Humouring both parties: a model of two-sided reputation*

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Abstract

We consider the problem of a certification agency that needs to attract both sellers and buyers in order to exercise its activity. By reporting more precise information on the seller's good, the agency boosts the market participation of buyers but may negatively impact the welfare of sellers who are also concerned with obtaining a positive recommendation. Therefore, its revenue may be maximized when it provides information that is neither too precise nor too noisy. In a dynamic setting, we examine how the desire to establish a reputation with respect to both sides of the market affects information production. Reputational concerns have an ambiguous effect on the precision of reports. When the perceived credibility of the agency is deficient, reputation has a disciplining effect and the precision of information improves. However, when the agency has a good reputation, it tends to be lenient in order to attract future sellers.

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1 Introduction

Recent spectacular failures in the functioning of the financial system have shed light on the activities of certification agencies. From the active role played by Arthur Andersen in the falsification of Enron's financial statements to the responsibility of rating agencies in the general underestimation of the risk of default of asset-backed securities, intermediaries which were supposed to improve the transparency of markets have proven deficient in many circumstances. The damages caused by inadequate transmission of information among market participants can be extremely severe: misallocation of capital, crisis of confidence, liquidity dry-up, fraud, not to mention repercussions on the economy as a whole. These episodes are all the more striking as the central role of certification agencies in capital markets stems for a large part from regulatory requirements.

The conflict of interest faced by certification agencies, and more generally by information intermediaries, has been an important concern for economists. Because information is by essence a public good, business models where buyers pay for information are difficult to sustain (Admati and Pfleiderer [1]). It is indeed hardly possible to preclude the free dissemination of information among buyers, so potential profits are not sufficient to provide incentives to acquire high-quality information. This explains the prevalence of a business model in which the seller pays in order to obtain certification. Although it solves the public good problem, this model carries an inherent conflict of interest: while intermediaries are supposed to provide reliable information, the bulk of their revenue comes from sellers who have a clear preference for obtaining favourable reports.

Reputation has often come as the main counter-argument of certification agencies. The value of an opinion is closely related to the reputation of the information provider, which needs to be built up over time through a substantial track record of accurate predictions. A seller would never hire an intermediary perceived as manipulating his recommendations, since buyers would strongly discount any positive opinion it would issue. The threat of losing one's reputational capital allegedly constitutes a

high-powered incentive to provide high-quality information.[1] This idea is formalized in models of reputation in which a strategic intermediary faces a trade-off between incentives to distort information to gain immediate monetary rewards and the long-term costs of losing credibility (Benabou and Laroque [5], Mathis, McAndrews and Rochet [26]). However, for this tension to be at play, it is critical that the short-term profits of the intermediary depend on its report: its immediate revenue should increase when it distorts information. This issue is in fact a central piece of the debate on rating agencies' business model. Rating agencies have so far received payments which are *de facto* contingent on the rating they issue. Their fee corresponds to a percentage of the amount of debt issued by their client. Since a better rating increases the chances for the issuer to launch a debt offering and impacts the size of the issuance, agencies have a direct incentive to bias their ratings upwards. However, such incentives should disappear as soon as contingent payments are restricted. The plan signed by the three main rating agencies and New York State Attorney General Andrew Cuomo, which imposes upfront payments, should therefore effectively alleviate the conflict of interest faced by agencies.

We argue that banning contingent payments might not prove sufficient to discipline intermediaries, because incentives to be lenient are pervasive through reputational concerns. We consider a model where the intermediary extracts the expected surplus it creates for the seller through an upfront payment. This surplus reflects conflicting expectations about the agency's behaviour. On the one hand, the seller prefers a more reputable intermediary because credibility helps convince buyers when the report is positive. On the other hand, a higher reputation for accuracy decreases the likelihood for the seller to get a good evaluation in the first place. As a result, the intermediary should avoid being perceived as too lenient or too strict, its ideal reputation lying exactly in the median point. In a static game, upfront payments ensure truth-telling since manipulating information does not affect the profit the intermediary can extract

[1] Often quoted as a precedent is the case of Arthur Andersen, which disappeared after their participation in the falsification of Enron's financial statements has been proved. This example notwithstanding, reputational punishments following misbehaviour loom relatively low, especially when compared with the associated welfare losses.

from issuers. However, things change dramatically as soon the game is repeated and reputational concerns come into play. Reputation indeed has an ambiguous effect. For agencies perceived as providing poor-quality information, reputation plays a disciplining role. These agencies need to enhance the credibility of future ratings. As a result, they increase the resources they spend in screening products and the overall quality of information rises. As reputation improves, sellers become more concerned with getting a good rating than with credibility, so that agencies actually want to dissipate their reputation and the quality of information falls to even lower than its level in a static game. Interestingly, both of these effects have nothing to do with the current seller the intermediary is dealing with, but are driven by the expectation of increased revenues from future sellers. These results suggest that despite the claim of most raters and auditors, reputational concerns do not always provide correct incentives, and these effects are all the more important as they would survive the elimination of direct incentives to distort information.

More generally, many intermediaries or platforms get paid by their clients for being able to attract some specific audience. Most business models in the media industry follow this pattern: financing mainly occurs through advertising, so that a given medium's revenue critically depends on its capacity to attract a large audience. The fact that a significant share of its profit stems from the advertisers' side creates a well-known conflict of interest: media may be tempted to cover up sensitive information in order not to alienate advertising business opportunities. Rational readers should be aware of such incentives and be somewhat skeptical about the reliability of news, which may cause a decline in readership. Overall, the medium's reputational incentives are conflicting as it would ideally like to develop a good reputation with respect to the two sides it faces.

1.1 Related literature

Our paper mainly relates to the literature on information transmission by an expert. After Sobel [36], many papers have considered information transmission in dynamic

settings in which reputation comes into play. Benabou and Laroque [5] show that reputational concerns do not fully prevent an insider with private information from repeatedly manipulating information or prices when public observers can only imperfectly monitor his behaviour. In a similar vein, Morris [29] shows that reputation may dwarf an expert's incentives to truthfully report his information, leading to possible welfare losses. Reputational concerns are therefore often not sufficient to ensure accurate information transmission. What is more, they may even provide wrong incentives that possibly lead to market breakdown, as shown by Ely and Välimaki [12], where the demand for services provided by an expert may collapse when he is expected to build up a reputation.

Recently, much attention was devoted to the market of rating and certification agencies. The disciplining role of reputation is central in Mathis, McAndrews and Rochet [26] and Mariano [25]. In the former paper, the fact that rating agencies get their revenue from issuers gives them a immediate incentive to provide excessively generous ratings. These incentives can only partly be counteracted by reputational concerns, especially when reputation is already well-established. The latter paper addresses the issue of competition among agencies and shows that competition increases temptation to follow public signals and to ignore private relevant information. Recently, a stream of papers has focused on the behaviour of issuers, in particular on the issue of rating shopping. Bolton, Freixas and Shapiro [7] emphasize that a monopoly is socially more efficient than a duopoly in case issuers shop for ratings. Sangiorgi, Sobokin and Spatt [34] focus on the winner's curse associated with rating shopping. In Skreta and Vedkamp [35], rating shopping arises when the complexity of assets increases since the agencies' predictions are more likely to differ. Finally, Faure-Grimaud, Peyrache and Quesada [15] examine contracting between an issuer and a rating agency. They show that the optimal renegotiation-proof contract can be implemented by transferring the ownership of the rating to the agency or to the issuer depending on the structure of the rating industry.

