When does strategic information disclosure lead to perfect consumer information?*

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Abstract

A firm chooses a price and how much information to disclose about its product to a consumer whose tastes are unknown to the firm. Full disclosure of product information is always part of a sequential equilibrium when types are independent. In that case, a necessary and sufficient condition on equilibrium payoffs is that they are at least as high as those under full revelation for all product types. We provide a sufficient, and to some extent, necessary condition for full disclosure of product information to be the *unique* equilibrium outcome whatever the priors (independent or not). That condition encompasses the condition that all consumers agree on the ranking of product types' quality as in a standard persuasion game but it also allows for different consumers to have different rankings of the potential product types.

Keywords: Persuasion game; informative advertising.

1 Introduction

What type of product information should we expect firms to disclose through advertising? A common perception is that advertising contains a limited amount of product information, if at all. This is for instance what is usually inferred from the content analysis literature in marketing that was initiated by Resnik and Stern (1977). This literature also suggests that the informativeness of ads varies across media. It should also be expected to differ significantly across markets. This suggests two related questions. For which products are firms more likely to provide more information in ads? Which product dimensions are more likely to be emphasized by firms?

We investigate these questions in the simplest possible framework, which is a generalization of the "persuasion game" of Grossman (1981) and Milgrom (1981). A monopolist sells one unit of a product to a consumer. The product's characteristics are known only to the firm while the consumer's tastes are his private information. The unit production cost may vary across product types. The firm makes a take it or leave it price offer and may provide certifiable information about

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the product before the consumer decides whether to purchase or not. The model is fully described by a "match function" that indicates the consumer's valuation as a function of the types of both parties.

Our framework accommodates as a special case the "persuasion game" where all consumer types have an identical ranking of valuations for the different products. This corresponds to products with different qualities on which all consumer types agree. This special case yields the stark result that full revelation of the product's type is the unique sequential equilibrium of the game. The main contribution of this paper is to provide a necessary and sufficient condition on the match function for full revelation of the product's type to be the unique equilibrium, independent of the prior beliefs on the firm and the consumer's types. Our condition, called "pairwise monotonicity", requires that for every pair of types of the firm and every pair of types of the consumer the matches can be ordered with respect to the firm's types or with respect to the consumer's type. This allows for products differentiated in terms of horizontal match in the sense that different consumer types rank products differently.

Recent theoretical works by Johnson and Myatt (2006) and Anderson and Renault (2006, 2009) have analyzed the disclosure of horizontal match information by firms. These authors find that it may be profit maximizing for a firm to disclose no information or partial information. They do not explicitly model the asymmetric information disclosure game but consider full information, no information or partial information modeled as a signal received by consumers that is correlated with their true valuation. All product types are implicitly assumed to be symmetric so that the profit maximizing solution is not type dependent. Johnson and Myatt (2006) consider a set of specific signals about the consumer's valuation that may be ranked unambiguously in terms of informativeness. They find that the monopolist's profit is maximized by using either the least informative or the most informative signal. Anderson and Renault (2006) also consider a monopoly problem. By allowing for general informative signals, they show that it is optimal for the firm to provide partial information that takes the form of a threshold on the consumer's valuation, where those consumers with valuations above the threshold learn this information but no more.[1]

Our analysis shows that the profit maximizing solution characterized in these papers is typically not the unique equilibrium outcome. In particular we show that if product types and consumer types are independent, full revelation of the product's type is always a sequential equilibrium. Furthermore, the type of information that can be transmitted to consumers about their valuation critically depends on the properties of the match function. Hence, the profit maximizing solution derived in Anderson and Renault (2006) or Johnson and Myatt (2006) might not be feasible given the characteristics of products and consumer tastes. Yet our results show that the most profitable outcome for the firm subject to this feasibility constraint is an equilibrium outcome. Indeed, when types are independent we show that a necessary and sufficient condition for an outcome to be supported by an equilibrium is that it yields a profit at least as large as full revelation for all product types.

[1]Although Anderson and Renault derive the result while allowing the consumer to acquire information through search, the result still holds if search is ruled out.

Related Literature. Nelson (1974), Milgrom (1981), Milgrom and Roberts (1986a,b), Lewis and Sappington (1994), Anderson and Renault (2006), Bergemann and Välimäki (2006), Johnson and Myatt (2006), Sun (2007), Milgrom (2008), Board (2009).

2 Model

Decision problem. A monopolist sells a single unit of its product to a consumer. The *match* between the characteristic of the firm's good $s \in S$ and the consumer's taste $t \in T$ is given by

$$r(s,t) \in \mathbb{R}.$$

The characteristic s of the product is privately known by the firm, and the consumer's taste t is privately known by the consumer. For technical simplicity the sets S and T are assumed to be finite. Let $\mu \in \Delta(S \times T)$ be the strictly positive prior probability distribution over the profile of types. When the type of the firm is independent of that of the consumer we denote respectively by $\sigma \in \Delta(S)$ and $\tau \in \Delta(T)$ the priors of the firm's types and the consumer's types, in which case $\mu(s,t) = \sigma(s)\tau(t)$.

