# Estimation of a Dynamic Auction with Participation

Joachim Groeger*

November 20, 2009

JOB MARKET PAPER

**Abstract**

We study participation decisions and bidding behaviour in Michigan Department of Transportation procurement auctions. Patterns in the bidding data suggest that bidders' participation behaviour has a forward looking component. To fully understand the extent of these effects on auction outcomes, we construct and estimate a dynamic asymmetric auction model with endogenous participation. We develop an estimation approach which builds on Guerre, Perrigne and Vuong (2000) and recently developed dynamic discrete game estimators. We then quantify the level of inefficiencies under the current auction rules and then consider how alternative auction rules affect efficiency. We also analyse the effect of ignoring dynamics in this market by estimating a static version of our model. This approach results in misleading conclusions concerning auction efficiency.

# 1 Introduction

Most of the literature on the structural estimation of auctions has looked at auctions in isolation, focusing on bidding behaviour and taking participation to be exogenous. However, the context in which an auction operates is as important as the rules governing bidding. This is especially true when considering the participation behaviour of asymmetric bidders, as discussed in Klemperer (2002)[1], who cites that different auction rules could explain large differences in auction revenues.

We analyse these issues in the context of procurement auctions run by the Michigan Department of Transportation. The data suggest there are dynamic linkages between auction rounds. Specifically, previous round competition has an effect on current procurement costs. There is also evidence that dynamic synergies in participation exist. These synergies effectively create an asymmetry between bidders that will have a bearing on auction efficiency. To fully

*Department of Economics, London School of Economics, Houghton Street, London WC2A 2AE, e-mail: j.r.groeger@lse.ac.uk. I would like to thank my advisors Martin Pesendorfer and Philipp Schmidt-Dengler for invaluable guidance and advice. I am also grateful to Pasquale Schiraldi for detailed discussions. I would also like to thank Sue Powers at the Michigan Department of Transportation for providing me with the data and Victor Ortego-Marti, Sorawoot Srisuma and Dimitri Szerman for helpful discussions. Any remaining errors are mine.

[1]Klemperer (2002) points out that sealed bid formats may favour weak bidders and encourage entry.

understand these features and their effect on auction outcomes, we construct and estimate a dynamic auction game. The level of inefficiencies due to auction rules is quantified and changes to the auction environment are considered. We then explore the effect of ignoring dynamics in estimation and find that this will result in misleading conclusions concerning efficiency.

This paper considers a dynamic auction game in which participation is decided in every period. Entry into an auction is costly and bidders must pay an information acquisition cost to learn their private completion costs,which are equated to the opportunity cost of preparing a bid. Bidders compare the expected profit stream from participation and non-participation to determine entry. There are information acquisition synergies between auction rounds. Specifically, a bidder can improve the information cost draws in the next period by participating in an auction in the current round. Once a bidder decides to enter an auction the bidders engage in a first price sealed bid procurement auction. We observe data on bids, auction characteristics and entry decisions. Private completion costs are inferred using a variation on Guerre *et al.* (2000) estimation approach. The distribution of entry costs for bidders is estimated using the asymptotic least squares estimator for dynamic games proposed by Pesendorfer & Schmidt-Dengler (2008).

Our paper complements a growing literature on estimating static auctions with endogenous participation[2], for example, Athey *et al.* (2008), Li & Zheng (2009), Bajari & Hortacsu (2003) and Krasnokutskaya & Seim (2005)[3] by incorporating dynamic considerations. The only other paper we are aware of that estimates a dynamic auction using highway procurement data is Jofre-Bonet & Pesendorfer (2003). That paper looks at how capacity constraints affect bidding behaviour and participation is taken to be exogenous. There are also a number of papers that look at entry into markets using dynamic frameworks, these include Collard-Wexler (2006) and Ryan (2006). Auguirregabiria & Mira (2007) also provide an application of their pseudo-maximum likelihood methods to firm entry and exit in local retail markets. Other papers that look at estimating dynamic games are Bajari *et al.* (2007) and Pakes *et al.* (2007). These papers all make use of the insights from Hotz & Miller (1993)[4]. Procurement auctions have been analysed by Porter & Zona (1993), Bajari & Ye (2003), Hong & Shum (2002) and Krasnokutskaya (2003). Paarsch (1992), Laffont *et al.* (1995) and Guerre *et al.* (2000) have developed empirical methods to estimate private information in static auction environments with exogenous participation.

[2]The theoretical literature on entry in auctions is more mature than the empirical one, with papers from Samuelson (1985), McAfee & McMillan (1987), Levin & Smith (1994). Hendricks *et al.* (2003) also consider first price auction models with endogenous participation. However, their paper is concerned with testing the implications of the theoretical model rather than estimating its primitives.

[3]These papers makes use of static participation games which suffer from multiple equilibria and the typical approach has been to select an equilibrium or to provide conditions that guarantee uniqueness. An alternative is to due to Manski & Tamer (2003) and Tamer (2003). These approaches estimate parameter bounds that are consistent with all equilibria implied by the model. As pointed out by Pesendorfer & Schmidt-Dengler (2008), our formulation as a Markovian game, avoids a priori selection of a specific equilibrium. The Markovian assumption therefore guarantees that a single time series has been generated by only one equilibrium.

[4]In particular, Hotz & Miller (1993) establish that conditional choice probabilities at each state can be used in an inversion to infer value functions.

The remainder of the paper is organised as follows. In Section 2 we provide a description of the data and reduced form analyses on the role of participation dynamics on auction outcomes. Section 3 outlines the theoretical model. We then show in Section 4 how the primitive parameters of our model can be estimated. Section 5 outlines the main estimation outputs and Section 6 summarises the results of our efficiency measurement and policy analysis. We then consider the effect of ignoring dynamics in participation by estimating a static version of our model in Section 7.

# 2 The Procurement Process in Michigan

In this section the data source and some aspects of the procurement process are described. Reduced form evidence of dynamic linkages between auction rounds is presented. In particular, we look at the effect of previous round competition on current round procurement costs. We then explore whether dynamic effects also exist when we look directly at an individual bidder's participation and bid level decisions. The results suggest that past round competition lowers procurement costs by a small amount in current round auctions. There is also evidence that potential dynamic synergies exist in bidders' participation behaviour and that past participation has an effect on bidding behaviour in current auctions.

*The Awarding Process:* Contracts are awarded by the Michigan State Department of Transportation (MDOT) in bi-monthly rounds using a first price sealed bid procurement auction. MDOT requires that bidders pre-qualify before bidding[5]. An average of 50 contracts are awarded in a round of bidding. The average project size is \$1.476 million and the maximum project size in our data set is \$165 million. The timing of contract rounds for the year is known in advance, however the contract characteristics are not fully known. Prior to the awarding of the contract, bidders can purchase plans for contracts, which detail the location of a project, the nature of the work and the estimated cost. The list of bidders who have purchased these plans, i.e. the plan holders, is publicly known and posted on the internet. However, some plans can be downloaded free of charge from the MDOT website, where bidders' identities are not revealed. Bids can be submitted in person or electronically. On the letting day, all bids are unsealed and ranked. The lowest total cost bidder wins the auction, i.e. the bottom line cost of the project determines the winner. Bidders must also provide a bid deposit that is a pre-determined percentage of the contract value, as determined by the engineer's estimate, prior to bidding. Once a contract has been awarded the primary contract winner is allowed to subcontract up to 60% of the contract value to subcontractors[6].

[5]This entails a check on the financial status of the firm. The information required for qualification includes: The identity of the owners, shareholder and managers of the company, any affiliations with other contractors, recently completed contracts and identity of clients, previous sales, an average of the firm's backlogs over the past three years, activities in other states, connections to other pre-qualified bidders, firm's Balance sheet. MDOT also has a disadvantaged business programme to encourage participation from smaller or disadvantaged firms, however this occurs only on a small fraction of contracts.

[6]We do not have any data on subcontracting and exclude this from our analysis.

Table 1: Summary Statistics

| | **Number Of Observations** | **Mean** | **Standard Deviation** | **Minimum** | **Maximum** |
|---|---|---|---|---|---|
| Number of Bidders | 4927 | 4.943 | 2.731 | 1 | 19 |
| Log of Engineer's Estimate | 4927 | 13.284 | 1.203 | 8.517 | 18.922 |
| $\frac{\text{Ranked2-Ranked1}}{\text{Ranked1}}$ | 4832 | 0.074 | 0.081 | 0 | 0.880 |
| $\frac{\text{Ranked1-Estimate}}{\text{Estimate}}$ | 4927 | -0.069 | 0.146 | -0.198 | 0.626 |

*Summary Statistics for all Bidders:* We focus our attention on general construction contracts. Table 1 and Table 2 provide summary statistics of the data. Table 1 reports that on average an auction attracts 5 bidders, with some contracts having only one bidder and others with 19 participating bidders. The third row of Table 1 presents data on "money left on the table", which can give an indication of the level of uncertainty in the market. This is the percentage difference between the winning bid and the lowest losing bid. On average there is a difference of about 7% with a standard deviation of 8%. This suggests that there are substantial informational asymmetries.

Table 2: Summary Statistics by Number of Bidders

| | Number of Bidders: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9-10 | 11-19 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Obs. | 91 | 742 | 999 | 781 | 618 | 534 | 376 | 272 | 286 | 228 |
| Estimate | | | | | | | | | | | |
| | Mean | 12.838 | 13.208 | 13.334 | 13.108 | 13.290 | 13.372 | 13.411 | 13.459 | 13.339 | 13.336 |
| | Standard Deviation | 1.030 | 1.093 | 1.176 | 1.297 | 1.286 | 1.262 | 1.221 | 1.087 | 1.157 | 1.053 |
| $\frac{\text{Ranked1-Est.}}{\text{Est.}}$ | | | | | | | | | | | |
| | Mean | 0.008 | -0.015 | -0.045 | -0.052 | -0.062 | -0.089 | -0.084 | -0.101 | -0.112 | -0.130 |
| | Standard Deviation | 0.128 | 0.133 | 0.152 | 0.161 | 0.163 | 0.1323 | 0.138 | 0.120 | 0.113 | 0.119 |
| $\frac{\text{Ranked2-Ranked1}}{\text{Ranked1}}$ | | | | | | | | | | | |
| | Mean | | 0.113 | 0.082 | 0.082 | 0.063 | 0.059 | 0.053 | 0.048 | 0.043 | 0.041 |
| | Standard Deviation | | 0.121 | 0.078 | 0.088 | 0.059 | 0.060 | 0.051 | 0.048 | 0.041 | 0.049 |

Table 2 summarises the data by number of bidders. The "Observations" row of the table reveals that 91 auctions attracted one bidder, 742 auctions attracted two bidders and so on. Table 2 also reproduces money left on the table data by number of bidders. It can be seen that money left on the table decreases with the number of bidders. However, the average amount of money left on the table is still substantial but is in line with other studies, see for example Krasnokutskaya (2003). Table 2 indicates that informational asymmetries matter in procurement competition. As the number of bidders increases the relative difference between the lowest bid and the engineer's estimate falls. Also, notice that larger projects do not necessarily attract more bidders, the degree of competition in an auction is not related to the engineer's estimate.

We now turn to the key question of whether procurement costs are affected by previous participation decisions and whether entry dynamics matter. To establish whether this feature is present in the data, we run a regression on the log winning bid and a set of covariates, which include the log number of previous round bidders, the engineer's estimate of the project cost and geographic variables. Table 3 summarises the results of an OLS regressions. The log number of previous round bidders and its squared component are individually statistically significant at 5% and are jointly significant at 5%. The regression results imply that increasing the number of previous round participants initially increases the winning bid level but after the number of previous round bidders exceeds roughly 3 bidders procurement costs start to decrease slightly, however the marginal effect of an additional bidder is very small. If we consider increasing the average number of bidders in a previous round by 10 bidders from 82 to 92 we see a 1% decrease in the procurement cost in a current auction.

Table 3: Regression Results of Winning Bid on Covariates

| **Variable**<br>Dependent Variable: Log Winning Bid | **Coefficient**<br>(Std. Err.)<br>OLS |
|---|---|
| Log of Engineer's Estimate | 1.0058 ** |
| | (0.0020) |
| Log Number of Bidders in Previous Round | 0.0338 * |
| | (0.0122) |
| Log Number of Bidders in Previous Round Squared | -0.0137 ** |
| | (0.0024) |
| Bay Region Dummy | 0.0093 |
| | (0.0090) |
| Grand Region Dummy | 0.0049 |
| | (0.0093) |
| Metro Region Dummy | 0.0068 |
| | (0.0089) |
| North Region Dummy | 0.0117 |
| | (0.0092) |
| Superior Region Dummy | 0.0408 ** |
| | (0.0097) |
| University Region Dummy | 0.0006 |
| | (0.0089) |
| Log Number of Auctions in Previous Round | 0.0441 ** |
| | (0.0090) |
| constant | -0.1913 ** |
| | (0.0303) |

Number of Observations: 4880

We next turn to the individual bidder's participation decisions. We first provide summary statistics on participation. To analyse an individual bidder's participation decision we estimate probit models of the probability of entry by an individual bidder on a set of covariates including a measure of previous round competition.

