Optimal Compensation Schemes in a Repeated Monitoring Relationship[*]

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Abstract

I study the design of optimal compensation schemes in a repeated principal-supervisor-agent hierarchy. While the principal observes a noisy signal about agent's effort and the supervisor's report, the agent and supervisor can perfectly observe each other's actions. In a repeated setting, this allows the employees to punish each other without error for shirking. The optimal compensation schemes are designed to trade off incentives to work in the current period against inducing the most severe punishments following a deviation. On the equilibrium path, optimal schemes provide incentives for the agent by compensating for favorable supervisory report, and off the equilibrium path by inducing severe minmax punishments. In contrast to a static setting where the likelihood ratios (the informational content of performance measures) shape the optimal schemes, repeated interaction can generate counterintuitive compensation schemes shaped by dynamic incentives that facilitate harsh punishments.

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1 Introduction

In a standard principal agent relationship, the principal deals with moral hazard by rewarding good performance and/or punishing bad performance. A similar result also holds in a repeated principal agent relationship.[1]

In reality, the principal can, and does, use more direct mechanisms to get around the informational disadvantage.[2] One such mechanism is supervision. Wage contracts often include clauses regarding evaluation by a supervisor. This is the subject of this paper.

I develop a repeated principal-supervisor-agent hierarchy. I assume that the agent and the supervisor can perfectly observe each other's actions whereas the principal observes only a noisy signal about the agent's effort (i.e., output levels, profits, stock prices etc.) and the supervisor's report. In a repeated game framework, a punishment phase between the agent and supervisor can be triggered without mistake when shirking is observed. This is because a punishment is triggered not by the principal who is informationally disadvantaged, but by the supervisor who is not.

In this setting, the optimal compensation schemes for the agent and the supervisor serve two purposes. First, wages must provide the right incentives to exert effort for both employees, and second, wages must induce severe punishments against deviation. When employees are sufficiently patient,

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[1]See Radner (1981, 1985) for a repeated principal agent model. The principal devises a statistical test with review and punishment phases. If the agent passes test, he is rewarded and the next review phase begins. If the agent fails, a punishment phase begins. See also Spear and Srivastava (1987) for a dynamic programming approach.

[2]In designing compensation schemes among identical employees, Green and Stokey (1983) shows that a relative performance scheme (i.e., tying the compensation of an employee to the performance measures of his peers) can only be attractive if there is a common noise component. In the absence of such noise, Lazear and Rosen (1981) show that tournmaments, a special form of a relative performance scheme perform as good as piece rates.

the latter dominates and hence optimal schemes are totally shaped by their ability to facilitate severe off equilibrium punishments. I show that these punishments can be as severe as minmax punishments even for low discount factors.

This argument suggests that dynamic interaction can generate compensation schemes that are qualitatively different from those that are optimal in a static setting. Whereas in a static setting, typically, it is optimal to reward the agent for good performance, in a dynamic setting instead, the optimal scheme is to reward the agent when his performance is low but the supervisor's report is favorable (non-monotonic wage scheme). This scheme sounds counter intuitive because it creates strong incentives to shirk in the current period. However the threat of subsequent severe punishments triggered by the supervisor outweighs, at least when employees are patient. While on the equilibrium path, the agent's incentives are provided by favorable supervisory reports, off the equilibrium they are provided by the lack of favorable supervisory reports.The supervisor can punish the agent severely by holding back favorable supervisory report. Hence the supervisory report has an important role in providing incentives.

Much of the analysis focuses on the best equilibrium for the principal (in the spirit of weak implementation). However, other equilibria may arise under the selected compensation scheme. Collusion-proofness, the idea that the supervisor and the agent could collude and coordinate to select an equilibrium that is more favorable to them, if it exists, is a particular concern. Under certain conditions, I find that the equilibrium under optimal compensation schemes is also collusion-proof.

These findings have provocative implications when applied to CEO compensation. Suppose the principal represents the shareholders, the supervisor the board of directors, and the agent the CEO. The stock price is a noisy signal of CEO's effort. Whereas in a static setting the CEO should be compensated for high stock prices, the model in this paper suggests that in a dynamic setting optimal compensation schemes could be different. In particular, the CEO could be compensated for low stock prices provided that the board's report is favorable. Such schemes are optimal because of their

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ability to facilitate severe off equilibrium punishments. Of course, this prediction depends on the details of my stylized model. However the general point is that in a dynamic setting optimal schemes may not necessarily be synonymous with monotone compensation schemes.

The model also predicts that when CEO-board relationship is long-term, tying CEO compensation primarily to the board's report may provide better incentives than merely rewarding him based on the stock price. While paying the CEO based on the supervisor's favorable report, and not based on the stock price may be interpreted as evidence of board capture by the CEO,[3] this paper shows that such schemes could in fact be in the best interest of the shareholders in terms of minimizing the wage bill and preventing collusion.

2   Related Literature

There is a small literature on supervised players (Tirole (1986), Laffont and Tirole (1991), Kofman and Lawarree (1993, 1996), Laffont and Meleu (1997), Strautz (1997),[4] Vafai (2005)). The majority of this literature is in a static setup,[5] and assumes that the agent and the supervisor can write down side contracts. The enforcement of side contracts is problematic in a static setup. In contrast, in this paper the employees are not allowed to write down side contracts but they can coordinate and collude through their effort choices. In a dynamic environment collusion and coordination arises as a self-enforcing behavior, hence there is no enforcement issue.

This paper is closely related to Che and Yoo (2001). Like Che and Yoo (2001), the explicit attention of this paper is the endogenous mechanisms

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[3]Bebchuk and Fried (2003, 2006) examine a large body of empirical evidence on executive compensation. They interpret the empirical evidence such as the design of stock option plans, resetting of option exercise prices, reloading options etc. as an indicator that executive compensation is not designed optimally, and that this empirical evidence is better explained by a managerial power approach in which executives have power to influence their own pay.

[4]Strautz (1997) argues that when the principal has a commitment problem, i.e., he cannot commit to monitoring or reveal the outcome of monitoring, delegation to a monitor may be beneficial even in the presence of collusion.

[5]Acemoglu (1994 and 1998) is an exception where collusion is possible due to unverifiability of supervisor's reports in a dynamic setting.

that generate implicit incentives for employees in a dynamic setting. Che and Yoo (2001) characterize optimal schemes for identical agents (horizontal relationship), whereas this paper characterizes optimal schemes in a hierarchical relationship (agent and supervisor) where there are natural complementarities among employees' efforts. Therefore, this paper has a different set of predictions on the design of compensation schemes in organizations. For example, in Che and Yoo (2001) it is never optimal to reward either employee for their own failure whereas it could be optimal in this setting.

Martimort (1997) analyze optimal schemes for two agents with hidden information in a dynamic setting. In Martimort (1997) the principal is restricted with anonymous contracts which are the source of collusion among agents. In this paper the principal is restricted with stationary (time-independent) but not anonymous contracts.

This work is also related to the dynamic contracting literature (Baker, Gibbons and Murphy (1994, 2002), Levin (2003), Meyer and Vickers (1997)). Unlike these articles which largely focus on the single-agent environment, current paper focuses on relationships among multiple agents, specifically supervisor-supervisee relationships. This paper is also related to the repeated game literature, but the payoff structure is determined endogenously through contract design.

3   The Model - Preliminaries

A hierarchy consists of an agent (he), a supervisor (she) and a principal. The agent supplies the productive effort and the supervisor supplies the costly monitoring effort. The supervisor is endowed with a monitoring technology that allows her to improve the quality of the information used in evaluating the agent. This technology (formal evaluation process) only works when she supplies monitoring effort.[6] The incentive problem stems from the players' unobservable effort decisions and environmental shocks.

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[6]Most of the literature treats the monitoring technology as exogenous in the sense that once the supervisor (monitor) is employed, the monitoring effort is automatic. For example, see Shapiro and Stiglitz (1984) for a model of imperfect monitoring.

Each period the agent ($i = 1$) and the supervisor ($i = 2$) makes a binary effort decision $e_i \in \{E, N\}$ $i = 1, 2$. The effort profile $e = (e_1, e_2)$ produces a publicly observable stochastic signal $\mathbf{x}(e)$ with generic element $x(e) = (x_1(e), x_2(e))$. The first signal x_1, which I call the 'output signal', can take values $x_1 = H$ (i.e., High output) and $x_2 = L$ (i.e., Low output). The second signal, x_2 which I call the 'supervisory signal', is the outcome of supervisor's monitoring effort. The monitoring process produces either a report with hard (verifiable) evidence ($x_2 = h$) or a report with no evidence at all ($x_2 = \varnothing$). So $x = (x_1, x_2) \in \{Hh, Lh, H\varnothing, L\varnothing\}$.

The probability distribution of $\mathbf{x}(\cdot) = (\mathbf{x_1}(\cdot), \mathbf{x_2}(\cdot))$ depends on the effort profile e and as well as an environmental shock. The shock is either favorable or unfavorable where the former event occurs with probability $\xi \in [0, 1]$. The shock is not observable to both players and the principal.

Suppose the environmental shock is unfavorable. Then the output signal x_1 is more likely to be high when the agent exerts effort. Specifically, the output signal is $x_1 = H$ with probability $p > 1/2$ when $e_1 = E$ and with probability $1 - p$ when $e_1 = N$. On the other hand, the supervisor is equipped with an imperfect monitoring technology. The costly monitoring effort requires expertise. It could involve reviewing, evaluating and assessing projects that the agent undertakes. The supervisory effort produces either hard evidence ($x_2 = h$) or does not produce anything at all ($x_2 = \varnothing$). Supervisor's report is verifiable in the following sense that she can convey her information to the principle in a credible way.[7] The report can be thought of as communication of the outcome of a quality test on the agent's product or an assessment of risk/profitability analysis of the projects the agent undertakes. I assume that it is easier for the supervisor to come up with evidence when the agent exerts effort as compared to when the agent does not exert effort. For example, it is harder for the supervisor to come up with evidence when the agent's reports/projects are not well organized etc. Specifically, the report is $x_2 = h$ with probability $q > 1/2$ when $e_1 = E$ and with probability $1 - q$ when $e_1 = N$, given that $e_2 = E$. I also assume

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[7]See Tirole (1986) for a similar supervised agency model in a static setting. In Tirole (1986), the supervisor's report is verifiable but the she can withold submitting the report.

SHOCK

Favorable   Unfavorable

(ζ) $(1\text{-}\zeta)$

$(x_1, x_2) = (Hh)$ **AGENT**

E   N

SUPERVISOR

E   N   E   N

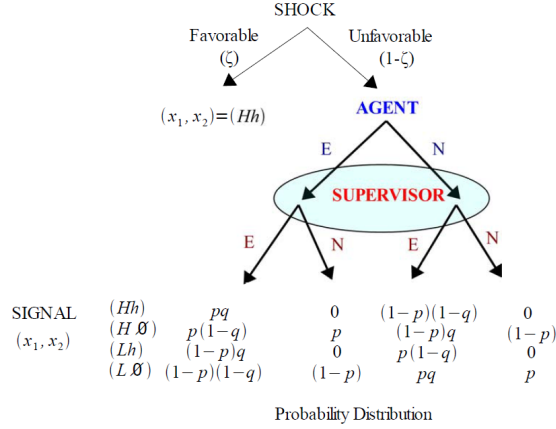| SIGNAL (x_1, x_2)					
(Hh)	pq	0	$(1-p)(1-q)$	0
$(H\varnothing)$	$p(1-q)$	p	$(1-p)q$	$(1-p)$
(Lh)	$(1-p)q$	0	$p(1-q)$	0
$(L\varnothing)$	$(1-p)(1-q)$	$(1-p)$	pq	p

Probability Distribution

Figure 1: Signal Distribution

that when the supervisor exerts low monitoring effort, he cannot produce any hard evidence at all. So $x_2 = \varnothing$ independent of the effort of the agent given that the supervisor shirks. Notice that under an unfavorable shock two signals are independently distributed.

Suppose the environmental shock is favorable. Then the signal realization is $x(\cdot) = Hh$ regardless of the effort choice of either player. The shock is a state of the world where the output is high whether the agent exerts effort or not. For example, suppose that x_1 is the stock price. The favorable shock could represent a market upswing or a stock market bubble. Under such circumstances, the supervisor's evaluation criteria may also be affected by the market upswing and hence it might be easier to find supporting evidence in favor of the agent.[8] Alternatively, the shock can also be interpreted as a biased (favorable) technological shock to the productivity of labor.

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[8] For example, before the mortgage crisis of 2008 in USA broke out, securities backed with mortgages were highly rated. The companies that held these securities made huge profits (without much effort) because these securities were profitable, at least till the crisis broke out. Interestingly, during the same times the ratings and evaluations on those same companies and their CEOs were very optimistic. For example, Lehman Brothers Holding Inc. got an 'A' rating from Standard and Poor's. Ironically, the bankruptcy of Lehman Brothers Inc. on September 2008 is the largest banktrupcy filing in US history so far. See Sprinzen and Azarch (2008) on "Why was Lehman Brothers rated 'A'?".