The utility of the consumer when he does not buy is zero. When consumer type $t \in T$ buys the product with characteristic $s \in S$ at price p, his utility is given by

$$r(s,t) - p.$$

The firm has a constant marginal cost $\gamma(s) \geq 0$ when its type is s, so its profit is zero when the consumer does not buy, and $p - \gamma(s)$ when he buys at price p. We assume that for every $s \in S$, there exists $t \in T$ such that $r(s,t) > \gamma(s)$, meaning that each type of the firm can potentially make a strictly positive profit with at least one type of the consumer.

Timing of the game.

- *Information stage.* The firm learns the characteristic of its good $s \in S$ and the consumer learns his taste $t \in T$;
- *Advertising (pricing and disclosure) stage.* The firm commits to an observable price $p \in \mathbb{R}$ and sends a message $m \in M(s)$, where $M(s)$ is a nonempty type dependent set;
- *Decision stage.* The consumer observes the price p and the message m, and chooses whether to buy the good or not;
- *Payoffs.* Players' payoffs are zero when the consumer does not buy the good; if the consumer buys the good his payoff is $r(s,t) - p$, and the firm's payoff is $p - \gamma(s)$.

Denote by $M = \bigcup_{s \in S} M(s)$ the set of all possible messages that can be sent in the advertising stage. For simplicity we assume that every subset of types of the firm is certifiable in the sense that for every $S' \subseteq S$, there is a message $m_{S'} \in M$ such that $m_{S'} \in M(s)$ if and only if $s \in S'$. In particular, full disclosure of the firm's type s (message m_s) and no disclosure (message m_S) are possible. This is particularly justified in the contest of advertising where lying about the characteristic of the good is prohibited, but is also justified if the firm can provide hard evidence about the product characteristic, or if advertised information can be verified by the consumer at no cost.

Strategies and Equilibrium A strategy[2] of the firm is a mapping $\varphi_F : S \to \mathbb{R} \times M$ such that $\varphi_F(s) \in \mathbb{R} \times M(s)$ for every $s \in S$. A strategy of the consumer is a mapping $\varphi_C : T \times \mathbb{R} \times M \to \{Buy, NotBuy\}$. A belief function of the consumer is a mapping $\beta : T \times \mathbb{R} \times M \to \Delta(S)$.

As a solution concept for this advertising game,[3] we use the sequential equilibrium.

Remark 1 Some care should be taken in defining strong belief consistency because the set of possible signals is infinite (any price $p \in \mathbb{R}$ is possible), so a strictly positive perturbed strategy cannot be defined as in Kreps and Wilson (1982) (they consider finite games). However, it is easy to avoid this problem by assuming that the set of possible prices is finite but fine enough (all results and examples below apply with a fine enough set of prices). Actually, this problem only appears with correlated types. Otherwise we can directly apply the notion of perfect Bayesian equilibrium defined in Fudenberg and Tirole (1991) which is equivalent in our model to the sequential equilibrium when types are independent, provided that we impose (due to the certifiability of information) that the consumer's belief about s is zero off the equilibrium path when $m \notin M(s)$. Anyway, our results are proved with beliefs off the equilibrium path that put probability one on a single firm type for every consumer types, so such beliefs are clearly strongly consistent whatever the (strictly positive) priors and the correlation.

3 Results

After the advertising stage, the optimal decision of the consumer of type t is to buy the product if and only if[4]

$$E[r(\mathbf{s}, \mathbf{t}) \mid \mathbf{t} = t, (p, m)] \geq p.$$

(Here a consumer buys when indifferent; else an equilibrium would fail to exist.) Therefore, the optimal choice (p, m) of the firm of type s in the advertising stage should satisfy

$$(p, m) \in \arg \max_{(p,m) \in \mathbb{R} \times M(s)} (p - \gamma(s)) D(p, m, s),$$

[2]For simplicity we only analyze pure strategy equilibria.

[3]The advertising game is a combination of a standard signaling game (Sobel, 2009) and a persuasion game (Grossman, 1981; Milgrom, 1981) since costless certifiable information can be provided in addition to the price signal.

[4]Bold letters denote random variables when there may be a risk of confusion. Expectations and probabilities are defined w.r.t. the prior μ, the firm's strategy φ_F and the consumer's belief β.

where

$$D(p, m, s) \equiv \Pr\left[E[r(\mathbf{s}, \mathbf{t}) \mid \mathbf{t}, (p, m)] \geq p \mid \mathbf{s} = s\right],$$

is the expected demand of the firm when its type is s and it sends the signal (p, m). Notice that $E[r(\mathbf{s}, \mathbf{t}) \mid \mathbf{t}, (p, m)]$ only depends on the realization of $\mathbf{s}$ via $\mathbf{t}$. Hence, if the type of the firm is not correlated with the type of the consumer then the demand of the firm given p and m does not depend on its actual type, i.e.,

$$D(p, m, s) = D(p, m) = \Pr\left[E[r(\mathbf{s}, \mathbf{t}) \mid \mathbf{t}, (p, m)] \geq p\right].$$

In that situation the preference of the firm over the consumer's belief is, conditionally on the price and the message sent, independent of its real type. Hence, starting from a strategy of full disclosure it is easy to prevent the firm from deviating by constructing worst case inferences (i.e., punishment beliefs off the equilibrium path) for the consumer that minimize the firm's demand at price p. This can be done independently of the firm's real type. This allows to get the following proposition.

