# Renegotiation-Proof Relational Contracts with Side Payments

Sebastian Kranz and Susanne Ohlendorf*

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**Abstract**

We study infinitely repeated two player games with perfect monitoring in which the players have the possibility to make monetary transfers to each other. We show that in order to find all Pareto-optimal subgame perfect payoffs for a given discount factor, one may restrict attention to a special class of subgame perfect equlibria which we call stationary contracts. We also examine different concepts of renegotiation-proofness and extend the result to renegotiation-proof payoffs. These results are useful because we arrive at simple conditions that characterize both Pareto-optimal and renegotiation-proof stationary contracts, which can therefore be found easily in many applications.

**JEL** classification: C73, L14

**Keywords**: renegotiation, infinitely repeated games, side payments, optimal penal codes

*Department of Economics, University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany. Emails: skranz@uni-bonn.de, sohlendo@uni-bonn. Financial support from Deutsche Forschungsgemeinschaft through SFB-TR 15 is gratefully acknowledged. We thank Andreas Blume and Paul Heidhues for helpful discussions.

# 1 Introduction

Relational contracts are self-enforcing informal agreements that arise in many long-term relationships, often in response to obstacles to write exogenously enforceable contracts. Examples include the non-contractible aspects of employment relations, illegal cartel agreements, or buyer-seller relations in which complete formal contracts are too costly to write. Agreements between countries also often have the nature of a relational contract, when there is no institution that is able or willing to enforce compliance with the agreed terms. In these examples, monetary transfers play a role in the relationships, be it in form of prices, bonuses or other compensation schemes, and could thus also be used to sustain the relational contract. Moreover, the relational contracts are drafted and negotiated, and meetings continue to take place as the relationship unfolds. In this paper, we analyze relational contracts under these circumstances: with renegotiation and the possibility to make monetary transfers.

As an illustration how side-payments can be used in a relational contract consider the case of collusive agreements. Cartels sometimes use compensation schemes to make sure that each firm in the cartel stays with the target (see Harrington (2006) for details[1]). A cartel member that violates the agreement is required to buy a certain quantity from a competitor, or to transfer a valuable customer to a competitor. Such compensation schemes seem more robust to renegotiation than threatening with an immediate price war after a violation of the agreement. Price wars are costly for all firms, and therefore cartel members will be tempted to agree to ignore the violation. In contrast, if a deviating firm must pay a fine, competitors gain from the punishment and have therefore no incentive to renegotiate the agreement. However, to induce a firm to pay the compensation there must be the threat of a real punishment in case no payment is made, i.e. a punishment that does not require the voluntary participation of the punished firm. Renegotiation may then play a role again.

The present paper investigates these issues and provides a characterization of renegotiation-proof relational contracts given arbitrary discount factors. We study infinitely repeated two player games with perfect monitoring where in each period

[1] For a list of cartels in which such compensation schemes have been used see the introduction of Harrington and Skrzypacz (2007), who also offer a theoretical analysis of collusion with imperfect monitoring.

players can make monetary transfers before they play a simultaneous move stage game. We translate Abreu's (1988) optimal penal codes to this set-up and show that every Pareto optimal subgame perfect payoff can be achieved using a class of simple strategy profiles, which we call *stationary contracts*. In a stationary contract, the same action profile is played in every period. A player who deviates is required to pay a fine to the other player, and after payment equilibrium play is resumed. If a player does not make a required payment, there is a one time play of a punishment action profile, followed by an adjusted payment, after which play continues as on the equilibrium path. Any feasible distribution of joint surplus can be achieved by incentive compatible up-front payments; equilibrium path payments in later periods can be used to balance incentives constraints between the two players.

In the second part of the paper, we characterize renegotiation-proof stationary contracts, and show that again, one can often restrict the analysis to the simple class of stationary contracts to find payoffs that survive renegotiation-proofness refinements. Since a period consists of two stages, a crucial question is at what times renegotiation is possible. In the existing literature, different assumptions have (often implicitly) been made. For example, Fong and Surti (2009) assume in their study of repeated Prisoner Dilemma games with side-payments that renegotiation is possible at every stage of the game, both before a payment is made and before an action is taken. In contrast, Levin (2003) assumes in his study of repeated principal-agent relationships that renegotiation is only possible at the beginning of a period, before payments are made.

Levin observes that the possibility of renegotiation before the payment stage does not alter the Pareto frontier of implementable payoffs. This observation easily extends to our set-up in which both players can take actions. The reason is that punishment at the payment stage takes the form of the deviator paying a fine to the other player immediately followed by a return to equilibrium play. Hence, in a Pareto efficient stationary contract, all continuation equilibria that start at the payment stage achieve the highest feasible joint continuation payoff. This means that if renegotiation is allowed only before payments can be made, the threat of inefficient continuation play (which is necessary to induce payment of the fine) is never subject to renegotiation.

In the main part of this paper, we also allow renegotiations before the action stage. Even having fixed the timing of renegotiation, there exist several concepts of renegotiation-proofness to choose from.

We first adapt strong perfection (see Rubinstein, 1980) to our setting. In a game with two players, a subgame perfect equilibrium is strong perfect if all its continuation payoffs lie on the Pareto frontier of subgame perfect continuation payoffs. In general, the set of strong perfect equilibrium payoffs is a subset of the Pareto frontier of subgame perfect payoffs, but it may well be empty. We show that every strong perfect payoff can be achieved by a stationary contract and derive simple conditions that allow to check for strong perfection. These conditions are then used to show that in simple principal-agent games strong perfect stationary contracts always exist, while in other examples they also often fail to exist, which only reflects that strong perfection should be considered a sufficient requirement for renegotiation-proofness rather than a necessary one.

We then analyse the concepts of weak renegotiation-proofness (WRP) and strong renegotiation-proofness (SRP) introduced by Farell and Maskin (1989).[2] An equilibrium is WRP if none of its continuation equilibria Pareto-dominate each other. This captures the idea that it must be a necessary condition for renegotiation-proofness that players never want to renegotiate to an alternative continuation equilibrium of the original contract. Strong renegotiation-proofness requires that all continuation equilibria lie on the Pareto frontier of weakly renegotiation-proof payoffs. We show that, if the discount factor is not below $\frac{1}{2}$, every Pareto-optimal WRP and every SRP payoff can be achieved by a stationary contract. SRP equilibria may not exist for discount factors in an intermediate range, but we provide simple sufficient conditions to check for existence. Finally, for discount factors below $\frac{1}{2}$, our definition of stationary contracts is not general enough to implement all possible Pareto optimal WRP payoffs. Instead, renegotiation-proofness can sometimes require some destruction of surplus, either by a nonstationary equilibrium path or by money burning on the equilibrium path, as we illustrate for a Prisoners' Dilemma game.

Our analysis is most closely related to the work of Baliga and Evans (2000), who study asymptotic behavior of SRP equilibria in a setting where payments and actions are chosen simultaneously. They establish that the set of SRP payoffs converges to the Pareto frontier of individually rational stage game payoffs as players become infinitely patient. Since with simultaneous choice of actions and payments

[2] Similar concepts have been developed by Bernheim and Ray (1989), but these are not addressed in this paper.

inefficient action profiles are subject to renegotiation, their set-up is more closely related to our analysis where renegotiation before play stage is possible than to the case where only renegotiation before payment stages is considered. The difference to our framework is that we take the discount factor as given when searching for SRP outcomes. When achievable outcomes are constrained by high discounting of the future, then players will want to use payments as rewards, and condition the payment on actual play in the period. Therefore, in our framework allowing the possibility of side-payments means adopting a sequential timing of payments and actions.

The same timing of side-payments and renegotiation as in the present paper is used by Fong and Surti (2009), who study infinitely repeated prisoners' dilemma games with side payments and for all combinations of discount factors of the two players. They find that restriction to stationary equilibrium paths is without loss of generality, and conjecture that this result should extend beyond the prisoners' dilemma. They also derive sufficient conditions under which Pareto-optimal subgame perfect payoffs can be implemented as a WRP equilibrium. The results in the present paper clarify their discussion on the role of side-payments in relational contracts by showing that a restriction to stationary contracts is possible even if optimal punishments are not equilibria of the stage game. While renegotiation-proofness is a difficult and technical issue in Fong and Surti's framework with a patient and an impatient player, in our framework with a common discount factor but a more general class of games it is often very simple to find the set of strong perfect, WRP, and SRP payoffs, and the definiton of stationary contracts helps to explain the technical conditions that describe these sets. Our result that for general two player stage games optimal subgame perfect equilibria can always have a stationary structure is also in line with the intuition from previous work on relational contracts, including models of principal agent relationships (Levin, 2003), business partnerships (Baker, Gibbons and Murphy, 2002, Doornik, 2006, or Blonski and Spagnolo, 2003) and collusion (Miklos-Thal, 2008).

The paper is organized as follows. In Section 2 we describe the model and introduce stationary contracts. Section 3 first establishes that all Pareto-optimal subgame perfect payoffs can be implemented by a stationary contract. We then explain a simple heuristic to characterize the Pareto-frontier of subgame perfect payoffs for all discount factors. Section 4 starts with noting that renegotiation only before payment stage does not restrict the set of Pareto-optimal subgame perfect payoffs. We then allow for renegotiation at both stages and use stationary contracts to derive simple conditions that characterize strong perfect payoffs. In a similar fashion, Section 5 characterizes weak and strong renegotiation-proof payoffs and exemplifies the derived conditions, e.g. using a Prisoners' Dilemma game and Bertrand duopolies with symmetric or asymmetric costs. Section 6 briefly summarizes the results. All proofs are relegated to the appendix.

# 2 Model and Stationary Contracts

## 2.1 The game

We consider an infinitely repeated two-player game with perfect monitoring and common discount factor $\delta \in [0, 1)$. Players are indexed by $i, j \in \{1, 2\}$ and we use the convention that $j \neq i$ if both $i$ and $j$ appear in the same expression. Every period $t$ comprises two substages, without discounting between the substages: a *payment stage* in which both players choose a nonnegative monetary transfer to the other player, and an *action stage* (or *play stage*) in which the players play a simultaneous move game.

The stage game of the action stage is given by a continuous payoff function $g : A_1 \times A_2 \rightarrow \mathbb{R} \times \mathbb{R}$, where the set $A_i$ is the compact action space of player $i$. We denote an action profile of this stage game by $a = (a_1, a_2)$ and the set of all action profiles by $A = A_1 \times A_2$. The joint payoff from an action profile $a$ is denoted by $G(a) = g_1(a) + g_2(a)$. The best reply or *cheating* payoff of player $i$ is denoted by $c_i(a) = \max_{\{\tilde{a} \in A \mid \tilde{a}_j = a_j\}} g_i(\tilde{a})$.

In the beginning of each period, each player may decide to make a monetary transfer to the other player. The players' endowment with money is assumed to be sufficiently large such that wealth constraints do not play a role, and we also assume that there is an upper bound on payments[3]. We denote by $p_i = \tilde{p}_i - \tilde{p}_j$ player $i$'s net payment given that player $i$ and $j$ make gross transfers $\tilde{p}_i$ and $\tilde{p}_j$. We will generally describe payments in the form of net payments, assuming that only the player with a positive net payment makes a monetary transfer. Clearly, simultaneous monetary transfers by both players will never be necessary to achieve

[3] Bounded payments ensure that payoffs are well-defined. However, the bound does not play a huge rule, because in a subgame perfect equilibrium payments must be bounded, as for example no payment above $\frac{3}{1-\delta} \max_a(|g_1(a)| + |g_2(a)|)$ could occur in an equilibrium.

a certain equilibrium payoff. Players are risk-neutral and utility is quasi-linear in money. Player $i$'s payoff in a period with net payments $p = (p_1, p_2)$ and play stage action profile $a$ is given by $g_i(a) - p_i$.

A history that ends before stage $k \in \{pay, play\}$ in period $t$ is a list of all transfers and actions that have occurred before this point in time. Let $H^k$ be the set of all histories that end before stage $k$. A strategy $\sigma_i$ of player $i$ in the repeated game maps every history $h \in H^{play}$ into an action $a_i \in A_i$, and every history $h \in H^{pay}$ into a payment. Every history $h \in H$ defines a subgame of the repeated game. We write $\sigma|h$ for the profile of continuation strategies following history $h$. We denote by $u_i(\sigma|h)$ player $i$'s average discounted continuation payoff, i.e. the discounted continuation payoff multiplied by $(1-\delta)$.[4] We use $u(\sigma) = (u_1(\sigma), u_2(\sigma))$ to denote the vector of continuation payoffs and denote the joint continuation payoff by $U(\sigma) = u_1(\sigma) + u_2(\sigma)$.

