Procurement and Debt[1]

Malin Arve[2]

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Abstract: This paper studies dynamic procurement design and the effect on this design of fundamental differences in the way firms fund their market participation. In a two-period model, at each period a procurement contract splits the production of a good between two firms and both sole and dual sourcing are a priori allowed. In this paper, firms differ in their ability to self-finance their presence in the market. I study the optimal financial contract for the firm in need of funding and the optimal procurement contract in a setting with both a self-financed and a cash-constrained firm. The main result is that it is optimal for the procurement agency to take into account the financial structure of the competing firms when choosing production quantities. However, what firm to favor is ambiguous and the different effects are identified in this paper.

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1 Introduction

The assumption that is often made in dynamic contract theory is that there is a fixed number of agents around to bid for all these contracts. This paper relaxes that assumption and studies the effect of a non-constant number of firms on optimal dynamic procurement. Ex ante, it is not sure that all firms existing on the market today will be around tomorrow. Some firms simply risk going bankrupt or equivalently risk being forced to leave the market by their investors.

The focus of this paper is on procurement and more specifically on public procurement. In fact public procurement is an important part of most countries' economic activity. In 2002 the value of public procurement was estimated to be about 16% of GDP in the EU[1] and around 20% in the United States[2]. Consider the case of procurement by the public health authorities. With regular intervals, the public health authorities need to provide new services or new facilities for the promotion or provision of public health. In many tenders by the public health authorities the number of potential providers is fairly low. The potential effect of a change in the number of competitors over time may have a huge effect on outcome and efficiency. Furthermore, procurement can in many cases result in dual sourcing. In the public health sector in the US, public procurement for drug abuse treatment has been growing rapidly (Schlesinger et al., 1990). Although market structure for drug abuse treatment varies widely, it is far from uncommon to have several providers of the same service. As pointed out by Commons et al. (1992) and McGuire and Riordan (1995), two-firm provision of drug abuse treatment is often the case. Dana and Spier (1994) present a normative analysis of the optimal market structure (dual sourcing, sole sourcing or government production) in public procurement.

However, dual sourcing is not only limited to the public health sector. Anton and Yao (1992) points out that in the public sector the application of this dual sourcing *runs the gamut from high-technology systems for telecommunications to their use in obtaining services such as refuse collection and street cleaning. In the private sector procurements of items such as customized computer chips and commercial aircraft have involved split awards.*

This paper analyzes the design of a dynamic procurement contract and the effect on this design of fundamental differences in the way competitors fund their participation in the market. There are two types of firms. On the one hand, there are self-financed

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[1]http://ec.europa.eu/internal_market/publicprocurement/index_en.htm
[2]Handbook of Procurement.

firms with "deep-pockets", who have enough capital to fund their participation in the market in every period. On the other hand, there are cash-constrained firms with little or no proper funding ("shallow-pocket" firms). These latter firms rely on external investors to fund their market participation and their existence in future markets depends on their performance in today's market and their financial contract. One could think of a "deep-pocket" firm as being a big (multi-)national firm and a "shallow-pocket" firm as being a local start-up. Both of these firms compete for the procurement of a good or service (which can potentially be split between producers). This paper is thus different from Laffont and Robert (1996) who study the optimal auction when all firms face the same, commonly known, financial constraint. It is also different from Maskin and Riley (2000) and McAfee and McMillan (1989) who study auctions with asymmetric bidders, but where the asymmetry is related to the cost distribution and not the financial situations of the firms.

The market or procurement setting in question is one where the procurement agency wishes to procure a good or a service in each period. However, the good or service in each period is not the same but similar enough so that the same firms have the competence and skill to provide both goods. In the case of treatments of drug abuse, this could be in-house (residential) treatment for abuse of a specific drug in one period and outpatient services in the second period. Alternatively, it could be different types of treatment programs in each period or going from one type of treatment program to a polydrug treatment program.[3] Formally, in each period a procurement agency (the public health authorities in my example) decides how to split the provision of a good or service between the two firms (or organizations). The procurement agency can freely choose the optimal split of production for dual sourcing (including degenerate splits that would be equivalent to sole sourcing). However, since some firms do not have the ability to self-finance their presence in the market, they need to sign a financial contract with an investor before entering the market. It is assumed that there is a fixed cost of participation in each period.[4] The actual number of firms at the second-period procurement stage will be endogenous and depends on both the financial contracts and the outcome of the first-period procurement. This paper studies the optimal financial contract for the firm in need of funding and the optimal procurement contract in a setting with self-financed and cash-constrained firms. It also compares the result

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[3]For instance, oploids abuse treatment program in the first period and alcohol abuse treatment program in the second period would require different approaches for treatment.

[4]This fixed cost could either be thought of as an investment in specialized production equipment for this specific good or as a cost related to the research and work needed for the firm to learn its cost for the specific project.

obtained in the environment where the procurement agency and the investor contract non-cooperatively with the firm(s) to situations without financial contracts, a cooperative[5] solution as well as a non-cooperative solution where firms are myopic[6].

In this paper there is a trade-off between the need to secure competition in the future and the optimal allocation of today's provision. The main result of this paper is that when the procurement agency and the investor behave non-cooperatively[7] and the financially constrained firm is not *efficient enough*, then the first-period optimal procurement contract should be biased. In other words one of the firms should be given a larger share of total production than in the case where both firms face the same constraints. But which firm to give an advantage is not clear and I identify three different effects; the *sampling* effect, the *rent* effect and the *convexity* effect. In fact there is a trade-off between more aggressive competition today[8] and the benefit of potential future dual sourcing and competition. On the one hand some firms will behave more aggressively today in order to get rid of its competitors in future periods. This allows the procurement agency to restrict its payment in the first period. But this has a negative effect on second-period surplus because the possibility for dual sourcing, and competition in general, decreases and second-period rents increase.

The reasons for engaging in dual sourcing are many and vary widely. Some of the arguments put forward in favor of dual sourcing are convex production costs (Auriol and Laffont, 1992 and McGuire and Riordan, 1995), learning (Anton and Yao, 1990, and Klotz and Chatterjee, 1995), reduction of risk of supply disruption (Kelle and Miller, 2001, and Jüttner, Peck and Christopher, 2003) and effective technological transfer (Daly and Schuttinga, 1982). In this paper I present a setting with convex costs, but ignore all other reasons for dual sourcing which would add to the argument presented in this paper. This paper is not a paper to motivate dual sourcing per se or a paper that comes up with a new justification for engaging in split awards. However, it uses a setting where dual sourcing may be optimal and studies the effect of the financial structure of firms and its effect on the production decision.

This paper allows for any kind of split, ranging from sole sourcing (degenerate splits) to all possible splits of total production. Anton and Yao (1992) analyze a split award procurement auctions in which the principal divides production between two

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[5]When the two principals jointly design a common contract.

[6]Firms only care about the current period.

[7]Which I believe is the case in public procurement because strict rule for competition in these tenders forbid the procurement agency to finance only a selected group of market participants.

[8]Through the self-financed firm accepting lower transfer for the first-period contract.

agents or awards all production to a single agent. They show that when agents have private information the static split award auction can lead to a Pareto improvement relative to the winner-takes-all auction. There are several differences between their analysis and this one. First, they study a static setting where the advantage of the split award auction comes from the fact that dual sourcing restricts coordination between bidders whereas this paper studies a the effect of firms' financial constraints in dynamic setting where dual sourcing is allowed for. Also, they consider a fixed 50-50 split while this paper solves for the optimal share for each firm.

The financial contract in this paper extends the result of Faure-Grimaud (2000) to the case where the realization of profits is endogenous and obtained using an equilibrium procurement mechanism. Because the probability of refinancing (and thus the probability of being on the market in the second-period) is increasing in the efficiency of the firm and increasing in profits, the financial structure of the firms has implications on the design of the optimal procurement contract. If the financially constrained firm doesn't perform well enough in the first period, he risks being liquidated by the investor. This gives incentives to the other firm to engage in predation. Earlier literature on predation (Bolton and Scharfstein 1990, Snyder 1996 and Faure-Grimaud 2000) consider situations where profits are private information but their value is exogenous. The level of profits and their distribution is taken as given by all firms and either profits are privately observed by the firm or they are observable but not verifiable. [9] It should be noted however that this paper focuses on the distribution of production and distortions on this distribution due to asymmetric information and differences in firms' financial structure.

This paper is organized as follows. The model is presented in Section 2. Section 3 solves for the optimal procurement contract when none of the firms need a financial contract. The "no-financial constraint" solution will serve as a first benchmark. Section 4 covers the analysis of the cooperative program where the procurement agency and the investor behave as a single entity. The non-cooperative solution is presented in Section 5. In this section the optimal financial contract as well as the optimal procurement contract both in the first and the second period are derived. This Section also discusses what the difference would be to a situation where the bidding firms are *myopic*. Section 6 briefly concludes.

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[9]By considering the financial contract as being a contract with a third party (and the procurement contract being the main contract) this paper also relates to the literature on games with side-contracts. For an introduction to this and related literature on games with third parties, see Gerratana and Kockensen (2009).

2   The model

• **Technology, information and preferences:** There are four types of players in this model; the investor, the procurement agency, the cash-constrained firm and the self-financed firm.

There exists a competitive market for investors. Therefore a firm seeking funding has all the bargaining power when it comes to the details of the financial contract. As pointed out in Faure-Grimaud (1997) when the investor has all the bargaining power the optimal contract has the same structure as in this case.

In each of the two periods, the procurement agency wants to divide the production of an amount $\bar{q}$ of a certain good between the two firms. He enjoys a gross surplus $S(\bar{q}) = \bar{S}$ from the provision of such a service. In general, the per period total quantities $\bar{q}_1$ and $\bar{q}_2$ do not need to be the same. However, for simplicity, $\bar{q} = \bar{q}_1 = \bar{q}_2$. Assuming that the procurement agency always wants a fixed quantity $\bar{q}$ of the good is the same as assuming that the demand function for this good is inelastic. For instance, when the public health authorities decide to provide a treatment facility for drug abuse, it seems reasonable that the public health authorities already knows how much of this good it needs. This facility is needed to cope with a growing public health problem and will, in general, not be very sensitive to production costs. So in this case it seems reasonable to assume that the total quantity demanded is either fixed or varies very little and the problem becomes one where the procurement agency needs to decide upon how much of this fixed quantity should be produced by each firm.[10] Denote by δ the discount factor which can also be interpreted as the relative importance of the second-period project.

The cash-constrained and the self-financed firms both have the ability to provide the goods to the procurement agency. There is a fixed cost D to be paid in each period for staying on the market and that the self-financed firm has got all the assets it needs to pay this fee whereas the cash-constrained firm has no such asset. D needs to be paid before each period by all firms who want to be present on the market. In other words, D is *not* a cost to enter the market, D is a fixed cost related to a project and needs to be paid for each new project.[11] Ex ante, the only difference between the firms is that the

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[10]Another example where quantities are fixed is the auctioning of broadcast permits for big events (such as the UEFA Champions League and the Olympic Games). In general there is a fixed quantity of licenses and the decision concerns to whom allocate these broadcasting rights and not how many licenses to allocate.

[11]This could also be thought of as an administrative cost associated with starting on a new project.

self-financed firm has "deep pockets" and does not need external financing to stay on the market. The cash-constrained firm however, has "shallow pockets" and need an investor to finance him in order to stay on the market.

In each period a firm's cost of procuring the required amount q of the good is $\theta q + \mu \frac{q^2}{2}$ where $\mu \geq 0$. Here costs are convex. In general, convex costs gives an intrinsic efficiency reason for using dual sourcing. Since costs increase with the production level, splitting the production between two (or more) producers allows the procurement agency to obtain the good at a lower total cost than if he used only one provider. The specification of the cost function also includes the case of linear cost (when $\mu = 0$). The parameter μ is common to all firms and is public knowledge. But θ is private information and independent across time and firms. Independence across time rules out all possibilities of learning. As mentioned in the Introduction, I focus on the effect of competition and predation in procurement contracts. Furthermore, costs are drawn from the same cumulative distribution function $F(\theta)$ with support $\Theta = [\underline{\theta}, \bar{\theta}]$. The associated density function is denoted $f(\theta)$. I assume that the hazard rate $\frac{F}{f}$ is increasing.