Because the principal does not observe effort decisions, he can only design wages depending on random signal $\mathbf{x}$. Let $w_i(x)$ denote the wage payment to agent i when signal x is observed. I assume $w_i \geq 0$ which may arise from worker's freedom to quit or institutional and/or legal constraints. Furthermore, I restrict attention to wage contracts in which the employees choose not to save or borrow, and instead consume their entire income in each period.

Throughout, I assume that the agent's work is sufficiently valuable so the principal prefers to induce the agent to exert effort. I also assume that the principal cannot credibly fire the players due to low performance. In firing an employee, firms incur procedural costs, which include creating any necessary documentation to substantiate the firing decision, severance payments, replacement costs such as search and training costs for new employees, and possible disruptions in the production process. Therefore, employees might not believe that the firm will carry out the (costly) termination threat. In addition, if the agent has developed relationship-specific investments (i.e., firm-specific or job-specific skills) with the firm, firing the agent will be a loss of firm-specific human capital which is costly to replace.[9]

4 The Repeated Game

To study the effects of dynamic interaction between the agent and the supervisor, I assume that the principal commits to a stationary wage scheme $w(\mathbf{x}) = (w_1(\mathbf{x}), w_2(\mathbf{x}))$ before the game starts, and no re-negotiations are allowed thereafter. This creates a repeated game between the agent and the supervisor. Denote this repeated game indexed by the the wage scheme w with $G^\infty(w)$.

I assume that the agent and supervisor can observe each other's past effort choices. Such mutual monitoring may be possible due to players' close interaction. Players' knowledge about each others' efforts is 'soft information' in the sense such knowledge cannot be used in the design of formal

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[9]See Williamson, O. (1973) for a description of relation-specific investments and Hart and Moore (1990) for a formal model.

contracts, i.e., not enforceable via courts. Hence the principal cannot design message games among employees.[10] However such knowledge can be used by players in a repeated game setting in an 'informal' punishment phase. On the other hand, the supervisory report and output signal are 'hard information' that can be used in the design of formal contracts. Hence the supervisor can convey only the supervisory reports to the principal in a credible way.[11]

All players are risk-neutral.[12] I model the players risk-neutral for two reasons: first, to ignore the standard trade-off between risk and incentives, and second, to ignore the intertemporal consumption smoothing effects. I want to de-emphasize the role of risk and incentives and monetary punishments but to emphasize the role of dynamic interaction among employees on the design of compensation schemes.

The per-period (expected) utility of agent i is $\mathbb{E}u_i(e; w_i) = \mathbb{E}(w_i(\mathbf{x}(e)) - c_i(e_i)$ for $i = 1, 2$ with $c_i(e_i = N) = 0$. The utility of outside option is zero for both players.[13] A strategy σ_i for each agent i is to choose an effort decision each period depending on the history of both players' effort choices. Agents are allowed to use pure strategies only.[14] The (expected) payoff of agent i under strategy profile $\sigma = (\sigma_1, \sigma_2)$ in repeated game $G^\infty(w)$, which is denoted by $\mathbb{E}U_i(\sigma; w_i)$, is the discounted sum of per-period utilities

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[10]If such message games were credible/enforceable, by penalizing inconsistent reports about past efforts, employeess can be induced to work at mininal cost. See Mae (1988) who devises a costless message game that implements the first-best outcome uniquely.

[11]Legal liabilities (i.e., prison sentences) may be sufficient for to deter false (manipulated) supervisory reports. For example, the shareholders can sue the board of directors on such grounds. See Acemoglu (1998) who argues that the imposition of potential legal liability on the auditor may deter collusion with the clients.

[12]When the agent is risk-neutral, the principal can simply 'sell the firm' to the agent which eliminates the moral hazard problem trivially. This remedy however is unsatistisfying when there are capital market imperfections that preclude the agent to raise the capital to buy the firm.

[13]This assumption is not without loss of generality, but is consistent with an environment where the agent's skills are firm-specific and thus not valuable outside the firm.

[14]This restriction is not without loss of generality. However, as the discount factor goes to one, I show that there is an equilibrium in pure strategies in which the principal extracts all the rent. So this assumption is without loss as $\delta \to 1$.
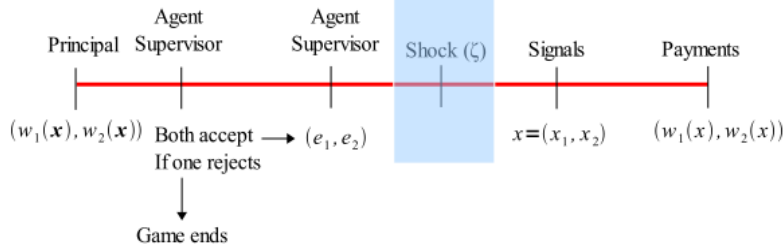
Figure 2: Timeline of events in the one-period game

normalized by $(1 - \delta)$ where $\delta \in (0, 1)$ is the discount factor. Formally,

$$\mathbb{E}U_i(\sigma; w_i) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}u_i(\mathbf{x}^t(\sigma))$$

where $\mathbf{x}^t(\sigma) = (\mathbf{x}_1^t(\sigma), \mathbf{x}_2^t(\sigma))$ is the signal tuple realized under the strategy profile σ.

The timing of the game is as follows: The principal offers a stationary contract $w(\mathbf{x})$. Both players decide whether to accept or reject the contract. If either player rejects, the game ends, and players receive their outside options. If the contract is accepted by both players in period 1, the agent and supervisor simultaneously choose their effort levels $e = (e_1, e_2)$. The signal $\mathbf{x}(e)$ is realized and the payments $w(\mathbf{x})$ are made. The game is repeated as in period 1 at any period $t > 1$ thereafter under contract $w(\mathbf{x})$.

I will consider the case where the principal is interested in implementing an equilibrium outcome in which both players exert effort every period, i.e., $(e_1^t, e_2^t)_{t=1}^{\infty} = (E, E)^{\infty} \equiv \mathbf{e}$. This effort profile $\mathbf{e}$ induces a stochastic signal path denoted by $(\mathbf{x}^t(\mathbf{e}))_{t=0}^{\infty}$. So the principal's problem is to design the lowest cost wage scheme that supports the outcome path $\mathbf{e}$ as a subgame-perfect equilibrium . Formally, the principal chooses $w(\cdot) = (w_1(\cdot), w_2(\cdot))$ to solve

$$\min_{w(\cdot)} \sum_{i=1}^{2} (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}w_i(\mathbf{x}^t(\mathbf{e})) \qquad (OF)$$

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subject to the incentive compatibility constraint

$$\mathbf{e} \text{ can be supported as an outcome of a SPE of } G^\infty(w) \qquad (IC)$$

and participation constraints

$$\mathbb{E}U_i(\mathbf{e}; w_i) \geq 0 \text{ for } i = 1, 2 \qquad\qquad (PC)$$

I call the problem defined by OF, IC and PC as the principal's problem.

Observe that under the limited liability assumption that the wages can not be negative, either player can guarantee himself at least zero payoff in equilibrium. Because the cost of shirking is zero, each player can guarantee at least zero payoff in the worst equilibrium. This implies that whenever IC_{SPE} holds, PC will hold automatically. So it is without loss of generality to ignore PC in solving the model.

To proceed further, I need an explicit characterization of subgame-perfect equilibrium. Abreu (1988) shows that any subgame-perfect equilibrium outcome can (also) be achieved by simple strategies. A simple strategy profile specifies an equilibrium path and a penal code for each player describing responses to deviations from equilibrium path. In particular, if a worst subgame-perfect equilibrium payoff exists for each player, then it can be used as an optimal penal code. In my model, the worst equilibrium payoff exists for each player.[15] For a given wage profile $w = (w_1, w_2)$ denote the (normalized) worst equilibrium payoff with $\underline{u}_i(w)$ for $i = 1, 2$. Then IC can be written as

$$\mathbb{E}U_i(\mathbf{e}; w_i) \geq (1 - \delta)\mathbb{E}u_i(e_i = N, e_{-i} = E; w_i) + \delta\underline{u}_i(w) \qquad (IC_{SPE})$$

for $i = 1, 2$.[16] IC_{SPE} is interpreted as follows: on the equilibrium path the

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[15] For any wage profile with $w \geq 0$ the expected utility of each agent is continuous and bounded from below. Furthermore, the action set is finite for each agent. Fudenberg and Levine (1983) shows that the set of subgame-perfect equilibria is compact in the product topology on strategies, and payoffs to strategies are continuous in this topology. So the worst equilibrium payoff exists for both players by Weistrass's Theorem.

[16] IC_{SPE} should hold for $t = 1, 2, ..$ I can drop the time scripts owing to the recursive structure of the subgame-perfect equilibrium.

player receives $\mathbb{E}U_i(\mathbf{e}; w_i)$. If (s)he deviates, s(he) receives $\mathbb{E}u_i(e_i = N, e_{-i} = E; w_i)$ in the current period but afterwards the game switches to the punishment phase where the deviator gets his/her worst equilibrium payoff on average at the rest of the play.

Although Abreu (1988) provides a nice recipe to support subgame-perfect equilibria, in my model the payoff structure arises endogenously through contract design. So the worst equilibrium payoff depends on the wage profile $w = (w_1, w_2)$. Without knowing $\underline{u}_i(w)$ (and the strategies that generate the worst equilibrium) it is not clear how to solve for optimal wage schemes. One method is to use the Abreu, Pearce and Stacchetti (1990) mechanism to characterize the worst equilibrium payoffs for any $\delta \in (0, 1)$. However since w is endogenous, this method is impractical. Therefore, I took a different path to solve the principal's problem. The next section introduces a relaxed problem and characterizes its solution.

4.1   The Relaxed Problem

Let $v_i(w)$ denote the (normalized) minmax payoff of agent i in $G^\infty(w)$. Notice that in any subgame-perfect equilibrium each player can guarantee herself/himself at least her/his minmax payoff. This follows from that any Nash equilibrium players can guarantee themselves their minmax payoffs, and the fact that a subgame-perfect equilibrium is a Nash equilibrium. In particular, the worst subgame-perfect equilibrium payoff satisfies

$$\underline{u}_i(w) \geq v_i(w)$$

for all $i = 1, 2$.

Because the minmax payoff constitutes a lower bound of the worst equilibrium payoff, any wage profile w that satisfies IC_{SPE} also satisfies the minmax incentive compatibility defined by

$$\mathbb{E}U_i(\mathbf{e}; w_i) \geq (1 - \delta)\mathbb{E}u_i(e_i = N, e_{-i} = E; w_i) + \delta v_i(w) \qquad (IC_M)$$

I denote the problem defined by OF and IC_M as the relaxed problem, and

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its solution by $w^R = (w_1^R, w_2^R)$.

Notice that any wage profile w that satisfies equation IC_M may not satisfy IC_{SPE}. The relaxed problem allows for a bigger set of wage profiles.

There are advantages of using the minmax payoff as punishment. First, the minmax payoff is easily computable given a wage scheme w. Second, the minmax payoff of each player depends only his/her own payoffs, and hence only his/her own wages w_i. In contrast, the worst equilibrium payoff depends on the wage profile w. Hence, I can write $v_i(w) = v_i(w_i)$ for $i = 1, 2$. Because the objective function is also separable in each player's wage scheme w_i, I can break the relaxed problem into two separate problems; one for the agent and one for the supervisor.

Before I get into more details, a few simplifications are in order. On the equilibrium path, the players exert effort forever, i.e., $\mathbf{e} = (E, E)^\infty$. Hence on the equilibrium path the probability distribution of signals is the same in each period . Furthermore, since the wage scheme is also time-invariant, the per period expected wage payment to player i is also time-invariant. That is,

$$\mathbb{E}w_i(\mathbf{x}^t(E, E)) = \mathbb{E}w_i(\mathbf{x}(E, E))$$

holds for all $t = 1, 2, \ldots$ So the (normalized) expected wage payment simplifies to

$$
\begin{aligned}
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}w_i(\mathbf{x}^t(\mathbf{e})) &= (1 - \delta) \frac{\mathbb{E}w_i(\mathbf{x}(E, E))}{1 - \delta} \\
&= \mathbb{E}w_i(\mathbf{x}(E, E))
\end{aligned}
$$

Analogously, on the equilibrium path each player's expected utility is simply

$$
\begin{aligned}
\mathbb{E}U_i(\mathbf{e}; w_i) &= (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}u_i(\mathbf{x}^t(\mathbf{e})) \\
&= (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}u_i(\mathbf{x}(E, E)) \\
&= \mathbb{E}u_i(\mathbf{x}(E, E)).
\end{aligned}
$$

4.2 The Static Game

It is useful to analyze the effects of static interaction on contract design in order to compare it with dynamic interaction. Suppose the players interact only for one period[17] and the principal wants to implement an equilibrium where both players exert effort. Thus the principal chooses w_i to minimize expected wage wage payment to both players subject to the Nash equilibrium condition

$$\mathbb{E}u_i(E, E; w_i) \geq \mathbb{E}u_i(e_i = N, e_{-i} = E; w_i) \qquad (SC)$$

for $i = 1, 2$. The Nash equilibrium requires that no player has an incentive to deviate from exerting effort given that the other player exerts effort. The static optimal wages are denoted by $w^{so} = (w_1^{so}, w_2^{so})$.

