{1-\gamma+2\mu}\leq-\frac{\partial K_1}{\partial\theta}\frac{1}{K_2}$) is always satisfied; (ii) condition $q^E(\theta_1,\mu)=q_i(0,\theta_1)$ implies $\theta_1<0$; (iii) condition $q^E(\theta_2,\mu)=q_i(0,\theta_2)$ finally implies $\theta_2=\hat{\theta}^{**}=\frac{2\alpha_1^*(1-\gamma+2\mu)}{1-2\gamma+\alpha_1^*+4\mu}$.■

Appendix 5.1

The candidate equilibrium under public common agency when the participation constraint does not bind

Consider the hypothetical game in which each firm i offers a quadratic price schedule

$$p_i(q_i,q_j)=\alpha_{1,i}q_i-\frac{\alpha_{1,i}}{2}q_i^2+\alpha_{3,i}q_j-\frac{\alpha_{3,i}}{2}q_j^2\qquad\text{for all }0\leq q_i,q_j\leq 1.$$

Given these schedules and assuming an interior solution, the consumer's optimal choice is

$$\check{q}_i(\theta)=\frac{(\theta-\alpha_{1,i}-\alpha_{3,j})(1-\alpha_{1,j}-\alpha_{3,i}-\gamma)-\gamma(\theta-\alpha_{1,j}-\alpha_{3,i})}{(1-\alpha_{1,i}-\alpha_{3,j}-\gamma)(1-\alpha_{1,j}-\alpha_{3,i}-\gamma)-\gamma^2}.$$

Hence, firm i's expected profit is

$$\pi_i=\int_{\check{\theta}}^{1}\left[\alpha_{1,i}\check{q}_i(\theta)-\frac{\alpha_{1,i}^*}{2}\left[\check{q}_i(\theta)\right]^2+\alpha_{3,i}\check{q}_j(\theta)-\frac{\alpha_{3,i}^*}{2}\left[\check{q}_j\right]^2\right]d\theta,$$

where $\check{\theta}$ is implicitly defined by the condition $\check{q}_i(\check{\theta}) = 0$.

The first order conditions are

$$\frac{\partial \pi_i}{\partial \alpha_{1,i}} = 0$$
$$\frac{\partial \pi_i}{\partial \alpha_{3,i}} = 0.$$

After imposing symmetry, i.e., $\alpha_{1,A} = \alpha_{1,B}$ and $\alpha_{3,A} = \alpha_{3,B}$, they reduce to

$$\gamma(2 - 3\alpha_1 - 3\alpha_3) - (1 - \alpha_1 - \alpha_3)(1 - 2\alpha_1 - \alpha_3) = 0$$
$$\gamma(2 - 3\alpha_1 - 3\alpha_3) - (1 - \alpha_1 - \alpha_3)(1 - \alpha_1 - 2\alpha_3) = 0.$$

The unique solution to this system is $\alpha_1 = \alpha_3 = \frac{1}{3}$.

Appendix 5.2

Proof of Proposition 5

We have to verify that the candidate equilibria satisfy the best response property in the unrestricted strategy space $\mathcal{S}_p$: provided firm j offers such schedules (p_j^{E***}, p_j^{C***}), it is indeed optimal for firm i to offer the same menu of schedules. The proof parallels that of Proposition 3 and is in two steps. As a first step we show that given p_j^{E***}, p_j^{C***}, there is not a profitable deviation for firm i to a exclusionary price schedule different from $p_i^{E***}(q_i)$. The proof of this step is identical to step 1 in the proof of Proposition 3 and is then omitted.

Second, we show that p_j^{E***}, p_j^{C***}, there is not a profitable deviation for firm i to a common agency price schedule different from p_j^{C***}. The argument is again very similar to the one in step 2 of the proof of Proposition 3. The main difference is that now the indirect utility in program $\mathcal{P}_i$ is

$$v_i(q_i, q_j, \theta) = \begin{cases} u(q_i, q_j, \theta) - \overline{p}_j^{C***}(q_j, q_i) & \text{if } \hat{q}_j^{***} \leq q_j \leq 1 \\ u(q_i, q_j, \theta) - \underline{p}_j^{C***}(q_j, q_i) & \text{if } 0 \leq q_j \leq \hat{q}_j^{***} \end{cases}$$

with $\hat{q}_i^{***} = \frac{2}{(5-6\gamma+12\mu)}$. Furthermore, now program $\mathcal{P}_i$ is a bilateral screening program between the multi-product firm i and the consumer with a reservation utility $U^E(\theta, \mu) = \frac{\theta^2}{2(1-\gamma+2\mu)}$ and associated quantity $q^E(\theta, \mu)$.

Proceeding as in the proof of proposition 3, one verifies that conditions PS, H, FP of Jullien (2001) are satisfied and that the program is weakly convex. Hence, the same arguments follow. [To expand.]

Appendix 5.3

Proof of Proposition 7

When $\mu = 0$, we calculate

$$
\begin{aligned}
E[\pi_i^*] &= \frac{(1-\alpha_1^*)\alpha_1^*}{3} \\
E[\pi_i^{***}] &= \frac{4(1-\gamma)(1-2\gamma)}{3(5-6\gamma)^2} \\
E\left[U^*(\theta)\right] &= \frac{(1-\alpha_1^*)^2}{3} \\
E\left[U^{***}(\theta)\right] &= \frac{13-4\gamma(5-\gamma)}{3(5-6\gamma)^2}
\end{aligned}
$$

Expected social welfare is $E[\pi_i] + E\left[U(\theta)\right]$. Using the definition of α_1^* (equation (1)), the Proposition follows immediately. ■

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