For ease of notation, denote by A the cash-constrained firm and B the self-financed firm.

• **Contracts:** In this model there are two contracts, a procurement contract and a financial contract.

The procurement contract is a long-term contract which stipulates transfers to both firms and the associated quantities to be produced by the two firms in each period. First-period transfers and quantities are only contingent on first-period announcements of θ_A and θ_B. However, second-period announcement are contingent on first-period history (such as who dropped out of the market between periods and first-period announcements) as well as second-period announcement of types.

A cash-constrained firm will have to finance its fixed costs by entering into a financial contract with an investor. The details of this contract is presented in Section 5.1.

• **Timing:** This paper analysis a two-period game where, in each period, a procurement agency wants to obtain a production of a good from the market participants. However, participation in the market requires the firms to pay, before each period, the fixed cost D. For the cash-constrained firm, paying the fixed cost D implies contracting beforehand with an investor who can finance the firm's market participation.

The specific timing considered in the different sections of this paper will be specified in detail at the beginning of each section.

Before presenting the benchmark and the results, some comments regarding the fixed payment, D, are in order. First, notice that the procurement agency can only make payments to the firms during the procurement stage. This rules out any upfront payment/subsidy to the firms (especially the cash-constrained firm). Although this could be one way of keeping cash-constrained firms on the market and thus be good for future competition, most rules governing public procurement (at least in Europe) would not allow for such *ex ante favoritism* and because the motivation of this paper has mainly focused on public procurement I rule out any such possibility.

Furthermore, the cash-constrained firm cannot falsely claim zero profits and default on its repayment to the investor in order to use its profit to finance the second-period fixed cost itself. Although profits are unobservable (except of course to the firm), claiming to have made zero profits and then being able to pay D can be seen as a way of revealing that profits where positive (at least for a cash-constrained firm) and can therefore be prohibited by law and punished severely.

Finally, I focus on the case of relatively small D. In fact, if D is large enough, a natural monopoly situation might arise. Here, I want to abstract from these issues and focus on situations where competition is beneficial (but fragile in the sense that some players are cash-constrained). Of course profits from the procurement stages will be endogenous, but I will in what follows assume that the (endogenous) expected value of participating in the market is higher than the up-front payment D.

• **First-Best Procurement:** Ignoring the difference in financial structure of the two firms and with symmetric information in both periods, the procurement agency's objective is to maximize the expected intertemporal social surplus subject only to the firms accepting the deal in each period. Here net surplus is constant since the total quantity $\bar{q}$ to be procured in each period is fixed, so maximizing expected intertemporal social surplus amounts to minimizing expected total cost of procurement for both periods.

With public information on the firms' production costs, the procurement agency will only pay the firms an amount equal to their costs. This means that for each unit of the good or service that is being provided the procurement agency will look at what firm can produce this additional unit at the lowest cost. In other words, if possible the procurement agency is going to choose to split the production so that marginal cost of each firm coincides. It is easy to see that if this is not the case, then the procurement

agency can reduce its payment by transferring a small amount of the provision from the firm with a high marginal cost to the one with a low marginal cost. However, sometimes[12] such a split is not possible because the marginal cost of one firm is above the marginal cost of the other firm for all possible splits the production. In this case the procurement agency is going to ask the most efficient firm (the one with the lowest marginal cost) to provide the entire production.

Formally, in each period k and for an interior solution this yields the following condition for the optimal solution for firm A's production $q_{kA}(\theta_{kA}, \theta_{kB})$

$$\theta_{kA} + \mu q_{kA}(\theta_{kA}, \theta_{kB}) = \theta_{kB} + \mu(\bar{q} - q_{kA}(\theta_{kA}, \theta_{kB})). \tag{1}$$

If one firm is inherently more efficient than the other so that (1) does not have a solution in $[0, \bar{q}]$, then the optimal strategy for the procurement agency is to select sole sourcing from the most efficient firm.

Assuming that there is an interior solution to (1)[13], this solution is such that at the optimal levels, $(q_{kA}(\theta_{kA}, \theta_{kB}), q_{kB}(\theta_{kA}, \theta_{kB}))$, both firms produce at the same marginal cost.

Firm B is obviously asked to produce the complimentary quantity $q_{kB}(\theta_{kA}, \theta_{kB}) = \bar{q} - q_{kA}(\theta_{kA}, \theta_{kB})$. It is immediate to see that the only difference between the provision rule in the first and the second period is that each rule uses the type of the firms in the associated period. I can therefore simplyfy the notation and write $q_A(\cdot)$ for both $q_{1A}(\cdot)$ and $q_{2A}(\cdot)$.

In what follows, because total quantity in each period is fixed, by definition of fixed there can be no distortion of total quantity and I will therefore focus on the allocative role of asymmetric information and financial structure on the distribution of $q_{kA}(\cdot)$ and $q_{kB}(\cdot)$, $k = 1, 2$. This first-best result as well as the result in the benchmark case reproduces the result in Auriol and Laffont (1992). The only difference between the models, is that here there is an fixed cost to be paid upon entering each period and in Auriol and Laffont's (1992) static model the fixed cost enters at the production stage.

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[12]For instance in the special case of linear costs ($\mu = 0$).

[13]This is the case when $\forall (\theta_{kA}, \theta_{kB})$, $0 \leq \frac{\bar{q}}{2} + \frac{\theta_B - \theta_A}{2\mu} \leq \bar{q}$.

3 Benchmark: No Financial Contract

Before studying the problem as it has been presented in the previous section, let us look at what happens when there is no need for financial contracts. In other words, this section characterizes the optimal split of the total production when the firms do not differ in the way they finance their participation in the market. In fact they both have enough cash to finance the fixed cost themselves.

In the absence of financial constraints both firms will be around in both periods and the procurement contract is a long-term contract which stipulates for each firm, transfers and quantities to be produced in each period. First-period transfers and quantities are only contingent on first-period announcements of θ_A and θ_B. However, second-period announcement are contingent on first-period history as well as second-period announcement of types. Formally this can be written as

$$\left\{ q_{1A}(\cdot), q_{1B}(\cdot), t_{1A}(\cdot), t_{1B}(\cdot), \left(\left\{ q_{2A}(\cdot), q_{2B}(\cdot), t_{2A}(\cdot), t_{2B}(\cdot) \right\} \right) \right\}.$$

The timing of the contract is as follows:

1. First period:

   - Firms pay the fixed costs D and privately learn their first-period cost θ_{1i}.

   - The procurement agency offers a contract to the firms. This contracts specifies first-period transfers and quantities as well as second-period transfers and quantities. These latter can be contingent on first-period outcome but a firm can always choose not to be active in the second period[14].
     Firms privately announce their first-period type to the procurement agency.

   - The outcome of the first-period procurement stage is realized and observed by both firms and the procurement agency.

2. Second period:

   - Firms pay the fixed cost D and privately learn their second-period cost.

   - The second-period procurement stage takes place and firms privately announce their type to the procurement agency.

   - Second-period transfers and quantities are realized.

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[14]I.e. we require ex post participation constraints in both periods.

- The game ends.

This timing is summarized in the following figure.

First period   **Second period**

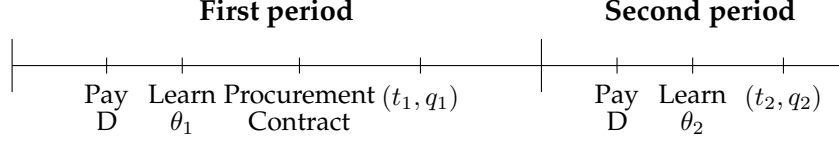| Pay | Learn | Procurement (t_1, q_1) | Pay | Learn | (t_2, q_2) |
| D | θ_1 | Contract | D | θ_2 | |

Figure 1: Timing- Benchmark without financial constraints

Since surplus is fixed, the only issue is how to optimally allocated the production between the firms. In other words, I will look for the split of production that minimizes procurement cost subject to participation and incentive constraints. To do so, I make use of the Revelation Principle (Myerson 1982).

Furthermore, to make notations more tractable, define $\theta_1 \equiv (\theta_{1A}, \theta_{1B})$ and $\theta_2 \equiv (\theta_{2A}, \theta_{2B})$.

At the beginning of the first period, the procurement agency chooses the menu of contracts that minimizes his intertemporal expected total cost subject to first- and second-period incentive compatibility and participation contraints. This writes

$$\min_{(t_{iA}(), t_{iB}(), q_{iA}(), q_{iB}())_{i=1,2}} E_{\theta_1} \left[t_{1A}(\theta_1) + t_{1B}(\theta_1) \right]$$
$$+ \delta E_{\theta_1, \theta_2} \left[t_{2A}(\theta_1, \theta_2) + t_{2B}(\theta_1, \theta_2) \right]$$

subject to incentive compatibility and participation constraints for both periods. Here it is clearly stated that second-period variables can be contingent on first-period types. In what follows and because types are independent across time it will become clear that this contingency is not necessary. Furthermore, it is straightforward to see that the problem is separable across periods. Therefore, I will first present the solution to the second-period problem and then turn to the first-period problem.

3.1 Second period

The firms' IC constraints are

$$U_{2i}(\theta_1, \theta_{2i}) = \max_{\tilde{\theta}_{2i}} U_{2i}(\theta_1, \theta_{2i}, \tilde{\theta}_{2i}) = \max_{\tilde{\theta}_{2i}} E_{\theta_{2j}} \left\{ t_{2i}(\theta_1, \tilde{\theta}_{2i}, \theta_{2j}) - \theta_{2i} q_{2i}(\theta_1, \tilde{\theta}_{2i}, \theta_{2j}) - \mu \frac{q_{2i}(\theta_1, \tilde{\theta}_{2i}, \theta_{2j})^2}{2} \right\},$$
$$(i, j) \in \{A, B\}^2, i \neq j.$$

The Envelope Theorem yields that $U(\cdot)$ is absolutely continuous and thus almost everywhere differentiable. Moreover at any differentiability point, we get:

$$\dot{U}_{2i}(\theta_1, \theta_{2i}) = -E_{\theta_{2j}}\left[q_{2i}(\theta_1, \theta_{2i}, \theta_{2j})\right]. \tag{2}$$

The local second-order necessary condition is $E_{\theta_j}\frac{\partial q_{2i}}{\partial \theta_{2i}}(\theta_1, \theta_{2i}, \theta_{2j}) \leq 0$ which will be checked ex post.

Observe that (2) implies that $U(\cdot)$ is non-decreasing in θ_{2i}, so that the participation constraint is binding at $\bar{\theta}$ only. Integrating (2) with respect to θ_{2i} yields

$$U_{2i}(\theta_1, \theta_{2i}) = E_{\theta_{2j}}\left\{t_{2i}(\theta_1, \theta_2) - \theta_{2i}q_{2i}(\theta_1, \theta_2) - \mu\frac{q_{2i}(\theta_1, \theta_2)^2}{2}\right\}$$

$$= E_{\theta_{2j}}\left\{\int_{\theta_{2i}}^{\bar{\theta}} q_{2i}(\theta_1, s, \theta_{2j})ds\right\}, \quad (i,j) \in \{A, B\}^2, i \neq j.$$

The procurement agency will therefore minimize expected total virtual cost where virtual, as defined by Myerson (1984), refers to the actual cost plus the adjustment (or rent) required for the mechanism to be incentive compatible. The procurement agency's optimization problem can thus be written

$$\min_{q_{2A}(\cdot), q_{2B}(\cdot)} E_{\theta_2}\left[\theta_{2A}q_{2A}(\theta_1, \theta_2) + \frac{F}{f}(\theta_{2A})q_{2A}(\theta_1, \theta_2) + \mu\frac{q_{2A}(\theta_1, \theta_2)^2}{2}\right.$$

$$\left. + \theta_{2B}q_{2B}(\theta_1, \theta_2) + \frac{F}{f}(\theta_{2B})q_{2B}(\theta_1, \theta_2) + \mu\frac{q_{2B}(\theta_1, \theta_2)^2}{2}\right]$$

subject to $q_{2B}(\theta_1, \theta_2) = \bar{q} - q_{2A}(\theta_1, \theta_2)$, $q_{2i}(\theta_1, \theta_2) \geq 0$ and $q_{2i}(\theta_1, \theta_2) \leq \bar{q}$ for $(i,j) \in \{A, B\}^2, i \neq j..$

The optimal quantity $q_{2A}(\theta_2)$ is independent of first-period types and, (unless it is a corner solution) is given by

$$-\theta_{2A} - \frac{F}{f}(\theta_{2A}) - \mu q_{2A}(\theta_2) + \theta_{2B} + \frac{F}{f}(\theta_{2B}) + \mu(\bar{q} - q_{2A}(\theta_2)) = 0 \tag{3}$$

If there exists a $q_{2A}(\theta_2) \in (0, \bar{q})$ that solves (3), then the optimal solution for the procurement agency is to adopt dual sourcing with firm A producing $q_{2A}(\theta_2)$ given by (3) and firm B producing $q_{2B}(\theta_2) = \bar{q} - q_{2A}(\theta_2)$. In fact this solution is such that the virtual marginal cost of firm A for producing the second-period good equals the virtual marginal cost of firm B for producing the same good. In the case where both firm have the same efficiency ($\theta_{2A} = \theta_{2B}$), the production is split into two equal parts. If firms differ in their efficiency, the split is not necessarily equal.