*Participation Decisions of Regular Bidders:* The tables presented here focus on the behaviour of large and small regular bidders. A regular bidder has participated in more than 100 auctions and a large bidder is one with more than six plants in Michigan State. We use six plants as a cutoff since plant sizes for bidders are clustered around one and two plants or more than six plants[7]. Participation rates are 7% of all auctions with the most frequent participating bidder entering 17% of the auctions of which they win on average 1.5%. Bidders on average are plan holders of around 11% of the contracts. *Participation in Auction Rounds:* Given that contracts are awarded in rounds we also provide some information on participation per round. Bidders participate in roughly 70% of the auction rounds. They enter in roughly 8% of auctions in a round, with an average of 50 contracts on offer per round which translates roughly to participation in four auctions in a round. Bidders participate in most of the auctions for which they hold plans but there is still uncertainty, being a plan holder does not guarantee participation. Figure 1 shows a plot of the number of plan holders for a project, i.e. bidders who have requested hard copies of plans and who then are listed on the webpage,versus the number of bidder who bid on the same contract. We might believe that there could be a backward bending relationship between the two, since bidders might be discouraged from bidding if they see a large number of plan holders. However, there is no such pattern. Moreover, we can see that sometimes the number of actual bidders exceeds the number of plan holders, i.e. there are data points above the $45^o$ line. This occurs when plans can be downloaded anonymously from the MDOT website, introducing uncertainty into the actual number of bidders.

Only one large bidder is active across Michigan whereas only 1/3 of small bidders are active across the entire state of Michigan. It therefore seems plausible that geographic factors might affect participation and ignoring them could be misleading. The majority of small bidders bid on all types of contracts, and only one large bidder bids on all types. Again, this suggests that ignoring contract characteristics for the participation decisions will lead to potentially misleading estimates. However, some contract types do not occur very often, for example New Construction contracts represent 1% of the total number of contracts offered, Traffic operations are only 8% of total contracts and Roadside Facilities are only 5%.

*Existence of Dynamic Synergies in Participation:* We now present reduced form evidence on the probability of participating in an auction. The descriptive analysis here seeks to determine the existence and qualitative features of dynamic synergies in participation. We estimate a set of Probits to determine whether participation in at least one auction in the previous period has an effect on participation in auctions in the current period. This would give an indication of whether there is evidence for synergies between rounds of auctions. The average participation rate in auctions is about 7.2%. We can see from Table 8 that participation in an auction in the

[7]This excludes the possibility that a bidder might have plants in neighbouring states and use those to mobilise equipment to complete a project. We will be investigating this in future.

Figure 1: Plan Holder versus Number of Actual Bidders


previous round increases the probability of participating in a current auction round from 7.2% to 9.5%. We also control for regular opponent's previous round entry behaviour. In particular, $\mathcal{N}_{\mathcal{S}}, \mathcal{N}_{\mathcal{L}}$ is the number of small and large regular bidders who participated in the previous round of bidding. $\mathcal{N}_{\mathcal{S}}, \mathcal{N}_{\mathcal{L}}$ are jointly significant in this specification. The effect persists when we include bidder fixed effects. However, the bidder fixed effects reduce the magnitude of the effect. The dynamic effects also persist when other contract characteristics are included such as the type of project and the location of the contract. Yet, the effect of state variables is reduced, while the effect of own participation remains at the same level. We also considered specifications including a bidder size variable and another specification with bidder fixed effects. In all specifications these dynamic effects persist in their statistical significance. Details of these specifications can be found in the appendix.

*Effect of Capacity Constraints on Participation:* As shown in Jofre-Bonet & Pesendorfer (2003) capacity constraints affect bidding behaviour as well as participation behaviour which could also have a bearing on participation in our market. We abstract from this issue, since capacity constraints would complicate estimation. Bajari & Ye (2003) discuss the possibility of a simplification which we will explore in future.

*Bid Level Decision:* Table 5 summarises results from Heckman estimates of the bid level decision. The engineer's estimate is clearly the strongest influence on the bid level. State variables do not have a statistically significant effect on bid levels. In particular, own participation status has no statistically significant effect on bid levels. The large bidder dummy has a negative sign, which indicates potential asymmetries between large and small bidders. We also consider a

Table 4: Probit Estimates

| Variable<br>Dependent Variable: Participation | Coefficient<br>(Std. Err.) |
|---|---|
| Participation in Previous Round | 0.1952** (0.0185) |
| constant | -1.5156** (0.0095) |
| Large Bidder Dummy | 0.2835** (0.0141) |
| *State Variables* | |
| $\mathcal{N}_{\mathcal{S}}$ | -0.004* (0.0014) |
| $\mathcal{N}_{\mathcal{L}}$ | -0.0185* (0.0078) |
| Number of Auctions: 4927 | |
| Number of Bidders: 30 | |

specification with bidder fixed effects for each regular type bidder. In this specification the own participation variable is significant and leads to a 7% increase in the bid level. This is evidence that there might be dynamic effects on bidding behaviour from previous round participation.

To summarise, the reduced form evidence suggests that dynamic linkages between auctions do exist and can have an impact on procurement costs for MDOT. Previous round competition has a statistically significant effect on current auction procurement costs. There is a positive effect of previous round participation on current auction participation, which indicates the existence of some form of synergies. Previous round competition and participation have little effect on the bid level decision. These results merely indicate patterns in the data and are by no means causal relationships. This prevents us from making stronger statements about the causal nature of these outcomes. To understand these effects more fully, we therefore proceed to construct a structural econometric model that will allow us to understand the dynamic strategic effects that might be operating in this market and quantify their effect. The next section outlines the theoretical framework we take to the data.

# 3 Participation and Auction Game

## 3.1 Setup and Assumptions

This section outlines the theoretical bidding model. The focus of the analysis is on risk neutral[8] long-lived bidders who participate in more than 100 auctions in our data set. Bidders who participate in fewer auctions are categorised as short lived fringe bidders. Throughout it is

[8]The assumption of risk-neutrality can be justified by realising that most regular bidders are large corporations and are active in various states. We can therefore use a portfolio diversification argument.

Table 5: Bid Level Estimates (Heckman)

| **Variable**<br>Dependent Variable: Log of Bid Level | **Coefficient**<br>(Std. Err.) |
|---|---|
| Log(Engineer Estimate) | 0.9751** <br>(0.0016) |
| Participation in Previous Round | 0.0166<br>(0.0106) |
| constant | 0.3862** <br>(0.0780) |
| Large Bidder Dummy | -0.0323 * <br>(0.0104) |
| *State Variables* | |
| $\mathcal{N}_{\mathcal{S}}$ | -0.0009<br>(0.0006) |
| $\mathcal{N}_{\mathcal{L}}$ | -0.0038<br>(0.0033) |
| Number of Auctions: 4927 | |
| Number of Bidders: 30 | |

assumed that participation is solely determined by information acquisition costs. These costs can be interpreted as bid preparation costs, i.e. the opportunity cost of having to discuss the completion of a project. It is also assumed that once private costs are known, a bidder will participate in the auction[9].

Time is discrete with an infinite horizon. There are two types of regular bidders, large bidders $\mathbf{N}_{\mathcal{L}} = \{1, ..., N_{\mathcal{L}}\}$, who have more than six plants in the state, small bidders $\mathbf{N}_{\mathcal{S}} = \{N_{\mathcal{L}} + 1, ..., N_{\mathcal{L}} + N_{\mathcal{S}}\}$, who have up to six plants, and a fixed set of fringe bidders $\mathbf{N}_{\mathcal{F}} = \{N_{\mathcal{L}} + N_{\mathcal{S}} + 1, ..., N_{\mathcal{F}} + N_{\mathcal{S}} + N_{\mathcal{L}}\}$. We will sometimes denote the total number of bidders as $N = N_{\mathcal{L}} + N_{\mathcal{S}} + N_{\mathcal{F}}$. A typical bidder of either type will be denoted by $i$. The number of bidders of each type does not vary over time.

*The Stage Game:* Each time period $t$ is broken down into two stages, the Participation Stage followed by the Auction Stage. The sequence of events is as follows:

1. Participation Stage
   (a) Each bidder $i$ receives a draw from a private information acquisition cost distribution at the beginning of period $t$
   (b) All bidders decide simultaneously whether to enter the auction

[9]An alternative participation model is by Samuelson where bidders already know their valuations when they make their entry decision but have to pay a cost to learn their values. Here the set of bidders will be a selected sample of bidders who have valuations above a certain threshold value. It is possible that the true participation decision is a hybrid between the Samuelson (1985) and Levin & Smith (1994) participation game. Li & Zheng (2009) estimate these different auction models in their paper.

2. Auction Stage

   (a) Contract characteristics $c_{0,t}$ are revealed to all bidders

   (b) Without observing the outcome of the first stage, each bidder learns their own completion cost privately

   (c) All participating bidders simultaneously submit bids without knowing the actual number of bidders participating

   (d) The contract is awarded to the low bidder

This setup is similar to Li & Zheng (2009)'s specifications of the entry game. Krasnokutskaya (2003) assumes that bidders know the identity of their opponents. We assume that bidders ignore plan holder information, since our data is from a period where it was also possible to download plan anonymously. The published plan holder list will therefore not always be complete and there is still uncertainty about the actual level of competition for a contract. However, if we did choose to include the plan holder information, the only difference from our model would be the inclusion of an extra stage prior to the participation stage, which determines plan holder status. This has been excluded for simplicity and, as described previously, the fact that plan holder status is not recorded for all bidders. We have also undertaken some reduced form analyses on plan holders and found that the dynamic synergies discussed in the previous section persist when controlling for plan holder status[10]. In addition, the inclusion of plan holder information would require the state space of the dynamic game to be expanded which will make the estimation more computationally burdensome. We have also assumed that bidders decide on entry prior to contract characteristics being revealed. However, in future we will correct these issues by allowing for plan holder information as well as some contract characteristics, such as project locations, to be used.

As mentioned before, the procurement auctions under investigation are run in rounds, where up to 100 construction contracts are auctioned off at once. This might introduce scope for possible synergies between contracts offered in the same round or possible exposure problems, where bidders win too many contracts at once. These possibilities are not considered in this model, due to the lack of further information on ex post costs and the department of transportation not offering contracts in packages. Moreover, only intertemporal substitution of auctions and not within round substitutions are considered. The rest of the setup is as follows:

*Reserve Price:* The Michigan Department of Transportation requires that the winning bid be lower than 110% of the engineer's estimate. If the Department wishes to accept a bid higher

[10]To establish this, we turn our attention to participation behaviour of bidders, given that they are plan holders. In particular we look at the probability of participation given state variables as before for the subset of bidders who have expressed an interest in the contract. The results suggests that the effects of that state variables persist with a reduced magnitude. Specifically, the probability of participating is increased by 10% if a bidder participated in the previous round of bidding. In other words, it does seem that even when controlling for plan holder status the between round synergies we posited still exist.

than this threshold, it is required to write a justification for doing so. The data have a number of projects being awarded for more than the threshold. This suggests that these restrictions do not come into effect very often. We follow Krasnokutskaya (2003) and assume there is no binding reserve price. We also follow Li & Zheng (2009) to address the issue that there is a chance a bidder will face no competitors and could then bid an infinite amount. To avoid this Li & Zheng (2009) suggest that when a bidder is the only entrant he must compete with the Department. We, therefore, assume that when a bidder is the sole participant, he will then face the DOT that draws a completion cost from a regular large bidder's cost distribution.

*Contract Characteristics* at time $t$ are denoted by $c_{0,t} \in \mathbf{C}_0$ and are drawn from the known exogenous distribution $F_0(.)$. Future contract characteristics are unknown to all bidders. The contract characteristics are the physical attributes of the contract. Our analysis restricts attention to the engineer's estimate of the project size.

*Private Completion Costs:* Bidder $i$ of type $j = \mathcal{L}, \mathcal{S}, \mathcal{F}$ draws private costs, $c_{i,t} \in \mathbf{C}_j$, independently and identically from the cost distribution $F_j(c_{i,t}|c_{0,t})$ on $[\underline{c}, \overline{c}]$, conditional on $c_{0,t}$. The assumption of independent private values can be justified by assuming that differences in cost estimates are due to firm specific factors such as differences in opportunity costs and input prices.

*An action for bidder $i$* in period $t$ is given by the participation decision and the bid submitted at the auction stage, $a_{i,t} \in A_i = \{0, 1\} \cup [0, \infty)$. Sometimes, the participation decision will be denoted separately by $d_{i,t} \in \{0, 1\}$ and the bid by $b_{i,t} \in [0, \infty)$.

*Public States for Regular Bidder $i$:* Bidder $i$ is characterised by a publicly observable state variable $s_{i,t} \in \mathbf{S}_i \equiv \{0, 1\}$ that affect its actions. The state is the participation status of a bidder in the previous round of bidding, i.e. $s_{i,t} = d_{i,t-1}$. The vector of all bidders' state variables is given by $\mathbf{s}_t = (s_{1,t}, ..., s_{N,t}) \in \mathbf{S} = \times_{k=1}^{N} \mathbf{S}_k$. We will sometimes use the notation $\mathbf{s}_{-i,t} = (s_{1,t}, s_{2,t}, ..., s_{i-1,t}, s_{i+1,t}, ...s_{N,t}) \in \mathbf{S}_{-i} = \times_{l \neq i} \mathbf{S}_l$ to denote the vector of state variables excluding bidder $i$. The cardinality of the state space $\mathbf{S}$ equals $m_s = 2^N$.