Definition 1 *Let* $\Pr(x \mid e_1, e_2)$ *denote the probability of signal x when the chosen effort profile is $e = (e_1, e_2)$. Then the likelihood ratio of signal x is defined as follows:*

$$LR(x) = \frac{\Pr(x \mid e_i = E,\ e_{-i} = E)}{\Pr(x \mid e_i = N,\ e_{-i} = E)}$$

The next lemma states how likelihood ratios shape the static wages.

Lemma 1 $w_i^{so}(x) > 0$ *if and only if* $x \in \arg\max_{\hat{x}} LR(\hat{x})$.

The principal's problem can be re-formulated as a standard linear programming problem and shown to have an extreme point solution. (See the technical appendix for details). Lemma 1 shows that in the one-period game, the optimal wages pay a strictly positive amount at a signal if and only if that signal is most likely to arise when the player exerts effort as compared to when s(he) does not exert effort. First, since the players are risk-neutral, it suffices to reward them only at one signal. Second, it is optimal to reward the agent at signals with the highest likelihood ratio. This result is in line with standard results in agency theory, absent risk aversion, and can

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[17]Note that this is also equivalent to the special case where IC_M holds with $\delta = 0$.

be interpreted that the forces that shape static optimal wages are likelihood ratios.

The next proposition explicitly characterizes the static optimal wages.

Proposition 1 *Suppose the principal implements* $(e_1, e_2) = (E, E)$ *in the one-period game.*

(a) *If $q > p$ there exists a level of shock $\xi_{q,p}$ such that for all $\xi > \xi_{q,p}$ the agent is paid zero everywhere except $w^{so}(Lh) = \frac{c_A}{(1-\xi)(q-p)}$, and paid zero everywhere except $w^{so}(Hh) = \frac{c_A}{(1-\xi)(p+q-1)}$ otherwise.*

(b) *If $p > q$ there exists another level of shock $\xi_{p,q}$ such that for all $\xi > \xi_{p,q}$ the agent is paid zero everywhere except $w^{so}(H\varnothing) = \frac{c_A}{(1-\xi)(p-q)}$, and paid zero everywhere except $w^{so}(Hh)$ otherwise.*

(c) *The supervisor is paid zero everywhere except $w^{so}(Lh) = \frac{c_2}{(1-\xi)(1-p)q}$ for any $\xi > 0$.*

(d) *The agent retains rent whereas the supervisor extracts no rent.*

Before interpreting Proposition 1, let's first consider the case where there is no environmental shock. In this case, signal $x = Hh$ would have the highest likelihood ratio for the agent, and therefore it would be optimal to reward the agent at signal $x = Hh$. Now consider the case where the probability of shock is increasing. This means that it becomes more likely that signal $x = Hh$ is due to the shock and not necessarily due to the fact that the agent exerts effort. There exists a level of shock beyond which the signal $x = Hh$ becomes too noisy, therefore either signal $x = H\varnothing$ (if $p > q$) or $x = Lh$ (if $q > p$) has higher likelihood. $p > q$ can be interpreted as a situation when the output signal is more 'accurate' than the supervisory signal when there is no environmental shock. The situation $q > p$ is interpreted analogously.

For the supervisor in the absence of shock both signals $x = Hh$ and Lh has equal likelihood ratio. Recall that the supervisor can not effect the output signal. Then in the presence of an infinitesimal probability of shock

on signal $x = Hh$, this indifference is broken in favor of the signal $x = Lh$. Therefore, when there is an infinitesimal probability of shock, the signal with the highest likelihood is $x = Lh$.

The supervisor can not extract any rent in this stylized model because she is paid at a signal that can only arise when she exerts effort. On the other hand, such a signal does not exist for the agent, and hence he extracts rent.

4.3   The Solution of the Relaxed Problem

Analogous to the solution of the static game, the relaxed problem can be formulated so that $w_i^R(x) > 0$ if and only if x maximizes the 'modified' likelihood ratio

$$\frac{\Pr(\ x \mid e_i = E,\ e_{-i} = E)}{(1-\delta)\Pr(\ x \mid e_i = N,\ e_{-i} = E) + \delta\Pr(\ x \mid \sigma^{i,v})}$$

where $\sigma^{i,v}$ is the strategy that yields the minmax payoff for agent i. Observe that the modified likelihood ratio is a function of the minmax strategy which is a function of the wage scheme w. So this likelihood ratio is determined endogenously by the wage scheme (whereas in the static game this ratio was independent of the wage scheme). The difficulty here is to determine how wages w_i shape the minmax payoff.

I took the following path to solve the relaxed problem. Consider a solution of the relaxed problem w_i^R. Suppose that w_i^R satisfies the static Nash equilibrium condition (SC). Then (E, E) is a Nash equilibrium of the one-period game under wage scheme w_i^R. Because either player can guarantee herself/himself at least her/his minmax payoff at a Nash equilibrium, we have

$$\mathbb{E}u_i(E, E; w_i^R) \geq v_i(w_i^R).$$

Together with (SC), this implies that the minmax incentive compatibility constraint IC_M is satisfied vacuously for any $\delta \in (0, 1)$. But then w_i^R is determined solely by (SC) which yields the static optimal wages w_i^{so}. Hence the next lemma follows.

Lemma 2 *Suppose a solution of the relaxed problem w_i^R satisfies (SC). Then the solution is unique and equals $w_i^R = w_i^{so}$ for $i = 1, 2$.*

Because the principal extracts all the rent from the supervisor under the static optimal wages, we can conclude that the solution of supervisor's relaxed problem is w_2^{so} by Lemma 2.

On the other hand, the agent extract rent at static optimal wages w_1^{so}. So there may be room for improvement for the principal at some other wage scheme. Suppose that w_1^R does not satisfy (SC). Then w_1^R must satisfy

$$\mathbb{E}u_1(E, E; w_1^R) < \mathbb{E}u_1(N, E = E; w_1^R) \qquad (DC)$$

which I call the 'dynamic constraint'. Observe that at any wage scheme that satisfies (DC), (E, E) can not be sustained as an equilibrium in the static game because the agent would certainly deviate and choose not to exert effort. Proposition 2 characterizes the solution of relaxed problem for wage schemes satisfying (DC) for the agent.

Proposition 2 *Suppose a solution of the relaxed problem for the agent satisfies (DC). Then there exists a strictly decreasing function $\tilde{\delta}(\xi)$ such that if $\delta < \tilde{\delta}(\xi)$ the agent is paid zero everywhere except $w_1^*(Hh) = \frac{c_1}{(1-\sigma)(p+q-1+\delta(1-p)(1-q))}$ and if $\delta > \tilde{\delta}(\xi)$ the agent is paid zero everywhere except $w_1^*(Lh) = \frac{c_1}{(1-\sigma)(q-p+\delta p(1-q))}$. If $p > q$ the result holds for $\frac{p-q}{(1-p)(1-q)} < \delta < 1$.*

What is left is to compare of the solutions that satisfy (DC) with the solutions that satisfy (SC). Suppose $q > p$ holds. First observe that $w_1^*(Hh)$ costs strictly less than $w_1^{so}(Hh)$ for any $\delta > 0$. In fact, the latter is a special case of the former with $\delta = 0$. So we conclude that the solution of the agent's relaxed problem is as characterized in Proposition 2 when $q > p$.

Corollary 1 *If $q > p$ holds, then the solution of the relaxed problem for the agent is $w_1^R = w_1^*$.*

Suppose now $p > q$ holds. It remains to compare $w_1^*(Lh)$ and $w^{so}(H\varnothing)$. It turns out that the former costs strictly less when the discount factor is sufficiently high. Hence the next corollary follows.

Corollary 2 *Assume $p > q$ and $\frac{p-q}{(1-p)(1-q)} < 1$. There exists[18] $\bar{\delta} \in (0, 1)$ such that if $\delta > \bar{\delta}$ then the solution of agent's relaxed problem is $w_1^R = w_1^*$.*

4.4 Unrelaxing the Problem

In this section, I show that at any solution of the relaxed problem, the minmax payoff of each player can be supported as a subgame-perfect equilibrium payoff. In order to prove this claim, I construct the strategies that support the minmax payoffs.

Suppose that the solution of the relaxed problem is given by the wage scheme

$$
\begin{aligned}
w^R & = (w_1^*(Hh), w_2^{so}(Lh)) \\
& = (\frac{c_1}{(1-\sigma)((p+q-1+\delta(1-p)(1-q)))}, \frac{c_2}{(1-\xi)(1-p)q})
\end{aligned}
\tag{1}
$$

(as characterized in Proposition 2) where with a slight abuse of notation, I mean that the agent is paid only at the Hh signal and the supervisor is paid only at the Lh signal. Recall that the solution of supervisor's relaxed problem is the static optimal wages w_2^{so}.

Under the wage scheme w^R as in (1), the stage game payoffs looks as in Figure 3 when $p > q$. The minmax payoffs are realized at the effort profiles

$$
(v_1(w^R), v_2(w^R)) = (\mathbb{E}u_1(N, N), \mathbb{E}u_2(E, E))
\tag{2}
$$

The following proposition constructs an equilibrium strategy for each player in which each player gets her/his minmax payoff.

Proposition 3 *Assume $p > q$ and $\frac{p-q}{(1-p)(1-q)} < \delta < 1$ and fix the wage scheme $w^R = (w_1^*(Hh), w_2^{so}(Lh))$.*

(a) *Let $T(\delta) \equiv \frac{\ln(\delta pq + (1-\delta)(p+q-1))}{\ln(\delta)}$. Consider the following strategy σ^A :*
Play (E, N) for $T(\delta) - 1$ periods and play (N, E) from period $T(\delta)$

---

[18] $\bar{\delta} = \max\{\frac{p(p-q)}{p(p-q)+q(1-p)(2p-1)}, 1 - (\frac{q-qp}{p-pq})^2\}$. Note that $\bar{\delta}$ is well-defined when $p > q$ and $\frac{p-q}{(1-p)(1-q)} < 1$.
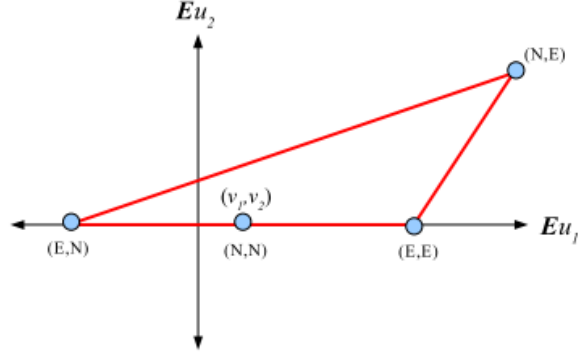
Figure 3: Stage game payoffs when $p > q$

onwards. *If either player deviates from σ^A then start playing σ^A again. Then σ^A constitutes a subgame-perfect equilibrium of game $G^\infty(w^R)$ in which the agent gets his minmax payoff.*[19]

(b) *Consider the following strategy σ^S : Play (E, E) forever unless anyone deviates. If the agent deviates, start playing σ^A. If the supervisor deviates, start playing σ^S again. Then σ^S constitutes subgame-perfect equilibrium of game $G^\infty(w^R)$ in which the supervisor gets her minmax payoff.*

Proposition 3 (a) shows that when $p > q$ the minmax payoff of the agent can be supported with a 'carrot-and-stick' type of strategy.[20] The strategy has two phases: the first serving to give the agent his lowest payoff in the game for a number of periods, and the second following the path of the best equilibrium payoff for the agent. The punishment phase is severe because the supervisor can hold back favorable supervisory reports by committing not to exert effort. The agent complies with the punishment because the

---

[19] The integer problem associated with $T(\delta)$ could be overcome by a public correlation device. When the agent deviates, the play switches to a play where each period agents play (E, N) with probability $P = \frac{p(1-q)}{\delta p(1-q)+q(1-p)}$ and (N, E) with probability $1-P$. When $q > p$, then $P \in (0, 1)$ for any $\delta > 0$. If $p > q$, we need $\delta > \frac{p-q}{p(1-q)}$ so that P is well-defined.

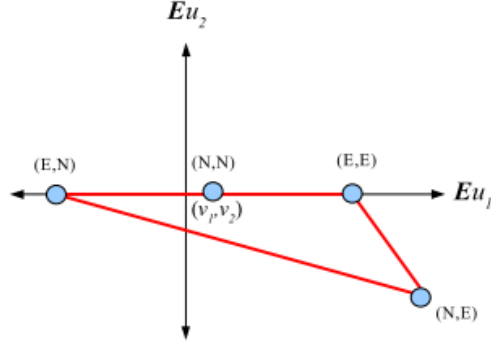[20] Abreu (1986) uses a similar type of strategy.

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Figure 4: Stage game payoffs when $q > p$

supervisor promises to reward the agent with favorable reports that are only possible if the supervisor exerts effort.

