
THE STRATEGIC USE OF DOWNLOAD LIMITS BY A MONOPOLY PLATFORM*

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ABSTRACT

We consider a heretofore unexplored explanation for why platforms, such as Internet service providers, might impose download limits on content consumers: doing so increases the degree to which those consumers view content providers' products as substitutes. This, in turn, intensifies the competition among providers, generating greater surplus for consumers. A platform, in turn, can capture this increased surplus by charging consumers higher access fees. Even accounting for congestion externalities, we show that a platform will tend to set the download limit at a lower level than would be welfare-maximizing; indeed, in some instances, so low that no download limit is welfare superior to the limit the platform would set. Somewhat paradoxically, we show that a platform will install more bandwidth when allowed to impose a download limit than when prevented from doing so. Other related phenomena are explored.

Keywords: two-sided markets, Internet, download limits (caps), congested platforms, network neutrality, price discrimination.

JEL Classification: L1, D4, L12, L13, C63, D42, D43.

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1 INTRODUCTION

Why do platforms, such as residential Internet service providers (ISPs) and mobile telephone companies, impose caps (limits) on how much content their customers can download each month? An immediate answer is that these caps may be part of a second-degree price discrimination scheme via quantity discounts. Another is that caps represent the platform's efforts at alleviating the congestion externality consumers impose on each other: reducing the externality raises consumer welfare, which the platform captures as higher access (hookup) fees. Although both answers are likely part of the story, in this paper we identify another effect that could motivate download caps: as customers become more limited in the amount they can download, the more they will see the digital products they acquire from different content providers as substitutes. This, in turn, will increase the competitive pressures on the content providers, who will respond by lowering their prices. Lower prices mean greater consumer surplus for customers, which the platform can capture via higher access fees. In essence, unlike a traditional price-discrimination analysis, in which a platform uses caps to appropriate surplus from end users, we show that a platform has an incentive to introduce caps to capture surplus from upstream providers.

The basic idea can be readily illustrated. Suppose there is a measure one of households (customers) and two content providers. A household's utility is

$$u = -H + \sum_{n=1}^2 (v - p_n) \chi_n,$$

where χ_n indicates whether a household purchases a unit of the n th content provider's product ($\chi = 1$) or not ($\chi = 0$), p_n is the n th content provider's price, and H is the hookup fee charged by the platform. Assume the content providers each have a constant marginal cost of 0 and no overhead or fixed costs. Assume the platform has no costs. If we assume the content providers set prices first, the platform sets H next, with households making their purchase decisions last, then it is readily seen that, in equilibrium, $p_1 = p_2 = v$ and $H = 0$.¹

Now consider an alternative game: for some reason (*e.g.*, a download restriction), each household is limited to one unit in total; that is, a household can buy from one content provider or the other, but not both. Otherwise the setting is as just described. The content providers are now effectively in Bertrand competition and the resulting equilibrium exhibits $p_1 = p_2 = 0$. Household surplus *gross* of the hookup fee is v , which means the platform can charge households a hookup fee of v .

Comparing the two games, the download restriction harms the content providers, which see their profits go from v each to zero; benefits the platform, which sees its profit go from 0 to v ; and reduces welfare (the sum of consumer surplus and platform and content providers' profits) from $2v$ to v . Because, in either scenario, the households' surplus is fully extracted, households are neither better nor worse off.

Although rudimentary, that example illustrates the basic tension we wish to explore: by restricting the aggregate amount that households can buy, the platform effectively induces greater competition among the content providers. This causes them

¹An arguably more realistic timing, which is equivalent in terms of the resulting equilibrium, is (i) the platform sets H ; (ii) households decide to connect or not; (iii) the content providers set prices; and (iv) the households decide what content to buy. There are some nuances, though, to this alternative timing, which we discuss later.

to lower their prices, which can raise household surplus gross of the hookup fee. The platform then captures this greater surplus via a higher hookup fee.

The reader's immediate response to this could be, "okay, it's a theoretical possibility, but is there any real-world evidence?" There is indeed some. For example, Reed Hastings, Netflix's CEO, arguing against the Canadian ISPs Rogers and Bell Canada's download caps, said, "It's an effective way to drive the bill up, that tends to be why caps are used."² Further, some reports suggest that ISPs are interested in imposing caps to enhance revenue because they are blocked from directly charging content providers (network neutrality).³ Moreover, evidence indicates ISPs might seek to impose caps even when congestion is not a significant problem.⁴

In the simple example above, the only reason for the platform to impose a download cap is rent extraction. As noted, an additional rationale for restrictions might exist if there is a congestion externality: by limiting total consumption, the platform could enhance welfare by reducing the congestion externality, even if the platform's motives are less than pure. Much of the analysis that follows—see, in particular, Sections 3–6—focuses on that issue. We find, in a static setting (fixed bandwidth), that, while it is possible that welfare is greater if the platform is free to impose a cap of its choosing than it would be absent any cap, there are many circumstances in which not imposing a cap is welfare superior to the overly tight cap the platform would choose.

In a dynamic setting (endogenous bandwidth), we find—somewhat paradoxically—that a platform's incentive to build bandwidth is *greater* when it can impose a download cap than when it cannot (see Proposition 4 in Section 5). Intuitively, the platform can extract more rent from content providers via a download cap the more content they sell in equilibrium. By expanding its bandwidth, the platform can expand the amount of content sold. Whether the benefits of greater bandwidth outweigh the static inefficiency caused by download caps is ambiguous, as we illustrate via an example.

We also explore alternatives to download caps, including allowing unlimited downloads during off-peak times (Section 6) and the use of two-part tariffs (a hookup fee plus a per-byte fee) by the platform (Section 7). We show that the platform does not have incentives to allow unlimited off-peak usage—consistent with actual practice (Proposition 6). This result further illustrates that it could well be rent extraction rather than congestion alleviation that motivates platforms' use of download caps. We find that two-part tariffs could yield greater profit than a download cap (Proposition 7), but the difference in profit between the two regimes tends to zero as the

²"Netflix says Internet download caps only in place to drive up bills," thespec.com, March 29, 2011 (accessed August 15, 2013 at URL <http://www.thespec.com/news-story/2202174-netflix-says-internet-download-caps-only-in-place-to-drive-up-bills/>). According to Van Gorp and Middleton (2010), "Canada is only one of four countries in the OECD where download caps were imposed on all the service plans studied by the OECD." That article also notes that Canada tends to have high prices for residential broadband Internet relative to other OECD countries, consistent with our model.

³See, *e.g.*, "If Net Neutrality Is Coming, So Is The End Of All-You-Can-Eat Internet Access," Dan Frommer, *Business Insider*, December 1, 2010 (accessed online August 15, 2013 at URL <http://www.businessinsider.com/if-net-neutrality-is-coming-so-is-the-end-of-all-you-can-eat-internet-access-2010-12>).

⁴See, *e.g.*, "AT&T puts broadband users on monthly allowance," Ryan Singel, *Wired*, March 15, 2011 (accessed online August 15, 2013 at URL <http://www.cnn.com/2011/TECH/web/03/15/att.broadband.allowance.wired/index.html>). A relevant quote from this article is "There's [*sic*] little data to demonstrate whether large ISPs actually are experiencing real issues with congestion."

number of content providers expands. Hence, a download cap may be a good substitute for a two-part tariff and might even be preferred by the platform given the non-trivial administrative costs likely associated with a two-part tariff.⁵

We also note, building on the well-known result that the statutory incidence of an excise tax is irrelevant to its actual incidence, that the analysis in Section 7 would apply if the content providers were the ones charged the per-byte fee. Hence, the analysis in that section speaks to the issue of whether download caps are a substitute for allowing the platform to charge the content providers directly for content delivery to consumers. Under current US policy—a regime broadly known as network neutrality—residential Internet service providers (ISPs) cannot charge content providers for so-called “last-mile” delivery of their content to households. The analysis in Section 7 suggests that US ISPs’ interests in imposing download caps could be a response to network neutrality, as download caps serve as a reasonable—and, in the limit, perfect—alternative to directly charging the content providers.⁶

Most of the paper assumes homogeneity across consumers (households) and content providers. In large part, those assumptions are necessary to have tractable models with which to explore many of the issues of interest. In Sections 8 and 9, we use simplified versions of our base model to explore issues that arise with heterogeneity, such as price discrimination across households and the mix of active content providers. In Section 8, we show that a platform would never offer a plan with unlimited downloads; that is, a desire to discriminate across households need not lead a platform to offer unlimited downloads to any household.⁷ In Section 9, we show that welfare can be greater, despite congestion externalities, with no cap than with the cap imposed by the platform even if the content providers are heterogeneous. That the platform would fail to set the welfare-maximizing cap arises for two reasons with heterogeneous content providers: one, the platform is more concerned about marginal effects than infra-marginal effects, so distortions arise (a result in the spirit of Spence, 1975, and others); and, two, the platform is seeking to extract rents from the content providers, as well as capturing consumer surplus. Hence, while the first reason makes it ambiguous as to whether the platform would set too liberal or too stringent a cap *vis-à-vis* the welfare optimum, the second reason leads it to set too stringent a cap.