*Private States for Regular Bidder $i$ of type $j$: Information Costs*, $\phi_{i,t} \in \Phi_j = [1, \infty)$, are drawn independently and identically from the conditional distribution $H_j(\phi_{i,t}|s_{i,t})$ with associated density $h_j(\phi_{i,t}|s_{i,t})$, and are unobserved by other bidders and the econometrician. Information costs have a Markov structure and have following transition probability:

$$h_j(\phi_{i,t}|s_{i,t}) = \alpha_j(s_{i,t})\phi_{i,t}^{-\alpha_j(s_{i,t})-1} \tag{1}$$

where $\log[\alpha_j(s_{i,t})] = \alpha_{0,j} + \alpha_{1,j} s_{i,t}$. Both parameters are unknown to the econometrician but known to the bidders. Rivals' actions and states do not affect the private costs of information acquisition. We have experimented with alternative specifications of the information cost distributions and estimated the model with them, including the exponential and half-logistic, however neither of these distributions comes close to matching the estimated choice probabilities. We, therefore, settled for the Pareto which provided a better fit. Note, that the structure of the information cost distribution is such that participating in an auction in this period will allow bidders to draw information costs from a more advantageous distribution, i.e. a distribution with lower mean, in the next period. It is possible to make the information cost depend on more than one previous period and to allow for cumulative cost advantages. However, this has been excluded in the current analysis but will be explored at a later date.

*Information Costs for Fringe Bidders:* Fringe bidder $i$ needs to pay information cost $K$ to learn her private completion costs.

*Discounting:* Bidders discount the future with common discount factor $\beta \in (0,1)$, known to the econometrician and the bidders and fixed over time. The annual discount factor equals $\beta = 0.9$. Note that this imposes forward-looking behaviour of regular bidders. Hendricks & Porter (2007) discuss possible strategies for identifying the discount factor using exogenous variation in the bidding environment.

*Conditional Independence:* As in Rust (1987), it is assumed that the unobserved information costs are conditionally independent of observable states. Also note that the structure of the problem already embodies the usual assumption that the private "shocks", here the information costs, are additively separable. For further discussion of these assumptions see Rust (1994).

*Bidder Strategies:* Strategies for bidder $i$ of type $j$ are restricted to be Markovian for the entry game. The strategy for bidder $i$ of type $j = \mathcal{L}, \mathcal{S}, \mathcal{F}$ consists of a participation strategy $d^{\sigma}_{i,j}(\mathbf{s}, \phi_i)$ and a bidding strategy $b^{\sigma}_{i,j}(\mathbf{s}, c_i, c_0)$ and will be denoted $\sigma_{i,j} = (d^{\sigma}_{i,j}(\mathbf{s}, \phi_i), b^{\sigma}_{i,j}(\mathbf{s}, c_i, c_0))$. Formally, a Markov strategy is a map, $\sigma_{i,j} : \mathbf{S} \times \Phi_j \times \mathbf{C}_j \times \mathbf{C}_0 \rightarrow A_i$. Fringe bidder strategies are denoted separately by $\sigma_{i,\mathcal{F}}$ and the set of all fringe strategies is denoted $\sigma_{\mathcal{F}} = \{\sigma_{i,\mathcal{F}} : i = 1, ..., N_{\mathcal{F}}\}$. Strategies consist of a bidding strategy and an entry strategy $d^{\sigma_{\mathcal{F}}}$.

*Beliefs on the Probability of Participation:* To form the necessary expectations and to compute the probability of a bidder winning an auction bidders' beliefs of the likely number of bidders based on the decision rules of bidders must be defined. Beliefs are

$$q_{i,j}(\mathbf{s}_t) \equiv \Pr(i \text{ of type } j \text{ enters} | s_{i,t}, \mathbf{s}_{-i,t}) = \int_1^{\infty} \mathbf{1}\{d^{\sigma}_{i,j}(\mathbf{s}_t, \phi) = 1\} h_j(\phi | s_{i,t}) d\phi \qquad (2)$$

The above is the expected behaviour of firm $i$ of type $j$ when $i$ follows its participation strategy in $\sigma$. The integration is over private information costs $\phi$.

*Characterisation of Payoffs for Regular Bidders:* A bidder decides whether to enter an auction and incur the information cost by comparing the value of participation and non-participation. Let $W^j_{i0}(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}})$ be the value of not participating at state $\mathbf{s}_t$ with all opponents following their strategies prescribed in $\sigma$ and $W^j_{i1}(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}})$ be the value of non participation. We will define these values more carefully after introducing some more notation. The Bellman equation for bidder $i$ of type $j$ is then:

$$W_i^j(\mathbf{s}_t, \phi_{i,t}; \sigma, \sigma_{\mathcal{F}}) = \max\left\{W_{i1}^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) - \phi_{i,t}, W_{i0}^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}})\right\} \tag{3}$$

Next define the *ex ante* value function as the integrated version of the above Bellman equation, where all private information is integrated out:

$$V_i^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) = \int_1^\infty W_i^j(\mathbf{s}_t, \phi; \sigma, \sigma_{\mathcal{F}})\alpha_j(s_{i,t})\phi^{-\alpha_j(s_{i,t})-1}d\phi, \ \forall \mathbf{s}_t \tag{4}$$

We can then write the choice specific values as:

$$\begin{aligned} W_{i1}^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) = E_{c,c_0}&\left[\max_b [b-c] \Pr(i \text{ wins}|s_{i,t}, \mathbf{s}_{-i,t}, b, c_0; \sigma, \sigma_{\mathcal{F}})\right] \\ &+\beta \sum_{\mathbf{s'}_{t+1}\in\mathbf{S}} \Pr(\mathbf{s'}_{t+1}|\mathbf{s}_t, d_{i,t}=1; \sigma, \sigma_{\mathcal{F}})V_i^j(\mathbf{s'}_{t+1}; \sigma, \sigma_{\mathcal{F}}) \end{aligned} \tag{5}$$

and the value of not participating

$$W_{i0}^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) = 0 + \beta \sum_{\mathbf{s'}_{t+1}\in\mathbf{S}} \Pr(\mathbf{s'}_{t+1}|\mathbf{s}_t, d_{i,t}=0; \sigma, \sigma_{\mathcal{F}})V_i^j(\mathbf{s'}_{t+1}; \sigma, \sigma_{\mathcal{F}}) \tag{6}$$

The value function can equivalently be written as

$$V_i^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) = q_{i,j}(\mathbf{s}_t)(W_{i1}^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) - E_\phi[\phi|\phi \leq \zeta^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}})]) + [1 - q_{i,j}(\mathbf{s}_t)]W_{i0}^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) \tag{7}$$

where $\zeta^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) = W_{i1}^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) - W_{i0}^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}})$ and we have made use of an individual bidder's decision rule, as defined in (2). This formulation will be useful for estimation later on. The probability of a bidder winning an auction is given by,

$$\Pr(i \text{ wins}) = \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{L}} \text{ potential } \mathcal{L} \\ \text{type bidders} \end{pmatrix} \times \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{S}} \text{ potential } \mathcal{S} \\ \text{type bidders} \end{pmatrix} \times \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{F}} \text{ potential } \mathcal{F} \\ \text{type bidders} \end{pmatrix} \tag{8}$$

where if bidder $i$ is of type $j$ and $k = 0, 1$ indexes the participation status of a bidder

$$\Pr\begin{pmatrix} i \text{ wins against } N_j \\ \text{potential } j \text{ type} \\ \text{bidders} \end{pmatrix} = \tag{9}$$

$$\sum_{\substack{n_{0,j}=0 \\ n_{1,j}=\mathbf{1}_{\{j=\mathcal{L}\}}}}^{\substack{N_{0,j}-\mathbf{1}_{\{s_{i,t}=0\}} \\ N_{1,j}-\mathbf{1}_{\{s_{i,t}=0\}}}} \prod_{k=0,1} \left[\mathbf{C}_{n_{k,j}}^{N_{k,j}-\mathbf{1}_{\{s_{i,t}=k\}}}\right] q_j(\mathbf{s}_{k,t})^{n_{k,j}}(1-q_j(\mathbf{s}_{k,t}))^{N_{k,j}-\mathbf{1}_{\{s_{i,t}=k\}}-n_{k,j}}[1-G_j(b_{i,t}|c_{0,t}, \mathbf{s}_{k,t})]^{n_{k,j}}$$

where $G_j(.|c_{o,t}, \mathbf{s}_k)$ is the equilibrium bid distribution at state $\mathbf{s}_{k,t} = (k, \mathbf{s}_{-i,t})$ where $k = 0, 1$ is the bidder's own participation status, $\mathbf{s}_{0,t}$ and $\mathbf{s}_{1,t}$ are then states for a bidder who did not participate in a previous round and a bidder that did participate, respectively. $\mathbf{C}_{n_{k,j}}^{N_{k,j}-\mathbf{1}_{\{s_{i,t}=1\}}} = \begin{pmatrix} N_{k,j} - \mathbf{1}_{\{s_{i,t}=k\}} \\ n_{k,j} \end{pmatrix}$ are the usual binomial coefficients, and $N_{k,j}$ is the total number of bidders who have participation status $k = 0, 1$ and $\mathbf{1}_{\{j=\mathcal{L}\}}$ is an indicator. The indicator captures the aforementioned assumption that a bidder will always face at least one large bidder, either an actual large bidder or the Department. $\mathbf{1}_{\{s_{i,t}=k\}}$ is an indicator that equals one if the bidder has participation status $k$. If a bidder $i$ is not of type $j$ then the above is simply:

$$\Pr\left(\begin{array}{c} i \text{ wins against } N_j \\ \text{potential } j \text{ type} \\ \text{bidders} \end{array}\right) =$$

$$\sum_{\substack{n_{0,j}=0 \\ n_{1,j}=\mathbf{1}_{\{j=\mathcal{L}\}}}}^{N_{0,j},N_{1,j}} \prod_{k=0,1} \left[\mathbf{C}^{N_{k,j}}_{n_{k,j}}\right] q_j(\mathbf{s}_{k,t})^{n_{k,j}}(1-q_j(\mathbf{s}_{k,t}))^{N_{k,j}-n_{k,j}}[1-G_j(b_{i,t}|c_{0,t},\mathbf{s}_{k,t})]^{n_{k,j}} \tag{10}$$

Note, that the main difference between (9) and (10), is that in (9) we are correcting for the number of opponent players that are of the same type as player $i$. For fringe bidders the equivalent expression is:

$$\Pr\left(\begin{array}{c} i \text{ wins against } N_{\mathcal{F}} \\ \text{potential } \mathcal{F} \text{ type} \\ \text{bidders} \end{array}\right) = \sum_{n_{\mathcal{F}}=0}^{N_{\mathcal{F}}} \Pr(n_{\mathcal{F}}|\mathbf{s}_t)[1-G_{\mathcal{F}}(b_{i,t}|c_{0,t},\mathbf{s}_t)]^{n_{\mathcal{F}}} \tag{11}$$

We discuss in detail how we specify the term $\Pr(n_{\mathcal{F}}|\mathbf{s}_t)$ in the next section.

*Markov Perfect Equilibria:* A MPE in this game is a set of strategy functions $\sigma^*$ such that for any $i$ of type $j$ and for any $(\mathbf{s}, \phi_i, c_i, c_0) \in \mathbf{S} \times \Phi_i \times \mathbf{C}_j \times \mathbf{C}_0$,

$$d^{\sigma^*}_{i,j}(\mathbf{s}, \phi_i) = \arg\max_{d_i \in \{0,1\}} \left\{d_i(W^j_{i1}(\mathbf{s}; \sigma^*, p^*_{\mathcal{F}}) - \phi_i) + (1-d_i)(W^j_{i0}(\mathbf{s}; \sigma^*, p^*_{\mathcal{F}}))\right\} \tag{12}$$

and

$$b^{\sigma^*}_{i,j}(\mathbf{s}, c_i, c_0) \in \arg\max_{b_i \in \mathbf{B}} [b_i - c_i] \Pr(i \text{ wins}|\mathbf{s}, b_i, c_0; \sigma^*, \sigma^*_{\mathcal{F}}) \tag{13}$$

where $\mathbf{B} = [0, \infty)$.

*Existence of Equilibrium in the Participation Game:* Following Auguirregabiria & Mira (2007) and Pesendorfer & Schmidt-Dengler (2008), the existence of equilibria of the participation game is analysed in probability space. A bidder will enter an auction if

$$W^j_{i1}(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) - \phi_{i,t} \geq W^j_{i0}(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) \tag{14}$$

(14) characterises the optimal decision rule. The above can be evaluated before the information acquisition costs are observed which yields the *ex ante* optimal choice probabilities, induced by $\sigma^*$, given perceptions of opponents' entry strategies, $\sigma$ and $\sigma_{\mathcal{F}}$.

$$p_{i,j}(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) = \int_1^{\infty} \mathbf{1}\{W^j_{i1}(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) - W^j_{i0}(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) \geq \phi\} dH_j(\phi|s_{i,t}) \equiv \Lambda_{i,j}(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) \tag{15}$$

$p_{i,j}$ is therefore the optimal choice probability induced by the MPE $\sigma^*$ defined previously. Equilibrium existence of the participation game is easily shown by looking at the ex ante optimal choice probabilities defined previously. Let the set of ex ante choice probabilities be given by $\Lambda(\mathbf{q}) = \{\Lambda_{i,j}(\mathbf{s}_t;\sigma) : i = 1, ..., N \;\&\; j = \mathcal{L}, \mathcal{S}\}$, these are the choice probabilities induced by the strategies in $\sigma$, where $\mathbf{q} = \{q_{i,j}(\mathbf{s}') : i = 1, ..., N \;\&\; j = \mathcal{L}, \mathcal{S} \;\&\; \mathbf{s}' \in \mathbf{S}\}$ is the set of entry decision rules for all regular bidders, as defined in 2. Equilibrium points are therefore fixed points, i.e. let $\mathbf{p}$ be the set of optimal choice probabilities for every state and every bidder then

$$\mathbf{p} = \Lambda(\mathbf{p}), \tag{16}$$

since beliefs in equilibrium are consistent. These choice probabilities are well defined and continuous in the compact set of bidder's choice probabilities. Brouwer guarantees that at least one set of beliefs exist for this system.

