Dynamic Duopoly with Inattentive Firms *

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July 2010

Abstract

This paper analyzes an infinite horizon dynamic duopoly with stochastic demand in which firms face planning costs. In this set-up we derive the following results. First, non-synchronized planning equilibria can only exist if products are strategic substitutes while synchronized planning equilibria can only exist if products are strategic complements. This result is in sharp contrast to the predictions obtained by models that suppose commitment power of firms. In addition, we show that for both classes of equilibria, alternating and synchronous planning, there exist multiple inattentiveness lengths that can be supported as an equilibrium.

Keywords: Inattentiveness, Planning costs

JEL Classification: C73, D43, D83

*We thank Matthias Doepke, Glenn Ellison, Philipp Kircher, George Mailath, Niko Matouschek, Monika Piazzesi, Sven Rady, and Martin Schneider for very helpful comments and suggestions. Markus Reisinger and Ludwig Ressner gratefully acknowledge financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15. Ludwig Ressner gratefully acknowledges financial support from the Bavarian Graduate Program in Economics.

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1 Introduction

It is a heavily discussed question in the economics literature whether oligopolistic firms choose to synchronize the adjustment of their prices (or quantities) or if they adjust them at different points in time. Since the seminal papers on dynamic duopoly by Maskin and Tirole (1987; 1988a;b) this question attracted considerable attention by researchers. The main body of this literature analyzed the question by considering how commitment power of firms shapes this decision, see e.g., the papers by Maskin and Tirole, De Fraja (1993) or Lau (2001). In these papers there is a physical friction that hinders firms from adjusting their plans each period. Thus, firms are either exogenously equipped with commitment power or can decide to be committed for some time period. The mechanism that then drives the results concerning synchronization versus non-synchronization is rooted in the mode of strategic interaction, that is, if firms' strategy variables are strategic complements or substitutes.

However, the approach relying on commitment power is not satisfactory in three important aspects: First, firms usually face a considerable amount of uncertainty with respect to demand and/or costs, which heavily influences their adjustment decision. This uncertainty is neglected in the above literature. Second, firms can often change their prices almost costlessly, and so the commitment assumption is somewhat artificial in many markets. Third, the length between two consecutive adjustments is prespecified in the above models to either one or two periods. However, firms are usually more flexible in choosing this length.

In this article, we approach this question by drawing on recent developments in the theory of rational inattention by Reis (2006b) and Hellwig and Veldkamp (2009). In particular, we consider a model in which firms' demand and costs, at each point in time, are hit by a shock. As in Reis (2006b) and Hellwig and Veldkamp (2009) a firm can choose to become informed about this shock and recalculate its optimal price. However, it has to incur a cost to acquire, absorb and process this information, i.e., it is costly for a firm to replan its optimal action.[1] If a firm chooses to plan, it sets a price path until its next planning date. The prices during this path are fully flexible, that is, they can differ at different points in time. This implies that firms have no commitment power.[2] Finally, a firm is free to choose the optimal

[1]This is a realistic assumption since, as e.g., Radner (1992) points out, absorbing and processing the relevant information for decision making is an important goal of many managerial occupations.

[2]This is line with recent empirical work. For example, the work by Bils and Klenow (2004) seems to contradict the finding that prices are adjusted only infrequently.

length for which it stays inattentive, i.e., the time period in which it does not plan. Overall, this framework seems to fit many industries in a better way than the commitment model since it is often much harder to determine the optimal price in an uncertain environment than to change the price. For example, Zbaracki et al. (2004), using data from a large U.S. manufacturing firm, find that the managerial planning costs of price adjustments are often much larger than the physical costs of price adjustments.

More specifically, we consider an infinite horizon continuous time model of competition between two firms who produce differentiated goods and – at each instant – face stochastic demands and costs. At the beginning of the game both firms are informed about the state of demand and costs and independently from each other choose an infinite sequence of dates at which they choose to plan, where planning means that they acquire information about demand and cost and re-optimize their prices. Since planning is costly, each firm chooses to plan only at some points in time and stays (rationally) inattentive in the meantime. During the inattentiveness period uncertainty builds up in the system. Thus, when choosing the sequence of planning dates each firm balances the costs of planning and the gains from having a re-optimized plan. At a planning date a firm observes the history of the game and the current shock realizations. Given this information it chooses a price path up to its next planning date. This implies that at each instant firms play a one-shot price competition game with potentially imperfect and different information. As a consequence, there is no commitment possibility. Instead, what matters for synchronization or non-synchronization of planning decisions is how the decision of one firm to plan affects the other firm's benefit from planning.

In this set-up we derive the following results. First, non-synchronized planning equilibria can only exist if products are strategic substitutes while synchronized planning equilibria can only exist if products are strategic complements. This result is in sharp contrasts to the predictions obtained by models that suppose commitment power of firms. As shown by Maskin and Tirole (1987; 1988b) and Lau (2001), in these models strategic complementarity leads to non-synchronization while strategic substitutability tends to lead to synchronization. The reason is that in the case in which products are strategic complements and decisions are synchronized each firm has an incentive to undercut the price of its rival which leads to low prices and profits. By contrast, in a sequential game the overall level of prices is higher. Thus, firms choose non-synchronization in equilibrium. The reverse argument holds if products are strategic substitutes.

The intuition behind our result is rooted in two different effects, the strategic effect and the externality effect. The strategic effect determines how a firm's incentive to plan at some instant changes if the rival plans at this instant. If firm i stays inattentive, its price is inaccurate compared to the full information price due to the variance of demand and costs. Now suppose that the rival firm j plans. If the demand realization is high, firm j sets a higher price than in case of non-planning. If products are strategic complements, firm i's optimal price is then larger than in the case in which firm j did not plan. By the same argument, if the demand realization is low, firm i's optimal price is lower than without planning of firm j. As a consequence, firm i's price when being inattentive becomes more inaccurate, which increases the incentive for firm i to plan. By the reverse argument, if products are strategic substitutes, planning of firm j lowers the incentive for firm i to plan. As a consequence, there is a tendency to synchronize planning decisions under strategic complementarity and to non-synchronize these decisions under strategic substitutability.

In addition, there is an effect that occurs because by acquiring information firm j exerts an externality on its rival. This is due to the fact that an informed firm's price reacts to the shocks and is thus a random variable. Now, by planning firm i induces firm j to change its price and therewith firm i can influence this externality. We refer to this effect as the "externality effect". The absolute value of this externality is, in expectation, the higher the larger is the difference between firm i's last planning date and the next planning date of firm j. Thus, firm i can reduce the extent of the expected externality by moving closer to firm j's planning date. This is the case because less time has elapsed since firm i's last planning date, which reduces the uncertainty about firm i's demand and cost. Now suppose that the externality is positive and products are strategic complements. In this case firm i prefers to set its planning dates relatively far away from the ones of firm j to increase the expected externality, thereby providing a tendency towards alternating planning. By a similar argument, if the externality is negative and products are strategic substitutes, there is a tendency towards synchronized planning decisions. In sum, we find that in our framework both effects in combination exclude any non-synchronous planning equilibria in case of strategic complementarity and any synchronous planning equilibria in case of strategic substitutability.

Interestingly, we show that for both classes of equilibria, alternating and synchronous planning, there exist multiple inattentiveness lengths that can be supported as an equilibrium. For synchronous planning this result is natural – and known from Hellwig and Veldkamp (2009) – since if both firms plan at the same instant, the objective function of each firm involves a discontinuity at this instant thereby giving rise to multiple equilibria. However, the result is new for the class of alternating equilibria. In this type of equilibrium firm j remains inattentive at a planning date of firm i. Thus, one may expect that firm i's incentive to exceed or shorten its inattentiveness period is the same. However, we show that this is not the case. The reason is that firm j expects firm i to set different planning dates in the future in the case in which firm i exceeded its inattentiveness lengths than in the case in which it shortened its inattentiveness length. Due to this, firm j will in the future react differently to the different changes of the inattentiveness length of firm i, which changes the objective function of firm i in a discontinuous way. As a consequence, multiple equilibria emerge even in the class of alternating equilibria.

We also characterize how the inattentiveness length changes with the degree of strategic complementarity. Here we find that the period lengths become shorter, the larger the degree of strategic complementarity. This is the case because if firm j chooses to plan it adjusts its price in the direction of the shock realizations, i.e., if demand or costs are high, firm j raises its price and vice versa. Now, by the argument explained above, if the degree of strategic complementarity rises, firm i's price when being inattentive becomes more inaccurate, and therefore, it has an incentive to plan earlier. As a consequence, the equilibrium inattentiveness lengths become shorter.