If the left-hand side (LHS) of (3) is always positive, $\forall q_{2A} \in (0, \bar{q})$, then sole sourcing by firm A is optimal and $q_{2A}(\theta_2) = \bar{q}$. In this case, the virtual marginal cost of firm A is, for all admissible quantities[15], lower than the virtual marginal cost of firm B.

If the left-hand side (LHS) of (3) is always negative, $\forall q_{2A} \in (0, \bar{q})$, then sole sourcing by firm B is optimal and $q_{2B}(\theta_2) = \bar{q}$. I.e. for all $q \in [0, \bar{q}]$, the virtual marginal cost of firm B is smaller than the virtual marginal cost of A.

To sum up these findings, given the *types* of the firms, the procurement agency chooses to split the production in the most efficient way between the two firms. If one firm is inherently more efficient than the other then the procurement agency will opt for sole sourcing. However, in the opposite case, when firms differ less in their *types*, the procurement agency will opt for dual sourcing since it allows to enjoy lower costs. In this case the procurement agency will ask the firms to produce quantities such that their respective virtual marginal costs are equal because if the procurement agency transfers some of the production from one firm to the other, the overall production cost increases.

Rearranging (3) we get

$$q_{2A}(\theta_2) = \frac{\bar{q}}{2} + \frac{\theta_{2B} + \frac{F}{f}(\theta_{2B}) - \theta_{2A} - \frac{F}{f}(\theta_{2A})}{2\mu}. \tag{4}$$

Since the hazard rate is increasing, it is straightforward to see that the most efficient firm produces more both under symmetric and asymmetric information. However, under asymmetric information the distortions are going to be such that the procurement agency will favor the more efficient firm even more in order to reduce the rent he has to pay for the firms to behave truthfully. In a standard principal-agent framework with a non-constant surplus function, quantities are reduced (distorted downward) to reduce the rent of the agents. Here total quantities are fixed, but by shifting some of the production from the less efficient firm to the most efficient one the procurement agency decreases the rent he has to pay to high types which again allows him to reduce the rent to more efficient types.[16] Therefore, when firm A is more efficient than firm B, $q_{2A}(\theta_2)$ is shifted upward compared to the first-best case. For the same reason, when firm B is more efficient than firm A, $q_{2A}(\theta_2)$ is shifted downward to allow more of the production of the efficient firm B.

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[15]$\forall q_{2A} \in (0, \bar{q})$.

[16]The more efficient you are the more rent you will require. But if the inefficient types get very little rent, then an efficient type will be more willing to accept lower levels of rent since this is still better than deviating.

Notice that for incentive compatibility and participation, the only thing that is required is that $E_{\theta_{2j}}\{t_{2i}(\theta_2)\} = E_{\theta_{2j}}\{\theta_{2i}q_{2i}(\theta_2) + \mu\frac{q_{2i}(\theta_2)^2}{2} + \int_{\theta_{2i}}^{\bar{\theta}}q_{2i}(s,\theta_{2j})ds\}$, but this leaves an infinity of ways for the procurement agency to define the actual $t_{2i}(\theta_2)$. This will be true throughout the paper, but since the focus of this paper is on distribution of production and its distortions, I will not be very preoccupied by this. Mookherjee and Reichelstein (1992) identify conditions under which there is no loss in replacing Bayesian incentive compatibility by the stronger requirement of incentive compatibility in dominant strategies. Their result apply to this setting and I will therefore focus on transfers that are not only Bayesian incentive compatible but are also dominant strategy incentive compatible. In this setting a dominant strategy incentive compatible transfer is such that $t_{2i}(\theta_2) = \theta_{2i}q_{2i}(\theta_2) + \mu\frac{q_{2i}(\theta_2)^2}{2} + \int_{\theta_{2i}}^{\bar{\theta}}q_{2i}(s,\theta_{2j})ds\}$.

3.2 First period

In the first period, firm i's incentive constraint writes

$$\theta_{1i} \in \arg\max_{\tilde{\theta}} E_{\theta_{1j}}\{t_{1i}(\tilde{\theta},\theta_{1j}) - \theta_{1i}q_{1i}(\tilde{\theta},\theta_{1j}) - \mu\frac{q_{1i}(\tilde{\theta},\theta_{1j})^2}{2} + \delta\left(\Pi^d - D\right)\},$$
$$(i,j) \in \{A,B\}^2, i \neq j.$$

Where $\Pi^d \equiv E_{\theta_2}\left(U_{2i}(\theta_2)\right)$ is the expected second-period profit of a firm before learning its type when there are still two firms around in the second period. The only difference with respect to the second period is the constant $\Pi^d - D$. Since Π^d is independent of θ_{1i} (first-period type), the first-period incentive compatibility constraint takes the same shape as the second-period incentive compatibility constraint.

the condition for incentive compatibility remains the same condition as previously and transfers are adjusted to take into account this constantt.

Applying the same techniques as previously, I get

$$U_{1i}(\theta_{1i}) = U_{1i}(\bar{\theta}) + \int_{\theta_{1i}}^{\bar{\theta}} E_{\theta_{1j}}q_{1i}(s,\theta_{1j})ds, (i,j) \in \{A,B\}^2, i \neq j.$$

The firms' participation constraints bind for the most inefficient type ($U_{1i}(\bar{\theta}) = 0$),

therefore the principal's minimization problem can be written

$$
\min_{q_{1A}(\cdot)} E_{\theta_1} \left\{ \theta_{1A} q_{1A}(\theta_1) + \frac{F}{f}(\theta_{1A}) q_{1A}(\theta_1) + \mu \frac{q_{1A}(\theta_1)^2}{2} - \delta \Pi^d + \delta D \right.
$$
$$
+ \theta_{1B}(\bar{q} - q_{1A}(\theta_1)) + \frac{F}{f}(\theta_{1B})(\bar{q} - q_{1A}(\theta_1)) + \mu \frac{(\bar{q} - q_{1A}(\theta_1))^2}{2}
$$
$$
\left. - \delta \Pi^d + \delta D \right\} + \delta E_{\theta_2} \left(t_{2A}(\theta_2) + t_{2B}(\theta_2) \right).
$$

The optimal quantity produced by firm A is given by the same formula as in the second period. To simplify notations, denote $q_A(\theta) \equiv q_{1A}(\theta) \equiv q_{2A}(\theta)$ where $\theta = (\theta_A, \theta_B)$ the decision resulting from this formula. In each period any interior solution is given by

$$
-\theta_A - \frac{F}{f}(\theta_A) - \mu q_A(\theta_A, \theta_B) + \theta_B + \frac{F}{f}(\theta_B) + \mu(\bar{q} - q_A(\theta_A, \theta_B)) = 0. \tag{5}
$$

In other words, for an interior solution, the virtual marginal cost of firm A equals that of firm B. So, given the *types* of the firms, the procurement agency splits the production in the most efficient way. If the efficiency of the firms are *not too different*, the procurement agency will opt for dual sourcing and split the production between the firms according to (5). Furthermore as described in the previous section, under dual sourcing, the quantity produced by the more efficient firm is increased to avoid paying unnecessary high rents. If one firm is significantly more efficient then the other, then the procurement agency will opt for sole sourcing by this firm.

Furthermore, since types are independent across time, the expected quantities and transfers in each period are independent of the other period and the whole problem reduces to two static problems.

Finally, this result can be viewed as an extension to the dynamic contracting problem of Baron and Myerson (1982) and Auriol and Laffont (1992), and as a multi-agent version of Baron and Besanko (1984). However, Baron and Besanko (1984) assumes that second-period individual rationality is relevant before the firm learn its second-period cost whereas here the second-period participation constraint should hold for the realized value of second-period cost. Therefore, with independence across time, they obtain the first-best outcome in the second period while I only obtain the second-best outcome for this period. However, this result is in line with findings in Laffont and Martimort (Chapter 8, 2002) where in the one-agent case and with ex post participation constraints in each period, the optimal dynamic contract is a sequence of one-shot optimal static contracts.

The findings in the case where firms have no financial constraints are summarized in the following Proposition.

Proposition 1 *When none of the firms face financial constraints, the optimal solution is the same in each period and is such that, if $\forall (\theta_A, \theta_B)$, $\frac{\bar{q}}{2} + \frac{\theta_B - \frac{F}{f}(\theta_B) - \theta_A - \frac{F}{f}(\theta_A)}{2\mu} \in [0, \bar{q}]$, then (non-degenerate) dual sourcing is optimal. Furthermore $q_A(\theta_A, \theta_B)$ is given by (5) and $q_B(\theta_A, \theta_B) = \bar{q} - q_A(\theta_A, \theta_B)$ and these production levels are such that each firm produces at the same virtual marginal cost.*

As described in Section 3.1, the condition (5) can straightforwardly be interpreted in terms of virtual marginal cost.

In the next section, the focus will be on another benchmark, where one firm faces a financial constraint but the procurement agency and the investor contracts *cooperatively* with the firms.

4   Cooperative Solution with Financial Constraints

In this section I study the cooperative case in which firm A needs external funding but the procurement agency and the investor are either the same entity or they decide to cooperate. In this case, which will be called the cooperative case, the results from the previous Proposition still hold. Recall that the focus of this paper is on public procurement. In general, rules and guidelines for public procurement forbids the procurement agency from giving financial assistance to the cash-constrained firm. Although this observation makes the problem addressed in this section less applicable to real world problems, it is useful to analyze this case from a pedagogical point of view. It makes it easier to understand the different effects in the non-cooperative environment come from that is presented in Section 5.

A contract in this setting will be defined in the same way as the contract above except that it will also include a refinancing decision between periods and second-period quantities and transfers are also contingent on the number of firms around in this period. In other words, a dynamic (cooperative) contract can be written as

$$\left\{ q_{1A}(\cdot), q_{1B}(\cdot), t_{1A}(\cdot), t_{1B}(\cdot), \beta(\cdot), \left(\{ q_{2A}(\cdot; n), q_{2B}(\cdot; n), t_{2A}(\cdot; n), t_{2B}(\cdot; n) \} \right)_{n=1,2} \right\},$$

where $\beta(\cdot)$ is the probability of non-liquidation of the cash-constrained firm between periods.

Note that here there is no need to specify two different payments, one transfer for the provision of the good and another to compensate the principal for funding D, between the principal and the cash-constrained firm. Instead of considering separately the repayment in the financial contract and the transfer for providing the good, I can (w.l.o.g) focus on the transfer (including both the provision payment and the repayment in the financial contract).

The timing of the contract is the same as in the previous section except that at the end of the first period there is an additional step where the principal decides upon whether to keep the cash-cosntrained firm or liquidated it. To summarize,

1. First period:

   - Firms pay the fixed costs D and privately learn their first-period cost θ.
   - The procurement agency offers a procurement contract to the firms. This contracts specifies first-period transfers and quantities as well as second-period transfers and quantities. These latter can be contingent on first-period outcome but a firm can always choose not to be active in the second period[17]. Firms privately announce their first-period type to the procurement agency.
   - The outcome of the first-period procurement stage is realized and observed by both firms and the procurement agency.
   - The principal decides upon the liquidation or not of the cash-constrained firm and everyone observes what firms are around for the second period.

