What do frictions mean for $Q-$theory?

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Abstract

This paper proposes an alternative approach to test investment equations based on a Q-theory model of investment with both real and financing frictions. The paper provides strong empirical evidence that the Q-theory significantly explains both firms' investment and external financing decisions. The empirical tests include both reduced form and structural estimation. In the model and in the data, (i) firms' investment and external financing policies are significantly related to both average Q and cashflows; (iii) firms' investment and external financing policies are weakly concave in average Q, and weakly convex in cashflows; (iv) investment is subject to non linear measurement error; and (v) investment is infrequently negative. The structural estimation quantifies firms' adjustment costs of investment, fixed costs of investment, and convex costs of external financing for the cross section of industries.

Keywords: Tobin's Q, financing frictions, real frictions, investment, financing frictions, cashflows.

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Introduction

Ever since Hayashi (1982) derived the frictionless neoclassical investment model referred as the Q-theory of investment, the literature has considered multiple theoretical extensions and alternative empirical approaches to improve on its empirical performance. The Q-theory predicts that the shadow price of capital, or marginal q, is a sufficient statistic for investment. Since marginal q is unobservable, Tobin's average Q, the ratio of equity plus debt value to the replacement cost of capital, is commonly used as an empirical proxy. Despite its strong theoretical underpinnings, Caballero (1997) surveys that the coefficients on average Q are frequently insignificant, while measures of liquidity such as cashflows enter positively and significantly in investment regressions.

The question of why average Q is frequently insignificant and not sufficient to explain investment remains central in financial economics. Since Fazzari, Hubbard and Petersen (1988), the empirical corporate finance literature has explored multiple approaches to test investment equations as a function of average Q and cashflows.[1] However, empirical practices are usually based on working assumptions which date back from Hayashi (1982), and which may not hold; for instance, that investment is linear in average Q, or that average Q is a good proxy for firms' growth opportunities. Similarly, Cochrane (1991) has derived an identity between investment and stock returns based on the Q-theory, which is heavily used by asset pricers to explain regularities in stock returns.[2] Yet it remains a puzzle why Cochrane's identity holds in the data, while the Q-theory does poorly on the investment side.

This paper proposes an alternative approach to test investment equations based on a Q-theory model of investment with both real and financing frictions. The paper provides strong empirical evidence that the Q-theory significantly explains both firms' investment and external financing decisions. The empirical tests include both reduced form and structural estimation. In the model and in the data, (i) firms' investment and external financing policies are significantly related to both average Q and cashflows; (iii) firms' investment and external financing policies are weakly concave in average Q, and weakly convex in cashflows; (iv) investment is subject to non linear measurement error; and (v) investment is infrequently negative. The structural estimation quantifies firms' unobservable costs of investment and external financing for the cross section of industries.[3]

The main contributions of this paper are three. First, the paper provides a tractable

[1] See Caballero (1997) for a survey, and also more recent publications such as Erickson and Whited (2000), Arellano (2003), and Almeida, Campello and Galvao (2010).

[2] See, for instance, Cochrane (1996), Zhang (2005), and Liu, Whited and Zhang (2010).

[3] The industry definition is based on the classification by Fama and French. See Appendix for details.

dynamic model of financed investment with several new testable implications. The model extends the framework by Abel and Eberly (1994) for all equity financed firms, to one in which firms are subject to convex adjustment costs of investment, fixed costs of investment, convex costs of external financing, and have defaultable long term debt. As in Gomes (2001), the paper elaborates on how external financing affects investment without discriminating between equity and debt issues. In contrast with Hayashi (1982) and Abel and Eberly (1994), the model predicts that marginal q *and* cashflows are sufficient statistics for investment when firms rely on external funds.

More importantly, the model adds to the literature as it provides testable implications on how firms' investment and external financing issues are *nonlinear.* Firms' investment and external financing policies are weakly concave in average Q, and weakly convex in cashflows. When firms rely on external funding, higher investment rates imply higher marginal costs of external funding; hence investment is less sensitive to Q and more sensitive to cashflows for higher levels of investment. Furthermore, due to firms' fixed costs of investment, firms' investment and external financing policies are lumpy, and more financially constrained firms are more reluctant to reinitiate investment.

The second main contribution is to derive an empirical approach that is consistent with the proposed theoretical framework. The paper proposes an easily implementable approach to test for non linear investment policies in the presence of frictions. Various studies consider theoretical extensions of non linear investment policies in the presence of either real or financing frictions;[4] yet these studies are usually silent about how to test their predictions empirically.[5] Conversely, the empirical literature has explored the empirical shortcomings of the Q-approach; yet these studies assume that investment is linear in q, and then focus on corrections for measurement error in q, endogeneity, and dynamic panel effects.[6]

The model predicts that investment is weakly concave in marginal q, and that in the presence of frictions marginal q is a noisy proxy of average Q. Put together, this implies

[4] Abel and Eberly (1994) derive a dynamic investment model with convex costs of adjustment and pseudo-fixed costs of investment. Caballero and Leahy (1996) elaborate on the impact of fixed costs on investment policies. Hennessy (2004) and Hennessy, Levy and Whited (2006) characterize how different financing frictions affect investment. Bolton, Chen and Wang (2010) consider a Q-theory model with implications for firms' investment, debt, equity, and cash policies.

[5] Two exceptions are Hennessy (2004) and Hennessy, Levy and Whited (2006), whose empirical approach is based on Erickson and Whited (2000).

[6] Erickson and Whited (2000) derive a higher order moments estimation approach based on the premises that investment is linear in q, and measurement error is non linear. Arellano (2003) surveys alternative econometric approaches to test investment models in panel data. Almeida, Campello and Galvao (2010) compare alternative econometric approaches to test Q-models.

that firms' investment policies are subject to non linear measurement error in q. In contrast with the existing literature on measurement error, however, the empirical approach to test the corresponding investment equation does not require instruments or the use of higher order moments estimators. The paper provides a simple expression in closed form for log likelihood function of the optimal investment policy with average Q as the dependent variable, and as a linear function of investment and cashflows.

The proposed log likelihood function provides a tool to test for investment policies in the presence of measurement error in q, real frictions, and financing frictions. With average Q as the dependent variable, measurement error affects the mean and variance of the estimation residuals.[7] The curvatures of firms' adjustment costs and firms' costs of external financing impose parameter constraints on the estimation coefficients. Finally, the log likelihood function accounts for firms' fixed costs of investment by incorporating a correction for truncation bias. Just as in truncated regression models, inertia regions imply that investment is not significantly related to average Q unless average Q is sufficiently high.[8]

The third and last main contribution of this paper relies on its empirical evidence. Results provide strong support on all testable predictions using multiple estimation techniques. These include non-parametric tests, OLS regressions, truncated regressions, and the structural estimation approach using maximum likelihood. Q-theory applies broadly to investment and external financing policies. Investment is weakly concave in Q, and weakly concave in cashflows. Net external issues are weakly concave in Q, and weakly concave in cashflows. Furthermore, the sensitivity of investment to Q increases in truncated regressions, which correct for the positive skewness of investment in the working sample.[9]

The structural estimation quantifies the curvature of firms' adjustment costs of investment; the curvature of firms' convex costs of external financing; and the magnitude of firms' fixed costs of investment. These point estimates are computed for multiple sample splits including all firms on average, financially constrained and unconstrained firms, and by industry. In the data, more financially constrained firms have higher costs of external financing. Results also show significant variation in firms' costs of investment and external financing for the cross section of industries.

The empirical section concludes by showing how to use the estimated parameters on frictions to construct proxies for firms' unobservable marginal q; the expected wedge between marginal q and average Q; firms' adjustment costs of investment; and firms' costs of external financing. The corresponding findings are that measurement error in q is significantly different

[7] See Arellano (2003) and Section III.

[8] This last statement also assumes asymmetric costs of investment and disinvestment. See Section II.

[9] Barnett and Sakellaris (1998) also find that investment is infrequently negative in their sample.

from zero. Industries with a higher marginal product of capital have higher investment rates. And industries with high dividend payout ratios, larger firms, and higher average indices of financial constraints have higher costs of external funding.[10]

The paper is structured as follows. The model of financed investment is constructed in Section I. I derive its main testable implications and the empirical approach in Section II. Section III provides the empirical evidence. Section IV concludes.

Related literature

The paper relates to numerous contributions on the economics and finance literatures. The model is most closely related to Abel and Eberly (1994) and Hennessy, Levy and Whited (2006). Abel and Eberly (1994) focus on all equity financed firms subject to real frictions; Hennessy, Levy and Whited (2006) introduce equity and debt related frictions to the standard Q-model. This paper combines both set-ups, and elaborates on the interaction between both types of frictions.

The model is also closely related to Gomes (2001) and Bolton, Chen and Wang (2010). An important difference with respect to Gomes (2001) is that this paper assumes convex (as opposed to linear) costs of external financing. In line with Bolton, Chen and Wang (2010), this paper shows that Q-theory applies broadly to investment and financing policies. In contrast with Bolton, Chen and Wang (2010), this model only focuses on investment and external financing, and its tractability is meant for the sake of empirical tests.

The result that both average Q and cashflows are sufficient statistics for investment when firms rely on external financing relates to Fazzari, Hubbard and Petersen (1988).[11] The predictions that Q is a noisy proxy of marginal q, and that investment is subject to nonlinear measurement error relate to Erickson and Whited (2000). An important difference with respect to Erickson and Whited (2000) is that investment is subject to non linear measurement error *because* it is weakly concave in q.[12] The model also relates to Almeida and Campello (2010), who observe that the sensitivity of cashflows to external financing issues depends on the interaction between investment and financing.

The empirical results on non linear investment policies relate to Barnett and Sakellaris (1998). Barnett and Sakellaris (1998) assess empirically the disinvestment, inaction, and investment regimes characterized in the model by Abel and Eberly (1994). Barnett and

[10] I consider the indices by Kaplan and Zingales (1997) and Whited and Wu (2006).

[11] Cooper and Ejarque (2004) and Abel and Eberly (2004) provide the complementary argument that cashflows proxy for impact of market power on investment equations.

[12] See Section II. Erickson and Whited (2000) assume that investment is linear in q.

Sakellaris (1998) do not account for the impact of financing frictions on investment, and use an alternative empirical approach. Yet in their robustness checks they report that higher orders of average Q are statistically significant for investment equations.

The structural estimation is most closely related to Cooper and Haltinwanger (2006) and Hennessy and Whited (2005). Cooper and Haltinwanger (2006) estimate firms' adjustment and fixed costs of investment from plant level data; I infer real and financing frictions at the firm level. Hennessy and Whited (2005) estimate firms' costs of debt and equity financing using simulated method of moments; this paper does not discriminate between debt and equity issues, applies maximum likelihood, and estimates real frictions. The structural estimation by Li and Liu (2011) on stock returns is also related to this paper; both studies quantify alternative real frictions for the cross section of industries.

I A model of financed investment

A The problem of the firm

The model builds upon the frameworks by Abel and Eberly (1994) and Hennessy, Levy and Whited (2006), and adopts their notation where possible. The firm is run by a manager who decides on its optimal investment policy to maximize the market value of the existing shares. Investors are risk neutral and discount cash flows at a constant risk free rate $r > 0$.[13] In contrast to the unlevered firm in Abel and Eberly (1994), the firm has issued a senior consol bond paying an infinite stream of constant payments b.[14] The firm may also raise additional external funds in the form of equity issuance to finance its future investments, subject to convex flotation costs of equity.

Anticipating, the assumptions on the capital structure of the firm are done for tractability and without loss of generality for the sake of empirical tests. This paper elaborates on how firms' net increases in external financing relate to investment, but makes no specific claim about equity or debt issues in particular. While the coupon b captures the effect of senior commitments on current investment, the convex flotation costs capture the stylized fact that firms' marginal costs of external financing are increasing in their funding needs. The financing frictions literature observes increasing marginal costs in both equity and debt issues.[15] I

[13] More precisely, r is the constant yield of a tax exempt risk free bond.

[14] The assumption of a consol bond is motivated by the desire to test for optimal investment decisions in firms that may have issued debt before their investment decision at time t. As noted by Hennessy (2004), the resulting structural investment equation holds for an arbitrary coupon b.

[15] Equity issues are subject to convex flotation costs of underwriting as documented by Altinkilic and Hansen (2000). Bond and Meghir (1994) and Gomes, Yaron and Zhang (2006) consider a convex interest schedule on

henceforth describe the firm in this paper as subject to convex costs of net external financing.

The gross operating profits $\pi(K_t, \epsilon_t)$ are a function of the current capital stock K_t and some diversifiable shock ϵ_t. The firm is a price-taker and its production function exhibits constant returns to scale, such that its gross profits are linear in capital. The function $\pi(K_t, \epsilon_t)$ is twice continuously differentiable and strictly increasing in all its arguments. The state variable ϵ_t captures innovations in both input and output prices, and evolves according to a diffusion process

$$d\epsilon_t = \mu\left(\epsilon_t\right)dt + \sigma\left(\epsilon_t\right)dW_t \tag{1}$$

where W_t is a standard Wiener process.

Capital is acquired by undertaking gross investment at a rate I_t, and the capital stock depreciates as a fixed proportional rate δ. The capital stock K_t evolves according to

$$dK_t = \left[I_t - \delta K_t\right]dt, \quad K_0 > 0 \tag{2}$$

When the firm undertakes gross investment, it incurs different types of costs. First, the firm incurs a direct cost of purchasing capital. I set the price of capital equal to 1 such that this cost is equal to I_t. Second, the firm incurs both variable adjustment costs of investment and fixed costs of investment, which I jointly denote by $C(I_t, K_t)$. In line with Abel and Eberly (1994), the function $C(I_t, K_t)$ is twice continuously differentiable, strictly convex, and homogeneous degree one in I_t and K_t such that

$$C(I_t, K_t) = \frac{\alpha}{2}K_t\left(\frac{I_t}{K_t} - \delta\right)^2 + fK_t\mathbf{1}_t^I \tag{3}$$

The parameter $\alpha > 0$ determines curvature, f is a fixed cost which is proportional to capital, and $\mathbf{1}_t^I$ indicates non-zero investment. The quadratic form in (3) provides tractability.[16]

The firm funds its investment through either internal cash or external financing. While internal financing with cash is costless, the firm is subject to convex costs of external financing if it raises external funds. Denote the external funding requirement of the firm X_t such that

$$X_t = I_t + C\left(I_t, K_t\right) + b - \pi\left(K_t, \epsilon_t\right) \tag{4}$$

At any time t, the firm either issues new securities to fund its investment when $X_t > 0$ or distributes a dividend $D_t > 0$ to its shareholders such that $D_t \equiv -X_t$. Denote the *net* increase in external finance of the firm each period by $\widetilde{X}_t = X_t - b$. I assume that the costs of

the debt obligations that is increasing in book leverage.

[16] I derive the more general case with convex adjustment costs of higher order in Appendix A.

external financing $H(X_t, K_t)$ are given by

$$H(X_t, K_t) = X_t + \frac{\theta}{2} K_t \left(\frac{\widetilde{X}_t}{K_t} \right)^2 \mathbf{1}_t^X \tag{5}$$

where $\theta > 0$ is a parameter determining curvature, and $\mathbf{1}^X$ equals one if $X_t > 0$ and zero otherwise.

The functional form in $H(X_t, K_t)$ follows closely that in Hennessy, Levy and Whited (2006) and is twice continuously differentiable, strictly increasing and weakly convex in X_t and decreasing and convex in K_t. In contrast with Hennessy, Levy and Whited (2006), however, I assume that marginal costs are increasing in their net external issuance $\widetilde{X}_t$.[17] This is analogous to the functional form in (3) for the adjustment costs of investment, and implies that the costs of external funding in period t are uniquely related to the additional funds raised in period t.

Given this set of assumptions, the decision to raise additional external financing is determined as a residual claim.[18] The vector (K_t, ϵ_t) captures all the relevant information at each instant t. At each point in time, the manager chooses the investment policy I_t that maximizes the current value of existing shares S_t. The manager also chooses an optimal time to default T, given that the value of equity equals zero in the event of default.[19] Bondholders recover a fraction $\gamma \leq 1$ of the undepreciated assets upon default. The manager then solves

$$S(K_t, \epsilon_t) = \max_{I_t, T} E_t \left[\int_0^T e^{-rs} \left[-H\left(X_s, K_s\right) \right] ds \right] \tag{6}$$

subject to (1), (2), (7), and $D_t \equiv -X_t$.