In our model we also characterize under which conditions an alternating planning equilibrium arises endogenously without imposing this structure of planning dates. To do so we allow one firm to set its first inattentiveness interval at a different length than the others which enables firm to reach a non-synchronized equilibrium. Thus, in this respect we go beyond most papers in the previous literature which impose that firms have already reached an alternating move structure and focus on the stationary equilibrium.

As mentioned, the previous literature on dynamic duopoly that analyzes if firms synchronize their decisions is mainly concerned how commitment power affects this decision. The seminal papers in this literature are the ones by Maskin and Tirole (1987; 1988a;b). The first two are on quantity competition (strategic substitutability) while the last one is on price competition (strategic complementarity). In the main parts of these papers Maskin and Tirole suppose that firms are committed to a particular price or quantity for two periods, and that the firms adjust their prices or quantities in a alternating manner. Maskin and Tirole then endogenize the timing by analyzing if firms indeed choose to adjust their variables in an alternating manner but keep the assumption that firms are committed for two periods. The authors find that this is indeed the case for price competition but not for standard quantity competition. The reason is that if products are strategic complements, the market is less competitive if one of the firms acts as the Stackelberg leader and the other one as the follower compared to the simultaneous move case. However, the reverse holds true for strategic substitutes.[3] Lau (2001) extends the price competition model by first allowing for differentiated goods and, second, by giving each firm the choice to set its commitment length for either one or two periods. He shows that even in this case firms prefer to move alternatingly and to be committed for two periods. By allowing for a commitment length of one period, Lau (2001) shows how firms can endogenously reach this alternating structure, i.e., one firm chooses a one-period commitment at the beginning and then sets a two-period commitment forever while the second firm chooses a two-period commitment from the beginning. Although the mechanism in these papers are very different to the ones we consider, the goal of the analyses – i.e., finding conditions for synchronization and non-synchronization – is similar. As mentioned, our framework yields opposing results compared to the literature relying on commitment.[4]

There are other papers that also address the question of synchronization versus non-synchronization but consider a continuum of players. These papers identify different aspects that can be of relevance for this decision. For example, Bhaskar (2002) develops a model with different industries each comprised of a continuum of firms that act non-strategically. There is aggre-

[3] De Fraja (1993) and Lau (1996) reach a similar conclusion in a model of wage setting in which the wages are fixed for two periods. Both papers find that the sum of the wage setters' profits is larger under staggered than under synchronized wage setting.

[4] For a general treatment of a dynamic duopoly with adjustment costs that provide an exogenous commitment, see Jun and Vives (2004).

gate strategic complementarity across industries but the degree of strategic complementarity within an industry is larger. Firms are committed to their price levels for two periods. Bhaskar (2002) shows that there is a (strict) Nash equilibrium that involves staggered price setting in which some firms adjust their prices in odd and others in even periods. This is the case since each firm prefers to set its price together with its industry rivals although the aggregate number of firms that adjusts its price in this periods may be relatively small. In a different framework Ball and Romer (1990) consider a model with aggregate and firm-specific shocks in which the firm-specific shocks arrive at different times for different firms, and firms can adjust their prices every two periods. They show that due to the difference in shock arrival, staggered price setting is a Nash equilibrium in which each firm adjusts its price at the time its demand is hit by a shock.[5]

The way we model rational inattentiveness of firms was developed by Reis (2006b).[6] He analyzes the optimal length of a monopolist's inattentiveness period and derives an approximate solution in a general setting.[7] He also tests the model's prediction by using US inflation data and finds that his recursive state-dependent approach fits the data better than previous state-contingent models do. The paper that is closest to ours is the one by Hellwig and Veldkamp (2009). They answer the question under which condition firms want to acquire the same information as their rivals. To do so they analyze a model with a continuum of firms in which costly information acquisition and processing is modeled as in Reis (2006b). The objective of each firm is to set its price close to a target price that consists of a weighted average of the shock realization and the average price of competitors. The higher is the degree of strategic complementarity between firms, the larger is the weight on the average price in this target price. In this framework Hellwig and Veldkamp (2009) show that there is a unique staggered planning equilibrium for any degree of strategic complementarity while there are multiple synchronized planning equilibria that only exist if prices are strategic complements. In contrast to Hellwig and Veldkamp (2009), we consider a duopoly model instead of a competitive monopolists model with a continuum of firms. In addition, the objective function of firms in our model is substantially different since we consider a standard duopoly model without a target price. Therefore, the strategic incentives in our model are different, e.g., the externality effect described above cannot be present in their model due

[5]For a paper with a similar structure, see Ball and Cecchetti (1988).

[6]For a model that focusses on rational inattentiveness of consumers, see Reis (2006a).

[7]For an extension of this analysis, see Jinnai (2007).

to the assumption of a continuum of firms. We also obtain different results concerning the alternating planning equilibrium, i.e., we find that there are multiple alternating planning equilibria while in their model it is unique, and show that the comparative static result concerning the degree of strategic complementarity are opposite to theirs. We will explain the differences and similarities to work of Hellwig and Veldkamp (2009) in detail repeatedly throughout this chapter.

This chapter proceeds as follows. The next section provides an easily accessible two-stage information-choice-then-pricing game that sheds light on how the nature of product market competition and uncertainty about the shocks determine whether the duopolists' decisions to acquire information are strategic substitutes or complements. Section 2.3 presents the dynamic model in which firms first choose their planning dates and then compete on the product market, and derives the conditions for existence of an alternating and a synchronous planning equilibrium, respectively. Section 2.4 concludes.

2 Two-Stage Information-Choice-then-Pricing Game

There are two firms denoted by 1 and 2 that produce differentiated goods and compete in prices. Each firm i faces the demand function

$$q_i = \alpha + \theta - p_i + \gamma p_j, \quad i \neq j,\ i,j = 1,2,$$

where the intercept α and the degree of strategic interaction γ are known constants, with $\alpha > 0$ and $-1 \leq \gamma \leq 1$. If $\gamma > 0$, firms produce substitutable goods and strategy variables are strategic complements, if $\gamma < 0$, firms produce complementary goods and strategy variables are strategic substitutes, and if $\gamma = 0$, firms' demands are unrelated. Demand is stochastic, which is captured by the random variable θ that represents a common shock to each firm's demand. Without loss of generality we assume that $E[\theta] = 0$ and $Var[\theta] = \sigma_\theta^2 > 0$. Each firm faces marginal production costs that consist of a deterministic part, c, and a stochastic part, ξ, which are also common to both firms.[8] We assume that $E[\xi] = 0$ and $Var[\xi] = \sigma_\xi^2 > 0$. Thus, the production costs of firm i are given by of $q_i(c+\xi)$. The two shocks can be correlated

[8]Arguably, the situation of common demand and cost shocks is a relatively realistic scenario since both firms are in the same market and are likely to procure the input from the same supply industry.

with correlation coefficient ρ. This implies that $E[\theta\xi] = \rho\sigma_\theta\sigma_\xi$. Therefore, the profit function of firm i for the case of full information, that is, when firm i knows the realization of both shocks, is given by

$$\Pi_i = (p_i - c - \xi)(\alpha + \theta - p_i + \gamma p_j). \tag{1}$$

Before competing in the product market each firm chooses whether to acquire information about the realizations of the shocks. The decision to acquire information comes at a cost of $K > 0$ for each firm.[9]

The timing of the game is as follows: In the first stage, both firms decide independently of each other whether to acquire information or not, and these decisions become publicly observable. In the second stage, firms choose their optimal prices conditional on the information choices in the first stage.

This timing and information structure is suitable to elucidate the effects that are at work in the dynamic game in an accessible way. In order to focus on the interplay of firms' information acquisition decisions, we assume in this two-stage framework that a firm knows the information status of its rival. A justification for the assumption in the two-stage game is that planning is likely to be a process that may need consultation of outside agencies and where, in large firms, several managerial layers are involved. Thus, firms may not be able to keep this decision secret.[10] It is important to note that this assumption is not needed in the dynamic framework in which a firm (more realistically) observes the history of the game when acquiring information but not the simultaneous planning choice of its rival.

We solve the game by backward induction and look for perfect Bayesian equilibria. In the following we denote by D_i the choice of firm i to acquire information in the first stage: $D_i = 1$ if firm i chooses to become informed and $D_i = 0$ otherwise. In the second stage three different scenarios can occur conditional on D_i and D_j, $i \neq j$, $i, j = 1, 2$: both firms are uninformed, $(D_i = 0, D_j = 0)$, both are informed $(D_i = 1, D_j = 1)$, and an asymmetric situation where firm i is informed while firm j is not $(D_i = 1, D_j = 0)$. We consider these

[9] For a model that considers costless information acquisition but in which firms can share their acquired information, see e.g., Raith (1996).