Proposition 3 (b) shows that the supervisor's minmax payoff can be achieved as an equilibrium payoff. Recall that the supervisor's minmax payoff is realized at effort profile (E, E). Although the supervisor does not have an incentive to deviate from (E, E), the agent does. Recall that w_1^R is a solution to agent's relaxed problem, so it satisfies (IC_M)

$$\mathbb{E}u_1(E, E; w_1^R) \geq (1 - \delta)\mathbb{E}u_1(N, E; w_1^R) + \delta v_1(w_1^R). \tag{3}$$

Because the agent's minmax payoff under wage scheme w_1^R is shown to be an equilibrium payoff in part (a) of Proposition 3, playing (E, E) forever can also be supported as an equilibrium payoff forever because (3) holds. Hence the supervisor's minmax payoff is also an equilibrium payoff.

Now assume that $q > p$ holds. Under the wage scheme w^R as in (1), the stage game payoffs looks as in Figure 4. Observing Figure 4 shows that (N, N) is a stage game Nash equilibrium. Hence its infinite repetition is a subgame-perfect equilibrium that yields the minmax payoff for the agent. For the supervisor, the payoff from (E, E) can be supported as an equilib-

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rium payoff using the same strategy in Proposition 3 (b). The next corollary summarizes this result.

Corollary 3 *Assume $q > p$ and fix the wage scheme $w^R = (w_1^*(Hh), w_2^{so}(Lh))$. Then the minmax payoff of each player can be supported as subgame-perfect equilibrium payoff of game $G^\infty(w^R)$ for any $\delta > 0$.*

What is left is to check what happens when the solution is given by the wage scheme

$$
\begin{aligned}
w^R & = (w_1^*(Lh), w_2^{so}(Lh)) && (4) \\
& = (\frac{c_1}{(1-\sigma)(q - p + \delta p(1-q))}, \frac{c_2}{(1-\xi)(1-p)q})
\end{aligned}
$$

When $p > q$ holds, the stage game payoffs look exactly like the as in Figure 3. So the conclusion of Proposition 3 follows. When $q > p$ holds, the stage game payoffs are as in Figure 4 , so the conclusion of Corollary 3 follows.

These results establish that the minmax payoff for each player is credible at any solution of the relaxed problem, i.e., the minmax payoff of each player constitutes an equilibrium payoff. Since the minmax payoff is a lower bound on the worst equilibrium payoff, we conclude that the worst equilibrium payoff equals the minmax payoff of each player. Therefore, the solution of the relaxed problem w^R is feasible in principal's problem. That is, w^R satisfies IC_{SPE}. Furthermore, there cannot be any other solution of principal's problem since the solution of the relaxed problem is unique. The next theorem summarizes this result.

Theorem 1 *Suppose the principal wants to implement $(E, E)^\infty$. Then the solution of the relaxed problem w^R is the unique solution of principal's problem.*

In the next section, I discuss some of the properties of optimal dynamic wage schemes.

4.5   Properties of Dynamic Wage Schemes

Likelihood Ratios vs Minmax Punishments

Theorem 1 implies that optimal wage schemes in the dynamic setting are determined by the minmax incentive compatibility constraint. This implies that in a dynamic setting wage schemes trade static incentives to deviate versus the threat of minmax punishments thereafter. By Lemma 1 static incentives are determined by likelihood ratios whereas the minmax punishments are determined by the strategic interaction among players which I call 'dynamic incentives'.

Hence dynamic interaction may create compensation schemes that are different than the ones in the static setting. For a demonstration of this claim, observe Figure 5. For the moment, assume that there is no shock. Then when the supervisor does not work, h signal is impossible according to my stylized model. Assume that the discount factor goes to one which means that the wages are merely determined by their ability to facilitate severe minmax punishments.

Consider a wage scheme that pays the agent only when the supervisor report is h, i.e., pay either $w_1(Hh)$ or $w_2(Lh)$ or a positive amount at both signals. We are interested in an equilibrium which yields the players the lowest possible minmax payoff under this wage scheme. Suppose there exists an equilibrium under which the supervisor does not work. For the supervisor, it is not costly to carry out a punishment. On the other hand, the agent is punished severely by the lack of favorable supervisory reports (due to a shirking supervisor) off the equilibrium. I show that, indeed, under the aforementioned wage scheme an equilibrium exists in which the agent gets zero payoff. When $q > p$, both players shirking becomes a static nash equilibrium, so it is trivially a subgame-perfect equilibrium. Notice that in this equilibrium the agent gets zero payoff because a favorable report h is never realized on the equilibrium path. When $p > q$, and q is close to p, I show that there exists an equilibrium which involves a "carrot-and-stick-type" of strategy as described in Proposition 3. This equilibrium involves a play that yields the lowest possible payoff in the game for a finite number of periods,
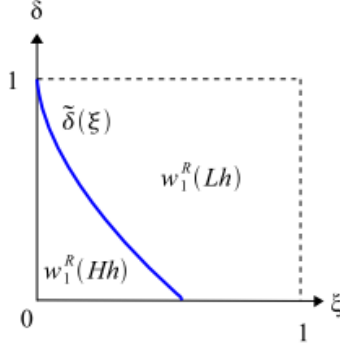
Figure 5: Dynamic Optimal Wages when $q > p$

and then switching to a play that yields the highest possible payoff in the game. Under such a strategy, I show that the agent's payoff is zero. So summarizing, there exists an equilibrium of the game under the aforementioned wage scheme which yields zero minmax payoff for both players.

Now when there is an infinitesimal amount of shock, if the agent is paid only at Hh, he can guarantee himself a strictly positive payoff at his worst equilibrium. This is basically due to the shock. On the other hand, if the agent is paid at Lh, the effect of the shock is shielded away, and I show that there exists an equilibrium under the wage scheme $w_1^*(Lh)$ that can keep the agent to zero payoff in the punishment phase.

While the static incentives favor the signal Hh because it has the highest likelihood ratio, the dynamic incentives instead favor the signal Lh in the presence of an infinitesimal shock. Since the wages are merely shaped by the dynamic incentives as the players are extremely patient, the wages are merely shaped by their ability to facilitate severe minmax punishments.

Full Rent Extraction

In the dynamic setting, the principal can lower the wage schemes in proportion to the discount factor. To see this observe (1) and (4). In fact,

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the principal extracts all the rent from the agent as the discount factor goes to one as shown in the next corollary.

Corollary 4 *As $\delta \to 1$ at the optimal solution the principal extracts all the rent from the agent and the supervisor.*

One implication of Corollary 4 is that in the limit a stationary wage scheme performs as good as a time-variant wage scheme. So the principal does not need to design complex, time-variant wage schemes.

Another implication of corollary 4 is that in the limit there is an equilibrium of the game in pure strategies in which the principal extracts all the rent. Even if the players were allowed to use mixed strategies, the principal could not do any worse. So as $\delta \to 1$ the restriction to pure strategies is without loss of generality.

Application

These findings have interesting implications when applied to CEO compensation. The principal represents the shareholders, the supervisor the board of directors, and the agent CEO. The stock price is a noisy signal of CEO's effort. Whereas in a static setting, the CEO should be compensated for high stock prices, the model predicts that in a dynamic setting compensation schemes could be different. In particular, the CEO could be compensated for low stock prices provided that the supervisor's report is favorable. Such schemes are optimal in the dynamic setting because of their ability to facilitate severe off-equilibrium punishments. Of course, this prediction depends on the details of my stylized model. However the general point is that optimal schemes may not necessarily be synonymous with monotone compensation schemes in a dynamic setting.

The model also predicts that when CEO-board relationship is long-term, tying CEO compensation primarily to the board's report may provide better incentives than rewarding him based on the stock price. While paying the CEO mostly based on the supervisor's favorable report, and not based on the stock price may be interpreted as evidence of board capture[21] by CEO,

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[21] See Bebchuck and Fried (2003,2006)

this paper shows that such schemes could in fact be in the best interest of the shareholders in terms of minimizing the wage bill and preventing collusion.

The results can also be applied to the literature on the behavior of wages over business cycles. The agent represents a productive worker (labor). x_1 is the output level that the worker produces. The favorable shock can be interpreted as an exogenous biased technological shock that increases the labor productivities. When the worker is paid at $x = Hh$, then his wage share (with respect to total output) will be positively correlated with the shock. That is, his wages will be procyclical. There is an extensive literature that confirms that wages behave procyclically (See Keane, Moffitt and Runkle (1988), Solon, Barsky and Parker (1994)). On the other hand, when the worker is paid at $x = Lh$, his wage shares will be negatively correlated with the shock, so his wages will be countercyclical. Young (2004) argues that biased technological changes could be an underlying source of countercyclical US labor shares.[22]

5 Collusion

In this section, I address the robustness of dynamically optimal wage schemes. Collusion-proofness, the idea that the supervisor and agent can collude and coordinate to select an equilibrium that is more favorable for them, if it exists, is a particular concern. At the optimal compensation schemes characterized in Section 4.4, there is multiplicity of subgame-perfect equilibria. Economic theory has not yet settled on a generally agreed-upon method for selecting equilibria of a repeated game. In order to circumvent this problem and ensure a unique prediction, I focus on the subgame-perfect equilibria that is most favorable to the players in the sense that the prescribed equilibrium maximize the player's joint payoff.[23]

Consider the case where the supervisory signals are more accurate than the output signal. That is, $q > p$. It turns out that at the optimal wage scheme (as characterized in Section 4.4) the prescribed equilibrium $(E, E)^\infty$

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[22] I thank Boyan Jovanovic for bringing this literature to my attention.

[23] Che and Yoo (2001) use the same criterion for collusion-proofness.

also maximizes players' joint payoff. To see this, observe Figure 4. Then the optimal wages characterized in Theorem 1 are collusion-proof. This implies that collusion-proofness comes at no additional cost to the principal.

Corollary 5 *Assume $q > p$. The dynamic optimal wage schemes are collusion-proof.*

In fact the equilibrium has even stronger properties. Observe Figure 4 to see that at the equilibrium $(E, E)^\infty$ the agent gets his highest equilibrium payoff. Thus the equilibrium does not require any side transfers among players.[24]

Furthermore, the collusion-proof scheme is 'strongly renegotiation proof' in the sense of Farrel and Maskin (1989). That is, none of the punishment payoffs (continuation payoffs) is strictly dominated by any other equilibrium payoff. This follows from the fact that the supervisor's set of equilibrium payoffs is a singleton (with a value of zero) when $q > p$.(See Figure 4) Therefore, the supervisor is indifferent between all her equilibrium payoffs. Hence, the results continue to hold even if I allow the agent and the supervisor to renegotiate.

On the other hand, when $p > q$ there exists equilibria where both players can strictly be better-off (See Figure 3). Thus the optimal wage schemes are not collusion-proof. It is generally difficult to characterize the collusion-proof schemes. For tractability, I assume that there is no shock. In the absence of shock when the supervisor does not work, he cannot produce any hard evidence, so the signals Hh and Lh are impossible.

In order to characterize the collusion-proof policies, first I solve the principal's problem without any restriction. This way I can utilize the methods developed in the previous sections. Interestingly, in this case there is a multiplicity of optimal schemes a subset of which is collusion-proof as the discount factor goes to one.

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[24]Given the wage scheme $w^r = (w_1^*(Hh), w_2^{so}(Lh))$, the players might find it profitable to play $(N, E)^\infty$ by using side transfers. However, this is not profitable when the ratio of cost of efforts satisfies $\frac{c_1}{c_2} < \frac{(q-p)pq}{(1-p)^2(1-q)}$.

Proposition 4 *Assume there is no shock. Let $p < q$ and $\frac{p-q}{(1-p)(1-q)} < 1$.*
Suppose the principal wants to implement $(\mathbf{e}_1, \mathbf{e}_2) = (E, E)^\infty$ in equilibrium.

(a) *As $\delta \to 1$ there is a multiplicity of optimal wage policies a subset of which is collusion-proof.*

(b) *Both players are paid whenever the supervisor produces evidence.*

(c) *In particular, the compensation scheme which is insensitive to the output signal (for both players) is also collusion-proof.*

(d) *For collusion-proofness, there is an upper bound how much the supervisor should be compensated for agent's failure.*

In the next section, I discuss some properties of collusion-proof wage schemes.

5.1 Properties of Collusion-proof Wage Schemes

Externalities

Under a collusion-proof scheme $w = (w_1, w_2)$ when a player exerts effort the other player gets a higher payoff. Hence, collusion-proof schemes create positive externalities. Formally, w satisfies

$$\mathbb{E}u_i(e_i = k, e_{-i} = E; w_i) \geq \mathbb{E}u_i(e_i = k, e_{-i} = N; w_i) \tag{5}$$

for $k \in \{E, N\}$ and $i = 1, 2$. Che and Yoo (2001) and Abdulkadiroglu and Chung (2003) reach a similar conclusion for two symmetric players.