Beside the issue of information transmission, our paper also borrows from the litera-

ture on two-sided markets.[2] In particular, it takes the view that the ability of platforms to attract two different partners with potential conflicting interests and to "keep both sides on board" is critical in these markets. The presence of participation externalities typically alters the pricing structure. Our paper does not focus on pricing issues but instead examines how reputation may be built up to attract both sides of the market, notably when there are restrictions on prices.

The question of strategic information transmission to multiple audiences has been raised in Farrell and Gibbons [14], who compare public and private cheap talk communication to two independent receivers in a static context. Other papers related to ours involve signalling to multiple audiences. For instance, Gertner, Gibbons and Scharfstein [17] examine the problem of a firm which tries to simultaneously signal its profitability to the capital and to the product markets. In a similar spirit, Austen-Smith and Fryer [3] consider the problem of an African-American who may want to invest in education to signal his ability to the job market but does not want to appear as "acting White" vis-à-vis his community. However, to the best of our knowledge, no paper has so far explicitly modeled reputation-building in a two-sided context.

2 The model

2.1 Players

We consider a setting with three players: a seller, a buyer and an intermediary. The seller owns a product of uncertain quality which he wishes to sell to a buyer (or a group of buyers) whose demand depends on the expected quality of the good. The intermediary (she) is an expert able to gather and transmit information on the quality of the product.[3]

[2] See Rochet and Tirole [33] for a review.

[3] On financial markets for instance, firms or banks usually need certification from a rating agency to convince potential investors of the quality of the security they issue. More generally, our setting captures any product market where sellers resort to certification agencies or standard-setting organizations (Lerner and Tirole [23]).

2.2 The intermediary: technology

The quality θ of the product is initially unknown to all players. It can be good ($\theta = \theta_G$) or bad ($\theta = \theta_B$) with $Pr(\theta = \theta_G) = q$. The intermediary receives a (potentially imperfect) signal $\sigma \in \{\varnothing, B\}$ on θ.

The precision of this signal depends on the amount of effort e she exerts. By raising e, she increases the probability of detecting a low-quality product. Formally,

$$Pr(\sigma = \varnothing | \theta = \theta_G) = 1 \text{ and } Pr(\sigma = B | \theta = \theta_B) = e.$$

Stated differently, a signal $\sigma = B$ perfectly reveals a bad quality, whereas no signal ($\sigma = \varnothing$) only imperfectly provides good news. A bad signal can be interpreted as hard evidence of bad quality, no such hard evidence existing for good products. The role of the intermediary is to devote resources in order to look for such pieces of evidence. When the product is of high quality, the level of effort is irrelevant because she always receives no signal ($\sigma = \varnothing$). However, providing effort e allows to detect hard information of bad quality with probability e.[4]

The intermediary has an intrinsic preference for reporting accurate information. She gets positive utility $u > 0$ if and only if the signal she reports matches the quality of the good.[5] However, information gathering comes at a cost which varies across intermediaries.[6] Effort has a convex cost $\frac{1}{2}ce^2$ where $c \in \{c_L, c_H\}$.[7] The cost of effort may be affected by investments in technologies or the hiring of qualified staff, which typically varies across intermediaries. Cost differences also reflect the opportunity cost of devoting scarce resources to some cases and giving up other business opportunities.[8]

[4]This asymmetry structure is not essential for the results, what matters for our theory is that the probabiliy of learning "good news" on the product is a decrasing function of the intermediary's effort, as will be made clear later.

[5]u can for instance capture the reputational concern of some employee of the agency on the labour market, in which more talented types have higher career prospects. In this respect, u can be thought as a reputational stake at a different level than the intermediary considered as an organization.

[6]Assuming that intermediaries differ across their intrinsic preference for telling the truth or across efficiency of effort would be formally equivalent.

[7]On that dimension, our model differs from reputational cheap talk models such as Benabou and Laroque [5] or Mathis, McAndrews and Rochet [26], where information comes at no cost but may be concealed for strategic reasons.

[8]The swift multiplication of asset-backed securities has for instance greatly widened the market of

The cost function of the intermediary (her type) is private information. Let ρ denote the probability that the intermediary is a low-cost type ($c = c_L$). We refer to ρ as the reputation of the intermediary.

We assume that both effort and the signal collected by the intermediary are unobservable by other players. What they observe is only the report $r \in \{B, G\}$ on θ made by the intermediary.

2.3 The intermediary: choice of effort in the static game

We first consider the intermediary's decision in the static game in which the benefits of effort, namely a higher probability of reporting the truth, are traded only against its cost. This serves both as a building block for the repeated game in which reputational concerns come into play and as a benchmark for the behaviour of the players in the absence of reputation effects.

Given that she has an intrinsic preference for accurate reports, the intermediary always follows her signal, i.e. reports $r = G$ after $\sigma = \varnothing$ and $r = B$ after $\sigma = B$.

An intermediary of type c_L thus maximizes

$$W^L(e_L) = qu + (1-q)e_L u - \frac{1}{2}c_L e_L^2.$$

An intermediary of type c_H maximizes

$$W^H(e_H) = qu + (1-q)e_H u - \frac{1}{2}c_H e_H^2.$$

Assuming interior solutions, we derive

$$0 < \frac{(1-q)u}{c_H} = e_H^* < e_L^* = \frac{(1-q)u}{c_L} < 1.$$

a few rating agencies with limited capacities.

2.4 Seller's surplus

Since low-quality goods are unattractive to buyers, sellers need certification from the intermediary in order to market their goods. Indeed, we assume that the expected quality of the seller's good is such that buyers have a priori no demand. Furthermore, since a bad rating identifies the product as being of low quality with certainty, sellers get positive surplus only after a report $r = G$, which occurs with probability $p(\rho)$. When the low-cost (resp. high cost) agency provides effort e_L (resp. e_H), this probability equals:

$$p(\rho) = q + (1-q)[\rho(1-e_L) + (1-\rho)(1-e_H)].$$

The surplus obtained by the seller is a function $\Pi(q_G)$, where q_G is the reliability of a report $r = G$:

$$q_G(\rho) = Pr(\theta = \theta_G | r = G) = \frac{q}{q + (1-q)[\rho(1-e_L) + (1-\rho)(1-e_H)]} = \frac{q}{p(\rho)}.$$