B Investment and marginal q

Denote the marginal product of capital by $q \equiv S_K$. The Bellman Equation corresponding to the optimization problem of the manager in (6) is given by

$$rS = -H\left(X, K\right) + \left(I - \delta K\right) S_K + \mu\left(\epsilon\right) S_\epsilon + \frac{\sigma\left(\epsilon\right)^2}{2} S_{\epsilon\epsilon} \tag{7}$$

The characterization of the investment rule by Abel and Eberly (1994) carries over to the current setting such that in the presence of fixed costs there exists a non degenerate inaction

[17] In Hennessy, Levy and Whited (2006), the quadratic term is a function of X such that $H\left(X, K\right)$ is homogeneous of degree one in X and K.

[18] This is in contrast with Hennessy, Levy and Whited (2006).

[19] The stopping time $T \in \mathcal{T}$ is such that $\mathcal{T}$ is the set of all stopping times defined with respect to the natural filtration. There are no deviations from absolute priority rule.

region of investment. The optimality condition determining investment in (7) is given by

$$q = H_X (1 + C_I) \tag{8}$$

if and only if $I > 0$. In the active region, firms invest up to the point in which the shadow value of a unit of installed capital equals the ratio of marginal cost of investment, and the marginal cost of external funding.[20]

Rearranging terms, the optimal investment policy in (8) is such that

$$\frac{I}{K} = \begin{cases} \delta - \frac{1}{\alpha} + \frac{1}{\alpha}\frac{q}{H_X} & \text{if } q \notin \Theta \\ 0 & \text{if } q \in \Theta \end{cases} \tag{9}$$

where Θ determines the range of values of marginal q for which it is optimal not to invest.

B.1. The active region of investment

Consider the predictions of the optimal investment policy in (9) for the active region of investment.

Proposition 1 *When $q \notin \Theta$, firms' optimal investment policy is such that: (i) $\frac{I}{K}$ is increasing in q $\forall X$, and increasing in $\frac{\pi}{K}$ if $X > 0$; (ii) $\frac{I}{K}$ strictly concave in q if $X > 0$, and linear in q otherwise; (iii) $\frac{I}{K}$ is strictly convex in $\frac{\pi}{K}$ if $X > 0$; and (iv) $\frac{\partial \frac{I}{K}}{\partial q}$ is decreasing in α $\forall X$, and θ and f.if $X > 0$.*

Proof. See Appendix B. ■

Proposition 1 contains several results that are different from the stylized predictions in the literature of the Q-theory. The first result which is in sharp contrast with the standard view in the literature is that marginal q need not be a sufficient statistic for investment. In this paper, the prediction by Hayashi (1982) that q is a sufficient statistic for investment only applies when firms finance their investment entirely with internal cashflows. More formally, I prove in Appendix B that if $q \notin \Theta$ the total derivative of $\frac{I}{K}$ in (9) is given by

$$d\left(\frac{I}{K}\right) = \frac{\partial \frac{I}{K}}{\partial q} dq + \frac{\partial I}{\partial \frac{\pi}{K}} d\frac{\pi}{K} \mathbf{1}^X \tag{10}$$

such that $\frac{I}{K}$ depends solely on marginal q if firms rely on internal funds, and $\frac{I}{K}$ depends both marginal q and π otherwise.

[20] Hayashi (1985), Hennessy, Levy and Whited (2006) and Bolton, Chen and Wang (2010) obtain this prediction in related set-ups.

When firms rely on internal cashflows ($X \leqslant 0$), investment is uniquely determined by marginal q, and the sensitivity of investment to marginal q is constant and equal to $\frac{1}{\alpha}$. However, when firms rely on external financing ($X > 0$), marginal q is not a sufficient statistic for investment; firms' investment decisions also depend on their available cashflows π. Proposition 1 relates to the empirical finding by Fazzari, Hubbard and Petersen (1988) as it predicts a significant relation between investment and cashflows for issuing firms. The underlying explanation is that the sensitivity of investment to marginal q or $\frac{1}{\alpha H_X}$ is itself a function of q and cashflows π when firms rely on external funds.

Proposition 1 also predicts that investment is non linear in marginal q when firms are subject to financing frictions. The general observation that investment is non-linear in q in the active region is not novel. The works by Hayashi (1985), Hennessy, Levy and Whited (2006) and Bolton, Chen and Wang (2010) elaborate on how firms' financing decisions generate multiple regimes in investment policies, and affect the sensitivity of investment to q. However, this paper adds as it shows that, when the marginal costs of external financing are increasing, investment is weakly concave in q. In the active region, investment is increasing and weakly concave in q such that

$$\frac{\partial \frac{I}{K}}{\partial q} > 0, \quad \frac{\partial^2 \frac{I}{K}}{\partial q^2} = 0 \text{ if } X \leqslant 0 \quad \text{and } \frac{\partial^2 \frac{I}{K}}{\partial q^2} < 0 \text{ if } X > 0 \tag{11}$$

The non linear sensitivity of investment to q in (11) stems directly from the convexity in firms' costs of external financing. When firms rely on internal cashflows ($X \leqslant 0$), investment in linear in q. However, when firms rely on external financing ($X > 0$), investment is strictly concave in q. All else equal, when firms rely on external funds, higher investment rates imply heavier reliance on external financing, and higher marginal costs of external funding. The sensitivity of investment to q is strictly decreasing in q when $X > 0$. Figure 1 illustrates these predictions numerically.

As a remark, note that the predicted concavity of investment in q depends both on the assumed convexity of firms' costs of external financing and firms' adjustment costs of investment. With quadratic adjustment costs as in (3), investment is linear in marginal q when firms do not rely on external funding. However, when firms rely on internal funds, investment is also concave in q if $C(I, K)$ is a polynomial of order higher than two.[21] The important difference between the cases of concavity due to real frictions (when $X \leqslant 0$) or financing frictions (when $X > 0$) relies on the impact of firms' cashflows π on investment decisions. The sensitivity of investment to q only depends on cashflows when firms rely on external financing.

[21] See Appendix A.

Proposition 1 also states that investment is weakly convex in cashflows. Hence not only the model predicts that cashflows enter significantly in the investment equation when firms rely on external funds; it also implies that higher order terms of firms' cashflows should also be significant. More formally, in the active region, investment is increasing and weakly convex in π since

$$\frac{\partial \frac{I}{K}}{\partial \frac{\pi}{K}} > 0 \text{ if } X > 0, \; \frac{\partial \frac{I}{K}}{\partial \frac{\pi}{K}} = 0 \text{ if } X \leqslant 0, \quad \text{and } \frac{\partial^2 \frac{I}{K}}{\partial \frac{\pi}{K}^2} > 0 \text{ if } X > 0 \tag{12}$$

Given that the marginal costs of external financing are decreasing in π, all else equal, the sensitivity of investment to π is increasing in π when firms rely on external funds. Figure 2 illustrates these results.

Finally, the model provides interesting comparative statics on how real and financing frictions affect the sensitivity of investment to q in the active region. These are shown in Figure 2. The sensitivity of investment to q is decreasing in θ; higher marginal costs of external financing reduce the optimal investment rate. The sensitivity of investment to q is decreasing in α; this is consistent with Hayashi (1982). Finally, and in contrast with existing models such as Abel and Eberly (1994), the sensitivity of investment to q in the active region is decreasing in firms' fixed costs of investment f. When firms have to finance their investment costs with external funds, higher fixed costs of investment imply higher marginal costs of external funding.

B.2. The inaction region of investment

Abel and Eberly (1994) show that fixed costs of investment induce a non-degenerate inaction region Θ in which firms find it optimal not to invest. Proposition 2 below contributes to their finding as it explores how such inaction region is affected by financing frictions.

Proposition 2 *The inaction region $\Theta \equiv [q_1, q_2]$ with $q_1 < 0$ and $q_2 > 0$ is such that: (i) when $X \leq 0$, the thresholds q_1 and q_2 are constant, and increasing in α and f; (ii) when $X > 0$, q_1 and q_2 are not constant, are increasing in α, f and θ, and decreasing in $\frac{\pi}{K}$.*

Proof. See Appendix C. ■

In line with Abel and Eberly (1994), the model predicts that firms invest only if the net payoff to investment is strictly positive. The inaction region of investment is given by $q \in [q_1, q_2]$ where $q_i \;\; i = 1, 2$ are the roots of the maximand $\Psi(q, K)$ reflecting the rewards to investment. The polynomial that solves for the optimal inertia region is then given by

$$\Psi(q, K) = (q-1) I - C(I, K) - \frac{\theta}{2} K \left(\frac{\widetilde{X}}{K} \right)^2 \mathbf{1}^X \tag{13}$$

where (13) collects all the terms in (7) which depend on investment.

Due to firms' convex costs of investment and external financing, $\Psi(q, K)$ is a polynomial of order two when $X \leq 0$, and of order four when $X > 0$. For the sake of comparison with Abel and Eberly (1994), consider then a first order Taylor approximation of the quadratic term in $H(X, K)$ around a vector $(\widetilde{x}, k)$ Using this Taylor expansion, I show in the Appendix that the cut-off values q_i are given by

$$q_i \cong h(1 - \delta\alpha) \pm h\sqrt{\delta^2\alpha^2 + 2a(f - \eta) - 2\alpha(h - 1)\frac{\pi}{K}} \tag{14}$$

where $h \equiv H_X(\widetilde{x}, k) > 0$ and $\eta \equiv -H_K(\widetilde{x}, k) > 0$.

When $X \leq 0$, the inertia region of investment Θ is such that (14) holds with strict equality, $h = 1$ and $\eta = 0$. The thresholds q_1 and q_2 are the same as those predicted by Abel and Eberly (1994). When $X > 0$, however, financing frictions affect the boundaries of the inertia region Θ. Conditional on firms having a given scale K, higher net funding requirements increase firms' marginal costs of external financing, and make them more reluctant to restart investment. Hence q_i is increasing in h.[22] Furthermore, when firms rely on external funding, the inaction region Θ shifts over time; the thresholds q_i depend on firms' cashflows, and vary each period depending on idiosyncratic shocks.

Figures 1 and 3 illustrate the main predictions of Proposition 2 numerically. Figure 1 shows how the convex flotation costs of external financing induce wider inertia regions of investment when firms rely on external financing. Figure 3 provides the comparative statics for the investment threshold q_2.[23] When $X \leq 0$, the predicted thresholds coincide with those in Abel and Eberly (1994), the inaction region is constant over time, and the inaction region does not depend on firms' cashflows π and marginal costs of external funding H_X. Conversely, when $X > 0$, the inaction region shifts each period, and depends on π and H_X. Firms are more reluctant to invest when their marginal costs of funding are high and their cashflows are low; the threshold q_2 is increasing in θ and decreasing in π.

As an additional result, Figure 3 shows how financing frictions magnify the effect of an increase in investment costs on the thresholds q_i. When firms rely on internal funding, increases in either curvature of the adjustment costs of investment α or the fixed costs of investment f also make firms more reluctant to reinitiate investment. When firms rely on external funding however, an increase in α or f also increases firms' marginal cost of external financing.

[22] I show that (14) is increasing in h in the Appendix. Note, however, that this analogy is not exact since both h and η are functions of X.

[23] These are based on the exact expression for the threshold q_2. The comparative statics for q_1 are symmetric.

C Net external financing and marginal q

The model determines firms' external financing requirements as a residual claim, and hence there is no optimal external financing policy *per se*. However, the model predicts how firms' external financing requirements depend on q using (4), (5) and (9). The resulting net external financing policy of the firm is then given by

$$\frac{\widetilde{X}}{K} = \begin{cases} \delta + f - \frac{1}{2\alpha} + \frac{1}{2\alpha}\left(\frac{q}{H_X}\right)^2 - \frac{\pi}{K} & \text{if } q \notin \Theta \\ \frac{\alpha\delta^2}{2} - \frac{\pi}{K} & \text{if } q \in \Theta \end{cases} \tag{15}$$

such that firms' net external financing decisions as a function of their available cashflows and marginal q.

Proposition 3 *Firms' net external financing policy is such that: (i) $\frac{\widetilde{X}}{K}$ depends jointly on $\frac{\pi}{K}$ $\forall I$ and q if and only if $q \notin \Theta$; (ii) $\frac{\widetilde{X}}{K}$ is increasing in q $\forall q \notin \Theta$, concave in q if $X > 0$ and convex in q otherwise; and (iii) $\frac{\widetilde{X}}{K}$ depends mechanically on $\frac{\pi}{K}$ $\forall I$ and $\forall q$.*

Proof. See Appendix D. ■

Proposition 3 shows that Q-theory applies broadly to both investment and financing decisions. This is consistent with Bolton, Chen and Wang (2010). Given an investment motive to raise funds, and in the presence of financing frictions, neoclassical investment models predict that both investment and financing policies depend on marginal q. Proposition 3 adds to Bolton, Chen and Wang (2010) as it characterizes non linear external financing policies for the sake of empirical tests.

The first important result in Proposition 3 is that q and $\frac{\pi}{K}$ are sufficient statistics for $\frac{\widetilde{X}}{K}$. Equation (16) shows that $\frac{\widetilde{X}}{K}$is always a function of $\frac{\pi}{K}$ by construction, and depends on marginal q when firms are actively investing. The total derivative of $\frac{\widetilde{X}}{K}$ in (9) is such that

$$d\left(\frac{\widetilde{X}}{K}\right) = \frac{\partial \widetilde{X}}{\partial \frac{\pi}{K}} d\frac{\pi}{K} + \frac{\partial \frac{\widetilde{X}}{K}}{\partial q} dq \mathbf{1}^I \tag{16}$$

The model also predicts that $\frac{\widetilde{X}}{K}$ is non-linear in marginal q. When firms are not actively investing, $\widetilde{X}$ is insensitive to marginal q, and firms may either distribute dividends or raise external funds to cover for low cashflows. Conversely, when firms are actively investing, $\widetilde{X}$ is sensitive to marginal q. In the active region of investment, I show formally in Appendix D that the sensitivity of $\widetilde{X}$ to q is such that

$$\frac{\partial \frac{\widetilde{X}}{K}}{\partial q} > 0, \quad \frac{\partial^2 \frac{\widetilde{X}}{K}}{\partial q^2} < 0 \text{ if } X > 0, \quad \text{and } \frac{\partial^2 \frac{\widetilde{X}}{K}}{\partial q^2} > 0 \text{ if } X \leqslant 0 \tag{17}$$

When firms are actively investing, $\widetilde{X}$ depends positively on q since a higher marginal product of capital q requires higher investment and, consequently, more external financing. When the firm is distributing dividends ($X \leqslant 0$), $\widetilde{X}$ is concave in q due to the firms' convex adjustment costs. The higher the investment rate, the higher the costs of investment and the lower the payout to shareholders. Conversely, when the firm is raising external funds ($X > 0$), firms are subject to convex costs of external funding, and $\widetilde{X}$ is concave in q. Figure 1 illustrates these predictions numerically.

Finally, the model shows that firms' external financing requirements are mechanically related to cashflows in a non linear fashion. The sign and curvature of the sensitivity of $\frac{\widetilde{X}}{K}$ to $\frac{\pi}{K}$ is such that

$$\begin{aligned} &\text{if } \tfrac{I}{K} = 0 \text{ or } X \leqslant 0, \quad \frac{\partial \frac{\widetilde{X}}{K}}{\partial \frac{\pi}{K}} < 0 \text{ and } \frac{\partial^2 \frac{\widetilde{X}}{K}}{\partial \left(\frac{\pi}{K}\right)^2} = 0 \\ &\text{if } \tfrac{I}{K} \neq 0 \text{ and } X > 0, \quad \frac{\partial \frac{\widetilde{X}}{K}}{\partial \frac{\pi}{K}} \gtrless 0 \text{ and } \frac{\partial^2 \frac{\widetilde{X}}{K}}{\partial \left(\frac{\pi}{K}\right)^2} > 0 \end{aligned} \tag{18}$$

When firms are not investing or invest with internal funds, $\frac{\widetilde{X}}{K}$ is linearly decreasing in $\frac{\pi}{K}$ by construction. When firms invest with external funds, and increase in $\frac{\pi}{K}$ both relaxes the requirement for external funding, but also increases firms' optimal investments, which may require additional funding. Hence net equity issues may be positively or negatively related to firms' available cashflows. Similarly, Almeida and Campello (2010) find empirically that the degree of complementarity between external financing and cashflows depends on the interaction between investment and financing decisions. Finally, when firms rely on external funds, $\frac{\widetilde{X}}{K}$ is convex in π since firms' marginal costs of external funding are decreasing in firms' available cashflows.