On the other hand, in my case collusion-proofness requires another condition: under the collusion-proof wage scheme agent's shirking should create 'severe negative externality' on the supervisor in the sense that

$$\mathbb{E}u_2(N, E; w^*(Lh)) < 0 \tag{6}$$

Or equivalently, a collusion-proof scheme should not reward the supervisor for the agent's shirking. If this condition does not hold, both players

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would strictly prefer to play (N, E) forever, and hence $(E, E)^\infty$ would not be collusion-proof. In particular, when supervisory signal is more accurate than the ouput signal,i.e., when $q > p$ holds, the supervisor is hurt by the decreased probability of favorable reports due to agent's shirking. To see this, recall that under $w^*(Lh)) = w^{so}(Lh)$

$$
\begin{aligned}
\mathbb{E}u_2(N, E; w^*(Lh)) &= (1 - \xi)p(1 - q)w^*(Lh) - c_2 \\
&= (1 - \xi)p(1 - q)\frac{c_2}{(1 - \xi)(1 - p)q} - c_2 \\
&= c_2\frac{p - q}{q - pq} < 0
\end{aligned}
$$

On the other hand, the opposite result is true when $p > q$, therefore the optimal wage scheme is not collusion proof in this case.

Insensitivity

In particular, an optimal scheme that is insensitive to the output signal (for both players) is also collusion-proof (given that there is no shock and $\delta \to 1$). It is as if the collusion-proof scheme ignores the output signal. This result is at odds with the standard results of static principal-agent problems which argue that the players are rewarded on the basis of sufficient statistic principle. Starting with Holmstrom (1979), the theory predicts that all the sufficient information should be used in the design of optimal incentive schemes. However when players are in a dynamic interaction, I show that this result no longer holds in the presence of collusion. In order to prevent collusion, optimal wage schemes may choose to ignore sufficient information. Tirole (1992) claims (but does not show) that "standard sufficient statistics principle does not hold in the presence of collusion".

While my results suggest giving up the output signal in designing optimal wages in a dynamic multi-agent environment, the results in the related literature suggest giving up the supervisory function in presence of collusion. Laffont and Meleu (1997) suggests giving up the supervisory function to a third party in a setting where better information among players may hurt

the principal. In accord, Martimort (1997) proposes a wage scheme that becomes less sensitive to players' reports through time. Frascatore (1998) advocates to ignore the supervisor at all. However, all these papers assume enforceable side contracts and study collusion in a different context. This paper shows that the opposite result may hold in a dynamic environment where collusion is possible through effort choices.

Non-monotonicity

In particular, paying both players positive amounts for agent's failure is part of a collusion-proof scheme. As discussed earlier, rewarding for failure is attractive because it can facilitate severe punishments. However there is an upper bound how much the supervisor should be rewarded for agent's failure. If the supervisor is paid 'too much', then both players get higher payoffs when the agent fails.[25] Therefore, the players can collude and can coordinate on the equilibrium where agent shirks (and the supervisor works) and achieve higher payoffs. So any collusion-proof scheme does not reward the supervisor for agent's failure by paying too much.

Another implication of Proposition 4 is that deterring collusion is not necessarily costly in a dynamic setting since the pricincipal can choose an optimal scheme that is collusion-proof. The collusion-proof constraint is not binding. Che and Kim (2006) comes to a similar conclusion with unknown types in a static setting. In contrast, in Tirole (1986), Kofman and Lawarree (1993) and Frascatore (1998) collusion deterrence reduces welfare in a static setting. The next corollary summarizes the result in the absence of shock.

Corollary 6 *When there is no shock, deterring collusion is not costly for the principal when $q > p$ for any $\delta > 0$. If $p > q$ the result holds for q close to p as $\delta \to 1$.*

---

[25]The upper bound on supervisor's wages is dictated by equation (6).

6 Concluding Remarks

The majority of principal-agent models are static, one-agent, one-signal models. This literature suggests designing the wage schemes according to the informational content of signals (i.e., likelihood ratios). However, in real life organizations are dynamic environments with multiple agents and with multiple sources of information. This paper contributes to the literature by studying the optimal design of incentives in a dynamic principal-supervisor-agent environment. In this setting, the principal can use two types of signals; standard output signal and the supervisory signal. The supervisor is not merely an information generating machine but she is an employee of the organization who interacts repeatedly with the agent. The wage schemes in this dynamic setting are, hence shaped by the nature of strategic interaction among employees. We show that the principal designs the wage schemes in order to create a game between employees in which it is easier for the supervisor to "*discipline*" the agent. Specifically, the wage schemes are shaped by their ability to facilitate severe off equilibrium punishments (minmax) against a deviation.

As a result, optimal wages here are different *qualitatively* and *quantitatively* than their counterparts in other settings. A particularly interesting result is that the agent may be rewarded for failure given that the supervisor's report is satisfactory. Although such schemes create strong incentives to shirk in the current period, this effect is overcome by the threat of severe subsequent punishments by the supervisor. Thus, the presence of a strategic supervisor plays a crucial role in shaping the dynamic optimal wages. One implication of this result is that optimal wage schemes should be designed differently than their static counterparts, and that any empirical analysis of compensation schemes that does not account for supervisory reports would be incomplete. This is because incentives are provided not only through compensating the agent for favorable outputs but also through favorable and the lack of supervisory reports.

The model is built on the assumption of perfect monitoring of efforts by the agent and the supervisor, the results can be extended to the case where

there is imperfect monitoring of efforts by players. Fudenberg, Levine and Maskin (1994) show that minmax payoffs[26] become approximately credible under certain conditions[27] when players are extremely patient. Hence, we expect similar results to hold in the imperfect monitoring case as well.

Although I have studied several aspects of compensation design in a hierarchical relationship, the results can generate insights that apply beyond the specific setting. In particular, the results can be applied to the design of compensation schemes among employees whose efforts are inherently complementary. Imagine that the supervisor is another productive agent. The agent and the supervisor produce a joint output (the report). If the supervisor shirks, the joint output is nil. The agent also produces an individual output that depends only his own effort. Assume that the employees interact repeatedly. Then the optimal compensation scheme rewards the agent when his individual production is low but the joint output is high. This implies that the joint task gets rewarded more often than the individual task. Therefore, in environments where there are complementarities among tasks, the compensation schemes should be designed accordingly.

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[26]Note that the minmax payoff in the imperfect monitoring is greater than or equal to the minmax payoff in the perfect monitoring case.

[27]The signal distribution in this paper satisfies the pairwise identifiability and full-rank conditions of Fudenberg, Levine and Maskin (1994) when there is no shock.

7 Appendix

Proof of Lemma 1. The principal's problem in the one-shot game can be reformulated as a standard linear programming problem for each player. From Lemma 4-7 in the technical appendix, there exists an extreme point solution to the problem which puts all the weight on one of the signals. That is, the wage scheme pays a strictly positive amount at most one of the signals is a solution to the principal's problem. Denote the solution that pays only at signal x^* with w^*. Now suppose on the contrary that x^* does not maximize the likelihood ratio, that is $x^* \notin \arg\max_{\hat{x}} LR(\hat{x})$. Then there must exist another signal $\tilde{x}$ such that

$$
\begin{aligned}
LR(\tilde{x}) &> LR(x^*) \qquad\qquad (7) \\
\frac{\Pr(\tilde{x}|e_i = E, e_{-i} = E)}{\Pr(\tilde{x}|e_i = N, e_{-i} = E)} &> \frac{\Pr(x^*|e_i = E, e_{-i} = E)}{\Pr(x^*|e_i = N, e_{-i} = E)}
\end{aligned}
$$

for players $i = 1, 2$. Note that w^* solves the principal's problem, therefore (SC) holds with equality. (See Lemma 4 in technical appendix for a proof). Solving (SC) for $w^*(x^*)$ gives

$$
w^*(x^*) = \frac{c_1}{\Pr(x^*|e_i = E, e_{-i} = E) - \Pr(x^*|e_i = N, e_{-i} = E)}
$$

Furthermore, because w^* is a solution, it must yield a wage payment at least as low as any other wage scheme that satisfies (SC), in particular the wage scheme that puts all the weight on signal $\tilde{x}$. Denote this wage scheme with $\tilde{w}$ Then, we must have

$$
\Pr(x^*|e_i = E, e_{-i} = E)w^*(x^*) \leq \Pr(\tilde{x}|e_i = E, e_{-i} = E)\tilde{w}(\tilde{x})
$$

and

$$
\frac{\Pr(x^*|e_i = E, e_{-i} = E)c_1}{\Pr(x^*|e_i = E, e_{-i} = E) - \Pr(x^*|e_i = N, e_{-i} = E)} \leq \frac{\Pr(\tilde{x}|e_i = E, e_{-i} = E)c_1}{\Pr(\tilde{x}|e_i = E, e_{-i} = E) - \Pr(\tilde{x}|e_i = N, e_{-i} = E)}
$$

which simplifies to

$$\frac{\Pr(\tilde{x}|e_i = E, e_{-i} = E)}{\Pr(\tilde{x}|e_i = N, e_{-i} = E)} \leq \frac{\Pr(x^*|e_i = E, e_{-i} = E)}{\Pr(x^*|e_i = N, e_{-i} = E)}$$

which contradicts with (7). Therefore, x^* be an argmax of the likelihood ratio. The claim that the wage scheme that pays a strictly positive amount only at the argmax of the likelihood ratio is an optimal wage scheme is proven analogously.

Proof of Proposition 1. The principal wants to minimize $\sum_{i=1}^{2} \mathbb{E}w_i(x(E, E))$ subject to the static Nash equilibrium conditions

$$\mathbb{E}u_1(E, E; w_1) \geq \mathbb{E}u_1(N, E; w_1) \tag{8}$$

for the agent and

$$\mathbb{E}u_2(E, E; w_2) \geq \mathbb{E}u_2(E, N; w_2) \tag{9}$$

for the supervisor. First observe that both (8) and (9) only depends on w_i and since the objective function is separable in w_i's, we can write-down two separate minimization problems for each player. We will ignore the participation constraint and then verify later that it is satisfied. ∎

To solve for agent's optimal wages, observe that LHS of equation (8) is

$$\begin{aligned}
\mathbb{E}u_1(E, E; w_1) &= \mathbb{E}w_1(x(E, E)) - c_1 \\
&= (\xi + (1 - \xi)pq)w_1(Hh) \\
&\quad + (1 - \xi)p(1 - q)w_1(H\varnothing) \\
&\quad + (1 - \xi)(1 - p)qw_1(Lh) \\
&\quad + (1 - \xi)(1 - p)(1 - q)w_1(L\varnothing) - c_1
\end{aligned}$$

and the RHS of equation (8) is

$$
\begin{aligned}
\mathbb{E}u_1(N, E; w_1) &= \mathbb{E}w_1(x(N, E)) \\
&= (\xi + (1 - \xi)(1 - p)(1 - q))w_1(Hh) \\
&\quad + (1 - \xi)(1 - p)qw_1(H\varnothing) \\
&\quad + (1 - \xi)p(1 - q)w_1(Lh) \\
&\quad + (1 - \xi)(1 - p)(1 - q)w_1(L\varnothing)
\end{aligned}
$$

so that (8) simplifies to

$$
\begin{aligned}
&(p + q - 1)w_1(Hh) + (p - q)w_1(H\varnothing) \\
&- (p - q)w_1(Lh) - (p + q - 1)w_1(L\varnothing) \\
&\geq \frac{c_1}{(1 - \xi)}
\end{aligned}
\tag{10}
$$

Given that $(p, q) \in (\frac{1}{2}, 1)^2$, (10) is decreasing in $w_1(L\varnothing)$ and increasing in $w_1(Hh)$. Given that the the objective function is increasing in w_1 and we seek to minimize the objective function, we should set $w_1(L\varnothing) = 0$. The coefficients of $w_1(Lh)$ and $w_1(H\varnothing)$ depends on the order of p and q. If $p > q$ we should set $w_1(Lh) = 0$ and if $q > p$ we should set $w_1(H\varnothing) = 0$. If $p > q$ the agent's problem reduces to a problem of two unknown variables, namely $w_1(Hh)$ and $w_1(H\varnothing)$. The slope of (10) becomes $-\frac{p-q}{p+q-1}$ whereas the slope of objective function $\mathbb{E}w_1(x(E, E))$ in agent's problem becomes $-\frac{(1-\xi)p(1-q)}{\xi+(1-\xi)pq}$. If (absolute value of) the slope of (10) is strictly greater than the slope of the objective function, it is optimal to set $w_1(H\varnothing) = \frac{c_A}{(1-\xi)(p-q)}$ and $w_1(Hh) = 0$. If the opposite holds, it is optimal to set $w_1(Hh) = \frac{c_A}{(1-\xi)(p+q-1)}$ and $w_1(H\varnothing) = 0$. The former results holds if and only if $\xi > \xi_{p,q}$ where $\xi_{p,q} = \frac{p(1-p)(2q-1)}{p(1-p)(2q-1)+p-q} \in (0, 1)$. The latter result holds if and only if $\xi < \xi_{p,q}$. In the case where $\xi = \xi_{p,q}$, the principal is indifferent between any wage scheme $(w_1(H\varnothing), w_1(Hh))$ that satisfies (10). Note that $\xi_{p,q} \in (0, 1)$ rests on the assumption that $p > q > \frac{1}{2}$. The case where $q > p$ holds is solved similarly.