So the expected surplus of the seller is

$$S(\rho) = p(\rho)\Pi[\frac{q}{p(\rho)}].$$

Buyers prefer high-quality goods, so $\Pi(.)$ is nondecreasing in q_G. Furthermore, since $e_L > e_H$, $p(\rho)$ is decreasing in ρ, whereas $q_G(\rho)$ increases with ρ. The effect of an increase in ρ on the surplus of sellers is therefore ambiguous. There are both a "credibility effect"and a "disclosure effect". More reputable agencies issue more credible ratings, which increases seller surplus in case they get a favourable recommendation; on the other hand, these agencies are more likely to detect low-quality projects and thus more likely to issue negative ratings, in which case sellers make no profit. The central assumption of the paper is that both effects alternatively prevail. For low values of the reputation, the intermediary issues good ratings with a high probability, but these ratings are unreliable and generate little surplus for the seller. Consequently, the seller would benefit from an increase in the intermediary's reputation. However, when reputation becomes high enough, the seller becomes more concerned with the probability of

getting a good evaluation rather than with the credibility of the intermediary's reports. Formally, we assume that $S(\rho)$ is first nondecreasing and then nonincreasing in ρ on $[0,1]$. This specification is of particular interest as it embeds a tension intermediaries are very commonly confronted with. They should not be seen as too close to buyers: by providing very accurate information, they might lose profit from sellers who are worried that their probability of getting a good report could fall. On the other hand, they should maintain a sufficient level of credibility to be able to raise the demand of buyers and generate profit for sellers.

We provide three illustrating examples in which the surplus created by the intermediary is non-monotonic in her reputation.

2.4.1 Example 1: Buyers with random outside option

The seller owns a good which he can sell at unit price to buyers. Following a report $r = G$, buyers have expected valuation

$$q_G(\rho)\theta_G + [1 - q_G(\rho)]\theta_B.$$

There is a continuum of buyers endowed with a random outside option ω, where ω has density f and cumulative distribution F. Upon observing a good report, buyers purchase one unit of the good with probability $F[q_G(\rho)\theta_G + (1 - q_G(\rho))\theta_L - 1]$. We assume that ω has decreasing density, which implies that it is increasingly difficult to attract additional buyers as the credibility of certification increases.

The seller's expected surplus equals

$$S(\rho) = p(\rho)F[\frac{q}{p(\rho)}(\theta_G - \theta_B) + \theta_B - 1].$$

When f is decreasing, the expected profit of the seller S is concave in ρ and, under simple assumptions, S is hump-shaped.

2.4.2 Example 2: Costly collateral pledging

Consider a risk-neutral firm endowed with a project that requires an investment cost I. The project returns $X + b$ with probability θ and 0 otherwise. X is the pledgeable income i.e. the proceeds that can be credibly promised to an outside investor as repayment for his initial outlay. However, b cannot be pledged to external investors. It can be interpreted as a reduced form for the rent that has to be left to the entrepreneur in a moral hazard setup (see Holmström and Tirole [19]).

The firm faces a credit constraint: $\theta X - I < 0$. This implies that the project cannot be financed unless the firm pledges some collateral. The firm has no cash at hand, but has illiquid assets in place that it values $\overline{A}$ under its management. However, the liquidation value of these assets is uncertain and can in particular be lower than operating value.[9] Specifically, we assume that the liquidation value is given by $\tilde{A} \in \left\{0, \overline{A}\right\}$, with $Pr(\tilde{A} = \overline{A}) = q$.

The rating agency publishes a report $r \in \{G, B\}$ on the recovery value $\tilde{A}$ in case of default, according to the technology described in 2.2. Given a favourable report $r = G$, the fraction α of assets the firm has to collateralize is such that

$$I = \theta X + \alpha(1-\theta)q_G(\rho)\overline{A},$$

and the expected value of the firm is[10]

$$\theta(b + \overline{A}) + (1-\theta)(1-\alpha)\overline{A} = \theta b + \overline{A} - \frac{I - \theta X}{q_G(\rho)}.$$

If instead the firm decides not to invest, the status-quo value of its assets is $\overline{A}$. Overall, recalling that $q_G(\rho) = \frac{q}{p(\rho)}$, the net expected surplus created by the rating agency is

$$S(\rho) = p(\rho)[\theta b - \frac{p(\rho)}{q}(I - \theta X)],$$

[9] Firm-specific assets are difficult to collect, sell (lemons problem) or redeploy. Therefore, their value to a lender often accounts for only a fraction of their value to the borrower (Coco [8]).

[10] Implicitly, the lender always liquidates in case of default, although he does not gain anything when the liquidation value is zero. Assuming that liquidation value is slightly positive in the bad state would make liquidation strictly optimal for the lender.

where $p(\rho)$ is the probability that the agency issues a report $r = G$. It is easily shown that S is concave in ρ. Besides, for properly chosen values of e_L and e_H, $S'(0) > 0 > S'(1)$, so that S is hump-shaped.

Intuitively, for low values of ρ, the marginal benefit of decreasing the amount of collateral the firm has to pledge, hence the liquidation cost, is high compared to the expected loss from decreasing the likelihood of a good rating. As the credibility of the agency improves, the firm becomes more concerned with financing the project and enjoying b in case of success than with the cost of financing.

2.4.3 Example 3: Credit rationing and cross-subsidies

Let us consider a similar project but assume now that the firm has no collateral and that there is uncertainty about the probability of success θ:

$$\theta \in \{\theta_G, \theta_B\} \text{ with } \theta_B < \theta_G \text{ and } Pr(\theta = \theta_G) = q.$$

We make the following set of assumptions:

$$\theta_B(X + b) > I \tag{1}$$

$$\theta_G X > I \tag{2}$$

$$[q\theta_G + (1 - q)\theta_B]X < I \tag{3}$$

Assumption (1) states that the project always generates positive social surplus, even when it is known to be of low quality. Assumption (2) states that good projects can be financed. Assumption (3) states the project cannot be financed *ex ante* because low-quality projects do not generate enough pledgeable income.

The rating agency issues a report $r \in \{G, B\}$ on the quality θ of the project. From (1) and (2), there exists a threshold $0 < \hat{q} < 1$ such that $[\hat{q}\theta_G + (1 - \hat{q})\theta_B]X = I$. Consequently, there exists $\hat{\rho}$ such that the project can be financed following a good rating if and only if $\rho > \hat{\rho}$.[11] Whenever $\rho \geq \hat{\rho}$, the expected profit of the issuer given

[11] Very simple assumptions on e_L and e_H ensure that $\hat{\rho} \in [0, 1]$.