D Marginal q and average Q

The model also characterizes the *average* product of capital or average Q. Denote average Q as the ratio of the total market value of the firm to its fixed assets. The model differs from Hayashi (1982) as it predicts a significant wedge between marginal q and average Q. In line with Hennessy (2004) and Hennessy, Levy and Whited (2006), the wedge between marginal q and average Q arises in the presence of financing frictions.

Proposition 4 *Average Q is such that $Q \equiv q + \upsilon$, where the wedge υ reflects the differential impact of frictions on firms' equity and asset values.*

Proof. See Appendix E. ■

As shown in Appendix E, the model predicts that the wedge υ relates exclusively to debt overhang effects. In line with Hennessy (2004), average Q overstates marginal q by incorporating the undepreciated value of assets accrued to bondholders upon default.[24] The related literature on Q-theory, however, shows that the wedge υ depends more generally on multiple real and financing frictions. For instance, Hayashi (1982) notes that real frictions such as taxes on investment and imperfect product market competition induce a wedge between q and Q. Hennessy, Levy and Whited (2006) predict that υ depends on the differential impact of financing frictions on equity and asset values. I therefore restrain from identifying the wedge between q and Q, and acknowledge that Q is a noisy proxy of q for the sake of empirical tests.

II Empirical implications

Propositions $1-4$ have important empirical implications which are the focus of this section. On one hand, the model characterizes how firms' investment and net external financing policies are non linear in average Q and firms' cashflows. On the other hand, these same predictions yield important implications on how to test the model empirically.

A Corollaries for firms' investment policies

Propositions $1-4$ provide a rich set of corollaries on the relation between firms' investment, average Q, real frictions, and financing frictions. The proofs of these corollaries are straightforward and omitted for the sake of brevity. All corollaries yield important empirical implications.

A.1. Investment and Q in the active region

Corollary 1 *In the active region of investment, $\frac{I}{K}$ is increasing and weakly concave in average Q.*

With quadratic adjustment costs, investment is concave in Q due to firms' increasing marginal costs of external funding. More formally, it follows from Propositions 1 and 4 that

$$\frac{\partial \frac{I}{K}}{\partial Q} > 0; \quad \frac{\partial^2 \frac{I}{K}}{\partial Q^2} = 0 \text{ if } X \leqslant 0; \quad \text{and } \frac{\partial^2 \frac{I}{K}}{\partial Q^2} > 0 \text{ if } X > 0 \tag{19}$$

if $q \notin \Theta$, and firms are subject to convex costs of external financing and quadratic adjustment costs of investment. Furthermore, if adjustment costs are of order higher than two, investment is also concave in Q when $X < 0$.[25]

[24] See Appendix for derivation.

[25] See Appendix A.

The empirical literature on Q-theory has typically interpreted the inverse of the sensitivity of investment rates to average Q as the inverse of the curvature of firms' adjustment costs of investment α.[26] Corollary 1 implies that such inference is biased, unless the estimation approach accounts for the concavity of investment to Q due to real and financing frictions.

More precisely, Corollary 1 predicts that the investment to capital ratio is positively and significantly related to average Q, and also negatively and significantly related to the square of average Q^2. As a result, the standard exercise in the empirical literature on Q-theory which tests for investment as a linear function of Q is subject to an omitted variable bias. The omitted higher order moments of Q in the investment equation lead to the underestimation of the (higher) sensitivity of investment to Q.

A.2. Non linear measurement error

Corollary 2 *In the active region of investment, $\frac{I}{K}$ is decreasing and convex in the unobservable wedge $\upsilon \equiv Q - q$.*

The model predicts that there exists a wedge υ such that average Q is a noisy proxy of marginal q. The empirical literature on Q-theory highlights that marginal q is measured with error due to multiple reasons.[27] Without loss of generality, all of these reasons can be attributed to the unobservable wedge between marginal q and average Q. The wedge υ is affected by both real and financing frictions, which are unobservable to the econometrician and may vary across firms.

In this context, Corollary 2 adds to the existing literature as it provides an economic rationale for the functional form of measurement error. When firms are subject to convex costs of external financing, firms' investment and financing policies are weakly concave in both average Q and the wedge υ. More formally, in the active region, it follows from Propositions 1 and 3 that

$$\frac{\partial \frac{I}{K}}{\partial \upsilon} > 0; \quad \frac{\partial \frac{I}{K}}{\partial \upsilon} = 0 \text{ if } X \leqslant 0; \quad \text{and } \frac{\partial \frac{I}{K}}{\partial \upsilon} < 0 \text{ if } X > 0 \tag{20}$$

such that investment is weakly convex in υ.

The inequality in (20) is related to the assumption by Erickson and Whited (2000) on non linear measurement error. Erickson and Whited (2000) derive a model in which investment is linear in average Q and cashflows, and then assume that measurement error is non linear to derive a higher order moments estimation approach. In contrast with their set-up, this paper

[26] See Hayashi (1982), and the survey by Caballero (1997).
[27] See Erickson and Whited (2000) for a survey.

predicts that investment is subject to non linear measurement error if and only if investment is concave in Q.

A.3. The inertia region of investment

Corollary 3 *Firms invest if $Q > q_2 + \upsilon$, disinvest if $Q < q_1 + \upsilon$, and the thresholds q_1 and q_2 depend mechanically on firms' cashflows $\frac{\pi}{K}$,α, f, δ and θ.*

Firms may find it optimal not to invest depending on the magnitude of their fixed costs of investment, and how financially constrained they are. This observation embodies two main empirical implications. The first is that the investment and disinvestment thresholds are affected by both real and financing frictions. So far, the empirical literature had focused on real frictions only.[28]

In the absence of financing frictions, Abel and Eberly (1994) predict constant thresholds q_1 and q_2; the corresponding empirical implication is that of three distinct regimes in firms' investment policies whose thresholds only depend on the (constant) underlying determinants of firms' adjustment costs of investment. The model in this paper predicts that the thresholds q_1 and q_2 depend on firms' underlying convex costs of adjustment, but also vary over time with firms' cashflows and binding financing constraints.

The second important implication of Corollary 3 is that there is truncation in the distribution of investment. Positive investment is truncated to those values of Q such that $Q > q_2 + \upsilon$. In other words, the probability that a firm invests a positive amount if $Q < q_2 + \upsilon$ is strictly zero. Likewise, the distribution of disinvestment is truncated to those values of Q such that $Q < q_1 + \upsilon$; the probability that a firm disinvests is strictly zero unless average Q is sufficiently low.

A.4. Investment and cashflows

Corollary 4 *In the active region, $\frac{I}{K}$ is weakly convex in $\frac{\pi}{K}$. Furthermore, if $X > 0$, firms with lower cashflows have lower investment rates (if $Q - \upsilon \notin \Theta$), or are more reluctant to invest (if $Q - \upsilon \in \Theta$).*

The model predicts that investment is strictly convex in firms' cashflows when firms rely on convex costs of external funding. When firms are subject to convex costs of external financing, higher cashflows alleviate the increasing marginal costs of funding for higher investment rates. This result is in contrast with Gomes (2001), whose model assumes that firms' costs of

[28] See, for instance, Barnett and Sakellaris (1998).

external financing are linear, and hence cashflows do not appear in the first order condition of investment.

Corollary 4 complements the existing literature which justifies the significant correlation between investment and cashflows with alternative arguments. Hayashi (1982) notes that, even in perfect financial markets, average Q is not a sufficient statistic for investment when the profit function is not linearly homogeneous. Cooper and Ejarque (2004) and Abel and Eberly (2004) show that cash flow proxies for the wedge between marginal q and average Q when firms have market power.

More importantly, Corollary 4 implies that the standard regression in Q-theory of investment on average Q should not only incorporate the first order moments of cashflows, but higher order moments of cashflows as well. When firms rely on external funding to finance their investments, the model predicts that the regular practice in the empirical literature of testing investment as a linear function of Q and cashflows is subject to omitted variable bias.

A.5. Investment and frictions

Corollary 5 *Firms subject to higher values of θ, α, f and δ are more reluctant to invest (if $Q - \upsilon \in \Theta$), or have lower investment rates (if $Q - \upsilon \notin \Theta$).*

All else equal, firms subject to higher frictions θ, α, f and δ have lower investment rates. Since these parameters are firm specific or industry specific at most, Corollary 5 is only testable in the context of structural estimation. I elaborate on the empirical approach to test for Corollary 5 below. The empirical findings by Hennessy and Whited (2005) on financing frictions are consistent with Corollary 5; they show that firms with higher marginal costs of external funding have lower investment rates.

B Alternative estimation approaches for investment policies

The testable implications for investment can be tested in three alternative ways.

B.1. Investment as a dependent variable

The first approach involves reduced-form tests with investment as a dependent variable. Denote by $m\left(q, \frac{\pi}{K}\right)$ a non-linear function such that $m_q = 0$ if $I = 0$; $m_q > 0$ if $I \neq 0$; $m_{qq} < 0$, $m_{\frac{\pi}{K}} > 0$ and $m_{\frac{\pi}{K}\frac{\pi}{K}} > 0$ if $X < 0$; $m_{qq} = 0$ and $m_{\frac{\pi}{K}} = 0$ if $X \leq 0$. The corresponding testable implication for the first order condition of investment for firm i, at any time t, and with investment as a dependent variable is given by

$$\frac{I}{K} = m\left(q; \frac{\pi}{K}\right) + u \tag{21}$$

where u is the error term; and the unobservable marginal q in (21) is proxied with error by Q such that

$$Q = q + \upsilon \tag{22}$$

where the wedge υ is assumed to be normally distributed, with non zero mean $\kappa > 0$ and variance σ_υ^2.[29]

The reduced-form tests of the model with investment as an explanatory variable allow the comparison with previous results in the literature. The empirical literature on Q-theory has regularly tested the first order condition of investment with investment as an dependent variable. Caballero (1997) documents poor performance of the Q-theory in explaining firm-level investment. Frequently, the coefficients on Q are low or insignificant, while cashflows and other measures of liquidity enter positively and significantly.

By inspection, equations (21) and (22) provide a rationale to the poor empirical performance documented by Caballero (1997). Investment is subject to non linear measurement error in q. Marginal q is not a sufficient statistic for investment when firms rely on external financing. The standard empirical tests of investment on Q and cashflows are subject to omitted variable bias; investment is weakly concave on average Q and weakly convex in cashflows. Finally, the fact that $m_q = 0$ if $I = 0$ implies that the standard OLS regression of investment on Q is subject to truncation bias. Since firms only engage in positive investments if Q is sufficiently high, investment is left truncated to positive values of Q.[30]

B.2. Investment as an explanatory variable

The empirical literature on Q-theory has regularly tested the first order condition in (9) with investment as an dependent variable. However, several of the difficulties addressed in the empirical estimation of this first order condition are bypassed when the first order condition of investment is tested in reduced-form with average Q as a dependent variable.

In the active region of investment, equation (8) implies that average Q is mechanically related to investment such that

$$Q = \phi_0 + \underset{(+)}{\phi_1}\frac{I}{K} + \underset{(-/0)}{\phi_2}\left(\frac{I}{K}\right)^2 + \underset{(-/0)}{\phi_3}\left(\frac{I}{K}\right)^3 + \underset{(+/0)}{\phi_4}\frac{\pi}{K} + \underset{(-/0)}{\phi_5}\frac{\pi}{K}\frac{I}{K} + \iota \tag{23}$$

[29] This is in line with Erickson and Whited (2000). The assumption of homoskedasticity in the wedge υ is done for the sake of notational simplicity throughout the section. As shown later on, it is straightforward to account for heteroskedasticity in υ when average Q is the dependent variable.

[30] I elaborate further on truncation below.

where $\iota \equiv u + \upsilon$, and the coefficients in (23) are explicit functions of α, δ, θ and f such that

$$\phi_0 = (1-\alpha\delta) + \theta(1-\alpha\delta)\left(2f + \alpha\delta^2\right)\mathbf{1}^X \tag{24}$$

$$\phi_1 = \alpha + \theta\left(1 + f\alpha - 2\alpha\delta + \frac{3}{2}\alpha^2\delta^2\right)\mathbf{1}^X \tag{25}$$

$$\phi_2 = \frac{3}{2}\alpha\theta(1-\alpha\delta)\mathbf{1}^X \tag{26}$$

$$\phi_3 = \frac{1}{2}\alpha^2\theta\mathbf{1}^X \tag{27}$$

$$\phi_4 = -\theta(1-\alpha\delta)\mathbf{1}^X \tag{28}$$

$$\phi_5 = -\alpha\theta\mathbf{1}^X \tag{29}$$

Conditional on firms being in the active region, the empirical approach to test (23) in reduced-form is straightforward. Average Q depends linearly on investment, cashflows and higher orders of these variables. In contrast with (21), there is no need to infer the function m from the data; the model provides an explicit characterization of coefficients ϕ_0 to ϕ_5 as a function of α, θ, δ and f. The wedge between marginal q and average Q enters linearly on the regression equation, such that measurement error is linear. Finally, linear measurement error in the dependent variable does not affect the consistency of the estimation; it only affects its efficiency.[31] Such loss in efficiency is mitigated by considering robust standard errors in the empirical tests.[32]

As a caveat, the unconditional reduced form tests of (23) do not account for the inertia region of investment. Firms invest if marginal q is sufficiently high, and firms disinvest if marginal q is sufficiently low. The corresponding empirical implication is that the probability that a firm invests at any time t equals

$$\Pr(q \geqslant q_2) = \Phi\left[\frac{\Delta^+ - Z'\zeta}{\sigma_\iota}\right] \tag{30}$$

where $\Delta^+ \equiv q_2 + \upsilon - \kappa$, $Z'\zeta$ summarizes in stack form the entire set of regressors in (23), and Φ is the cumulative normal distribution function. Similarly, the probability that a firm disinvests at any time t yields

$$\Pr(q \leq q_1) = 1 - \Phi\left[\frac{\Delta^- - Z'\zeta}{\sigma_\iota}\right] \tag{31}$$

where $\Delta^- \equiv q_1 + \upsilon - \kappa$.

[31] See Arellano (2003).

[32] In the context of the model, the wedge υ is a function of firms' underlying characteristics. Hence accounting for heteroskedasticity in the error term mitigates the loss in efficiency due to measurement error.

B.3. Log likelihood function and structural estimation

Equations (23), (30) and (31) can be assessed jointly using structural estimation. To do so, I derive the log likelihood function of the first order condition in (8). Such derivation is greatly simplified by two additional working assumptions.

Assumption 1 *Firms are subject to a fixed cost of investment f^+, and a fixed cost of disinvestment f^-, such that $f^+ \neq f^-$, $f^+ > 0$ and $f^- \longrightarrow \infty$.*

Assumption 1 acknowledges that there is irreversibility in firms' investment decisions. Assumption 1 simplifies the analysis as it implies that the probability of dinsinvestment in (31) is not significantly different from zero. The corresponding investment rule is that $I = 0$ if $q \leq q_2$ and $I > 0$ otherwise.

The assumption of asymmetric costs of investment and disinvestment is standard in the literature, and is consistent with the empirical evidence. Abel, Eberly and Pyndick (1996) and Barnett and Sakellaris (1998) consider a wedge in the purchase and resale prices of capital to explain inertia regions in investment policies. Similarly, Zhang (2005) shows how the neoclassical investment model rationalizes the value premium when firms' adjustment costs of investment and disinvestment are asymmetric. More importantly, the stylized prediction of strictly positive investment is consistent with the empirical distribution of investment at the firm level. Barnett and Sakellaris (1998) and Section III show that firm-level investment is infrequently negative.[33]

Assumption 2 *Δ^+ is constant.*

The model predicts that both q_2 and the wedge υ are time varying and depend on firms' binding real and financing frictions. Similarly, since the wedge υ depends on firms' underlying characteristics, its expected value κ need not be the same for all firms. Assumption 2 simplifies the analysis as it imposes that the unobservable threshold Δ^+ is constant across firms; note however that this does not imply necessarily that each of the terms in Δ^+ is constant.

Assumption 2 is done without much loss of generality as long as the estimation approach acknowledges that Δ^+ and $Z'\zeta$ are interrelated.[34]

[33] Note however that even though zero or negative investment is rarely observed at the firm level, aggregation may mask inaction and disinvestment at other levels. Cooper, Haltinwanger, and Power (1995) document substantial evidence of zero investment at the plant level. Aggregation over different types of capital goods can also lead to observing positive firm investment at all times, even though the firm undertakes no investment in some types of capital. Time to build may also conceal inaction.

[34] I elaborate on the estimation approach later on.