Proposition 1 *Assume that the consumer's type is not correlated with the firm's type. Then, the advertising game has a fully revealing equilibrium.*

Proof. Consider a complete disclosure strategy of the firm such that a message $m_s \in M(s)$ is sent by each type of the firm, with $m_s \notin M(s')$ for all $s' \neq s$. When types are not correlated, the demand for such a message at price p is given by

$$D(p, m_s) = \Pr[r(s, \mathbf{t}) \geq p].$$

For every $m \in M$ and $p \in \mathbb{R}$, let

$$wct(p, m) \in \arg \min_{s \in M^{-1}(m)} D(p, m_s),$$

be the (worst case) type of the firm whose demand at price p when it reveals its type to the consumer is the lowest one among all types that can send message m. For any signal $(p, m) \in \mathbb{R} \times M$ of the firm, consider the belief of the consumer that puts probability one on $wct(p, m)$ whatever the consumer's type.[5] This belief system is clearly consistent. Along the equilibrium path, the firm gets

$$\max_{p_s}(p_t - \gamma(s))D(p_s, m_s).$$

This profit is larger than what it gets by deviating to $(p, m) \in \mathbb{R} \times M(s)$ in the advertising stage, which by construction is equal to

$$(p - \gamma(s)) \min_{s' \in M^{-1}(m)} D(p, m_{s'}).$$

[5] That is, $\beta(wct(p, m) \mid t, p, m) = 1$.

This completes the proof of Proposition 1. ■

The appendix describes an example (Example 6) with correlated types where there is no fully revealing equilibrium. Notice that in the example, as well as in the proofs of our results, we require strong belief consistency (see Remark 1). If we allow arbitrary beliefs off the equilibrium path, existence of a fully revealing equilibrium is immediate even with correlated types. It suffices to consider beliefs off the equilibrium path that put probability one on $\arg\min_{s\in M^{-1}(m)} r(s,t)$, which are inconsistent when they depend on t.

It is also worth noticing that when the cost of the firm is not type dependent, the proof of Proposition 1 can be simplified by considering price-independent beliefs off the equilibrium path, with the following worst case type:

$$wct(m) \in \arg\min_{s\in M^{-1}(m)} \max_{p} (p-\gamma)D(p,m_s).$$

This is not possible when γ depends on the firm's type since in that case the max above would depend on the actual type of the firm. This is illustrated in Example 7 in the Appendix.

In the next two examples we show that even when Proposition 1 applies (i.e., the consumer's type is not correlated with the firm's type), all equilibria are not necessarily fully revealing. Non revealing or partially revealing equilibria may exist, that are strictly preferred by the firm whatever its type, to the fully revealing equilibrium. Later we will provide further conditions for the fully revealing equilibrium to exist and to be *unique* whatever the priors, even with correlation of types. In both examples, the prior probability distribution is assumed to be uniform over type profiles, and the cost is zero whatever the firm's type. They also share the property that all types of consumers and firms have the same interim beliefs about the realization of the match. That is, the random variable $\boldsymbol{r}$ coincides with $\boldsymbol{r} \mid s$ and $\boldsymbol{r} \mid t$ for every s and t. Such a "common beliefs" assumption is made in Anderson and Renault (2006) as well as Johnson and Myatt (2006).

Example 1 (*A non revealing equilibrium strictly prefered by all types of the firm.*) With the following match function

$$r = \boxed{\begin{array}{ccc} & t_1 & t_2 \\ s_1 & 4 & 2 \\ s_2 & 2 & 4 \end{array}}$$

there is a fully revealing equilibrium that gives a profit equal to 2 whatever the firm's type, and a non revealing equilibrium that gives a higher profit equal to 3 whatever the firm's type.[6]

Example 2 (*A partially revealing equilibrium strictly prefered by all types of the firm.*) Consider

[6]Notice that there are non-revealing equilibria that are less efficient for the firm. Any price $p \in [2,3]$ with no disclosure is an equilibrium, by considering degenerated beliefs on s_1 or s_2 for any disclosure and price $p' \neq p$ sent off the equilibrium path.

the following match function, which is inspired by Table 1 in Anderson and Renault (2006):

$$r = \begin{array}{|cccc|} \hline & t_1 & t_2 & t_3 \\ s_1 & 3 & 2 & -2 \\ s_2 & 3 & -2 & 2 \\ s_3 & 2 & 3 & -2 \\ s_4 & -2 & 3 & 2 \\ s_5 & 2 & -2 & 3 \\ s_6 & -2 & 2 & 3 \\ \hline \end{array}$$

The fully revealing equilibrium leads to a profit of 4/3 whatever the firm's type. No information revelation yields a payoff of 1, so it is not an equilibrium. However, there is a partially revealing equilibrium in which the firm's disclosure strategy yields the partition $\{\{s_1, s_3\}, \{s_2, s_5\}, \{s_4, s_6\}\}$, the price is equal to 5/2 and the profit is equal to 5/3 whatever the firm's type.