To solve for supervisor's optimal wages observe that (9) reduces to

$$pqw_2(Hh) - pqw_2(H\varnothing) \tag{11}$$
$$+(1-p)qw_2(Lh) - (1-p)w_2(L\varnothing)$$
$$\geq \quad \frac{c_2}{(1-\xi)}$$

Observe that the objective function is increasing in both $w_2(H\varnothing)$ and $w_2(L\varnothing)$ whereas (11) constraint is decreasing in these variables. Since we want to minimize the objective function, we must set $w_2(H\varnothing) = w_2(L\varnothing) = 0$. Then the slope of (11) becomes $-\frac{1-p}{p}$ whereas the slope of the objective function is $-\frac{(1-\xi)(1-p)q}{\xi+(1-\xi)pq}$. As long as $\xi > 0$, the (absolute value) of the slope of the constraint is always strictly greater than the slope of the objective function. So the optimal solution is at the boundary where $w_2(Lh) = \frac{c_2}{(1-\xi)(1-p)q}$ and $w_2(Hh) = 0$. On the other hand, if there is no shock, i.e. $\xi = 0$, then any wage scheme $(w_2(Hh), w_2(Lh))$ that satisfies (11) is an optimal wage scheme.

What remains is to check that both participation constraints are satisfied at these proposed wage schemes. ∎

Lemma 3 *Suppose the solution of the relaxed problem w_1^R satisfies DC. Then the minmax payoff for the agent is given by*

$$v_1(w_1^R) = \max\{\mathbb{E}u_1(E, N; w_1^R), \mathbb{E}u_1(N, N; w_1^R)\}$$

Proof of Lemma 3. We will suppress w_1^R for expositional clarity. The minmax payoff is given by

$$
\begin{aligned}
v_1 &= \min\{\max\{\mathbb{E}u_1(E, E), \mathbb{E}u_1(N, E)\}, \max\{\mathbb{E}u_1(E, N), \mathbb{E}u_1(N, N)\}\} \\
&= \min\{\mathbb{E}u_1(N, E), \max\{\mathbb{E}u_1(E, N), \mathbb{E}u_1(N, N)\}\}
\end{aligned}
$$

where the second line follows because w^R satisfies DC. If

$$\mathbb{E}u_1(N, E) < \max\{\mathbb{E}u_1(E, N), \mathbb{E}u_1(N, N)\},$$

then $v_1 = \mathbb{E}u_1(N, E)$. Since w^R is a solution, it must satisfy the minmax incentive compatibility (IC_M)

$$
\begin{aligned}
\mathbb{E}u_1(E, E) &\geq (1 - \delta)\mathbb{E}u_1(N, E) + \delta v_1 \\
\mathbb{E}u_1(E, E) &\geq (1 - \delta)\mathbb{E}u_1(N, E) + \delta\mathbb{E}u_1(N, E) \\
\mathbb{E}u_1(E, E) &\geq \mathbb{E}u_1(N, E)
\end{aligned}
$$

But then this contradicts with DC. If

$$
\mathbb{E}u_1(N, E) = \max\{\mathbb{E}u_1(E, N), \mathbb{E}u_1(N, N)\}
$$

then $v_1 = \mathbb{E}u_1(N, E)$ and the same result follows. Then it must be that

$$
\mathbb{E}u_1(N, E) > \max\{\mathbb{E}u_1(E, N), \mathbb{E}u_1(N, N)\}
$$

holds at the solution, and the conclusion follows. ∎

Proof of Proposition 2. As an implication of the lemma 3, without loss of generality, we can restrict ourselves to wage schemes w_1 that satisfy $v_1(w_1) = \max\{\mathbb{E}u_1(E, N; w_1), \mathbb{E}u_1(N, N; w_1)\}$. We will examine three separate subcases where the minmax equal either $\mathbb{E}u_1(E, N; w_1)$ or $\mathbb{E}u_1(N, N; w_1)$ or both.

Case 1: $v_1(w_1) = Eu_1(N, N; w_1)$ Under this case IC_M becomes

$$
\mathbb{E}u_1(E, E; w_1) \geq (1 - \delta)\mathbb{E}u_1(N, E; w_1) + \delta\mathbb{E}u_1(N, N; w_1)
$$

and simplifies to

$$
\begin{aligned}
w_1(Hh)(\delta pq &+ (1 - \delta)(p + q - 1)) \qquad\qquad (12)\\
&+ w_1(H\varnothing)(p - q - \delta(1 - p)(1 - q)) \\
&+ w_1(Lh)(\delta p(1 - q) - (p - q)) \\
&+ w_1(L\varnothing)(\delta p(q - 1) + 1 - p - q) \\
\geq \quad & \frac{c_1}{(1 - \sigma)}
\end{aligned}
$$

Since the coefficient of $w_1(L\varnothing)$ is negative given that $p + q > 1$, at the solution we must set $w_1(L\varnothing) = 0$. Observe that (12) and objective function are linear in the wage scheme. So there exists an extreme point solution. In the wage scheme space, it can be shown that the extreme wage schemes are those that pay a strictly positive amount at most one signal. If the solution is at the extreme point with $w_1(Hh) > 0$, then we should set $w_1^*(Hh) = \frac{c_1}{(1-\sigma)(\delta pq + (1-\delta)(p+q-1))}$. If the solution is at the extreme point with $w_1(Lh) > 0$, we should set $w_1^*(Lh) = \frac{c_1}{(1-\sigma)(\delta p(1-q)-(p-q))}$. If the solution is at the extreme point with $w_1(H\varnothing) > 0$, we should set $w_1^*(H\varnothing) = \frac{c_1}{(1-\sigma)(p-q)-\delta(1-p)(1-q))}$.

If $\frac{p-q}{(1-p)(1-q)} < 1$, at the solution we must have $w_1(H\varnothing) = 0$. If the solution is $w_1^*(Hh)$, then the expected wage payment to the agent is

$$\mathbb{E}w_1^*(Hh) = c_1 \frac{\xi + (1-\xi)pq}{(1-\xi)(\delta pq + (1-\delta)(p+q-1))}$$

whereas if the solution is $w_1^*(Lh)$ then

$$\mathbb{E}w_1^*(Lh) = c_1 \frac{q(1-p)}{\delta p(1-q) - (p-q)}$$

and $w_1^*(Lh)$ is the unique solution if and only if

$$\begin{aligned}
\mathbb{E}w_1^*(Lh) &< \mathbb{E}w_1^*(Hh) \\
\delta &> \frac{(p-q)(\xi + (1-\xi)pq) + q(1-\xi)(1-p)(p+q-1)}{(1-q)(q(1-\xi)(2p-1) + p\xi)} \\
&\equiv \tilde{\delta}(\xi)
\end{aligned}$$

Observe that $\tilde{\delta}(\xi = 0) = 1$, $\lim_{\xi \to 1} \tilde{\delta}(\xi) = \frac{p-q}{p(1-q)}$ and $\tilde{\delta}(\xi)$ is continuous and strictly decreasing in $\xi > 0$. The derivative of $\tilde{\delta}(\cdot)$ with respect to ξ is

$$\frac{d}{d\xi}\tilde{\delta}(\xi) = \frac{(p-1)(2p-1)q^2}{(\xi p + (1-\xi)q(2q-1))^2(1-q)} < 0$$

is strictly negative, so we conclude that $\tilde{\delta}(\cdot)$ is strictly decreasing in ξ. To

find the limit

$$\lim_{\xi \to 1} \tilde{\delta}(\xi) = \frac{\lim\limits_{\xi \to 1}(p-q)(\xi + (1-\xi)pq) + q(1-\xi)(1-p)(p+q-1)}{\lim\limits_{\xi \to 1}(1-q)(q(1-\xi)(2p-1) + p\xi)}$$

since the denominator is non-zero and both limits exist

$$= \frac{(p-q)\lim\limits_{\xi \to 1}(\xi + (1-\xi)pq) + (1-p)(p+q-1)q\lim\limits_{\xi \to 1}q(1-\xi)}{(2p-1)(1-q)q\lim\limits_{\xi \to 1}(1-\xi) + (1-q)p\lim\limits_{\xi \to 1}\xi}$$

$$= \frac{p-q}{p(1-q)}$$

We showed that for any $\xi > 0$ there exists a $\tilde{\delta}(\xi)$ which is well-defined. What remains is to check if at the proposed solution the minmax payoff equals $v_1 = \mathbb{E}u_1(N, N)$. First, consider the wage scheme $w_1^*(Hh)$

$$\mathbb{E}u_1(N, E; w_1^*(Hh)) - \mathbb{E}u_1(E, E; w_1^*(Hh)) = \frac{\delta(1-p)(1-q)}{\delta pq + (1-\delta)(p+q-1)}c_1 > 0$$

$$\mathbb{E}u_1(N, N; w_1^*(Hh)) - \mathbb{E}u_1(E, N; w_1^*(Hh)) = c_1 > 0$$

and

$$\mathbb{E}u_1(N, E; w_1^*(Hh)) - \mathbb{E}u_1(N, N; w_1^*(Hh)) = \frac{(1-p)(1-q)}{\delta pq + (1-\delta)(p+q-1)}c_1 > 0$$

so $v_1(w_1^*(Hh)) = \mathbb{E}u_1(N, N; w_1^*(Hh))$ as was supposed. Second, consider the wage scheme $w_1^*(Lh)$

$$\mathbb{E}u_1(N, E; w_1^*(Lh)) - \mathbb{E}u_1(E, E; w_1^*(Lh)) = \frac{\delta p(1-q)}{\delta p(1-q) - (p-q))}c_1 > 0$$

$$\mathbb{E}u_1(N, N; w_1^*(Lh)) - \mathbb{E}u_1(E, N; w_1^*(Lh)) = c_1 > 0$$

where the first line is true if $\delta > \frac{p-q}{p(1-q)}$[28] and

$$\mathbb{E}u_1(N, E; w_1^*(Lh)) - \mathbb{E}u_1(N, N; w_1^*(Lh)) = \frac{p(1-q)}{\delta pq + (1-\delta)(p+q-1)}c_1 > 0$$

---

[28] If $p > q$ and $\delta < \frac{p-q}{p(1-q)}$, the same result will follow.

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so that $v_1(w_1^*(Lh)) = \mathbb{E}u_1(N, N; w_1^*(Lh))$ as was supposed.

If $\frac{p-q}{(1-p)(1-q)} > 1$ at the solution we could have $w_1(H\varnothing) > 0$. So we need to compare $w_1^*(H\varnothing)$ against $w_1^*(Hh)$ and $w_1^*(Lh)$ Then the expected wage payment under this wage scheme is

$$\mathbb{E}w_1^*(H\varnothing) = c_1 \frac{p(1-q)}{p-q-\delta(1-p)(1-q)}$$

Observe that as $\delta \to 1$

$$\mathbb{E}w_1^*(H\varnothing) \to c_1 \frac{p(1-q)}{p(1-q)-(1-p)} > 1$$

whereas $\mathbb{E}w_1^*(Lh) \to c_1$. So there exists $\hat{\delta} \in (0, 1)$ such that for all $\delta \geq \hat{\delta}$[29] paying at signal Lh costs strictly less than paying at signal $H\varnothing$. Similarly for $\delta \geq \hat{\delta}$ paying at signal Hh costs strictly less than paying at signal $H\varnothing$.

So we conclude that for any $\delta \geq \hat{\delta}$ there exists $\tilde{\delta}(\xi)$ such that if $\delta > \tilde{\delta}(\xi)$ then paying only at the signal Lh is the unique solution and if $\delta < \tilde{\delta}(\xi)$ then paying at the signal Hh is the unique solution to the relaxed problem of the agent. (If $q > p$ holds the result holds for any $\delta \in (0, 1)$ as $\frac{p-q}{(1-p)(1-q)} < 1$ vacuously holds)

Case 2: $v_1(w_1) = Eu_1(E, N; w_1)$ Under this case IC_M becomes

$$\mathbb{E}u_1(E, E; w_1) \geq (1 - \delta)\mathbb{E}u_1(N, E; w_1) + \delta\mathbb{E}u_1(E, N; w_1)$$

and simplifies to

$$
\begin{aligned}
w_1(Hh)(\delta pq + (1 - \delta)(p + q - 1)) &\quad (13)\\
+w_1(H\varnothing)(p - q - \delta(p + pq - q)&\\
+w_1(Lh)(\delta p(1 - q) - (p - q))&\\
+w_1(L\varnothing)(\delta(p + pq - 1) + 1 - p - q)&\\
\geq (1 - \delta)\frac{c_1}{1 - \sigma}&
\end{aligned}
$$

[29] $\hat{\delta} = \frac{p(p-q)}{p(p-q)+q(1-p)(2p-1)} < 1$

First, observe that the coefficients of $w_1(Hh)$ and $w_1(Lh)$ are the same as in case (12), so the same conclusions follow about these wages. Second, observe that the coefficient of $w_1(L\varnothing)$ is negative for any $\delta \in (0,1)$.[30] So without loss of generality, we can restrict attention to wage schemes with $w_1(L\varnothing) = 0$. If $q > p$ then the coefficient of $w_1(H\varnothing)$ is negative. To see this

$$p - q - \delta(p - q + pq) < 0 \Leftrightarrow \delta > \frac{p - q}{p - q + pq}$$