[10] This two-stage game structure is also standard in several oligopoly models. A prominent example is Singh and Vives (1984) in which firms first publicly announce their strategy variable (price or quantity) and then compete in the chosen strategy variables.

scenarios in turn. In the following we denote by $p_i(D_i, D_j)$ and $\Pi_i(D_i, D_j)$, the equilibrium price and the equilibrium profit of firm i for given information choices.

Case 1: $D_i = 0, D_j = 0$

First, consider the case in which no firm is informed. Since $E[\theta] = E[\xi] = 0$ and $E[\theta\xi] = \rho\sigma_\theta\sigma_\xi$, from (1) we have that the expected profit of firm i can be written as

$$E[\Pi_i] = (p_i - c)(\alpha - p_i + \gamma p_j) - \rho\sigma_\theta\sigma_\xi.$$

Deriving the Nash equilibrium yields equilibrium prices of

$$p_i(0,0) = p_j(0,0) = p(0,0) = \frac{\alpha + c}{2 - \gamma}.$$

Inserting these prices into the profit function, we obtain the expected profit in case that both firms abstain from acquiring information in the first stage:

$$\Pi_i(0,0) = \Pi_j(0,0) = \Pi(0,0) = \frac{(\alpha - c(1-\gamma))^2}{(2-\gamma)^2} - \rho\sigma_\theta\sigma_\xi.$$

Case 2: $D_i = 1, D_j = 1$

Second, we look at the case in which both firms choose to acquire information. In this case, the realizations of θ and ξ are observed by both firms. Therefore, in the second stage each firm maximizes

$$\Pi_i = (p_i - c - \xi)(\alpha + \theta - p_i + \gamma p_j).$$

Deriving the Nash equilibrium yields equilibrium prices of

$$p_i(1,1) = p_j(1,1) = p(1,1) = \frac{\alpha + c}{2 - \gamma} + \frac{\theta + \xi}{2 - \gamma}. \tag{2}$$

Substituting these prices into the profit function, simplifying and taking expectations from the perspective of stage 1 we obtain the expected profit in the case in which both firms choose to acquire information:

$$\Pi_i(1,1) = \Pi_j(1,1) = \Pi(1,1) = \frac{(\alpha - c(1-\gamma))^2}{(2-\gamma)^2} + \frac{\sigma_\theta^2 + \sigma_\xi^2(1-\gamma)^2 - \rho\sigma_\theta\sigma_\xi(2-\gamma)}{(2-\gamma)^2}.$$

Case 3: $D_1 = 1, D_2 = 0$

Finally, we consider the asymmetric case in which one firm is informed while the other one is not. Let us denote by firm 1 the firm that is informed and by firm 2 the one that abstained from acquiring information. In the second stage the profit function of firm 1 is then given by

$$\Pi_1 = (p_1 - c - \xi)(\alpha + \theta - p_1 + \gamma p_2)$$

while the expected profit of firm 2 is

$$E[\Pi_2] = E[(p_2 - c - \xi)(\alpha + \theta - p_2 + \gamma p_1)].$$

The reaction functions of firm 1 and firm 2 can then be written as

$$p_1(p_2) = \frac{\alpha + c + \theta + \xi + \gamma p_2}{2}$$

and

$$p_2(E[p_1]) = \frac{\alpha + c + E[\theta] + E[\xi] + \gamma E[p_1]}{2},$$

respectively. Since firm 1 is informed, it conditions its price on the shock realizations, while firm 2 cannot do so due to its lack of information. As a consequence, firm 2 must form expectations about the shock realizations and about the price of firm 1. This is due to the fact that this price is a random variable for firm 2 because it depends on the shock realizations which firm 2 cannot observe. Solving for the Nash equilibrium and using that $E[\theta] = E[\xi] = 0$ we obtain

$$p_1(1,0) = \frac{\alpha + c}{2 - \gamma} + \frac{\theta + \xi}{2} \tag{3}$$

and

$$p_2(0,1) = \frac{\alpha + c}{2 - \gamma}.$$

Thus, from the perspective of firm 2 the expected equilibrium price of firm 1 is $E[p_1] = (\alpha + c)/(2 - \gamma)$.

We can now determine the profits of both firms. Inserting the equilibrium prices into the profit function of firm 1 and taking expectations we obtain

$$\Pi_1(1,0) = \frac{(\alpha - c(1-\gamma))^2}{(2-\gamma)^2} + \frac{\sigma_\theta^2 + \sigma_\xi^2 - 2\rho\sigma_\theta\sigma_\xi}{4}.$$

Since firm 2 is the uninformed firm its profit can be written as

$$E[\Pi_2] = (p_2 - c)\left(\alpha - p_2 + \gamma E[p_1]\right) - E[\xi(\theta + \gamma p_1)]. \tag{4}$$

It is evident from the last term on the right-hand side, $E[\xi(\theta + \gamma p_1)]$, that the covariance between ξ and p_1 affects firm 2's expected profits. This is due to the fact that p_1 depends on the realizations of θ and ξ and, therefore, $E[\xi p_1]$ depends on the variance of ξ and the covariance between θ and ξ. Inserting the equilibrium prices into (4) yields the expected profit of firm 2:

$$\Pi_2(0,1) = \frac{(\alpha - c(1-\gamma))^2}{(2-\gamma)^2} + \frac{\gamma(\sigma_\theta\sigma_\xi\rho + \sigma_\xi^2) + 2\rho\sigma_\theta\sigma_\xi}{2}. \tag{5}$$

We are now in a position to proceed to the information acquisition stage.

Information acquisition stage

We set out by determining the benefit from acquiring information for a firm conditional on the information acquisition decision of its rival. Let $\Delta(1) := \Pi_i(1,1) - \Pi_i(0,1)$ and $\Delta(0) := \Pi_i(1,0) - \Pi_i(0,0)$. Suppose that firm j is informed. Then firm i's benefit from acquiring information is

$$\Delta(1) = \frac{2(\sigma_\theta^2 + \rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - \gamma^2(2-\gamma)(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)}{2(2-\gamma)^2}. \tag{6}$$

If instead firm j is not informed, firm i's benefit is given by

$$\Delta(0) = \frac{\sigma_\theta^2 + \sigma_\xi^2 + 2\rho\sigma_\theta\sigma_\xi}{4} > 0. \tag{7}$$

Taking the difference between (6) and (7), we see that the benefit of information acquisition is larger in the case in which the other firm is also informed if and only if

$$\frac{\gamma\left(4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)-\gamma(\sigma_\theta^2+6\rho\sigma_\theta\sigma_\xi+5\sigma_\xi^2)+2\gamma^2(\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)\right)}{4(2-\gamma)^2}>0. \tag{8}$$

Rearranging (8) yields the following result:

Proposition 1 *Information acquisition decisions are strategic complements, i.e.,* $\Delta(1)-\Delta(0)>0$*, if and only if*

$$\gamma>0 \quad \text{and} \quad \rho>\max\left[-\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)},-1\right] \tag{9}$$

or

$$\gamma<0 \quad \text{and} \quad \rho<\max\left[-\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)},-1\right]. \tag{10}$$

Information acquisition decisions are strategic substitutes, i.e., $\Delta(1)-\Delta(0)<0$*, if and only if neither* (9) *nor* (10) *hold.*

Proposition 1 implies that, if (9) or (10) hold, there is strategic complementarity in information acquisition decisions. This implies that there exists a range of acquisition costs K, i.e.,

$$\frac{\sigma_\theta^2+\sigma_\xi^2-2\rho\sigma_\theta\sigma_\xi}{4}<K<\frac{2\sigma_\theta^2+\sigma_\xi^2(2-2\gamma^2+\gamma^3)-\rho\sigma_\theta\sigma_\xi(4-2\gamma^2+\gamma^3)}{2(2-\gamma)^2}, \tag{11}$$

such that either both firms are informed or no firm is informed.[11] If none of the two conditions hold, then information acquisition decisions are strategic substitutes. In this case there is a range of K, i.e.,

$$\frac{2\sigma_\theta^2+\sigma_\xi^2(2-2\gamma^2+\gamma^3)-\rho\sigma_\theta\sigma_\xi(4-2\gamma^2+\gamma^3)}{2(2-\gamma)^2}<K<\frac{\sigma_\theta^2+\sigma_\xi^2-2\rho\sigma_\theta\sigma_\xi}{4}, \tag{12}$$

in which an asymmetric equilibrium emerges, that is, one firm acquires information while the other one does not.

[11] Clearly, if K is larger than the term on the right-hand side of (11), it is optimal for both firms not to acquire information, while if K is smaller than the term on the left-hand side, acquiring information is optimal for both firms.

Whether an asymmetric equilibrium emerges or not depends on how a firm's decision to acquire information changes the other firm's incentive to become informed. There are two effects that change this incentive.