2. Second period:

   - Firms pay the fixed cost D and privately learn their second-period cost.
   - The second-period procurement stage takes place and firms privately announce their type to the procurement agency.
   - Second-period transfers and quantities are realized.
   - The game ends.

This timing is summarized in the following figure.

First period						Second period		
Pay D	Learn θ_1	Procurement Contract	(t_1, q_1)	β		Pay D	Learn θ_2	(t_2, q_2)

---

[17]I.e. we require ex post participation constraints in both periods.

Figure 2: Timing- Cooperative solution with financial constraint

Compared to the situation studied previously, nothing changes with regard to the second-period procurement. Since the funding of the cash-constrained firm (if it takes place) is sunk before this stage is played, I get the same results as previously. Notice however, that if there is only one firm (the self-financed firm) around in the second period, its expected profits are bigger than when there are two firms around. Define the expected duopoly profit as $\Pi^d = E_{\theta_2}\left(\frac{F}{f}(\theta_{2i})q_{2i}(\theta_2)\right)$ and the expected monopoly profit $\Pi^m = \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\bar{\theta}} \bar{q}\,ds\,dF(\theta) = \bar{q}\left(\bar{\theta} - E_{\theta_{1i}}(\theta_{1i})\right)$.

For future use, let us also define the procurement agency's expected total second-period payment when there is one firm (resp. two firms) left on the market as $T_2^m = \bar{\theta}\bar{q} + \mu\frac{\bar{q}^2}{2}$ (resp. $T_2^d = E_{\theta_2}[t_{2A}(\theta_2) + t_{2B}(\theta_2)]$).

In the first period, the expected payment required by a firm depends on its underlying financial situation through the refinancing variable $\beta(\cdot)$. For the cash-constrained firm I have[18],

$$E_{\theta_{1B}}\left(t_{1A}(\theta_1)\right) = E_{\theta_{1B}}\left\{\theta_{1A}q_{1A}(\theta_1) + \mu\frac{q_{1A}(\theta_1)^2}{2} + \int_{\theta_{1A}}^{\bar{\theta}} q(s,\theta_{1B})ds - \delta\beta(\theta_1)\Pi^d\right\},$$

whereas for the self-financed firm the expected transfer is

$$E_{\theta_{1A}}\left(t_{1B}(\theta_1)\right) = E_{\theta_{1A}}\left\{\theta_{1B}q_{1B}(\theta_1) + \mu\frac{q_{1B}(\theta_1)^2}{2} + \int_{\theta_{1B}}^{\bar{\theta}} q_{1B}(s,\theta_{1A})ds\right.$$
$$\left. -\delta\beta(\theta_1)\Pi^d - \delta(1-\beta(\theta_1))\Pi^m - \delta D\right\}.$$

In fact, the refinancing variable $\beta(\cdot)$ modifies the continuation valuation for the firms in different ways and thus it also modifies the transfer they will require to satisfy participation (and incentive) constraints in the initial procurement stage.

When taking into account the value of the respective transfers, the principal's relaxed problem becomes

$$\min_{q_A(\cdot),\beta(\cdot)} E_{\theta_1}\left\{\theta_{1A}q_{1A}(\theta_1) + \frac{F}{f}(\theta_{1A})q_{1A}(\theta_1) + \mu\frac{q_{1A}(\theta_1)^2}{2} - \delta\beta(\theta_1)\Pi^d\right.$$
$$+ \theta_{1B}(\bar{q} - q_{1A}(\theta_1)) + \frac{F}{f}(\theta_{1B})(\bar{q} - q_{1A}(\theta_1)) + \mu\frac{(\bar{q} - q_{1A}(\theta_1))^2}{2}$$
$$- \delta\beta(\theta_1)\Pi^d - \delta(1-\beta(\theta_1))\Pi^m + \delta D + \delta\beta(\theta_1)(T^d + D)$$
$$\left. + \delta(1-\beta(\theta_1))T^m\right\}$$

---

[18]Assuming that the merged principal pays the sunk cost for this firm in each period, if, of course, this firm is allowed to continue.

From the minimization problem above, it is straightforward to observe that there are no terms with both $\beta(\cdot)$ and $q_{1A}(\cdot)$. The problem is therefore separable in $\beta(\cdot)$ and $q_{1A}(\cdot)$.

Looking first at the first-order condition with respect to $q_{1A}(\cdot)$, it is immediate that the optimal quantity $q_{1A}(.)$ is still given by Equation 5. It follows that when both firms are around in each period the decision rule does not change (but is of course dependent on the relevant period's private information).

To completely solve the optimization problem, the optimal value of $\beta(\cdot)$ remains to be determined. Denote by $V(q_{1A}, \beta)$ the expression that the procurement agency is minimizing.

$$\frac{1}{\delta} \frac{\partial V(q_{1A}, \beta)}{\partial \beta)} = -2\Pi^d + T^d + D + \Pi^m - T^m. \tag{6}$$

Define $V^m = S(\bar{q}) - T^m$ (respectively $V^d = S(\bar{q}) - T^d$) and the previous expression can be written as $V^m + \Pi^m - (V^d - 2\Pi^d - D)$. This is the difference in expected total surplus with only one firm in the second period and the equivalent surplus with two firms left in the second period. Replacing these expressions by their value and rearranging term yield

$$\frac{1}{\delta} \frac{\partial V(q_{1A}, \beta)}{\partial \beta} = E_{\theta_2} \left\{ q_{2A}(\theta_2)(\theta_{2A} - \theta_{2B}) + \mu(q_{2A}(\theta_2)^2 - \bar{q} q_{2A}(\theta_2)) + D \right\}. \tag{7}$$

The second-term in this expression is negative. Note that $q_{2A}(\cdot)$ increases when firm A is relatively more efficient than firm B. In other words, the coefficient $q_{2A}(\cdot)$ is more important when $\theta_{2A} < \theta_{2B}$. The second term is therefore also negative. To finish the discussion on the optimal choice of $\beta(\cdot)$, note that if D is small enough, $\frac{\partial V(q_{1A}, \beta)}{\partial \beta} \leq 0$. The optimal choice of β is thus one.

This implies that it is always optimal to keep firm A in the second period. Keeping the cash-constrained firm not only allows a more efficient split of the second-period production, but it also reduces the rent that the principal has to pay for second-period incentive compatibility. It is therefore optimal for the merged principals to always refinance the cash-constrained firm, and the solution obtained is the same as the one presented without financial contracts.

These findings are summarized in the following Proposition.

Proposition 2   • *It is always optimal to refinance the financially constrained firm.*

   • *The optimal quantities in both periods are the same as when firms do not need external investment and when procurement agency and investor cooperate.*

5 Non-cooperative solution

In this section, I study the optimal dynamic procurement problem when one out of two firms is financially constrained. Here financially constrained refers to the fact that the firm in question does not have enough cash to finance the fixed cost D and therefore needs to contract with an outside investor. In the previous section, it was assumed that the procurement agency and the investor were in fact the same entity[19]. In many cases, especially in public procurement, the procurement agency does not offer financial support for potential service providers. Firms in lack of financial resources therefore have to contract separately with their investors. This section derives the optimal contract both for the investor and the procurement agency in a setting where the two principals contract non-cooperatively with the firm(s), i.e. each principal contracts individually and without coordinating or communicating with the other principal.

In this setting I need to characterize two distinct contracts; the optimal long-term procurement contract and the optimal financial contract.

The procurement contract is a long-term contract which stipulates for each firm, transfers and quantities to be produced in each period. First-period transfers and quantities are only contingent on first-period announcements of θ_{1A} and θ_{1B}. However, second-period announcement are contingent on first-period history (such as who dropped out of the market between periods and first-period announcements) as well as second-period announcement of types. Formally, the procurement contract can be written

$$\left\{ q_{1A}(\cdot), q_{1B}(\cdot), t_{1A}(\cdot), t_{1B}(\cdot), \left(\left\{ q_{2A}(\cdot; n), q_{2B}(\cdot; n), t_{2A}(\cdot; n), t_{2B}(\cdot; n)\right\}\right)_{n=1,2} \right\},$$

where n represents the number of firms (one or two) left in the second period and is observable (and verifiable) between periods.

As in Bolton and Scharfstein (1990) and Faure-Grimaud (1997, 2000), a financial contract stipulates repayment schemes for the first period and a non-liquidation probability function contingent on the cash-constrained firm's realized profit in the first period as well as a repayment scheme for the second period[20]. Denoting by $R_i(\cdot)$ the repayment scheme in period i and by $\beta(\cdot)$ the non-liquidation probability function, a financial contract can be defined as follows.

$$\left\{ (R_1(\cdot), R_2(\cdot), \beta(\cdot) \right\}$$

---

[19]Or at least acted that way.

[20]For further explanation regarding the financial contract and its structure see Section 5.1.

For future use, define the cash-constrained firm's first-period realized profit as $\Pi_1(\theta_1) = t_{1A}(\theta_1) - \theta_{1A}q_{1A}(\theta_1) - \mu\frac{q_{1A}(\theta_1)^2}{2}$.

Compared to previous sections, now the cash-constrained firm has to enter into a contractual relationship with an investor at the beginning of the game and at the end of each period the financial contract will stipulate a repayment (and a refinancing decision in the first period). The game therefore unfolds as follows:

1. First-period:

   - First, if required[21], a financial contract is negotiated. This contract is non-verifiable by outside players[22].

   - Then firms pay the fixed costs D and privately learn their first-period cost θ.

   - The procurement agency offers a procurement contract to the firms. This contracts specifies first-period transfers and quantities as well as second-period transfers and quantities. These latter can be contingent on first-period outcome (number of participant left in period two etc.) but a firm can always choose not to be active in the second period[23].
     Firms privately announce their first-period type to the procurement agency.

   - The outcome of the first-period procurement stage is realized and observed by both firms and the procurement agency (but not by the investor).

   - The cash-constrained firm announces its profits to the investor and make its first-period repayment. Depending on its announcement of realized profits, this firm is forced by the investor to leave the market or not.

2. Second period:

   - Firms that are still active on the market pay the fixed cost D and privately learn their second-period cost.

   - The second-period procurement stage takes place and firms privately announce their type to the procurement agency.

   - Second-period transfers and quantities are realized. These transfers and quantities are only observed by the firms and the procurement agency.

   - If the cash-constrained firm is still on the market, he makes the second-period repayment in accordance with the financial contract.

---

[21]Recall that this is only required for the cash-constrained firm.
[22]But these external players can of course deduce the optimal contract.
[23]I.e. we require interim participation constraints in both periods.

- The game ends.

This timing is summarized in the following figure.

First period **Second period**

Financial Pay Learn Procurement (t_1, q_1) Repayment   Pay Learn (t_2, q_2)
Contract D θ_1 Contract   and   D θ_2
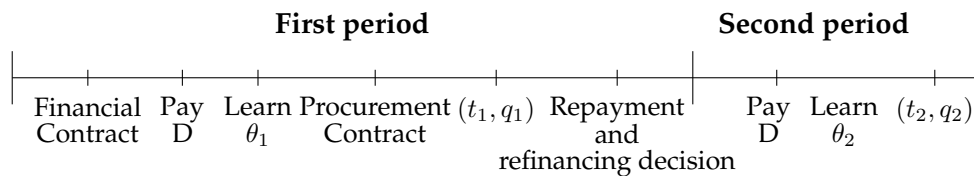   refinancing decision

Figure 3: Timing

Notice that because there is no communication among players between the offering of the two contracts. Therefore the timing of the financial contract and the first procurement contract is such that it is strategically equivalent to a simultaneous game. When two principals contract simultaneously with an agent, but each principal (investor and procurement agency) only controls part of the agent's activity, then the Revelation Principle does not necessarily hold. Martimort (1992) and Martimort and Stole (2002) shows that in situations where several principals control the agent's activity through non linear tariffs, the Revelation Principle does not apply, and needs to be replaced by a weaker concept; the Delegation Principle.