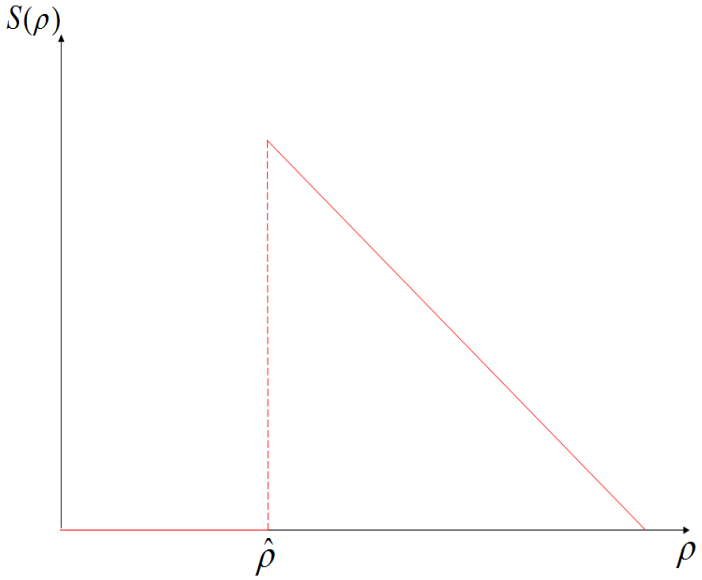
a good rating is

$$[q_G(\rho)\theta_G + (1 - q_G(\rho))\theta_B][X + b] - I,$$

and from an *ex ante* perspective, the expected surplus is therefore

$$p(\rho)\left\{[q_G(\rho)\theta_G + (1 - q_G(\rho))\theta_B][X + b] - I\right\}.$$

Recalling that $q_G(\rho) = \frac{q}{p(\rho)}$ and that $\theta_B(X + b) > I$, it is easy to check that this expression is decreasing in ρ.



When $\rho < \hat{\rho}$, the expected surplus is zero. However, it would be easy to make this expected surplus increasing on $[0, \hat{\rho}]$ without affecting its value in case $\rho \geq \hat{\rho}$. For instance, one can assume that the firm holds stochastic cash reserves that are *ex ante* unknown. In such a case, a marginal increase in the credibility of ratings relaxes the credit rationing constraint in a continuous way.

For low values of ρ, an increase in the reputation enhances the credibility of a good rating, which relaxes the financial constraint. However, once ρ is sufficiently high for good projects to cross-subsidize bad projects, marginal improvements in credibility have no effect on the financial constraint. The only remaining effect is that issuers have

a lower probability of getting a good rating, which reduces their expected welfare.[12]

2.4.4 Diminishing returns of credibility

One sufficient condition to obtain a hump-shaped surplus function is to assume that $\Pi(.)$ is concave in q_G. This captures the idea of diminishing returns: as confidence about the quality of the product becomes higher, its marginal impact on demand and hence on seller surplus decreases. This effect can be generated by regulations or corporate charters. Some categories of investors (mutual funds, pension funds) are for instance restricted to investing in securities with a risk of default below a certain level. For instance, obtaining an "investment-grade" rating allows a security issuer to reach a significantly larger pool of investors, in which case the liquidity of the claim and the cost of credit improve dramatically. Consequently, the incremental benefit from reaching this particular risk level is much higher than the benefit derived from further improvements in risk profile. Likewise, products are often required to meet minimal levels of safety or environmental standards to be approved for market sale. Alternatively, a firm may need a minimal quality to convince buyers to switch from an existing technology, but may not be able to charge for improvements beyond the level that triggers adoption (Farhi, Lerner, Tirole [13]).

The concavity of $\Pi(.)$ readily translates into the concavity of $S(.)$. If $\Pi(.)$ is twice differentiable on $[q, 1]$, then

$$\frac{\partial S}{\partial \rho}(\rho) = (e_H - e_L)[\Pi(q_G(\rho)) - \Pi'(q_G(\rho))q_G(\rho)]$$

and

$$\frac{\partial^2 S}{\partial \rho}(\rho) = (e_H - e_L)^2 \frac{q_G^2(\rho)}{p(\rho)} \Pi''(q_G(\rho)) < 0.$$

We assume that $\Pi(q_G(0)) > 0$, so that even if the intermediary is known to be a high-cost type, the level of effort e_H she provides is still high enough to generate

[12] This reduction in social surplus is reminiscent of the Hirshleifer effect [18], according to which an increase in the precision of information may prevent efficient risk-sharing.

positive surplus for sellers.[13] Furthermore, we assume that:[14]

$$\frac{\Pi'[q_G(1)]q_G(1)}{\Pi[q_G(1)]} < 1 < \frac{\Pi'[q_G(0)]q_G(0)}{\Pi[q_G(0)]} \tag{4}$$

which implies

$$S'(1) < 0 < S'(0) \tag{5}$$

2.5 Payments

We assume that the intermediary is unable to charge both sides and that her only source of income is fees paid by sellers.[15] Importantly, the payment is paid upfront by the seller and thus does not depend on the report made by the intermediary. Furthermore, we assume that the intermediary is able to capture all the expected surplus of the seller given his belief about the intermediary's type.[16]

We restrict attention to upfront payments because payments that are contingent on a good report give an obvious motive for the intermediary to provide inaccurate information. It actually drives the incentive problem of rating agencies in Mathis, McAndrews and Rochet [26] or in Bolton, Shapiro and Freixas [7], where information distortion occurs because the rating agency obtains higher fees when she grants a good rating. In this regard, considered or implemented regulatory reforms aim at suppressing contingent payments in the rating industry, following this claim that they create clear conflicts of interest. The Cuomo plan proposes for instance to ban contingent payments from security issuers to rating agencies. The Sarbanes-Oxley Act pursues similar objectives by imposing a rotation of auditors for public firms. This limits the scope for future business relationships that constitute *de facto* another form of contin-

[13] Alternatively, there could exist a region where the reputation of the intermediary is too low to generate any surplus even following a good rating. This would not qualitatively affect our results.

[14] The function $\frac{\Pi'(q_G)q_G}{\Pi(q_G)}$ can be seen as the elasticity of the seller's demand for credibility, considering that the implicit price for increased credibility lies in a lower probability of being certified. Our assumption states that the demand for credibility is inelastic for low reputations and elastic for high ones.

[15] This is the case for rating agencies that cannot charge investors for free-riding reasons.

[16] This is only for simplicity: assuming that she obtains only a fraction or even any non-decreasing transformation of the seller's expected utility would not affect the results.

gent reward. We deliberately consider the polar case of non-contingent payments in order to examine whether banning conditional payments is sufficient to provide good incentives to the intermediary.

3 Equilibrium in the repeated game

We consider the two-period version of the game.

3.1 Timing

The timing of the game is as follows:

In each period,

1. The seller goes to the intermediary and pays the upfront fee.
2. The intermediary makes an effort choice, learns σ and reports r.
3. Whenever $r = G$, the seller makes a profit $\Pi(q_G)$.

Sellers and buyers live only one period, so that the upfront payment charged to sellers equals their expected payoff in the current period. What changes across periods is the intermediary's reputation: period-2 buyers and sellers observe both the report of the past period and whether this report has proved correct, so they update their beliefs on the intermediary's type in the end of period 1.

3.2 Period 2

In period 2, each type of intermediary selects its static level of effort: as the fees she is able to charge have been paid upfront, her incentive is only driven by the tradeoff between intrinsic preference for accuracy and cost of effort, which yields levels of effort e_L^* and e_H^*.