This paper is part of the emerging literature on two-sided markets (see *e.g.*, Roson, 2005, Rochet and Tirole, 2006, and Rysman, 2009, for surveys). In particular, it is part of the literature on monopoly platform practices, especially those of Internet service providers (ISPs).⁸ In terms of the model employed, we build most directly on Hermalin and Katz (2007) and Economides and Hermalin (2012), although neither considers

⁵It could also be that, because consumers don’t have a good sense of how many bytes various downloads represent, there would be consumer resistance to per-byte charges. That noted, there are platforms that utilize such tariffs: the UK mobile telephone provider Three offers data plans under which consumers pay 1p per megabyte.

⁶It is possible that a platform might wish to do both: charge content providers directly and impose download caps. Given length considerations, we do not explore that extension in this paper.

⁷In this regard, we note that the Canadian ISP Rogers, which offers many plans, only offers plans with download caps. Source: <http://www.rogers.com> (accessed on September 17, 2013).

⁸A partial list of papers on this topic includes Hermalin and Katz (2007), Choi and Kim (2010), Krämer and Wiewiorra (2010), Cheng et al. (2011), Economides and Tåg (2012), Economides and Hermalin (2012), and Choi et al. (2013). None of these papers, however, consider download caps.

download caps. As discussed in the next section, tractability requires us to make stronger functional-form assumptions than either Hermalin and Katz or Economides and Hermalin. In two related articles, Dai and Jordan (2013a,b) consider download caps in the context of price discrimination by an ISP in the residential market. Unlike us, they don't model price setting by the content providers and do not consider the effect download caps have on that pricing. Beyond the Dai and Jordan articles, the academic literature on download caps appears limited and focused on legal and regulatory issues (see, *e.g.*, Van Gorp and Middleton, 2010).

2 BASELINE MODEL

Households want to engage with some or all of N content (or application) providers. To reach households, the providers' content must pass through a "pipe" controlled by a monopoly platform, herein called the ISP for concreteness. The pipe has a capacity (bandwidth) of B ; that is, B units (*e.g.*, bytes) can go from the content providers to the households per unit of time.⁹

The assumed sequence of play is that the ISP moves first, announcing its policies and prices. Next, households decide whether to purchase access. The content providers then announce their prices. Finally, households decide how much content to purchase from each content provider.

This game admits multiple equilibria. One is a degenerate equilibrium in which households expect the content providers to set such exorbitant prices that they are deterred from purchasing access. Because households don't then acquire access and there are, thus, no households to which to sell, it is a weak best response for the content providers to indeed set exorbitant prices. In what follows, we ignore that equilibrium and focus instead on the equilibrium in which households acquire access. Households anticipate the prices that will emerge in the subgame that follows if they acquire access. Provided they can attain non-negative surplus, they will acquire access. Because the ISP can make money only if the households acquire access, it will set its access fee so that household surplus will be non-negative.¹⁰

We assume that content providers are limited to linear pricing. Denote content provider n 's price by p_n . We further assume the content providers are not in direct competition and that each has a marginal cost of 0.

Initially, we limit the ISP to charging households a hookup fee, H . In Section 7, we allow the ISP to also charge a per-unit (*e.g.*, per byte or packet) price. Consistent with actual practice, we rule out the ISP's setting access charges that are contingent on the prices announced by the content providers.

There is a measure one of households. Each household has quasi-linear utility:

$$U = \sum_{n=1}^N \int_0^{x_n} \mu(xL(X|B))dx + y, \quad (1)$$

where x_n is the amount of content acquired from the n th content provider, y is the numéraire good, and $L(X|B)$ reflects the congestion loss that arises when X total

⁹There are some nuanced issues concerning time, which we address later.

¹⁰An alternative timing, which would yield similar results, is the ISP sets its policies, but not its access price; the content providers announce their prices; the ISP responds by announcing its access price; and, finally, consumers simultaneously decide whether to acquire access and what content to purchase.

content is transmitted. The function $\mu(\cdot)$ gives the *marginal* contribution to utility of a unit with “quality” $L(X|B)$. We assume diminishing marginal utility; that is, $z > z'$ implies $\mu(z) < \mu(z')$. The utility function in (1) is similar to the ones used by Economides and Hermalin (2012) and Hermalin and Katz (2007). We assume that content consumption by a household is never so great as to consume all income; that is, if I is household income, we assume

$$0 < y = I - \sum_{n=1}^N p_n x_n \tag{2}$$

always holds.

For low levels of total platform usage, it is possible that congestion is irrelevant, in which case we set $L(X|B) = 1$. But we assume there is an $\underline{X} \in \mathbb{R}_+$ (possibly zero), such that (i) $L(X|B) = 1$ for all $X \leq \underline{X}$; (ii) $L(X|B) > 1$ for all $X > \underline{X}$; and (iii) $L(\cdot|B)$ is an increasing function on (\underline{X}, ∞) . Assume that $L(\cdot|B)$ is everywhere continuous and, except possibly at \underline{X} , everywhere differentiable.

Because each household is negligible, it rationally does not take into account its consumption decisions on congestion. Although one could imagine that content providers recognize their effects on congestion, it seems more plausible that they do not and we limit attention to that case. We note that both assumptions enhance the possibility of our finding that the ISP’s imposition of download restrictions is welfare improving. Hence, conclusions in the analysis that follows that such restrictions are not welfare improving can be viewed as fairly robust.

2.1 HOUSEHOLD DECISION MAKING

Assuming a household has decided to connect to the platform, it then chooses the amount of content to purchase—the bundle (x_1, \dots, x_N) —so as to maximize its utility, expression (1). It may possibly be subject to a total download restriction:

$$\sum_{n=1}^N x_n \leq \bar{x}, \tag{3}$$

where \bar{x} is the cap on total downloads. Substituting for y according to (2), the first-order condition for a household’s maximization program is

$$\mu(x_n L(X|B)) - p_n - \lambda = 0, \tag{4}$$

where λ is the Lagrange multiplier on (3).

Tractability, unfortunately, requires functional-form restrictions: assume the marginal-utility function is linear:

$$\mu(z) = \alpha - z. \tag{5}$$

Setting the slope to -1 is a convenient normalization and imposes no further loss of generality. Given (5) the solution to the household’s maximization program is

$$x_n = \frac{\alpha - p_n}{L(X|B)} \tag{6}$$

if there is no download limit (or it doesn’t bind); and

$$x_n = \frac{\bar{x}}{N} + \frac{\sum_{j \neq n} p_j - (N-1)p_n}{NL(X|B)} \tag{7}$$

if there is a binding download limit.¹¹ Notice that the imposition of a download cap effectively turns the previously independent goods into substitute goods (at least from the perspective of the content providers' strategies).

3 BENCHMARK: NO CONGESTION

As a benchmark, we consider the case in which there is no congestion externality; that is, $L(X|B) \equiv 1$.

If there is no download limit, then, from (6), the profit-maximizing price for the content providers is $p_n = \alpha/2$. A household's consumer surplus from its consumption of a given content provider's content is

$$\int_{\alpha/2}^{\alpha} (\alpha - p) dp = \frac{\alpha^2}{8}.$$

Consequently, the ISP will charge a hookup fee of

$$H_{\text{NOCAP}} = \frac{N\alpha^2}{8}.$$

If there is a binding download limit, then, from (7), each content provider sets its price to maximize

$$p_n \left(\frac{\bar{x}}{N} + \frac{\sum_{j \neq n} p_j - (N-1)p_n}{N} \right).$$

The corresponding first-order condition is equivalent to

$$\bar{x} + \sum_{j \neq n} p_j - 2(N-1)p_n = 0,$$

which yields the best-response function

$$p_n = \frac{\bar{x} + \sum_{j \neq n} p_j}{2(N-1)}.$$

The unique Nash equilibrium is

$$p_1 = \dots = p_N = \frac{\bar{x}}{N-1}.$$

Hence, from (7), each household consumes \bar{x}/N from each content provider in equilibrium. Its consumer surplus from its consumption of a given content provider's content is, consequently,

$$\int_0^{\bar{x}/N} (\alpha - x) dx - \underbrace{\frac{\bar{x}}{N-1}}_{p_n} \frac{\bar{x}}{N} = \bar{x} \frac{\alpha}{N} - \frac{(3N-1)\bar{x}^2}{2(N-1)N^2}.$$

¹¹To be precise, the formulæ in (6) and (7) are valid only when non-negative. For our purposes, with only one household type, we do not need to make explicit reference to that fact. If there were multiple household types, the non-negativity condition could require explicit attention. However, extending this version of the model to multiple household types proves intractable. For the more limited model we use to explore heterogeneous households in Section 8, this issue also does not arise.