Proposition 5 *Given Assumptions* 1 *and* 2*, the log likelihood function* $L\left(\zeta,\Delta^{+}\middle|\,Z\right)$ *that corresponds to (23) and (30) is given by*

$$\ln L\left(\zeta,\Delta^{+}\middle|\,Z\right)=\ln\left(\phi\left(\frac{Q-Z'\zeta}{\sigma_{\iota}}\right)\right)-\ln\left(\sigma_{\iota}\right)-\ln\left(1-\Phi\left(\frac{\Delta^{+}-Z'\zeta}{\sigma_{\iota}}\right)\right) \tag{32}$$

where ϕ *is the normal distribution density,* $Z'\zeta$ *summarizes in stack form the regressors in (23),* $\sigma_{\iota}\equiv\sigma_{u}+\sigma_{\nu}$*, and* Φ *is the cumulative normal distribution function.*

Proof. See Appendix F. ■

Equation (32) contributes to the existing literature as it provides a tool to test the first order condition of investment in the presence of measurement error, real frictions, and financing frictions. The log likelihood in (32) resembles that of the truncated regression models regularly studied in the economics literature.[35] In line with truncated regression models, (30) implies that a significant relation between average Q and investment is not observed unless average Q exceeds a given threshold Δ^{+}. In contrast with these models, the truncation point Δ^{+} is unobserved, and needs to be estimated.[36]

The log-likelihood in (32) can be estimated with or without the parameter constraints implied by equations (24) to (29). If (32) is estimated without parameter restrictions, the model predicts the sign and significance of coefficients ϕ_1 to ϕ_5 as indicated in (23). Given the parameter restrictions and the identity in (7), the structural estimation yields point estimates for the unobserved parameters α,θ and f.

C Corollaries for net external issues

The model yields predictions on firms' net external financing policies. The structural estimation with parameter constraints accounts jointly for the predictions on investment and net external financing. This is because the parameter constraints in (24)-(29) stem directly from the definition of the net external financing requirement of the firm in (4).

In addition, the model provides explicit corollaries on firms' net external issues $\widetilde{X}>0$ which can be assessed in reduced form. I provide the corresponding corollaries below. As a remark, I focus on the predictions for positive net external issuance ($\widetilde{X}>0$); the model is not realistic otherwise. The model assumes that firms raise external funds if $\widetilde{X}>0$, and that firms pay dividends to their shareholders if $\widetilde{X}<0$. In practice, however, firms need not distribute dividends when $\widetilde{X}<0$; firms may retain earnings, repurchase debt, or repurchase equity.

[35] See Hayashi (2000).

[36] Since Δ^{+} is unobserved, the structural estimation of (32) involves multiple stages. I enlarge on this later.

C.1. The Q-theory of net external issues

Corollary 6 *In the active region, firms' net external issues are weakly convex in* $\frac{\pi}{K}$*; concave in* Q *if* $\frac{I}{K} > 0$*; and are subject to non-linear measurement error if* $\frac{I}{K} > 0$*.*

The model shows that net external issues are mechanically related to cashflows irrespective of firms' investment decisions. Net external issues are convex in cashflows if firms are actively investing. Furthermore, firms' net external financing policies are weakly concave in υ when firms are actively investing such that

$$\frac{\partial \frac{\widetilde{X}}{K}}{\partial(Q-\upsilon)} > 0, \quad \frac{\partial^2 \frac{\widetilde{X}}{K}}{\partial(Q-\upsilon)^2} < 0 \text{ if } X > 0 \tag{33}$$

Corollary 6 implies that net external issues are positively and significantly related to cashflows for all firms, to average Q if firms are actively investing (i.e. Q is sufficiently high), negatively and significantly related to Q^2, and positively and significantly related to squared cashflows.

C.2. Pecking order and the inertia region

Corollary 7 *Firms' net external issues are significantly related to* Q *only if* Q *is sufficiently high.*

Given that firms are subject to fixed costs of investment, firms invest if and only if average Q is sufficiently high. Furthermore, firms raise external financing only if their investment requirement exceeds its available internal funding. Put together, net external issues are sensitive to Q only if Q is sufficiently high.

C.3. Net external issues and frictions

Corollary 8 *Firms with low* $\frac{\pi}{K}$ *and/or subject to high* θ*,* α*,* f *and* δ *are less likely to raise external funds, or raise lower levels of* $\widetilde{X}$*.*

All else equal, firms subject to higher θ, α, f and δ have lower net external issues to capital ratios. Since these parameters are firm specific or industry specific at most, Corollary 8 is only testable in the context of the structural estimation of the model.

III Empirical evidence

The empirical analysis applies both reduced form tests and structural estimation to assess all the testable predictions of the model. The empirical results provide strong support to all predictions. The structural estimation results further contribute to the existing literature in quantifying firms' real and financing frictions by industry.

A Working sample

The working sample is drawn from the merged CRSP-COMPUSTAT database, considering the 1980-2008 period for $4,839$ US manufacturing firms. I provide the details on database construction in Appendix G. I follow Barnett and Sakellaris (1998) and define I as the difference between capital expenditures and sales of property, plant and equipment. Given that capital is homogeneous in the model, Barnett and Sakellaris (1998) and this paper consider a modified measure of gross investment that substracts sales of capital from purchases.[37] I denote $\widetilde{X}$ as the sum of the changes in net equity and net debt. I define b as interest expenses, B as long term debt, and denote D as dividends on common stock.

Table I provides the summary statistics for the whole working sample, by issuing and non issuing firms, and by different measures of financing constraints regularly used in the empirical literature. On average, firms that rely on external financing have significantly higher investment rates, higher average Q, higher cash flows, lower book leverage, and are larger in size. On average, financially constrained firms have lower cashflows, rely less heavily on external financing, pay less dividends, and are smaller in size.

Table II shows that average firm characteristics vary significantly across industries in the working sample. The industry definition is based on that by Fama and French.[38]

Figure 4 illustrates the empirical distribution of investment and average Q in the working sample. The upper left chart in Figure 4 reflects that investment is left truncated towards positive values. Only 1% of the observations in the working sample are negative, and only 0.12% equal zero.[39] This is consistent with the Assumption 1 of asymmetric costs of investment and disinvestment.

Figure 4 compares the empirical distribution of investment and average Q to the normal density function. The difference between the empirical distribution of investment and the normal density function suggests that parametric tests of investment may be subject to truncation biases. Similarly, average Q is left truncated towards positive values, and its distribution is non-normal.

[37] In untabulated results, I did all tests using *gross* capital expenditures (i.e. the usual capital expenditures measure, as reported in COMPUSTAT). Results for net and gross capital expenditures are all very similar, and are available upon request.

[38] I elaborate on industry definition in the Appendix.

[39] Barnett and Sakellaris (1998) also observe infrequent disinvestment and inaction at the firm level in their working sample.

B Non parametric results

I consider non parametric tests to assess the sensitivity of investment to Q and cashflows, and of net external issues to Q and cashflows. Figure 5 plot a local, non parametric polynomial smooth kernel functions of investment and net external issues on both Q and cashflows.

Figure 5 shows that investment is concave in Q, and weakly convex in cashflows. Net external issues are weakly concave in average Q, and weakly convex in cashflows. All results are aligned with the predictions of the model.

C Reduced form tests on investment

C.1. Investment as a dependent variable

Consider first the reduced-form empirical tests with investment as a dependent variable. To focus on the null hypothesis that investment is non-linear in Q and cashflows, assume for now that average Q is fairly good proxy of marginal q, such that measurement error in q is negligible. I restate the non-linear investment policy in (21) as a bivariate Taylor expansion of order two in Q and π, and estimate investment as a linear function of average Q, $\frac{\pi}{K}$ and higher order terms of these variables.

Table III provides the corresponding empirical results. Panel A shows the estimation results of the benchmark regression of investment on Q and cashflows. Panel B incorporates cashflows, as well as the higher order terms of Q and cashflows corresponding to the bivariate Taylor expansion of (21). Notably, the coefficient on Q is significantly higher than that in Panel A, and the coefficient on Q^2 is negative and highly significant. This supports the prediction in Corollary 1 that investment is concave in Q. Panels C and D add firm fixed effects, and both firm fixed effects and lagged investment to the regression in Panel B. The coefficients on Q and Q^2 are slightly lower, but the coefficient on Q is still significantly higher than the corresponding coefficient in Panel A.

Panel E estimates (21) with a truncated regression, in which the minimum investment to capital ratio in the working sample is used as the left truncation point. Dubinin and Vardeman (2003) show that when truncation points are unknown and there is no measurement error, the most efficient truncation point is the sample minimum. The truncation threshold is Panel E is set to the minimum investment rate -0.162.

In the context of the model, results in Panel E suggest that accounting for the skewness of investment significantly improves the goodness of fit. The R^2 in Panel E is significantly higher than the corresponding coefficient in Panels A and B. Similarly, the coefficient on Q in Panel E is significantly higher than the corresponding coefficient in Panels A and B. Results suggest that the OLS estimates are subject to truncation bias given that investment is infrequently negative.

Panel F considers the same truncated regression in Panel E but for the subsample of non issuing firms. In the model, all higher order moments of Q and cashflows are only significant for issuing firms. In the data, higher order moments of Q and cashflows are also significant for non issuing firms. The model predicts significant higher order moments for average Q for non issuing firms with convex adjustment costs of order higher than two.[40] However, the model is silent about why higher order moments of cashflows are significant for non issuing firms.

Panel G adds additional controls to the truncated regression in Panel E as a robustness check. The third order moment of Q is significant; this suggests that the Taylor approximation of order two does not capture all the non-linearity in Q. For all firms on average, higher order moments of cashflows are not significant. Finally, book leverage adds little explanatory power; it is only significant at $p < 0.1$.

C.2. Investment as an explanatory variable

Table IV provides the reduced form tests of investment with average Q as a dependent variable. With Q as the dependent variable, measurement error affects the efficiency but not the consistency of the estimation.[41]

Panel A provides the benchmark regression with investment and cashflows as regressors. Panel B adds the higher order terms on investment, and the interaction of investment and cashflows to the regressors in Panel A, as predicted in (23). Consistent with the model, the additional controls in Panel B increase the explanatory power, and the coefficient on the third order moment is negative. Panels C and D add firm fixed effects and a lagged Q; the R^2 drops in Panel C, and the third order moment of $\frac{I}{K}$ becomes insignificant.

Panel E estimates the model in Panel B using the truncated regression of (23). I follow a two-stage procedure to estimate the coefficients and the truncation threshold Δ^+. First, I run multiple truncated regressions of (23) for different values of Δ^+. Second, I choose the threshold Δ^+ that minimizes the distance between the actual distribution of Q and the fitted values of the estimation.

The coefficients of the truncated regression in Panel E are significantly higher than the ones of the OLS estimation of Panel B. The corresponding R^2 is also significantly higher. The estimated truncation threshold $\widehat{\Delta}^+$ for average Q equals -1.68. The sign of $\widehat{\Delta}^+$ is easily interpreted by inspecting Figure 4; the fitted values of Q using OLS predict much higher values

[40] See Appendix A.
[41] See Arellano (2003).

of Q than the ones observed in practice.

Panel F considers the same estimation approach as in Panel E for the subsample of non-issuing firms. Results suggest that firms' convex costs of external financing need not be the only plausible explanation for the non linear relation between investment and Q. Panel G adds additional controls to the truncated estimation in Panel E. The significance of such controls in Panel G suggests that firms' adjustment costs of investment or convex costs of external financing may be of order higher than two.

D Reduced form tests on net external issues

Consider the null hypotheses that net external issues are non-linear in Q and cashflows. Assume that average Q is fairly good proxy of marginal q, such that measurement error in q is negligible. I restate the non-linear net external issuance policy in (15) as a bivariate Taylor expansion of order two in Q and π, and estimate net external issues as a linear function of average Q and $\frac{\pi}{K}$, and higher order terms of Q and $\frac{\pi}{K}$.

Panels A to C in Table V provide the corresponding empirical results. Panel A shows the estimation results of a panel regression of investment on average Q and cashflows. Panel B estimates the bivariate Taylor expansion of (15) using OLS. The coefficient on average Q is always positive and significant; this confirms that Q-theory also applies to net external financing policies. Furthermore, the coefficient on Q in Panel B is much higher than the corresponding coefficient in A. The coefficient on Q^2 in Panel B is negative and highly significant. Hence net external issues are concave in average Q. Panel C adds firm fixed effects to control unobserved heterogeneity; the qualitative results remain unchanged.

Panels D to F in Table V consider the reduced form tests for net external issues with average Q as a dependent variable. Panel D considers the benchmark OLS regression. Panel E adds firm fixed effects. Panel F corrects for left truncation in Q. In all cases, net external issues are positively and significantly related to average Q, and to the cross product of investment and net external issues. Results in Panel F show that correcting for truncation in Q significantly improves the goodness of fit of the model.

E Structural estimation

The reduced form tests in Tables III to V provide strong empirical support for the model. Investment is concave in Q and weakly concave in cashflows. Net external issues are concave in Q and weakly concave in cashflows. Furthermore, correcting for truncation significantly improves the goodness of fit; this suggests that firms' inertia regions of investment are economically significant. Given these results, I use log likelihood function in Proposition 5 as a tool to infer the magnitude of the unobserved parameters α, θ and f.

I perform the structural estimation in alternative (sub)samples. I first estimate the log likelihood function subject to (24)-(29) and (7) for the whole working sample. I then estimate the model using alternative sample splits for financial constraints. Finally, I estimate the model by industry, and compare the magnitude of frictions by industry.

E.1. The average magnitude of α, θ and f

Consider first the structural estimation of α, θ and f for all firms on average. Results are reported in Table VI. The estimated parameters α, θ and f are significantly different for zero, and they are all jointly significant according to the Wald test. The R^2 of the estimation is higher than the corresponding R^2 of the OLS regression in Panel A of Table IV; similarly, the log likelihood is higher than the log likelihood in Panel A of Table III.

On average, the curvature of firms' adjustment costs of investment α equals 6.094; this implies that firms' adjustment costs of investment amount to 7.7% of capital on average. The reported estimate of α lies in the range of point estimates of previous studies. The corresponding point estimates range from over 20 (Hayashi (1982)) to as low as 3 (Gilchrist and Himmelberg (1995)).

The curvature of firms' costs of external financing θ in Table IV equals 0.783; this estimate implies that firms' external costs of financing amount to 6.6% of capital on average. Similarly, firms' average marginal costs of external financing are 15.6%. In a related paper, Hennessy and Whited (2005) estimate marginal equity flotation between 5% and 11%. Yet these point estimates are not strictly comparable; the underlying model in Hennessy and Whited (2005) is different, and discriminates between debt and equity issues.

Table IV also estimates firms' fixed costs as 37.6% of capital. This is significantly higher than fixed costs estimated by Cooper and Haltinwanger (2006) at 3.9% of capital at the plant level. I show later on that the estimated parameter f is highly correlated with firms' operating expenses in the cross section of industries. Table IV also reports the average unobservable marginal product of capital q for all firm-year observation in the working sample, equal to 1.467.

Finally, the structural estimation suggest that measurement error is economically significant. Results in Table IV report an significant expected wedge κ of 1. This confirms that there exists a significant wedge between marginal q and average Q in the working sample. Similarly, results in Table VI imply that the sensitivity of investment to q or $\frac{1}{\alpha H_X}$ for all firms in the sample equals 0.146 on average. The implied sensitivity of investment to q is significantly higher than the sensitivity of investment to Q reported in Panel E of Table IV.

E.2. Sample splits for financing constraints

I perform the structural estimation for alternative sample splits of financially constrained and unconstrained firms. I consider firm size and dividend payout as alternative measures of firms' financing constraints. I also consider the indices of financing constraints by Kaplan and Zingales (1997) (hereafter, the KZ index), and by Whited and Wu (2006) (hereafter, the WW index). I provide the details on the construction of these indices in the Appendix. All results are reported in Table VII.

In the alternative sample splits, the structural estimation successfully identifies higher costs of external financing in firms that are financially constrained. Financially constrained firms are subject to higher average costs of external financing than non-financially constrained firms. The average costs of external financing for less financially constrained firms range from 0.6% of capital to 3.1%. The average costs of external financing for more financially constrained firms range from 7.9% to 22.5%.

Table VII also shows that the estimated truncation threshold $\Delta^+$is larger for less financially constrained firms. Firms with higher payout, lower WW index, lower KZ index, and larger firms have relatively higher truncation thresholds. This suggests that the underlying distribution of marginal q is skewed towards more positive values for less financially constrained firms. This is consistent with the prediction that financially constrained firms are more reluctant to reinitiate investment, or have lower investment rates.