Note that this partially revealing equilibrium implements a threshold of 2 along the lines of the optimal solution described in Anderson and Renault (2006): all consumer types with willingness to pay above 2 learn this with no additional information. Also note that this solution implements a socially first-best outcome since a consumer buys if and only if his match exceeds marginal cost. This is also the profit maximizing solution since the firm extracts all of the consumer's expected surplus through its price of 5/2.[7]

The next proposition provides necessary and sufficient conditions for a disclosure strategy to be an equilibrium of the game when types are independent. The condition simply states that this disclosure strategy should induce an (interim) payoff for the firm which is not smaller than the payoff the firm would get at the fully revealing equilibrium whatever the firm's type.

Proposition 2 *Assume that the consumer's type is not correlated with the firm's type. A disclosure strategy induces an equilibrium iff whatever the firm's type the induced interim expected payoff for the firm is not smaller than its fully revealing equilibrium payoff.*

Proof. *Necessity.* This part is obvious.[8] If the disclosure strategy of the firm is such that $\varphi_F(s) = (p, m)$ with

$$(p - \gamma(s))D(p, m) < (p_s - \gamma(s))D(p_s, m_s),$$

where (p_s, m_s) is the signal sent by the firm at the fully revealing equilibrium, then (simply by subgame perfection) the firm can profitably deviate from φ_F at s by sending the signal (p_s, m_s).

Sufficiency. Consider a disclosure strategy φ_F of the firm such that $\varphi_F(s) = (p, m)$ and

$$(p - \gamma(s))D(p, m) \geq (p_s - \gamma(s))D(p_s, m_s), \quad (1)$$

[7]In Anderson and Renault (2006) where consumers may acquire full product information through costly search before buying, the coincidence of profit maximization and the first-best socially optimum outcome only arises if search costs are large enough.

[8]Notice that this part also applies with correlated types.

where (p_s, m_s) is the signal sent by the firm of type s at the fully revealing equilibrium. Consider the same beliefs for the consumer off the equilibrium as in the proof of Proposition 1. If the firm deviates from $\varphi_F(s) = (p, m)$ to (p', m') it gets

$$\begin{aligned}&(p' - \gamma(s)) \min_{s' \in M^{-1}(m')} D(p', m_{s'}) \leq \max_{p_s}(p_s - \gamma(s)) \min_{s' \in M^{-1}(m')} D(p_s, m_{s'}) \\ &\leq \max_{p_s}(p_s - \gamma(s)) D(p_s, m_s) \text{ because } s \in M^{-1}(m'),\end{aligned}$$

which by (1) is smaller than the payoff it gets without deviating from $\varphi_F(s)$. ■

The proposition does not extend to correlated types since we already observed in Example 6 in the Appendix that full revelation may not be an equilibrium when types are correlated.

We now give a condition for a fully revealing equilibrium to exist and to be *unique* whatever the priors, even correlated.

We say that the match is *statewise monotonic with respect to the firm's type*, if $r(s', t) > r(s, t)$ for all $t \in T$ and $s' > s$. This assumption is made in the standard persuasion games of Milgrom (1981) or Milgrom and Roberts (1986b), where S is the quality of the product, T is a singleton, and the price is fixed exogenously. Statewise monotonicity with respect to the firm's type is also satisfied with the following match:

$r =$

	t_1	t_2	t_3
s_1	9	5	7
s_2	8	3	6
s_3	6	2	0

It is easy to show that existence and uniqueness of the fully revealing equilibrium is guaranteed under statewise monotonicity with respect to the firm's type, even with correlated types.

Proposition 3 *If the match is statewise monotonic with respect to the firm's type, then the advertising game has a unique sequential equilibrium outcome, which is fully revealing.*

Proof. Existence is obtained by considering the following price independent worst case type:

$$wct(p, m) = \min M^{-1}(m),$$

since in that case $D(p, m, s) \leq D(p, m_s, s)$ for all $m \in M(s)$ and $p \in \mathbb{R}_+$. To show that there is no other equilibrium outcome assume by way of contradiction that types in $S' \subseteq S$, with $|S'| \geq 2$, send the same signal (p, m). Let s' be the highest type in S'. If s''s profit is zero, then by certifying its type it can apply a price higher than $\gamma(s')$ and get a strictly positive profit, a contradiction. If s''s profit is strictly positive, then by statewise monotonicity with respect to the firm's type we have $E[r(\mathbf{s}, t) \mid \mathbf{s} \in S'] < r(s', t)$ for all $t \in T$, so the firm can strictly increase its profit when it certifies its type by increasing its price and by increasing demand (or at least keeping its demand constant), which yields again a contradiction. ■

Notice that if the match is statewise monotonic with respect to the firm's type only for a subset of types in S, then those types do not necessarily separate in equilibrium. For instance, in Example 3 below, type s_3 strictly dominates type s_1, but those types do not necessarily separate in equilibrium.