It follows that the ISP will charge a hookup fee of

$$H_{\text{CAP}} = \bar{x}\alpha - \frac{(3N - 1)\bar{x}^2}{2(N - 1)N}. \quad (8)$$

To maximize its profit, the ISP will set the download limit to maximize (8); hence,

$$\bar{x} = \frac{\alpha(N - 1)N}{3N - 1}. \quad (9)$$

Note this solution is relevant only if the \bar{x} given by (9) is less than $N\alpha/2$, which is the total content downloaded absent a limit (or if the limit does not bind). Because the \bar{x} given by (9) is always less than $N\alpha/3$, it follows the limit binds. Expressions (8) and (9) yield the equilibrium access (hookup) fee:

$$H_{\text{CAP}}^* = \frac{N\alpha^2}{8} \times \frac{4(N - 1)}{3N - 1}. \quad (10)$$

We can conclude:

Proposition 1. *Given the functional forms assumed, the ISP strictly prefers to impose a download limit if there are four or more content providers; is indifferent between a limit and no limit if there are three content providers; and prefers no limit if there are two or fewer content providers.*

Intuition for Proposition 1 can be gained from Figure 1. The download limit effectively induces competition among the content providers. Hence, their prices fall, which will increase household's consumer surplus *ceteris paribus* (the dark rectangle in the figure). At the same time, though, the content purchased from each content provider falls, which reduces household surplus *ceteris paribus* (the light triangle). However, as the figure suggests—and the algebra confirms—the increase due to competition can outweigh the loss due to reduced consumption. Not surprisingly, the competition effect is greater the more content providers there are, which helps explain why the ISP finds a download limit profitable when there are many content providers, but not when there are only a few.

Because the content acquired from each content provider is reduced and there are no congestion externalities, the download limit must reduce welfare, as illustrated in Figure 1.

4 CONGESTION EXTERNALITIES

If there is no download limit, then, from (6), the profit-maximizing price is again $p_n = \alpha/2$. There is an adding-up constraint:

$$X = \sum_{n=1}^N x_n = \sum_{n=1}^N \left(\frac{\alpha - \overbrace{\alpha/2}^{p_n}}{L(X|B)} \right) = \frac{N\alpha}{2L(X|B)}. \quad (11)$$

Because (i) the leftmost side of (11) is zero at $X = 0$, while the rightmost side is positive; (ii) the leftmost side increases without bound in X while the rightmost side is non-increasing; and (iii) $L(\cdot|B)$ is continuous, it follows that a unique solution to (11) exists. Call it $X(B)$.

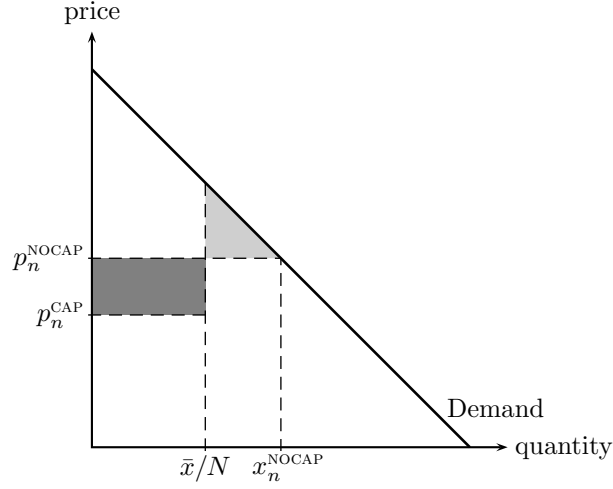


Figure 1: Rationale for download limits: surplus extraction from a single content provider n . By effectively inducing the content providers to compete, household surplus increases by the area of the dark rectangle less the areas of the light triangle.

Each content provider sells $X(B)/N$ in content, so household surplus per content provider is

$$\begin{aligned} \int_0^{X(B)/N} (\alpha - xL(X(B)|B)) dx - \frac{\alpha}{2} \frac{X(B)}{N} \\ = \frac{X(B) (\alpha N - X(B)L(X(B)|B))}{2N^2} = \frac{\alpha X(B)}{4N}, \end{aligned}$$

where the last equality follows from (11). It follows the hookup fee charged by the ISP is

$$H_{\text{NOCAP}} = \frac{\alpha X(B)}{4}.$$

Total surplus (welfare) is

$$\frac{3\alpha X(B)}{4}.$$

If there is a binding download limit, then, from (7), each content provider sets its price to maximize

$$p_n \left(\frac{\bar{x}}{N} + \frac{\sum_{j \neq n} p_j - (N-1)p_n}{NL(\bar{x}|B)} \right).$$

The corresponding first-order condition is equivalent to

$$\bar{x}L(\bar{x}|B) + \sum_{j \neq n} p_j - 2(N-1)p_n = 0,$$

which yields the best-response function

$$p_n = \frac{\bar{x}L(\bar{x}|B) + \sum_{j \neq n} p_j}{2(N-1)}.$$

The unique Nash equilibrium is

$$p_1 = \dots = p_N = \frac{\bar{x}L(\bar{x}|B)}{N-1}. \quad (12)$$

Expression (12) might, at first, seem counterintuitive: equilibrium price is *increasing* in the distaste for congestion *ceteris paribus*. This result can, however, be understood by considering (7). The greater distaste, the less sensitive household demand is to relative prices *ceteris paribus*; hence, the lower are the competitive pressures arising from the download limit and, hence, the greater the price the content providers feel able to charge.

From (7), each household consumes \bar{x}/N from each content provider in equilibrium. Its consumer surplus from its consumption of a given content provider's content is, therefore,

$$\int_0^{\bar{x}/N} (\alpha - xL(\bar{x}|B)) dx - \frac{\bar{x}L(\bar{x}|B)}{N-1} \frac{\bar{x}}{N} = \bar{x} \frac{\alpha}{N} - \frac{(3N-1)L(\bar{x}|B)\bar{x}^2}{2(N-1)N^2}.$$

Consequently, the ISP will charge a hookup fee of

$$H_{\text{CAP}} = \bar{x}\alpha - \frac{(3N-1)}{2(N-1)N} L(\bar{x}|B)\bar{x}^2. \quad (13)$$

Accounting for the content providers' profits, welfare is

$$W_{\text{CAP}} = \bar{x}\alpha - \left(\frac{(3N-1)}{2(N-1)N} - \frac{2N}{2(N-1)N} \right) L(\bar{x}|B)\bar{x}^2. \quad (14)$$

Observe that H_{CAP} and W_{CAP} have a common "revenue-like" term, but the former has a "cost-like" term that has a higher margin. Hence, by the usual comparative statics, it must be that the \bar{x} the ISP would set to maximize its profit is *lower* than the cap that would maximize welfare. To summarize:

Proposition 2. *Assuming the households' marginal utility functions are given by (5), a profit-maximizing ISP will set a household download limit (cap) that is lower than the limit that would maximize welfare.*

Although too stringent *vis-à-vis* the cap that would be welfare maximizing, is the cap chosen by the ISP welfare superior to no cap at all? In general, the answer is complicated because of the number of forces at work:

1. because neither content providers nor households take into account the congestion externality, there will be a tendency to transmit too much content *ceteris paribus*;
2. but, because the content providers exercise market power, too little content would get traded if there were no congestion externality; and,
3. as noted, the ISP has incentives to set less than the welfare-maximizing cap.

The first two points indicate the theory of the second best is at work absent any download caps: the reduced trade due to the exercise of market power partially offsets the congestion externality (or, conversely, because of the congestion externality, the welfare loss from the exercise of market power is reduced).

To study the question, suppose that the loss-from-congestion function is

$$L(X|B) = \Lambda \left(\frac{X}{B} \right)^\theta, \quad (15)$$

where $\Lambda > 0$ and $\theta \geq 0$ are constants. The analysis of Section 3 corresponds to $\Lambda = 1$ and $\theta = 0$.

Given the functional form assumed in (15), the solution to (11) is

$$X = \left(\frac{N\alpha B^\theta}{2\Lambda} \right)^{\frac{1}{\theta+1}},$$

which implies that, absent a cap,

$$x_1 = \dots = x_N = \left(\frac{\alpha B^\theta}{2\Lambda N^\theta} \right)^{\frac{1}{\theta+1}}, \quad H_{\text{NOCAP}} = \left(\frac{N\alpha^{\theta+2} B^\theta}{2^{2\theta+3}\Lambda} \right)^{\frac{1}{\theta+1}},$$

and $W_{\text{NOCAP}} = 3 \left(\frac{N\alpha^{\theta+2} B^\theta}{2^{2\theta+3}\Lambda} \right)^{\frac{1}{\theta+1}}. \quad (16)$

Suppose there is a binding cap. Maximizing the ISP's profit, expression (13), with respect to \bar{x} yields

$$\bar{x} = \left(\frac{2\alpha(N-1)NB^\theta}{\Lambda(3N-1)(\theta+2)} \right)^{\frac{1}{\theta+1}}.$$

Substituting yields:

$$H_{\text{CAP}} = \frac{\theta+1}{\theta+2} \left(\frac{2N\alpha^{\theta+2} B^\theta (N-1)}{\Lambda(3N-1)(\theta+2)} \right)^{\frac{1}{\theta+1}} = H_{\text{NOCAP}} \frac{\theta+1}{\theta+2} \left(\frac{2^{2\theta+4}(N-1)}{(3N-1)(\theta+2)} \right)^{\frac{1}{\theta+1}}. \quad (17)$$