*Existence of Equilibrium in the Auction Game:* The auction game resembles existing static auctions considered in the theoretical literature. First note that the following hold: the completion cost space $\mathbf{C}_j$ is a separable metric space with measurable partial order, the joint density of types is bounded and atomless, action space is compact, and payoffs are continuous for every $c \in [\underline{c}, \overline{c}]$. In our case the interim payoffs are log supermodular and therefore single crossing holds. Following Reny (2008), the auction game we consider has an equilibrium in monotone pure strategies.

# 4 Estimation

Our data consists of repeated observations of bids, participation decisions for all players and contract characteristics for $T$ periods.

$$\text{data} = \{b_{i,t}, d_{i,t}, c_{0,t} : i = 1, ..., N;\; t = 1, 2, ..., T\} \tag{17}$$

In this section we show how private costs can be inferred from observed bids. We then show how the participation model can be estimated using an asymptotic least squares estimator. The first step requires the estimation of auxiliary parameters of the model, i.e. the optimal choice probabilities of participation $p_{i,j}(\mathbf{s}_t)$ and the equilibrium bid distributions. We then use the first order condition for an optimal bid to compute an expression for privately known costs as a function of the submitted bid, the equilibrium bid distribution and the optimal choice probabilities. We then outline an optimal minimum distance estimator that finds parameters which minimise the distance between the non-parametrically estimated choice probabilities and the choice probabilities implied by our model.

## 4.1 Optimal Choice Probabilities

Per period profits do not depend on the identity of the bidder but merely on the number of each type of bidder, i.e. the number of large and small bidders, all relevant information in $\{d_{i,t-1} : i = 1, 2, ..., N\}$ can be captured in a bidder's own participation status $d_{i,t-1}$ and the number of competitors of each type, defined as $\mathcal{N}_{\mathcal{L},t} = \sum_{k \in \mathbf{N}_{\mathcal{L}} \setminus i}^{N_{\mathcal{L}}} d_{k,t-1}$ & $\mathcal{N}_{\mathcal{S},t} = \sum_{k' \in \mathbf{N}_{\mathcal{S}} \setminus i}^{N_{\mathcal{S}}} d_{k',t-1}$. Following Auguirregabiria & Mira (2007), the optimal conditional choice probabilities at some state $\mathbf{s}^m = (\widetilde{s}, \widetilde{\mathcal{N}}_{\mathcal{L}}, \widetilde{\mathcal{N}}_{\mathcal{S}})$ are estimated by simple frequency estimators, for states not observed the nearest neighbour is used. The estimator has the following form:

$$\widehat{p}_j(\widetilde{s}, \widetilde{\mathcal{N}}_{\mathcal{L}}, \widetilde{\mathcal{N}}_{\mathcal{S}}) = \frac{\sum_{i \in \mathbf{N}_j} \sum_t^T \mathbf{1}\{d_{i,t} = 1, s_{i,t} = \widetilde{s},\ \mathcal{N}_{\mathcal{L},t} = \widetilde{\mathcal{N}}_{\mathcal{L}},\ \mathcal{N}_{\mathcal{S},t} = \widetilde{\mathcal{N}}_{\mathcal{S}}\}}{\sum_{i \in \mathbf{N}_j} \sum_{k=0,1} \sum_t^T \mathbf{1}\{d_{i,t} = k, s_t = \widetilde{s}_i,\ \mathcal{N}_{\mathcal{L},t} = \widetilde{\mathcal{N}}_{\mathcal{L}},\ \mathcal{N}_{\mathcal{S},t} = \widetilde{\mathcal{N}}_{\mathcal{S}}\}} \tag{18}$$

Ideally, we would make use of kernels to smooth over states that we do not observe and are exploring possible approaches to this. We use these probabilities as inputs into the computation of the transition matrices and the period payoffs. The estimation routine does not require us to use nearest neighbours. We could equally only use choice probabilities for observed states and then estimate the structural parameters. However, this would be inconsistent with the auction game, since bidders take expectations over all possible number of bidders and not just over observed bidder configurations. At a later stage we will explore alternative methods that will not require the use of unobserved states.

*Fringe Bidders:*Fringe bidders enter with same probability $p^*$ conditional on state variables, as specified by their entry strategy in $\sigma^*_{\mathcal{F}}$. With a large number of potential bidders, we model the number of fringe bidders in an auction as a poisson process with parameter $\delta$ depending on $\mathbf{s}$, similar to Bajari & Hortacsu (2003). In other words, the probability of observing $n_{\mathcal{F}}$ fringe bidders is given by:

$$\Pr(n_{\mathcal{F}} | \widetilde{\mathbf{s}}) = \frac{e^{-\delta(\widetilde{\mathbf{s}})} \delta(\widetilde{\mathbf{s}})^{n_{\mathcal{F}}}}{n_{\mathcal{F}}!} \tag{19}$$

where $\log[\delta(\widetilde{\mathbf{s}})] = \delta_0 + \delta_1 \widetilde{\mathcal{N}}_{\mathcal{S}} + \delta_2 \widetilde{\mathcal{N}}_{\mathcal{L}}$.

## 4.2 Bid Distributions

Following Athey *et al.* (2008) and Jofre-Bonet & Pesendorfer (2003), bid distributions are estimated parametrically and assumed to follow Weibull distributions. We assume that the shape parameter, $\psi_{1,j}$ and scale parameter, $\psi_{2,j}$, depend on the contract size and the state variables. The bid distributions have the following functional form,

$$G_j(b_{i,t} | c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t}) = 1 - \exp\left[ -\left( \frac{\log(b_{i,t} + 1)}{\psi_{2j}(c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t})} \right)^{\psi_{1j}(c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t})} \right] \tag{20}$$

where $\psi_{k,j}(.)$ is given by $\ln \psi_{k,j}(c_0, s_{i,t}, \mathbf{s}_{-i,t}) = \psi_{k,0j} + \psi_{k,1j} c_{0,t} + \psi_{k,2j} s_{i,t} + \psi_{k,3j} \mathcal{N}_{\mathcal{L},t} + \psi_{k,4j} \mathcal{N}_{\mathcal{S}}$, for $j = \mathcal{L}, \mathcal{S}$ & $k = 1, 2$.

*Fringe Bidders:* We assume that the fringe bid distribution is also described by a Weibull distribution.

$$G_{\mathcal{F}}(b_{i,t} | c_{0,t}, \mathbf{s}_t) = 1 - \exp\left[ -\left( \frac{\log(b_{i,t} + 1)}{\psi_{2\mathcal{F}}(c_{0,t}, \mathbf{s}_t)} \right)^{\psi_{1\mathcal{F}}(c_{0,t}, \mathbf{s}_t)} \right] \tag{21}$$

where $\log \psi_{k,\mathcal{F}}(c_{0,t}, \mathbf{s}_t) = \psi_{k,\mathcal{F},0} + \psi_{k,\mathcal{F},1} c_{0,t} + \psi_{k,\mathcal{F},2} \mathcal{N}_{\mathcal{S}} + \psi_{k,\mathcal{F},3} \mathcal{N}_{\mathcal{L}}$ for $k = 1, 2$, where the last two terms in the expression are the number of large and small bidders who participated in an auction in the previous round.

## 4.3 Private Costs

The first order condition for an optimal bid can be re-written to yield an expression for private information $c_{i,t}$, in terms of observables:

$$c_{i,t} = b_{i,t} - [\eta_{\mathcal{L}}(b_{i,t}, \mathbf{s}_t, c_{0,t}; \sigma^*) + \eta_{\mathcal{S}}(b_{i,t}, \mathbf{s}_t, c_{0,t}; \sigma^*) + \eta_{\mathcal{F}}(b_{i,t}, \mathbf{s}_t, c_{0,t}; \sigma^*_{\mathcal{F}})]^{-1} \tag{22}$$

where

$$\eta_j(b_{i,t}, \mathbf{s}_t, c_{0,t}; \sigma^*) = \left[ \frac{\partial \Pr \left( \begin{array}{c} i \text{ wins against } N_j \\ \text{potential } j \text{ type bidders} \end{array} \middle| b_{i,t}, \mathbf{s}_t, c_{0,t}; \sigma^* \right) / \partial b_{i,t}}{\Pr \left( \begin{array}{c} i \text{ wins against } N_j \\ \text{potential } j \text{ type bidders} \end{array} \middle| b_{i,t}, \mathbf{s}_t, c_{0,t}; \sigma^* \right)} \right] \tag{23}$$

The second term in (22) is the markup term. The exact form of (23) can be found in the appendix. The above closely follows Guerre *et al.* (2000), except that expectations are taken over the number of actual competitors with the number of potential competitors constant over time. Quasi-valuations can be computed by estimating the equilibrium bid distributions $G_j(.|c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t})$ and substituting these into expression (22). This allows for point-wise estimation of the private cost distribution.

## 4.4 Parameters of the Participation Game

The primitives are estimated by finding parameters that minimise the distance between the non parametrically estimated choice probabilities and the choice probabilities implied by the model. Values can be computed from the data conditional on the structural parameters, these can then be used to compute optimal choice probabilities for our model. Let $\theta = (\alpha_{0,\mathcal{L}}, \alpha_{1,\mathcal{L}}, \alpha_{0,\mathcal{S}}, \alpha_{1,\mathcal{S}})$. Given the Pareto distribution assumption on information costs this allows for a simple form of the value function. We can use equation (7) together with the Pareto assumption to re-write the value function for bidder of type $j$ as:

$$V^j(s_{i,t}, \mathbf{s}_{-i,t}; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) = \tag{24}$$

$$\gamma^j(\mathbf{s}_t, \sigma^*) W_1^j(s_{i,t}, \mathbf{s}_{-i,t}; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) + [1 - \gamma^j(\mathbf{s}_t, \sigma^*)] W_0^j(s_{i,t}, \mathbf{s}_{-i,t}; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) + \frac{\alpha_j(s_{i,t})}{1 - \alpha_j(s_{i,t})}$$

where $\gamma^j(\mathbf{s}_t, \sigma^*) = \frac{p^j(\mathbf{s}_t) - \alpha_j(\mathbf{s}_{i,t})}{1 - \alpha_j(\mathbf{s}_{i,t})}$[11]. The next step involves substituting the expression for an optimal bid into the expected period payoff function. Specifically, changing the variable of

[11] This result follows if one computes the conditional expectation defined in (7) using the Pareto assumption. Specifically, in equilibrium $p^j(\mathbf{s}) E_\phi(\phi | \phi \leq \zeta^j(\mathbf{s}_t)) = \int_1^{\zeta_j(\mathbf{s}_t)} \phi \alpha_j \phi^{-\alpha_j - 1} d\phi = \alpha_j/(1 - \alpha_j)(W_1^j - W_0^j)^{-\alpha_j + 1} - \alpha_j/(1 - \alpha_j)$. The result follows if we then note that $1 - p_j = (W_1^j - W_0^j)^{-\alpha}$

integration from cost, $c$ to bids $b$ yields

$$\pi^j(s_{i,t}, \mathbf{s}_{-i,t}; \sigma^*, \sigma^*_{\mathcal{F}}) = E_{c_0}\left[\int_0^{\bar{b}} \frac{\Pr(i \text{ wins}|s_{i,t}, \mathbf{s}_{-i,t}, b, c_0; \sigma^*, \sigma^*_{\mathcal{F}})}{\eta_{\mathcal{L}} + \eta_{\mathcal{S}} + \eta_{\mathcal{F}}} g_j(b|s_{i,t}, \mathbf{s}_{-i,t}, c_0) db\right] \quad (25)$$

where $\eta_L$, $\eta_S$ and $\eta_{\mathcal{F}}$ are as defined in (23). The above can be used in the ex-ante value function which is then given by

$$V^j(s_{i,t}, \mathbf{s}_{-i,t}; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) = b^j_1(\mathbf{s}_t, \sigma^*)\pi^j(s_{i,t}, \mathbf{s}_{-i,t}; \sigma^*, \sigma^*_{\mathcal{F}}) \quad (26)$$

$$+\beta\left[\gamma^j(\mathbf{s}_t, \sigma^*) \sum_{\mathbf{s}'_{t+1} \in \mathbf{S}} \Pr(\mathbf{s}'_{t+1}|\mathbf{s}_t, d_{i,t} = 1; \sigma^*) + [1 - \gamma^j(\mathbf{s}_t, \sigma^*)] \Pr(\mathbf{s}'_{t+1}|\mathbf{s}_t, d_{i,t} = 0; \sigma^*)\right]$$

$$\times V^j(\mathbf{s}'_{t+1}; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) + \frac{\alpha_j(s_{i,t})}{1 - \alpha_j(s_{i,t})}$$