Example 3 Consider the following match function, where the prior probability distribution is assumed to be uniform over type profiles, and the marginal cost is zero whatever the firm's type:

$$r = \begin{array}{|ccc|} & t_1 & t_2 \\ s_1 & 6 & 0 \\ s_2 & 3 & 9 \\ s_3 & 10 & 4 \\ s_4 & 5 & 11 \end{array}$$

The demand of the firm when its type index is high first order stochastically dominates the demand of the firm when its type index is lower. The demand of the firm when its type is s_3 (s_4, resp.) also strictly dominates type s_1 (s_2, resp.). However, there is a non-revealing equilibrium in which the firm's price and profit is $24/4 = 6$. This equilibrium is supported by any *degenerated* belief off the equilibrium path. The point is that a high index firm's type would like the consumer to believe that its type belong to $\{s_3, s_4\}$, but not that its type is exactly s_3 or s_4. Considering degenerated beliefs prevents the firm to deviate from the NRE and to disclose $\{s_3, s_4\}$.

We will see now that statewise monotonicity with respect to the firm's type is a too strong condition for uniqueness, and provide a much weaker condition. To motivate this weakest condition we consider the two following examples.

Example 4 (Statewise Monotonicity with Respect to the Consumer's Type) Consider the following match function with zero cost for the firm:

$$r = \begin{array}{|ccc|} & t_1 & t_2 \\ s_1 & 1 & 4 \\ s_2 & 2 & 3 \end{array}$$

This match function is not statewise monotonic with respect to the firm's type, but it is statewise monotonic *with respect to the consumer's type* in the sense that $r(s, t') > r(s, t)$ for all $s \in S$ and $t' > t$, and $r(s, t) \neq r(s', t)$ for all $s \neq s'$, $t \in T$. It is easy to see that whatever the (strictly positive) priors (correlated are not) the unique equilibrium is fully revealing. To get existence it suffices to consider the following worst case type for $m \in M(s_1) \cup M(s_2)$:

$$wct(p,m) = \begin{cases} s_2 & \text{if } p > 2, \\ s_1 & \text{if } p \leq 2. \end{cases}$$

To get uniqueness, suppose on the contrary that s_1 and s_2 pool. If both t_1 and t_2 buy then the profit is given by $p \in (1,2)$ and type s_2 can profitably deviate by revealing its type and applying the price $p' = 2$, in which case both types of the consumer still buy and the profit is $p' = 2 > p$. If only t_2 buys then the expected profit of type s_1 is given by $\Pr(t_2 \mid s_1)p$ with $p \in (3,4)$ and type s_1 can profitably deviate by revealing its type and applying the price $p' = 4$, in which case consumer t_2 still buys.

Existence and uniqueness of the fully revealing equilibrium is also guaranteed under statewise monotonicity with respect to the consumer's type whatever the priors.

Proposition 4 *If the match is statewise monotonic with respect to the consumer's type, then the advertising game has a unique sequential equilibrium outcome, which is fully revealing.*

Proof. Existence is obtained by considering the (price dependent) worst case type that minimizes the *number* of consumer types buying at price p:

$$wct(p,m) = \arg \min_{s \in M^{-1}(m)} |\{t \in T : r(s,t) \geq p\}|.$$

Since consumer's types are ordered in their willingness to pay independently of the firm's type, we have

$$\{t \in T : r(s,t) \geq p\} \subseteq \{t \in T : r(wct(p,m),t) \geq p\},$$

and thus $D(p,m,s) \leq D(p,m_s,s)$ for every $s \in S$, $m \in M^{-1}(m)$ and $p \in \mathbb{R}_+$. To show that there is no other equilibrium outcome assume by way of contradiction that types in $S' \subseteq S$, with $|S'| \geq 2$, send the same signal (p,m). Then, the firm's type $s \in S'$ with the highest match with the marginal consumer can strictly increase its price and keep all consumer types by disclosing its type, a contradiction. ∎

Example 5 Consider the following match function with zero cost for the firm:

$r =$

	t_1	t_2	t_3
s_1	5	4	3
s_2	6	1	2

This match function is neither statewise monotonic with respect to the firm's type nor statewise monotonic with respect to the consumer's type. Nevertheless, the unique equilibrium is fully revealing whatever the priors, but the argument is a little more complicate than in the previous example. To see that there is a fully revealing equilibrium it suffices to consider the following worst case type for $m \in M(s_1) \cup M(s_2)$:

$$wct(p,m) = \begin{cases} s_2 & \text{if } p \leq 4, \\ s_1 & \text{if } p > 4. \end{cases}$$

To see that this equilibrium outcome is unique, suppose on the contrary that s_1 and s_2 pool. If t_3 buys, then to prevent s_1 from revealing its type the price should not be smaller than 3, a contradiction with the fact t_3 buys the good. If only t_1 and t_2 buy, then to prevent s_1 from revealing its type the price should not be smaller than 4, a contradiction with the fact t_2 buys the good. If only t_1 buys, then s_2 deviates by revealing its type and choosing a price equal to 6. Finally, if only t_2 buys, then s_1 deviates by revealing its type and choosing a price equal to 4. Hence, the unique equilibrium is fully revealing.