E.3. Real and financing frictions in the cross section of industries

I estimate α, θ and f for the cross section of industries to assess the interaction between real and financing frictions. The approach is consistent with Kaplan and Zingales (1997), who observe that firms' marginal cost of external financing are affected by their production technology. Results in Table VIII show significant variation in firms' real and financing frictions across industries. The goodness of fit of the model varies across industries, but remains significantly high in all cases.

Consider the results related to the curvature of adjustment costs α. Table IX reports the corresponding adjustment costs of investment by industry. The industries with the highest α and those in which the implied sensitivity of $\frac{I}{K}$ to average Q is the lowest.[42] This is illustrated in Figure 6. The correlation between the industrial series of α and the average $\frac{I}{QK}$ by industry is negative and equal to -0.649.

[42] To see this, consider the estimated α for industries 7 (Drugs, tobacco and cosmetics) and 15 (retail stores). While both industries have the same average investment rate, the Q of industry 7 is more than three times higher than the Q of industry 15. The corresponding α of industry 7 is more than three times larger.

Denote the operating expenses of the firm by Σ. Table IX reports the point estimates for firms' fixed costs of investment f by industry, and also reports the operating expenses per unit of capital $\frac{\Sigma}{K}$. Notably, those industries with the highest fixed costs are those with the highest operating expenses per unit of capital. Even if the structural estimation does not account for operating expenses explicitly, the correlation between f and $\frac{\Sigma}{K}$ across industries is 0.773. This is illustrated in Figure 6.

Consider now the curvature of firms' costs of external financing θ across industries. Notably, the only industry for which parameter θ is not significantly different from zero is that of financial services.[43] In the cross section of industries, parameter θ is negatively correlated to the average dividend to capital ratio (-0.542). This is illustrated in Figure 6. Table X reports the corresponding implications for firms' average and marginal costs of external financing by industry.

Finally, the structural estimation in Table IX can be used to predict the unobservable marginal q, average adjustment costs, average costs of external financing, and marginal costs of external financing for each firm-year observation in the working sample. Table XI reports the correlation of each these variables with observable industry characteristics in the working sample. Industries with higher marginal q have higher investment rates, and hence higher costs of adjustment on average. And industries with high dividend payout ratios, larger firms, a higher KZ index, and a higher WW index have higher costs of external funding.

IV Conclusions

This paper proposes a rationale to the poor cross sectional performance of the Q-theory of investment by examining the impact of real and financing frictions on corporate decisions. The predictions of the model are different from the stylized predictions by Hayashi (1982) and Abel and Eberly (1994) in several ways. This paper proves that marginal q is not a sufficient statistic for investment when firms rely on external financing, and proves that investment is weakly concave in q and weakly convex in cashflows. The model also rationalizes the existence of non-linear measurement error in q.

The paper considers an alternative approach to test investment equations in the presence of real and financing frictions, and then provides strong empirical evidence that the Q-theory of investment significantly explains both firms' investment and external financing decisions. The proposed log likelihood function provides a simple tool to test Q-models with real and financing frictions, and infer the magnitude of these frictions structurally. In the data, investment is

[43] Financial Services corresponds to industry 16.

concave in Q and weakly convex in cashflows, and net external issues are concave in Q, and weakly convex in cashflows. The correction of truncation bias also increases the goodness of fit; this suggests that firms' inertia regions of investment are economically significant.

Finally, and as a more broader contribution, the estimated magnitude of firms' real and financing frictions can be used to compute proxies for relevant unobservable firm characteristics. Using the estimates provided in the paper and COMPUSTAT tapes, it is straightforward to construct proxies for firms' unobservable marginal q; the expected wedge between marginal q and average Q; firms' adjustment costs of investment; and firms' costs of external financing.

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A Extension with non quadratic adjustment costs

Consider the more general cost function $C(I,K) = \frac{\alpha}{\eta} K \left(\frac{I}{K} - \delta\right)^{\eta} + \Phi^I f K$. For any $\eta > 1$, the optimal investment policy is then given by $q = H_X \left[1 + \alpha \left(\frac{I}{K} - \delta\right)^{\eta - 1}\right]$ in the active region. This already shows that marginal q is a non linear function of investment when $\eta > 1$. Rearranging terms, the optimal investment policy equals

$$\frac{I}{K} = \begin{cases} \delta + \left[\frac{1}{\alpha}\left(\frac{q}{H_X} - 1\right)\right]^{\frac{1}{\eta - 1}} & \text{if } q \notin \Theta \\ 0 & \text{if } q \in \Theta \end{cases}$$

where the boundaries of Θ are such that $\Psi(q,K) = 0$ and $\Psi(q,K)$ is defined as in (13).

B Proof of Proposition 1

Given (9) and (4), in the active region investment the total derivative of $\frac{I_t}{K_t}$ yields

$$\frac{d\left(\frac{I}{K}\right)}{dt} = \frac{\partial \frac{I}{K}}{\partial q}\frac{dq}{dt} + \frac{\partial I}{\partial \frac{\pi}{K}}\frac{d\frac{\pi}{K}}{dt}\mathbf{1}^X \tag{34}$$

Multiplying both sides of (34) by dt yields (10). Given (9) and (7), the sensitivity of $\frac{I}{K}$ to q is such that

$$\frac{\partial \frac{I}{K}}{\partial q} = \left[\alpha H_X + H_{XX}\left(\frac{q}{H_X}\right)^2\right]^{-1} > 0 \tag{35}$$

$$\frac{\partial^2 \frac{I}{K}}{\partial q^2} = -\alpha H_{XX}\left(\frac{\partial \frac{I}{K}}{\partial q}\right)^2\left(1 + 2\frac{\partial \frac{I}{K}}{\partial q}\right)\mathbf{1}^X \leqslant 0 \tag{36}$$

where (36) equals zero if $X \leqslant 0$ and is strictly negative if $X > 0$. Given (9) and (7), the sensitivity of $\frac{I}{K}$ to $\frac{\pi}{K}$ is such that

$$\frac{\partial \frac{I}{K}}{\partial \frac{\pi}{K}} = \frac{\theta \mathbf{1}^X}{\alpha H_X}; \quad \text{and } \frac{\partial^2 \frac{I}{K}}{\partial \left(\frac{\pi}{K}\right)^2} = \frac{\theta \mathbf{1}^X}{\alpha H_X^2} \tag{37}$$

C Proof of Proposition 2

Whenever $X \geq 0$, the maximand $\Psi(q,K)$ is homogeneous of degree one in K, and its roots are in line with Abel and Eberly (1994) since

$$q_i = 1 - \delta\alpha \pm \sqrt{\delta^2\alpha^2 + 2f\alpha} \tag{38}$$

Whenever $X > 0$, the firm is subject to convex costs of financing. Since these costs are quadratic, $\Psi(q,K)$ is a polynomial of order four. For the sake of comparison with (38), consider a first order Taylor approximation of the third term in (13) around the vector $(\widetilde{x}, k)$, such that

$$\frac{\theta}{2}K\left(\frac{\widetilde{X}}{K}\right)^2 \approx (h-1)\widetilde{X} - \eta K \tag{39}$$

where (39) is homogeneous of degree one, $h \equiv H_X(\widetilde{x}, k) > 0$, $\eta \equiv -H_K(\widetilde{x}, k) > 0$. The sensitivity of investment to q is by $(\alpha h)^{-1}$ .The roots in (14) correspond to the maximand

$$\Psi(q,K) \approx (q-h)I - hC(I,K) + (h-1)\frac{\pi}{K} + \eta K \tag{40}$$

D Proof of Proposition 3

Given (15), the sensitivity of $\frac{\widetilde{X}}{K}$ to q is such that

$$\frac{\partial \frac{\widetilde{X}}{K}}{\partial q} = \frac{1}{\alpha} \frac{q}{H_X} \frac{\partial \frac{I}{K}}{\partial q} > 0 \tag{41}$$

$$\frac{\partial^2 \frac{\widetilde{X}}{K}}{\partial q^2} = \left(\frac{\partial \frac{I}{K}}{\partial q} \right)^2 + \frac{1}{\alpha} \frac{q}{H_X} \frac{\partial^2 \frac{I}{K}}{\partial q^2} \lesseqgtr 0 \tag{42}$$

When $X \leq 0$, (42) is strictly positive since (36) is zero. When $X > 0$, (42) is strictly negative if and only if

$$\frac{qH_{XX}}{H_X} - 1 + 2 \frac{\partial \frac{I}{K}}{\partial q} \frac{qH_{XX}}{H_X} > 0 \tag{43}$$

Rearranging terms, (43) is equivalent to

$$\left(\frac{qH_{XX}}{H_X} - 1 \right) \left(\alpha H_X + \frac{q}{H_X} \right) + \frac{q}{H_X} \left(\frac{qH_{XX}}{H_X} - 1 \right)^2 + 2 \left(\frac{qH_{XX}}{H_X} \right) > 0$$

A sufficient conditions such that (43) holds is then

$$\left(\frac{qH_{XX}}{H_X} - 1 \right) \left(\alpha H_X + \frac{q}{H_X} \right) + \frac{q}{H_X} \left(\frac{qH_{XX}}{H_X} - 1 \right)^2 + 2 \left(\frac{qH_{XX}}{H_X} - 1 \right) > 0$$

which holds for any parameter value since

$$\alpha H_X + H_{XX} \left(\frac{q}{H_X} \right)^2 + 2 > 0$$

such that (42) is strictly negative when $X > 0$. Given (15), the sensitivity of $\frac{\widetilde{X}}{K}$ to $\frac{\pi}{K}$ is such that

$$\frac{\partial \frac{X}{K}}{\partial \frac{\pi}{K}} = \frac{\theta}{\alpha^2} \frac{q}{H_X^2} \mathbf{1}^I \mathbf{1}^X - 1 \lesseqgtr 0; \quad \text{and} \quad \frac{\partial^2 \frac{X}{K}}{\partial \left(\frac{\pi}{K} \right)^2} = \frac{\theta^2 q}{\alpha^2 H_X^3} \mathbf{1}^I \mathbf{1}^X \gtreqless 0 \tag{44}$$

E Proof of Proposition 4

The derivation follows Hennessy, Levy and Whited (2006). Define the Dynkin operator $A(f)$ on a twice differentiable function f, as

$$A(f) = dKS_K + \mu(\epsilon) f_\epsilon + \frac{\sigma(\epsilon)^2}{2} f_{\epsilon\epsilon}$$

Then our Bellman equation equals

$$rS = -H + A(S) \tag{45}$$

The Bellman equation holds pointwise, implying that partial derivatives of both sides of (45) must match. Differentiating (45) with respect to K yields

$$rq = -H_X X_K - H_K + q(I_K - \delta) + A(q)$$

From Ito's lemma and the optimality condition on I it follows that

$$KA(q) - \delta qK = A(qK) - qI = A(qK) - H_X(I + C_I I)$$

Substituting and invoking the properties of functions π, C and H yields

$$rqK = -H + b + A(qK) \tag{46}$$

Substracting (46) from (45) and rearranging terms, we then have that

$$A(S - qK) - r(S - qK) = b$$

The smooth pasting conditions for optimal default are $S_T = q_T = 0$. Using Dynkin's formula and replacing by the smooth pasting conditions upon default, the expression above is given by

$$q_t K_t = S_t + E\left[\int_t^T e^{-rs} b ds\right]$$

Denote B_t the market value of debt, and also $Q_t \equiv \frac{S_t + B_t}{K_t}$. Then the wedge between average Q and marginal q relates to debt overhang effects as in Hennessy (2004) since

$$Q_t = q_t + \frac{1}{K_t} E_t\left[e^{-r(T-t)} \gamma K_T\right] \tag{47}$$

F Proof of Proposition 5

Given (23), (30) and assumptions $1 - 2$, the post-truncation density of the average Q of any firm at time t, defined over $\left(\Delta^+, \infty\right)$ is given by

$$f\left(Q \middle| Z; \zeta, \Delta^+\right) = \frac{\frac{1}{\sigma_\iota} \phi\left(\frac{Q - Z'\zeta}{\sigma_\iota}\right)}{1 - \Phi\left(\frac{Q - Z'\zeta}{\sigma_\iota}\right)} \tag{48}$$

such that (48) is the density (conditional on Z) of observing Q for any firm at time t in the working sample. The conditional log likelihood in (32) obtains by taking logs to (48).

G Data Appendix

The working sample is drawn from the merged CRSP-COMPUSTAT database, considering the 1980-2008 period for $4,839$ US manufacturing firms and $53,230$ firm-year observations. The sample is filtered for missing data, or for observations where total assets, the gross capital stock or sales are either zero or negative. All firms with less than five consecutive years of accounting data are not included in the sample. I also delete any firm that experienced a merger accounting for more than 15% of the book value of its assets. I follow Barnett and Sakellaris (1998) and consider investment I as a measure of net capital expenditures. This stands as the difference in CRSP-COMPUSTAT items CAPXV and SPPE. Total depreciation corresponds to item DP. The gross capital stock is item PPEGT. The depreciation rate δ equals DP over PPEGT. The replacement value of capital K is computed using the perpetual inventory method described in Whited (1992), which sets K equal to PPEGT for the first firm-year observation, and then for the remaining observations of the same firm the subsequent values of K are computed using

$$K_t = \left[K_t\left(\frac{P_t^k}{P_{t-1}^k}\right) + \text{CAPXV}\right]\left[1 - \frac{2 * \text{DP}}{\text{PPEGT} + \text{CAPXV}}\right] \tag{49}$$

where P_t^k is the annual deflator for non residential investment reported by the Bureau of Economic Analysis.

Cashflows π are the sum of items IB and DP. The market value of equity S is the product of item PRCC_F times item DPC. Net debt issuance is the difference between item DLTIS and item DLTR. Net equity issuance is the difference between item SSTK and item PRSTKC. Net extenal issuance $\widetilde{X}$ is the sum of net debt issuance and net equity issuance. The total dividends paid D are the sum of item DVC and item DVP. Interest expenses b equal item XINT. The operating expenses Σ are item XOPR. Total long term debt B is the sum of item

DLTT and item DLC. Average Q is computed as the ratio of the market value of equity S, plus the book value of debt B, divided by the replacement value of capital. All variables are winsorized at 1%.

The WW index is computed following Whited and Wu (2006). The KZ index by Kaplan and Zingales (1997) is computed as in Lamont et al (1996). Both indices are normalized such that its range goes from 0 to 1. Higher values of each index correspond to more financially constrained firms.

The industry definition is based on the 17-industry groups proposed by Fama and French. Fama and French group SIC codes into 17 industries. The 17-industry groups by Fama and French are posted on Prof. French's online data library. To gain intuition on the cross sectional differences among industries, I further split the 17th industry in Fama and French into Services (industry 17 in Table II, including SIC codes 7000 to 8999), and other firms (industry 18 in Table II, including the remaining SIC codes). This yields 18 industry groups.

H Parameter choice in Figures 1-3

The benchmark parameter choice in Figures $1-3$ is $\alpha = 3; \theta = 2; \delta = 0.3; \pi = 4; K = 10; f = 0.25; b = 1$. In Figure 2, high θ is $\theta = 4$;high f is $f = 0.35$;high α is $\alpha = 4$; and low $\frac{\pi}{K}$ is $\pi = 2$ and $K = 10$.

Figure 1. Main predictions in comparison with Abel and Eberly (1994)

This figure illustrates the main predictions of the model for investment policies in comparison with those in Abel and Eberly (1994). The red lines correspond to the case in this paper in which $\theta > 0$ and $X > 0$. The blue lines correspond to the case in which firms are not subject to convex costs of external financing, either because $\theta > 0$ and $X < 0$ (as in this paper), or because $\theta = 0$ as in Abel and Eberly (1994). When firms are not subject to convex costs of external financing, investment is linear in marginal q, and the inertia region corresponds to the one derived by Abel and Eberly (1994). When $\theta > 0$ and $X > 0$, investment is concave in q, and firms are more reluctant to (dis)invest.