The upfront payment charged by the intermediary equals $S(\rho_2) = p(\rho_2)\Pi[\frac{q}{p(\rho_2)}]$, where

$$p(\rho_2) = q + (1-q)[\rho_2(1-e_L^*) + (1-\rho_2)(1-e_H^*)] = \frac{q}{q_G(\rho_2)}.$$

Continuation utilities of type $c = c_H$ and $c = c_L$ when reputation is ρ_2 in the beginning of period 2 are given by:

$$V^H(\rho_2) = S(\rho_2) + W^H(e_H^*)$$

and

$$V^L(\rho_2) = S(\rho_2) + W^L(e_L^*).$$

3.3 Period 1

Incentives to look for information change in the first period because forward-looking intermediaries now take into account the impact of their current actions on their continuation utilities through sellers' and buyers' beliefs.

3.4 Updating of beliefs

Let ρ_1 denote the probability that the intermediary is of type $c = c_L$ at the beginning of the first period. Period 2 updated probability ρ_2 is a function of the levels of effort provided by both types (e_H and e_L) and of ρ_1. How beliefs are updated also depends on the report made in period 1 and on the accuracy of this report (observed *ex post*). Assuming that she always reports a bad signal when she learns it (which is true in equilibrium), the intermediary's reputation is computed in the following way (see Figure 1):

- When the intermediary reports a good signal and the good is of high quality, $\rho_2 = \rho_1$. No intermediary can ever receive a signal, independently of her type, when $\theta = \theta_G$, so no information is learnt.

- When the intermediary reports a good signal and the product is of low quality, $\rho_2 = \rho^-(\rho_1, e_L, e_H) = \dfrac{\rho_1(1-e_L)}{\rho_1(1-e_L) + (1-\rho_1)(1-e_H)}$.

- When the intermediary reports low quality, $\rho_2 = \rho^+(\rho_1, e_L, e_H) = \dfrac{\rho_1 e_L}{\rho_1 e_L + (1-\rho_1)e_H}$.

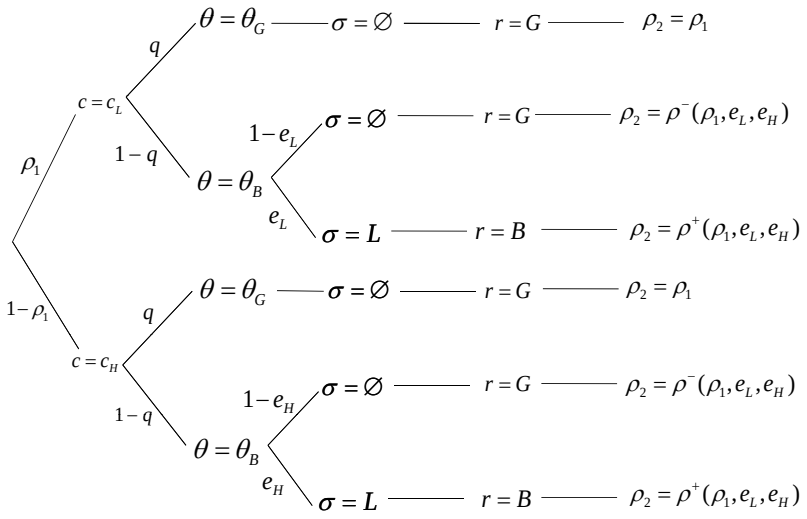


Figure 1: Belief updating

3.5 Equilibrium

Expected payoffs given beliefs ρ^+ and ρ^- are:

$$\tilde{W}_1^H(e_H) = W_1^H(e_H) + q\delta V^H(\rho_1) + (1-q)\delta[e_H V^H(\rho^+) + (1-e_H)V^H(\rho^-)]$$

and

$$\tilde{W}_1^L(e_L) = W_1^L(e_L) + q\delta V^L(\rho_1) + (1-q)\delta[e_L V^L(\rho^+) + (1-e_L)V^L(\rho^-)]$$

For any prior reputation $\rho_1 \in [0,1]$, a perfect Bayesian equilibrium of the game is defined by effort choices e_L^{**} and e_H^{**} and beliefs $\rho^+(\rho_1, e_L^{**}, e_H^{**})$ and $\rho^-(\rho_1, e_L^{**}, e_H^{**})$ such that:

- e_H^{**} solves $\max_{e_H} \tilde{W}_1^H(e_H)$ given $\rho^+(\rho_1, e_L^{**}, e_H^{**})$ and $\rho^+(\rho_1, e_L^{**}, e_H^{**})$[17]

[17]Varying e_L^{**} has no direct impact on type c_H's incentives since it only affects her welfare through beliefs, which she takes as given.

- e_L^{**} solves $\max_{e_L} \tilde{W}_1^L(e_L)$ given $\rho^+(\rho_1, e_L^{**}, e_H^{**})$ and $\rho^+(\rho_1, e_L^{**}, e_H^{**})$

- $\rho^+(\rho_1, e_L^{**}, e_H^{**})$ and $\rho^-(\rho_1, e_L^{**}, e_H^{**})$ are derived from Bayes' rule:

  - $\rho^+(\rho_1, e_L^{**}, e_H^{**}) = \dfrac{\rho_1 e_L **}{\rho_1 e_L^{**} + (1-\rho_1) e_H^{**}}$

  - $\rho^-(\rho_1, e_L^{**}, e_H^{**}) = \dfrac{\rho_1(1-e_L^{**})}{\rho_1(1-e_L^{**}) + (1-\rho_1)(1-e_H^{**})}$.

3.5.1 Equilibrium analysis with one strategic type

In this section, we solve the simpler case where only type $c = c_H$ behaves strategically, the other type being committed to playing its static strategy e_L^* in each period and to reporting her signal. Dropping superscripts for notational simplicity, we derive the following proposition:[18]

Proposition 1 For c/δ and u sufficiently high, the perfect Bayesian equilibrium of the game is unique. The equilibrium level of effort is given by a function $e_H^{**}(\rho)$ such that:

- $e_H^{**}(\rho_1)$ is continuous in ρ_1

- $e_H^{**}(0) = e_H^{**}(1) = e_H^*$

- There exists a threshold $\overline{\rho} \in (0,1)$ such that:

  - $e_H^{**}(\rho_1) > e_H^*$ for $\rho_1 \in (0; \overline{\rho})$,

  - $e_H^{**}(\rho_1) = e_H^*$ for $\rho_1 = \overline{\rho}$,

  - $e_H^{**}(\rho_1) < e_H^*$ for $\rho_1 \in (\overline{\rho}; 1)$.

Furthermore, the intermediary always reports the signal she gets.

Proof In the Appendix.

Three forces are at work in the intermediary's decision problem. Beside her static preference for telling the truth (short-run incentive), she faces reputational incentives

[18] The following result extends to any continuation function V C^1, bounded, increasing and then decreasing, concave and such that V' is bounded.