From (17), it can be seen that $H_{\text{CAP}} > H_{\text{NOCAP}}$ for all $N > 3$ regardless of θ . If $\theta > 0$, then that inequality holds for all $N \geq 3$. If $\theta \geq .295$, then that inequality holds for all $N \geq 2$. Substituting \bar{x} into the statement for welfare:

$$\begin{aligned} W_{\text{CAP}} &= \left(\frac{2\alpha^{\theta+2}(N-1)NB^\theta}{\Lambda(3N-1)(\theta+2)} \right)^{\frac{1}{\theta+1}} \left(\frac{(3N-1)(\theta+2) - (N-1)}{(3N-1)(\theta+2)} \right) \\ &= W_{\text{NOCAP}} \frac{1}{3} \left(\frac{2^{2\theta+4}(N-1)}{(3N-1)(\theta+2)} \right)^{\frac{1}{\theta+1}} \left(\frac{(3N-1)(\theta+2) - (N-1)}{(3N-1)(\theta+2)} \right). \quad (18) \end{aligned}$$

There are values for θ and N such that $W_{\text{CAP}} > W_{\text{NOCAP}}$ and such that $W_{\text{CAP}} < W_{\text{NOCAP}}$ (an example of the former are $\theta = 7$ and $N = 6$, an example of the latter would be $\theta = 6$ and $N = 6$). In other words, allowing the ISP to impose a download cap of its choosing can be welfare superior to prohibiting it from imposing any cap at all; but it can also be welfare inferior—the answer depends on the parameters.

To investigate the parameters more systematically, observe, from (18), that the ratio $W_{\text{CAP}}/W_{\text{NOCAP}}$ is increasing in N . In the limit, as $N \rightarrow \infty$, that ratio is

$$\frac{3^{-\frac{3+2\theta}{1+\theta}} \left(\frac{4^{2+\theta}}{2+\theta} \right)^{\frac{1}{1+\theta}} (5+3\theta)}{2+\theta}.$$

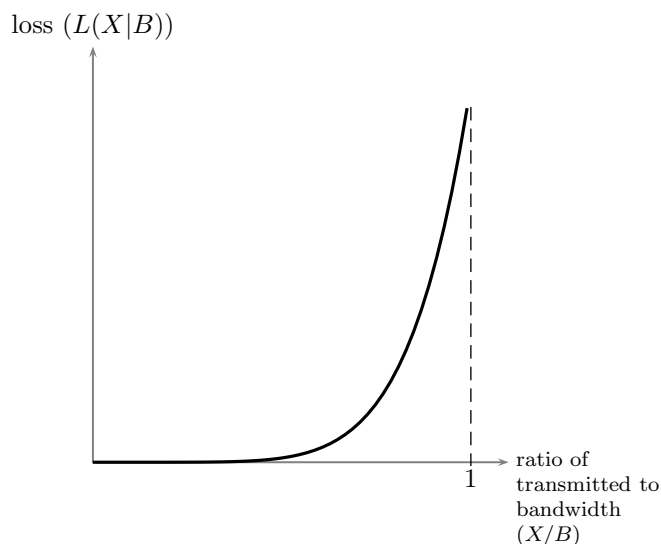


Figure 2: A highly convex disutility/loss of congestion function.

If $\theta < 6.4687$ (approximately), then that limit is less than one: welfare is greater if the ISP is barred from imposing a download cap of its choosing. If $\theta > 6.4687$, then the limit exceeds one and allowing the ISP to set a cap of its choosing is welfare superior to no cap. This yields the following:

Proposition 3. *Assume bandwidth is fixed. Then, given the functional forms assumed, prohibiting the ISP from imposing a download restriction of its choosing is welfare superior to allowing it to impose such a restriction unless the disutility/loss from congestion function is highly convex (specifically, unless $\theta \geq 6.4687$).*

Figure 2 plots $z^{6.4687}$, which illustrates how convex it is. It is plausible that such convexity is consistent with actual preferences: presumably marginal disutility/loss is very small at low levels of congestion: going from no freezes in an on-demand video to the occasional freeze due to a small increase in congestion is presumably less costly to people than going from the occasional freeze to constant freezing, as might incur with an increase in congestion at a higher level of congestion. In a sense, Figure 2 can be seen as approximating a backward-L-shaped curve—a “breaking-point” model in which congestion is acceptable to a certain point, but thereafter almost wholly unacceptable.

Once $\theta > 6.4687$, the consequences for welfare from allowing the ISP to impose a download restriction of its choosing depend on the number of content providers. In particular, the greater is θ , the lower is the number of content providers necessary to make welfare greater with the cap than without.

5 BANDWIDTH DECISIONS

The analysis to this point has treated the bandwidth, B , as fixed. It is possible, over some time horizons at least, that the ISP can change the bandwidth. We briefly consider

the consequences of endogenizing the bandwidth in this section.¹² Our analysis maintains the previously made functional-form assumptions. We assume $\theta > 0$ and $N \geq 3$, so that $H_{\text{CAP}} > H_{\text{NOCAP}}$. From expression (17), this entails $\partial H_{\text{CAP}}/\partial B > \partial H_{\text{NOCAP}}/\partial B$ if the latter partial derivative is positive. That the latter is indeed positive is immediate from expression (16). This establishes the following:

Proposition 4. *Assume (i) the ISP's cost of installing bandwidth is everywhere differentiable; and (ii) that, if barred from imposing a download cap or restriction, the ISP's choice of bandwidth is an interior solution to the problem of maximizing profit with respect to bandwidth. Then, given the functional forms assumed, an ISP able to choose a download restriction will install more bandwidth than one barred from imposing a cap.*

Proposition 4 may, at first, seem counter-intuitive: one might have expected download restrictions to be a substitute for expanding bandwidth, as both address the congestion externality. Although true, there are additional effects. First, the expanded bandwidth is used. This is the recongestion effect identified by Economides and Hermalin (2012) and familiar to anyone who has seen physical highway expansion fail to end bumper-to-bumper traffic. But greater use increases the potential surplus the ISP can capture from inducing competition among the content providers. Moreover, as discussed in conjunction with expression (12), induced competition is fiercer the lower is $L(\bar{x}|B)$ *ceteris paribus*. When these other effects are taken into account, it follows that the ISP's investment incentives are greater when it can impose a download cap than when it cannot.

Whether the welfare benefits of greater bandwidth outweigh the potential welfare loss from too tight a download cap is, in general, ambiguous. As an example, suppose that the ISP's cost of installing B bandwidth is kB , k a positive constant. Continue to suppose that $L(X|B)$ is defined as in (15). Let $\Lambda = \theta = 1$. Note, because $\theta < 6.4687$, static welfare (*i.e.*, for a fixed B) is greater if the ISP is barred from imposing a download limit. In contrast, in a dynamic setting (*i.e.*, when B is endogenous), welfare can be greater if the ISP is allowed to set a limit. Specifically, it can be shown that

$$B_{\text{NOCAP}} = \frac{N\alpha^3}{128k^2} \text{ and } B_{\text{CAP}} = \frac{2(N-1)N\alpha^3}{27(3N-1)k^2} = \frac{256}{27} \frac{N-1}{3N-1} B_{\text{NOCAP}}.$$

Welfare, including the cost of installing the bandwidth, is

$$W_{\text{NOCAP}} = \frac{5}{128} \frac{N\alpha^3}{k} \text{ and } W_{\text{CAP}} = \frac{2(5N-1)(N-1)N\alpha^3}{27(3N-1)^2k} = \frac{256}{135} \frac{(5N-1)(N-1)}{(3N-1)^2} W_{\text{NOCAP}}.$$

It follows that if $N < 11$, then welfare is greater if the ISP is barred from imposing a download cap, but if $N > 11$, then welfare is greater if the ISP is permitted to impose a cap of its choosing (if $N = 11$, welfare is the same in the two regimes.)

6 RESTRICTIONS BY TIME OF DAY

To the best of our knowledge, ISPs and similar platforms that impose download caps or limits (*e.g.*, cellular networks with data plans for smartphones) do so on a monthly

¹²Other articles that have explored ISPs incentives to expand bandwidth include Choi and Kim (2010), Krämer and Wiewiorra (2010), Cheng et al. (2011), and Economides and Hermalin (2012). None of these, however, investigate the relation between bandwidth and download caps.

basis. This suggests that the unit of time in our model is a month. On the other hand, network congestion is generally not a constant throughout the month: there are peak and off-peak hours. For example, watching on-demand video might be something done in the evening rather than during the day (when members of the household are at work). One could, nonetheless, justify our model by imagining the households make monthly usage decisions and the loss function L reflects some average congestion disutility that they expect given the usage pattern they've chosen.

Alternatively, we could imagine that the households have to consume some content at peak hours (*e.g.*, intensive residential Internet usage can occur only in the evenings after work). That is, off-peak usage is necessarily limited and minor (*e.g.*, a quick check of email before heading to work in the morning); hence, peak hours are effectively all that matter.