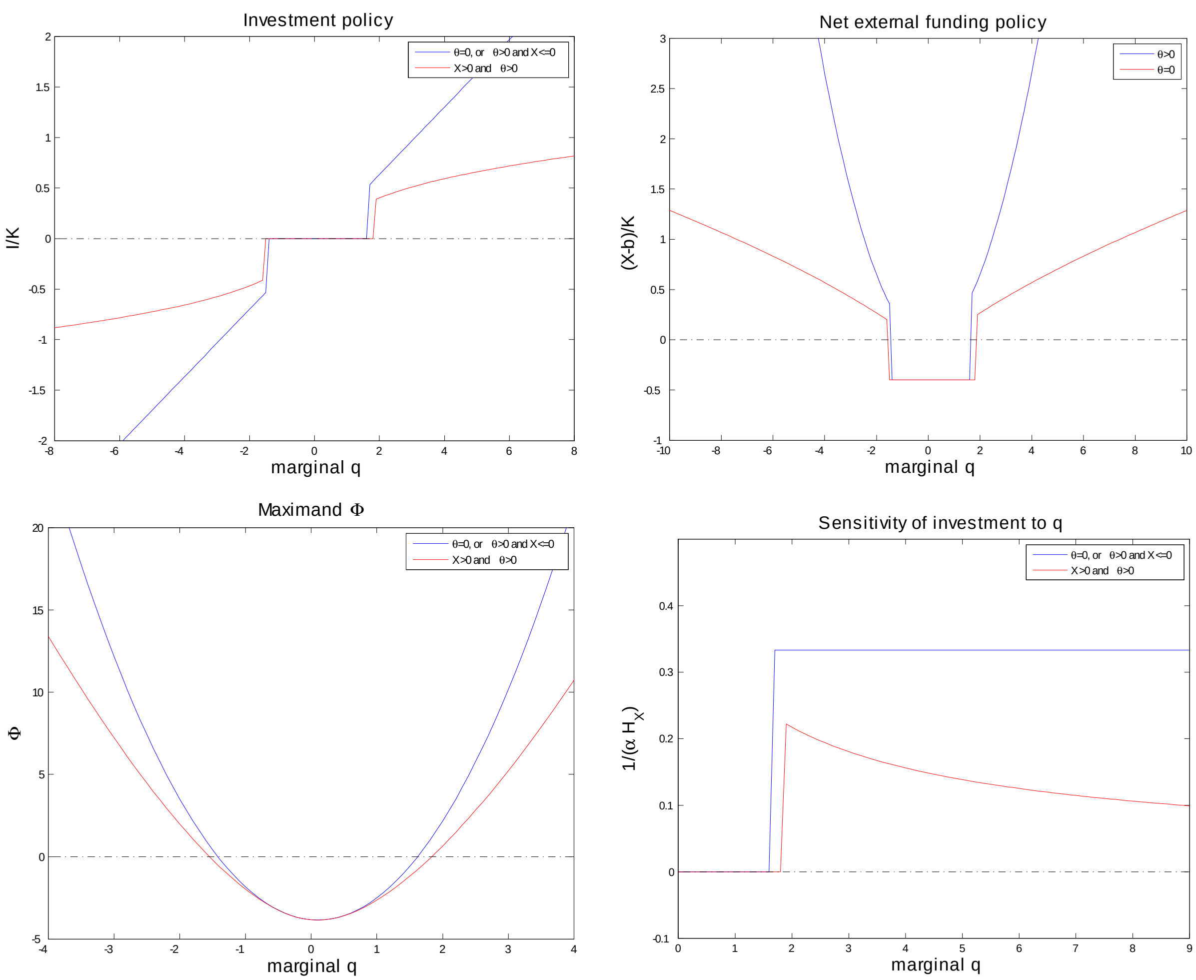

Figure 2: Impact of real and financing frictions on the sensitivity of investment to q when $\mathbf{X} > 0$

This figure provides the comparative statics of the sensitivity of investment to marginal q. In Abel and Eberly (1994), $\theta = 0$ and the sensitivity of investment to marginal q is constant and lower for higher values of α. Furthermore, the threshold q_2 increases with α and f. When $\theta > 0$ and $X > 0$, this paper shows that in the active region the sensitivity of investment to q decreases for higher levels of investment, and decreases with α, θ, f. The sensitivity of investment to q also increases with $\frac{\pi}{K}$. In the inertia region, firms are more reluctant to invest for high levels of θ, α, f and low levels of $\frac{\pi}{K}$.

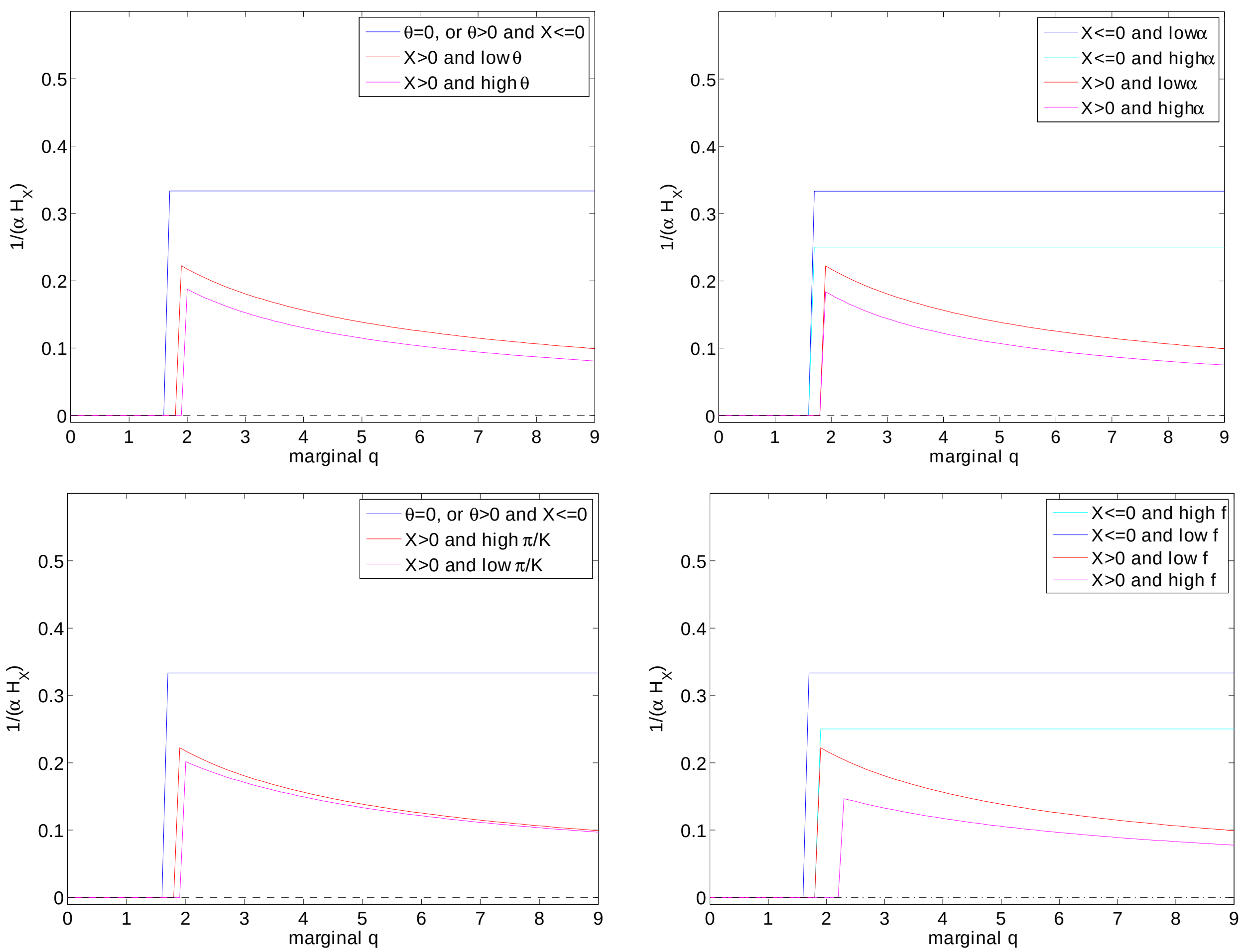

Figure 3: Impact of real and financing frictions on the investment threshold $\mathbf{q_2}$

This figure provides the comparative statics of the investment threshold $q_2 > 0$ when firms are subject to real and financing frictions. The red lines correspond to the case in which $\theta > 0$ and $X > 0$. The blue lines correspond to the case in which $\theta > 0$ and $X < 0$, or $\theta = 0$. The threshold $q_2 > 0$ is increasing in θ, α and f, and decreasing in $\frac{\pi}{K}$. The reverse holds for the disinvestment threshold $q_1 < 0$.

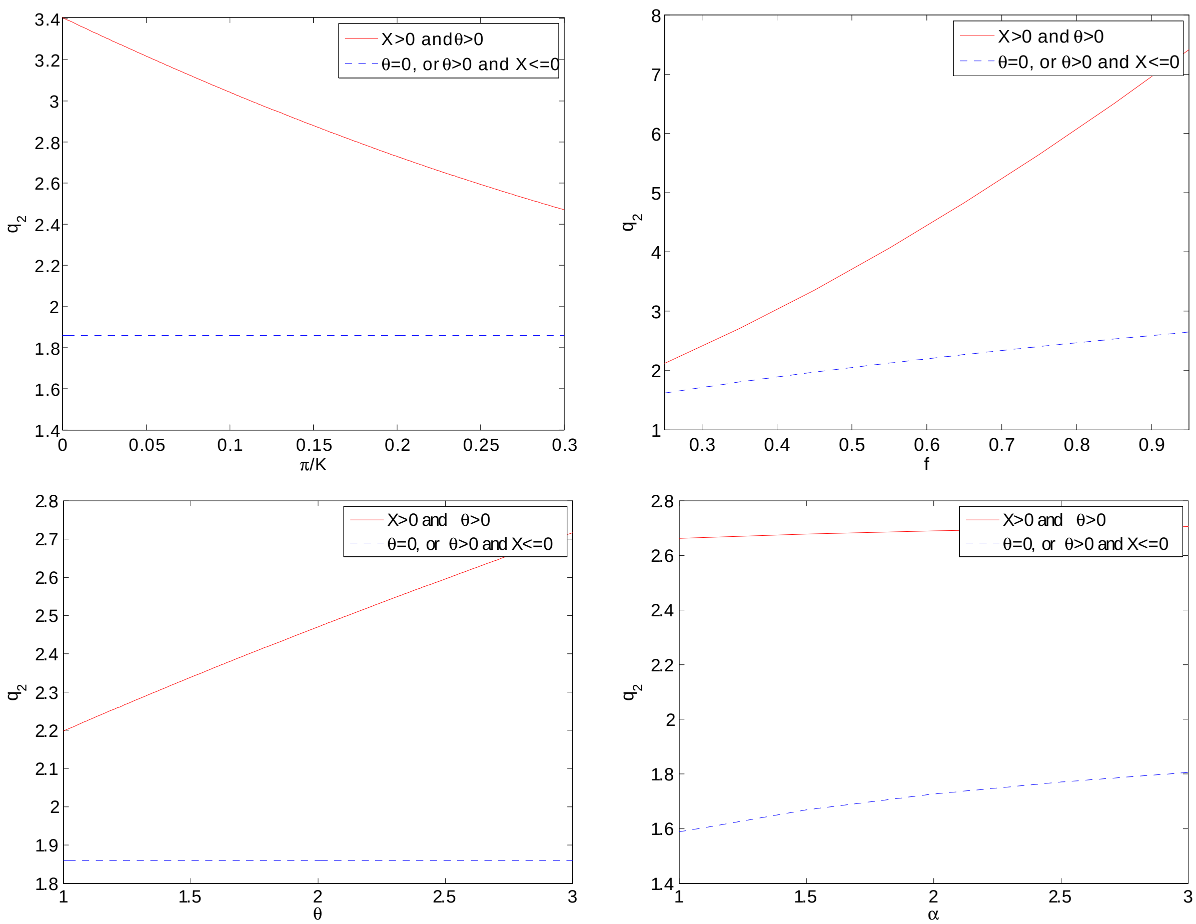

Figure 4: Actual and fitted distributions of $\frac{I}{K}$ and Q

This figure elaborates on the distribution of investment and average Q in the working sample. The left hand side charts compare the empirical distribution of investment and Q with a normal density function with their same mean and standard deviation. The empirical distribution of investment and Q are non-normal, and skewed towards positive values. The right hand side charts compare the empirical distribution of investment and Q with the distribution of the fitted values in the empirical tests. For investment, the blue line corresponds to the OLS regression in Panel B of Table III. The red line corresponds to the truncated regression of Panel E of Table III. For average Q, the blue line corresponds to OLS regression in Panel B of Table IV. The red line corresponds to the truncated regression in Panel E of Table IV. The green line corresponds to the structural estimation in Table VI. Investment and Q are better proxied by truncated regressions, and the structural estimation in Table VI.

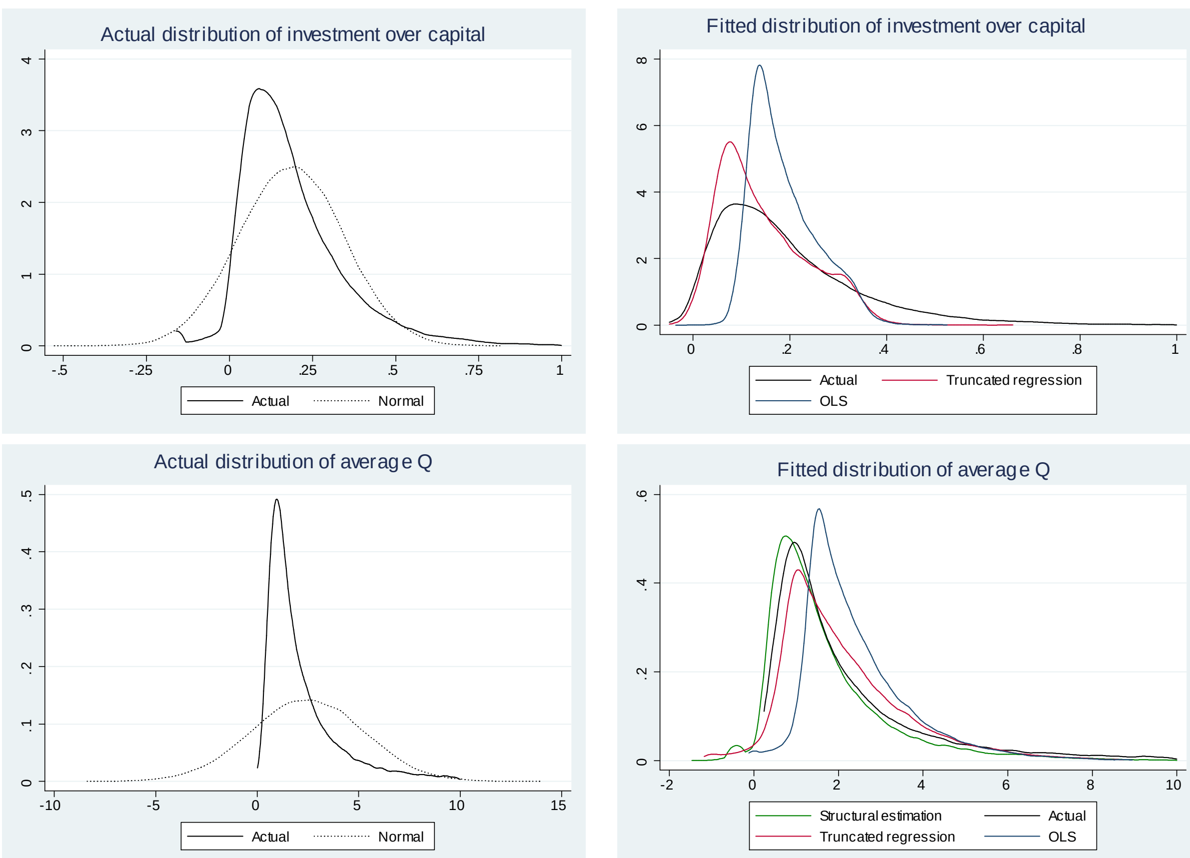

Figure 5: Conditional correlation of $\frac{I}{K}$ and $\frac{\widetilde{X}}{K} > 0$ with respect to Q and $\frac{\pi}{K}$ in the working sample

These graphs illustrate the sensitivity of investment to capital $\frac{I}{K}$ and net external issues to capital $\frac{\tilde{X}}{K} > 0$, with respect to average Q and cashflows $\frac{\pi}{K}$ in the working sample. The figures plot a local, non parametric polynomial smooth kernel function of either $\frac{I}{K}$ or $\frac{X}{K} > 0$, on either Q or $\frac{\pi}{K}$. The kernel function used is Epanechnikov. The shaded areas correspond to the 95 per cent confidence interval. Investment is concave in Q, and weakly convex in cashflows. Net external issues are weakly convex in Q, and weakly convex in cashflows.