Looking at the non-cooperative solution to this problem I first derive best-replies for the procurement agency and investor. This can be done in three steps. On the one hand, I need to characterize the investor's financial contract. On the other hand, I need to study the behavior of the procurement agency. In fact, as seen in the cooperative case, the contract offered by the procurement agency can be split into two separate parts, one related to the first period and another related to the second period. In Subsection 5.2.1 where the second-period part of the optimal contract is presented, we will see that I am in fact computing the same optimal contract as in the no financial constraint case where firms on the market in this period are, ex ante, symmetric. Having obtained the best-reply contracts for each principal, I characterize a Nash equilibrium of this game.

This section starts by deriving the optimal financial contract for the cash-constrained firm. Then I present the optimal procurement contract and the equilibrium conditions for this game. At the end of this section, I consider another benchmark where bidders are myopic and compare my results to this outcome.

5.1 Optimal financial contract

Without asymmetric information, the financial contract between the investor and the cash-constrained firm would simply be a sharing rule of realized profits between the investor and the firm in exchange for the investor paying D. However, when profits are privately observed by firms we need a more sophisticated mechanism. Here, I make the assumption of private information on realized profits because costs are privately observed by firms and also the announcements in the procurement stage remain private.[24] Furthermore, for the investor, the outcome (quantities and dual/sole sourcing decision) in the procurement contract is non contractible. Once a firm has realized some profits, it needs to be induced to repay the investor rather than strategically defaulting by pretending not to have made any profits. Following Bolton and Scharfstein (1990) and Faure-Grimaud (2000), I study contracts with a repayment schedule and a "reward" function. The financial contract stipulates a repayment transfer (as a function of realized profits) and a reward which in my case is the refunding in the following period. If a firm claims to have made no profits in the first-period just in order not to pay back the investor, he can be punished by not being refinanced in the second period. This penalizes default deviations and helps the investor in monitoring the firm.

A financial contract consists of repayment schemes for each period and probabilities of the firm being refinanced in the second period. A priori, if the identity of the producers chosen to produce a non-negative quantity in the first-period is publicly announced, the financial contract could be contingent on this information. However, in the case of dual sourcing, knowing the type of one particular firm is not sufficient to deduce the profits earned by the cash-constrained firm. Indeed, the investor must also learn the type of the other firm (which the investor has no contact with). Or equivalently, the investor needs to know the quantity produced by the cash-constrained firm. In fact, if the investor knows both firms' type he can deduce the quantities and if he knows one firm's type and the quantity he can deduce the other firm's type in the case of dual sourcing. With sole sourcing the other firm's type becomes irrelevant for the investor. Notice however that the only reason for which the investor wants this information is so that he can deduce the profit level of the cash-constrained firm. Having this in mind, I will therefore take a different approach and assume that the financial con-

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[24]Bolton and Scharfstein (1990) point out different reasons for contracts not being profit-contingent. First, the managers of the firm might be able to divert profits (cash-diversion argument). Second, the firm in question might be related to another firm and therefore has some flexibility in the (joint) allocation of costs and revenues between these firms. These reasons for non-contractibility carry over to this paper.

tract is contingent on the (announced) profit level of the cash-constrained firm rather than on its type and quantities produced. To simplify the exposition I will assume that the investor cannot make the contract contingent on which and how many producers are chosen. The reason being either that this information is not public (or verifiable) or that the investor only cares about profits.

Formally, a financial contract is a menu

$$\left\{ (R_1(\hat{\Pi}_1), (R_2(\hat{\Pi}_1, \hat{\Pi}_2)), \beta(\hat{\Pi}_1)) \right\},$$

where $R_1(\hat{\Pi})$ is the repayment in period 1 for a firm announcing profit level $\hat{\Pi}_1$, $R_2(\hat{\Pi}_1, \hat{\Pi}_2)$ is the repayment in period 2 for a firm announcing profit level $\hat{\Pi}_1$ in the first period and $\hat{\Pi}_2$ in the second period, and, finally, $\beta(\hat{\Pi}_1)$ is the probability of being refinanced in the second period following this annoucement.

A financial contract is a menu of repayment and refinancing probabilities that satisfy three types of constraints: incentive compatibility (IC), limited liability (LL) and individual rationality (IR^I) constraints. The two first types of constraints are related to the firm whereas the latter is related to the investor (hence the superscript I). In fact, since I assume that there is perfect competition between investors, the financial contract will maximize the firm's expected inter-temporal profit subject to the participation constraint of the investor (and, of course, incentive compatibility).

For ease of notation I denote by Π^d the ex ante expected profit in the second-period procurement contract (given that both firms are still on the market). Both Π_1 and Π^d are endogenous and will be determined by the procurement stage. Define $G(\Pi_1) = Prob\{(\theta_A, \theta_B), \Pi_1(\theta_A, \theta_B) \le \Pi_1\}$ and $g(\Pi_1)$ the associated density function. The support of $\Pi_1(\theta)$ is $[\Pi_1(\bar{\theta}, \underline{\theta}), \Pi_1(\underline{\theta}, \bar{\theta})]$ where $\Pi_1(\bar{\theta}, \underline{\theta}) = 0$ and, abusing notation, I use $\bar{\Pi}_1$ instead of $\Pi_1(\underline{\theta}, \bar{\theta})$.

It is assumed that firms are protected by limited liability. In other words, none of the repayments stipulated in the financial contract can exceed the gains of the firm (from this project). This implies that, given that the true type and hence the profits of the firm are unobservable, a second-period repayment can never be bigger than the first-period profits of the firm net of its first-period repayment. This is because the firm can always strategically default in the second period and avoid any excessive payment. In fact, there is no possibility for the investor to screen second-period profits.

The remaining limited liability (LL) constraints are therefore, for all Π_1,

$$R_1(\Pi_1) \leq \Pi_1$$

$$\delta R_2(\Pi_1, \Pi_2) \leq \Pi_1 - R_1(\Pi_1), \forall \Pi_2$$

Taken Π_1 and Π^d as given, incentive compatibility (IC) requires, for all Π_1[25],

$$\Pi_1 \in \arg \max_{\hat{\Pi}_1 \leq \Pi_1} \Pi_1 - R_1(\hat{\Pi}_1) + \delta\beta(\hat{\Pi}_1)\big[\Pi^d - R_2(\hat{\Pi}_1)\big]\big].$$

This incentive-compatibility constraint can be replaced by its first-order condition.

Lemma 1 *(Faure-Grimaud 1997, 2000) The incentive compatibility constraints is binding and $\forall \Pi_1$,*

$$-R_1(\Pi_1) + \delta\beta(\Pi_1)\Pi^d = C$$

The individual rationality of the investor, (IR^I), can be written as

$$\int_0^{\bar{\Pi}_1} \big[R_1(\Pi_1) + \delta\beta(\Pi_1)\big(R_2(\Pi_1) - D\big)\big]dG(\Pi_1) \geq D$$

Assuming that there is perfect competition on the financial market, this constraint will be binding.

The firm's maximization problem can be written as

$$\max_{\{R_1(), R_2(), \beta()\}} \int_0^{\bar{\Pi}_1} \left[\Pi_1 - R_1(\Pi_1) + \delta\beta(\Pi_1)[\Pi^d - R_2(\Pi_1)] \right]dG(\Pi_1)$$

subject to (IC), (LL) and (IR^I).

Before solving the above optimization problem, let us define what I mean by a debt contract.

Definition 1 *A debt contract is a financial contract $\{(R_1(\hat{\Pi}_1), \beta(\hat{\Pi}_1))\}$ with a fixed first-period repayment and where upon repayment the firm is always refinanced. If the firm cannot repay the required fixed amount, he has to give his entire profit to the investor and risks liquidation $(\beta(\hat{\Pi}_1) < 1)$.*

---

[25]This model follows Faure-Grimaud (2000) and only allows the firm to announce below its realized profits. The justification for this restriction on announcements is simply that announcing a certain level of profit consists of showing this amount to the principal. Other models such as Gale and Hellwig (1984) and Townsend (1979) also only allow announcements below the realized level of profits whereas Bolton and Scharfstein does not make this restriction.

The shape of the repayment scheme of a debt contract is illustrated in the figure below.
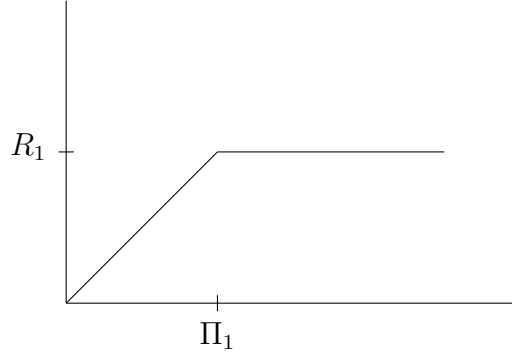

Figure 4: Debt Contract Repayment Scheme

The solution to the Lagrangian optimization is summarized in the following Proposition.

Proposition 3 *(Faure-Grimaud, 2000) The optimal debt contract $\left\{ \left(R_1(\hat{\Pi}_1), R_2(\hat{\Pi}_1), \beta(\hat{\Pi}_1) \right) \right\}$ takes the form of a debt contract with all $R_2(.)$ equal to zero.*

- *If the firm makes high enough profits, it reimburses a fixed amount Π^8 and is never liquidated.*
  $\forall \Pi_1 \geq \Pi_1^*, \beta(\Pi_1) = 1 \text{ and } R_1(\Pi_1) = \Pi_1^*.$

- *If profits are not high enough, the firm has to repay all it has and the probability of refinancing is less than one.*
  $\forall \Pi_1 \leq \Pi_1^*, \beta(\Pi_1) = 1 - \frac{\Pi_1^* - \Pi_1}{\delta \Pi^d} < 1 \text{ and } R_1(\Pi_1) = \Pi_1.$

The fixed repayment is given by the following equation

$$\int_0^{\bar{\Pi}_1} \left[R_1(\Pi_1) - \delta \beta(\Pi_1) D \right] dG(\Pi_1) = D$$

or equivalently

$$\Pi_1^* - \frac{\Pi^d - D}{\Pi^d} \int_0^{\Pi_1^*} G(\Pi_1) d\Pi_1 = (1 + \delta)D \tag{8}$$

Proof: See Appendix.

Here the solution is such that $R_2(\cdot) = 0$. In fact, the more general requirement for the optimal financial contract is that $R_1(\Pi_1) + \delta R_2(\Pi_1) = \Pi_1^*$. The solution in Proposition

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3 is just one way of splitting the repayment between the periods. However, to make sure that the firm will not retract its announcement (claim that it overstated the first-period profit and/or exert cash-diversion between the periods) and refuse to pay the second repayment, it is better for the investor to receive the entire repayment in the first period.

Notice that the financial contract is the same as in Faure-Grimaud (2000). If the cash-constrained firm is sufficiently efficient, it is always refinanced and does not pay his entire first-period gain. If the firm is less efficient, but it produces in the first-period, it is refinanced with a certain probability and has to pay its entire first-period gain from the production. The difference with Faure-Grimaud (2000) is that profits, Π_1 are endogenous and are determined by the procurement contracts. Recall that the financial contract is a best response to a given profit distribution. Proposition 3 characterize the optimal financial contract for a given distribution of profits, but this distribution will be determined by the (best-response) contract offered by the procurement agency. After having characterized the optimal procurement agency, this paper determines the endogenous distribution of profits that occur at equilibrium.

The debt contract described in the above Proposition also resembles the "smoothed debt" contract presented in Hart and Moore (1998). In that model, the focus is slightly different from here and the authors focus on a situation with symmetric unverifiable information and renegotiation. In their setting they derive sufficient conditions for debt contracts to be optimal.

5.2   Optimal procurement contract

The optimal procurement contract is a menu of transfers and quantities for both firms contingent on the types in the corresponding period[26] and the number of active firms in this period. In previous sections, the two firms were active in both periods. Here, there is a possibility that only one firm is active in the second period. However, second-period transfers and quantities are contingent on the current number of firms and therefore separability across periods still hold. I can therefore solve for the optimal second-period part of the contract when there is one active firm left, the optimal second-period part of the contract when there are two active firms left and the optimal first-period part of the contract separately.