On the other hand, this discussion calls into question why ISPs and similar platforms impose a monthly limit rather than a limit that applies during peak hours only (*e.g.*, a plan with unlimited downloads in the wee hours of the morning, but with limits during evenings or other congested periods). Although modeling time-of-day consumption complicates an already complex model, we are able to examine that possibility, to an extent, in this section. We find that an ISP has higher profits with an "all-the-time" (*e.g.*, monthly) cap than with a time-of-day cap.

As a somewhat crude analysis of such time-of-day issues and their implications, consider a model in which there are two periods per day. One is a high-usage period (h) and the other a low-usage period (ℓ). Let the gross utility a household gains from x_n units of the n th content provider's product be

$$\int_0^{x_n} \left(t(x)(\alpha - xL(X_h|B)) + (1 - t(x))(\alpha - \eta x) \right) dx, \quad (19)$$

where X_h is total consumption in the h period; $t(x) \in \{0, 1\}$ reflects the time of day the x th unit is consumed, with $t(x) = 1$ corresponding to the h period and $t(x) = 0$ corresponding to the ℓ period; and where $\eta > 1$ reflects the reduced benefit from consuming during the less-preferred period (*e.g.*, during the day or late at night).¹³ Although a more general model would allow for the possibility that congestion is also a problem during the ℓ period, we assume it is not for the sake of tractability.

We assume that the content providers cannot engage in time-of-day pricing; that is, each content provider's price is the same in both the ℓ and h periods.

A household chooses $t(x)$ to maximize its utility. Absent any download cap, it follows from (19) that a household's timing decision satisfies

$$t(x) = \begin{cases} 1, & \text{if } L(X_h|B) < \eta \\ 0, & \text{if } L(X_h|B) > \eta \end{cases}$$

for all $x \in [0, x_n]$. Hence, a household is willing to consume during *both* periods only if $L(X_h|B) = \eta$.

Only equilibria in which there is h -period consumption are of interest. Let X again denote total consumption over both periods and let X_h^e denote equilibrium consumption during the h period. We restrict attention to equilibria in which

$$X_h^e = \begin{cases} X, & \text{if } L(X|B) \leq \eta \\ L^{-1}(\eta|B), & \text{if } L(X|B) > \eta \end{cases}.$$

¹³Or it could reflect the nuisance of having to remember, for example, to download a video at a low-usage time, even if it will be watched at a high-usage time.

That is, either all consumption is during the high-usage period because, even given the loss from congestion, households find consumption in the low-usage period too distasteful; or consumption occurs in the high-usage period until the point that the loss from congestion just equals the distaste for off-peak consumption, with all consumption beyond that being in the low-usage period.

Absent a download cap, a content provider's demand is proportional to $\alpha - p_n$ regardless of the timing of household purchase decisions. Hence, its price is $\alpha/2$. Recall $X(B)$, which is the solution to (11). If $L(X(B)|B) > \eta$, then the imposition of a download limit during the h period has no effect on content providers' prices:

Proposition 5. *Under the assumptions of this section, if households consume positive amounts in the low-usage (ℓ) period absent download restrictions, then the content providers' equilibrium prices remain unaffected if a download restriction is imposed for the high-usage (h) period.*

Proof: By supposition, marginal consumption is ℓ -period consumption. So, if x_n is total consumption of the n th provider's content, then x_n maximizes

$$\int_0^{x_n^h} (\alpha - xL(X_h^e|B))dx + \int_{x_n^h}^{x_n} (\alpha - \eta x)dx - p_n x_n,$$

where x_n^h is h -period consumption of the n th provider's content. Imposition of peak-time download limits could affect X_h^e and x_n^h , but would not affect the optimal x_n . It follows that marginal demand is

$$\frac{1}{\eta}(\alpha - p_n).$$

So the n th content provider's pricing problem is unaffected and it will thus continue to choose $p_n = \alpha/2$. ■

Intuitively, the content providers don't care when their content is consumed. Hence, measures by the ISP that shift the timing of consumption, but don't affect the total amount consumed, can have no bearing on the content providers' pricing.

Suppose, absent a cap, there is no consumption during the ℓ period. For the moment, suppose that households cannot move consumption to the ℓ period following a cap. The analysis of the previous sections would then apply. In particular, consider the prices given by (12) and denote them as \hat{p}_n , $n = 1, \dots, N$: if

$$\hat{p}_n \geq \alpha - \eta \frac{\bar{x}}{N},$$

then the equilibrium is unchanged if households were now allowed to buy in the ℓ period. The reason is as follows: those equilibrium prices satisfy $\hat{p}_n < \alpha/2$. Hence, on the margin, a content provider wants to increase its price above \hat{p}_n if demand is proportional to $\alpha - p$. It cannot, therefore, benefit a content provider to induce consumption in the ℓ period by lowering its price further. Similar reasoning applies if

$$\hat{p}_n < \alpha - \eta \frac{\bar{x}}{N}. \tag{20}$$

Because $\hat{p}_n < \alpha/2$, the content provider wants to raise its price—the benefit from getting a higher price on ℓ -period sales outweighs the loss from fewer sales. Hence, the equilibrium prices when the content providers can sell in the ℓ period and (20) holds will be greater than the prices given by (12).

Putting all this analysis together, we have

Proposition 6. *Under the assumptions of this section, a time-of-day download limit during the high-usage (h) period never induces a greater competition effect among the content providers than an all-the-time download limit, but can induce less of a competition effect.*

Because the ISP makes greater profit by inducing more intense competition among the content providers, Proposition 6 offers a possible explanation for why ISPs and mobile networks do not relax download restrictions for content accessed off peak.

7 ISP UTILIZES A TWO-PART TARIFF

Suppose that rather than impose a download cap, the ISP could utilize a two-part tariff in which a household pays $H + \tau x$ if it downloads x total content.

By now familiar reasoning, each content provider would set a price of $(\alpha - \tau)/2$. Total content downloaded would need to satisfy the analog of (11):

$$X = \sum_{n=1}^N \left(\frac{\alpha - \tau - \frac{1}{2}(\alpha - \tau)}{L(X|B)} \right) = \frac{N(\alpha - \tau)}{2L(X|B)}. \quad (21)$$

If L is given by (15), then the total amount downloaded is

$$X = \left(\frac{N(\alpha - \tau)B^\theta}{2\Lambda} \right)^{\frac{1}{\theta+1}}. \quad (22)$$

Household surplus from consuming the n th provider's content is, thus,

$$\int_0^{X/N} (\alpha - xL(X|B))dx - \left(\frac{\alpha - \tau}{2} + \tau \right) \frac{X}{N} = \frac{X(\alpha - \tau)}{4N},$$

where the equality follows, in part, from (21). The hookup fee is, therefore,

$$H = \frac{X(\alpha - \tau)}{4}.$$

The ISP's profit is thus

$$\frac{X(\alpha - \tau)}{4} + \tau X = \frac{1}{4}X(\alpha + 3\tau) = \frac{1}{4} \left(\frac{N(\alpha - \tau)B^\theta}{2\Lambda} \right)^{\frac{1}{\theta+1}} (\alpha + 3\tau), \quad (23)$$

where the second equality follows from (22). Maximizing with respect to τ yields

$$\tau = \frac{\alpha(2 + 3\theta)}{3(2 + \theta)}.$$

Substituting that back into (23) reveals the ISP's profit to be

$$\frac{\theta + 1}{\theta + 2} \left(\frac{2N\alpha^{\theta+2}B^\theta}{3\Lambda(\theta + 2)} \right)^{\frac{1}{\theta+1}}. \quad (24)$$

Comparing (24) with H_{CAP} given in (17), it follows, because

$$\frac{1}{3} > \frac{N-1}{3N-1}, \quad (25)$$

that the ISP's profit is greater when it can employ a two-part tariff than when it must rely on a download cap. On the other hand, the limit of the righthand side of (25) as $N \rightarrow \infty$ is $1/3$; hence, if there are a large number of content providers, the ISP's loss from utilizing a download cap rather than a two-part tariff is minor. If there are significant transaction costs in administering a two-part tariff, then the ISP could prefer a download cap. To summarize:

Proposition 7. *Given the functional forms assumed, the ISP makes greater profit with a two-part tariff and no download cap than utilizing just a download cap and a hookup fee. However, as the number of content providers gets large, this difference shrinks; in the limit, profits are the same under the two regimes. Hence, if there are significant transaction fees associated with administering a two-part tariff, then the ISP would prefer a download cap and a hookup fee if the number of content providers is large enough.*

As we observed in the Introduction, the analysis of this section applies equally well if the content providers were the ones charged τ , given that τ can be viewed as an excise tax paid to the ISP and, as is well known, the statutory incidence of such a tax is irrelevant to its actual incidence (effect). In many places—the US in particular—residential ISPs cannot currently charge content providers for delivering content to their residential customers (this is part of a broad set of policies and customs commonly known as network neutrality).¹⁴ The analysis of this section thus suggests that an ISP's motive to impose a download cap will be greater in a network-neutrality regime than in a regime in which it could directly charge the content providers.