We denote by $\Sigma^k_{SGP}$ the set of subgame perfect (continuation) equilibria that start in stage $k$. If $\sigma$ is a subgame perfect equilibrium, we call $u(\sigma)$ a subgame perfect payoff. We say a set of subgame perfect equilibria $\Sigma$ *implements* a set of payoffs $\mathcal{U}$ if $\mathcal{U} = \{u(\sigma) | \sigma \in \Sigma\}$. All continuation payoffs of a given strategy profile $\sigma$ at stage $k$ are denoted by $\mathcal{U}^k(\sigma) = \{u(\sigma|h)|\ h \in H^k\}$. The set of subgame perfect payoffs at stage $k$ is denoted by $\mathcal{U}^k_{SGP} = \{u(\sigma)|\ \sigma \in \Sigma^k_{SGP}\}$.

Note that we have restricted the game to pure strategies. Similar to Farell and Maskin (1989) and Baliga and Evans (2000), one can allow for mixing in the stage game by letting the action space $A$ contain all mixed strategies of the original stage game and the payoff function $g(a)$ describe the expected payoffs. It is then assumed that a player can ex-post observe the other player's mixing probabilities and not only the realized outcome.

It will be convenient to assume that the stage game has a Nash equilibrium in $A$. Our main results would also hold without this assumption given that the discount factor $\delta$ is sufficiently large, such that a subgame perfect equilibrium of the repeated game exists.

[4] We assume that payoffs are directly realized after each stage within a period. This means that if a history $h$ ends before play stage in period $t$ then the transfers made in the pay stage of period $t$ do not appear in the continuation payoff $u_i(\sigma|h)$.

## 2.2 Stationary contracts

In the following, we define a class of simple stationary strategy profiles which are helpful to characterize the Pareto-frontier of subgame perfect payoffs and to study the effects of different renegotiation-proofness requirements.

**Definition 1** *A stationary strategy profile is characterized by a triple of action profiles* $(a^e, a^1, a^2)$*, called an action structure, and a payment scheme in the following way:*

*In the payment stage of period 0, there are up-front payments* $p^0$*.*

*Whenever a player makes the prescribed payment in the payment stage, the equilibrium actions* $a^e$ *are played in the next play stage.*

*Whenever there is no (or a bilateral) deviation from* $a^e$*, equilibrium payments* $p^e$ *are conducted in the next payment stage.*

*If player* $i$ *unilaterally deviates from a prescribed action, he pays a fine* $F^i$ *to the other player in the subsequent payment stage.*

*If player* $i$ *deviates from a required payment, the punishment profile* $a^i$ *is played in the next play stage and adjustment payments* $p^i$ *are made in the subsequent payment stage.*

The structure of a stationary strategy profile is illustrated in Figure 1. Note that the payments in a stationary strategy profile are described by the vectors of the net payments. For example, if player 1 pays a fine $F^1$ to player 2, then at this pay stage player $j$ makes no transfer to player $i$, but receives the fine. Net payments are then given by $(F^1, -F^1)$. Similarly, up-front payments $p^0 = (p_1^0, p_2^0)$ are net payments with the property that $p_1^0 = -p_2^0$.

One can express stationary strategy profiles also in terms of simple strategy profiles as defined by Abreu (1988). A simple strategy profile for two players prescribes play of the initial path $Q^0$, while any unilateral deviation from the prescribed paths by player $i$ is followed by play along the punishment path $Q^i$. In our setting, a stationary strategy profile consists of the initial path

$$Q^0 = (p^0, a^e, p^e, a^e, p^e, ...)$$

and two punishment paths for player $i$, depending on whether the deviation occurred in the payment stage or in the play stage:

$$Q^i_{pay} = (F^i, a^e, p^e, a^e, p^e, ...)$$
$$Q^i_{play} = (a^i, p^i, a^e, p^e, a^e, p^e, ...).$$

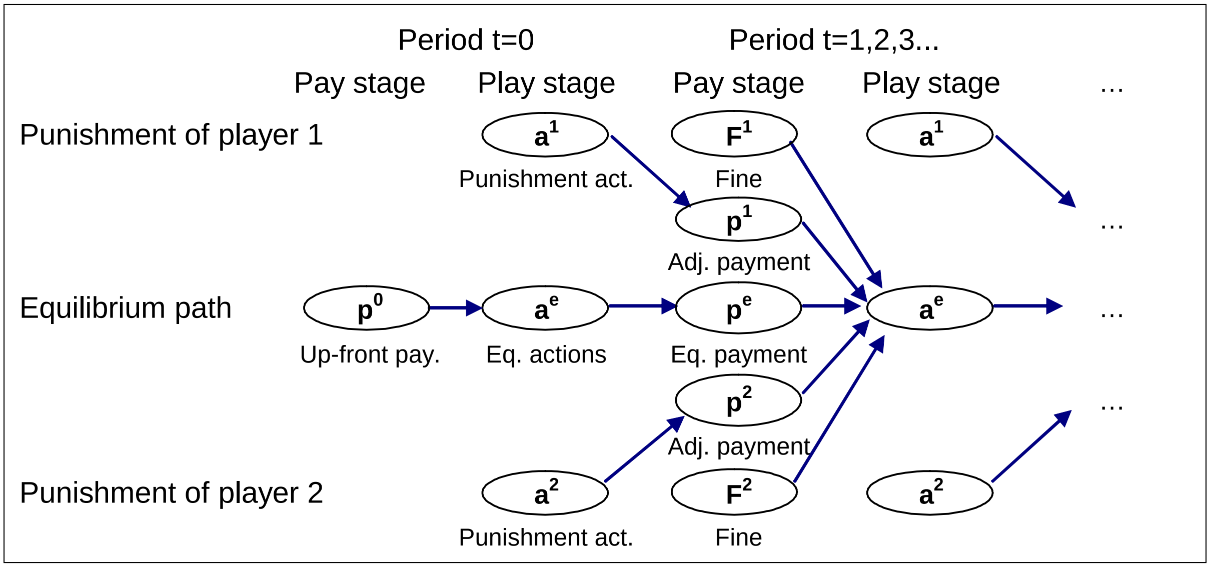


Figure 1: Structure of stationary strategy profiles. Arrows indicate continuation play if no player deviates (or a bilateral deviation took place). If player 1 (2) unilateraly deviates then the top (bottom) row will be played in the next stage.

Abreu (1988) is build around the now familiar idea that for subgame perfection the punishment does not need to fit the crime. Any unilateral deviation from a prescribed path can be punished by the same continuation equilibrium, namely the worst possible subgame perfect equilibrium for that player. The optimal penal codes, as such worst play paths are called in Abreu's work, then often have a "stick and carrot" structure: they begin with the worst possible action for the punished player, and may reward him for complying with the punishment further along the path. In our framework, the punishment paths have a similar structure: chosen optimally, the action $a^i$ must have a low enough cheating payoff $c_i(a^i)$ to deter a deviation by player $i$. The adjustment payment $p^i$ is used to fine-tune the punishment and to guarantee that both the punished player $i$ and the punishing player $j$ have no incentives to deviate from the punishment profile $a^i$.

Recall that there are two different punishment paths because a punishment can start in the payment or play stage. We fix for all stationary strategy profiles the adjustment payment $p^i$ such that both punishment paths yield the same payoff for the punished player:

$$p_i^i = \frac{F^i + g_i(a^i) - g_i(a^e) + \delta p_i^e}{\delta}. \tag{1}$$

It turns out that fixing adjustment payments in this way does not restrict the ability of stationary strategy profiles to characterize optimal subgame perfect and renegotiation-proof payoffs.

**Definition 2** *A stationary strategy profile that constitutes a subgame perfect equilibrium is called a* stationary contract.

In the following, we find conditions that imply subgame perfection of a stationary strategy profile. It is often more convenient to think about a stationary contract in terms of the continuation payoffs it defines, and not in terms of the actual payments that have to be made. We denote player $i$'s continuation payoff before a play stage on the equilibrium path by

$$u_i^e = g_i(a^e) - \delta p_i^e. \tag{2}$$

Player $i$'s continuation payoff when punished is called *punishment payoff* and is given by

$$u_i^i = -(1-\delta)F^i + u_i^e. \tag{3}$$

To verify that a given stationary strategy profile is a subgame perfect equilibrium, it is sufficient to check that there are no profitable one-shot deviations.[5] We first consider the punishment of player $i \in \{1,2\}$. Irrespective of the stage at which the punishment starts, player $i$'s payoff is $u_i^i$ if he complies with the punishment. If he deviates once and complies afterwards, player $i$'s payoffs are $u_i^i$ or $c_i(a^i)(1-\delta) + \delta u_i^i$ depending on whether the punishment started in the payment stage or play stage. Therefore, player $i$ will not deviate from his punishment whenever

$$u_i^i \geq c_i(a^i). \tag{4}$$

We now turn to the role of player $j$ in player $i$'s punishment. We do not only have to ensure that player $j$ does not deviate from the punishment profile $a^i$ but also that he pays the adjusted payment (in case $p_j^i > 0$). Both conditions are fulfilled if and only if

$$(1-\delta)G(a^i) + \delta G(a^e) - u_i^i \geq (1-\delta)c_j(a^i) + \delta u_j^j. \tag{5}$$

Note that the left hand side of this condition is player $j$'s continuation payoff when punishing player $i$ at play stage. Since punishment payoffs are the same for

[5] The one-shot deviation principle holds in our setting if payments are bounded. Subgame perfection should be equivalent to no profitable one-shot deviations and bounded payments.

the payment and the play stage, play along the path starting with the action is more difficult to induce than play along the same path starting with the following payment.

On the equilibrium path, compliance with both the actions $a^e$ and the payments $p^e$ is achieved if and only if for each player $i = 1, 2$

$$u_i^e \geq (1-\delta)c_i(a^e) + \delta u_i^i. \tag{6}$$

Finally, an up-front payment $p^0$ is subgame perfect whenever for both players $i = 1, 2$

$$p_i^0 \leq F^i. \tag{7}$$

To summarize, a stationary strategy profile with action structure $(a^e, a^1, a^2)$, payments $p^0, p^e$, and fines $F^1, F^2$ constitutes a stationary contract if conditions (4), (5), (6), and (7) are satisfied for both players.

**Remark 1** *Let $\sigma$ be a stationary contract with equilibrium actions $a^e$ and punishment payoffs $u_1^1$ and $u_2^2$. The set of of those stationary contracts that differ from $\sigma$ at most in the upfront-payments $p^0$ implements the payoffs on the line from $(u_1^1, G(a^e) - u_1^1)$ to $(G(a^e) - u_2^2, u_2^2)$.*

In particular, this result means that for given equilibrium actions and punishment payoffs, the set of feasible distributions of the joint payoff $G(a^e)$ is independent of the equilibrium payments $p^e$. The intuition is simple: if a player makes lower equilibrium payments, he is willing to make higher up-front payments that can counterbalance the distributive effects of the equilibrium payments.

Hence, equilibrium payments $p^e$ can be chosen for the sole purpose of smoothing the incentives not to deviate from the equilibrium path. In fact, whenever the sum of the inequalities in (6) holds, then $p^e$ can be chosen such that the individual conditions hold for both players. Furthermore, if we are merely interested in subgame perfection, we can set fines to the maximal level such that punishment payoffs are given by $u_i^i = c_i(a^i)$.[6] By selecting equilibrium payments and fines appropriately, we arrive at simple conditions for checking whether a stationary contract with some specific action structure exists:

[6]These maximal fines are given by $F^i = \frac{1}{(1-\delta)}(u_i^e - c_i(a^i))$. The maximal fines become very large as the game's surplus rises. Such extreme values are not necessary, but convenient in our search for all sustainable equilibrium payoffs.