We first turn to the effect to which we will refer as the "strategic effect" in the remainder. Suppose that firm j has acquired information. Firm j adjusts its price in direction of the shock realizations, that is, it sets a high price if the shock to the demand or to the cost is high and a low one if the reverse holds true. This can be seen from the equilibrium prices of an informed firm, which are given by

$$p(1,1) = \frac{\alpha + c}{2 - \gamma} + \frac{\theta + \xi}{2 - \gamma} \quad \text{and} \quad p(1,0) = \frac{\alpha + c}{2 - \gamma} + \frac{\theta + \xi}{2}, \tag{13}$$

where $p(1,1)$ is the price for the case in which the other firm is informed as well while $p(1,0)$ is the price for the case in which the other firm is not informed. Now suppose for the sake of exposition that the shock to the cost is relatively small. Thus, firm j's price follows the demand shock. Since the shock is a common shock, firm i when being informed would also set a high price if θ is large and a low one if θ is small. Now, if products are strategic complements, i.e., $\gamma > 0$, firm i's optimal price is higher if θ is large compared to the case without strategic interaction because firm j sets a high price if θ is large. Similarly, if θ is low, firm i's optimal price with strategic interaction is lower than without. Taken together, this implies that the variance of firm i's full information price has gone up. As a consequence, firm i's price when being uninformed becomes more inaccurate, which renders information acquisition more profitable for firm i. Thus, if products are strategic complements, firm i's incentive to acquire information increases if firm j chooses to become informed due to the strategic effect. By a similar argument, if product are strategic substitutes, the reverse holds true, i.e., firm i's benefit from becoming informed is lower if firm j chooses to acquire information.

In addition to this strategic effect, there is a second effect that we will refer to as the "externality effect". This effect occurs because a firm exerts by acquiring information an externality on its rival. In the remainder we refer to this externality as the "information-induced externality". In order to illustrate it, suppose that firm j does acquire information. In this case firm j exerts – by planning – an externality on firm i. This is due to the fact that now firm j's price is a random variable. Thus, in the profit function of firm i firm j induces a covariance between ξ and p_j, $E[\xi p_j]$, and therewith an information-induced externality that is given by $-\gamma E[\xi p_j]$.

We first characterize how the sign of $E[\xi p_j]$ is determined by the sign of the correlation coefficient and the relation between σ_ξ and σ_θ. The next corollary follows immediately from inspection of an informed firm's optimal price.

Corollary 1 *Suppose that firm j acquires information. Then, $E[\xi p_j] > 0$, if either $\rho > 0$ or $\rho < 0$ and $\sigma_\theta < \sigma_\xi$. If $\rho < 0$ and $\sigma_\theta > \sigma_\xi$, then $E[\xi p_j] < 0$. If the shocks are uncorrelated, i.e., $\rho = 0$, then $E[\xi p_j] = 0$.*

By acquiring information firm i can alter the information-induced externality that firm j exerts. If firm i acquires information it affects firm j's price. As can be seen from comparing the two prices in (13), for $\gamma > 0$ the denominator of the second term in both expressions is smaller for $p(1,1)$ than for $p(1,0)$. This implies that firm j's equilibrium price reacts more strongly to the realization of the shocks if firm i is informed than if firm i is not informed. This is the case because, as explained above, if products are strategic complements each firm amplifies the reaction of the other firm. The opposite holds true for strategic substitutes.

We can now determine how this effect changes firm i's expected profit. Suppose for example that $\gamma > 0$ and that $E[\xi p_j] < 0$. Since the expected externality is given by $-\gamma E[\xi p_j]$, it is positive in this case. Now if firm i acquires information, we know from above that firm j's optimal price reacts more strongly on the realizations of the shocks. Since, from Corollary 1, $E[\xi p_j] < 0$ can only occur if $\sigma_\theta > \sigma_\xi$ and $\rho < 0$, we have that $E[\xi p_j]$ becomes more negative if firm i acquires information. Thus, firm i increases the positive information-induced externality if it acquires information. Therefore, in this case the strategic and the externality effect go in the same direction.

However, as can be seen from Proposition 1 if ρ is sufficiently negative, information acquisition decision are strategic substitutes although $\gamma > 0$. Thus, in this case both effects have to work in opposite directions and the externality effect has to dominate the strategic effect.

To see this note first that the situation

$$-1 \leq \rho < -\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)}$$

can only occur if $\sigma_\xi > \sigma_\theta$. From Corollary 1 we know that the covariance between ξ and p_j is positive if $\rho < 0$ and $\sigma_\xi > \sigma_\theta$. This implies that the information-induced externality is negative. Now, if products are strategic complements, firm i can ameliorate this externality by choosing to stay uninformed. In addition, if the correlation between the two shocks is highly negative, an informed firm's price in each state is relatively close to the optimal price of an uninformed firm. But this implies that the component on which the strategic effect operates is relatively small. Therefore, the externality effect dominates the strategic effect in this case, and information acquisition decision are strategic substitutes. A similar effect occurs if $\gamma < 0$, $\sigma_\xi > \sigma_\theta$ and ρ is highly negative. Then, information acquisition decision are strategic complements although prices are strategic substitutes.

The strategic effect of information acquisition is at the heart of the analysis of Hellwig and Veldkamp (2009). This is due to the fact that their framework considers a continuum of players which implies that the information-induced externality that a planning firm exerts on all other firms is negligible. Therefore, the externality effect cannot arise in their environment and it is solely the strategic effect that determines the mode of strategic interaction in the information choice stage.

We will now go on and analyze a fully dynamic model with an infinite number of planning dates and show that the main insights obtained in this simple two-stage game carry over and how the interplay between strategic and externality effect plays out in a dynamic framework.

3 Dynamic Information Choices

3.1 The Model

We consider two firms, each producing a non-storable good in continuous time. Each instant firms face the linear demand system

$$q_i(t) = \alpha + \theta(t) - p_i(t) + \gamma p_j(t), \tag{14}$$

$$q_j(t) = \alpha + \theta(t) - p_j(t) + \gamma p_i(t), \tag{15}$$

where again α and γ are known constants, with $\alpha > 0$ and $-1 \leq \gamma \leq 1$. The random variable $\theta(t)$ follows a Brownian motion with zero drift. More specifically, we assume that

$$\theta(t) = \sigma_\theta Z(t), \tag{16}$$

where $\sigma_\theta > 0$ and $Z(t)$ is a standard Wiener process. Without loss of generality, we set $\theta(0) = 0$. The instantaneous profit function of firm i is given by

$$\Pi_i(t) = \Big(\alpha + \theta(t) - p_i(t) + \gamma p_j(t)\Big)\Big(p_i(t) - (c + \xi(t))\Big), \tag{17}$$

where $c > 0$ denotes the firm's constant marginal cost of production and $\xi(t)$ denotes a cost shock. The cost shock's evolution is described by

$$\xi(t) = \sigma_\xi Y(t), \tag{18}$$

where $\sigma_\xi > 0$ and $Y(t)$ is a standard Wiener process. Again we set $\xi(0) = 0$ without loss of generality. The processes $Z(t)$ and $Y(t)$ are (potentially) correlated and we denote the instantaneous correlation coefficient by ρ, where $-1 \leq \rho \leq 1$. The statistical properties of the processes $\theta(t)$ and $\xi(t)$ are common knowledge.[12]

We follow Reis (2006a;b) and Hellwig and Veldkamp (2009) in the way in which we incorporate the feature of costly information processing into our setting: A firm incurs a fixed cost

[12]For reasons that will become clear when we turn to a firm's objective function we do not impose non-negativity constraints on (expected) prices or (expected) quantities.

$K > 0$ each time it chooses to process the available information, i.e. to plan, in order to (re-)compute its optimal actions. As a consequence, time will be endogenously partitioned into planning and non-planning dates. Let $D_i(n) : \mathbb{N}_0 \to \mathbb{R}^+$ denote the process that determines the dates at which firm i chooses to process information, with $D_i(0) = 0$, $i = 1, 2$.

We distinguish between firm 1 and firm 2. At the outset of the game both firms simultaneously and irrevocably choose the length of their inattentiveness periods. This choice cannot be observed by the rival. Firm 1 chooses its inattentiveness interval $d_1 \in \mathbb{R}^+$ which induces an infinite sequence of planning dates, given by $\{d_1, 2d_1, 3d_1, ...\}$. Firm 2 also chooses its inattentiveness interval $d_2 \in \mathbb{R}^+$ and, simultaneously, has to decide between two planning modes:

$$(A) \qquad \left\{\frac{d_2}{2}, \frac{3d_2}{2}, \frac{5d_2}{2}, ...\right\},$$

$$(S) \qquad \{d_2, 2d_2, 3d_2, ...\}.$$

A planning mode translates d_2 into a sequence of planning dates.