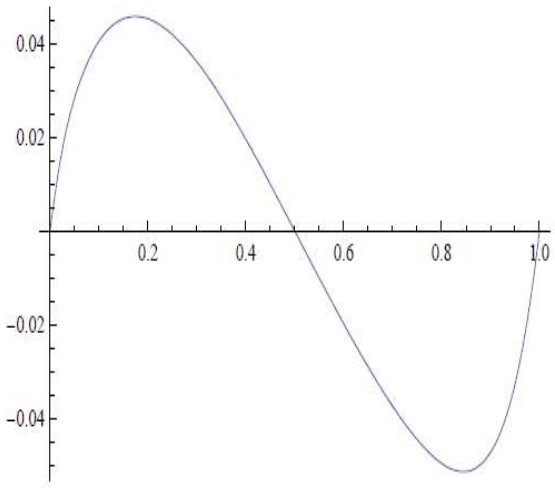


Figure 2: The function $e_H^{**}(\rho_1) - e_H^*$

the direction of which is ambiguous: she would like to have a reputation for being an efficient low-cost type which conveys credible information. Meanwhile, since she creates surplus for sellers only when publishing favourable reports, she would also like to appear as a high-cost type in order to extract more from sellers. Unlike in standard reputations models where short-term and reputational incentives are conflicting, our model displays situations in which short-term and long-run interests are indeed aligned: when she is perceived as a high-cost type, the intermediary needs to establish a reputation for providing accurate reports in order to foster demand on the buyer's side; this leads her to provide more effort. However, in the region where she tends to be perceived as efficient, she cuts effort in order to please future sellers, whose willingness to pay increases as her reputation decreases. Importantly, our results are strictly driven by the tension between long-term interests, i.e. by the fact that the intermediary wishes to have an intermediate reputation in order to increase her profit. Therefore, the assumption of an intrinsic preference for accuracy is innocuous to the extent that the nature of the results would not be affected if we instead had assumed no preference for accuracy at all or, conversely, a preference for always reporting good grades. In this regard, contingent payments would clearly increase the temptation to shirk in

our model (it would be equivalent to a diminution of u), but the shape of the effort function would remain unchanged as long as a significant fraction of the payment is paid upfront.

Another result that sharply contrasts with standard reputation models is that the impact of reputation on effort decision for intermediate values of ρ is very small. In one-sided reputation models, distortion from the static preferred action is usually more important in the intermediate region of beliefs where prior uncertainty is highest because attempts to influence others' beliefs are more effective. Here, because the intermediary is basically indifferent between being seen as close to buyers or equivalently close to sellers, the two reputational forces offset each other so that incentives are only driven by short-term preferences.

3.5.2 Two strategic players

We examine in this section whether our results remain valid when the low-cost agency also behaves in a strategic way.

In the second period, expected utilities are given by

$$V^L(\rho_2) = S(\rho_2) + W_2^L(e_L^*) \text{ and } V^H(\rho_2) = S(\rho_2) + W_2^H(e_H^*).$$

$V^L(.)$ and $V^H(.)$ differ only by a constant $W_2^L(e_L^*) - W_2^H(e_H^*)$, which greatly simplifies the analysis. For given beliefs ρ^+ and ρ^-, the expected payoffs are given by

$$\tilde{W}_1^H(e_H) = qu + (1-q)\left\{e_H[\delta V^H(\rho^+) + u] + (1-e_H)\delta V^H(\rho^-)\right\} - \frac{1}{2}c_H e_H^2$$

and

$$\tilde{W}_1^L(e_L) = qu + (1-q)\left\{e_L[\delta V^L(\rho^+) + u] + (1-e_L)\delta V^L(\rho^-)\right\} - \frac{1}{2}c_L e_L^2.$$

Taking c_L and c_H sufficiently large to obtain interior solutions, it must be the case

in any candidate equilibrium that

$$(1-q)\left\{\delta V^H(\rho^+) + u - \delta V^H(\rho^-)\right\} = c_H e_H^{**} \tag{6}$$

$$(1-q)\left\{\delta V^L(\rho^+) + u - \delta V^L(\rho^-)\right\} = c_L e_L^{**} \tag{7}$$

Because $V^H(.)$ and $V^L(.)$ differ only by a constant, the left-hand side of (6) and (7) must be equal in equilibrium, which implies $c_H e_H^{**} = c_L e_L^{**}$. The equilibrium level of effort of type c_H is a fixed fraction of that of type c_L. Using this property, the following proposition obtains:

Proposition 2 For c_L/δ and c_H/δ sufficiently large, the perfect Bayesian equilibrium of the game is unique. The equilibrium levels of effort are given by two continuous functions $e_L^{**}(.)$ and $e_H^{**}(.)$ such that $e_L^{**}(0) = e_L^{**}(1) = e_L^*$ and $e_H^{**}(0) = e_H^{**}(1) = e_H^*$.

- $e_L^{**}(\rho_1) > e_L^*$ and $e_H^{**}(\rho_1) > e_H^*$ for $\rho_1 \in (0; \overline{\rho})$,
- $e_L^{**}(\rho_1) = e_L^*$ and $e_H^{**}(\rho_1) = e_H^*$ for $\rho_1 = \overline{\rho}$,
- $e_L^{**}(\rho_1) < e_L^*$ and and $e_H^{**}(\rho_1) < e_H^*$ for $\rho_1 \in (\overline{\rho}; 1)$.

Proof In the Appendix.

Proposition 2 first provides a robustness check that our results hold in a situation where both types behave in a strategic way. Although the magnitude of effort varies, the shape of the effort function remains the same for the high-cost agency. In addition, reputational concerns impact the low-cost intermediary's effort as well. The latter provides an equilibrium level of effort which accounts for a constant multiple of the inefficient type's effort. Intuitively, the efficient intermediary wants to separate from the high-cost type when her reputation is low and wants to mimic her when her reputation is high. The equilibrium condition $c_H e_H^{**} = c_L e_L^{**}$ illustrates the strategic complementarity between efforts of both types. When the continuation profit is, say, increasing in ρ, a higher level of effort from the low-cost type increases the marginal reputational benefit

When $\rho_1^A \in [\rho_a, \rho_c]$, the prospect of being perceived as more reliable than B in period 2 boosts intermediary A's incentives and effort increases compared to its level e_H^* in the static game. Similarly, if $\rho_1^A \in [\rho_b, \rho_d]$, period 2 profits increase following an incorrect report so that A underprovides effort.

Recalling that $\overline{\rho}$ is the interior value of the reputation such that the intermediary provides her static level of effort in the absence of competition, the next result establishes a comparison with the monopoly case:

Corollary The level of effort is strictly lower than the monopoly level of effort if $\rho_1^A < \overline{\rho}$ and larger if $\rho_1^A > \overline{\rho}$.

Proof From the definition of ρ_b and ρ_c, $\overline{\rho}$ must be in $[\rho_c, \rho_c]$. The comparison is then straightforward when $\rho_1^A \in [0, \rho_a] \cup [\rho_c, \rho_b] \cup [\rho_d, 1]$. Suppose that $\rho_1^A \in [\rho_a, \rho_c]$, then $V[\rho^+(\rho_1^A, e_c^*(\rho_1^A))] - V(\rho^B) - c[e_H^c(\rho_1^A) - e_H^*] = 0$. This implies $V[\rho^+(\rho_1^A, e_H^c(\rho_1^A))] - V[\rho^-(\rho_1^A, e_H^c(\rho_1^A))] - c[e_H^c(\rho_1^A) - e_H^*] > 0$. Since $V[\rho^+(\rho_1^A, e)] - V[\rho^-(\rho_1^A, e)] - c(e - e_H^*)$ is decreasing in e, $e_H^c(\rho) < e_H^*(\rho)$.