**Lemma 1** *There exists a stationary contract with action structure* $(a^e, a^1, a^2)$ *if and only if*

$$G(a^e) \geq (1-\delta)(c_1(a^e) + c_2(a^e)) + \delta(c_1(a^1) + c_2(a^2)) \qquad \text{(SGP-a}^{\text{e}}\text{)}$$

*and for both players* $i = 1, 2$

$$(1-\delta)G(a^i) + \delta G(a^e) \geq (1-\delta)(c_1(a^i) + c_2(a^i)) + \delta(c_1(a^1) + c_2(a^2)). \qquad \text{(SGP-a}^{\text{i}}\text{)}$$

# 3 Optimal Subgame Perfect Contracts

## 3.1 Main Result

This section shows that in our setting every Pareto-optimal subgame perfect payoff can be achieved by stationary contracts and we illustrate how stationary contracts allow a simple characterization of these payoffs. We denote the weak Pareto frontier of the set of subgame perfect payoffs by $\mathcal{P}(\mathcal{U}_{SGP}^{pay})$[7]. Furthermore, let

$$\bar{U}_{SGP} := \sup_{u \in \mathcal{U}_{SGP}^{pay}} u_1 + u_2$$

be the supremum of joint payoffs in subgame perfect equilibria, and

$$\bar{u}_{SGP}^i := \inf_{u \in \mathcal{U}_{SGP}^{pay}} u_i$$

be the infimum of player $i$'s payoffs in subgame perfect equilibria. Note that these values would not change if the range of payoffs $\mathcal{U}_{SGP}^{pay}$ was replaced by $\mathcal{U}_{SGP}^{play}$, the set of subgame perfect continuation payoffs at the play stage. We say an action profile $\widetilde{a}$ is *admissible* for a given discount factor $\delta$ if there exists some subgame-perfect equilibrium in which $\widetilde{a}$ is played on the equilibrium path. For a given discount factor, we call an action profile $\overline{a}^e$ *optimal* if it is admissible and $G(\overline{a}^e) = \bar{U}_{SGP}$. Similarly, we call an action profile $\overline{a}^i$ a *strongest punishment* for player $i$ if it is admissible and $c_i(\overline{a}^i) = \bar{u}_{SGP}^i$. For the remainder of this paper, the labels $\overline{a}^e$ and $\overline{a}^i$ will always refer to an optimal action-profile and strongest punishment, respectively.

[7] The weak Pareto frontier is defined as $\mathcal{P}(\mathcal{U}_{SGP}^{pay}) = \{(v_1, v_2) : v_1 \geq u_1, v_2 \geq u_2$ for all $u \in \mathcal{U}_{SGP}^{pay}\}$.

**Proposition 1** *An optimal action profile $\overline{a}^e$ and strongest punishments $\bar{a}^1, \bar{a}^2$ always exist. The Pareto-frontier of subgame perfect payoffs is linear and can be implemented by stationary contracts with action structure $(\overline{a}^e, \overline{a}^1, \overline{a}^2)$ and maximal fines.*

Note that a stationary contract with strongest punishments and maximal fines uses optimal penal codes in the sense of Abreu (1988).

**Lemma 2** *An action profile $\widetilde{a}$ is admissible if and only if*

$$G(\widetilde{a}) - (c_1(\widetilde{a}) + c_2(\widetilde{a})) \geq \frac{\delta}{1-\delta}(c_1(\overline{a}^1) + c_2(\overline{a}^2) - G(\overline{a}^e)). \tag{8}$$

## 3.2 Finding optimal action profiles and strongest punishments

Given Proposition 1, characterizing the Pareto-frontier of subgame perfect payoffs boils down to finding an optimal action profile $\overline{a}^e$ and strongest punishments $\overline{a}^1, \overline{a}^2$ for a given discount factor. The Pareto frontier of subgame perfect payoffs is then given by the line from $(c_1(a^1), G(a^e) - c_1(a^1))$ to $(G(a^e) - c_2(a^2), c_2(a^2))$. Recall from Lemma 1 that a stationary contract with action structure $(a^e, a^1, a^2)$ exists if and only if

$$c_1(a^e) + c_2(a^e) - G(a^e) \leq \frac{\delta}{1-\delta}(G(a^e) - (c_1(a^1) + c_2(a^2))) \tag{SGP-a$^\text{e}$}$$

and for both players $i = 1, 2$

$$c_1(a^i) + c_2(a^i) - G(a^i) \leq \frac{\delta}{1-\delta}(G(a^e) - (c_1(a^1) + c_2(a^2))) \tag{SGP-a$^\text{i}$}$$

These conditions have a convenient structure: More efficient equilibrium play, i.e. higher levels of $G(a^e)$, always relaxes conditions (SGP-a$^\text{i}$) and thereby facilitates stronger punishments. Similarly, a stronger punishment of player $i$, i.e. lower levels of $c_i(a^i)$, always facilitates a stronger punishment of player $j$ and more efficient equilibrium play.

Consequently, there is a simple iterative procedure for games with a finite action space that yields a list of optimal action profiles and optimal punishments for all discount factors. For each round $n = 0, 1, 2, ...$ we define an action structure $(a^e(n), a^1(n), a^2(n))$, starting with $a^e(0) \in \arg\max_{a \in A} G(a)$ and $a^i(0) \in \arg\min_{a \in A} c_i(a)$.

This means we start with the most efficient action profile and the harshest punishments of the stage game.

For $k \in \{e, 1, 2\}$, let $\delta^k(n)$ denote the minimal discount factor for which the action structure of round $n$ fulfills condition (SGP-a$^{\mathrm{k}}$) and let $\delta^*(n) \equiv \max_k \delta^k(n)$.[8] It will be true that $\delta^*(0) < 1$ (the folk theorem holds in our setting), hence $a^e(0)$ is an optimal action profile and $a^1(0)$, $a^2(0)$ are strongest punishments for every discount factor $\delta \in [\delta^*(0), 1]$.

If $\delta^e(n) = \delta^*(n)$, i.e, the equilibrium path constraint was binding in round $n$, then we choose in round $n+1$ an action-profile $a^e(n+1)$, which maximizes $G(a)$ among all action profiles $a$ that satisfy

$$G(a) - (c_1(a) + c_2(a)) > G(a^e(n)) - (c_1(a^e(n)) + c_2(a^e(n))).$$

If there is more than one action with this property, then we take the one with the lowest value of $c_1(a)+c_2(a)$. Similarly, if $\delta^i(n) = \delta^*(n)$, we choose an action-profile

$$a^i(n+1) \in \arg\min_a \{c_i(a) : G(a) - (c_1(a)+c_2(a)) > G(a^i(n)) - (c_1(a^i(n)) + c_2(a^i(n)))\}.$$

Whenever there are multiple candidates for a punishment profile $a^i$ that have the same cheating payoff $c_i(a^i)$, one should choose an action-profile that maximizes $G(a^i) - c_j(a^i)$ in order to relax condition (SGP-a$^{\mathrm{i}}$) as much as possible. If instead $\delta^k(n) < \delta^*(n)$, then we don't need to relax condition (SGP-a$^{\mathrm{k}}$) and keep the old action-profile, i.e. $a^k(n+1) = a^k(n)$.

The next step is to see whether the new action profiles are indeed admissible. If in round $n$ we find $\delta^*(n) < \min_{m<n}\{\delta^*(m)\}$, then $a^e(n)$ is an optimal action profile and $a^i(n)$ are optimal punishments for all discount factors $\delta \in [\delta^*(n), \min_{m<n}\{\delta^*(m)\})$. The procedure iterates until in some round $n^*$ all selected action-profiles are a Nash equilibrium of the stage game, which implies $\delta^*(n^*) = 0$. Clearly, the procedure will always terminate and yields a list of optimal action profiles and strongest punishments for every discount factor.

## 3.3 Example: Simplified Cournot Game due to Abreu

We now illustrate the procedure above, for a simplified Cournot game due to Abreu (1988). Two firms simultaneously choose either low (L), medium (M), or high (H)

[8] We normalize $\delta^k(n) = 1$ if no discount factor $\delta < 1$ fulfills condition (SGP-a$^{\mathrm{k}}$).

output and stage game payoffs are given by the following matrix:

| | | Firm 2 L | M | H |
|---|---|---|---|---|
| Firm 1 | L | $10, 10$ | $3, 15$ | $0, 7$ |
| | M | $15, 3$ | $7, 7$ | $-4, 5$ |
| | H | $7, 0$ | $5, -4$ | $-15, -15$ |

Joint payoffs are maximized if firms choose (L,L), the unique Nash equilibrium of the stage game is (M,M), and high output minimizes the cheating payoff of the other firm. Abreu considered the case without side-payments and constructed optimal penal codes that support collusive play of (L,L) for any discount factor $\delta \geq \frac{4}{7}$, while the threat of an infinite repetition of the stage game equilibrium can sustain collusion only if $\delta \geq \frac{5}{8}$.

For the case with side payments, the first candidate for an optimal action profile is clearly the collusive outcome, i.e. $a^e(0) = (L, L)$. A harshest punishment of the stage game requires that the punisher chooses high output. Maximization of $G(a^i) - c_j(a^i)$ requires that the punished player chooses medium output, i.e. we have $a^1(0) = (M, H)$ and $a^2(0) = (H, M)$. While it would be more efficient for both players if the punished player chooses low output, the choice of medium output substantially reduces the punishers' incentives to deviate from the punishment and therefore makes the punishment easier to implement.

With this action structure, condition (SGP-a$^e$) holds for all $\delta \geq \frac{1}{3}$ while conditions (SGP-a$^i$) hold for all $\delta \geq \frac{3}{13}$. Hence, the collusive outcome can be sustained for all discount factors $\delta \geq \frac{1}{3}$. To characterize Pareto-optimal payoffs for lower discount factors, we relax condition (SGP-a$^e$) by choosing either $a^e(1) = (L, M)$ or $a^e(1) = (M, L)$ and keep the previous punishment profiles. Condition (SGP-a$^e$) then holds for all $\delta \geq \frac{2}{11}$ while conditions (SGP-a$^i$) hold for all $\delta \geq \frac{1}{4}$. Thus, for all $\delta \in [\frac{1}{4}, \frac{1}{3})$, a partial collusive equilibrium play of $(L, M)$ or $(M, L)$ can be sustained. Note that the corresponding stationary contracts require positive equilibrium payments from the firm that chooses medium output to the firm that chooses low output.[9] Continuing the procedure, we find that for lower discount factors only an infinite repetition of the stage game equilibrium can be sustained.

[9] Using condition (6) (see Section 2.2) one finds that these payments have to lie in the interval $[\frac{4-7\delta}{\delta}, 15]$.

### 3.4 Example: Prisoners' Dilemma

For another example, consider a Prisoners' Dilemma game with payoff matrix:

| | | Player 2 C | Player 2 D |
|---|---|---|---|
| Player 1 | C | $(1,1)$ | $(S-d,d)$ |
| | D | $(d,S-d)$ | $(0,0)$ |

where $d > 1 > \frac{S}{2}$. The first candidates for strongest punishments are $\overline{a}^1 = \overline{a}^2 = (D, D)$. Since this strongest punishment is a Nash equilibrium of the stage game, conditions (SGP-$a^i$) hold for all discount factors. The first candidate for optimal equilibrium actions is mutual cooperation $(C, C)$ and condition (SGP-$a^e$) shows that it can be sustained for all $\delta \geq \frac{d-1}{d}$. If $S > 1$ we find that the asymmetric equilibrium action-profiles $(C, D)$ and $(D, C)$ are optimal for all $\delta \in [\frac{d-S}{d}, \frac{d-1}{d})$. For lower discount factors, only the Nash equilibrium of the stage game $(D, D)$ can be sustained.

# 4 Renegotiation-Proofness: Strong Optimality and Strong Perfection

## 4.1 Definitions and Main Results

In many relationships it seems reasonable that players have the possibility to meet and renegotiate their existing relational contract. If players anticipate such a renegotiation, a subgame-perfect equilibrium may cease to be stable, however. There are different concepts of renegotiation-proofness that intend to refine the set of subgame perfect equilibria to those equilibria that are robust against this criticism.

We first consider the concept of *strong optimality* that Levin (2003) applies in his study of repeated principal-agent games. Levin implicitly assumes that renegotiation can only take place at the beginning of a period, i.e. before the payment stage but not before the action stage. A minimal requirement for a successful renegotiation at this stage is that there exists a new contract that is subgame perfect and creates some surplus compared to the existing contract, in the sense of achieving a higher joint payoff. Consequently, there is never scope for renegotiation if all continuation equilibria already achieve the highest joint payoff $\overline{U}_{SGP}$ that is possible in a subgame perfect equilibrium.

**Definition 3** *A subgame perfect equilibrium $\sigma$ is* strongly optimal *(w. r. t. renegotiations at payment stages) if $U(\sigma|h) = \overline{U}_{SGP}$ for all $h \in H^{pay}$.*

Levin introduces this concept because strong optimality is easily fulfilled in his setting. Similarly, in our setting an analogous result directly follows from Proposition 1:

**Corollary 1** *Every stationary contract with optimal equilibrium action profile $\overline{a}^e$ is strongly optimal. The Pareto-frontier of subgame perfect and strongly optimal payoffs coincides.*

The reason that every stationary contract with optimal equilibrium actions is strongly optimal is that in every continuation equilibrium starting at a payment stage, the required payments will always be conducted and afterwards the optimal actions $\overline{a}^e$ are played in all subsequent periods. Since by assumption there is no renegotiation directly before a play stage, continuation equilibria that require the play of a punishment profile $a^i$ are never subject to renegotiation.