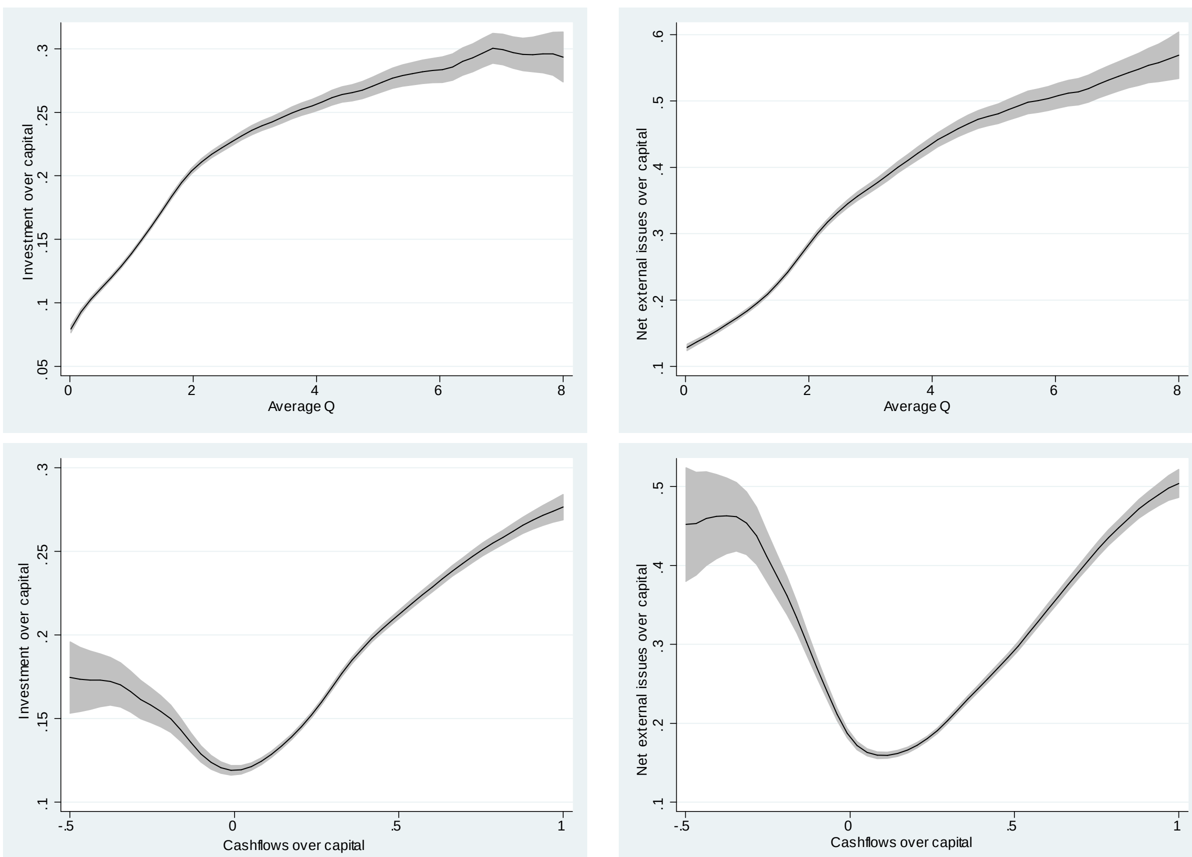

Figure 6: Correlation between observable industry characteristics and estimates α, f, and θ

These graphs illustrate the correlation between the estimated curvature of adjustment costs α, the fixed costs of investment f, and the curvature of costs of external financing θ, and observable industry characteristics. The correlation between α and the investment rate to Q ratio $\frac{I}{QK}$ is -0.649. The correlation between f and the ratio of operating expenses to capital ratio $\frac{\Sigma}{K}$ is 0.773. The correlation between θ and the dividend to capital ratio $\frac{D}{K}$ is -0.543 per cent.

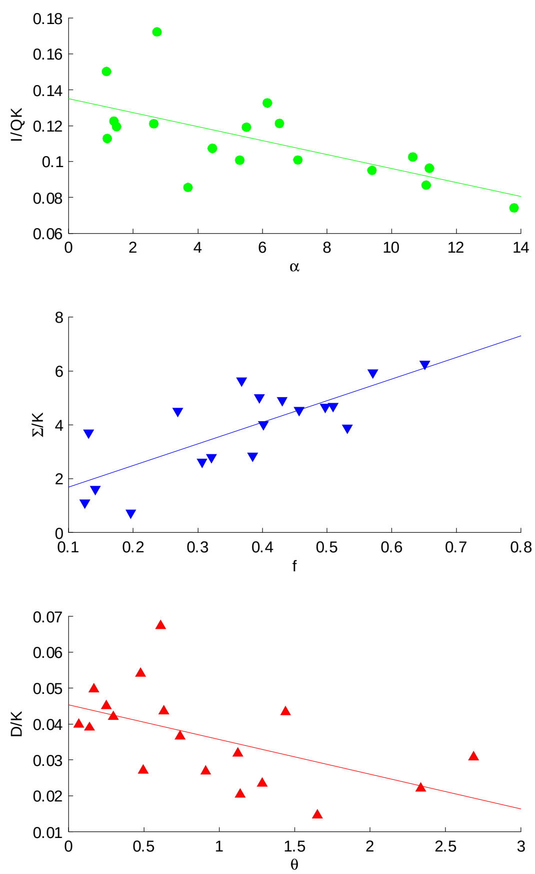

Table I: Summary statistics for all firms,
by net external issuance, and by financing constraints

This table summarizes the average firm characteristics for all firms on average, issuing firms, non issuing firms, and alternative sample splits of financially (un)constrained firms. $\frac{I}{K}$ is the investment to capital ratio, $\frac{\pi}{K}$ is cashflows to capital, $\frac{\tilde{X}}{K}$ is net external issues to capital, $\frac{b}{K}$ is interest expenses to capital, $\frac{B}{K}$ is book leverage, $\frac{D}{K}$ is dividends to capital, ln(S) is the logarithm of the market value of equity, and δ is depreciation rate. The KZ index corresponds to the index of financing constraints by Kaplan and Zingales (1987), and is normalized such that a higher value of the index denotes more financially constrained firms. The WW index correspondes to the index of financing constraints by Whited and Wu (2004), and is also normalized such that a higher value indicates more financially constrained firms.

Subsample	$\frac{I}{K}$	Q	$\frac{\pi}{K}$	$\frac{\tilde{X}}{K}$	$\frac{b}{K}$	$\frac{B}{K}$	$\frac{D}{K}$	ln(S)	δ	N
All firms	0.188	2.467	0.401	0.122	0.080	0.321	0.033	4.988	0.157	53230
Issuing firms	0.221	2.742	0.482	0.302	0.073	0.306	0.035	5.264	0.153	35237
Non issuing firms	0.125	1.929	0.241	-0.230	0.092	0.350	0.030	4.447	0.165	17993
Small firms	0.176	1.894	0.310	0.076	0.092	0.312	0.018	3.138	0.158	26610
Large firms	0.200	3.041	0.492	0.168	0.068	0.330	0.049	6.838	0.156	26611
Low payout	0.204	2.562	0.350	0.119	0.090	0.323	0.000	4.040	0.173	26609
High payout	0.172	2.372	0.452	0.125	0.069	0.319	0.066	5.936	0.140	26612
Low KZ Index	0.193	2.614	0.552	0.154	0.073	0.219	0.061	5.436	0.171	26611
High KZ Index	0.183	2.320	0.250	0.090	0.086	0.423	0.005	4.540	0.142	26610
Low WW Index	0.183	2.585	0.475	0.141	0.072	0.348	0.053	6.676	0.147	26611
High WW Index	0.193	2.349	0.327	0.103	0.087	0.294	0.013	3.300	0.166	26610

Table II: Summary statistics by industry

This table summarizes the average firm characteristics of a firm in a given industry, and of all firms in the working sample in general. The industry definition is based on the 17-industry groups by Fama and French. $\frac{I}{K}$ is the investment to capital ratio, $\frac{\pi}{K}$ is cashflows to capital, $\frac{\tilde{X}}{K}$ is net external issues to capital, $\frac{b}{K}$ is interest expenses to capital, $\frac{B}{K}$ is book leverage, $\frac{D}{K}$ is dividends to capital, ln(S) is the logarithm of the market value of equity, and δ is depreciation rate. The upperscripts +,- or = indicate whether a given mean for a given subsample is significantly above, equal, or below the corresponding mean of the reminder of the firms in the working sample (p<0.05). There are significant differences across firms in the cross section of industries.

Industry	Subsample	$\frac{I}{K}$	Q	$\frac{\pi}{K}$	$\frac{X}{K}$	$\frac{b}{K}$	$\frac{B}{K}$	$\frac{D}{K}$	ln(S)	δ	N
	All industries	0.188	2.467	0.401	0.122	0.080	0.321	0.033	4.988	0.157	53230
1	Food	0.164^{-}	2.405^{-}	$0.401^{=}$	0.119	0.079	0.339^{+}	0.044^{+}	5.414^{+}	0.132^{-}	2457
2	Mining and Minerals	0.143^{-}	1.947^{-}	0.168^{-}	0.096^{-}	0.040^{-}	0.290^{-}	0.027^{-}	5.540^{+}	0.098^{-}	1015
3	Oil and Petroleum Products	0.191	1.615^{-}	0.176^{-}	0.107^{-}	0.039^{-}	0.352^{+}	0.015^{-}	5.019	0.116^{-}	3382
4	Textiles and Apparel	0.170^{-}	1.695^{-}	0.580^{+}	0.133	0.118^{+}	0.280^{-}	0.045^{+}	4.451^{-}	0.153^{-}	1687
5	Consumer Durables	0.174^{-}	1.914^{-}	0.480^{+}	0.094^{-}	0.104^{+}	0.307^{-}	0.042^{+}	4.334^{-}	0.164^{+}	2184
6	Chemicals	0.143^{-}	2.054^{-}	0.379^{-}	0.086^{-}	0.065^{-}	0.354^{+}	0.050^{+}	6.045^{+}	0.125^{-}	1460
7	Drugs, Cosmetics and Tobacco	0.216^{+}	4.158^{+}	0.497^{+}	0.158^{+}	0.095^{+}	0.253^{-}	0.067^{+}	5.361^{+}	0.167^{+}	1488
8	Construction	0.149^{-}	1.873^{-}	0.368^{-}	0.098^{-}	0.087^{+}	0.314^{-}	0.032	4.631^{-}	0.133^{-}	2144
9	Steel Works	0.132^{-}	1.406^{-}	0.292^{-}	0.095	0.059^{-}	0.338^{+}	0.023^{-}	5.307^{+}	0.101^{-}	1490
10	Fabricated Products	0.153^{-}	1.817^{-}	0.462^{+}	0.107	0.086^{+}	0.311^{-}	0.043^{+}	4.486^{-}	0.145^{-}	1425
11	Machinery and Equipment	0.203^{+}	2.550^{+}	0.469^{+}	0.138^{+}	0.088^{+}	0.245^{-}	0.031^{-}	4.539^{-}	0.189^{+}	8523
12	Automobiles	0.191	2.061^{-}	0.531^{+}	0.144^{+}	0.092^{+}	0.317	0.054^{+}	5.429^{+}	$0.157^{=}$	900
13	Transportation	0.167^{-}	1.850^{-}	0.296^{-}	0.109^{-}	0.056^{-}	0.406^{+}	0.022^{-}	5.547^{+}	0.123^{-}	2756
14	Utilities	0.098^{-}	1.258^{-}	0.166^{-}	0.054^{-}	0.046^{-}	0.496^{+}	0.037^{+}	6.695^{+}	0.061^{-}	2602
15	Retail stores	0.215^{+}	1.754^{-}	0.390^{-}	0.134^{+}	0.073^{-}	0.327^{+}	0.020^{-}	5.055^{-}	0.140^{-}	5183
16	Financial Services	0.162^{-}	3.704^{+}	0.323^{-}	0.089^{-}	0.076	0.330	0.040^{+}	3.914^{-}	0.133^{-}	1015
17	Services	0.242	4.148	0.469	0.147	0.095	0.333	0.027	4.445	0.222	4992
18	Other	0.210	3.230	0.465	0.138	0.091	0.309	0.039	5.039	0.190	8527

Table III: Empirical tests with $\frac{I}{K}$ as a dependent variable

This table provides the reduced form tests for investment policies with investment over capital $\frac{I}{K}$ as a dependent variable. $\frac{\pi}{K}$ is the cashflow to capital ratio, and $\frac{B}{K}$ is the book leverage ratio. Panel A provides the benchmark OLS regression of $\frac{I}{K}$ on average Q and $\frac{\pi}{K}$. Panel B adds the interaction of Q and $\frac{\pi}{K}$, and higher order terms of these to the regression in Panel A. The coefficient on Q is higher in Panel B than in Panel A, and the coefficient on Q^2 is negative and significant. Panel C adds firm fixed effects to the regression in Panel B. Panel D considers both fixed effects and lagged investment. Panel E estimates the model in Panel B using truncated maximum likelihood. The lower threshold on investment is set to the minimum value of $\frac{I}{K}$ in the sample. Panel F performs the same truncated regression for the subsample of non-issuing firms. Panel G adds additional controls to the truncated regression in Panel E. All regressions include year dummies, and standard errors are clustered by industry.

	(A)	(B)	(C)	(D)	(E)	(F)	(G)
Q_t	0.0159***	0.0361***	0.0304***	0.0228***	0.0491***	0.0472***	0.0625***
	(0.000669)	(0.00194)	(0.00340)	(0.00078)	(0.00281)	(0.00361)	(0.00430)
Q_t^2		-0.00122***	-0.00104***	-0.00081***	-0.00165***	-0.00191***	-0.00536***
		(0.000145)	(0.000167)	(0.000043)	(0.000188)	(0.000256)	(0.000637)
$\frac{\pi_t}{K_t}$	0.0665***	0.0760***	0.0506***	0.0502***	0.109***	0.0755***	0.0643***
	(0.00641)	(0.00957)	(0.00866)	(0 00278)	(0.0134)	(0.0102)	(0.0101)
$\left(\frac{\pi_t}{K_t}\right)^2$		0.00185	-0.00013	-0.00180*	-0.00092	0.00761***	0.00263
		(0.00195)	(0.00189)	(0.00096)	(0.00313)	(0.00257)	(0.00221)
$Q_t * \frac{\pi_t}{K_t}$		-0.00389***	-0.00251***	-0.00261***	-0.00642***	-0.00507***	-0.00236***
		(0.000827)	(0.000764)	(0.000307)	(0.00103)	(0.00115)	(0.00079)
$\frac{I_{t-1}}{K_{t-1}}$				0.2547***			
				(0.00430)			
Q_t^3							0.0001***
							(0.00002)
$\left(\frac{\pi_t}{K_t}\right)^3$							-0.0005
							(0.00041)
$\frac{B_t}{K_t}$							-0.0233*
							(0.01351)
N	53,230	53,230	53,230	43,910	53,230	17,392	53,230
R^2	0.170	0.197	0.085	0.150	0.668	0.634	0.714
R^2-adj.	0.169	0.198	0.084	0.149	0.667	0.633	0.712
lnL	27,647	28,550			29,847	13,991	30,208

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table IV: Empirical tests with Q as a dependent variable

This table provides the reduced form tests for investment to capital ratios $\frac{I}{K}$ with average Q as a dependent variable. $\frac{\pi}{K}$ is the cashflow to capital ratio, and $\frac{B}{K}$ is the book leverage ratio. Panel A provides the benchmark OLS regression of $\frac{I}{K}$ and $\frac{\pi}{K}$ as regressors. Panel B adds the higher order terms of $\frac{I}{K}$, and the interaction of $\frac{I}{K}$ and $\frac{\pi}{K}$ to the regressors in Panel A. These additional regressors add explanatory power, and the coefficient on $(\frac{I}{K})^3$ is negative and significant. Panel C adds firm fixed effects to the regression in Panel B. Panel D considers both fixed effects and lagged Q as a regressor. Panel E estimates the model in Panel B using a truncated regression. The lower threshold on investment is -1.68. Panel F performs the same truncated regression for the subsample of non-issuing firms. Panel G adds controls to the truncated regression in Panel E. All regressions include year dummies, and standard errors are clustered by industry.