Now let $\Pi^j(\sigma^*, \sigma^*_{\mathcal{F}}) = [\pi^j(\mathbf{s}'; \sigma^*, \sigma^*_{\mathcal{F}})]_{\mathbf{s}' \in \mathbf{S}}$ be the vector of expected period payoffs , $V^j(\sigma^*, \sigma^*_{\mathcal{F}}, \theta) = [V^j(\mathbf{s}'; \sigma^*, \sigma^*_{\mathcal{F}}, \theta)]_{\mathbf{s}' \in \mathbf{S}}$ be the vector of values, $P^j(\mathbf{s}_t) = [p_j(d_{i,t} = 1|\mathbf{s}'; \sigma^*)]_{\mathbf{s}' \in \mathbf{S}}$ is the vector of choice probabilities, $\Gamma^j(\sigma^*) = \text{diag}([\gamma^j(s', \sigma^*)]_{s' \in \mathbf{S}_i})$ and $M^j_1(\sigma^*)$ is the $m_s \times m_s$ transition matrix induced by participation in the current round of auctions. In other words, row $s \in \mathbf{S}$ of the transition matrix is given by $[\Pr(s'|s, d = 1; \sigma^*)_{s' \in \mathbf{S}}]$. And let $A^j = \text{diag}([\alpha_j(s')]_{s' \in \mathbf{S}_i})$ be a $m_s \times m_s$ diagonal matrix. The matrix equation for the value function can then be written as follows:

$$V^j(\sigma^*, \sigma^*_{\mathcal{F}}; \theta) = \Gamma^j(\sigma^*)\Pi^j(\sigma^*, \sigma_{\mathcal{F}}) + \beta[\Gamma^j(\sigma^*)(M^j_1(\sigma^*) - M^j_0(\sigma^*)) + M^j_0(\sigma^*)]V^j(\sigma^*, \sigma^*_{\mathcal{F}}; \theta) \quad (27)$$

$$+(I - A^j)^{-1}A^j \cdot \iota$$

where $\iota$ is a $m_s \times 1$ vector of ones. The above can be re-arranged to yield

$$V^j(\sigma^*, \sigma^*_{\mathcal{F}}; \theta) = [I - \beta\Gamma^j(\sigma^*)(M^j_1(\sigma^*) - M^j_0(\sigma^*)) - \beta M^j_0(\sigma^*)]^{-1} \quad (28)$$

$$\times[\Gamma^j(\sigma^*)\Pi^j(\sigma^*, \sigma^*_{\mathcal{F}}) + (I - A^j)^{-1}A^j \cdot \iota]$$

To compute the value function estimates of the choice probabilities, defined above, and the period payoffs are required. The period payoffs can be computed by numerically integrating the expression in (25). To compute the expectation with respect to $b_{i,t}$ in (25) Gaussian quadrature methods are applied as outlined in Judd (1998) and with respect to contract characteristics the payoffs are evaluated at a fixed random sample of 50 contract characteristics and simple averages are taken. Given the above we can then compute the optimal choice probabilities implied by our model. The probability that a bidder enters an auction, given her participation in the previous round of bidding, is given by

$$p_j(s_{i,t}, \mathbf{s}_{-i,t}; \sigma^*, \sigma^*_{\mathcal{F}}) = \quad (29)$$

$$H_j\left\{\left[\pi^j(\mathbf{s}_t;\sigma^*,\sigma^*_{\mathcal{F}})+\beta\sum_{\mathbf{s}'_{t+1}\in\mathbf{S}}[\Pr(\mathbf{s}'_{t+1}|\mathbf{s}_t,d_{i,t}=1;\sigma^*)-\Pr(\mathbf{s}'_{t+1}|\mathbf{s}_t,d_{i,t}=0;\sigma^*)]V^j(\mathbf{s}'_{t+1};\sigma^*,\sigma^*_{\mathcal{F}})\right]\middle| s_{i,t}\right\}$$

Stacking the above expression over states and bidder types yields $\mathbf{p} = \Lambda(\mathbf{p}; \theta)$. The estimator forces the equality constraints $\mathbf{p} - \Lambda(\mathbf{p}; \theta) = 0$ to estimate the structural parameters. In other words the estimator given first stage estimates is given by

$$\min_{\theta} \left(\widehat{\mathbf{p}} - \Lambda(\widehat{\mathbf{p}}; \theta)\right)' \widehat{W}(\theta) \left(\widehat{\mathbf{p}} - \Lambda(\widehat{\mathbf{p}}; \theta)\right) \tag{30}$$

where $W$ is the optimal weight matrix which depends on the covariance matrix of auxiliary parameters and the bid distributions and the derivatives of the estimating equations with respect to the auxiliary parameters and the parameters of the bid distribution functions[12]. Specifically, the optimal $W$ is given by

$$W(\theta) = \left(\left[(\mathbf{I}:\mathbf{0}) - \nabla_{(\mathbf{p},\Psi)'}\Lambda(\mathbf{p};\theta)\right] \Sigma \left[(\mathbf{I}:\mathbf{0}) - \nabla_{(\mathbf{p},\Psi)'}\Lambda(\mathbf{p};\theta)\right]'\right)^{-1} \tag{31}$$

$\mathbf{0}$ is a $(2{*}m_s)\times m_\psi$ matrix of zeros, where $m_\psi$ is the number of parameters in the bid distribution function, $\Psi$ is the vector of bid distribution parameters, and $\Sigma$ is the variance covariance matrix of the choice probability estimator and of the bid distribution parameters. The optimality of this weight matrix follows from the conditions presented in Gourieroux & Monfort (1995). Asymptotic normality of this estimator is also established there. Our estimator is different from Pesendorfer & Schmidt-Dengler (2008) since payoffs are known and computed in the first stage of estimation. As a result, our weight matrix will not only depend on the variance covariance matrix of the optimal choice probabilities but also on the bid distribution estimates. Given that our first stage estimates are consistent and asymptotically normal we can directly apply the results in Gourieroux & Monfort (1995).

## 4.5 Identification

Identification of the latent values of bidders follows directly from the conditions from Guerre *et al.* (2000). In particular as pointed out by Athey & Haile (2007), the identification result from Guerre *et al.* (2000) can be re-interpreted as being conditional on the realisation of auction specific covariates and state variables. For identification we require monotonicity of the markup term in (22) conditional on auction covariates and state variables.

The parameters of the dynamic game are overidentified. This can be established following Pesendorfer & Schmidt-Dengler (2008). In particular, we can also write an equilibrium characterization linear in the unknown parameters for a bidder that is just indifferent between entering and not. This equation system will have more equations than unknowns. In our case period payoffs are known, except for the bid distribution and the optimal choice probabilities.

[12]Note that the special feature of auction participation games as opposed to standard market entry games, is that the payoffs do not contain any unknown parameters. As a result, for certain information cost specifications we can directly compute the information cost parameters, for example this is the case with the exponential distribution.

The only unknown parameters of the dynamic game are therefore those of the information cost distribution. The best estimator for our problem is the asymptotic least squares estimator outlined above making use of the optimal weight matrix.

# 5 Results

This section presents results of the estimation. We begin by summarising the estimation of the auxiliary parameters followed by estimation results on period payoffs and the structural parameters. In each section we provide evidence on the goodness of fit of each estimator.

## 5.1 Optimal Choice Probabilities

We estimate choice probabilities by frequency estimators, as shown in (39), and use these two compute the transition matrices. *Goodness of Fit*: To test the goodness of fit we compare the average number of large and small bidders across all auctions in our data with simulated numbers computed using our choice probability estimates. For each realisation of the state variables observed in the data, we select the associated choice probability and draw a uniform random variable on $[0, 1]$, if the choice probability is greater than the uniform variable the bidder enters. We complete this procedure for large and small bidders separately. We compute the mean simulated number of large and small participants across all auctions and compare with the data. We find that the mean actual number of bidders is 0.457 and the simulated mean is 0.448, with standard deviations given by 0.656 and 0.637, respectively. The means are not statistically different at 99% confidence. The average number of small bidders is 1.723 with standard deviation 1.754. The simulated number is 1.7290 with standard deviation 1.251. The means are not statistically different at 99% confidence.

## 5.2 Bid Distribution Estimates

The parameters of the bid distributions are shown in Table 6. *Goodness of Fit*: To test the goodness of fit of the bid distributions, we follow Jofre-Bonet & Pesendorfer (2003) and Athey *et al.* (2008) and compute the mean and standard deviation of the observed bids across all auctions and compare with means and standard deviations of bids generated by the estimated distribution. The computation of the means proceeds as follows. First we extract the bids, the number of bids submitted in each auction and the associated auction covariates including the state variables from the data. We compute the mean and standard deviation of all bids observed in our data. We then use the data on the number of bids submitted in an auction and the covariates to draw bids from our estimated distribution. We then compute the mean and standard deviation across all drawn bids. This computation is done separately for large and small bidders.

For large bidders the mean of observed log bids is 13.503 and the simulated mean is 13.5116, with standard deviations given by 1.1298 and 1.1232. The difference between the two means is statistically not significant at 99% confidence. Similar results are found for the small bidder distributions, with an observed mean of 13.512 and a simulated mean of 13.596, with standard deviations 1.1232 and 1.2287, respectively. The two means are not statistically different at 99% confidence. We conduct similar tests for the fringe bidder distributions and find similar results.

We also simulate a low bid from the distributions. This test is more appropriate since the minimum bid determines the procurement costs of MDOT. We take the minimum bid in each auction by large and small bidders across all auctions, the number of large and small bids submitted and the associated auction covariates. We compute the mean and standard deviation across the entire set of minimum bids for each type of bidder separately. We then draw the same number of bids as observed in the data from our estimated distribution for each set of auction. We then compute the minimum bid in each auction and compute the mean and standard deviation across all auctions.

For this test, we follow Athey *et al.* (2008) and we consider the means and distribution of the bid residuals. Specifically, given the Weibull distribution we can write a bid from bidder $i$ of type $j$ as $b_{i,t} = \psi_{2,j}(c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t})\epsilon_{i,t}(c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t})$ which can be re-arranged as $b_{i,t}/\psi_{2,j}(c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t}) = \epsilon_{i,t}(c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t})$. We then compute the bid residuals for the observed minimum bids and for our simulated minimum bids. We find that for large bidders the means of the residuals are given by 0.994 and 0.988, for the actual and simulated bids, respectively. The standard deviation for the observed bids is 0.013 and for the simulated it is 0.020. The means are not statistically different at 95% confidence. For small bidders the observed mean is 0.985 with standard deviation 0.019 and the simulated mean is 0.939 with standard deviation 0.068. The means are statistically not different at 95% confidence. We therefore find that the fit is good for both distributions.

*Effect of Individual Variables on Large Bid Distribution:* We compute the mean of our estimated Weibull distribution for the average engineer's estimate and value of the state variables.

*Log of Engineer's Estimate:* Increasing the engineer's estimate by 0.01 increases the mean from by 1%, the variance decreases from 0.0453 to 0.0452.

*State Variables:* Increasing $\mathcal{N}_{\mathcal{L}}$ by one increases the mean of the distribution by 0.8% and the variance increases to 0.0535. Increasing $\mathcal{N}_{\mathcal{S}}$ by one increases the mean by 6.8% and the variance increases to 0.2623. Very similar results are found for the small bidders distribution.

## 5.3 Estimation of Private Costs and Markups

Recall that markups over costs are defined in (22). We substitute the estimated density and bid distribution into this expression evaluated at the observed bid and its associated covariates. On average the observed markup for large bidders is equal to 21.46%. The average markup for a winning bid is equal to 25.89%. The average markup for a small bidders is 21.08%. The average markup for a winning small bid is 23.76%. The average markup for a fringe bidder is equal to 24.27%.

Table 6: Bid Distribution Estimates Whole Sample

| **Variable** | | **Coefficient** (Std. Err.) | |
|---|---|---|---|
| | Large | Small | Fringe |
| $\psi_{10}$, constant | 1.8385 | 1.8351 | 1.4342 |
| | (0.1062) | (0.0408) | (0.0020) |
| $\psi_{11}$, engineer | 0.1897 | 0.1898 | 0.2122 |
| | (0.0092) | (0.0033) | (0.0002) |
| $\psi_{20}$, constant | 1.6591 | 1.6593 | 1.6488 |
| | (0.0104) | (0.0221) | (0.0001) |
| $\psi_{21}$, engineer | 0.0702 | 0.0702 | 0.0710 |
| | (0.0007) | (0.0014) | (0.0000) |
| *State Variables* | | | |
| $\psi_{12}$, i's state, $s_{i,t} = d_{i,t-1}$ | 0.0299 | 0.0263 | |
| | (0.0698) | (0.0263) | |
| $\psi_{13}$, # of large bidders in previous round, $\mathcal{N}_{\mathcal{L}}$ | 0.0071 | 0.0069 | 0.0003 |
| | (0.0029) | (0.001) | (0.0001) |
| $\psi_{14}$, # of small bidders in previous round, $\mathcal{N}_{\mathcal{S}}$ | -0.0602 | -0.0573 | -0.0363 |
| | (0.0316) | (0.0071) | (0.0004) |
| $\psi_{22}$, i's state, $s_{i,t} = d_{i,t-1}$ | 0.0018 | 0.0018 | |
| | (0.0023) | (0.003) | |
| $\psi_{23}$, # of large bidders in previous round, $\mathcal{N}_{\mathcal{L}}$ | -0.0002 | -0.0002 | 0.0001 |
| | (0.0001) | (0.0003) | (0.0001) |
| $\psi_{24}$, # of small bidders in previous round, $\mathcal{N}_{\mathcal{S}}$ | 0.0010 | 0.0010 | -0.0008 |
| | (0.0014) | (0.0015) | (0.0000) |

Number of Auctions= 4927
Number of Bidders= 30

Krasnokutskaya (2003) estimates her model using MDOT data as well, however for a different time period, and finds markups in the order of 8% and for a winning bid the markup is 16%. The main difference between these results and ours is the inclusion of unobserved heterogeneity which reduces the amount of variation due to private information. It is possible, as in Athey *et al.* (2008), to include parametric unobserved heterogeneity into the bid distributions here and estimate a Weibull-Gamma mixture. We aim to implement this feature in the future. Jofre-Bonet & Pesendorfer (2003) find markups that exclude dynamic effects on average roughly equal to 20%. Bajari *et al.* (2006) and Bajari *et al.* (2004) estimate markups that are equal to 6%.