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[26]Because types are independent across time, second-period transfers and quantities which can be contingent on first-period types, will at the optimum be independent of first-period types and this contingency is therefore ignored. This is discussed and proved in Section 3.

5.2.1 Optimal second-period procurement contract

• **Both firms are present in the second period:** If both firms are still present on the market and given the repayment scheme for the second period (which I have now shown is always equal to zero), the problem is the same as in Section 3 simply because the repayment scheme for the second period is equal to zero. .

To summarize, an interior solution, $q_{2A}(\theta_2) \in (0, \bar{q})$, is given by

$$-\theta_{2A} - \frac{F}{f}(\theta_{2A}) - \mu q_{2A}(\theta_2) + \theta_{2B} + \frac{F}{f}(\theta_{2B}) + \mu(\bar{q} - q_{2A}(\theta_2)) = 0 \qquad (9)$$

The interpretation of this solution (regardless of whether it is an interior or a corner solution) is exactly the same as in the benchmark case.

• **Only the self-financed is present in the second period:** If only the self-financing firm is left on the market in the second period, it is immediat that itt provides $\bar{q}$ and thus

- $q_{2B}(\theta_{2B}) = \bar{q}$,

- $t_{2B} = \bar{\theta}\bar{q} + \mu\frac{\bar{q}^2}{2}$,

- Profit when being of type θ_{2B}: $\Pi^m(\theta_{2B}) = (\bar{\theta} - \theta_{2B})\bar{q}^{27}$,

- Ex ante (before learning the type) expected profit: $\Pi^m = \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta_{2B}}^{\bar{\theta}} \bar{q} ds dF(\theta_{2B}) \geq \Pi^d = E_{\theta_{2j}} \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta_{2i}}^{\bar{\theta}} q_{2i}(s, \theta_j) ds dF(\theta_{2i})$.

Clearly the principal's surplus when there is only one firm left on the market in the second period is lower than when there are two firms. This is simply because the firm's rent when he is in a monopoly situation is higher than when there is competition.

Note that here the only remaining firm gets the entire production of $\bar{q}$.[28] Here, it is as if demand is inelastic and the cost does not matter. For instance, it is not because only one institution is available to treat drug abusers that the public sector wants to reduce the availability of the treatment. Drug abuse is a serious problem, that needs to be dealt with, regardless of the competition for the provision of this service.

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[27]Here I assume that it is not optimal to shut down the production for the least efficient types.
[28]Because of the assumption that $\bar{q}$ is fixed, there is no room in this model for strategies such that when only one provider is available total provision is reduced.

5.2.2 First-period optimal procurement contract

Incentive compatibility of the first-period part of the contract requires that the procurement contract, $\{q_{1A}(\hat{\theta}_1), q_{1B}(\hat{\theta}_1), t_{1A}(\hat{\theta}_1), t_{1B}(\hat{\theta}_1)\}$, needs to satisfy the following incentive constraints.

$$\theta_{1A} \in \arg\max_{\tilde{\theta}} E_{\theta_{1B}}\Big\{t_{1A}(\tilde{\theta}, \theta_{1B}) - \theta_{1A}q_{1A}(\tilde{\theta}, \theta_{1B}) - \mu\frac{q_{1A}(\tilde{\theta}, \theta_{1B})^2}{2}$$
$$-R_1(\Pi_1(\theta_1, \tilde{\theta})) + \delta\beta(\Pi_1(\theta_1, \tilde{\theta}))\Pi^d\Big\}.$$

Where $\Pi_1(\theta_{1A}, \theta_{1B}, \tilde{\theta}) = t_{1A}(\tilde{\theta}, \theta_{1B}) - \theta_{1A}q_{1A}(\tilde{\theta}, \theta_{1B}) - \mu\frac{q_{1A}(\tilde{\theta},\theta_{1B})^2}{2}$ for the cash-constrained firm (firm A).

Recall from Lemma 1 that $-R_1(\Pi_1) + \delta\beta(\Pi_1)\Pi^d = C$. I can therefore rewrite the firm's incentive constraint as

$$\theta_{1A} \in \arg\max_{\tilde{\theta}} E_{\theta_{1B}}\Big\{t_{1A}(\tilde{\theta}, \theta_{1B}) - \theta_{1A}q_{1A}(\tilde{\theta}, \theta_{1B}) - \mu\frac{q_{1A}(\tilde{\theta}, \theta_{1B})^2}{2} + C\Big\}.$$

The cash-constrained firm completely internalizes the effect of the financial contract and only considers the effect of his first-period type announcement on first-period profits.

Incentive compatibility for the self-financed firm (firm B) can be written as

$$\theta_{1B} \in \arg\max_{\tilde{\theta}} E_{\theta_{1A}}\Big\{t_{1B}(\theta_{1A}, \tilde{\theta}) - \theta_{1B}q_{1B}(\theta_{1A}, \tilde{\theta}) - \mu\frac{q_{1B}(\theta_{1A}, \tilde{\theta})^2}{2} + \delta\Pi(\theta_{1A}, \tilde{\theta})\Big\}$$

where

$$E_{\theta_{1A}}(\Pi(\tilde{\theta})) = E_{\theta_{1A}}\Big\{\beta(\Pi_1(\theta_{1A}, \tilde{\theta}, \theta_{1A}))\Pi^d + (1 - \beta(\Pi_1(\theta_{1A}, \tilde{\theta}, \theta_{1A}))\Pi^m\Big\} - D$$

It is clear that the expected profit of the self-financed firm depends on the financial contract of the cash-constrained firm. If the self-financed firm, through its behavior in the first-period, can reduce the value of $\beta(\theta)$, it can increase its own expected profit from the second-period.

So even if the cash-constrained firm internalizes the effect of the financial contract, its competitor, the self-financed firm, does not and a predatory effect from the financial contract appears in firm B's first-period incentives. This will be of crucial importance in the next Proposition.

The optimal procurement contract also needs to satisfy the participation constraints. For the cash-constrained firm this writes

$$U_{1A}(\theta_{1A}) = E_{\theta_{1B}}\Big\{t_{1A}(\theta_1) - \theta_{1A}q_{1A}(\theta_1) - \mu\frac{q_{1A}(\theta_1)^2}{2} + C\Big\} \geq 0.$$

And the participation constraint of the self-financed firm:

$$U_{1B}(\theta_{1B}) = E_{\theta_{1A}}\left\{t_{1B}(\theta_1) - \theta_{1B}q_{1B}(\theta_1) - \mu\frac{q_{1B}(\theta_1)^2}{2} + \delta\Pi(\theta_1)\right\} \geq 0.$$

Differentiating the incentive compatibility conditions yields, at any differentiability point or $(i,j) \in \{A,B\}^2, i \neq j$,

$$\dot{U}_{1i}(\theta_{1i}) = -E_{\theta_{1j}}q_{1i}(\theta_1)$$

Since $\dot{U}_{1i}(\theta_{1i})$ is decreasing, the participation constraint will be binding for the least efficient type only.

Now the minimization problem can be written as follows

$$\min_{t_{1A}(\cdot),t_{1B}(\cdot),q_{1A}(\cdot),q_{1B}(\cdot)} E_{\theta_1,\theta_2}\Big[t_{1A}(\theta_1) + t_{1B}(\theta_1) + \delta\beta(\Pi_1(\theta_1))\left(t_{2A}(\theta_2) + t_{2B}(\theta_2)\right)$$
$$+ \delta(1 - \beta(\Pi_1(\theta_A,\theta_B)))\left(\bar{\theta} + \mu\frac{\bar{q}^2}{2}\right)\Big]$$

subject to

$$\dot{U}_{1i}(\theta_{1i}) = -E_{\theta_{1j}}q_{1i}(\theta_1) \qquad\qquad (i,j) \in \{A,B\}, i \neq j$$
$$U_{1i}(\bar{\theta}) = 0 \qquad\qquad i \in \{A,B\}$$
$$\bar{q} = q_{1A}(\theta_1) + q_{1B}(\theta_1)$$

Integrating the local first-order conditions yields, $\forall(i,j) \in \{A,B\}^2, i \neq j$,

$$U_{1i}(\theta_{1i}) = U_{1i}(\bar{\theta}) + E_{\theta_{1j}}\int_{\theta_{1i}}^{\bar{\theta}} q_{1i}(s,\theta_{1j})ds.$$

Furthermore, recall that the definitions of expected second-period transfer when there is only one firm left on the market, T_2^d, and when there are two firms around, T_2^d.

$$T_2^m = \bar{\theta}\bar{q} + \mu\frac{\bar{q}^2}{2}$$
$$T_2^d = E_{\theta_2}[t_{2A}(\theta_2) + t_{2B}(\theta_2)]$$

The relaxed minimization problem can thus be rewritten

$$\min_{q_{1A}(\cdot),q_{1B}(\cdot)} E_{\theta_1}\Big\{\theta_{1A}q_{1A}(\theta_1) + \frac{F}{f}(\theta_{1A})q_{1A}(\theta_1) + \mu\frac{q_{1A}(\theta_1)^2}{2}$$
$$+ \theta_{1B}(\bar{q} - q_{1A}(\theta_1)) + \frac{F}{f}(\theta_{1B})(\bar{q} - q_{1A}(\theta_1)) + \mu\frac{(\bar{q} - q_{1A}(\theta_1))^2}{2}$$
$$- \delta\beta(\Pi_1(\theta_1))(\Pi^d - \Pi^m - T^d + T^m) - \delta\Pi^m + \delta T^m + \delta D.\Big\}$$

Recall that $\beta(\cdot)$, the probability of non-liquidation, depends on realized first-period profits and therefore on first-period quantities and transfers. However, so far this probability has been defined as a function of realized profits which are endogenous. Replacing the endogenous variable Π_1 by its expression in terms of firm A's first-period utility yields

$$\Pi_1 = U_{1i}(\theta_1) - C. \tag{10}$$

Furthermore, the condition $\Pi_1 \geq \Pi_1^*$ can be replaced by $\theta_{1A} \leq \theta^*(\theta_{1B})$ where $\theta^*(\theta_{1B})$ is such that $\Pi_1(\theta^*(\theta_{1B}), \theta_{1B}) = \Pi_1^*$. The probability of non-liquidation can therefore be defined in the following way.

$$\beta(\theta_1) = \begin{cases} 1 & \text{if } \theta_{1A} \leq \theta^*(\theta_{1B}), \\ 1 - \frac{\Pi_1^* - U_{1i}(\theta_{1i}) + C}{\delta D} & \text{otherwise.} \end{cases}$$

Using this,

$$E_{\theta_1}\beta(\theta_1) = \int_{\theta_1, \Pi_1(\theta_1) \geq \Pi_1^*} dF(\theta_{1A})dF(\theta_{1B}) + \int_{\theta_1, \Pi_1(\theta_1) < \Pi_1^*} \left(1 - \frac{U_{1i}(\theta^*(\theta_{2i}))}{\delta D}\right) dF(\theta_{1A})dF(\theta_{1B})$$
$$+ \frac{1}{\delta D} \int_{\theta_1, \Pi_1(\theta_1) < \Pi_1^*} U_{1i}(\theta_{1A})dF(\theta_{1A})dF(\theta_{1B}).$$

Integrating the third term by parts and rearranging terms yield

$$E_{\theta_1}\beta(\theta_1) = 1 - \frac{\Pi^d}{D} - \frac{1}{\delta D}E_{\theta_{1B}}\left(\int_{\theta^*(\theta_{1B})}^{\bar{\theta}} \frac{F}{f}(\theta_{1A})q_{1A}(\theta_1)dF(\theta_{1A})\right).$$

Ignoring all constant terms, the maximization problem can now be written as

$$\min_{q_{1A}(\cdot), q_{1B}(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left(\theta_{1A}q_{1A}(\theta_1) + \frac{F}{f}(\theta_{1A})q_{1A}(\theta_1) + \mu\frac{q_{1A}(\theta_1)^2}{2}\right.$$
$$\left. + \theta_{1B}(\bar{q} - q_{1A}(\theta_1)) + \frac{F}{f}(\theta_{1B})(\bar{q} - q_{1A}(\theta_1)) + \mu\frac{(\bar{q} - q_{1A}(\theta_1))^2}{2}\right) dF(\theta_{1A})dF(\theta_{1B})$$
$$+ \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta^*(\theta_{1B})}^{\bar{\theta}} \left(\frac{1}{D}\frac{F}{f}(\theta_{1A})q_{1A}(\theta_1)\left(\Pi^d - \Pi^m - T^d + T^m\right)\right) dF(\theta_{1A})dF(\theta_{1B}).$$

Piecewise optimization yields the following two conditions for interior solution. If $\theta_{1A} \leq \theta^*(\theta_{1B})$ then the optimal quantity $q_{1A}(\theta_1)$ satisfies:

$$-\theta_{1A} - \frac{F}{f}(\theta_{1A}) - \mu q_{1A}(\theta_1) + \theta_{1B} + \frac{F}{f}(\theta_{1B}) + \mu(\bar{q} - q_{1A}(\theta_1)) = 0, \tag{11}$$

If $\theta_{1A} > \theta^*(\theta_{1B})$ then the optimal quantity $q_{1A}(\theta_1)$ satisfies:

$$-\theta_{1A} - \frac{F}{f}(\theta_{1A}) - \mu q_{1A}(\theta_1) + \theta_{1B} + \frac{F}{f}(\theta_{1B}) + \mu(\bar{q} - q_{1A}(\theta_1))$$
$$+ \frac{1}{D}\frac{F}{f}(\theta_{1A})(\Pi^d - \Pi^m - T^d + T^m) = 0, \tag{12}$$

In the case where the cash-constrained firm is sufficiently efficient compared to the self-financed firm, the condition giving the optimal quantity is the same as when firms are symmetric in their financial structure.