	(A)	(B)	(C)	(D)	(E)	(F)	(G)
$\frac{I_t}{K_t}$	4.491***	1.561**	1.312***	0.412**	3.542***	3.796***	2.484***
	(0.654)	(0.693)	(0.188)	(0.153)	(1.337)	(1.075)	(1.140)
$\left(\frac{I_t}{K_t}\right)^2$		7.906***	1.134*	-1.805**	7.515***	16.10***	11.33***
		(1.831)	(0.650)	(0.536)	(2.272)	(4.254)	(4.072)
$\left(\frac{I_t}{K_t}\right)^3$		-5.968***	-0.752	0.009	-6.439***	-15.79***	-6.829***
		(1.306)	(0.577)	(0.505)	(1.593)	(4.701)	(2.182)
$\frac{I_t}{K_t} * \frac{\pi_t}{K_t}$		1.344	0.505*	0.602***	0.432	-0.650	-0.306
		(0.784)	(0.265)	(0.101)	(0.820)	(0.802)	(0.773)
$\frac{\pi_t}{K_t}$	1.639***	1.285***	1.425***	1.008***	2.000***	0.832***	0.974***
	(0.258)	(0.155)	(0.150)	(0.0328)	(0.319)	(0.260)	(0.301)
Q_{t-1}				0.497***			
				(0.0044)			
$\left(\frac{I_t}{K_t}\right)^4$							-7.568***
							(3.227)
$\left(\frac{\pi_t}{K_t}\right)^2$							0.813***
							(0.112)
$\frac{B_t}{K_t}$							-1.576***
							(0.335)
N	53,230	53,230	53,230	43,910	53,230	17,988	53,230
R^2	0.230	0.248	0.165	0.387	0.570	0.532	0.632
R^2-adj.	0.230	0.247	0.164	0.386	0.568	0.532	0.631
lnL	-123,563	-117,659			-113,676	-35,197	-116,563

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table V: Empirical tests for $\frac{\tilde{X}}{K} > 0$

This table provides the reduced form tests for net external issues to capital ratios $\frac{\tilde{X}}{K} > 0$. $\frac{\pi}{K}$ is the cashflow to capital ratio, and $\frac{I}{K}$ is the investment to capital ratio. Panels A to C consider regressions with $\frac{\tilde{X}}{K}$ as the dependent variable. Panel A provides the benchmark OLS regression of $\frac{\tilde{X}}{K}$ on average Q and cashflows to capital $\frac{\pi}{K}$. Panel B adds the interaction of Q and cashflows, and higher order terms of these to the regression in Panel A. Consistent with the model, cashflows are highly significant, the coefficient on Q is higher in Panel B than in Panel A, and the coefficient on Q^2 is negative and significant. Panel C adds firm fixed effects to the regression in Panel B. The magnitude of the coefficients is similar to that in Panel A. Panels D to F consider regressions with average Q as the dependent variable. Panel D provides the OLS regression of average Q on $\frac{\tilde{X}}{K}$ and the interaction term of $\frac{\tilde{X}}{K}$ and investment to capital $\frac{I}{K}$. Panel E adds fixed effects to the estimation in Panel A. Panel F considers the truncated regression of the model in Panels A and B. The truncation point equals zero. The explanatory power of the model is higher in Panel F than in Panels D and E. All regressions include year dummies, and standard errors are clustered by industry.

	(A)	(B)	(C)	(D)	(E)	(F)
Q_t	0.0298***	0.0682***	0.0800***			
	(0.00256)	(0.00471)	(0.00503)			
Q_t^2		-0.00191***	-0.00251***			
		(0.000245)	(0.000237)			
$\frac{\pi_t}{K_t}$	0.185***	0.205***	0.156***			
	(0.0246)	(0.0243)	(0.0190)			
$\left(\frac{\pi_t}{K_t}\right)^2$		0.0310***	0.0171***			
		(0.00545)	(0.00427)			
$Q_t * \frac{\pi_t}{K_t}$		-0.0145***	-0.00647***			
		(0.00174)	(0.00106)			
$\frac{\widetilde{X}_t}{K_t}$				2.147***	1.277***	2.839***
				(0.303)	(0.124)	(0.501)
$\frac{\widetilde{X}_t}{K_t} * \frac{I_t}{K_t}$				3.543***	1.146***	3.832***
				(0.959)	(0.217)	(1.138)
N	35,233	35,233	35,080	35,237	35,084	35,237
R^2	0.215	0.253	0.121	0.195	0.072	0.536
R^2-adj.	0.214	0.252	0.121	0.193	0.072	0.535
lnL	-5,421	-4,558		-83,827		-81,250

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table VI: Baseline structural estimation

This table provides the baseline structural estimation of the model using maximum likelihood. The table reports the estimated curvature of adjustment costs α, fixed costs of investment per unit of capital f, and curvature of costs of external financing θ. Robust standard errors are reported in parentheses. All parameters are individually significant at p<0.01. σ_ι is the standard deviation of the residuals, δ the depreciation rate, and Δ^+ is estimated lower truncation threshold. The p-value of the Wald test shows that all parameters are jointly significant. The R^2 of the estimation is significantly higher than the OLS regressions reported in Table IV. The estimation quantifies firms' average adjustment costs of investment per unit of capital $C(\frac{I}{K},1)$, average costs of external financing per unit of capital $\frac{H(X,K)}{K}$, average marginal costs of external financing H_X, marginal q, and the expected wedge between average Q and marginal q κ.

α	f	θ	σ_ι	Δ^+
6.094	0.376	0.783	3.025	-1.680
(0.089)	(0.003)	(0.007)	(1.008)	
N	R^2	lnL	χ^2	δ
53221	0.482	-116997	0.001	0.157
$C(\frac{I}{K},1)$	$\frac{H(X,K)}{K}$	H_X	q	κ
0.076	0.094	1.221	1.451	1.017

Table VII: Structural estimation for subsamples of more and less financially constrained firms

This table reports structural estimation results for subsamples of less and more financially constrained firms. The table reports the estimated curvature of adjustment costs α, fixed costs of investment per unit of capital f, and curvature of costs of external financing θ. σ_ι is the standard deviation of the residuals, δ the depreciation rate, and Δ^+ is estimated lower truncation threshold. Robust standard errors are reported in parentheses. All Wald tests have p-values equal to zero and are omitted for brevity. Larger firms, firms with higher dividend payout, a lower KZ index, and a lower W index are less financially constrained and have a lower θ.

Subsample	α	f	θ	σ_ι	N	R^2	lnL	δ	Δ^+	$C(\frac{I}{K},1)$	$\frac{H(X)}{K}$	H_X
Small firms	1.565	0.300	1.923	2.098	26610	0.520	-53551.1	0.158	-2.200	0.023	0.169	1.350
	(0.022)	(0.004)	(0.012)	(1.014)								
Large firms	11.395	0.459	0.334	3.509	26611	0.487	-60230	0.156	-1.100	0.121	0.028	1.073
	(0.151)	(0.004)	(0.003)	(1.009)								
Low payout	2.589	0.314	2.113	3.280	26609	0.473	-60847.6	0.173	-1.800	0.045	0.225	1.467
	(0.224)	(0.005)	(0.093)	(1.012)								
High payout	10.880	0.462	0.097	2.575	26612	0.501	-51135.8	0.140	-0.400	0.084	0.006	1.017
	(0.085)	(0.004)	(0.002)	(1.009)								
Low KZ index	8.522	0.565	0.351	2.995	26611	0.466	-55662.4	0.171	-0.900	0.089	0.031	1.077
	(0.115)	(0.004)	(0.004)	(1.009)								
High KZ index	5.150	0.178	0.944	3.067	26610	0.461	-58130.2	0.142	-1.400	0.076	0.079	1.171
	(0.238)	(0.004)	(0.026)	(1.012)								
Low WW index	11.999	0.457	0.192	2.832	26611	0.507	-53397.1	0.147	-0.400	0.101	0.014	1.037
	(0.106)	(0.004)	(0.002)	(1.009)								
High WW index	3.792	0.280	1.152	3.038	26610	0.453	-57531.5	0.166	-1.300	0.064	0.115	1.240
	(0.254)	(0.004)	(0.045)	(1.012)								

Table VIII: Structural estimation for the cross section of industries

The table reports the curvature of adjustment costs α, fixed costs f, and the curvature of costs of external financing θ by industry. Robust standard errors are reported in parentheses. σ_ι is the standard deviation of the residuals, δ the depreciation rate, and Δ^+ is estimated lower truncation threshold. All Wald tests have p-values equal to zero and are omitted for brevity.

Industry	α	f	θ	σ_ι	N	lnL	R^2	δ	Δ^+
1	7.097	0.430	0.631	2.553	2457	-4694	0.525	0.132	-0.350
	(0.853)	(0.011)	(0.053)	(1.028)					
2	5.298	0.141	0.910	1.844	1015	-1878	0.610	0.098	-1.480
	(1.557)	(0.014)	(0.194)	(1.062)					
3	1.173	0.125	1.649	1.485	3382	-5490	0.645	0.116	-0.880
	(0.441)	(0.006)	(0.380)	(1.038)					
4	6.151	0.651	0.251	1.550	1687	-2506	0.622	0.153	-0.250
	(0.440)	(0.020)	(0.019)	(1.033)					
5	5.503	0.497	0.296	1.889	2183	-3551	0.556	0.164	-0.250
	(0.406)	(0.014)	(0.031)	(1.031)					
6	9.385	0.385	0.167	2.066	1460	-2566	0.547	0.125	-0.560
	(1.174)	(0.016)	(0.045)	(1.038)					
7	11.067	0.457	0.612	5.008	1488	-3893	0.421	0.167	-2.000
	(2.792)	(0.027)	(0.066)	(1.030)					
8	4.452	0.395	1.123	2.076	2142	-4007	0.525	0.133	-1.185
	(0.715)	(0.014)	(0.094)	(1.048)					
9	2.638	0.321	1.285	1.106	1490	-2016	0.705	0.101	-0.550
	(0.139)	(0.012)	(0.034)	(1.053)					
10	1.485	0.531	1.440	1.609	1425	-2311	0.675	0.145	-0.695
	(0.543)	(0.017)	(0.156)	(1.039)					
11	1.196	0.509	2.686	2.666	8521	-17791	0.577	0.189	-1.290
	(0.039)	(0.009)	(0.026)	(1.016)					
12	6.524	0.571	0.479	1.969	900	-1586	0.595	0.157	-0.650
	(0.843)	(0.024)	(0.041)	(1.069)					
13	1.403	0.306	2.335	1.961	2756	-4921	0.608	0.123	-0.730
	(0.194)	(0.009)	(0.124)	(1.039)					
14	3.694	0.197	0.739	0.552	2602	-1863	0.873	0.061	0.095
	(0.093)	(0.003)	(0.021)	(1.040)					
15	2.737	0.368	1.138	1.472	5182	-8555	0.652	0.140	-1.050
	(0.105)	(0.008)	(0.027)	(1.024)					
16	13.781	0.131	0.067	5.239	1013	-2615	0.302	0.133	-1.450
	(4.373)	(0.028)	(0.067)	(1.038)					
17	11.163	0.269	0.495	5.205	5311	-13785	0.361	0.213	-1.650
	(1.062)	(0.012)	(0.030)	(1.017)					
18	10.650	0.401	0.138	4.134	8207	-19408	0.353	0.195	-1.185
	(0.539)	(0.009)	(0.011)	(1.014)					

Table IX: Quantitative implications for real frictions by industry

This table provides the quantitative implications of the structural estimation for real frictions in the cross section of industries. The left hand side columns indicate the predictions of the model for the unobservable α, costs of adjustment, and fixed costs of investment f. $C(\frac{I}{K},1)$ are the average adjustment costs per unit of capital, $\frac{I}{K}$ is the investment to capital ratio, and $\frac{\Sigma}{K}$ is the ratio of operating expenses to capital.

	Industry	α	$C(\frac{I}{K},1)$	f	$\frac{I}{K}$	Q	$\frac{I}{QK}$	$\frac{\Sigma}{K}$
1	Food	7.097	0.056	0.430	0.164	2.405	0.101	5.118
2	Mining and Minerals	5.298	0.049	0.141	0.143	1.947	0.101	1.567
3	Oil and Petroleum Products	1.173	0.020	0.125	0.191	1.615	0.150	1.162
4	Textiles and Apparel	6.151	0.058	0.651	0.170	1.695	0.133	6.480
5	Consumer Durables	5.503	0.055	0.497	0.174	1.914	0.119	4.709
6	Chemicals	9.385	0.063	0.385	0.143	2.054	0.095	2.915
7	Drugs, Cosmetics and Tobacco	11.067	0.156	0.457	0.216	4.158	0.087	5.181
8	Construction	4.452	0.044	0.395	0.149	1.873	0.107	5.748
9	Steel Works	2.638	0.019	0.321	0.132	1.406	0.121	2.784
10	Fabricated Products	1.485	0.013	0.531	0.153	1.817	0.120	4.011
11	Machinery and Equipment	1.196	0.016	0.509	0.203	2.550	0.113	4.859
12	Automobiles	6.524	0.061	0.571	0.191	2.061	0.121	5.969
13	Transportation	1.403	0.016	0.306	0.167	1.850	0.123	2.784
14	Utilities	3.694	0.010	0.197	0.098	1.258	0.086	0.722
15	Retail stores	2.737	0.038	0.368	0.215	1.754	0.172	5.973
16	Financial Services	13.781	0.247	0.131	0.162	3.704	0.074	3.712
17	Services	11.163	0.226	0.269	0.242	4.148	0.096	4.853
18	Other	10.650	0.149	0.401	0.210	3.230	0.103	4.268

Table X: Quantitative implications for costs of external financing by industry

This table provides the quantitative implications of the structural estimation for financing frictions in the cross section of industries. The left hand side columns indicate the predictions of the model for the unobservable θ and costs of external financing by industry. $\frac{H(X,K)}{K}$ are the average costs of external financing per unit of capital, H_X are marginal costs of financing, $\frac{D}{K}$ is the dividend to capital ratio, $ln(S)$ the logarithm of the market value of equity, and KZ and WW are indices of financing constraints.

	Industry	θ	$\frac{H(X,K)}{K}$	H_X	$\frac{D}{K}$	$ln(S)$	KZ	WW
1	Food	0.631	0.038	1.110	0.044	5.414	0.753	0.421
2	Mining and Minerals	0.910	0.048	1.140	0.027	5.540	0.793	0.428
3	Oil and Petroleum Products	1.649	0.075	1.248	0.015	5.019	0.832	0.487
4	Textiles and Apparel	0.251	0.025	1.056	0.045	4.451	0.707	0.489
5	Consumer Durables	0.296	0.024	1.055	0.042	4.334	0.733	0.510
6	Chemicals	0.167	0.009	1.025	0.050	6.045	0.740	0.348
7	Drugs, Cosmetics and Tobacco	0.612	0.076	1.159	0.067	5.361	0.687	0.473
8	Construction	1.123	0.092	1.202	0.032	4.631	0.765	0.472
9	Steel Works	1.285	0.068	1.197	0.023	5.307	0.794	0.397
10	Fabricated Products	1.440	0.115	1.272	0.043	4.486	0.733	0.493
11	Machinery and Equipment	2.686	0.312	1.653	0.031	4.539	0.737	0.539
12	Automobiles	0.479	0.043	1.106	0.054	5.429	0.721	0.395
13	Transportation	2.335	0.134	1.378	0.022	5.547	0.815	0.396
14	Utilities	0.739	0.009	1.050	0.037	6.695	0.808	0.236
15	Retail stores	1.138	0.080	1.221	0.020	5.055	0.792	0.467
16	Financial services	0.067	0.006	1.012	0.040	3.914	0.754	0.590
17	Services	0.495	0.058	1.122	0.027	4.445	0.797	0.551
18	Other	0.138	0.015	1.032	0.039	5.039	0.753	0.480

Table XI: Inter industry correlation of marginal q and costs of external financing with industry characteristics

This table reports the correlation of observable average industry characteristics with the average marginal q, adjustment costs, and costs of external financing by industry as in Table X. $C(\frac{I}{K}, K)$ is the average adjustment cost of investment per unit of capital, $\frac{H(X,K)}{K}$ is the average cost of external financing per unit of capital, H_X is the average marginal cost of financing, $\frac{I}{K}$ is the average investment to capital ratio, $\frac{D}{K}$ is the average dividend to capital ratio, $ln(S)$ is the average of the logarithm of the market value of equity, and KZ and WW are the average indices of financing constraints.

	q	$C(\frac{I}{K}, 1)$	$\frac{H(X,K)}{K}$	H_X
q	1	0.2502	-0.1536	0.5498
$\frac{H(X,K)}{K}$	0.5762	-0.1536	1	0.9807
H_X	0.5498	-0.2438	0.9807	1
$\frac{I}{K}$	0.5468	0.4652	0.2555	0.2065
$\frac{D}{K}$	0.0506	0.2842	-0.2704	-0.3882
ln(S)	-0.0904	-0.4022	-0.2139	-0.1409
KZ	-0.0976	-0.2083	0.0012	0.145
WW	0.3228	0.5229	0.2714	0.1973