Corollary 1. *Given the functional forms assumed, the ISP would earn greater profit if it could directly charge content providers for delivering their content (with no download caps) than utilizing a download cap. However, as the number of content providers gets large, the difference in profits between the regimes shrinks, with profits being the same in the limit.*

8 HETEROGENOUS HOUSEHOLDS AND PRICE DISCRIMINATION

Next, we explore the relation between download caps and second-degree price discrimination by the ISP. Price discrimination necessarily entails different household types, which raises the issue of what constitutes a “type” in this setting: is it different benefits from all content or only from some content?

An additional issue is that, when there are different household types and the ISP discriminates by offering different packages with varying access prices and download limits, the pricing subgame among the content providers will often have solutions in mixed-strategies only. The intuition for why is that, depending on the parameter values, competition among the content providers when seeking to cater to all types could be fierce. Consequently, a content provider could be tempted to “drop out” of the competition to sell to all types and simply seek to sell to a subset, but at a greater price. But it cannot be an equilibrium for all content providers to do so: by shaving its price, a content provider could pick up a considerable share of the remaining

¹⁴There is a small literature that analyzes the pros and cons of network neutrality in terms of economic welfare: see, *e.g.*, Hermalin and Katz (2007), Choi and Kim (2010), Krämer and Wiewiorra (2010), Cheng et al. (2011), Economides and Tåg (2012), Economides and Hermalin (2012), and Choi et al. (2013).

types' business. Even for our simple model and eschewing congestion effects, the analysis quickly becomes intractable, especially if different definitions of type are to be considered.

To avoid having to solve directly for any mixed-strategy pricing equilibria of the content providers' pricing subgame and also to keep the definition of type relatively straightforward, our analysis of heterogenous households and price discrimination by the ISP is restricted to a more limited model than heretofore considered. To wit, assume there are two household types, 0 or 1. We use β to denote an arbitrary element of $\{0, 1\}$. Let proportion $\phi \in (0, 1)$ of households be type 1, which can be considered the "high" type. Let a type-1 household's utility be

$$U = y + v_1 \sum_{n=1}^N \chi_n,$$

where $\chi_n = 0$ indicates no purchase from the n th content provider and $\chi_n = 1$ indicates the purchase of a unit. Observe that a household wants at most one unit of content from any provider. Observe, too, that we are not considering the effects of congestion (*i.e.*, $L(X|B) \equiv 1$).

The preferences of type-0 households are slightly different. Let \mathcal{N} denote the set of all content providers. Type-0 households only want content from content providers in a set \mathcal{N}_0 , $\mathcal{N}_0 \subseteq \mathcal{N}$ (note we are allowing for the possibility that $\mathcal{N}_0 = \mathcal{N}$). Content providers know if they are in \mathcal{N}_0 or not. Because the ordering of content providers is arbitrary, assume content provider n is in \mathcal{N}_0 if $n \leq N_0$, N_0 being the size of \mathcal{N}_0 , and it is not in \mathcal{N}_0 if $n > N_0$. Assume $N_0 \geq 2$. Let a type-0 household's utility be

$$U = y + v_0 \sum_{n=1}^{N_0} \chi_n,$$

where χ_n has the same interpretation as before.

For the sake of brevity, we limit attention to situations in which the high-type households would realize greater benefit from unfettered access than low-type (type-0) households. In fact, to speed the analysis, we impose a slightly stronger condition:

$$N_0 v_0 < (N - 1) v_1. \quad (26)$$

We assume the content providers cannot distinguish which households are which type. Hence, given the assumed utility functions, the content providers are necessarily limited to uniform pricing.

Absent download caps, there is no equilibrium in which the ISP earns a positive profit. To see this, note that without caps there is no mechanism by which the ISP can discriminate: it must charge a uniform hookup price, H . If only one household type, β , obtains access in equilibrium, then the content providers will necessarily charge v_β : those households will obtain no surplus and, hence, be willing to purchase access only if $H = 0$. If both household types obtain access in equilibrium,¹⁵ then profit maximization by the content providers entails $p_n \geq \min\{v_0, v_1\}$ if $n \leq N_0$ and $p_n = v_1$ if $n > N_0$. It follows that at least one type is earning no surplus; so, if both are to obtain access, it must again be that $H = 0$. To summarize:

¹⁵We could allow mixing by households, but this doesn't change the analysis. Hence, for the sake of brevity, we do not consider this.

Lemma 1. *Absent download caps, the ISP earns zero profit given the assumptions of this section.*

If $\phi v_1 > v_0$, then all content providers price at $p = v_1$ in the absence of download caps. In this case, low-type households would be shut out of the market and only high-type households would acquire access. If $\phi v_1 \leq v_0$ and a content provider in \mathcal{N}_0 believes all households have purchased access, then it prices as follows:

$$p_n = \begin{cases} v_0, & \text{if } (1 - \phi)v_0 > v_1 \\ \min\{v_0, v_1\}, & \text{otherwise} \end{cases} .$$

A content provider not in \mathcal{N}_0 prices at v_1 . Welfare is maximized if content providers in \mathcal{N}_0 price at $\min\{v_0, v_1\}$ and falls short of the maximum otherwise (as, then, some household types are priced out of the content offered by content providers in \mathcal{N}_0).

Suppose the ISP imposes a uniform download cap of $\bar{n} < N_0$ on all households. The situation is analogous to that considered in the Introduction: the content providers become, effectively, Bertrand competitors, which leads them to price at zero, from which it follows that a household that obtains access realizes a surplus of $v_\beta \bar{n}$ gross of the hook-up fee. It further follows that, under such a uniform cap, the ISP will set

$$H = \begin{cases} \bar{n}v_1, & \text{if } \phi v_1 > v_0 \\ \bar{n}v_0, & \text{if } (1 - \phi)v_0 > v_1 \\ \bar{n} \min\{v_0, v_1\}, & \text{otherwise} \end{cases} . \quad (27)$$

Because $\bar{n} < N$, welfare is, necessarily, lower with a cap than without. In the first and third pricing cases in (27), the set of households who obtain access is unaffected by the imposition of the cap *vis-à-vis* what it would be absent the cap. That is not true in the second: there is a further reduction of welfare in this case because the high-type households would be priced out of the market. It is readily seen that, conditional on $\bar{n} < N_0$, the ISP maximizes profit by setting $\bar{n} = N_0 - 1$. To summarize:

Proposition 8. *Relative to a situation with no caps, the imposition of a uniform download cap binding on all household types (i.e., $\bar{n} < N_0$) reduces welfare and either leaves the set of households who obtain access unaffected or reduces it. In this scenario, the ISP's profit-maximizing cap is $\bar{n} = N_0 - 1$.*

What about a uniform cap that would bind only on high-type households (i.e., $N_0 \leq \bar{n} < N$)? Maximum welfare in this scenario is

$$(1 - \phi)N_0v_0 + \phi\bar{n}v_1 ,$$

which is achievable only if all households obtain access. But if all type-0 households obtain access, then a content provider in \mathcal{N}_0 can guarantee itself an expected profit of at least $(1 - \phi)v_0$ by pricing at v_0 . Consequently, conditional on \bar{n} , the ISP's expected profit, π_{ISP} , in any equilibrium satisfies

$$\pi_{\text{ISP}} \leq \phi\bar{n}v_1 . \quad (28)$$

The righthand side of (28) is increasing in \bar{n} and is a maximum when $\bar{n} = N - 1$. Moreover, the ISP can obtain the righthand side in that case with certainty: suppose it sets $H = (N - 1)v_1$. Given condition (26), type-0 households would not obtain access even if they anticipated getting content for free. Consequently, only type-1 households would possibly choose to obtain access. The content providers would, thus, find themselves virtual Bertrand competitors and price at zero. Type-1 households would, thus, just be willing to obtain access. To conclude:

Proposition 9. *Conditional on the ISP's setting a uniform cap that would bind only on high-type households, there is an equilibrium in which the ISP sets a cap of $N-1$ and prices access equal to the resulting consumer surplus high-type households will receive from buying content (i.e., $H = (N-1)v_1$). Only high-type households acquire access and the content providers price at zero.*

Comparing Propositions 8 and 9 yields the following:

Proposition 10. *Suppose the ISP is limited to setting a uniform download cap, \bar{n} . Its equilibrium choice of cap and hookup fee, H , are*

$$(\bar{n}, H) = \begin{cases} (N_0 - 1, (N_0 - 1)v_0), & \text{if } \frac{v_0}{v_1} > \frac{1}{1-\phi} \max \left\{ 1, \frac{\phi(N-1)}{N_0-1} \right\} \\ (N_0 - 1, (N_0 - 1) \min\{v_0, v_1\}), & \text{if } v_1 \in \left[(1-\phi)v_0, \frac{v_0}{\phi} \right] \\ (N - 1, (N - 1)v_1), & \text{otherwise} \end{cases} \quad (29)$$

Observe that an ISP limited to a single cap will set a relatively tight cap ($N_0 - 1$) if it seeks to provide access to all households or only households that place a large value on the content they want, but who want a limited amount of content (i.e., $v_0 > v_1$, but $N_0 < N$). It will set a looser cap ($N - 1$) if it seeks to provide access only to high-type households (households that want more content *ceteris paribus*, each unit of which they may value more).