Thus, we restrict each firm to a once-and-for-all decision about the length of its respective inattentiveness interval. As will become clear below this restriction does not alter the equilibrium outcome of the game. This is due to the fact that we formulate the model so that it is stationary. Hence, it would indeed not be profitable for a firm to vary its optimal inattentiveness period, even if it had the possibility to do so at the outset.

The reason for giving firm 2 the opportunity to choose between two planning modes is the following. We are particularly interested whether equilibria exist in which firms plan in an alternating and sequential order. In our setting firms can only attain this planning pattern if firm 2 chooses planning mode (A) since we assume that both firms acquire information simultaneously at $t = 0$. In this mode, the first inattentiveness period of firm 2 is exactly half as long as its future ones. A consequence of the assumptions concerning the planning modes is that we require the firms to reach the alternating planning scenario in a single step. Therefore, if (stationary) alternating and sequential planning equilibria exist in this setting, they will very likely exist as well if we allow for more sophisticated convergence patterns.

At a planning date a firm sets a sequence of prices for each instant until its consecutive planning date. These prices are set so that they are measurable with respect to the available information. At a non-planning date a firm does not process information and sets the previously determined price.

Now, we characterize the information that is available for firm i at a planning date $D_i(n)$. We denote by $H_{D_i(n)}$ the set of all possible histories up to, but not including instant $D_i(n)$. An element $h_{D_i(n)} \in H_{D_i(n)}$ includes all past prices $p(s) = (p_1(s), p_2(s))$, $0 \leq s < D_i(n)$, all past planning dates $D_i(m) < D_i(n)$ and $D_j(k) < D_i(n)$, and all past shock realizations $\theta(s)$ and $\xi(s)$, $0 \leq s < D_i(n)$. The information available to a firm i which plans at instant $D_i(n)$ is

$$\Xi_{D_i(n)} = \left(h_{D_i(n)}, \theta(D_i(n)), \xi(D_i(n))\right).$$

Put differently, at a planning date the planning firm observes the current shock realizations in addition to the complete history. Since a firm is inattentive in between its planning dates, it cannot update its available information. Hence, $\Xi(t) = \Xi(D_i(n))$ at any instant t, such that $D_i(n) \leq t < D_i(n+1)$. As a consequence, the firms' histories are of unequal length if the latest planning dates of the firms differ.

We restrict firms to (pure) stationary Markov strategies with the firms' beliefs about the realizations of $\theta(t)$ and $\xi(t)$ as the state variables since these variables are the only payoff relevant ones. Because the latest planning dates of both firms determine their beliefs about the realization of the shocks at date t, the economy is at each instant t characterized by these planning dates. If firm i last planned at date $D_i(n)$, it enters date $t > D_i(n)$ with the information set $I_t = I_{D_i(n)} = \left(\theta_{D_i(n)}, \xi_{D_i(n)}\right)$. If firm i plans at the current date t, its new information set contains the shock realizations of the current date: $I_t = I_{D_i(n+1)} = \{\theta_{D_i(n+1)}, \xi_{D_i(n+1)}\}$. Thus, given that the latest planning dates of firm i and j are $D_i(n)$ and $D_j(m)$, respectively, the Markov state of firm i is

$$\omega_i = \begin{cases} \left(I_{D_i(n)}, I_{D_j(m)}\right) & \text{if} \quad D_i(n) > D_j(m), \\ I_{D_i(n)} & \text{if} \quad D_i(n) \leq D_j(m). \end{cases} \tag{19}$$

If firm i is better informed than firm j, i.e., if $D_i(n) > D_j(m)$, firm j's information set is included in the Markov state of firm i since firm i can observe the complete history. Instead, if firm i and firm j are equally informed or if firm i is worse informed than firm j, i.e., $D_i(n) \leq D_j(m)$, only the information set of firm i constitutes its Markov state. Let Ω_i be the set of all possible Markov states. Then, a stationary Markov strategy p_i for firm i is defined as $p_i : \Omega_i \rightarrow \mathbf{R}$, that is, it maps the state space into actions.

We adopt the solution concept of Markov Perfect Bayesian Equilibrium (MPBE). The reason for using this solution concept is that it is inherent in the assumption of costly information processing that a firm cannot observe its rival's actions while it is inattentive. A feature of the MPBE is that a firm's beliefs about the future behavior of its rival are arbitrary once it observed an out-of-equilibrium action of its competitor. By restricting each firm's strategies to irrevocably choosing one inattentiveness interval at the outset of the game we get a grip on the arbitrariness of out-of-equilibrium beliefs to a certain degree. This is due to the fact that once firm i observes that firm j has chosen an inattentiveness length of d_j', although the equilibrium called for a length of d_j, firm i knows that firm j will set d_j' forever.

However, this construction cannot restrict the firm's beliefs at the planning dates at which it detects a deviation but does not observe its rival's actual out-of-equilibrium strategy. This happens, if e.g., firm i plans at some instant and observes that firm j has not planned yet although firm j should have done so in equilibrium. At these planning dates we assume that firm i believes that firm j chose an inattentiveness interval that induces the same consecutive planning date as the inattentiveness interval that was prescribed in equilibrium. Although, this is only one possible belief, we argue that it is among the most reasonable. This is due to the fact that this out-of-equilibrium belief formalizes mistakes of the following kind: in equilibrium the planning unit is supposed to meet every Wednesday. However, the announcement is mistaken and states that the meeting takes place every other Wednesday.

The objective of firm i is to minimize the loss function

$$\mathcal{L}_i = \mathbb{E}_0 \left\{ \sum_{n=0}^{\infty} \left(\int_{D_i(n)}^{D_i(n+1)} e^{-rt} \left(\Pi_i^{FI}(t) - \Pi_i(t) \right) dt + e^{-rD_i(n+1)} K \right) \right\}, \tag{20}$$

via prescribing an infinite sequence of planning dates at the outset of the game, taking as given the sequence of its rival. It follows from our formulation of the strategy space that firms differ in the way in which they determine their respective sequences: firm 1 induces its infinite sequence of planning dates by choosing one inattentiveness interval, denoted by d_1. Firm 2 establishes its sequence by simultaneously setting a planning mode in addition to an inattentiveness interval, denoted by d_2. In (20), $\Pi_i^{FI}(t)$ denotes the (hypothetical) full information profit that firm i would earn if it planned at instant t, for given actions of firm j. The firms' common discount rate is denoted by $r > 0$.

We set up the firms' problem in terms of a loss function which we construct by scaling a firm's instantaneous expected profit by the firm's (hypothetical) instantaneous expected full information profit. This objective function delivers the same results in terms of instantaneous optimal prices and the equilibrium pattern of planning dates as a formulation which considers only the firm's profit function. This is due to the fact that $\Pi_i^{FI}(t)$ is a constant at a given instant. One way to think about the instantaneous loss function is that it measures, for given actions of its rival, a firm's expected instantaneous opportunity cost of choosing its prescribed actions, i.e., information status and associated price, relative to processing information.

The merit of working with the loss and not with the profit function is that the former specification guarantees that the model is stationary. This is the case because a firm's expected loss is zero at any planning date, irrespective of the absolute time that elapsed between a planning date and $t = 0$. Thus, a firm's expected loss depends at each instant only on the time that passed since its latest planning date and its rival's actions. Therefore a firm's incentive to plan is invariant with respect to absolute time. To the contrary, a firm's expected instantaneous profit at a given instant depends on the absolute time distance between this instant and the starting point of the model. This is due to the fact that at the outset firms have to evaluate the sequence of expected profits that their respective sequence of planning dates delivers. Thus, firms have to form expectations from the perspective of instant $t = 0$. Since the second moments of the stochastic processes increase linearly in time, expected instantaneous profits and therewith the firms' incentive to plan depend on absolute time.

The fact that the model is stationary makes the analysis tractable. In addition, it implies that we can, without loss of generality, restrict the strategy space so that each firm irrevocably chooses exactly one inattentiveness interval. As mentioned before, given the stationarity of planning incentives, a firm would not find it optimal to set inattentiveness periods of unequal length, even if we explicitly allowed for this.

To sum up, at the outset, both firms observe the initial state $\theta_0 = 0$ and $\xi_0 = 0$. Each firm can then calculate the per-instant price equilibrium for any combination of planning dates. Given this, firm 1 chooses it inattentiveness interval d_1 to minimize the loss function $\mathcal{L}_1$, taking as given the sequence of firm 2, while firm 2 simultaneously chooses its inattentiveness interval d_2 and its planning mode in order to minimize the loss function $\mathcal{L}_2$, taking as given the sequence of firm 1.

3.2 Equilibrium Prices and Loss Functions

We solve the model by backward induction. First, we determine the optimal prices and corresponding expected losses for any combination of planning dates chosen by the firms at the outset. Second, we derive the equilibrium planning pattern and the respective inattentiveness intervals.