Assuming that no renegotiation is allowed before a play stage has certain appeal in situations where payments can be organized quickly compared to the time it requires to renegotiate a contract. The reason is that then there is a point in time at which it is still possible to pay the fine but there is no longer sufficient time to renegotiate future actions. In renegotiations in which the focal disagreement point is always the original equilibrium, and if "not paying" is a reversible action, the nondeviating party will expect to receive the fine as soon as the negotiations stop.

A more stringent test of renegotiation results if one does consider the possibility of renegotiation at all stages within a period. This is assumed, for example, by Fong and Surti (2009) who study repeated Prisoners' Dilemma games with side payments. The strictest concept of renegotiation-proofness would be a modification of strong optimality that requires for every continuation equilibrium — including those starting at a payment stage— that the sum of continuation payoffs is equal to the highest possible value $\overline{U}_{SGP}$. Since punishment actions typically require some efficiency loss, this condition is too strong to allow for much insightful analysis, however. A slightly weaker requirement follows from adapting *strong perfection* (see Rubinstein, 1980) to our set-up.

**Definition 4** *A subgame perfect equilibrium $\sigma$ is* strong perfect *at both stages if $\mathcal{U}^k(\sigma) \subset \mathcal{P}(\mathcal{U}^k_{SGP})$ for all $k \in \{pay, play\}$.*

Strong perfection requires for both stages that no continuation payoff is strictly Pareto dominated by another subgame perfect continuation payoff of the same stage. Strong perfect equilibria may fail to exist, but the concept provides a useful sufficient condition for renegotiation-proofness. If there is no subgame perfect continuation equilibrium that makes both players better off, then one may feel confident that renegotiation is deterred. Let $\overline{u}^i_{SP}$ denote the infimum of player $i$'s payoffs in strong perfect equilibria, in case such equilibria exist.

**Proposition 2** *Every strong perfect payoff can be implemented by a stationary contract with optimal equilibrium action profile $\bar{a}^e$. The set of strong perfect payoffs is either empty or given by the line from $(\overline{u}^1_{SP}, G(\bar{a}^e) - \overline{u}^1_{SP})$ to $(G(\bar{a}^e) - \overline{u}^2_{SP}, \overline{u}^2_{SP})$.*

Clearly, a strong perfect stationary contract requires optimal equilibrium actions $\bar{a}^e$, but it is not generally the case that optimal penal codes can be used, since the corresponding continuation payoffs may be Pareto-dominated. We now derive results that help to find strong perfect equilibria for a given game and discount factor or to verify their non-existence.

**Proposition 3** *A stationary contract with action structure $(\bar{a}^e, a^1, a^2)$ and punishment payoffs $u^1_1$ and $u^2_2$ is strong perfect if and only if for both players $i = 1, 2$ and for all admissible $\tilde{a}$ with $G(\tilde{a}) > G(a^i)$ it holds that either*

$$(1-\delta)G(a^i) - u^i_i \geq (1-\delta)\left(G(\tilde{a}) - c_i(\tilde{a})\right) - \delta c_i(\bar{a}^i) \text{ or} \tag{SP1}$$

$$u^i_i \geq (1-\delta)(G(\tilde{a}) - c_j(\tilde{a})) + \delta G(\bar{a}^e) - \delta c_j(\bar{a}^j) \tag{SP2}$$

Intuitively, conditions (SP1) and (SP2) concern the punishment for player $i$ at play stage. Condition (SP1) ensures that there exist no subgame perfect continuation equilibria that give a higher payoff to the punishing player $j$, i.e., that the punisher has no incentive to renegotiate the punishment. Should such continuation equilibria exist, condition (SP2) ensures that they would make the punished player $i$ worse-off. That one cannot always restrict attention to stationary contracts with maximal fines is due to the fact that we do not allow for correlated strategies. In a stationary contract that ceases to be strong perfect if the maximal fines are used, the punishment payoff is always dominated by a convex combination of payoffs in $\mathcal{U}^{play}_{SGP}$.

We now derive two Corollaries of Proposition 3 that facilitate the analysis in many examples. We say a strongest punishment $\bar{a}^i$ of player $i$ is an *optimal strongest punishment* if there exists no other strongest punishment of player $i$ with a higher joint payoff than $G(\bar{a}^i)$.

**Corollary 2** *A stationary contract with optimal strongest punishments $\bar{a}^1, \bar{a}^2$, maximal fines, and optimal equilibrium action profile is strong perfect if and only if for all admissible action profiles $\tilde{a}$ and both players $i = 1, 2$*

$$G(\bar{a}^i) - c_i(\bar{a}^i) \geq G(\tilde{a}) - c_i(\tilde{a}). \tag{9}$$

**Corollary 3** *There exists no strong perfect stationary contract with action structure $(\bar{a}^e, a^1, a^2)$ if for both players $i = 1, 2$ we have $G(\bar{a}^e) > G(a^i)$ and*

$$(1-\delta)\left(G(\bar{a}^e) - c_i(\bar{a}^e)\right) - \delta c_i(\bar{a}^i) > (1-\delta)G(a^i) - c_i(a^i). \tag{10}$$

These two corollaries can be used to show that in Abreu's simple Cournot example from the last section there is no strong perfect equilibrium, except for the Nash equilibrium of the stage game in case $\delta < \frac{1}{4}$. Instead of exercising this non-existence in detail, we now present two examples in which strong perfect equilibria (at least sometimes) exist.

## Example: Principal-Agent Game

Assume that only player 1 (the agent) chooses an action $a \in \mathbb{R}_0^+$. The action creates a non-positive payoff $g_1(a)$ for player 1 and a nonnegative benefit $g_2(a)$ for player 2 (the principal). One interpretation is that player 1 is a supplier who delivers a product of a certain quality, where higher quality is more expensive. Another interpretation is that player 1 is a worker who can exert work effort $a$, which can be observed by the employer. The agent can choose a 'do-nothing' action $a = 0$ that yields zero payoff for both players.

Clearly, $\overline{a}^2 = 0$ is a strongest punishment for the principal. Since the agent's cheating payoff in play stage is always 0, every action $\overline{a}^1 \in \mathbb{R}_0^+$ is by definition a strongest punishment for the agent. In particular, also the optimal equilibrium actions $\overline{a}^e$ constitute a strongest punishment for the agent. Using these strongest punishments, we find from conditions (SGP-a$^{\text{e}}$) and (SGP-a$^{\text{i}}$) that the optimal equilibrium actions $\overline{a}^e$ solve $\max_{a^e \in A} G(a^e)$ subject to $\delta g_2(a^e) \geq -g_1(a^e)$.

Using Corollary 2, we find that the stationary contracts with action structure $(\overline{a}^e, \overline{a}^e, 0)$ are strong perfect, if and only if for every admissible $\tilde{a}$ the condition $G(\tilde{a}) - g_2(\tilde{a}) \leq 0$ holds. Since $g_1(\tilde{a}) \leq 0$, this condition is always fulfilled. Hence, in this simple complete information game, we confirm the intuition of Levin (2003) that when the incentive problem is one-sided, optimal SGP payoffs can be implemented in a renegotiation-proof way.

## Example: Strong Perfection in the Prisoners' Dilemma

Recall the Prisoners' Dilemma game from Section 3.4. It is instructive to first consider the case $S < 1$ and $\frac{d-1}{d} \leq \delta < \frac{d-S}{d-S+2}$. Then optimal equilibrium actions are $\overline{a}^e = (C, C)$, strongest punishments are $\bar{a}^1 = \bar{a}^2 = (D, D)$, and Lemma 2 implies that $(C, D)$ and $(C, D)$ are not admissible. Corollary 2 then says that there exists a strong perfect stationary contract if and only if $d \geq 2$.

These strong perfect stationary contracts rely on punishments at play stage in which the inefficient action-profile $(D, D)$ is played for one period. It may seem surprising that such continuation equilibria can lie on the Pareto-frontier of subgame perfect continuation equilibria, and not be Pareto dominated by continuation equilibria in which $(C, C)$ is played in every period. The intuitive reason is that play of $(D, D)$ allows more asymmetric payoffs, i.e. larger payments from the punished player to the punisher than after play of $(C, C)$. If $d \geq 2$, there is a large incentive to deviate from $(C, C)$; the maximum possible payments after play of $(C, C)$ are then so low that it is better for the punisher to play $(D, D)$ for one period and to get a relatively large payment afterwards. The punisher then has no incentive to renegotiate the punishment.

For a complete characterization of strong perfect equilibria, one can use Lemma 2 to get the parameter ranges for which $(C, C)$, $(D, C)$, and $(C, D)$, respectively, are admissible. It is clear that punishment profiles in any strong perfect equilibrium must be $(D, D)$ if $S < 0$ and $(C, D)$ respectively $(D, C)$ if $S > 0$. If $(C, C)$ is admissible, Corollary 2 implies that strong perfect equilibria exist if and only if $d \geq 2 - \max(S, 0)$. For the case that $(C, D)$ and $(D, C)$ are admissible but $(C, C)$ is not, one finds that there always exist strong perfect contracts that play $(C, D)$ or $(D, C)$ on the equilibrium path. If only $(D, D)$ is admissible, the infinite repetition of the stage game equilibrium is trivially strong perfect.

# 5 Weak and Strong Renegotiation-Proofness

## 5.1 Definitions and Main Results

Strong perfection is a very strict criterion; in a strong perfect equilibrium every continuation payoff must survive comparison to all subgame perfect equilibria, including those that are not renegotiation-proof themselves. In this section, we analyze two concepts that only consider renegotiation to continuation equilibria that are renegotiation-proof themselves, namely weak and strong renegotiation-proofness defined by Farell and Maskin (1989). An equilibrium is weakly renegotiation-proof if none of its own continuation equilibria is strictly Pareto dominated by another continuation equilibrium. Strong renegotiation-proofness requires stability against renegotiation to any weakly renegotiation-proof continuation equilibrium. The formal definitions, allowing for renegotiation within a period, are as follows:

**Definition 5** *A SGP equilibrium $\sigma$ is weakly renegotiation-proof (WRP) if for no stage $k$ there are two continuation payoffs $u, u' \in \mathcal{U}^k(\sigma)$ such that $u$ is strictly Pareto-dominated by $u'$.*

WRP equilibria always exist but the concept often does not have much restricting power. For example, it is always a WRP equilibrium to play in every period the same Nash equilibrium of the stage game and to never conduct any payments.

Let $\Sigma_{WRP}$ denote the set of WRP equilibria and $\mathcal{U}^k_{WRP} = \bigcup_{\sigma \in \Sigma_{WRP}} \mathcal{U}^k(\sigma)$ the set of all WRP payoffs of stage $k$.

**Definition 6** *A WRP equilibrium $\sigma$ is strongly renegotiation-proof (SRP) if for no stage $k$ and $u \in \mathcal{U}^k(\sigma)$ there exists another WRP payoff $u' \in \mathcal{U}^k_{WRP}$ such that $u$ is strictly Pareto-dominated by $u'$.*

It follows directly from this definition that the set of SRP payoffs is a subset of the Pareto-frontier of all WRP payoffs, but in general the two sets do not coincide. In fact, for intermediate discount factors SRP equilibria often do not even exist. In the following, we show that stationary contracts can be used to characterize the Pareto-frontier of WRP payoffs and the set of SRP payoffs. The results are derived for the case $\delta \geq \frac{1}{2}$, i.e. for the case that future payoffs have a larger weight than present payoffs. We discuss the case $\delta < \frac{1}{2}$ afterwards.

For ease of exposition we further restrict the class of stationary contracts:

**Definition 7** *A stationary contract* $\sigma$ *with action structure* $(a^e, a^1, a^2)$ *is called* regular *if it has maximal fines, and for both players* $i = 1, 2$, $G(a^e) \geq G(a^i)$, *and either* $c_i(a^i) < c_i(a^e)$ *or* $a^i = a^e$.

Clearly, for finding Pareto optimal subgame perfect payoffs one may restrict attention to regular stationary contracts, and the same is true for the following analysis of the Pareto frontier of WRP payoffs.

**Lemma 3** *A regular stationary contract with action structure* $(a^e, a^1, a^2)$ *and equilibrium payoffs* $u^e$ *is WRP if and only if for both players* $i = 1, 2$

$$(1-\delta)G(a^i) + \delta G(a^e) - c_i(a^i) \geq u_j^e \tag{11}$$

Hence, for regular stationary contracts weak renegotiation-proofness means that player $j$ would not agree to a switch from the play stage in player $i$'s punishment to the play stage on the equilibrium path.