Finally, we consider the possibility of the ISP's offering two packages, (\bar{n}_0, H_0) and (\bar{n}_1, H_1) , intended for the type-0 and type-1 households, respectively. It is convenient for what follows to define $N_1 = N$, $\phi_1 = \phi$, and $\phi_0 = 1 - \phi$. If $\bar{n}_\beta \geq N_\beta$, then type- β households have no download cap (their package allows "all you can eat").

Lemma 2. *There is no equilibrium in which the ISP gains by offering a package with all you can eat (i.e., a package with $\bar{n}_\beta \geq N_\beta$).*

Proof: Suppose not. Maximum possible total surplus is

$$S = \sum_{\beta=0}^1 \phi_\beta \min\{N_\beta, \bar{n}_\beta\} v_\beta. \quad (30)$$

If $\bar{n}_\beta \geq N_\beta$ for a given type β , then content providers that cater to that type can guarantee themselves an expected profit of at least $\phi_\beta v_\beta$ by pricing at v_β . Hence, in aggregate, those content providers must earn expected profits of at least $\phi_\beta N_\beta v_\beta$. Because $S \geq \pi_{\text{ISP}}$, the second term being the ISP's expected profit, it follows from (30) that, if $N_\beta \leq \bar{n}_\beta$ for both β , the ISP can earn no expected profit. As seen (Propositions 8 and 9), the ISP can earn a positive profit by offering a single package with a binding download cap. Hence, $N_\beta > \bar{n}_\beta$ for at least one β .

Suppose $N_1 > \bar{n}_1$, but $N_0 \leq \bar{n}_0$. It follows from (30) and the previously given logic that

$$\pi_{\text{ISP}} \leq \phi_1 \bar{n}_1 v_1;$$

but the ISP can achieve a profit of $\phi_1(N-1)v_1$ by setting a uniform download cap of $N-1$ and pricing access at $(N-1)v_1$; that is, the ISP does weakly better offering that than two packages, one of which provides type-0 households all they can eat.

Suppose $N_0 > \bar{n}_0$, but $N_1 \leq \bar{n}_1$. Similar logic entails

$$\pi_{\text{ISP}} \leq \phi_0 \bar{n}_0 v_0;$$

but the ISP can achieve a profit of at least $\phi_0(N_0 - 1)v_0$ by setting a uniform download cap of $N_0 - 1$ and pricing access at $(N_0 - 1)v_0$. ■

Suppose the ISP offers two packages and that the content providers expect each type of household to choose the package intended for it. In light of Lemma 2, the ensuing pricing game among the content providers will be Bertrand-like, with equilibrium prices equal to zero. Hence, gross of the hookup fee, a type-1 household's surplus from purchasing the package intended for a type- β household is $\bar{n}_\beta v_1$ and a type-0 household's surplus from purchasing such a package is $\min\{\bar{n}_\beta, N_0\}v_0$. The ISP's problem is then

$$\max_{\{H_0, H_1, \bar{n}_0, \bar{n}_1\}} (1 - \phi)H_0 + \phi H_1 \quad (31)$$

subject to

$$\bar{n}_0 v_0 - H_0 \geq \min\{\bar{n}_1, N_0\}v_0 - H_1, \quad (\text{IC-0})$$

$$\bar{n}_1 v_1 - H_1 \geq \bar{n}_0 v_1 - H_0, \quad (\text{IC-1})$$

$$\bar{n}_0 v_0 - H_0 \geq 0, \text{ and} \quad (\text{IR-0})$$

$$\bar{n}_1 v_1 - H_1 \geq 0. \quad (\text{IR-1})$$

Because the ISP could “offer” the package $(0, 0)$, there is no loss of generality in assuming both household types participate (*i.e.*, the individual rationality, IR, constraints will hold for both types).

Suppose $v_0 > v_1$. Given (26), this entails $N_0 < N - 1$. It is readily seen that the solution to the program (31) is

$$(\bar{n}_0, H_0) = ((N_0 - 1), (N_0 - 1)v_0) \text{ and } (\bar{n}_1, H_1) = ((N_1 - 1), (N_1 - 1)v_1).$$

Observe this solution means the ISP achieves the maximum possible profit; in particular, no surplus is left to the households.

The remaining case is $v_0 \leq v_1$ and $N_0 \leq N$, with at least one inequality strict. This corresponds to the textbook second-degree price discrimination situation (see *e.g.*, Tirole, 1988, Chapter 3). Hence, we know that if the ISP markets two distinct packages, then, in equilibrium,

- $\bar{n}_0 < \bar{n}_1$;
- $H_0 = \bar{n}_0 v_0$;
- high-type households receive an information rent of $R = \bar{n}_0 v_1 - H_0 = \bar{n}_0(v_1 - v_0)$; and, hence,
- $H_1 = \bar{n}_1 v_1 - R = \bar{n}_1 v_1 - \bar{n}_0(v_1 - v_0)$.

If the ISP were to market two packages with $\bar{n}_1 > \bar{n}_0$, its profit would be

$$\phi H_1 + (1 - \phi)H_0 = \phi(\bar{n}_1 - \bar{n}_0)v_1 + \bar{n}_0 v_0. \quad (32)$$

Clearly, (32) is increasing in \bar{n}_1 , from which we can conclude $\bar{n}_1 = N - 1$. Expression (32) is nondecreasing in \bar{n}_0 if $\phi v_1 \leq v_0$, which means the ISP maximizes its profit by setting $\bar{n}_0 = N_0 - 1$. Expression (32) is decreasing in \bar{n}_0 if $\phi v_1 > v_0$, which means the ISP maximizes its profit by setting $\bar{n}_0 = 0$ (*i.e.*, it offers a single package intended for high-type households only). To conclude

Proposition 11. *In equilibrium, if $\phi v_1 > v_0$, then the ISP will offer a single package with a download cap of $N - 1$ and hookup fee equal to $(N - 1)v_1$. In this case, only high-type households acquire access. If $v_1 \geq v_0 \geq \phi v_1$, then the ISP will offer two packages: one with a download cap of $N_0 - 1$ and a hookup fee of $(N_0 - 1)v_0$ intended for low-type households and the other with a download cap of $N - 1$ and a hookup fee of $(N - N_0)v_1 + (N_0 - 1)v_0$ intended for high-type households.¹⁶ Finally, if $v_0 > v_1$, then the ISP will also offer two packages: the download caps will be the same, but the hookup fees will be $(N_0 - 1)v_0$ and $(N - 1)v_1$ for the low-type and high-type packages, respectively. In the latter two cases, both household types acquire access.*

We observe that Proposition 11 is consistent with what we see when residential ISPs utilize download caps (*e.g.*, as in Canada—see footnote 7 *supra*). Although they may price discriminate across customers by offering different packages, all packages have download limits.

An important implication of Proposition 11 is the following. Given the assumptions of this section, allowing the ISP to impose a download cap is always welfare reducing (although the relative reduction in welfare is arguably minor when the number of content providers, N , is large). In particular, the ISP's pricing scheme never expands access beyond the set of households who would obtain access were download caps prohibited. Under such a prohibition, the content providers would price to exclude low-type households if $\phi v_1 > v_0$ and serve all households otherwise. Similarly, if $\phi v_1 > v_0$, the ISP prices to exclude low-type households, whereas otherwise it caters to all households.

In a model with more standard demand (*e.g.*, closer to the model of Section 3), such a result might not hold. As is well known, price discrimination can sometimes expand the number of populations served (see, *e.g.*, Tirole, 1988, Chapter 3) *vis-à-vis* the number served under uniform (non-discriminatory) pricing. On the other hand, the forces identified earlier would also apply: to better capture rents from the content providers, the ISP will have a tendency to set the download caps too low. In contrast, in this section, although it sets the cap below the welfare-maximizing level, that effect is relatively small when N is large.

A further issue, also omitted from our analysis, is an ISP's ability to discriminate via other means. In particular, many residential ISPs discriminate on the basis of connection or download speed (*i.e.*, engage in second-degree price discrimination via quality distortions or versioning). Although an ISP might wish to use both instruments (*i.e.*, download speed and limits) solely for the purpose of price discrimination, it is possible that download speed is a sufficient instrument for discrimination and, hence, the additional imposition of download limits could be driven primarily by incentives to extract rents from content providers, as modeled in this paper.¹⁷

9 HETEROGENEOUS CONTENT PROVIDERS

Finally, we briefly consider the situation if the content providers are heterogenous. This is not easily done with either model used so far; hence, we utilize the following

¹⁶If $N = N_0$, then these are the same package.