We set out by characterizing the firms' equilibrium prices. As will become evident below, each firm's best response is linear in the shock realizations. In conjunction with our assumption that the stochastic processes have zero drift, it will turn out that each firm's optimal price is constant over time until the subsequent planning date of any firm.

We start with the case in which firms' planning dates are asynchronous. Denote by v the latest and by v' the subsequent planning date of firm i, i.e., $v := D_i(n)$ and $v' := D_i(n+1)$, and by w the latest and by w' the next planning date of firm j, i.e., $w := D_j(m)$ and $w' := D_j(m+1)$.

Lemma 2 *Suppose that firm i planned for the last time at v and that firm j planned for the last time at date w, where $w < v$. Firm j believes (correctly) that firm i plans at v. In the Markov Perfect Bayesian Equilibrium, prices at $v \leq t < \min\{v', w'\}$ are*

$$p_i^\star(t) = \frac{\alpha + c + \theta(v) + \xi(v)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(v) - \theta(w) + \xi(v) - \xi(w)\Big), \tag{21}$$

$$p_j^\star(t) = \frac{\alpha + c + \theta(w) + \xi(w)}{2-\gamma}. \tag{22}$$

The firms' full information prices are

$$\begin{aligned} p_i^{FI}(t) &= \frac{\alpha + c + \theta(t) + \xi(t)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(t) - \theta(w) + \xi(t) - \xi(w)\Big), \qquad (23) \\ p_j^{FI}(t) &= \frac{\alpha + c + \theta(t) + \xi(t)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(t) - \theta(v) + \xi(t) - \xi(v)\Big) + \\ &\quad + \frac{\gamma^2}{4(2-\gamma)}\Big(\theta(v) - \theta(w) + \xi(v) - \xi(w)\Big). \qquad (24) \end{aligned}$$

Proof We start with the optimization problem of firm i. Firm i's expected instantaneous profit for $v \leq t < \min\{v', w'\}$ is

$$E\Big[\Big(\alpha + \theta(t) - p_i(t) + \gamma p_j(t)\Big)\Big(p_i(t) - (c + \xi(t))\Big)\Big| I_v, I_w\Big],$$

which can be written as

$$(\alpha + \theta(v) - p_i(t) + \gamma p_j(t))(p_i(t) - c) - \xi(v)(\alpha - p_i(t) + \gamma p_j(t)) - E[\theta(t)\xi(t)|I_v, I_w]. \quad (25)$$

From (25) it becomes evident that $p_j(t)$ is, from the perspective of firm i at v, a non-random variable. This is due to the fact that firm j's optimal price is based on information that is known to firm i at v. Differentiating (25) with respect to $p_i(t)$ yields firm i's reaction function

$$p_i(t) = \frac{\alpha + c + \gamma p_j(t) + \theta(v) + \xi(v)}{2}. \qquad (26)$$

Now we turn to firm j. The expected instantaneous profit of firm j at date t is

$$E\Big[\Big(\alpha + \theta(t) - p_j(t) + \gamma p_i(t)\Big)\Big(p_j(t) - (c + \xi(t))\Big)\Big| I_w\Big]. \qquad (27)$$

Firm j's information set contains all shock realizations up to and including its latest planning date w. Now, taking into account that firm j observed $\theta(w)$ and $\xi(w)$ when it planned for the last time, it follows from (16) and (18) that $E[\theta(t)|I_w] = \theta(w)$ and $E[\xi(t)|I_w] = \xi(w)$. Thus (27) can be represented as

$$\Big(\alpha + \theta(w) - p_j(t) + \gamma E[p_i(t)|I_w]\Big)\Big(p_j(t) - c\Big) - \xi(w)\Big(\alpha - p_j(t)\Big) - E\Big[\xi(t)\Big(\theta(t) + \gamma p_i(t)\Big)\Big| I_w\Big]. \qquad (28)$$

From the perspective of firm j with information set I_w, the price that firm i sets at time t is a random variable. This is due to the fact that firm i updated its information set for the last time at $v > w$. Thus, firm i acts on more recent information which firm j has to infer.

Differentiating (28) with respect to $p_j(t)$ and rearranging yields

$$p_j(t) = \frac{\alpha + c + \gamma E[p_i(t)|I_w] + \theta(w) + \xi(w)}{2}. \tag{29}$$

Now, we derive the belief that firm j holds about firm i's price at date t. Though firm j does not observe $\theta(t)$ and $\xi(t)$, it knows that firm i's reaction function is given by (26). Forming expectations about $\theta(t)$ and $\xi(t)$ reveals that firm j expects the following reaction function of firm i at date t:

$$E[p_i(t)|I_w] = \frac{\alpha + c + \gamma p_j(t) + \theta(w) + \xi(w)}{2}. \tag{30}$$

Solving (29) and (30) for $E[p_i(t)|I_w]$ and $p_j(t)$ yields

$$E[p_i(t)|I_w] = p_j^\star(t) = \frac{\alpha + c + \theta(w) + \xi(w)}{2 - \gamma}, \tag{31}$$

which is (22). Firm i knows firm j's rationale which implies that firm i knows that firm j sets its price according to (31). Then, inserting (31) in (26) yields (21).

Finally, we derive the full information prices. We start with $p_i^{FI}(t)$, the price that firm i would optimally set, if it planned at t. The full information profit of firm i at date t is given by

$$(\alpha + \theta(t) - p_i(t) + \gamma p_j(t))(p_i(t) - (c + \xi(t))) \tag{32}$$

Differentiating (32) with respect to $p_i(t)$ and rearranging yields

$$p_i(t) = \frac{\alpha + c + \gamma p_j(t) + \theta(t) + \xi(t)}{2}. \tag{33}$$

Now, inserting $p_j^\star(t)$ as given in (22) yields (23).

By the same logic the full information price of firm j is given by

$$p_j(t) = \frac{\alpha + c + \gamma p_i(t) + \theta(t) + \xi(t)}{2}. \tag{34}$$

Inserting $p_i^\star(t)$ as given in (21) yields (24). ■

Next, we characterize prices for the case of synchronous planning.

Lemma 3 *Suppose that firm j and firm i planned for the last time at date v, and each firm believes that the other also planned at v. Then, in the Markov Perfect Bayesian Equilibrium, prices at $v \leq t < \min\{v', w'\}$ are*

$$p_i^\star(t) = p_j^\star(t) = \frac{\alpha + c + \theta(v) + \xi(v)}{2-\gamma}, \tag{35}$$

while full information prices are

$$p_i^{FI}(t) = p_j^{FI}(t) = \frac{\alpha + c + \theta(t) + \xi(t)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(t) - \theta(v) + \xi(t) - \xi(v)\Big). \tag{36}$$

Proof Firm i's expected instantaneous profit at t is given by (25). Thus, firm i's reaction function is (26). Solving (26) for $p_i(t)$ using symmetry yields (35). Similarly the full information price is derived analogously to (23). ■

Before we move on with the analysis we point out an important observation that follows immediately from Lemma 2 and Lemma 3.

Corollary 2 *Suppose that the latest planning dates of firms i and j are v and w, respectively, with $v \geq w$. Then, firm j's optimal price path $\{p_j^\star(t) : v \leq t < w'\}$ remains constant until firm j's consecutive planning date.*

Put differently, a firm sets identical prices in two situations that are informationally not equivalent: equal and worse information. The reason is the following: If firm j has more outdated information than firm i, it forms expectations about the shock realizations that firm i will observe at its next planning date. Since the stochastic processes have zero drift, firm j's best estimates of future shock realizations are the ones that it observed at its latest planning date. Thus, from the perspective of firm j, the firms' information sets are identical in expectation. In addition, the optimal price of each firm is linear in the shock realizations.

Therefore, firm j expects firm i to set the same price as in the case in which firm j has the same information as firm i. This implies that firm j sets the same price in both situations.