In many examples, the stage game is symmetric and one wants to check weak renegotiation-proofness for regular stationary contracts that have a symmetric action structure $(a^e, a^1, a^2)$, i.e. $a_1^e = a_2^e$, $a_1^1 = a_2^2$ and $a_2^1 = a_1^2$. In this case, positive equilibrium payments $p_i^e$ are not necessary for subgame perfection or weak renegotiation-proofness. Lemma 3 therefore implies that there exist WRP regular stationary contracts with that action structure if and only if

$$(1-\delta)G(a^1) - c_1(a^1) \geq (1-2\delta)\, g_1(a^e). \tag{12}$$

Lemma 4 deals with the general case.

**Lemma 4** *If the set of regular stationary contracts with action structure* $(a^e, a^1, a^2)$ *is non-empty, it contains WRP contracts if and only if for both players* $i = 1, 2$

$$(1-\delta)G(a^i) + \delta G(a^e) - c_i(a^i) \geq (1-\delta)c_j(a^e) + \delta c_j(a^j) \tag{WRP-i}$$

$$(1-\delta)(G(a^1) + G(a^2)) + (2\delta - 1)G(a^e) \geq c_1(a^1) + c_2(a^2). \tag{WRP-Joint}$$

If $\delta \geq \frac{1}{2}$, regular stationary contracts allow us to characterize the Pareto-frontier of WRP payoffs:

**Proposition 4** *Let* $\delta \geq \frac{1}{2}$. *For every WRP equilibrium* $\sigma$ *there exists a regular WRP stationary contract with an action structure* $(a^e, a^1, a^2)$ *such that for all*

$u \in \mathcal{U}^{play}(\sigma)$: $G(a^e) \geq u_1 + u_2$, $c_i(a^i) \leq u_i$ *and* $G(a^i)(1-\delta) + \delta G(a^e) - c_i(a^i) \geq u_j$. *Every payoff on the Pareto frontier* $\mathcal{P}(\mathcal{U}^{pay}_{WRP})$ *can be implemented by a regular WRP stationary contract.*

We say an action profile $\widehat{a}^e$ is *WRP-optimal* if $G(a^e) \geq U(\sigma)$ for all $\sigma \in \Sigma_{WRP}$ and there exists a regular WRP stationary contract where $\widehat{a}^e$ is played on the equilibrium path. Let $\overline{u}^i_{SRP}$ denote the infimum of player $i$'s payoffs in SRP equilibria, if such equilibria exist.

**Proposition 5** *Let* $\delta \geq \frac{1}{2}$. *Every SRP payoff can be implemented by a stationary contract with WRP-optimal equilibrium actions* $\widehat{a}^e$. *The set of SRP payoffs is either empty or given by the line from* $(\overline{u}^1_{SRP}, G(\widehat{a}^e) - \overline{u}^1_{SRP})$ *to* $(G(\widehat{a}^e) - \overline{u}^2_{SRP}, \overline{u}^2_{SRP})$.

Proposition 5 is only helpful if we know whether SRP equilibria exist at all. We have the following sufficient condition for an optimal WRP stationary contract to be SRP:

**Proposition 6** *Let* $\delta \geq \frac{1}{2}$. *A regular WRP contract* $\sigma$ *with WRP-optimal equilibrium action* $\widehat{a}^e$ *and punishment profiles* $a^1, a^2$ *is SRP if there is no regular WRP stationary contract with an action structure* $(\tilde{a}^e, \tilde{a}^1, \tilde{a}^2)$ *such that for one player* $i \in \{1, 2\}$

$$G(\tilde{a}^i)(1-\delta) + \delta G(\tilde{a}^e) - c_i(\tilde{a}^i) > G(a^i)(1-\delta) + \delta G(a^e) - c_i(a^i). \qquad (13)$$

## 5.2 Remarks on the case $\delta < \frac{1}{2}$

To see what is different for the case $\delta < \frac{1}{2}$, go back to condition (WRP-Joint) in Lemma 4 to see that is relaxed for lower joint equilibrium payoffs $G(a^e)$. This fact illustrates the two sided nature of weak renegotiation-proofness: While one way to avoid renegotiation of punishments is to choose more efficient punishments that guarantee the punisher a higher payoff than on the equilibrium path, the other way is to choose sufficiently inefficient equilibrium play.

Since the other subgame perfection and WRP conditions require a sufficiently large joint equilibrium payoff $G(a^e)$, it may be optimal to have a small degree of inefficiency that requires to alternate between different action profiles. While for $\delta \geq \frac{1}{2}$ our results ensure that such alternation is not required to achieve Pareto-optimal WRP or SRP payoffs, this does not generally hold true for $\delta < \frac{1}{2}$.

For an example, assume the stage game is a Prisoners' Dilemma with payoff matrix

| | C | D |
|---|---|---|
| C | $(1,1)$ | $(-\frac{6}{10},\frac{11}{10})$ |
| D | $(\frac{11}{10},-\frac{6}{10})$ | $(0,0)$ |

.

A stationary contract with $a^e=(C,C)$ exists whenever $\delta \geq \frac{1}{11}$, but in a regular WRP stationary contract mutual cooperation can be sustained if and only if $\delta \geq \frac{1}{3}$, where this critical discount factor is determined by condition (WRP-Joint). In fact, it can be shown that for $\delta < \frac{1}{3}$ the only stationary contract that is WRP repeats in every period the stage game equilibrium $(D,D)$. However, for $\delta = \frac{3}{10}$ there exists a WRP equilibrium that alternates between $(C,C)$ and $(D,C)$ on the equilibrium path.[10]

An alternative possibility to relax the WRP conditions is to extend the game and stationary contracts by allowing players to burn money in payment stages. Skipping any details, we simply note that for $\delta = \frac{3}{10}$ there would then exist a WRP stationary contract with $a^e=(C,C)$ where each player burns $\frac{5}{12}$ units of money in every period on the equilibrium path. Since a requirement to burn money seems at odds with any intuitive idea of renegotiation-proofness, we abstain from the attempt to use stationary contracts with money burning for a characterization of optimal WRP and SRP payoffs for $\delta < \frac{1}{2}$.

In many other examples, the characterization of Section 5.1. extends also to the case $\delta < \frac{1}{2}$. Here are two helpful sufficient conditions.

**Proposition 7** *Let $\delta < \frac{1}{2}$. If all action structures $(a^e,a^1,a^2) \in A^3$ that satisfy for $i=1,2$ conditions (SGP-$a^e$), (SGP-$a^e$), (WRP-i), and $G(a^e) > G(a^i)$ also satisfy condition (WRP-Joint), then the results in Propositions 4, 5, 6 also apply for $\delta$.*

[10] Consider the paths

$$\begin{aligned}
Q^0 &= (p^0,(C,C),p^{DC},(D,C),p^{CC},(C,C),p^{DC},(D,C),p^{CC},...)\\
Q^i_{pay} &= (F^i,(C,C),p^{DC},(D,C),p^{CC},(C,C),p^{DC},(D,C),p^{CC},...)\\
Q^i_{play} &= (a^i,p^i_i,(C,C),p^{DC},(D,C),p^{CC},(C,C),p^{DC},(D,C),p^{CC},...)
\end{aligned}$$

with $p^0 = p^{DC} = (0.6,-0.6)$, $p^{CC} = (0.823,-0.823)$, $a^1=(C,D)$, $a^2=(D,C)$, $F^i = \frac{1-\delta}{1-\delta^2}\left(1-\delta p_1^{DC}+\delta d-\delta^2 p_1^{CC}\right)$ and $p^i_i = F^i + \frac{g_i(a^i)}{\delta}$. Let $\sigma$ be a simple strategy profile where play follows $Q^0$ whenever there was no unilateral deviation in the past and (re-)starts with $Q^i_{pay}$ ($Q^i_{play}$) directly after any unilateral deviation of player $i$ in play (pay) stage. It can be easily checked that $\sigma$ constitutes a WRP equilibrium for $\delta = \frac{3}{10}$.

**Proposition 8** *There does not exist a WRP equilibrium with a joint equilibrium payoff of $U^*$, if there exists no action structure $(a^e, a^1, a^2)$ with $G(a^e) \geq U^*$ that fulfills conditions (SGP-$a^e$), (SGP-$a^i$), (WRP-i) and*

$$(G(a^1) + G(a^2))(1-\delta) + 2\delta U^* - c_1(a^1) - c_2(a^2) \geq U^*.$$

## 5.3 Example: Abreu's simple Cournot Game

For a first example for WRP and SRP equilibria, recall Abreu's Cournot game from Section 3.3. Using Proposition 7, we find that the results of Proposition 4, 5 and 6 hold for all discount factors $\delta \geq 0$. Recall that collusive outcomes under subgame perfection are sustained for the largest set of discount factors if $a^1 = (M, H)$ and $a^2 = (H, M)$ are used as punishment. However, if weak renegotiation-proofness is required, the more efficient punishments $a^1 = (L, H)$ and $a^2 = (H, L)$ can sustain collusive play for a larger range of discount factors. Using this punishment, the optimal regular WRP equilibria have the collusive equilibrium play $a^e = (L, L)$ for all $\delta \geq \frac{1}{3}$ and the partial collusive equilibrium play of $a^e = (M, L)$ for all $\delta \in [\frac{4}{13}, \frac{1}{3})$. For $\delta < \frac{4}{13}$, the only WRP stationary contract prescribes an infinite repetition of the Nash equilibrium of the stage game. Using Proposition 6 we find that these regular WRP equilibria are also SRP in the corresponding range of discount factors. In contrast, stationary contracts with punishments $a^1 = (M, H)$ and $a^2 = (H, M)$ are never SRP.[11]

Hence, we find that the requirement of renegotiationproofness restrict the set of possible punishment profiles, and for $\delta \in [\frac{1}{4}, \frac{4}{13})$ also reduce the maximally achievable equilibrium payoffs.

## 5.4 Example: Prisoners' Dilemma

First consider the case $S \leq 0$. Using the results of Section 5.1, we find that SRP equilibria always exist and that all of them use the punishment profiles $a^1 = a^2 = (D, D)$. For the implementation of cooperative equilibrium play $a^e = (C, C)$, WRP and SRP tighten the original subgame perfection condition to $\delta \geq \max\{\frac{1}{2}, \frac{d-1}{d}\}$; for smaller discount factors no equilibrium is WRP or SRP other than an infinite repetition of the stage game equilibrium.

[11] A WRP equilibrium with action structure $((L, L), (L, M), (M, L))$ exists if and only if $\delta \geq \frac{9}{19}$ and with action structure $((M, L), (L, M), (M, L))$ if and only if $\delta \geq \frac{1}{2}$.

Assume $S > 0$. If $\delta \leq \min\{\frac{d-S}{d}, \frac{d-1}{d}\}$ only the Nash equilibrium of the stage game be sustained as a subgame perfect equilibrium, which then trivially is SRP. Otherwise the results of Section 5.1 allow us to characterize SRP equilibria only for the case $\delta \geq \frac{1-S}{2-S}$. SRP equilibria then always exist, a SRP stationary contract must punish with $a^1 = (C, D)$ and $a^2 = (D, C)$ and has optimal equilibrium actions $\overline{a}^e$ as characterized in Section 3.4.

## 5.5 Example: Bertrand competition with symmetric costs

We now investigate the case of a Bertrand duopoly with side payments. To have a compact strategy space and well defined cheating payoffs, we assume that prices $a_i$ are chosen from a finite grid $M = \{m\varepsilon\}_{m=0}^{\overline{m}}$, where $\varepsilon > 0$ measures the grid size and $\overline{m}$ is a sufficiently large upper bound. Firm $i$'s profits are given by

$$g_i(a) = \begin{cases} (a_i - k)D(a_i) & \text{if } a_i < a_j \\ (a_i - k)\frac{D(a_i)}{2} & \text{if } a_i = a_j \\ 0 & \text{if } a_i > a_j \end{cases},$$

where $D(.)$ is a weakly decreasing, non-negative market demand function and $k \in M$ denotes the constant marginal costs that are identical for both firms. Clearly, marginal cost pricing is an optimal punishment for both firms. Furthermore, in every stationary contract that yields an equilibrium price between marginal cost and the monopoly price, it holds true that $c_i(a^e) = G(a^e) - \psi_i(\varepsilon)$, where $\psi_i(\varepsilon)$ is some non-negative function that converges to 0 as $\varepsilon \to 0$.[12]

For the limit $\varepsilon \to 0$, condition (SGP-a$^e$) implies that any such collusive price is sustainable if and only if $\delta \geq \frac{1}{2}$. (Note that it does not matter whether both firms supply the market equally or only one firm supplies the market and compensates the other firm.).