The above match function satisfies what we call pairwise monotonicity: for every pair of types of the firm and pair of types of the consumer, the match restricted to this 2×2 type space is either statewise monotonic with respect to the firm's type or statewise monotonic with respect to the consumer's type. More precisely:

Definition 1 The match function $r(\cdot,\cdot)$ is *pairwise monotonic* if for every pair of types of the firm $(s,s') \in S^2$, $s \neq s'$, and every pair of types of the consumer $(t,t') \in T^2$, $t \neq t'$, one of the following conditions hold:

$$(i) \begin{cases} r(s,t) > r(s',t) \\ r(s,t') > r(s',t'); \end{cases} \qquad (ii) \begin{cases} r(s',t) > r(s,t) \\ r(s',t') > r(s,t'); \end{cases}$$

$$(iii) \begin{cases} r(s,t) > r(s,t') \\ r(s',t) > r(s',t'); \end{cases} \qquad (iv) \begin{cases} r(s,t') > r(s,t) \\ r(s',t') > r(s',t). \end{cases}$$

Of course, any match which is statewise monotonic in the firm's type or in the consumer's type is pairwise monotonic. The match function of Example 5 also satisfies pairwise monotonicity, as well as the following match functions:

$r =$

	t_1	t_2
s_1	5	8
s_2	6	7
s_3	4	1
s_4	3	2

, $r =$

	t_1	t_2	t_3
s_1	8	6	3
s_2	6	5	4
s_3	7	1	2

$r =$

	t_1	t_2	t_3
s_1	5	11	10
s_2	6	7	8
s_3	4	1	9
s_4	3	2	7

The following theorem is the main result of our paper. We say that the match is *generic* if $r(s,t) \neq r(s',t)$ for every $s \neq s'$.

Theorem 1 *If the match is pairwise monotonic and generic, then the advertising game has a unique sequential equilibrium outcome, which is fully revealing.*

Proof. *Uniqueness.* Assume that there exists an equilibrium where a set of types $S' \subseteq S$, with $|S'| \geq 2$, pool (i.e., choose the same signal (p,m)). Let $T' \subseteq T$ be the set of consumer's types that buy the good after the signal (p,m). Since for every $s \in S$ there exists $t \in T$ such that $r(s,t) > \gamma(s)$, we necessarily have $T' \neq \emptyset$ and $p > \gamma(s)$ for all $s \in S'$. When $|T'| = 1$, the standard unravelling argument shows that pooling is impossible, so let $|T'| \geq 2$. Denote by $R' = (r(s,t))_{(s,t) \in S' \times T'}$ the matrix of matches restricted to types in $S' \times T'$. Notice that the price applied by firms in S' is such that

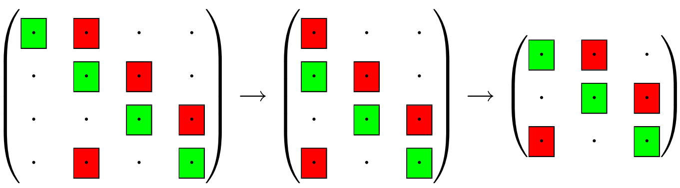
$$p \leq \min_{t \in T'} E(r(\boldsymbol{s},t) \mid \boldsymbol{s} \in S').$$

For every $s \in S'$ and $t \in T'$ let

$$\underline{t}(s) = \arg\min_{t \in T'} r(s,t) \quad \text{and} \quad \bar{s}(t) = \arg\max_{s \in S'} r(s,t).$$

That is, $\underline{t}(s)$ is (possibly a selection of) the smallest match in the s line of R' (call it a red cell) and $\bar{s}(t)$ is the highest match in the t column of R' (call it a green cell). Equilibrium conditions imply that those (green and red) cells cannot by confounded; that is, there is no pair (s,t) such that $(s,t) = (\bar{s}(t), \underline{t}(s))$. Otherwise, firm s can profitably deviate by revealing its type and applying the price $r(s,t) = r(\bar{s}(t),t) = \max_{s' \in S'} r(s',t) > p$, for which all consumers in T' buy since $r(s,t) = r(s,\underline{t}(s)) \leq r(s,t')$ for every $t' \in T'$.

Now, in the matrix R' we delete iteratively any line without a green cell, i.e., any line s such that $s \neq \bar{s}(t)$ for every $t \in T'$, and any column without a red cell, i.e., any column t such that $t \neq \underline{t}(s)$ for every $s \in S'$. Denote by R^* the remaining matrix of matches, and S^* and T^* the corresponding sets of types of the firm and of the consumer. Notice that this matrix has at least two lines and two columns because green and red cells cannot be confounded. This procedure is illustrated below.