After having characterized the optimal and full information prices, we are in a position to derive the instantaneous expected losses under synchronous and asynchronous planning. We obtain the following result:

Lemma 4 *Suppose that firm i planned for the last time at v and that firm j planned for the last time at w, where $w \leq v$. Firm j (correctly) believes that firm i plans at v. The instantaneous expected loss of firm i at instant t with $v \leq t < \min\{v', w'\}$ is given by*

$$L_i := E\Big[\Pi_i^{FI}(t) - \Pi_i(t)|I_v, I_w\Big] = \frac{1}{4}\Big(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(t-v), \tag{37}$$

and the instantaneous expected loss of firm j for $v \leq t < \min\{v', w'\}$ is given by

$$L_j := E\Big[\Pi_j^{FI}(t) - \Pi_j(t)|I_w\Big] = \frac{1}{4}\Big(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(t-w) + \frac{\gamma(4+\gamma)}{16}\Big(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(v-w). \tag{38}$$

Proof First, we derive firm i's profit under full information. Using (23) and (21) in (32) yields

$$\Pi_i^{FI}(t) = \frac{1}{4}\Big(\alpha - c + \theta(t) - \xi(t) + \frac{\gamma(a + c + \theta(w) + \xi(w))}{2-\gamma}\Big)^2.$$

Thus, the expected full information profit of firm i at instant t with $v \leq t < \min\{v', w'\}$ is given by

$$E\Big[\Pi_i^{FI}(t)|I_v, I_w\Big] = \frac{1}{4}\Big(\sigma_\theta^2 - 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(t-v) + \chi, \tag{39}$$

where

$$\chi := \left(\frac{\alpha + \theta(v) - c - \xi(v) + \gamma c - \frac{\gamma}{2}\Big(\theta(v) - \xi(v) - (\theta(w) + \xi(w))\Big)}{2-\gamma}\right)^2.$$

Next, using (21) and (22) in (25) yields firm i's expected profit if it planned for the last time at $v \leq t$:

$$E\Big[\Pi_i(t)|I_v, I_w\Big] = -\rho\sigma_\theta\sigma_\xi + \frac{(\theta(v) - \xi(v))^2}{4} + \chi. \tag{40}$$

Subtracting (40) from (39) yields (37).

Now we turn to firm j. Using (21) and (24) in the full information profit that is given by

$$(\alpha + \theta(t) - p_j(t) + \gamma p_i(t))(p_j(t) - (c + \xi(t)))$$

and taking expectations from the perspective of date w yields

$$\begin{aligned} E\Big[\Pi_j^{FI}(w)|I_w\Big] &= \frac{1}{4}\Big(\sigma_\theta^2 - 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(t-w) + \frac{\gamma}{4}\Bigg(\sigma_\theta^2 - \sigma_\xi^2 + \frac{\gamma}{4}\Big(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)\Bigg)(v-w) \\ &+ \left(\frac{\alpha + \theta(w) - c - \xi(w) + \gamma(c + \xi(w))}{2-\gamma}\right)^2. \end{aligned} \tag{41}$$

If instead firm j planned for the last time at w, then its expected profit at t is given by

$$\begin{aligned} E\Big[\Pi_j(t)|I_w\Big] &= \Big(\alpha + \theta(w) - p_j(t) + \gamma E[p_i(t)|I_w]\Big)(p_j(t) - c) \\ &- \xi(w)\Big(\alpha - p_j(t)\Big) - E\Big[\xi(t)\Big(\theta(t) + \gamma p_i(t)\Big)|I_w\Big] \end{aligned} \tag{42}$$

We already derived that

$$E_j[p_i(t)|I_w] = p_j(t) = \frac{\alpha + c + \theta(w) + \xi(w)}{2-\gamma}.$$

Using

$$p_i(t) = \frac{\alpha + c + \theta(v) + \xi(v)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(v) - \theta(w) + \xi(v) - \xi(w)\Big),$$

in (42) and taking expectations delivers that

$$\begin{aligned} E\Big[\Pi_j(t)|I_w\Big] &= -\rho\sigma_\theta\sigma_\xi(t-w) - \frac{\gamma}{2}\Big(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi\Big)(v-w) \\ &+ \left(\frac{\alpha + \theta(w) - c - \xi(w) + \gamma(c + \xi(w))}{2-\gamma}\right)^2. \end{aligned} \tag{43}$$

Subtracting (43) from (42) yields (38). ■

After having characterized the expected instantaneous loss functions, we can determine how a firm affects the loss of its rival via planning. A comparison of (37) and (38) then reveals the following result:

Corollary 3 *Suppose that the latest planning dates of firms i and j are v and w, respectively, with $w \leq v$, and that firms i and j plan for the next time at v' and w', respectively, with $v < w' < v'$. At w' firm i's instantaneous expected loss increases discretely if the firms' strategy variables are strategic complements, i.e., if $\gamma > 0$. If firms compete in strategic substitutes, i.e., $\gamma < 0$, the instantaneous expected loss of firm i decreases by a discrete amount at w'.*

The intuition for this result is that in the considered situation only the strategic effect is at work. This is the case because the strategic effect, unlike the externality effect, is triggered solely by the rival's planning decision: when firm j plans, it changes its price in accordance with the realizations of the shocks. Whether this move decreases or increases the inaccurateness of the price that the uninformed firm i sets at this instant depends on the mode of strategic interaction: the inaccurateness increases if $\gamma > 0$, whereas it decreases if $\gamma < 0$. Suppose for example that $\gamma > 0$. In this case firm j raises its price if the realizations of θ and ξ are large. But since the shocks are common to both firms, this implies that, due to the strategic complementarity, firm i's full information price is now even larger as compared to the case in which firm j did not plan. By a similar argument, if the realizations of θ and ξ are low, firm i's full information price is lower than in the case in which firm j did not plan. Thus, the price that firm i sets when being uninformed becomes more inaccurate. The reason why the externality effect does not appear is that by construction of the loss function the action of the rival is held constant. Thus, firm i does not change the information-induced externality.

3.3 Structure of Planning Dates

After having determined the expected instantaneous losses for every pattern of planning dates, we now derive the equilibrium structure of planning dates and the respective inattentiveness periods. Here our aim is to characterize the conditions under which an alternating or synchronized equilibrium in planning dates exist. First, we turn to the analysis of an alternating planning equilibrium.

3.3.1 Alternating Planning

In our framework the issue of existence of an alternating planning equilibrium is particularly interesting. This is due to the fact that each firm's first planning date is at $t = 0$. Thus, in the beginning firms are by assumption in a synchronous planning pattern. As a consequence, we characterize under which conditions an alternating planning equilibrium arises endogenously without imposing this structure of planning dates.[13] Given that an alternating planning equilibrium exists, we determine whether it is unique.

We restrict our attention to the case of symmetric alternating planning equilibria, that is, to equilibria in which the inattentiveness period denoted by d is the same for both firms and each firm plans exactly in the middle of the inattentiveness period of the other firm. Put differently, the time that elapses between a planning date of firm i and a planning date of firm j is $d/2$.[14] Given our assumption on the strategy space such an equilibrium can only arise if firm 1's sequence of planning dates is given by $\{d, 2d, 3d, ...\}$ and firm 2 chooses planning mode (A), that is, its sequence of planning dates is

$$\left\{\frac{d}{2}, \frac{3d}{2}, \frac{5d}{2}, ...\right\}.$$

Before we proceed with the analysis we introduce some notation. Let

$$\begin{aligned}
\Lambda_1 &:= \frac{\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2}{4}, \\
\Lambda_2 &:= \frac{\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2}{(2-\gamma)^2}, \\
\Gamma_1 &:= \frac{2(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - \gamma^2(2-\gamma)(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)}{2(2-\gamma)^2}, \\
\Gamma_2 &:= \Lambda_2\left(1 - \frac{\gamma}{2}\right) - \frac{\gamma^2\left(2(1-\gamma)(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi) + \sigma_\xi^2 - \sigma_\theta^2\right)}{8(2-\gamma)^2}.
\end{aligned}$$

[13] The only other paper that pursues a similar goal is Lau (2001). However, as mentioned above, he uses a deterministic framework in which each firm can choose to be committed to a price for one or two periods. Thus, an asynchronous equilibrium implies that each firm chooses a commitment length of two periods and adjusts its price exactly in periods when the rival does not.

[14] Most papers in the literature focussed on this equilibrium adjustment pattern. For example, Hellwig and Veldkamp (2009) consider a similar planning structure in their model. Maskin and Tirole (1988a;b) and Lau (2001) also confine their attention to the case of symmetric commitment lengths.

Now, suppose that $d_a^\star$ is the length of an inattentiveness interval in an alternating planning equilibrium. We first analyze for which $d_a^\star$ neither firm 1 nor firm 2 have an incentive to deviate by marginally shortening or extending this interval to $d_a^\star - \epsilon$ or $d_a^\star + \epsilon$.