The threat of entry attenuates reputational effects. Competition dissipates part of and sometimes all the rent the intermediary can extract from sellers, so reputation becomes less of a concern. This lowers the disciplinary effect of reputation. For low values of ρ^A, the intermediary still provides more effort than in the static case in an attempt to improve his reputation, but effort falls below its monopoly level. Conversely, competition makes the clientele effect less prevalent although underprovision of effort still occurs.

5 Discussion

The general view on the impact of reputation for certification agencies is that it provides incentives to acquire or report high-quality information. Profits for these intermediaries are based on sellers' beliefs in the accuracy of their reports. Given that a reputation for providing correct information needs to be established over time, the

quest for credibility should have a disciplining effect. We do not argue against this effect of reputation, which actually shows in our results. Indeed, intermediaries with a reputation for providing low-quality information tend to raise their effort above the level they would naturally choose absent reputational concerns. Reputation-building may therefore improve the quality of information provided to the market.

However, we differ from standard reputation models by showing that this disciplining effect is sometimes dominated by a clientele effect driven by the desire to attract future sellers. Sellers do not only care about the credibility of ratings, but also about the probability to obtain certification in the first place. When credibility is sufficiently high, their primary concern becomes the likelihood of being rated. In such a scenario, intermediaries have an incentive to develop a reputation for being lenient. This contrasts with models in which the incentives to distort information are traded against reputational losses. In those models, reputation does not allow to perfectly discipline intermediaries as long as their actions are observed with some noise, although it improves information transmission. Our model suggests that reputation might actually cause certifiers to reduce the accuracy of their reports below the level they would intrinsically prefer. This occurs despite the fact that the intermediary has an intrinsic preference for providing accurate information.

The effect we uncover is likely to be persistent. First, it is not driven by the presence of short-term incentives to decrease the quality of information transmission, such as contingent payments. As a result, it would survive any attempt to prevent the seller from paying for good ratings. We are not making a case against the restriction of contingent fees which emerges as a central recommendation in Bolton, Freixas and Shapiro [7], and was part of the agreement between New York State Attorney General Andrew Cuomo and the three main rating agencies. It is actually easy to show in our setting that making payments contingent on favourable reports would unambiguously decrease the quality of information transmission. However, we raise the issue that these restrictions might not be sufficient to alleviate the conflict of interest faced by rating or certification agencies. Second, the clientele effect prevails even in the absence of any kind of repeated interaction between the intermediary and a given seller, in which

the latter could threaten the former to cut on future business relationships. Rather, the clientele effect goes through as long as the actions of the intermediary are publicly observed. The intermediary wants to be too lax with current clients not to please them, but to signal to future ones that their chances to get a favourable report are high. Regulatory reforms such as the Sarbanes-Oxley act, which prevents firms from keeping up with the same auditor more than five years may help relieve auditing firms from the direct pressure they are confronted with, but indirect incentives to be too lenient with clients may remain pervasive through reputational effects.

Our analysis underlines the challenge raised by fundamental changes in the business model of rating or auditing firms that would go beyond the restriction on conditional payments or rewards. Mathis, McAndrews and Rochet [26] argue that insulating rating agencies from issuers could be achieved through a platform that would levy a fee from the issuer, select a rater and pay him an unconditional payment in exchange for a report. The efficiency of this solution relies on the premise that buyers (investors) and sellers (issuers) ultimately have a common interest in improving the reliability of ratings. As a result, a profit-maximizing platform would eliminate the undesirable effects generated by the price structure and the distortions in information transmission would vanish. This would not be the case in our framework since the platform would still have to accommodate two sides of the market with conflicting interests. Furthermore, trying to dissociate the expected payment of the intermediary from her reputation by imposing for instance a flat fee is a two-edged sword. On the one hand, it may deter the intermediary from degrading the quality of information when her reputation is high, but it would on the other hand annihilate implicit incentives to raise information quality for lower values of his reputation.

6 Conclusion

This paper considers the case of a certification agency that needs to attract both sellers and buyers in order to exercise its activity. In a dynamic setting, building up a reputation for providing accurate information boosts the demand of buyers but

may decrease the willingness to pay of sellers who also care about the likelihood of obtaining a positive recommendation. In contrast with the existing literature, we show that reputational concerns have an ambiguous effect on the quality of information production. When the perceived reliability of reports is deficient, reputation has a disciplining effect and the precision of information improves. However, agencies with a good reputation vis-à-vis buyers dissipate their reputation in order to increase the revenue they derive from future sellers, and therefore tend to be lenient. This occurs even though payments to the agency are not contingent on a favourable report.

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7 Appendix

7.1 Proof of proposition 1

When type c_L always makes effort e_L^*, the derivative of $W_1^H(e_H)$ reads

$$(1-q)\left\{u+\delta V(\rho^+(\rho_1,e_L^*,e_H^{**}))-\delta V(\rho^-(\rho_1,e_L^*,e_H^{**})\right\}-c_He_H^{**}.$$

Assuming that

$$\forall\rho_1, u+\delta V(\rho^+(\rho_1,e_L^*,0))-\delta V(\rho^-(\rho_1,e_L^*,0))>0,$$

and remarking that

$$(1-q)\left\{u+\delta V(\rho^+(\rho_1,e_L^*,e_L^*)-\delta V(\rho^-(\rho_1,e_L^*,e_L^*))\right\}-c_He_L^*=(1-q)u-c_He_L^*<0,$$

the solution of the problem is interior and given by:

$$(1-q)\left\{u+\delta V(\rho^+(\rho_1,e_L^*,e_H^{**}))-\delta V(\rho^-(\rho_1,e_L^*,e_H^{**}))\right\}-c_He_H^{**}=0.$$

Let $f(\rho_1,e)=(1-q)\left\{u+\delta V(\rho^+(\rho_1,e_L^*,e))-\delta V(\rho^-(\rho_1,e_L^*,e))\right\}-c_He$.

We derive $f_2(\rho_1,e)=(1-q)\delta\left\{V'(\rho^+(\rho_1,e_L^*,e))\frac{\partial\rho^+}{\partial e}-V'(\rho^-(\rho_1,e_L^*,e))\frac{\partial\rho^-}{\partial e}\right\}-c_H$.

f_2 is bounded once one has assumed that V' is bounded, so by taking c_H large enough, we can guarantee that the solution of the problem is unique and that $f_2(\rho_1,e_L^*,e_H^{**}(\rho_1))<0$. Since f is C^1 in all its arguments, the solution $e_H^{**}(\rho_1)$ is continuous.

If $\rho_1\in\{0,1\}, \rho^+(\rho_1,e_L^*,e)=\rho^-(\rho_1,e_L^*,e_H)$ for all e so the solution of the problem is the same as in the static problem.

Consider another possible $\overline{\rho}$ such that $e_H^{**}(\overline{\rho})=e_H^*$.