A discount factor of $\frac{1}{2}$ is also the minimal discount factor to sustain collusive prices as a subgame perfect equilibrium in a Bertrand duopoly without side payments, i.e. if only subgame perfection is considered this result may suggest that side-payments do not facilitate collusion. However, Lemma 4 implies that for all $\delta \geq \frac{1}{2}$ these collusive prices can also be sustained by a weakly renegotiation-proof

[12] One may want to neglect $\psi_i(\varepsilon)$ and simply approximate $c_i(a^e) \approx G(a^e)$. This would be harmless as long as one studies subgame perfection, weak or strong renegotiation-proofness. However, with this approximation Corollary 2 would suggest that every subgame perfect payoff can be achieved by a strong perfect stationary contract, which is not the case.

stationary contract. Moreover, using Proposition 6, one can establish that the monopoly price can be sustained even by a strongly renegotiation-proof stationary contract that uses maximal fines and marginal cost pricing as punishment.[13]

For a Bertrand duopoly without side payments, Farell and Maskin (1989) establish that only marginal cost pricing can be sustained in a WRP equilibrium in pure strategies. Based on this result and Blume (1994), McCutcheon (1997) argues that small fines for meetings where prices are discussed can facilitate collusion, since renegotiation becomes harder. Although for very large discount factors, collusive outcomes can be sustained as a WRP equilibrium if one allows for mixed strategies (Farell and Maskin, 1989) or if prices must be choosen from a sufficiently coarse grid (Andersson and Wengström, 2007), the possibility for renegotiation-proof collusion for intermediate discount factors is generally reduced if WRP is required. Our example shows that with side payments, neither weak or strong renegotiation-proofness restricts the set of discount factors for which perfect collusion is possible. Thus, while the effect of meetings in smoke filled rooms on collusion may be ambiguous, this result makes clear that collusion is facilitated if participants of such meetings can easily swap briefcases filled with cash.

## 5.6 Example: Bertrand competition with asymmetric costs

Miklos-Thal (2008) shows that cost asymmetries facilitate the existence of collusive subgame perfect equilibria in repeated Bertrand competition if side payments are possible. We use our general characterization to replicate her results for a Bertrand duopoly and then show that weak renegotiation-proofness does not restrict the set of equilibrium payoffs.

There are two firms $i = 1, 2$ with constant marginal cost $k_1$ and $k_2$ with $k_1 < k_2$. We will characterize optimal subgame perfect and WRP contracts for the game considering the limit of continuous payments $\varepsilon \to 0$.

Let $\pi_1(a_1) = (a_1 - k_1)D(a_1)$ denote firm $i$'s profits if it serves the whole market at a price $a_1$. As punishment profiles we choose $a^i = (k_i, k_i + \varepsilon)$, which guarantees a cheating payoff of $c_i(a^i) \approx 0$ to the punished firm. In the punishment of firm 2, firm 1 it gets a positive profit of $\pi_1(k_2)$, while firm 2 makes zero profits in the

[13] To see this, note that for $\delta \geq \frac{1}{2}$ and sufficiently small $\varepsilon$ the expression $(1-\delta)G(\tilde{a}^i) - c_i(\tilde{a}^i)$ is non-positive for all $\tilde{a}^i \in A$, and for marginal cost pricing this expression is zero. Hence there can exist no WRP stationary contract where the left hand side of the condition in Proposition 6 is strictly bigger than the right hand side.

punishment of firm 1.

It follows from condition (SGP-a$^e$) that collusion is easiest to sustain if the low cost firm 1 supplies the whole market and compensates the high cost firm 2 with side-payments.[14] We consider equilibrium action profiles $a^e = (a_1^e, a_1^e + \varepsilon)$ where $a_1^e$ is a price above firm 1's marginal cost and weakly below firm 1's monopoly price. For small $\varepsilon$, corresponding cheating payoffs for firm 1 and 2 are $c_1(a^e) = \pi_1(a_1^e)$ and $c_2(a^e) \approx \phi(a_1^e)\pi_1(a_1^e)$ where $\phi(a_1^e) \equiv \frac{a_1^e - k_2}{a_1^e - k_1}$ is the ratio of firm 2's markup to firm 1's markup. Condition (SGP-a$^e$) thus implies that an equilibrium price $a_1^e$ is sustainable if and only if

$$\delta \geq \frac{\phi(a_1^e)}{1 + \phi(a_1^e)}. \tag{14}$$

Since $\phi(a_1^e) < 1$, this critical discount factors is smaller $\frac{1}{2}$, which means cost asymmetries indeed facilitate collusion. Moreover, since $\phi$ is continuous and $\phi(k_2) = 0$ some collusive markup above $k_2$ can be sustained for every discount factor $\delta > 0$.

Such contracts are also always weakly renegotiation-proof. The WRP condition (WRP-i) for firm 1 turns out to be directly equivalent to the subgame perfection condition (14). Condition (WRP-Joint) and condition (WRP-i) for firm 2 coincide and become

$$\delta \geq \frac{\pi_1(a_1^e) - \pi_1(k_2)}{2\pi_1(a_1^e) - \pi_1(k_2)}. \tag{15}$$

For a completely inelastic demand function $D(.)$, condition (15) is identical to the subgame perfection condition (14) and since $D(.)$ is weakly decreasing, condition (15) is weaker than condition (14).

# 6 Summary

We have shown that Pareto optimal subgame perfect payoffs and renegotiation-proof payoffs can generally be found by restricting attention to a simple class of stationary contracts. These stationary contracts prescribe play of the same action profile in every period on the equilibrium path, and fines are used to construct one-period-punishments that allow returning to the equilibrium play after a punishment has been carried out. While it is not surprising that one can restrict attention to equilibria with a stationary equilibrium path, the first part of our

[14] Joint payoffs $G$ are maximized if firm 1 conducts the whole production. Since cheating payoffs result from marginally undercutting the equilibrium price, they do not depend on who serves the market (at least not in the limit of continuous prices $\varepsilon \to 0$).

paper contributes to the existing literature by establishing simple conditions that allow an easy characterization of the Pareto-frontier of subgame perfect payoffs for all discount factors in general two player stage games with side payments.

In the second part of the paper, we compared and characterized different concepts of renegotiation-proofness for intermediate discount factors. First we established that if renegotiation-proofness can take place only before the payment stage, every Pareto optimal subgame perfect payoff can always be implemented in a renegotiation-proof way. Then we assumed renegotiation is possible at all stages, and used stationary contracts to characterize strong perfect payoffs. We derived simply conditions to check for the existence of strong perfect equilibria. Afterwards, we investigated the less restrictive concepts of weak and strong renegotiation-proofness. While in many examples Pareto-optimal subgame perfect payoffs can be implemented as WRP or even SRP equilibria, this is not always the case: Pareto-optimal subgame perfect equilibria that rely on very inefficient punishments can fail to be renegotiation-proof. In general, optimal WRP equilibria may require a fine-tuned degree of inefficiency on the equilibrium path that can be achieved by alternating between different action profiles, or by burning money on the equilibrium path. If $\delta \geq \frac{1}{2}$ holds (or another sufficient condition), we have shown that such contractual features will never be necessary to achieve optimal WRP or SRP payoffs.

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# Appendix: Proofs

**Lemma 1:** *There exists a stationary contract with action structure* $(a^e, a^1, a^2)$ *if and only if*

$$G(a^e) \geq (1-\delta)(c_1(a^e) + c_2(a^e)) + \delta(c_1(a^1) + c_2(a^2)) \tag{SGP-a$^e$}$$

*and for both players* $i = 1, 2$

$$(1-\delta)G(a^i) + \delta G(a^e) - c_i(a^i) \geq (1-\delta)c_j(a^i) + \delta c_j(a^j). \tag{SGP-a$^i$}$$

Proof of Lemma 1: We are interested in finding conditions on $a^e, a^1, a^2$ that make it possible to define the equilibrium transfer $p^e$ and fines $F^1$ and $F^2$ such that conditions (6), (4), (5) for subgame perfection are fulfilled. Note that there are three conditions that bound $u_i^i$, $i = 1, 2$ from above but only condition (4) bounds it from below. Therefore, these conditions hold for some $u_i^i$ if and only if they hold for the lowest possible punishment payoffs $u_i^i = c_i(a^i)$, which are achieved by maximal fines.

Equilibrium transfers $p^e$ then only appear in the conditions (6):

$$g_i(a^e) - \delta p_i^e \geq c_i(a^e)(1-\delta) + \delta c_i(a^i) \quad \text{for } i \in \{1, 2\}$$

Choosing $\delta p_1^e = g_1(a^e) - c_1(a^e)(1-\delta) - \delta c_1(a^1)$, these conditions bind exactly for player 1, and the condition for player 2 becomes condition (SGP-a$^e$).■

**Proposition 1:** *An optimal action profile* $\bar{a}^e$ *and strongest punishments* $\bar{a}^1, \bar{a}^2$ *always exist. The Pareto-frontier of subgame perfect payoffs is linear and can be implemented by stationary contracts with action structure* $(\bar{a}^e, \bar{a}^1, \bar{a}^2)$ *and maximal fines.*

Proof of Proposition 1: Consider three sequences of subgame perfect equilibria $\sigma^e(n), \sigma^1(n), \sigma^2(n)$ in $\Sigma_{SGP}^{play}$with $U(\sigma^e(n)) \to \overline{U}_{SGP}$ and $u_i(\sigma^i(n)) \to \overline{u}_{SGP}^i$. Let $a^k(n)$ be the first action profile on the equilibrium path of $\sigma^k(n)$ for $k \in \{e, 1, 2\}$. Then $a^k(n)$ is a sequence in the compact set $A$, and as such must have convergent subsequences with limits in $A$. We assume w.l.o.g. that these convergent subsequences are already given by $a^k(n)$ and denote their limits by $\bar{a}^e, \bar{a}^1$ and $\bar{a}^2$, respectively. In the following we use the properties of $\sigma^e(n), \sigma^1(n), \sigma^2(n)$ to make inferences about $\bar{a}^e, \bar{a}^1$ and $\bar{a}^2$. First, if we decompose $\sigma^e(n)$ into current play and future payoff, we have that

$$U(\sigma^e(n)) \leq (1-\delta)G(a^e(n)) + \delta \overline{U}_{SGP}. \tag{16}$$

Since $G$ is continuous, taking the limit $n \to \infty$ yields

$$\overline{U}_{SGP} \leq G(\bar{a}^e). \tag{17}$$

Second, subgame perfection of $\sigma^i(n)$ implies

$$u_i(\sigma^i(n)) \geq (1-\delta)c_i(a^i(n)) + \delta\overline{u}^i_{SGP}. \tag{18}$$

Since $c_i$ is continuous, taking the limit $n \to \infty$ yields

$$\overline{u}^i_{SGP} \geq c_i(\bar{a}^i). \tag{19}$$

Third, summing up the subgame perfection conditions of players 1 and 2 for $\sigma^e(n)$ yields

$$\overline{U}_{SGP} \geq (1-\delta)\left(c_1(a^e(n)) + c_2(a^e(n))\right) + \delta(\overline{u}^1_{SGP} + \overline{u}^2_{SGP}). \tag{20}$$

In the limit, and using (17) and (19), this becomes

$$G(\bar{a}^e) \geq (1-\delta)\left(c_1(\bar{a}^e) + c_2(\bar{a}^e)\right) + \delta(c_1(\bar{a}^1) + c_2(\bar{a}^2)). \tag{21}$$

Last, we exploit the subgame perfection condition

$$u_j(\sigma^i(n)) \geq c_j(a^i(n))(1-\delta) + \delta\overline{u}^j_{SGP} \tag{22}$$

as well as

$$G(a^i(n))(1-\delta) + \delta\overline{U}_{SGP} \geq U(\sigma^i(n))$$

to get

$$G(a^i(n))(1-\delta) + \delta\overline{U}_{SGP} - \overline{u}^i_{SGP} \geq c_j(a^i(n))(1-\delta) + \delta\overline{u}^j_{SGP}. \tag{23}$$

In the limit, and using (17) and (19), this becomes

$$G(\bar{a}^i)(1-\delta) + \delta G(\bar{a}^e) - c_i(\bar{a}^i) \geq c_j(\bar{a}^i)(1-\delta) + \delta c_j(\bar{a}^j). \tag{24}$$

Equations (21) and (24) together with Lemma 1 now tell us that there is a stationary contract with action structure $(\bar{a}^e, a^1, a^2)$, with joint payoff $G(\bar{a}^e) = \overline{U}_{SGP}$ and punishment payoffs $c_i(\bar{a}^i) = \overline{u}^i_{SGP}$, i.e. $\bar{a}^e$ is an optimal action profile and $\bar{a}^i$ are optimal punishments. Recall from Remark 1 that different up-front payment can be used to achieve all payoffs on the line between $(c_1(\bar{a}^1), G(\bar{a}^e) - c_1(\bar{a}^1))$ and $(G(\bar{a}^e) - c_2(\bar{a}^2), c_2(\bar{a}^2))$. Since player $i$ will never get a lower payoff than