However, when the cash-constrained firm is not sufficiently efficient compared to its competitor, the condition changes slightly. But it can be interpreted in the same way as the previous conditions for q_{2A}. If there exists a $q_{1A}(\theta_1) \in (0, \bar{q})$ that solves (12), then the optimal solution for the procurement agency is to adopt dual sourcing with firm A producing $q_{1A}(\theta_1)$ given by (12) and firm B producing $q_{1B}(\theta_1) = \bar{q} - q_{1A}(\theta_1)$. Furthermore, this means that the virtual marginal cost of firm A equals that of firm B plus an extra term. If the left-hand side (LHS) of (12) is positive $\forall q_{1A} \in (0, \bar{q})$, then sole sourcing by firm A is optimal and $q_{1A}(\theta_1) = \bar{q}$. If the left-hand side (LHS) of (12) is negative $\forall q_{1A} \in (0, \bar{q})$, then sole sourcing by firm B is optimal and $q_{1B}(\theta_1) = \bar{q}$.

Define the cash-constrained firm's modified virtual marginal cost as its virtual marginal cost plus the bias term (which can be positive or negative. When $\theta_{1A} \leq \theta^*(\theta_{1B})$, the virtual marginal cost and the modified virtual marginal cost coincide.

Proposition 4 summarizes these findings.

Proposition 4   • *Suppose that one of the firms (here, firm A) faces a financial constraint and that there is an interior solution to the procurement agency optimization problem. Then, in the first period, the procurement agency chooses $q_{1A}(\theta_1)$ according to (11) when the cash-constrained firm is relatively efficient. When the cash-constrained firm is not sufficiently efficient, it is optimal for the procurement agency to slightly bias the procurement rule following (12).*

$q_{1B}(\theta_1)$ follow from $q_{1B}(\theta_1) = \bar{q} - q_{1A}(\theta_1)$.

The optimal $q_{1A}(\theta_1)$ and $q_{1B}(\theta_1)$ are such that the (possibly modified) virtual marginal cost of firm A equals the virtual marginal cost of firm B.

• *In the second-period, the static optimal solution remains optimal.*

If firm A is sufficiently efficient, the split of production remains the same as in the benchmark case without financial contracts. This is because when the profits of the cash-constrained firm is high enough he is always refinanced. However, when the self-financed firm is not efficient enough, condition (12) takes into account the effect of the split of production on the financial contract (or more precisely on the probability of refinancing) and thus the effect on second-period surplus and profits. The sign of the

bias term is ambiguous (even in the linear case). If the effect on net gain for the procurement agency in the second period is higher than the effect from firm B being more aggressive (requiring a lower transfer in the first period), then firm A will be asked to produce at a higher level than in the benchmark. This will increase its probability of being refinanced as well as the expected surplus of the procurer in the second period because the probability of more competition increases. In the opposite case, when the gain from firm B being more aggressive today outweighs the gains from increased competition tomorrow, then the interior solution for $q_{1A}(\theta_1)$ is such that the virtual cost of B is lower than the virtual cost of firm A and firm A will therefore be asked to produce at a lower level in the first procurement contract.

In fact, I identify three different effects that influence the trade-off between favoring firm A or firm B.

Define

$$P \equiv - \{\Pi^d - \Pi^m - T^d + T^m\}$$
$$\equiv V^m + \Pi^m - (V^d + \Pi^d).$$

If P is positive, then the procurement contract will favor[29] firm B, but if P is negative firm A will be favored. This can be seen in the graph below.
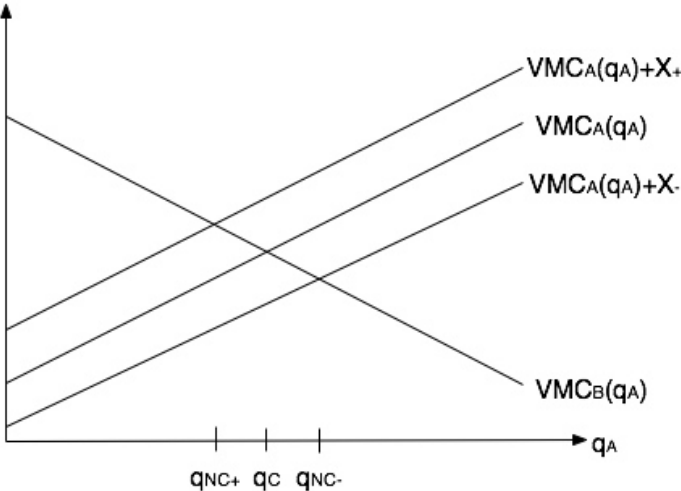

Figure 5: Virtual Marginal Costs

To determine the effects in P, I use the fact that V^i is equal to expected surplus minus expected transfer(s) and expected profits are equal to expected transfers minus

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[29]Here favored means increased production by the favored firm.

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expected costs.

$$P = \int_{\Theta^2} \left\{ S(\bar{q}) - t_{2B}(\bar{q}) + t_{2B}(\bar{q}) - \theta_{2B}\bar{q} - \mu\frac{\bar{q}^2}{2} - \right.$$
$$\left[S(\bar{q}) - t_{2A}(q_{2A}(\theta_2)) - t_{2B}(\bar{q} - q_{2A}(\theta_2)) + t_{2B}(\bar{q} - q_{2A}(\theta_2)) \right.$$
$$\left. \left. - \theta_{2B}(\bar{q} - q_{2A}(\theta_2)) - \mu\frac{(\bar{q} - q_{2A}(\theta_2))^2}{2} \right] \right\} dF(\theta_{2A})dF(\theta_{2B})$$
$$= \int_{\Theta^2} \left\{ \left(\theta_{2A} - \theta_{2B} + \frac{F}{f}(\theta_{2A}) \right) q_{2A}(\theta_2) + \mu q_{2A}(\theta_2)(q_{2A}(\theta_2) - \bar{q}) \right\} dF(\theta_{2A})dF(\theta_{2B})$$

Denote $p(\theta_2)$ the integrand of this integral. If the solution for $q_{2A}(\theta_2)$ is interior, using (9) I get

$$p(\theta_2) = \frac{F}{f}(\theta_{2B})q_{2A}(\theta_2) - \mu q_{2A}(\theta_2)^2.$$

I call the first term in this expression the *rent* effect and the second term the *convexity* effect. It is easy to see that the second term is negative. Thus the convexity effect favors firm A. Since convex costs in itself is a reason to consider dual sourcing, favoring firm A increases the probability that both firms are active in the second period and thus increases the probability that the dual sourcing option is available in this period.

In the definition of P, the procurement agency's expected surplus and firm B's expected profits are taken into account. Firm A's expected profit does not enter into this expression. Having both the procurement agency's expected surplus and firm B's expected profits in the equations means that the procurement agency does not care about the amount of rent given to this firm (the terms cancel out) because this amount is recovered by the procurement agency through firm B's more aggressive bidding in the first period. This is not the case for firm A, and the principal takes into account that giving up rent to this firm cannot be recovered and is therefore costly. The rent effect is therefore positive and favors firm B.

If the solution is such that $q_{2A}(\theta_2) = 0$, then $p(\theta_2) = 0$. However, if the solution is such that $q_{2A}(\theta_2) = \bar{q}$ then

$$p(\theta_2) = \frac{F}{f}(\theta_{2B})\bar{q} + (\theta_{2A} - \theta_{2B})\bar{q}.$$

Again there is a rent effect, Here, it is actually stronger because since $q_{2A}(\theta_2) = \bar{q}$. The second effect is an *sampling* effect, which is negative because $q_{2A}(\theta_2)$ is equal to $\bar{q}$ only when firm A is more efficient than firm B. Therefore the effect favors firm A. In fact, to make sure that there is competition in the second period, which itself implies that the

probability of drawing a low marginal cost for the industry is higher, the procurement agency is willing to bias the procurement in favor of firm A.

Clearly the result in Proposition 4 departs from the Modigliani-Miller Theorem (Modigliani and Miller, 1958) which asserts that, under certain conditions, the value of the firm is independent of its financial structure. In this paper, it is the interaction between the financial contract and the procurement contracts that allows us to deviate from the Modigliani-Miller world[30]. For a detailed introduction to financial contracting and its references, see Harris and Raviv (1992).

Notice also that if the procurement agency changes from one period to the other, this implies that $V^m = V^d = 0$. In this case, the first-period procurement agency cares only about the outcome of this period. It will not take into account the effect on future surplus since it does not benefit from it (another agency will) and does only care about gain from firm B being more aggressive and accepting a lower first-period transfer. It will therefore lower the production of the cash-constrained firm in order to decrease the probability of refinancing, which leads the self-financed firm to accept a lower first-period transfer. If one assume that the different procurement agencies are responsible for the procurement of goods for different public sectors, it is straightforward to conclude that with the dynamic view of procurement that is presented in this paper, there are gains to be made from centralizing procurement (simple externality argument).

Corollary 1 *Centralized procurement can be beneficial.*

5.2.3 Equilibrium

As mentioned at the beginning of this section, the financial contract and the procurement contract are best-replies in the game developed in this paper. However, in the presentation of the optimal procurement contract above I have already taken into account the interaction with the financial contract. This has been done by replacing variables and function related to the financial contract by their corresponding values in the financial contract and by using Lemma 1. As pointed out in Section 5.1, the optimal financial contract is characterized for a given distribution of realized profits. But these profits are endogenous and depend on the optimal procurement contract. In order to fully characterize the Nash equilibrium of this game, it remains to characterize this distribution function.