$f(\overline{\rho},e_H^*)=0$ which is equivalent to

$$V(\rho^+(\overline{\rho},e_L^*,e_H^*))=V(\rho^-(\overline{\rho},e_L^*,e_H^*)).$$

Since V admits one maximum by assumption, this implies that

$$V'(\rho^+(\overline{\rho}, e_L^*, e_H^*)) < 0 < V'(\rho^-(\overline{\rho}, e_L^*, e_H^*)).$$

By the implicit function theorem,

$$\frac{\partial e_H^{**}}{\partial \rho_1}(\rho_1) = -\frac{f_1(\rho_1, e_L^*, e_H^{**}(\rho_1)))}{f_2(\rho_1, e_L^*, e_H^{**}(\rho_1)))} = -\frac{V'(\rho^+(\rho_1, e_L^*, e_H^{**}(\rho_1)))\frac{\partial \rho^+}{\partial \rho_1} - V'(\rho^-(\rho_1, e_L^*, e_H^{**}(\rho_1)))\frac{\partial \rho^-}{\partial \rho_1}}{f_2(\rho_1, e_L^*, e_H^{**}(\rho_1)))}.$$

Furthermore,

$$\frac{\partial \rho^+}{\partial \rho_1}(\rho_1, e_L^*, e) = \frac{e_L^* e}{(\rho_1 e_L^* + (1-\rho_1)e)^2}$$

and

$$\frac{\partial \rho^-}{\partial \rho_1}(\rho_1, e_L^*, e) = \frac{(1-e_L^*)(1-e)}{(\rho_1(1-e_L^*) + (1-\rho_1)(1-e))^2}.$$

This implies, recalling that $e_H^{**}(0) = e_H^{**}(1) = e_H^*$ and that $f_2(\rho_1, e_H^{**}(\rho_1))) < 0$:

$$\frac{\partial e_H^{**}}{\partial \rho_1}(0, e_L^*, e_H^{**}(0)) = -\frac{V'(0)(\frac{e_L^*}{e_H^*} - \frac{1-e_L^*}{1-e_H^*})}{f_2} > 0$$

and

$$\frac{\partial e^*}{\partial \rho_1}(1, e_L^*, e_H^{**}(1)) = -\frac{V'(1)(\frac{e_H^*}{e_L^*} - \frac{1-e_H^*}{1-e_L^*})}{f_2} > 0.$$

By continuity of $e_H^{**}(\rho_1)$ and from $e_H^{**}(0) = e_H^{**}(1) = e_H^*$, there exists at least one $\overline{\rho}$ such that $e_H^{**}(\overline{\rho}) = e_H^*$. Furthermore, from $V'(\rho^+(\overline{\rho}, e_H^*)) < 0 < V'(\rho^-(\overline{\rho}, e_H^*))$, we derive that $\frac{\partial e_H^{**}}{\partial \rho_1}(\overline{\rho}) < 0$ for any $\overline{\rho}$ such that $e_H^{**}(\overline{\rho}) = e_H^*$. So $\overline{\rho}$ is unique. From uniqueness of $\overline{\rho}$ and from $\frac{\partial e_H^{**}}{\partial \rho_1}(\overline{\rho}) < 0$, we derive that $e_H^{**}(\rho_1) \geq e_H^* \iff \rho_1 \leq \overline{\rho}$.

The fact that truthful reporting after a signal $\sigma = B$ is optimal comes from the fact that the agency gets the benefit u only if her report is correct. In an equilibrium in which the probability of reporting $r = G$ whenever $\sigma = B$ is strictly positive, the intermediary would at least weakly prefer not finding out the truth, so that the marginal benefit of effort is at most zero. This yields to an equilibrium effort of zero, which we rule out.

7.2 Proof of proposition 2

Let us assume that

$$(1-q)[\sup_{\rho} V(.) + u - V(0)] < \frac{c_L}{\delta}. \tag{8}$$

(8) implies that $e_L^{**} < 1$ and therefore $e_H^{**} < 1$ for any ρ_1.

If $\rho_1 \in \{0;1\}$, $e_H^{**} = e_H^* > 0$ and $e_L^{**} = e_L^* > 0$. Suppose now that $\rho_1 \in (0;1)$ and there exists an equilibrium such that $e_H^{**} = 0$. Then we must have $\delta V^H(\rho^+) + u - \delta V^H(\rho^-) < 0$, which implies $\delta V^L(\rho^+) + u - \delta V^L(\rho^-) > 0$, which in turn implies $e_L^{**} = 0$. But if $e_L^{**} = e_H^{**} = 0$, then $\delta V^H(\rho^+) + u - \delta V^H(\rho^-) = u > 0$. A contradiction.

Therefore, if (8) holds, the equilibrium levels of effort must be interior. This implies that $c_H e_H^{**} = c_L e_L^{**}$, and for any prior reputation ρ_1, it suffices to derive the equilibrium level of effort of type c_H. It solves

$$(1-q)\left[\delta V^H(\rho^+) + u - \delta V^H(\rho^-)\right] - c_H e_H^{**} = 0.$$

We drop the subscript on V for simplicity.

Since $c_H e_H^{**} = c_L e_L^{**}$, beliefs following a bad rating do not computationally depend on the equilibrium levels of effort:

$$\rho^+ = \frac{\rho_1}{\rho_1 + (1-\rho_1)\dfrac{c_L}{c_H}}.$$

As a result, $\delta V(\rho^+) + u$ does not depend on e_H^{**} and we restrict attention to $g(e) = (1-q)\delta V(\rho^-) + c_H e_H$, where

$$\rho^- = \frac{\rho_1(1 - \dfrac{c_H}{c_L}e_H)}{\rho_1(1 - \dfrac{c_H}{c_L}e_H) + (1-\rho_1)(1-e_H)}.$$

Then $g'(e) = (1-q)\delta V'(\rho^-)\dfrac{\partial \rho^-}{\partial e} + c_H$.

$\dfrac{\partial \rho^-}{\partial e} < 0$. From (8), $1 - \dfrac{c_H}{c_L}e_H > \epsilon > 0$ in equilibrium for any ρ_1, therefore there

exists $\hat{\rho}$ such that $V'(\rho^-) < 0$ for any $e < (1-\epsilon)\frac{c_E}{c_e}$. For $\rho \geq \hat{\rho}$, $g'(.)$ is positive, and for $\rho < \hat{\rho}$, $-\frac{\partial \rho^-}{\partial e}$ is bounded above, $V'(\rho^-) < V'(0)$ so that by taking c_e/δ sufficiently large, $g'(.)$ is also positive.

This establishes the uniqueness of the equilibrium. The other properties of $e_H^{**}(.)$ are derived along the lines of the proof of proposition 1, while the properties for $e_L^{**}(.)$ are straightforward consequences of the equilibrium property that $c_H e_H^{**} = c_L e_L^{**}$. The fact that $e_H^{**}(\rho_1) = e_H^*$ and $e_L^{**}(\rho_1) = e_H^*$ for $\rho_1 = \overline{\rho}$ simply comes from the fact that $c_L e_L^* = c_H e_H^*$.