$c_i(\bar{a}^i) = \overline{u}^i_{SGP}$ in any SGP equilibrium, this line constitutes the Pareto-frontier of SGP payoffs. ■

**Lemma 2:** *An action profile $\widetilde{a}$ is admissible if and only if*

$$(1-\delta)G(\widetilde{a}) + \delta G(\overline{a}^e) \geq (1-\delta)(c_1(\widetilde{a}) + c_2(\widetilde{a})) + \delta(c_1(\overline{a}^1) + c_2(\overline{a}^2)).$$

<u>Proof of Lemma 2:</u> First, let $\widetilde{a}$ be admissible. Let $\widetilde{\sigma}$ be a subgame-perfect equilibrium in which $\widetilde{a}$ is played in the first period. Let $\widetilde{u}^e$ and $\widetilde{u}^i$ denote the continuation payoffs in the payment stage of period 2 after no deviation or a unilateral deviation of player $i$, respectively. Subgame perfection requires

$$(1-\delta)g_i(\widetilde{a}) + \delta\widetilde{u}^e_i \geq (1-\delta)c_i(a^e) + \delta\widetilde{u}^i_i \text{ for } i = 1, 2$$

Summing up the two conditions yields

$$(1-\delta)G(\widetilde{a}) + \delta\left(\widetilde{u}^e_1 + \widetilde{u}^e_2\right) \geq (1-\delta)(c_1(a^e) + c_2(a^e)) + \delta(\widetilde{u}^1_1 + \widetilde{u}^2_2).$$

Since $\widetilde{u}^e_1 + \widetilde{u}^e_2 \leq \overline{U}_{SGP} = G(\overline{a}^e)$ and $\widetilde{u}^i_i \geq \overline{u}^i_{SGP} = c_i(\overline{a}^i)$ this condition can be fulfilled only if condition (8) holds.

To prove the other direction, we define extended stationary strategy contracts, a subclass of subgame perfect equilibria that encompasses our stationary contracts. An extended stationary contract shall have the same structure as stationary contracts, with the only exception that the initial path of play is given by $Q^0 = (p^0, \widetilde{a}, \widetilde{p}, a^e, p^e, a^e, p^e, ...)$, i.e. equilibrium actions in the first period are $\widetilde{a}$ instead of $a^e$ and the directly following payments are $\widetilde{p}$ instead of $p^e$. Let now $\widetilde{a}$ be an action in $A$ that fulfills condition (8). It can then be shown that there is an extended stationary contract with first period action profile $\widetilde{a}$, optimal equilibrium actions $\overline{a}^e$, maximal fines, and strongest punishments $\overline{a}^i$. Payments $p^e_1$ and $\widetilde{p}_1$ can for example be chosen such that the incentives constraints not to deviate from $\overline{a}^e$ and $\widetilde{a}$ are exactly binding for player 1. ■

**Proposition 2:** *Every strong perfect payoff can be implemented by a stationary contract with optimal equilibrium actions $\bar{a}^e$. The set of strong perfect payoffs is either empty or given by the line from $(\overline{u}^1_{SP}, G(\bar{a}^e) - \overline{u}^1_{SP})$ to $(G(\bar{a}^e) - \overline{u}^2_{SP}, \overline{u}^2_{SP})$.*

<u>Proof of Proposition 2:</u> Assume that a strong perfect equilibrium exists. It is easy to verify that a player's lowest strong perfect continuation payoffs before play and pay stage coincide, i.e. $\overline{u}^i_{SP} = \inf_{u \in \overline{U}^{play}_{SP}} u_i = \inf_{u \in \overline{U}^{pay}_{SP}} u_i$. For both players $i = 1, 2$, let $\bar{u}^i$ be a tuple in the closure of $\overline{U}^{play}_{SP}$ with $\bar{u}^i_i = \overline{u}^i_{SP}$. Since punishments

with continuation payoffs $\bar{u}_i^i$ must be able to sustain at least one optimal action profile $\bar{a}^e$, it must hold that

$$G(\bar{a}^e) \geq (c_1(\bar{a}^e) + c_2(\bar{a}^e))(1-\delta) + \delta(\bar{u}_1^1 + \bar{u}_2^2). \tag{25}$$

By similar steps as in the proof of Proposition 1, we find that for both players $i = 1, 2$ there must exist $a^i \in A$ with $\bar{u}_i^i \geq c_i(a^i)$ and

$$G(a^i)(1-\delta) + \delta G(\bar{a}^e) - \bar{u}_i^i \geq \bar{u}_j^i \geq c_j(a^i)(1-\delta) + \delta \bar{u}_j^j. \tag{26}$$

Analogous to Lemma 1, conditions (25) and (26) imply that there must exist a stationary contract $\sigma$ with action structure $(\bar{a}^e, a^1, a^2)$ and punishment payoffs $\bar{u}_1^1$ and $\bar{u}_2^2$. In this stationary contract, all continuation equilibria (at payment or play stage) either have total payoff $\overline{U}_{SGP}$, or a continuation payoff of $u^i$ with $u_i^i = \bar{u}_i^i$ and $u_j^i \geq \bar{u}_j^i$ (the latter follows from condition (26)). Thus $\sigma$ is strong perfect, and by varying the up-front transfer it can be used to implement the whole set of strong perfect payoffs.■

**Proposition 3:** *A stationary contract with action structure $(\bar{a}^e, a^1, a^2)$ and punishment payoffs $u_1^1$ and $u_2^2$ is strong perfect if and only if for both players $i = 1, 2$ and for all admissible $\tilde{a}$ with $G(\tilde{a}) > G(a^i)$ it holds that either*

$$(1-\delta)G(a^i) - u_i^i \geq (1-\delta)(G(\tilde{a}) - c_i(\tilde{a})) - \delta c_i(\bar{a}^i) \text{ or} \tag{SP1}$$

$$u_i^i \geq (1-\delta)(G(\tilde{a}) - c_j(\tilde{a})) + \delta G(\bar{a}^e) - \delta c_j(\bar{a}^j) \tag{SP2}$$

Proof of Proposition 3: First, we show that a stationary contract $\sigma$ with action structure $(\bar{a}^e, a^1, a^2)$ and punishment payoffs $u_1^1$ and $u_2^2$ is strong perfect given that the conditions listed in the proposition hold. Clearly, continuation equilibria that start in the payment stage or before play of $\overline{a}^e$ cannot be Pareto-dominated. We only have to show that no continuation equilibria in which a player is punished in play stage is Pareto dominated. Assume to the contrary that there exists a continuation equilibrium $\tilde{\sigma} \in \Sigma_{SGP}^{play}$ that strictly Pareto-dominates the punishment for player $i$. The first action $\tilde{a}$ of $\tilde{\sigma}$ is admissible and since

$$G(a^i)(1-\delta) + \delta G(\bar{a}^e) < U(\tilde{\sigma}) \leq G(\tilde{a})(1-\delta) + \delta G(\bar{a}^e)$$

it must hold that $G(\tilde{a}) > G(a^i)$, hence either inequality (SP1) or (SP2) holds. In the equilibrium $\tilde{\sigma}$ player $j$'s payoff is bounded by the joint payoff $U(\tilde{\sigma})$ minus player i's minimum payoff $(1-\delta)c_i(\tilde{a})+\delta\overline{u}^i_{SGP}$. Hence, strict Pareto dominance of $\tilde{\sigma}$ implies that

$$(1-\delta)G(a^i)+\delta G(\bar{a}^e)-u^i_i<u_j(\tilde{\sigma})\leq(1-\delta)G(\tilde{a})+\delta G(\bar{a}^e)-(1-\delta)c_i(\tilde{a})-\delta\overline{u}^i_{SGP}$$

and

$$u^i_i<u_i(\tilde{\sigma})\leq G(\tilde{a})(1-\delta)+\delta G(\bar{a}^e)-(1-\delta)c_j(\tilde{a})-\delta\overline{u}^j_{SGP},$$

which leads to a contradiction to the fact that either (SP1) or (SP2) has to hold.

Next we assume that $\sigma$ is a strong perfect stationary contract with action structure $(\bar{a}^e,a^1,a^2)$ and punishment payoffs $u^1_1,u^2_2$ and show that the conditions stated in the proposition have to hold. Assume to the contrary that there exists an admissible action profile $\tilde{a}$ with $G(\tilde{a})>G(a^i)$ and

$$(1-\delta)G(\tilde{a})-(1-\delta)c_i(\tilde{a})-\delta\overline{u}^i_{SGP}>(1-\delta)G(a^i)-u^i_i \tag{27}$$

as well as

$$(1-\delta)G(\tilde{a})+\delta G(\bar{a}^e)-(1-\delta)c_j(\tilde{a})-\overline{u}^j_{SGP}>u^i_i. \tag{28}$$

for some player $i$. Because $\tilde{a}$ is admissible, there exists an extended stationary contract (see the proof of Lemma 2) with equilibrium path

$$\tilde{Q}:=(\tilde{a},\tilde{p},\bar{a}^e,p^e,\bar{a}^e,....)$$

and the optimal penal codes as punishments for some payments $\tilde{p}$. Since $G(\tilde{a})>G(a^i)$, this equilibrium has a higher joint payoff than the continuation payoff of our stationary contract $\sigma$ at the punishment of player $i$ in play stage. Moreover, conditions (27) and (28) then imply that payments after play of $\tilde{a}$ can be chosen such that each player gets strictly more than in that punishment phase of $\sigma$, i.e. $\sigma$ is not strong perfect.■

**Corollary 2:** A stationary contract with optimal strongest punishments $\bar{a}^1,\bar{a}^2$, maximal fines and optimal equilibrium actions is strong perfect if and only if for all admissible action profiles $\tilde{a}$ and both players $i=1,2$

$$G(\bar{a}^i)-c_i(\bar{a}^i)\geq G(\tilde{a})-c_i(\tilde{a}).$$

Proof of Corollary 2: Assume first that there is an admissible action profile $\tilde{a}$ and a player $i\in\{1,2\}$ such that $G(\bar{a}^i)-c_i(\bar{a}^i)<G(\tilde{a})-c_i(\tilde{a})$. Since $c_i(\bar{a}^i)\leq c_i(\tilde{a})$, it must hold that $G(\tilde{a})>G(\bar{a}^i)$. Condition (SP1) of Prop. 3 takes the form

$G(\tilde{a}) - c_i(\tilde{a}) \leq G(\bar{a}^i) - c_i(\bar{a}^i)$ and therefore does not hold. Condition (SP2), together with admissibility of $\tilde{a}$, implies that

$$c_i(\tilde{a})(1-\delta) + \delta c_i(\bar{a}^i) \leq G(\tilde{a})(1-\delta) + \delta G(a^e) - c_j(\tilde{a})(1-\delta) - \delta c_j(\bar{a}^j) \leq c_i(\bar{a}^i).$$

This could only be true if $c_i(\tilde{a}) = c_i(\bar{a}^i)$, but then $G(\tilde{a}) > G(\bar{a}^i)$ contradicts the fact that $\bar{a}^i$ is an optimal strongest punishment.