In Figure 3 we plot the cost distributions for the sample average of the engineer's estimate and at the most frequently observed state variables, which occurs in 13% of the auctions. We can see that the large bidder distribution has a lower mean, which we would expect given that a large bidder is more able to mobilise equipment across the state of Michigan. Note, that this was not imposed during the estimation. The average cost for a large bidder is $521,130.04 and for a small bidder the mean is $836,069.91. Moreover, the difference in costs is statistically significant at 99% confidence.

*Bidding Function:* Given our estimates we can compute the bidding function. We use (22) to compute an optimal bid for different completion costs. The bid function is plotted by holding the state variables fixed, taking an average of the engineer's estimate and varying the completion cost. The results can be seen in Figure 2. The bid function approaches the 45° line. We also find that for bids at the lower end of the bid support, we compute negative costs which we find implausible. When we find negative costs we set these equal to zero, as in Jofre-Bonet & Pesendorfer (2003).

We can also compare how bidding behaviour changes as a bidder's own participation status changes, i.e. when $s_{i,t} = 0$ and when $s_{i,t} = 1$ and all other state variables remain unchanged. The results are shown in Figure 4. It can be seen that a bidder who has $s_{i,t} = 1$ bids higher. Participation in a previous round improves a bidder's cost position. As a result, bidders who did not participate in a previous round of bidding, even if they are of the same type as their opponent, are more likely to have worse entry cost draws. They will then bid more aggressively. Recall that from the reduced form analysis we found a positive effect of own participation on the bid level. The effect of own participation status was stronger and statistically significant when including bidder fixed effects. This would be in line with our previous intuition and the computed bidding functions.

## 5.4 Period Payoffs

We compute ex ante expected period payoffs using (25) yielding payoffs as a function of public states. These results will be used as inputs in the estimation of the structural parameters. The average period payoff over all possible states for a large bidder is $111,901.14 and for a small

Figure 2: Bid Function for Large Bidders

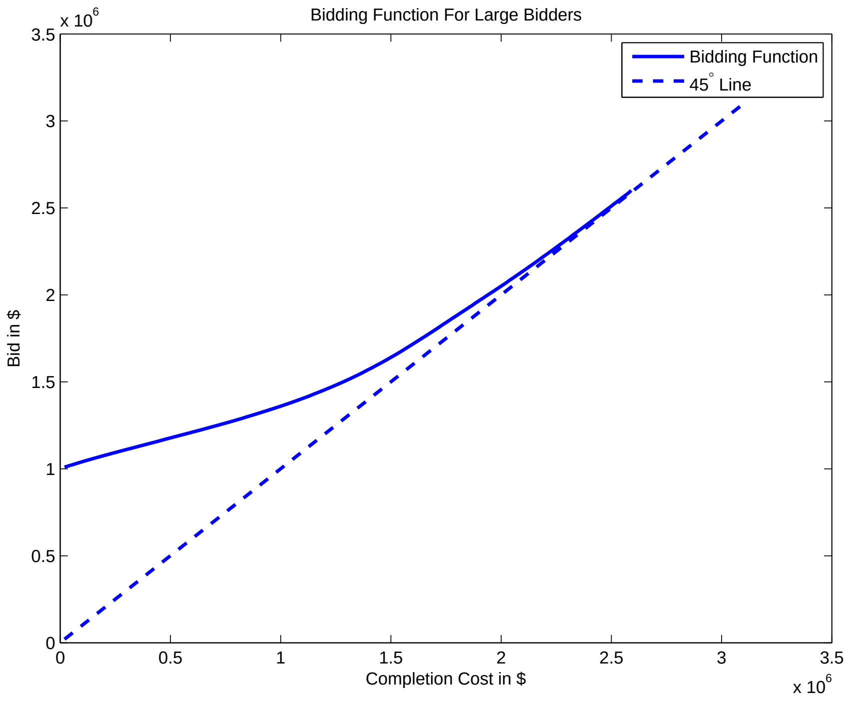



Figure 3: Completion Cost Distributions For Most Frequently Realised State

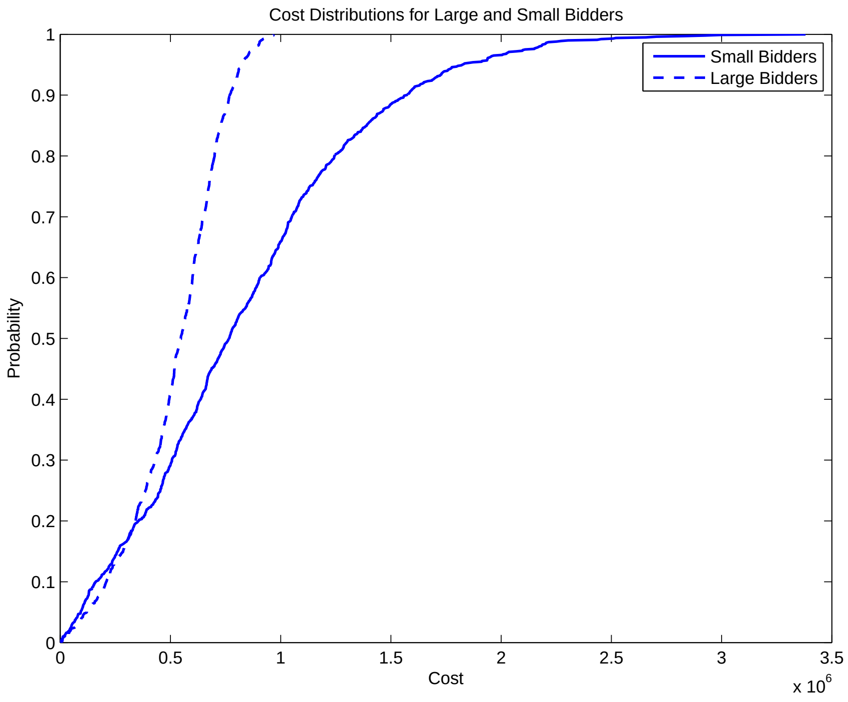
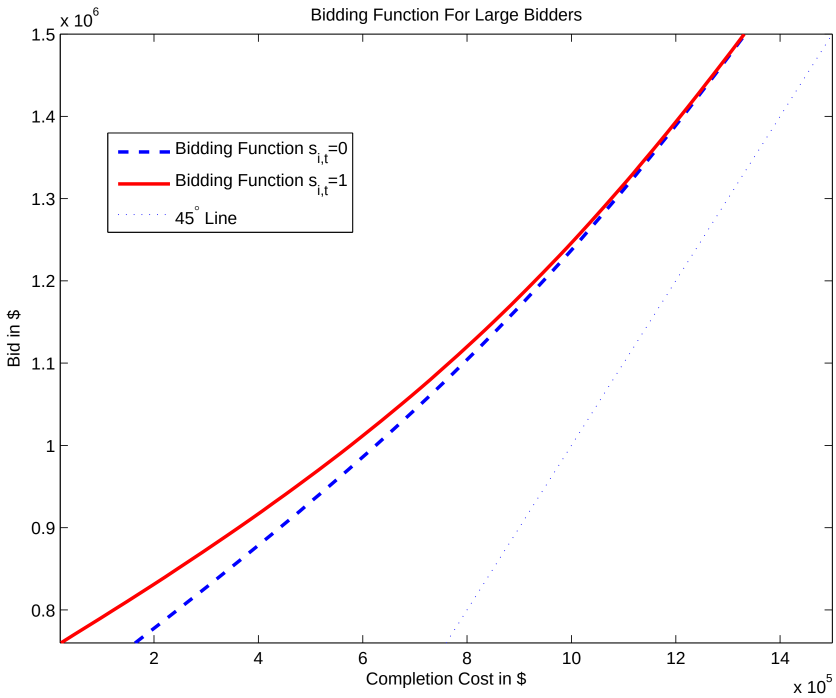


Figure 4: Bid Function for Large Bidders




bidder it is \$43,539.92.

## 5.5 Information Costs

The parameters for the participation game are presented in Table 7. Given the parameters we can compute the average expected sunk information cost, i.e. $E_\phi(\phi|\phi \leq W_1^j(\mathbf{s}_t;\sigma^*,\sigma_{\mathcal{F}}^*) - W_0^j(\mathbf{s}_t;\sigma^*,\sigma_{\mathcal{F}}^*))$ averaged over all states. The information cost amounts to \$9212.10 for a large bidder and \$3979.90 for a small bidder. For states where a bidder participated in the previous round the average information cost is then \$8052.30 and for states where a bidder did not participate is given by \$10,371.97. For small bidders the average information cost for states where they participated in the previous round is given by \$4043.30 and for states they did not the average information cost is \$3916.50. Average information cost sunk by smaller bidders are lower than large bidders, since small bidders require very low draws for the information costs in order to enter. As shown in the previous section, small bidders' period payoffs are on average much lower than large bidders, therefore a small bidder requires a much lower information cost to induce participation, since the expected profits from participation are not as high as a large bidder.

*Goodness of Fit:* To test the goodness of fit of our model we simulate the number of entrants for all observed states and compute the average number of bidders across all auctions. We draw information costs from our estimated distribution for $\phi$ and compare this to the value of entering given by the difference $W_1^j(\mathbf{s}_t;\sigma^*,\sigma_{\mathcal{F}}^*) - W_0^j(\mathbf{s}_t;\sigma^*,\sigma_{\mathcal{F}}^*)$. The average number of large bidders is 0.4572 with standard deviation 0.6555. The model predicts an average number of

Table 7: Structural Parameter Estimates

| Variable | Coefficient (Ltd. Err.) |
|---|---|
| $\alpha_{\mathcal{L},0}$ | -4.8346 |
| | (0.0101) |
| $\alpha_{\mathcal{L},1}$ | 0.2245 |
| | (0.0064) |
| $\alpha_{\mathcal{S},0}$ | -5.2267 |
| | (0.003) |
| $\alpha_{\mathcal{S},1}$ | 0.1322 |
| | (0.0001) |

0.3920 with standard deviation 0.5930. The model underpredicts the number of entrants on average. On average there are 1.7233 small bidders with standard deviation 1.7537. The model predicts an average number of entrants equal to 1.4210 with standard deviation 1.1528.

# 6 Inefficiencies

## 6.1 Inefficiencies due to Auction Format

When bidders are asymmetric it is possible that a first price sealed bid auction will lead to inefficient outcomes. In our model there are two potential sources of asymmetry. The first is through the size of the bidder which affects the completion costs and the other is through the dynamic synergies in participation. To determine whether this is the case in the MDOT data we use the primitives to estimate how often the low bidder does not win the auction, holding the entry process fixed.

We take a random sample of 250 contract characteristics and compute the simulations for the same set of contracts 1000 times. The steps of the simulation are as follows. We draw information costs from our estimated information cost distribution and use the estimated strategies of large and small bidders to determine entry. For fringe bidders we use the estimated poisson model to determine the number of entrants. After the entrants have been determined we draw bids and use the inverse bidding strategy to compute costs. We then compute the fraction of auctions the low bid does not correspond to the lowest cost. We find that this occurs on average in 10.84% of the auctions. The average difference between the low bidder and the winner's cost is 4.31% of the engineer's estimate. Krasnokutskaya (2003) estimates the average probability of inefficient outcomes in her MDOT data at 5%. Jofre-Bonet & Pesendorfer (2003) find a 32% average probability of the low bidder not winning.

## 6.2 Changing the Auction Format: Open Auctions

Given that there are inefficiencies due to auction format we consider changing the auction rule. The open auction rule can lead to efficient outcomes, when bidders play the dominant strategy equilibrium in which they bid their costs truthfully. We exclude fringe bidders in this analysis, since we have not explicitly estimated the primitives that drives fringe participation behaviour. As mentioned before, we focus on the dominant strategy equilibrium where all bidders bid their costs. We then look at the change in procurement costs relative to the original format and the presence of fringe bidders. We simulate this auction using the same random sample of 250 contracts used in the efficiency measurement exercise.

As mentioned before, conditional on entry, the auction stage game can be easily simulated given that the strategies we focus on take the simple of form of bidding one's costs. We draw costs for the entrants and compute the auction price and profits. If there is only one bidder in an auction, they must compete against the MDOT who draw their costs from a regular large bidder completion cost distribution. The next step is to compute an equilibrium for the entry game. The main challenge we face here is that there are potentially multiple entry equilibria, as described in Doraszelski & Satterthwaite (2009). We initially search for equilibria using the estimated entry choice probabilities as starting values.