$R' =$  $= R^*$

Let

$$r^* = \max_{(s,t)\in S^*\times T^*} r(s,t),$$

be the highest match in the matrix R^*. By definition r^* necessarily corresponds to a green cell, i.e., $r^* = r(\bar{s}(t^*), t^*)$ for some $t^* \in T^*$. Let $s^* = \bar{s}(t^*)$ so that $r^* = r(s^*, t^*)$. By the construction of R^* (any column having a red cell and any line having a green cell) there exists $s' \in S^*$ such that $\underline{t}(s') = t^*$ and $t' \in T^*$ such that $\bar{s}(t') = s'$ as illustrated below:

$$\begin{pmatrix} \boxed{r^* = r(s^*, t^*)} & \boxed{\cdot} & r(s^*, t') \\ \cdot & \boxed{\cdot} & \boxed{\cdot} \\ \boxed{r(s', t^*)} & \cdot & \boxed{r(s', t')} \end{pmatrix}$$

Finally, $r(s^*, t') < r(s^*, t^*) \equiv \max_{(s,t)\in S^*\times T^*} r(s,t)$, so the match is not pairwise monotonic.

Existence. Consider a complete disclosure strategy of the firm such that a message $m_s \in M(s)$ is sent by each type of the firm, with $m_s \notin M(s')$ for all $s' \neq s$. We construct a worst case type function such that no firm's type has an incentive to deviate from this complete disclosure strategy. Consider a signal (p, m) off the equilibrium path, and consider the matrix of matches restricted to firm types in $M^{-1}(m)$, with $|M^{-1}(m)| \geq 2$. By the first part of the proof we know that if the match is pairwise monotonic, then the minimum match of some firm's type coincides with the maximum match of some consumer's type; that is, at least one red cell coincides with some green cell. Denote by (s_1, t_1) the corresponding cell, and $r_1 = r(s_1, t_1)$ the corresponding value. If $p > r_1$ then type t_1 never buys whatever his beliefs, and we apply the same construction to the matrix of matches without t_1. If $p \leq r_1$ then type s_1 has no incentive to deviate from full disclosure to (p, m) and we let $wct(p, m) \neq s_1$ (i.e., we consider a belief system such that $\beta(s_1 \mid t, m, p) = 0$ for every t). If $|M^{-1}(m) \backslash \{s_1\}| = 1$ the argument is complete. Otherwise we consider the new matrix of matches without the s_1 line. We apply the same reasoning as above to this new matrix: let (s_2, t_2) be a green and red cell of this matrix, let $r_2 = r(s_2, t_2)$ be the corresponding value, and construct the worst case types as above. If $p > r_2$ then type t_2 never buys whatever his beliefs, and we apply the same construction to the matrix of matches without t_2. If $p \leq r_2$ then type s_2 has no incentive to deviate from full disclosure to (p, m) and we let $wct(p, m) \neq s_2$. If $|M^{-1}(m) \backslash \{s_1, s_2\}| = 1$ the argument is complete. Otherwise, we apply the same construction up to (s_k, t_k) such that $|M^{-1}(m) \backslash \{s_1, s_2, \ldots s_k\}| = 1$. ■

The converse of the proposition is also true in the following sense:

Proposition 5 *Assume that the match is positive and generic but not pairwise monotonic for some pairs of consumer and firm' types, and that there is no cost. Then, there exist independent and strictly positive priors such that the advertising game has an equilibrium outcome which is not fully revealing, and strictly better for the firm than the fully revealing equilibrium.*

Proof. If the match is generic and strictly positive but not pairwise monotonic then there exist two pairs $(s_1, t_1) \in S \times T$ and $(s_2, t_2) \in S \times T$, with $r(s_1, t_1) = a$, $r(s_1, t_2) = b$, $r(s_2, t_1) = c$ and $r(s_2, t_2) = d$ such that $a > b, c \geq 0$ and $d > b, c \geq 0$, as below:

$$\begin{pmatrix} a & > & b \\ \vee & & \wedge \\ c & < & d. \end{pmatrix}$$

For simplicity, we put all positive probabilities on $\{s_1, s_2\}$ and $\{t_1, t_2\}$ but the proof clearly extend with small enough probabilities on all other type profiles. Consider the following priors:

$$\sigma(s_1) = 1 - \sigma(s_2) = \sigma = \frac{d - c}{a + d - c - b} \in (0, 1),$$

and

$$\tau(t_1) = 1 - \tau(t_2) = \tau = \frac{d}{a + d} \in (0, 1).$$

Assume that types s_1 and s_2 pool and choose the price

$$p = \frac{ad - cb}{a + d - c - b}.$$

Since $p = a\sigma + c(1 - \sigma) = b\sigma + d(1 - \sigma)$ both types t_1 and t_2 of the consumer buy. The firm does not deviate by revealing its type if

$$p \geq \max\{b, c, a\tau, d(1 - \tau)\} = \max\{b, c, \frac{ad}{a + d}\}.$$

It is easily shown that this inequality is satisfied by $a > b, c$ and $d > b, c$. The inequality is even strict so that the equilibrium is strictly better for the firm than the FRE. ■

Notice that if the match is not pairwise monotonic for some negative matches then the unique equilibrium may be fully revealing. For example, when

$$r = \begin{array}{|ccc|} & t_1 & t_2 \\ s_1 & 1 & -2 \\ s_2 & -2 & 1 \end{array}$$

then unique equilibrium is fully revealing whatever the independent priors.