Note that $p - q + pq > 0$ given that $p > \frac{1}{2}$. So if $q > p$ we must set $w_1(H\varnothing) = 0$ as well. However the payoff at (N, N) is

$$\mathbb{E}u_1(N, N; w_1) = \xi w_1(Hh) + (1 - \xi)(1 - p)w_1(H\varnothing)$$

whereas the payoff at (E, N) is

$$\mathbb{E}u_1(E, N; w_1) = \xi w_1(Hh) + (1 - \xi)pw_1(H\varnothing) - c_1$$

So if $w_1(H\varnothing) = 0$, then the payoff at (N, N) becomes strictly greater than the payoff at (E, N) which in turn implies that the minmax payoff can not equal to $\mathbb{E}u_1(E, N; w_1)$ in the first place. So for the minmax payoff to equal the payoff at the strategy profile (E, N) we must have

$$
\begin{aligned}
\xi w_1(Hh) + (1 - \xi)pw_1(H\varnothing) - c_1 &\geq \xi w_1(Hh) + (1 - \xi)(1 - p)w_1(H\varnothing) \\
w_1(H\varnothing) &\geq \frac{c_1}{(1 - \xi)(2p - 1)}
\end{aligned}
$$

So it follows the wage payment at the signal $H\varnothing$ must be greater than a threshold. If $q > p$ at the solution we set $w_1(H\varnothing) = \frac{c_1}{(1-\xi)(2p-1)}$ as we can

---

[30]

$$\delta(p + pq - 1) + 1 - p - q < 0 \rightarrow \delta < \frac{p + q - 1}{p + pq - 1} > 1$$

not reduce $w_1(H\varnothing)$ any further. Replacing this into (13)

$$w_1(Hh)(\delta pq + (1 - \delta)(p + q - 1)) \tag{14}$$
$$+ w_1(Lh)(\delta p(1 - q) - (p - q))$$
$$\geq \frac{c_1}{1 - \sigma}\left(\frac{\delta(1 - p)(1 - q) + p + q - 1}{2p - 1}\right)$$

Observe that the last term on the RHS becomes strictly greater than 1 when $q > p$, so the solution in this case will yield strictly more costly wage schemes than the wages defined by (13).

If $p > q$ setting $w_1(H\varnothing) = \frac{c_1}{(1-\xi)(2p-1)}$ is optimal if $\delta > \frac{p-q}{p-q+pq}$. In addition, the last term on the RHS of (14) becomes strictly greater than 1 when $\delta > \frac{p-q}{(1-q)(1-p)}$. So the wages defined by (13) costs strictly less since the coefficients of $w_1(Hh)$ and $w_1(Lh)$ are the same in both situations if $\frac{p-q}{(1-q)(1-p)} < 1$.

Consider the last case with $v_1(w_1) = \mathbb{E}u_1(N, E; w_1) = \mathbb{E}u_1(N, N; w_1)$. Continuing from our argument in the second case, this case implies that $w_1(H\varnothing) = \frac{c_1}{(1-\xi)(2p-1)}$. But then this reduces (14) and the same conclusion follows. ∎

Proof of Corollary 2. Suppose that $p > q$ and $\frac{p-q}{(1-q)(1-p)} < 1$. The static optimal wages are

$$\mathbb{E}w^{so}(Hh) = \frac{c_1}{1-\xi}\frac{\xi + (1-\xi)pq}{p+q-1} \quad \text{if } \xi < \xi_{p,q}$$
$$\mathbb{E}w^{so}(H\varnothing) = c_1\frac{p-pq}{p-q} \quad \text{if } \xi \geq \xi_{p,q}$$

First observe that $\mathbb{E}w_1^*(Hh) < \mathbb{E}w^{so}(Hh)$ for any $\delta > 0$. $\mathbb{E}w_1^*(Lh) < \mathbb{E}w^{so}(H\varnothing)$ iff

$$\delta > 1 - \left(\frac{q - qp}{p - qp}\right)^2$$

Observe that the RHS is in the region $(0, 1)$ as long as $p > q$. Define $\bar{\delta} = \max\{1 - (\frac{q-qp}{p-qp})^2, \hat{\delta}\}$. If $\delta > \bar{\delta}$ then the solution satisfies DC. ∎

Proof of Proposition 3.

Proof of part (a): For any $\delta \in (0, 1)$ choose $T(\delta)$ such that

$$(1 - \delta^T)\mathbb{E}u_1(E, N) + \delta^T\mathbb{E}u_1(N, E) = \mathbb{E}u_1(N, N)$$

holds. That is, set $T(\delta) \equiv \frac{\ln(\delta pq + (1-\delta)(p+q-1))}{\ln(\delta)}$ and examine figure 3 to verify that this can be done. When the discount factor is taken as $\delta > \frac{p-q}{(1-p)(1-q)}$ then $T(\delta) > 0$. If $T(\delta) > 0$. If $T(\delta) < 1$ or is not an integer, the same strategy can be implemented by using a public correlating device where each period the players play (E, N) with probability $P = \frac{(1-p)(1-q)}{\delta(1-p)(1-q)+pq} \in (0, 1)$ and (N, E) with the remaining probability. Note that this probability is well-defined since $p > \frac{1}{2}$ and $q > \frac{1}{2}$. If a public correlating device is not available, then a more complex strategy that involves playing (E, N) and (N, E) can still be constructed so that the payoff at profile (N, N) is approximately attainable.

σ^A constitutes a SPE of the game $G^\infty(w^R)$ if and only if the following (i)-(ii) holds. Let's examine each case in detail. We will suppress using w^R in the utility function all the time for expositional purposes.

(i) Under σ^A we must show that the supervisor has no incentive to deviate. In the play of σ^A, there are two different types of subgames: The prescribed play is (E, N) for $T(\delta) - 1$ periods and (N, E) from period $T(\delta)$ onwards. First of all, observe that the supervisor will not deviate in the subgame that prescribes the play of (N, E) forever, because he achieves his highest possible payoff in the game $G^\infty(w^R)$. In the subgame that prescribes the play (E, N) for $T(\delta) - 1$ periods, it suffices to check if the agent has no incentive to deviate at the beginning of the $(T(\delta) - 1)^{th}$ period since the incentives to deviate are strongest in the beginning of the play (E, N) since $\mathbb{E}u_2(E, N) < \mathbb{E}u_2(N, E)$.

In the subgame that starts with the play (E, N), the supervisor will not deviate since

$$\mathbb{E}U_2(\sigma^A) \geq (1 - \delta)\mathbb{E}u_2(E, E) + \delta\mathbb{E}U_2(\sigma^A)$$

$$(1 - \delta^T)\underbrace{\mathbb{E}u_2(E, N)}_{0} + \delta^T\mathbb{E}u_2(N, E) \geq (1 - \delta)\underbrace{\mathbb{E}u_2(E, E)}_{0} + \delta(\delta^T\mathbb{E}u_2(N, E))$$

$$\mathbb{E}u_2(N, E) \geq \delta\mathbb{E}u_2(N, E)$$

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holds for $\delta \in (0, 1)$.

(ii) We must show that the agent has no incentive to deviate from the σ^A. He will not deviate in the subgame where the play is (N, E) because he gets his highest equilibrium payoff. The agent will not in the subgame where (N, E) is played since

$$
\begin{aligned}
\mathbb{E}U_1(\sigma^A) &\geq (1 - \delta)\mathbb{E}u_1(N, N) + \delta\mathbb{E}U_1(\sigma^A) \\
\mathbb{E}u_1(N, N) &\geq (1 - \delta)\mathbb{E}u_1(N, N) + \delta\mathbb{E}u_1(N, N) \\
\mathbb{E}u_1(N, N) &\geq \mathbb{E}u_1(N, N)
\end{aligned}
$$

vacuously holds. Combining (i) and (ii) completed the proof for part (a).

Proof of part (b): σ^S is a subgame-perfect equilibrium if and only if (i)-(iii) holds.

(i) The supervisor has no incentive to deviate since

$$
\begin{aligned}
\mathbb{E}U_2(\sigma^S) &\geq (1 - \delta)\mathbb{E}u_2(E, N) + \delta\mathbb{E}U_2(\sigma^S) \\
\mathbb{E}u_2(E, E) &\geq (1 - \delta)\mathbb{E}u_2(E, N) + \delta\mathbb{E}u_2(E, E) \\
0 &\geq 0
\end{aligned}
$$

vacuously holds.

(ii) From part (a) we know that σ^A is an equilibrium strategy. Then the agent has no incentive to deviate when

$$
\begin{aligned}
\mathbb{E}U_1(\sigma^S) &\geq (1 - \delta)\mathbb{E}u_1(N, E) + \delta\mathbb{E}U_1(\sigma^A) &\qquad (15) \\
\mathbb{E}u_1(E, E) &\geq (1 - \delta)\mathbb{E}u_1(N, E) + \delta\mathbb{E}u_1(N, N)
\end{aligned}
$$

holds. Recall that these payoffs depends on the wage profile w^R which is a solution of the relaxed problem. By definition w^R we must satisfy the minmax incentive compatibility constraint

$$
\mathbb{E}u_1(E, E; w^R) \geq (1 - \delta)\mathbb{E}u_1(N, E; w^R) + \delta v_1(w^R)
$$

but since at w^R the minmax payoff of the agent is $v_1(w^R) = \mathbb{E}u_1(N, N; w^R)$

(15) is satisfied. This concludes the proof. ■

Proof of Theorem 1. In proposition 3 we showed that exerting effort every period constitutes a subgame perfect equilibrium at the wage scheme w^R and the minmax payoffs can be supported as equilibrium payoffs. So under w^R the worst equilibrium payoff of each agent is given by their respective minmax payoffs. Then w^R is feasible in principal's problem, that is w^R satisfies (IC_{SPE}). Now suppose on the contrary that w^R is not a solution to the general problem. Then there must exist another stationary wage scheme $w \neq w^R$ that solves principal's problem with

$$\sum_{i=1}^{2} \mathbb{E}w_i < \sum_{i=1}^{2} \mathbb{E}w^R$$

However since w^R solves the relaxed problem we must have

$$\sum_{i=1}^{2} \mathbb{E}w_i \geq \sum_{i=1}^{2} \mathbb{E}w^R$$

which is a contradiction. Now suppose that w^r is not the unique solution of principal's problem. Then there exist another wage scheme $w \neq w^R$ that solves principal's problem with

$$\sum_{i=1}^{2} \mathbb{E}w_i = \sum_{i=1}^{2} \mathbb{E}w^R \tag{16}$$

then w is also feasible in the relaxed problem by construction of the relaxed problem, that is w satisfies . Because w^R is the unique solution of the relaxed problem, we must have

$$\sum_{i=1}^{2} \mathbb{E}w_i > \sum_{i=1}^{2} \mathbb{E}w^R \tag{17}$$

but then we have a contradiction with (16). ■

Proof of Corollary 4. When $\delta \to 1$, the optimal wage scheme is given by $w_1^*(Lh) = \frac{c_1}{(1-\sigma)(q-p+\delta p(1-q))}$ for the agent . The (normalized) expected

wage payment is given by

$$
\begin{aligned}
&= \quad \mathbb{E}u_1(E, E; w_1^*(Lh)) + c_1 + \mathbb{E}u_2(E, E; w_2^*(Lh)) + c_2 \\
&= \quad \frac{(1-\delta)(1-q)p}{\delta(1-q)p - (p-q)} c_1 + c_1 + c_2 \\
&= \quad c_1 \frac{q(1-p)}{\delta p(1-q) + q - p} + c_2
\end{aligned}
$$

It is easy to verify that the first term above goes to c_1 as $\delta \to 1$. ■

Proof of Proposition 4. When we ignore the additional constraint that the principal wants to implement the best equilibrium for the players, the problem is a special case of the problem in section 4 with $\sigma = 0$ and $\delta \to 1$. By Fudenberg and Maskin (1986) when $\delta \to 1$ the minmax payoffs can be credibly used as punishments and thus the optimal wage schemes for each agent can be solved separately. When $\sigma = 0$, the static optimal wages for the supervisor is any wage scheme $w^{so} = (w_2(Hh), w_2(Lh))$ that satisfies

$$
\begin{aligned}
\mathbb{E}u_2(E, E; w_2) &\geq \mathbb{E}u_2(E, N; w_2) \quad\quad\quad (18) \\
pqw_2(Hh) + (1-p)qw_2(Lh) &\geq c_2
\end{aligned}
$$

and the supervisor earns no rent at the equilibrium. Notice that when $\sigma = 0$ there is multiplicity of optimal wages for the supervisor.