The first step in this exercise is to compute the expected period payoffs for the auction stage game at all states for a player of type $j$.

$$\pi_j^{II}(\mathbf{s}) = \sum_{n_{\mathcal{F}}}^{N_{\mathcal{F}}} \sum_{n_{\mathcal{S}}}^{N_{\mathcal{S}} - \mathbf{1}_{\{j=\mathcal{S}\}}} \sum_{n_{\mathcal{L}}}^{N_{\mathcal{L}} - \mathbf{1}_{\{j=\mathcal{L}\}}} \Pr(n_{\mathcal{F}}, n_{\mathcal{S}}, n_{\mathcal{L}}) E_{c0,c,x}[(x-c)\mathbf{1}(c<x)|\mathbf{s}] \tag{32}$$

where $x = \min\{c_k\}_{k=1}^{n_{\mathcal{F}}+n_{\mathcal{S}}+n_{\mathcal{L}}}$, is the lowest cost of the participating opponent bidders. To evaluate the above integral, we apply Monte Carlo techniques. Given the payoffs we can then search for the optimal set of choice probabilities that are fixed points of the aforementioned equilibrium condition

$$\mathbf{p} = \Lambda(\mathbf{p}) \tag{33}$$

There are potentially multiple fixed points for the above, however for this counterfactual exercise we follow Ryan (2006) and will focus on only one. *Procurement Costs:* We now compare the procurement costs in the simulated open auction with the realised procurement costs. Using the same intuition as in Athey *et al.* (2008), we would expect the open auction to discourage entry from small bidders, since their chances of winning are reduced as large bidders no longer shade their costs more than small bidders. As a result, this could then also lead to an increase in the procurement costs. In our case the procurement cost in the open auction are on average 133.33% of the engineer's estimate with standard deviation 24% and the observed procurement costs are 95.51% with standard deviation 14%. The mean procurement costs are statistically different. We also compare the procurement costs if we made information costs zero, excluded fringe bidders and used an open auction format. In this environment all bidders enter since participation is costless and the dynamic effects have been removed in this market. In this case the average procurement costs are 11.94% of the engineer's estimate.

# 7 A Brief Comparison to a Static Participation Model

In this section we present and estimate a static version of our dynamic game in order to determine the effect of ignoring dynamic linkages in these auctions. The estimation approach we take here is similar to the dynamic game estimator presented previously. However, the simple structure of these auction games, specifically the fact that period payoffs are known and do not contain unknown parameters aside from the bid distributions and entry probabilities, allows for an even more straightforward estimation approach.

The static model echoes our dynamic participation game. Bidder's participation decision is again completely determined by the information cost which is unobserved by the other players. Bidders compare the information acquisition cost with the expected profit from entering the auction, if this difference is positive they enter, if not they stay out.

## 7.1 Setup

Bidders are risk-neutral and are categorised as large ($\mathcal{L}$), small ($\mathcal{S}$) and fringe ($\mathcal{F}$) bidders, as in the main paper. Denote the set of bidders of type $j$ as $\mathbf{N}_j$. The game is structured as before.

*Private Completion Costs:* Bidder $i$ of type $j = \mathcal{L}, \mathcal{S}, \mathcal{F}$ draws private costs, $c_i \in \mathbf{C}_j$, independently and identically from the cost distribution $F_j(c_i|c_0)$ on $[\underline{c}, \overline{c}]$, conditional on $c_{0,t}$.

*Information Costs for type $j$ bidder:* All types of bidders draw information costs from a Pareto distribution with density

$$\widetilde{h}_j(\phi_i) = \widetilde{\alpha}_j \phi_i^{-\widetilde{\alpha}_j - 1} \tag{34}$$

Let $\widetilde{H}_j$ denote the distribution of $\phi$ for type $j$.

*Strategies* consist of a bidding strategy and an entry strategy. A bidding strategy maps from the completion cost to a bid. The entry strategy specifies a cut-off value for entry costs. Denote the participation choice separately as $d_j$ for bidder of type $j$. Denote the set of strategies for each type separately $\widetilde{\sigma} = \{\sigma_{i,j},\ i = 1, ..., N\ \& j = \mathcal{L}, \mathcal{F}, \mathcal{S}\}$. Let $q_j$ be the ex ante expected entry probability for a bidder of type $j$, in other words

$$\widetilde{q}_j = \int_1^{\infty} \mathbf{1}\{d_j^{\widetilde{\sigma}}(\phi) = 1\} \widetilde{h}_j(\phi) d\phi \tag{35}$$

The above is the entry probability of a bidder if they follow their strategies specified in $\widetilde{\sigma}_j$.

*Payoffs:* A bidder's expected profit conditional on entry is given by:

$$\widetilde{\pi}^j(c_0, c_i, \widetilde{\sigma}) = \max_b [b - c_i] \Pr(i \text{ wins}|b, c_0; \widetilde{\sigma}) \tag{36}$$

where

$$\Pr(i \text{ wins}) = \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{L}} \text{ potential } \mathcal{L} \\ \text{type bidders} \end{pmatrix} \times \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{S}} \text{ potential } \mathcal{S} \\ \text{type bidders} \end{pmatrix} \times \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{F}} \text{ potential } \mathcal{F} \\ \text{type bidders} \end{pmatrix} \tag{37}$$

and where

$$\Pr\left(\begin{array}{c} i \text{ wins against } N_j \\ \text{potential } j \text{ type} \\ \text{bidders} \end{array}\right) = \tag{38}$$

$$\sum_{n_j=\mathbf{1}_{\{j=\mathcal{L}\}}}^{N_j-\mathbf{1}_{\{i\in\mathbf{N}_j\}}} \binom{N_j-\mathbf{1}_{\{i\in\mathbf{N}_j\}}}{n_j} \widetilde{q}_j^{n_j}(1-\widetilde{q}_j)^{N_j-\mathbf{1}_{\{i\in\mathbf{N}_j\}}-n_j}[1-G_j(b_{i,t}|c_{0,t})]^{n_j}$$

where $\mathbf{1}_{\{i\in\mathbf{N}_j\}}$ is an indicator that equals one if bidder $i$ is of type $j$

*Equilibrium:* A type symmetric equilibrium is a collection of bidding strategies for each type of bidder and entry strategies with property that, bidder's bid strategy maximises profits conditional on entering and each bidder only enters if the expected profits, prior to revelation of contract characteristics, exceeds information acquisition cost. Denote the equilibrium strategies as $\widetilde{\sigma}^*$. Denote the equilibrium entry probability for a bidder of type $j$ as $p_j$.

## 7.2 Estimation

### 7.2.1 Entry Probability

We estimate the entry probability for each regular bidder type separately and non-parametrically by a simple frequency estimator.

$$\widehat{p}_j = \frac{\sum_{i\in\mathbf{N}_j}\sum_t^T \mathbf{1}\{d_{i,t}=1\}}{\sum_{k=0,1}\sum_t^T \mathbf{1}\{d_{j,t}=k\}} \tag{39}$$

Entry probabilities for large players is given by 11.43% and for small players 6.63%. We test the goodness of fit of these estimates by assessing whether the estimator can reproduce the average number of entrants of each type. We find that they indeed can.

For fringe bidders we estimate entry probabilities by assuming that the number of fringe bidders follows a poisson process with distribution given by:

$$\Pr(n_{\mathcal{F}}) = \frac{e^{-\widetilde{\delta}}\widetilde{\delta}^{n_{\mathcal{F}}}}{n_{\mathcal{F}}!} \tag{40}$$

### 7.2.2 Private Costs

We assume that all bidder types have the same functional form for their bid distributions. All types have bid distributions that follow a Weibull distribution, with shape and scale parameters functions of the contract size.

$$\widetilde{G}_j(b_{i,t}|c_{0,t}) = 1-\exp\left[-\left(\frac{\log(b_{i,t}+1)}{\psi_{2j}(c_{0,t})}\right)^{\psi_{1j}(c_{0,t})}\right] \tag{41}$$

for $j=\mathcal{F},\mathcal{L},\mathcal{S}$ where $\log\psi_{k,j}(c_{0,t}) = \psi_{k,j,0}+\psi_{k,j,1}c_{0,t}$ for $k=1,2$. The fit of the bids is tested following the same procedure outlined in the main paper. We find that the bid distributions fit well. We then use the first order condition for an optimal bid to infer private costs as described previously. In other words private costs can be estimated by computing:

$$c_{i,t} = b_{i,t} - [\widetilde{\eta}_{\mathcal{L}}(b_{i,t},c_{0,t};\widetilde{\sigma}^*) + \widetilde{\eta}_{\mathcal{S}}(b_{i,t},c_{0,t};\widetilde{\sigma}^*) + \widetilde{\eta}_{\mathcal{F}}(b_{i,t};\widetilde{\sigma}^*)]^{-1} \tag{42}$$

where and

$$\widetilde{\eta}_j(b_{i,t}, c_{0,t}; \widetilde{\sigma}^*) = \left[ \frac{\partial \Pr \left( \begin{array}{c} i \text{ wins against } N_j \\ \text{potential } j \text{ type bidders} \end{array} \middle| b_{i,t}, c_{0,t}; \widetilde{\sigma}^* \right) / \partial b_{i,t}}{\Pr \left( \begin{array}{c} i \text{ wins against } N_j \\ \text{potential } j \text{ type bidders} \end{array} \middle| b_{i,t}, c_{0,t}; \widetilde{\sigma}^* \right)} \right] \tag{43}$$

The above expression will have a similar structure to the dynamic version.

### 7.2.3 Entry Cost Distribution

Given the first stage estimates we can directly compute the parameters of the entry cost distribution. The first step involves computing the expected profits from an auction, prior to contract characteristics being revealed. This involves substituting the expression for an optimal bid into the payoff function in (36). We then integrate over private costs and contract characteristics. A change of variable of integration from $c$ to $b$ yields

$$\pi^j(\widetilde{\sigma}^*) = E_{c_0} \left[ \int_0^{\bar{b}} \frac{\Pr(i \text{ wins}|b, c_0; \widetilde{\sigma}^*)}{\widetilde{\eta}_{\mathcal{L}} + \widetilde{\eta}_{\mathcal{S}} + \widetilde{\eta}_{\mathcal{F}}} \widetilde{g}_j(b|c_0) db \right] \tag{44}$$

We use Gaussian quadrature to evaluate the integral with respect to bids and take a random sample of 50 contract characteristics and take simple averages. A bidder will enter an auction if

$$\widetilde{\pi}^j(\widetilde{\sigma}^*) \geq \phi_i \tag{45}$$

This optimality condition can be evaluated before the information costs are known, by integrating over information cost acquisitions, which yields the ex ante optimal entry probabilities

$$p_j = \int_1^\infty \mathbf{1}\{\pi^j(\widetilde{\sigma}^*) \geq \phi\} \widetilde{h}_j(\phi) d\phi = \widetilde{H}(\widetilde{\pi}^j(\widetilde{\sigma}^*)) = 1 - [\widetilde{\pi}^j(\widetilde{\sigma}^*)]^{-\widetilde{\alpha}_j} \tag{46}$$

We can now simply rearrange the above expression to infer the value for $\alpha_j$ for each type of bidder, which yields

$$\widetilde{\alpha}_j = -\frac{\log(1 - p_j)}{\log(\widetilde{\pi}^j(\widetilde{\sigma}^*))} \tag{47}$$

Everything on the right hand side is known and we can therefore infer the value of $\widetilde{\alpha_j}$.

## 7.3 Results and Comparison to Dynamic Model

We summarise the main results of the estimation below.

- *Markups* are comparable to the dynamic model, average markups across all auctions for large bidders is 25.08% and for small bidders is 24.13%.

- *Completion costs* are on average higher for small bidders than for large bidders. We compute costs for a bidder at the average engineer's estimate. We find that average costs for a large bidder are \$475,376.40 and \$570,297.98 for a small bidder, with standard deviations \$134,688.55 and \$568,837.87 for large and small players respectively. The means are statistically different at 95% confidence.

- *Entry costs* sunk by bidders are on average \$3553.28 for large bidders, and \$4283 for small bidders.
- *Inefficiencies:* Holding the entry process fixed, we simulate auctions for the same random sample of 250 auction covariates used previously, and repeat the experiment 1000 times. We test the efficiency of the auctions using the same procedure outlined in the main paper. It turns out that only 0.1% of the auctions are inefficient with an average cost difference of 0.29% of the engineer's estimate.

To summarise, the estimated entry costs are on average lower than the dynamic model. Average markups are roughly in line with the dynamic estimates. However, the major discrepancy between the two approaches are in the measurement of inefficiencies due to the first price auction rule. The static estimates imply virtually zero inefficiencies. One possible reason for the low level of inefficiencies is due to the fact that the asymmetry between bidders who participated in a previous round of bidding has been removed from the static framework. Recall the discussion of the bidding function in the previous section. In Figure 4 we saw that bidders of the same type bid differently for participation statuses $s_{i,t} = 1$ and $s_{i,t} = 0$. The static model, on the other hand, does not allow for this asymmetry between bidders of the same type. Ignoring this asymmetry between bidders who are of the same type but have different states would make the first price auction seem less inefficient than it actually may be. Unfortunately, we cannot compare these results with other papers that consider static auctions with participation such as Athey *et al.* (2008) or Li & Zheng (2009) since they either do not measure inefficiencies due to auction rules or only consider symmetric auction models.

# 8 Conclusion

This paper demonstrates the feasibility of estimating a dynamic auction game with participation and that dynamics in participation should not be ignored. This paper analyses procurement auctions of the Michigan State Department of Transportation. We find evidence of dynamic linkages between auction rounds. In particular, past participation has an influence on current procurement costs. We also find evidence for participation synergies between rounds of bidding. To fully account for these features we propose an estimation procedure for auctions with endogenous participation in a dynamic framework. Private completion costs and parameters affecting information acquisition costs and participation are estimated. We then quantify the level of inefficiencies due to the use of a first price auction rule. The probability of an inefficient outcome is roughly 11%. We find that changing the auction rule from a first price auction rule to an open auction, increases the average procurement costs, however it does eliminate the inefficiencies caused by asymmetries of bidders in first price auctions. We find that the driving force behind higher procurement costs, is that smaller bidders are discouraged from entering. There is also scope to analyse the efficiency of entry in this setup. We could, for example, consider limiting potential competition. These policy simulations will be pursued at a later date.