To show sufficiency, assume that for all admissible action profiles $\tilde{a}$ and both players $i = 1, 2$ condition (9) holds. Then condition (9) in particular holds for all admissible action profiles $\tilde{a}$ for which $G(\tilde{a}) > G(\bar{a}^i)$ and condition (SP1) of Proposition 3 is true.■

**Corollary 3:** *There exists no strong perfect stationary contract with action structure* $(\bar{a}^e, a^1, a^2)$ *if for both players* $i = 1, 2$ *we have* $G(\bar{a}^e) > G(a^i)$ *and*

$$(1-\delta)\left(G(\bar{a}^e) - c_i(\bar{a}^e)\right) - \delta c_i(\bar{a}^i) > (1-\delta)G(a^i) - c_i(a^i).$$

Proof of Corollary 3: For a proof by contradiction, assume that there is a strong perfect stationary contract $\sigma$ with action structure $(\bar{a}^e, a^1, a^2)$ and punishment payoffs $u_1^1$ and $u_2^2$ such that for both players $i = 1, 2$ condition (10) holds and $G(\bar{a}^e) > G(a^i)$. We consider Proposition 3 for $\tilde{a} = \bar{a}^e$. Condition (SP1) does not hold for any player $i = 1, 2$. Therefore, condition (SP2) must hold for both players, and in sum these conditions imply that $u_1^1 + u_2^2 \geq G(\bar{a}^e)$. This can only be fulfilled if $u_1^1 + u_2^2 = G(\bar{a}^e)$, and in this case the condition also implies that $G(\bar{a}^e) = c_1(\bar{a}^1) + c_2(\bar{a}^2)$. This means that $\mathcal{P}(\mathcal{U}_{SGP}^{pay})$ and $\mathcal{P}(\mathcal{U}_{SGP}^{play})$ consist of just one point with joint payoff $\overline{U}_{SGP}$, hence $\mathcal{P}(\mathcal{U}_{SGP}^{play})$ does not contain the punishment payoff in $\sigma$, which cannot be strong perfect. ■

**Lemma** 3: *A regular stationary contract with action structure* $(a^e, a^1, a^2)$ *and equilibrium payoffs* $u^e$ *is WRP if and only if for both players* $i = 1, 2$

$$(1-\delta)G(a^i) + \delta G(a^e) - c_i(a^i) \geq u_j^e \tag{29}$$

Proof of Lemma 3: Let $\sigma$ be a WRP regular stationary contract with action structure $(a^e, a^1, a^2)$ and equilibrium payoffs $u^e$. If it were true that

$$(1-\delta)G(a^i) + \delta G(a^e) - c_i(a^i) < u_j^e,$$

then it must hold that $c_i(a^i) \geq u_i^e$, i.e., $c_i(a^i) \geq c_i(a^e)$. This implies $a^i = a^e$ and thus we arrive at a contradiction:

$$G(a^i) + \delta G(a^e) - c_i(a^i) = G(a^e) - c_i(a^e) \geq G(a^e) - u_i^e = u_j^e.$$

Next, assume that for an action structure $(a^e, a^1, a^2)$ and equilibrium payoff $u^e$ inequality (11) holds. Since $G(a^e) \geq G(a^i)$ this implies that the payoff when player $i$ is punished and the equilibrium payoff $u^e$ cannot be Pareto-ranked. Moreover, $(1-\delta)G(a^i) + \delta G(a^e) - c_i(a^i) \geq u_j^e \geq c_i(a^i)$, and therefore the two punishments cannot be Pareto-ranked, either.■

**Lemma** 4: *If the set of regular stationary contracts with action structure $(a^e, a^1, a^2)$ is non-empty, it contains WRP contracts if and only if for both players $i = 1, 2$*

$$(1-\delta)G(a^i) + \delta G(a^e) - c_i(a^i) \geq (1-\delta)c_j(a^e) + \delta c_j(a^j) \tag{WRP-i}$$

$$(1-\delta)(G(a^1) + G(a^2)) + (2\delta - 1)G(a^e) \geq c_1(a^1) + c_2(a^2). \tag{WRP-Joint}$$

Proof of Lemma 4: Conditions (WRP-i) and (WRP-Joint) follow from condition 11 and subgame perfection. For the other direction, assume there exists regular stationary contract with action structure $(a^e, a^1, a^2)$, which fulfills (WRP-i) and (WRP-Joint). These conditions and the subgame perfection conditions imply that there exist net-payments $p^e$ such that

$$G(a^e) - (1-\delta)c_2(a^e) - \delta c_2(a^2) \geq g_1(a^e) - p_1^e \geq c_1(a^e)(1-\delta) + \delta c_1(a^1),$$

$$(1-\delta)G(a^2) + \delta G(a^e) - c_2(a^2) \geq g_1(a^e) - p_1^e \geq (1-\delta)(G(a^e) - G(a^1)) - c_1(a^1).$$

Using these inequalities, Lemma 1, and Lemma 4, it is straightforward to verify that a WRP regular stationary contract $\sigma$ with action structure $(a^e, a^1, a^2)$ and equilibrium payments $p^e$ exists.■

**Proposition 4:** *Let $\delta \geq \frac{1}{2}$. For every WRP equilibrium $\sigma$ there exists a regular WRP stationary contract with an action structure $(a^e, a^1, a^2)$ such that for all $u \in U^{play}(\sigma)$: $G(a^e) \geq u_1 + u_2$, $c_i(a^i) \leq u_i$ and $G(a^i)(1-\delta) + \delta G(a^e) - c_i(a^i) \geq u_j$. Every payoff on the Pareto frontier $P(U^{pay}_{WRP})$ can be implemented by such a regular WRP stationary contract.*

Proof of Proposition 4: Let $\sigma$ be any WRP equilibrium and let

$$\bar{U} = \sup_{u \in \mathcal{U}^{play}(\sigma)} u_1 + u_2, \quad \text{and} \quad \bar{u}_i^i = \inf_{u \in \mathcal{U}^{play}(\sigma)} u_i.$$

We take $\bar{u}^e$ to be a payoff tuple in the closure of $\mathcal{U}^{play}(\sigma)$ such that $\bar{u}_1^e + \bar{u}_2^e = \bar{U}$. We also define $\bar{u}^i$ as the tuple in the closure of $\mathcal{U}^{play}(\sigma)$ that maximizes player $j$'s payoff among all such tuples with payoff $\bar{u}_i^i$ for player $i$. We then have that

$\bar{u}_i^i \leq u_i$ and $\bar{u}_j^i \geq u_j$ for all $u \in \mathcal{U}^{play}(\sigma)$. Let $u(\sigma|h^e(n))$ be a sequence in $\mathcal{U}^{play}(\sigma)$ with limit $\bar{u}^e$ and for $i = 1, 2$ let $u(\sigma|h^i(n))$ be a sequence with limit $\bar{u}^i$. Let furthermore $a^k(n)$ be the w.l.o.g. convergent sequences of the first action profiles of the continuation equilibria $\sigma|h^k(n)$, $k \in \{e, 1, 2\}$. Completely analogous to the subgame perfection case (see the proof of Proposition 1) we have for the limits of these sequences, denoted by $a^e, a^1, a^2$, that $G(a^e) \geq \bar{U}$, $c_i(a^i) \leq \bar{u}_i^i$,

$$\bar{U} \geq (c_1(a^e) + c_2(a^e))(1-\delta) + \delta(\bar{u}_1^1 + \bar{u}_2^2),$$

$$G(a^i)(1-\delta) + \delta\bar{U} - \bar{u}_i^i \geq \bar{u}_j^i \geq c_j(a^i)(1-\delta) + \delta\bar{u}_j^j,$$

as well as

$$G(a^i)(1-\delta) + \delta\bar{U} - c_i(a^i) \geq \bar{u}_j^e \geq c_j(a^e)(1-\delta) + \delta c_j(a^j),$$

which also implies

$$(G(a^1) + G(a^2))(1-\delta) + 2\delta\bar{U} - c_1(a^1) - c_2(a^2) \geq \bar{U}.$$

Since we assumed that $\delta \geq \frac{1}{2}$, these conditions are relaxed if we replace $\bar{U}$ by $G(a^e)$. Next, define $\tilde{a}^e \in \{a^e, a^1, a^2\}$ such that $G(\tilde{a}^e) = \max\{G(a^e), G(a^1), G(a^2)\}$, and $\tilde{a}^i = a^i$ if $c_i(a^i) < c_i(\tilde{a}^e)$ and $\tilde{a}^i = \tilde{a}^e$ else. It is straightforward to show that all conditions still hold:

$$G(\tilde{a}^e) \geq (1-\delta)(c_1(\tilde{a}^e) + c_2(\tilde{a}^e)) + \delta(c_1(\tilde{a}^1) + c_2(\tilde{a}^2)),$$

$$G(\tilde{a}^i)(1-\delta) + \delta G(\tilde{a}^e) - c_i(\tilde{a}^i) \geq \max(c_j(\tilde{a}^e), c_j(\tilde{a}^i))(1-\delta) + \delta c_j(\tilde{a}^j),$$

and

$$(G(\tilde{a}^1) + G(\tilde{a}^2))(1-\delta) + 2\delta G(\tilde{a}^e) - (c_1(\tilde{a}^1) + c_2(\tilde{a}^2)) \geq G(\tilde{a}^e).$$

Because of Lemma 4 there is a WRP regular stationary contract with action structure $(\tilde{a}^e, \tilde{a}^1, \tilde{a}^2)$ that has the properties as stated in the first part of proposition. It follows that for any WRP payoff $u(\sigma), \sigma \in \Sigma_{WRP}^{pay}$ there is a stationary contract that weakly Pareto dominates it, which then implies the last sentence of the proposition.

**Proposition 5:** *Let $\delta \geq \frac{1}{2}$. Every SRP payoff can be implemented by a stationary contract with WRP-optimal equilibrium actions $\widehat{a}^e$. The set of SRP payoffs is either empty or given by the line from $(\overline{u}_{SRP}^1, G(\widehat{a}^e) - \overline{u}_{SRP}^1)$ to $(G(\widehat{a}^e) - \overline{u}_{SRP}^2, \overline{u}_{SRP}^2)$.*

Proof of Proposition 5: Since no payoff in $\mathcal{U}_{SRP}^{play}$ Pareto dominates the other, one can show as in the WRP case that there exists a WRP stationary contract $\sigma$ with action structure $(a^e, a^1, a^2)$ such that $G(a^e) \geq u_1 + u_2$, $c_i(a^i) \leq u_i$, and $G(a^i)(1-\delta)+\delta G(a^e)-c_i(a^i) \geq u_j$ for all $u \in \mathcal{U}_{SRP}^{play}$. Since $\sigma$ cannot Pareto dominate the SRP equilibria it follows that $G(a^e) = \max_{u \in \mathcal{U}_{SRP}^{play}}\{u_1+u_2\}$. Because the worst SRP payoffs must be able to sustain $a^e$ it follows that there is a SRP equilibrium with action structure $(a^e, a^1, a^2)$ and punishment payoffs $u_i^i = \min_{u \in \mathcal{U}_{SRP}^{play}} u_i$.

■

**Proposition 6:** *Let $\delta \geq \frac{1}{2}$. A regular WRP contract $\sigma$ with WRP-optimal equilibrium action $\widehat{a}^e$ and punishment profiles $a^1, a^2$ is SRP if there is no regular WRP stationary contract with an action structure $(\tilde{a}^e, \tilde{a}^1, \tilde{a}^2)$ such that for one player $i \in \{1,2\}$*

$$G(\tilde{a}^i)(1-\delta) + \delta G(\tilde{a}^e) - c_i(\tilde{a}^i) > G(a^i)(1-\delta) + \delta G(a^e) - c_i(a^i). \tag{30}$$

Proof of Proposition 6: Assume that $\sigma$ is not SRP. Since $\sigma$ is an optimal WRP stationary contract, it can only be dominated in the punishment phase, that is, there must be $i \in \{1,2\}$ and a WRP equilibrium $\tilde{\sigma}$ such that $u_i(\tilde{\sigma}|h) > c_i(a^i)$ and

$$u_j(\tilde{\sigma}|h) > G(a^i)(1-\delta) + \delta G(a^e) - c_i(a^i)$$

for some $h \in H^{play}$. Because of Prop. 4 there exists a regular WRP stationary contract with action structure $(\tilde{a}^e, \tilde{a}^1, \tilde{a}^2)$ that fulfills

$$G(\tilde{a}^i)(1-\delta) + \delta G(\tilde{a}^e) - c_i(\tilde{a}^i) \geq u_j(\tilde{\sigma}|h).$$

■

**Proposition 7:** Let $\delta < \frac{1}{2}$. If all action structures $(a^e, a^1, a^2) \in A^3$ that satisfy for $i = 1, 2$ conditions (SGP-a$^e$), (SGP-a$^i$), (WRP-i), and $G(a^e) \geq G(a^i)$ also satisfy condition (WRP-Joint), then the results in Propositions 4, 5, 6 also apply for $\delta$.

Proof of Proposition 7:This result follows immediately because Propositions 5 and 6 assume that $\delta \geq \frac{1}{2}$ only because they rely on Proposition 4. It is obvious from its proof that Proposition 4 also holds for $\delta < \frac{1}{2}$ if the joint WRP condition (WRP-Joint) is implied by the other conditions.

■

**Proposition 8:** *There does not exist a WRP equilibrium with a joint equilibrium payoff of $U^*$, if there exists no action structure $(a^e, a^1, a^2)$ with $G(a^e) \geq U^*$*

*that fulfills conditions (SGP-$a^{\mathrm{e}}$), (SGP-$a^{\mathrm{i}}$), (WRP-i) and*

$$(G(a^1) + G(a^2))(1-\delta) + 2\delta U^* - c_1(a^1) - c_2(a^2) \geq U^*.$$

.

Proof of Proposition 8: Straightforward, given the proof of Proposition 4.■