4 Appendix

Example 6 (*Non-existence of a fully revealing equilibrium due to correlated types.*) Assume that costs are zero ($\gamma(s_1) = \gamma(s_2) = 0$) and consider the following match function and correlation matrix, where we assume $0 < \rho < 2\varepsilon < 2$:[9]

$$r = \begin{array}{|ccc|} \hline & t_1 & t_2 \\ s_1 & \rho & 2 \\ s_2 & 2 & \rho \\ \hline \end{array} \qquad \mu = \begin{array}{|ccc|} \hline & t_1 & t_2 \\ s_1 & \frac{1-\varepsilon}{2} & \frac{\varepsilon}{2} \\ s_2 & \frac{\varepsilon}{2} & \frac{1-\varepsilon}{2} \\ \hline \end{array}$$

If the firm fully reveals its type to the consumer, then it sets the price $p = 2$ and gets a profit equal to 2ε whatever its type. We show that if ε is small enough, then the firm has an incentive to deviate from full revelation. We have to show that whatever the consumer's belief after a deviation by the firm, s_1 or s_2 gets a profit which is strictly larger than 2ε.

Notice that the consumer's belief off the equilibrium path may depend on the observed price. This allows a large flexibility to punish the firm if it deviates from complete information disclosure. In this example, however, a deviation to the same price $p = 2\varepsilon + (1 - \varepsilon)\rho$ will be profitable for at least one of the firm's type whatever the consistent belief of the consumer. The idea is that this price is accepted by one type of the consumer with large probability $(1 - \varepsilon)$ whenever a type s_i of the firm makes the consumer believe that it is the other type s_{-i}, for $i = 1, 2$. Under some conditions on the game, the fact that two types want to imitate each others is sufficient to prevent full revelation of information (see, e.g., the "single crossing" property in Giovannoni and Seidmann, 2007), but is not sufficient in our framework since we also have to consider non-degenerated beliefs off the equilibrium path. For example, when $\rho = 0$, by setting the price to $2(1 - \varepsilon)$, each type of the firm would be strictly better off when the consumer believes that it is the other type, but a fully revealing equilibrium can be constructed by setting the consumer's belief off the equilibrium path to his prior belief.

We first observe that if t_1 or t_2 buys the good at price $p = 2\varepsilon + (1 - \varepsilon)\rho$, then the expected profit of one of the two types of the firm is at least

$$\Pi = (1 - \varepsilon)(2\varepsilon + (1 - \varepsilon)\rho),$$

so the deviation would be profitable for one of those types whenever $\Pi > 2\varepsilon$, i.e., $\rho > \frac{2\varepsilon^2}{(1-\varepsilon)^2}$. This is possible under the assumption that $\rho < 2\varepsilon$ whenever ε is small enough (take $\frac{\varepsilon}{(1-\varepsilon)^2} < 1$).

It remains to check that at least one of the consumer's type t_1 or t_2 always accepts to buy at price p off the equilibrium path. Let $m \in M(s_1) \cap M(s_2)$ be a message available to the firm whatever its type, and let μ_i be the consumer's belief that the firm's type is s_1 when the consumer's type is t_i and he observes the signal (p, m) off the equilibrium path. The maximum price under which the consumer accepts to buy the good his $\bar{p}_1 = 2(1 - \mu_1) + \rho\mu_1$ when his type is t_1 and $\bar{p}_2 = 2\mu_2 + \rho(1 - \mu_2)$ when his type is t_2. The firm does not deviate from full revelation by sending (p, m) only if $\bar{p}_1$ and $\bar{p}_2$ are both smaller than p, which yields $\mu_1 > 1 - \varepsilon$ and $\mu_2 < \varepsilon$. However this belief system is not consistent since it cannot be obtained by Bayes' rule whatever the strategy of

[9] Notice that this match is not pairwise monotonic. Otherwise, a fully revealing equilibrium would always exist by our theorem.

the firm.

Example 7 (*Price-dependent inferences off the equilibrium path.*) Consider the following match function

$$r = \boxed{\begin{array}{ccc} & t_1 & t_2 \\ s_1 & 3 & 0 \\ s_2 & 2 & 2 \end{array}}$$

with a uniform prior probability distribution and type dependent costs $\gamma(s_1) = \gamma_1 < 3$ and $\gamma(s_2) = \gamma_2 < 2$. Under full information disclosure, the firm of type s_1 sets its price to 3 and gets a profit equal to $\frac{3-\gamma_1}{2}$, and the firm of type s_2 sets its price to 2 and gets a profit equal to $2 - \gamma_2$. Consider a belief μ on s_1 for the consumer off the equilibrium path that does not depend on the price chosen by the firm,[10] so that consumer t_1's willingness to pay is $2 + \mu$ and consumer t_2's willingness to pay is $2 - 2\mu$. For the firm not to deviate we should have

$$\frac{3-\gamma_1}{2} \geq (2-2\mu) - \gamma_1 \text{ and } 2 - \gamma_2 \geq \frac{2+\mu-\gamma_2}{2},$$

i.e., $\frac{1-\gamma_1}{4} \leq \mu \leq 2 - \gamma_2$, which is impossible whenever $\gamma_1 < 4\gamma_2 - 7$.

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[10] Since types are independent, belief consistency requires that the consumer's belief is independent of his own type t.

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