We then compare our results to a static participation model. The estimation output leads to misleading conclusions concerning the inefficiency of the auctions. In particular, we find that the probability of an inefficient auction is close to zero. This divergence can be attributed to the removal of a source of asymmetry between bidders in the static model, namely synergies in participation between rounds. This result emphasizes the importance of considering potential dynamic effects in auction markets.

# A Appendix

## A.1 Computation of Markups

To compute the expressions in (23) we require the derivative of the probability of a bidder. Let $g_j(.|c_{0,t}, s_{i,t}, \mathbf{s}_{-i,t})$ be the associated density of the equilibrium bid distribution. When bidder $i$ is of type $j$ this derivative equals

$$-\frac{\partial \Pr\begin{pmatrix} i \text{ wins against } N_j \\ \text{potential } j \\ \text{type bidders}\end{pmatrix}}{\partial b_{i,t}} =$$

$$\sum_{\substack{N_{0,j}-\mathbf{1}_{\{s_{i,t}=0\}} \\ N_{1,j}-\mathbf{1}_{\{s_{i,t}=0\}}}}^{} \sum_{\substack{n_{0,j}=0 \\ n_{1,j}=\mathbf{1}_{\{j=\mathcal{L}\}}}} \sum_{k=0,1} \left[\mathbf{C}_{n_{k,j}}^{N_{k,j}-\mathbf{1}_{\{s_{i,t}=k\}}}\right] p_j(\mathbf{s}_k)^{n_{k,j}}(1-p_j(\mathbf{s}_k))^{N_{k,j}-\mathbf{1}_{\{s_{i,t}=k\}}-n_{k,j}}[1-G_j(b_{i,t}|c_{k,t},\mathbf{s}_{k,t})]^{n_{k,j}}$$

$$\times\frac{n_{k,j}g_j(b_{i,t}|c_{0,t},\mathbf{s}_{k,t})}{1-G_j(b_{i,t}|c_{0,t},\mathbf{s}_{k,t})}$$

$$\times\left[\mathbf{C}_{n_{-k,j}}^{N_{-k,j}}\right] p_j(\mathbf{s}_{-k})^{n_{-k,j}}(1-p_j(\mathbf{s}_{-k}))^{N_{-k,j}-n_{-k,j}}[1-G_j(b_{i,t}|c_{0,t},\mathbf{s}_{-k,t})]^{n_{-k,j}} \tag{A-1}$$

for regular bidders and if $i$ is *not* of type $j$ then the above is equal to:

$$-\frac{\partial \Pr\begin{pmatrix} i \text{ wins against } N_j \\ \text{potential } j \\ \text{type bidders}\end{pmatrix}}{\partial b_{i,t}} =$$

$$\sum_{\substack{n_{0,j}=0 \\ n_{1,j}=\mathbf{1}_{\{j=\mathcal{L}\}}}}^{N_{0,j},N_{1,j}} \sum_{k=0,1} \left[\mathbf{C}_{n_{k,j}}^{N_{k,j}}\right] p_j(\mathbf{s}_k)^{n_{k,j}}(1-p_j(\mathbf{s}_k))^{N_{k,j}-n_{k,j}}[1-G_j(b_{i,t}|c_{k,t},\mathbf{s}_{k,t})]^{n_{k,j}-1}n_{k,j}g_j(b_{i,t}|c_{k,t},\mathbf{s}_{k,t})$$

$$\times\left[\mathbf{C}_{n_{-k,j}}^{N_{-k,j}}\right] p_j(\mathbf{s}_{-k})^{n_{-k,j}}(1-p_j(\mathbf{s}_{-k}))^{N_{-k,j}-n_{-k,j}}[1-G_j(b_{i,t}|c_{-k,t},\mathbf{s}_{-k,t})]^{n_{-k,j}} \tag{A-2}$$

and for fringe bidders

$$-\frac{\partial \Pr\begin{pmatrix} i \text{ wins against } N_{\mathcal{F}} \\ \text{potential } \mathcal{F} \\ \text{type bidders}\end{pmatrix}}{\partial b_{i,t}} = \sum_{n_{\mathcal{F}}=0}^{N_{\mathcal{F}}} \Pr(n_{\mathcal{F}}|\mathbf{s})n_{\mathcal{F}}[1-G_{\mathcal{F}}(b_{i,t}|c_{0,t},\mathbf{s}_t)]^{n_{\mathcal{F}}}\frac{g_{\mathcal{F}}(b_{i,t}|c_{0,t},\mathbf{s}_t)}{1-G_{\mathcal{F}}(b_{i,t}|c_{0,t},\mathbf{s}_t)} \tag{A-3}$$

## A.2 Reduced Form Analysis

Table 8: Probit Estimates

| **Variable** | **Coefficient** | | | | |
|---|---|---|---|---|---|
| Dependent Variable: Participation | (Std. Err.) | | | | |
| Participation in previous round | 0.1952** (0.0185) | 0.1925** (0.0185) | 0.1267* (0.0203) | 0.1152* (0.0205) | 0.1811** (0.0192) |
| constant | -1.5156** (0.0095) | -1.5927** (0.0011) | -2.6129** (0.0797) | -2.7573** ( 0.0800) | -2.4729** (0.0279) |
| Large bidder dummy | 0.2835** (0.0141) | | | | |
| # Auctions won in previous round | 0.2835** (0.0141) | | | 0.0158* (0.0034) | 0.0119 * (0.0027) |
| Size of bidder | | 0.0444** (0.0021) | | | 0.0492** (0.0021) |
| $\mathcal{N}_{\mathcal{S}}$ | -0.004 * (0.0014) | -0.0034* (0.0014) | -0.0036* (0.0015) | -0.0004 (0.0015) | -0.0056 * (0.0014) |
| $\mathcal{N}_{\mathcal{L}}$ | -0.0185 * (0.0078) | -0.0218* (0.0077) | -0.0126 (0.0081) | -0.0121 (0.0082) | -0.0167 * (0.0079) |
| Bidder Fixed Effects | NO | NO | YES | YES | NO |
| Auction Covariates | NO | NO | YES | YES | YES |

Number of Auctions: 4927
Number of Bidders: 30

## A.3 Notation Summary

Table 9: Notation

| **Label** | **Variable** |
|---|---|
| $j = \mathcal{L}, \mathcal{S}, \mathcal{F}$ | Type of bidder |
| *Actions* | |
| $b_{i,t} \in [0, \infty)$ | Bid |
| $d_{i,t} \in \{0, 1\}$ | Participation |
| *State Variables* | |
| $s_{i,t} \in \mathbf{S}_i$ and $s_{i,t} = d_{i,t-1}$ | Public state for bidder $i$ |
| $\mathbf{s}_t \in \mathbf{S} = \times_{k=1}^{N} \mathbf{S}_k$ | Vector of public states for all bidders |
| $\phi_{i,t} \in \Phi_j$ | Private state for bidder $i$, (bid preparation cost) |
| $c_{0,t} \in \mathbf{C}_0$ | Contract characteristics |
| $c_{i,t} \in \mathbf{C}_j$ | Completion cost for bidder $i$ |
| *Distributions* | |
| $F_j(.\vert c_0)$ | Distribution of completion costs |
| $H_j(.\vert s_{i,t})$, with param. $\alpha_j(s_{i,t})$ | Distribution of bid prep. costs |
| $G_j(.\vert c_0, s_{i,t})$ | Distribution of equilibrium bids |

# References


Athey, S., & Haile, P. 2007. *Nonparametric Approaches to Auctions*. Handbook of Econometrics,Heckman J.J. and Leamer E. (eds.), vol. 36. Elsevier. Chap. 60, pages 3847–3965.

Athey, S., Levin, J., & Seira, E. 2008. Comparing Open and Sealed Bid Auctions: Theory and Evidence from Timber Auctions. *NBER Working Paper Number W14590.*

Auguirregabiria, V., & Mira, P. 2007. Sequential Estimation of Dynamic Discrete Games. *Econometrica*, **75**, 1–53.

Bajari, P., & Hortacsu, A. 2003. The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions. *RAND Journal of Economics*, **34**(2), 329–55.

Bajari, P., & Ye, L. 2003. Deciding Between Competition and Collusion. *The Review of Economics and Statistics*, **85**, 971–989.

Bajari, P., Hong, H., & Ryan, S. 2004. Identifcation and Estimation of a Discrete Game of Complete Information. *NBER Technical Working Paper Number T0301.*

Bajari, P., Houghton, S., & Tadelis, S. 2006. Bidding for Incomplete Contracts: An Empirical Analysis. *NBER Working Paper Number W12051.*

Bajari, P., Benkard, C.L., & Levin, J. 2007. Estimating Dynamic Models of Imperfect Competition. *Econometrica*, **75**, 1331–1370.

Collard-Wexler, A. 2006. *Plant Turnover and Demand Fluctuations in the Ready–Mix Concrete Industry.* technical report, New York University.

Doraszelski, U., & Satterthwaite, M. 2009. *Computable Markov-Perfect Industry Dynamics.* mimeo.

Gourieroux, C., & Monfort, A. 1995. *Statistics and Econometric Models Vol.1.* Cambridge, U.K.: Cambrige University Press.

Guerre, E., Perrigne, I., & Vuong, Q. 2000. Optimal Nonparametric Estimation of First Price Auctions. *Econometrica*, **68**, 525–574.

Hendricks, K., & Porter, R. H. 2007. *An Empirical Perspective on Auctions.* Handbook of Industrial Organization, Mark Armstrong and R. Porter(eds.), vol. 3. Elsevier. Chap. 32, pages 2073–2143.

Hendricks, K., J., Pinkse, & Porter, R.H. 2003. Empirical Implications of Equilibrium Bidding in First–Price, Symmetric, Common Value Auctions. *Review of Economic Studies*, **70**, 115–145.

Hong, H., & Shum, M. 2002. Increasing Competition and the Winner's Curse: Evidence from Procurement. *Review of Economic Studies*, **69**, 871–898.

Hotz, V. J., & Miller, R. A. 1993. Conditional Choice Probabilities and the Estimation of Dynamic Models. *Review of Economic Studies*, **60**, 497–529.

Jofre-Bonet, M., & Pesendorfer, M. 2003. Estimation of a Dynamic Auction Game. *Econometrica*, **71**, 1443–1489.

Judd, K. 1998. *Numerical Methods in Economics.* Cambridge: MIT Press.

KLEMPERER, P. 2002. What Really Matters in Auction Design. *Journal of Economic Perspectives*, **16**, 169–189.

KRASNOKUTSKAYA, E. 2003. *Identification and Estimation in Highway Procurement Auctions under Unobserved Auction Heterogeneity.* mimeo.

KRASNOKUTSKAYA, E., & SEIM, K. 2005. *Preferential treatment Program and Participation Decisions in Highway Procurement.* mimeo.

LAFFONT, J-J., OSSARD, H., & VUONG, Q. 1995. Econometrics of First Price Auctions. *Econometrica*, **63**, 953–980.

LEVIN, D., & SMITH, J.L. 1994. Equilibrium in Auctions with Entry. *American Economic Review*, **84**, 585–599.

LI, T., & ZHENG, X. 2009. Entry and Competition Effects in First-Price Auctions: Theory and Evidence from Procurement Auctions. *Review of Economic Studies*, **76**, 1397–1429.

MANSKI, C.F., & TAMER, E. 2003. Inference on Regressions with Interval Data on a Regressor or Outcome. *Econometrica*, **70**, 519–546.

MCAFEE, R.P., & MCMILLAN, J. 1987. Auctions with Entry. *Economic Letters*, **23**, 343–347.

PAARSCH, H. 1992. Deciding between the Common and Private Value Paradigms in Empirical Models of Auctions. *Journal of Econometrics*, **51**, 191–215.

PAKES, A., OSTROVSKY, M., & BERRY, S. 2007. Simple Estimators for the Parameters of Discrete Dynamic Games, With Entry/Exit Examples. *RAND Journal of Economics*, **38**, 373–399.

PESENDORFER, M., & SCHMIDT-DENGLER, P. 2008. Asymptotic Least Squares Estimators for Dynamic Games. *Review of Economic Studies*, **75**, 901–928.

PORTER, R.H., & ZONA, J. 1993. Detection of Bid Rigging in Procurement Auctions. *Journal of Political Economy*, **101**, 518–538.

RENY, P.J. 2008. *On the Existence of Monotone Pure Strategy Equilibria in Bayesian Games.* mimeo.

RUST, J. 1987. Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher. *Econometrica*, **55**, 999–1033.

RUST, J. 1994. *Estimation of Dynamic Structural Models, Problems and Prospects, Part I: Discrete Decision Processes.* Advances in Econometrics: Sixth World Congress, Vol. 2,C. Sims and J. Laffont(eds.). Cambridge: Cambridge University Press. Pages 119–170.

RYAN, S. 2006. *The Costs of Environmental Regulation in a Concentrated Industry.* technical report, MIT.

SAMUELSON, W.F. 1985. Competitive Bidding with Entry Costs. *Economics Letters*, **17**, 53–57.

TAMER, E. 2003. Incomplete Simultaneous Discrete Response Model with Multiple Equilibria. *Review of Economic Studies*, **70**, 147–165.