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[30]For other papers where the financial structure fo the firm matters, see the literature starting with Kraus and Litzenberg (1973) and Scott (1976)

Sections 5.2.1 and 5.2.2 pin down the optimal split of production in each period, but because incentive compatibility is required before the firm learns the type of its competitor there exists an infinity of solutions for the associated transfers. This was discussed briefly in Section 3. To be able to characterize an equilibrium I focus on transfers that are not only Bayesian incentive compatible but are also dominant strategy incentive compatible (Mookherjee and Reichelstein, 1992). In this setting a dominant strategy incentive compatible transfers are such that

$$t_{1A}(\theta_1) = \theta_{1A}q_{1A}(\theta_1) + \mu\frac{q_{1A}(\theta_1)^2}{2} + \int_{\theta_{1A}}^{\bar{\theta}} q_{1A}(s, \theta_{1B})ds - C,$$

$$t_{2B}(\theta_1) = \theta_{1B}q_{1B}(\theta_1) + \mu\frac{q_{1B}(\theta_1)^2}{2} + \int_{\theta_{1B}}^{\bar{\theta}} q_{1B}(s, \theta_{1A})ds - \delta\Pi(\theta_1),$$

$$t_{2i}(\theta_2) = \theta_{2i}q_{2i}(\theta_2) + \mu\frac{q_{2i}(\theta_2)^2}{2} + \int_{\theta_{2i}}^{\bar{\theta}} q_{2i}(s, \theta_{2j})ds.$$

Using these transfers, the first-period realized profits of the cash-constrained firm can be written

$$\Pi_{1A}(\theta_1) = \int_{\theta_{1A}}^{\bar{\theta}} q_{1A}(s, \theta_{1B})ds - C$$

Recall the definition of $G(\Pi)$,

$$G(\Pi) \equiv Prob\{(\theta_1), \Pi_{1A}(\theta_1) \leq \Pi\}.$$

This yields

$$G(\Pi) = \int_{\underline{\theta}}^{\bar{\theta}} Prob\left\{\int_{\theta_{1A}}^{\bar{\theta}} q_{1A}(s, \theta_{1B})ds - C \leq \Pi\right\} dF(\theta_{1B}).$$

Define $\hat{\theta}(\theta_{1B}, \Pi)$ as the solution to the following equation

$$\int_{\hat{\theta}_{1A}(\theta_{1B},\Pi)}^{\bar{\theta}} q_{1A}(s, \theta_{1B})ds = \Pi + C.$$

Since $\int_{\theta_{1A}}^{\bar{\theta}} q_{1A}(s, \theta_{1B})ds$ is a decreasing function of θ_{1i} I can rewrite $G(\Pi)$ as

$$G(\Pi) = \int_{\underline{\theta}}^{\bar{\theta}} Prob\left\{\theta_{1A} \geq \hat{\theta}_{1A}(\theta_{1B}, \Pi)\right\} dF(\theta_{1B}).$$

Rearranging this expression yields

$$G(\Pi) = 1 - \int_{\underline{\theta}}^{\bar{\theta}} F(\hat{\theta}_{1A}(\theta_{1B}, \Pi))dF(\theta_{1B}). \tag{13}$$

An equilibrium of this game is thus characterized by the optimal contracts presented in previous sections and the distribution of realized first-period profits given by (13).

Proposition 5 *At equilibrium and with transfers that are dominant-strategy incentive compatible, the distribution of realized first-perid profits is characterized by (13).*

• **Example:** In the case where θ_{ij} are distributed uniformly on $[\underline{\theta}, \bar{\theta}]$ the distribution of realized first-period profits is given by

$$G(\Pi) = 1 + \frac{\underline{\theta}}{\bar{\theta} - \underline{\theta}} - \frac{1}{(\bar{\theta} - \underline{\theta})^2} \int_{\underline{\theta}}^{\bar{\theta}} \hat{\theta}_{1A}(\theta_{1B}, \Pi) d\theta_{1B}, \tag{14}$$

where $\hat{\theta}_{1A}(\theta_{1B}, \Pi)$ solves

$$\hat{\theta}_{1A}(\theta_{1B}, \Pi)^2 - \left(\mu\bar{q} + 2\theta_{1B} - P\right)\hat{\theta}_{1A}(\theta_{1B}, \Pi) + \left((\mu\bar{q} + 2\theta_{1B} - P)\bar{\theta} - \bar{\theta}^2 + 2\mu(\Pi - C)\right) = 0$$

5.3 Myopic Firms

In the remaining of this section, I will compare the above results to an environment where the firms are myopic, meaning that they only care about the current-period profit. There are several reasons why I do this. First it is an interesting (second) benchmark that points out the importance of firms having a dynamic vision for my results to apply. This is very useful if the result are to be used correctly in more policy oriented work. Myopia can also be explained by the managers of the competing firm taking a short-term view on profits. In other words, managers care about current results, because it shows how good managers they are. They care less about long-term goals and profits and therefore fail to take into account the kind of *predation* that would allow self-financed firms to get rid of future competitors. Finally, myopia could come from the shareholders in the firm who care about the current dividends only and who do not allow the firm or its managers to sacrifice current profit to increase future expected profits.

In a setting with myopic firms it is straightforward to show that the behavior and outcome of the second-period part of the procurement contract will be the same as in the model(s) presented previously. However, since the firms do not take into account the second-period gains when competing in the first-period, the outcome of the first-period procurement contract changes. In fact, since the self-financed firm is no longer

competing aggressively, there is no gains from increasing his production (it will not give away second-period gains in the first-period transfer). So the "new" term in condition (12) is now positive and the first-period contract will be biased in favor of the cash-constrained firm in order to increase future expected surplus.

When looking at the constraints related to the financial contract presented in Subsection 5.1, it can be seen that the constraints are not affected by the myopic behavior in the procurement stage and thus the optimal financial contract remains the same. I assume of course that when players write a financial contract they are aware of the two periods. Otherwise a financial contract in itself would be impossible, since the the firm would then systematically strategically default on its repayment.

Therefore, the only difference (compared to the previous analysis) when bidders are myopic is that the sign of the bias in the first-period procurement contract is unambiguously positive and favors the cash-constrained firm.

6   Conclusion

This paper studies dynamic procurement design and the impact of firm's financial structure and strength on the optimal procurement design. In a two-period model, at each period a procurement contracts splits the production of a good between two firms. Both sole and dual sourcing are allowed and I show when dual sourcing is occurs. In the current model, firms differ in their ability to self-finance their presence in the market. Before entering the market, a cash-constrained firm need to sign a financial contract with an investor. The actual number of competitors in the second-period procurement stage will be endogenous and depends on both the financial contracts and the outcome of the first period. I study the optimal financial contract for the cash-constrained firm and the optimal procurement contract in a setting with both self-financed and cash-constrained firms. The main result is that it is optimal for the procurement agency to take into account the financial structure of the competing firms. However, which firm to favor in the first-period procurement is ambiguous and depends on three different effects which I have called the *sampling* effect, the *rent* effect and the *convexity* effect.

One possible extension of this paper is to reduce the commitment power of the procurement agency. So far I have assumed that the principal is able to offer a two-period contract. It would be interesting to weaken this assumption and study what happens

if a spot contract is offered in each period. This would also allow new strategies for the firms. A self-financed firm would in this case have the option of being inactive in the first-period (so the procurement agency cannot extract all of its future rent in the first period) and this would in turn affect the procurement agency's provision rule.

Another extension would be to analyze whether the self-financed firm would prefer or not to acquire debt or other external funding in these kinds of environments. Since-which firm to favor in the first-period procurement is ambiguous, it is not straightforward to conclude that the self-financed firm would also like to acquire debt in order to make the procurement symmetric. In the case where the procurement procedure is biased towards the self-financed firm[31], it can be shown that this is not the case. In the opposite case and if this bias is sufficiently large, it might be optimal for the self-financed firm to acquire debt so that the procurement is no longer biased towards the other firm. In the current paper, this possibility has not been explored. This is partly because it is not enough for the self-financed firm to acquire debt, it also needs to credibly commit to leave the market after the first period if its profits are not high enough. However, there are positive expected profits to be earned in the second period and the profit-maximizing self-financed firm needs to credibly commit not to go for these profits under some circumstances and it is not clear how a profit-maixmizing firm can commit to this.

It would also be interesting to extend this paper to a situation where firms have different weights in the welfare function. For political reasons a procurement agency or a government may be more favorable towards certain firms than other. Another interesting extension would be to change the timing of the game so that one of the principals become a Stackelberg leader and analyze how this effects the optimal procurement design.

Appendix

Proof of Proposition 3:
To characterize the optimal financial contract, the maximization of expected firm surplus[32] is subject to limited liability and incentive compatibility as well as individual

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[31]Because the procurement agency can extract enough rent from this firm in the first-period to make up for efficiency losses in the second period.

[32]Because of competition on the financial market the firm has all the bargaining power concerning the financial contract. See Bolton and Scharfstein (1990) for the optimal financial contract under the alternative assumption where the investor has all the bargaining power.

rationality by the investor.

Notice that any potential repayment in the second period must come from the first-period profit since there is no incentives for the firm to report truthfully its second-period profits if its repayment is contingent on this. This is because there is no possibility of using a threat of non-refinancing the firm in future periods to act as a incentive device to truthfully report profits in the current period. In the second period, a firm will always strategically default on its repayment. And so any repayment in the second period can without loss of generality be included in the corresponding first-period repayment and $R_2(\cdot) = 0$.

Given that the adverse selection parameter concerns the profits of the firm, it is impossible to falsify the claim "upward". In other words, the firm cannot claim to have higher profits than it actually has since it might not be able to make the repayment associated with this claim.

Recall from Lemma 1 that $-R_1(\Pi_1) + \delta\beta(\Pi_1)\Pi^d = C$[33]

Using Lemma 1, I can define the Lagrangian as

$$L(C, \beta(\Pi_1)) = \big[\Pi_1 + C + \alpha(-C + \delta\beta(\Pi_1))(\Pi^d - D)\big]g(\Pi_1)$$
$$- \lambda_0\Big[\beta(\Pi_1) - \frac{\Pi_1 + C}{\delta\Pi^d}\Big] - \lambda_1\big[\beta(\Pi_1) - 1\big] + \lambda_2\beta(\Pi_1)$$

where α is the multiplier associated with the individual rationality constraint. λ_0 is the multiplier associated with the limited liability constraint. λ_1 (respectively λ_2) is the multiplier associated with the requirement $\beta(\Pi_1) \leq 1$ (respectively $\beta(\Pi_1) \geq 0$).

The first-order conditions are

$$\frac{\partial L}{\partial C} = (1 - \alpha)g(\Pi_1) + \frac{\lambda_0}{\delta\Pi^d} = 0$$
$$\frac{\partial L}{\partial \beta(\Pi_1)} = \alpha\delta(\Pi^d - D)g(\Pi_1) - \lambda_0 - \lambda_1 + \lambda_2 = 0$$

Notice first that $\alpha \neq 0$. In fact, $\alpha = 0$ would imply that $\frac{\partial L}{\partial C} > 0$ which is impossible.

Furthermore, it is not possible to have $\lambda_0 = \lambda_1 = 0$ because that would imply that $\frac{\partial L}{\partial \beta(\Pi_1)} > 0$ (since $\alpha \neq 0$). So either $\beta(\Pi_1) = 1$ or $\beta(\Pi_1) = \frac{\Pi_1 - C}{\delta\Pi^d}$.

Define Π_1^* to be such that $\Pi_1^* = \delta\Pi^d - C$.

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[33]Th eproof of this is immediate, but a detailed proof of this Lemma can be found in Chapter 3 of Faure-Grimaud's Ph.D. Thesis (University of Toulouse, 1995).

$\forall \, \Pi_1 < \Pi_1^*$, it is impossible to have $\beta(\Pi_1) = 1$ since it would violate limited liability. In fact, limited liability implies $\Pi_1^* - \Pi_1 \leq 0$. However, since $\Pi_1 < \Pi_1^*$ this gives a contradiction. So $\beta(\Pi_1) = \frac{\Pi_1 - C}{\delta \Pi^d}$. By definition of Π_1^*, I finally get $\beta(\Pi_1) = 1 - \frac{\Pi_1^* - \Pi_1}{\delta \Pi^d} < 1$.

$\forall \, \Pi_1 > \Pi_1^*$, limited liability is always satisfied for the same reason that it did not hold for $\Pi_1 < \Pi_1^*$. So $\lambda_0 = 0$ which implies that $\beta(\Pi_1) = 1$ (since λ_0 and λ_1 cannot simultaneously be equal to zero). From the definition of Π_1^* I get $R_1(\Pi_1) = \Pi_1^*$.

To complete the proof, I need to find C as big as possible or equivalently $\hat{\Pi}_1$ as big as possible) such that individual rationality is satisfied.

Formally, Π_1^* is such that

$$\int_0^{\bar{\Pi}_1} \left[R_1(\Pi_1) - \delta \beta(\Pi_1) D \right] dG(\Pi_1) = D,$$

or, equivalently

$$\Pi_1^* - \frac{\Pi^d - D}{\Pi^d} \int_0^{\Pi_1^*} G(\Pi_1) d\Pi_1 = (1 + \delta)D.$$

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