For the agent the principal needs to perform better than the static wages since the agent extracts rent in the static game. As I show in Theorem 1, the static wages for the agent can not be optimal as $\delta \to 1$. The optimal wage schemes in the repeated game as $\delta \to 1$ are characterized with

$$
\mathbb{E}u_1(E, E; w_1) \geq v_1(w_1) \quad\quad\quad (19)
$$

when $\mathbb{E}u_1(E, E; w_1) < \mathbb{E}u_1(N, E; w_1)$. When the discount factor is sufficiently high, the minmax payoff is approximately attainable as an equilibrium payoff. As was shown in Lemma 3, the pure strategy minmax payoff

of the agent is given by

$$v_1(w_1) = \max\{\mathbb{E}u_1(E, N; w_1), \mathbb{E}u_1(N, N; w_1)\}$$

If $v_1(w_1) = \mathbb{E}u_1(N, N; w_1)$ then (19) becomes

$$
\begin{aligned}
& pqw_1(Hh) + (1-p)qw_1(Lh) \\
& +(p(1-q) - (1-p))w_1(H\varnothing) \\
& +(p(q-1) + 1 - p - q)w_1(L\varnothing) \\
\geq \quad & c_1
\end{aligned}
$$

We should set $w_1(L\varnothing) = 0$ since its coefficient is negative for any $p > \frac{1}{2}$ and $q > \frac{1}{2}$. Furthermore, the coefficient of $w_1(H\varnothing)$ is negative as long as $1 - q > \frac{1-p}{p}$.[31] By setting $w_1(H\varnothing) = 0$, (19) reduces to

$$pqw_1(Hh) + (1-p)qw_1(Lh) = c_1 \tag{20}$$

Observe that at any wage scheme $w_1 = (w_1(Hh), w_1(Lh))$ satisfying (20) has $\mathbb{E}u_1(E, E; w_1) = 0$, so the participation constraint is also satisfied with equality. Now we will show that at any wage scheme that satisfies (20) the minmax payoff is indeed at (N, N). To see this, at such w_1 we have $\mathbb{E}u_1(N, N; w_1) = 0$ and $\mathbb{E}u_1(N, E; w_1) = -c_1 < 0$. Furthermore

$$\mathbb{E}u_1(N, E; w_1) = \frac{(1-q)}{(1-p)q}(w_1(Hh)q(1 - 2p) + pc_1) > 0$$

where the last inequality follows from the fact that the highest value that $w_1(Hh)$ can get is $\frac{c_1}{pq}$. So $\mathbb{E}u_1(N, E; w_1) > \mathbb{E}u_1(E, E; w_1)$. Then this shows that $v_1(w_1) = \mathbb{E}u_1(N, N; w_1)$. We can conclude that this is indeed an optimal wage scheme without checking the other cases since any wage scheme that costs strictly less will violate the participation constraint. Denote the optimal wage schemes that satisfy (20) as $\hat{w}_1$. At $\hat{w}_1$ the order of the agent's

---

[31] If $q > p$ the coefficient is negative regardless. To see this, note that the coefficient is negative as long as $\frac{2p-1}{p} < q$. Define $f(p) \equiv \frac{2p-1}{p}$. Note that $f(\frac{1}{2}) < \frac{1}{2}$, $f(1) = 1$ and $f(\cdot)$ is strictly increasing. Then it follows that $f(p) < p < q$.

payoff is given by

$$\mathbb{E}u_1(E, N; \hat{w}_1) < \mathbb{E}u_1(N, N; \hat{w}_1) \le \mathbb{E}u_1(E, E; \hat{w}_1) < \mathbb{E}u_1(E, E; \hat{w}_1)$$

At supervisor's optimal wages w^{so} we have

$$\mathbb{E}u_2(E, E; w^{so}) = \mathbb{E}u_2(N, N; w^{so}) = \mathbb{E}u_2(E, N; w^{so}) = 0$$

so that $v_2(w^{so}) = 0$. However the sign of $\mathbb{E}u_2(N, E; w^{so})$ could be positive or negative depending on w^{so}. If $\mathbb{E}u_2(N, E; w^{so}) > 0$ then the supervisor can not get his best equilibrium payoff at (E, E) since he earns zero at that scheme and (N, E) gives a strictly higher utility. Then we must have $\mathbb{E}u_2(N, E; w^{so}) \le 0$ for collusion-proofness. This condition is satisfied at w^{so} if

$$\mathbb{E}u_2(N, E; w^{so}) \quad \le \quad 0 \tag{21}$$
$$(1-p)(1-q)w^{so}(Hh) + p(1-q)w^{so}(Lh) \quad \le \quad c_2$$
$$w^{so}(Lh) \quad \le \quad \frac{c_2 - (1-p)(1-q)w^{so}(Hh)}{p(1-q)}$$

on the other hand by (18) we have

$$w^{so}(Hh) = \frac{c_2 - (1-p)qw_2^s(Lh)}{pq}$$

and replacing this into (21) gives

$$w^{so}(Lh) \quad \le \quad \frac{c_2 - (1-p)(1-q)(\frac{c_2 - (1-p)qw_2^s(Lh)}{pq})}{p(1-q)}$$
$$w^{so}(Lh) \quad \le \quad c_2 \frac{p + q - 1}{q(1-q)(2p-1)} \equiv \bar{w}_2(Lh)$$

and we conclude that if $w^{so}(Lh) \le \bar{w}(Lh)$ then $\mathbb{E}u_2(N, E; w^{so}) \le 0$. So at optimal wage scheme $\hat{w} = (\hat{w}_1, w^{so})$ with $w^{so}(Lh) \le \bar{w}(Lh)$, both players will be getting their highest equilibrium payoffs. This implies that at such wage profiles, the equilibrium will be collusion-proof. In particular $w^{so}(Lh) = 0$

is a also collusion proof. A simple calculation shows that $\bar{w}_2(Lh) > c_2$. ∎

8   Technical Appendix

Let $I = \{1, 2, .., n\}$, $a_i \in \mathbb{R}_{++}$ and $b_i \in \mathbb{R}$ for all $i \in I$. Consider the following linear constrained minimization problem where $a_i > 0$ for all $i \in I$, $K \in \mathbb{R}_{++}^n$ and there is at least one $j \in I$ such that $b_j > 0$.

$$\min_{(x_1,..,x_n) \in \mathbb{R}_+^n} \sum_{i \in I} a_i x_i \qquad \text{(SLP)}$$

subject to constraints

$$\sum_{i \in I} b_i x_i \geq K$$
$$x_i \geq 0 \text{ for all } i \in I$$

We will refer the first inequality as the "constraint" and the second set inequalities as the "non-negativity constraints".

Throughout we will refer to this problem as the standard linear programming (SLP) problem. Let $x = (x_1, .., x_n) \in \mathbb{R}_+^n$, $a = (a_1, .., a_n) \in \mathbb{R}_{++}^n$ and $b = (b_1, .., b_n) \in \mathbb{R}_+^n$. We can also re-write the SLP problem as

$$\min_x a \cdot x^T$$

subject to

$$b \cdot x^T \geq K, \quad x \geq 0$$

where x^T is the transpose of x and $a \cdot b^T$ is the dot product of vectors $a, b \in \mathbb{R}^n$.

We will make use of the following lemmas frequently throughout the paper.

Lemma 4 *At an optimal solution of the SLP problem, the constraint holds with inequality.*

Proof. Suppose on the contrary that at the optimal solution, say $x \in \mathbb{R}^n_+$, the constraint does not hold with equality. Then we must have $\sum_{i \in I} b_i x_i > K$. If this is the case, since we have $x_i \geq 0$ for all $i = 1..n$, there must be an $l \in I$ such that $x_l > 0$ and $b_l > 0$. For an infinitesimal $\varepsilon > 0$, define $\hat{x}_l = x_l - \varepsilon > 0$, then the constraint still continues to hold with strict inequality at $\hat{x}_l$. But the value of the objective function is strictly less than its value at x_l since $a_l > 0$. This is a contradiction. QED ∎

Lemma 5 *Assume there exists an $k \in I$ such that $b_k < 0$. Then at an optimal solution, if it exists, we must have $x_k = 0$.*

Proof. Suppose on the contrary that at the at the optimal solution, we have $x_k > 0$. From Lemma 4 we know that , at an optimal solution the constraint will hold with equality. Pick $j \in I$ such that $b_j > 0$ and re-write the constraint as;

$$x_j = \frac{K - \sum_{i \neq j} b_i x_i}{b_j}$$

Replacing x_j gives the following value for the objective function;

$$\sum_{i \neq j} b_i x_i + a_j \left(\frac{K - \sum_{i \neq j} b_i x_i}{b_j} \right)$$

$$= a_j \frac{K}{b_j} + (b_k - a_k \frac{K}{b_k}) x_i + \sum_{i \neq j} (b_i - a_i \frac{K}{b_i}) x_i$$

Since $b_k < 0$ and $a_k > 0$, the coefficient of the of x_k is strictly positive. But then setting $x_k = 0$ gives a lower value for the objective function which is in contradiction with our assumption was at the optimal solution $x_k > 0$. QED. ∎

Lemma 6 *The SLP problem has a solution.*

Proof. From Lemma 1, we know that at an optimal solution, if it exists, we must have $x_k = 0$ for all k such that $b_k < 0$. Then without loss of generality,

we can restrict attention to the case where $b_i > 0$ for all $i \in I$. Furthermore from Lemma 4, we know that at an optimal solution, the constraint will hold equality. Then the solution of the SLP problem coincides with the solution of the following problem, adjusted for the fact that $x_k = 0$ for all k such that $b_k < 0$);

$$\min_x a \cdot x^T$$

subject to

$$b \cdot x^T = K, \quad x \geq 0$$

with $b \in \mathbb{R}^n_{++}$, $a \in \mathbb{R}^n_{++}$ and $K > 0$.

Now define $C = \{x \in \mathbb{R}^n_+ : b \cdot x^T = K\}$ where and $f : C \to \mathbb{R}$ by $f(x) = a \cdot x$ Observe that C is a compact and convex subset of $\mathbb{R}^n_+$ since $b \in \mathbb{R}^n_{++}$ and $K > 0$. Furthermore, f is a continuous function on C. Then by Weierstrass's Theorem, we can conclude that f attains its minimum on C. QED. ∎

Definition 2 *Let C be a convex set in $\mathbb{R}^n$. x is an extreme point of C if and only if there does not exist distinct points $x_1, x_2, .., x_n$ in C such that $x = \alpha_1 x_1 + .. + \alpha_n x_n$ for all $\alpha_i \in [0, 1]$ with $\sum\limits_{i=1}^{n} \alpha_i = 1$.*

Lemma 7 *If the SLP has a solution, then there exists an extreme point at which the minimum is attained.*

Proof. Consider the following set

$$C^{ext} = \{(x_1, .., x_n) \in C : \text{ There is an } i \in I \text{ st. } x_i > 0 \text{ and } x_j = 0 \text{ for all } j \neq i\}$$

First, we claim that C^{ext} is the set of extreme points of C. To see this, note that if $x \in C^{ext}$, then x is an extreme point of C. What remains to be shown is that there does not exist any other extreme point of C. Take any $y \in C \backslash C^{ext}$. Let $y = (y_1, .., y_n)$. We will show that y can be written as a convex combination of points in C^{ext}.

Notice that if $x^i \in C^{ext}$, then $x_i^i = \frac{K}{b_i} > 0$ and $x_j^i = 0$ for all $j \neq i$. Define $\alpha_i = \frac{y_i b_i}{K}$ for all i. Notice that since each $b_i > 0$, $K > 0$ and $y_i \geq 0$, we have

$\alpha_i \geq 0$.(Also since there is at least one i with $y_i > 0$, we have at least one $\alpha_i > 0$). Furthermore, since $y \in C$, we have $\sum_{i=1}^{n} y_i b_i = K$. This implies both $\alpha_i \in [0, 1]$ and

$$\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \frac{y_i b_i}{K} = 1.$$

However, we also have

$$\begin{aligned} \sum_{i=1}^{n} \alpha_i x^i &= \alpha_1(\frac{K}{b_1}, 0, .., 0) + .. + \alpha_n(0, 0, .., \frac{K}{b_n}) \\ &= (\alpha_1 \frac{K}{b_1}, .., \alpha_n \frac{K}{b_n}) \\ &= (\frac{y_1 b_1}{K} \frac{K}{b_1}, .., \frac{y_n b_n}{K} \frac{K}{b_n}) \\ &= (y_1, .., y_n) \end{aligned}$$

which shows that y can be written as a convex combination of distinct points in C^{ext}. But then any $y \in C \backslash C^{ext}$ can not be an extreme point of C. So we showed that C^{ext} is the set of extreme points of C. ∎

Returning to the proof of Lemma 7, assume that SLP problem has a solution. Suppose on the contrary that at none of the extreme points the minimum is attained. That is, assume that $y \notin C^{ext}$. Denote the minimum solution as $y \in C$. Then we must have $f(x^i) > f(y)$ for all $x^i \in C'$. Then by using Claim 1, we can show that y can be written as a convex combination of elements in C^{ext}. That is, there exists some $\alpha_i \in [0, 1]$ with $\sum_{i=1}^{n} \alpha_i = 1$ such that $\sum_{i=1}^{n} \alpha_i x^i = (y_1, .., y_n)$. Observe that, since $f(\cdot)$ is linear, we must have;

$$\begin{aligned} f(\sum_{i=1}^{n} \alpha_i x^i) &= f(y_1, .., y_n) \\ \sum_{i=1}^{n} \alpha_i f(x^i) &= f(y) \end{aligned} \tag{22}$$

But since we assumed that $f(x^i) > f(y)$ for all x^i, we must also have $\sum_{i=1}^{n} \alpha_i f(x^i) > f(y)$ which is in contradiction with (10). This completes

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the proof.

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