¹⁷Somewhat the flip side: slow downloads could also cause households to see different content providers as substitutes. This is an issue, however, for future research.

variant: a household's utility is

$$U = \int_0^N v(n, X) \chi_n dn + y,$$

where $\chi_n \in \{0, 1\}$ indicates whether the household has acquired a unit of the n th content provider's content. Note, as in the previous section, a household wants, at most, one unit of any content provider's content and we are assuming a measure N of content providers. The quantity $v(n, X)$ is the contribution to total household utility from a unit of the n th content provider's content given X units of total content are being transported. We again assume congestion is detrimental: $v(n, \cdot)$ is a decreasing function for all n . We assume a fixed order of preference for the different content; specifically, assume that $n > n'$ implies $v(n, X) < v(n', X)$ for all X . Assume $v(N, N) \geq 0$ (i.e., even with maximum congestion, a household gains utility from its least preferred content). At the same time, we maintain the assumption that limits on congestion can be welfare enhancing: let the function

$$U(M) \mapsto \int_0^M v(n, M) dn$$

be concave in M and assume $U'(N) < 0$.

Absent any download limit, all content providers will operate and content provider n sets a price of $v(n, N)$. There is no household surplus, so $H_{\text{NOCAP}} = 0$. Welfare is $U(N)$.

Suppose a download restriction of $M < N$ is imposed. The following is an equilibrium, as is readily verified:

- All content providers $n \geq M$ set their prices to 0.
- A content provider n , $n < M$, sets its price to $v(n, M) - v(M, M)$.

Household surplus is $Mv(M, M)$; hence, $H_{\text{CAP}} = Mv(M, M)$.

The ISP will set M to maximize H_{CAP} . The first-order condition is

$$v(M, M) + M \left(\frac{\partial v(M, M)}{\partial n} + \frac{\partial v(M, M)}{\partial X} \right) = 0. \quad (33)$$

In contrast, welfare maximization entails setting M to solve

$$v(M, M) + \int_0^M \frac{\partial v(n, M)}{\partial X} dn = 0. \quad (34)$$

Clearly, expressions (33) and (34) are different, from which it follows that the download limit that is profit maximizing for the ISP is, generically at least, not the limit that would be welfare maximizing. Further examination of these expressions reveals that a familiar tension exists: the monopolist (the ISP) is concerned with marginal values, while welfare depends on infra-marginal values (see, e.g., Spence, 1975). In particular, we have the following result.

Proposition 12. *Under the assumptions of this section, if the marginal disutility of congestion is not less for content of lower value than higher value (i.e., if $\partial(\partial v/\partial X)/\partial n \leq 0$), then the ISP will set a download cap that is below the welfare-maximizing cap.*

Proof: By the intermediate value theorem (34) equals

$$v(M, M) + M \frac{\partial v(\hat{n}, M)}{\partial X} \quad (35)$$

for some $\hat{n} \in [0, M]$. By assumption, (35) is not less than

$$v(M, M) + M \frac{\partial v(M, M)}{\partial X},$$

which in turn strictly exceeds (33). The result follows. \blacksquare

For example, if $v(n, X) = K - nX$, $K \geq N^2$, then the ISP will impose a cap of $\sqrt{K/3}$, while welfare maximization entails a cap of $\sqrt{2K/3}$. Indeed, as long as the ISP wishes to impose a cap (*i.e.*, whenever $N > \sqrt{K/3}$), welfare with no cap exceeds welfare under the ISP's preferred cap:

Proposition 13. *Under this section's assumptions and assuming $v(n, X) = K - nX$, $K \geq N^2$, welfare is greater with no download cap than with the cap the ISP would choose.*

Proof: Observe $U(n) = Kn - n^3/2$ is a concave function in n . At $n = \sqrt{K}$ (the upper bound):

$$U\left(\sqrt{\frac{K}{3}}\right) = \frac{5K^{3/2}}{6\sqrt{3}} < \frac{K^{3/2}}{2} = U(\sqrt{K}).$$

The derivative of $U(n)$ evaluated at $n = \sqrt{K/3}$ is $K/2 > 0$. Hence, $U(n) > U(\sqrt{K/3})$ for all $n \in (\sqrt{K/3}, \sqrt{K}]$. \blacksquare

Proposition 12 assumed that the disutility from congestion was greater for less desired content than more desired content. It is, of course, possible the opposite is true (*e.g.*, movies on demand could be both highly valued and marginal disutility from congestion high relative to material from online periodicals). Even in such a setting, it is possible that the ISP will wish to set a cap below the welfare-maximizing cap due to the direct effect of inducing competition among the content providers (*i.e.*, reflecting the $\partial v(M, M)/\partial n$ term in (33)). To illustrate this, suppose

$$v(n, X) = \frac{1}{\sqrt{X}}(K - n), \quad (36)$$

where $K \geq N$. Observe $\partial(\partial v/\partial X)/\partial n > 0$. It is readily see that

$$U(M) = K\sqrt{M} - \frac{M^{3/2}}{2}.$$

The welfare-maximizing cap is, therefore, $M_W^* = 2K/3$. Solving (33) yields $M_{\text{ISP}}^* = K/3$.

Proposition 14. *Under this section's assumptions and assuming $v(n, X)$ is given by (36), welfare is greater with no download cap than with the cap the ISP would choose.*

Proof: Observe $U(n)$ is a concave function in n . At $n = K$ (the upper bound):

$$U\left(\frac{K}{3}\right) = \frac{5K^{3/2}}{6\sqrt{3}} < \frac{K^{3/2}}{2} = U(K).$$

The derivative of $U(n)$ evaluated at $n = K/3$ is $\sqrt{3K}/4 > 0$. Hence, $U(n) > U(K/3)$ for all $n \in (K/3, K]$. \blacksquare

10 CONCLUSIONS AND FUTURE WORK

This paper has considered how a platform, such as a residential Internet service provider (ISP) or mobile telephone company, can profit from the imposition of download caps on its customers even absent motives of price discrimination and congestion alleviation. Download caps intensify the competitive pressures on content providers, which causes them to reduce the prices they charge consumers. Because, *ceteris paribus*, consumers would realize greater surplus, the platform can raise its access charge, thereby increasing its profit.

Beyond demonstrating that effect, the paper has shown it could be sufficiently strong that the platform has incentives to set the cap so low that, even if there were welfare benefits to be had from congestion alleviation, welfare would be higher with no cap than with the overly tight cap the platform would choose. The implications of this effect for both the platform's capacity (bandwidth) decisions and time-of-day practices were considered. Somewhat paradoxically, it was shown that allowing the platform to impose a download cap increases its incentives to expand capacity. It was also shown that the profits generated by a download cap undermine incentives the platform might have to shift consumption to off-peak times (at least to shift them by relaxing the caps during off-peak times).

We considered other means by which the platform could extract rents, specifically by charging households or content providers direct per-unit (*e.g.*, per byte) fees. We showed those means generate greater profit than download caps. Critically, though, the difference in the platform's profits between these other means and the utilization of download caps shrinks as the number of content providers increases. When the number of content providers is large, the platform could be close to indifferent between these other means and download caps; hence, once transaction costs, consumer attitudes, or prevailing regulations are taken into account, the platform may choose to use download caps rather than those other means.

We extended the analysis to allow for heterogeneous content providers. Although tractability limited us to a less-general model than used for most our analysis, our results suggest that our conclusions continue to hold when the value consumers place on the content of different providers varies. We also extended our analysis to allow for heterogeneous consumers (households). Again, issues of tractability limited the analysis, but we were able to demonstrate that the logic of our earlier results was not dependent on our assumption of homogenous consumers.

Future work remains. First, there is the question of quantifying the size of the effect of download caps. In this regard, comparing the US with Canadian residential Internet markets could be instructive. The former is currently characterized by few download caps, while download caps are prevalent in the latter. If the former market is characterized by higher content prices, but lower access fees, relative to the latter,¹⁸ then this would provide both support for our model, as well as a means of quantifying the effect.

Second, our analysis has been limited to a monopoly platform. Although platform competition in the relevant markets is often limited, it does exist and the consequences of oligopolistic competition among platforms on the use of download caps to be explored. Among the issues worth exploring is the extent to which the platforms are tempted to free ride on each other: because platform A benefits from the reduction in

¹⁸As noted earlier, Van Gorp and Middleton (2010) report evidence that Canadian access fees tend to exceed those of other OECD countries.

content prices induced by platform B's download caps, A might be tempted to offer less stringent caps to gain a competitive advantage *vis-à-vis* B. On the other hand, as a consequence, A's would be the more congested platform.

Another competitive issue arises when the platform is also a content provider. For instance, a cable TV company could provide broadband Internet and compete directly with Internet-based purveyors of on-demand movies. A download limit could harm such rivals, but because it also makes them fiercer competitors, it could lower the cable TV's profits from its own sale of on-demand movies.

Download caps could also affect how content providers operate. Beyond, for instance, giving them incentives to employ better compression algorithms, it could affect how they see the balance between directly charging consumers and generating revenue through other means, such as advertising. To the extent content providers respond to caps by lowering the quality of their content (transmitting lower-resolution images or having more ads), consumer surplus would be reduced *ceteris paribus*, which in turn could affect the platform's incentives to impose caps in the first place. All of these extensions are, however, beyond the scope of the current paper and remain work for the future.

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