Lemma 5 *Suppose that an alternating planning equilibrium exists and let $d_a^\star$ denote the common equilibrium inattentiveness period. Firm 1 has no incentive to deviate marginally from $d_a^\star$ if and only if $\underline{d}_a^1 \leq d_a^\star \leq \bar{d}_a^1$, where $\underline{d}_a^1$ is the unique solution to*

$$\frac{\underline{d}_a'\Lambda_2(8-4\gamma+\gamma^2)}{8e^{r\underline{d}_a'}} - \frac{re^{r\underline{d}_a'}K}{(e^{r\underline{d}_a'}-1)^2} - \frac{\Lambda_1 e^{r\underline{d}_a'/2}+\Gamma_1}{r(e^{r\underline{d}_a'}-1)(e^{r\underline{d}_a'/2}+1)} + \frac{\Gamma_2\underline{d}_a'(2e^{r\underline{d}_a'}-1)}{e^{r\underline{d}_a'}(e^{r\underline{d}_a'}-1)^2} = 0, \tag{44}$$

and $\bar{d}_a^1$ is the unique solution to

$$\frac{\bar{d}_a'\Lambda_1(8+4\gamma+\gamma^2)}{8e^{r\bar{d}_a'}} - \frac{re^{r\bar{d}_a'}K}{(e^{r\bar{d}_a'}-1)^2} - \frac{\Lambda_1 e^{r\bar{d}_a'/2}+\Gamma_1}{r(e^{r\bar{d}_a'}-1)(e^{r\bar{d}_a'/2}+1)} + \frac{\Gamma_2\bar{d}_a'(2e^{r\bar{d}_a'}-1)}{e^{r\bar{d}_a'}(e^{r\bar{d}_a'}-1)^2} = 0. \tag{45}$$

Firm 2 has no incentive to deviate marginally from $d_a^\star$ if and only if $\underline{d}_a^2 \leq d_a^\star \leq \bar{d}_a^2$, where $\underline{d}_a^2$ is the unique solution to

$$\begin{aligned} &\frac{\Lambda_1\underline{d}_a'}{2e^{r\underline{d}_a'}} - \frac{\Lambda_1 e^{r\underline{d}_a'/2}(e^{r\underline{d}_a'/2}-1)}{r(e^{r\underline{d}_a'}-1)} - \frac{rKe^{r\underline{d}_a'}}{e^{r\underline{d}_a'}-1} - \frac{\Gamma_1(e^{r\underline{d}_a'/2}-1)}{r\left(e^{r\underline{d}_a'}-1\right)} - \\ &-\Gamma_2\underline{d}_a'\left(\frac{1}{e^{r\underline{d}_a'}(e^{r\underline{d}_a'}-1)} + \frac{\ln(1-e^{-r\underline{d}_a'/2})-\ln(1+e^{-r\underline{d}_a'/2})}{e^{-r\underline{d}_a'/2}}\right) = 0, \end{aligned} \tag{46}$$

and $\bar{d}_a^2$ is the unique solution to

$$\frac{\Lambda_1\bar{d}_a'}{2e^{r\bar{d}_a'}} - \frac{\Lambda_1 e^{r\bar{d}_a'/2}(e^{r\bar{d}_a'/2}-1)}{r(e^{r\bar{d}_a'}-1)} - \frac{\Gamma_1(2e^{r\bar{d}_a'/2}-2-r\bar{d}_a')}{2re^{r\bar{d}_a'/2}(e^{r\bar{d}_a'}-1)} - \frac{rKe^{r\bar{d}_a'}}{e^{r\bar{d}_a'}-1} = 0. \tag{47}$$

Proof See the Appendix.

In Lemma 5 we derive for each firm lower and upper bounds on the potential equilibrium inattentiveness period $d_a^\star$. More specifically, $d_a^\star$ must be larger than $\underline{d}_a^i$ since otherwise firm i would have an incentive to deviate to $d_a^\star+\epsilon$. Similarly, the upper bound $\bar{d}_a^i$ prevents deviations to $d_a^\star - \epsilon$. Since the two firms have to choose different planning sequences in order to attain the alternating pattern, the bounds for firm 1 and firm 2 differ.

An interesting observation is that for each firm the conditions that determine the upper and the lower bound on potential equilibrium inattentiveness lengths differ. Due to fact that firm j is inattentive at a planning date of firm i one may expect that firm i's incentive to marginally shorten or extend its inattentiveness length are identical. However, the proposition shows that this is not the case. The reasons for this result are inherent in the dynamic nature of the model and in the fact that firms are strategic players in our set-up. Consider the case in which firm 1 deviates from an inattentiveness length d to $d+\epsilon$. This deviation is detected by firm 2 at its next planning date. Due to our assumption on the firms' strategy spaces, firm 2 knows that firm 1's inattentiveness length will be $d+\epsilon$ for the rest of the game. This implies that the time that elapses between a planning date of firm 2 and the next planning date of firm 1 becomes longer and longer over time. Thus, after firm 2 detected firm 1's deviation, it knows that it will be the better informed firm over a longer time horizon, i.e., one which is longer than $d/2$. As a consequence, firm 2 is the worse informed firm for a shorter time. The reverse is true in the case in which firm 1 deviated to $d-\epsilon$. As can be seen from (21) and (22), a firm sets different prices when it is better informed than when it is worse informed than the competitor. This implies that firm 2 reacts, after it detects a deviation at its next planning date, differently to the two deviations. Hence, marginally shortening or prolonging the inattentiveness period are differently profitable for firm 1. In other words, the streams of expected losses that both deviations induce differ discretely. Therefore, the inattentiveness length that prevents one or the other deviation must differ as well.

A necessary condition for the existence of alternating planning equilibria is that for each firm $\underline{d}_a^i \leq \bar{d}_a^i$. The following lemma states the conditions under which this is the case.

Lemma 6 *For firm 1, $\underline{d}_a^1 < \bar{d}_a^1$ for all ρ and $\gamma \neq 0$, while $\underline{d}_a^1 = \bar{d}_a^1$ if $\gamma = 0$. For firm 2,*

$$\bar{d}_a^2 \gtreqqless \underline{d}_a^2 \quad \text{if} \quad \rho \gtreqqless \max\left[\hat{\rho}, -1\right],$$

where $\hat{\rho}$ is the unique solution to

$$-\frac{\Lambda_1}{2(1-e^{-rd_a})} - \frac{\Gamma_2\left(\ln(1+e^{-rd_a/2}) - \ln(1-e^{-rd_a/2})\right)}{e^{-rd_a/2}} = 0, \tag{48}$$

and d_a is the solution to (46) and (47) at $\rho = \hat{\rho}$.[15]

[15]Note that the left-hand side of (48) depends on ρ since Λ_1 and Γ_2 depend on ρ.

Proof See the Appendix.

The result stated in Lemma 6 shows that unless $\gamma = 0$ there exists a range of potential equilibrium inattentiveness periods for firm 1 that are robust against a marginal deviation. This is not true for firm 2: If ρ is sufficiently negative, this range might be empty. The reason for this result is the following. Generally, a negative correlation implies longer equilibrium inattentiveness intervals because the shocks partially offset each other. Now, this effect has a different impact on the bounds of firm 2 since its first planning interval is only half the length of its future ones under planning mode (A). Thus, firm 2 has a stronger incentive to extend its planning period than to reduce it. This implies that the lower bound shifts upward more strongly than the upper bound as the correlation between the shocks decreases. Now, if the correlation is sufficiently negative, it may happen that the range of potential equilibrium inattentiveness intervals is empty.

Moreover, it is important to note that for $\gamma = 0$, i.e., the case in which the firms' demands are unrelated, there is a range of potential optimal inattentiveness periods for firm 2, whereas for firm 1 there is a unique optimal solution. For firm 1 this result is a natural implication of the fact that $\gamma = 0$. Since the firm is a monopolist, there is no interaction with firm 2 on the product market. Therefore, its deviation incentives are the same for marginally extending or shortening its inattentiveness period. In this situation, i.e., for $\gamma = 0$, the firms are in principle alike. Thus, we should expect the same result for firm 2. However, in order to reach the alternating planning pattern we oblige firm 2 to choose planning mode (A). This implies that for firm 2, even though there is no strategic interaction with firm 1, there are multiple inattentiveness periods that are robust against a marginal deviation. This finding highlights the importance of considering non-marginal deviations in our set-up. In this particular case, i.e. $\gamma = 0$, we get the result that firm 2 will choose the same inattentiveness period as firm 1 and the planning mode (S).

Finally, we combine the existence conditions for both firms. This is done in the next lemma:

Lemma 7 *Marginal deviations do not preclude the existence of an alternating planning equilibrium if and only if* $\rho \geq \max[\check{\rho}, -1]$ *where* $\check{\rho} \geq \hat{\rho}$ *solves*

$$\Gamma_2 d_a \left(\frac{2}{e^{rd_a}(e^{rd_a}-1)^2} + \frac{\ln(1-e^{-rd_a/2}) - \ln(1+e^{-rd_a/2})}{e^{-rd_a/2}} \right) - \frac{\Lambda_1 d_a (8 + 4\gamma + \gamma^2 - e^{rd_a}(2+\gamma)^2)}{8e^{rd_a}(e^{rd_a}-1)} = 0, \tag{49}$$

and d_a *is the solution to* (46) *and* (47) *at* $\rho = \check{\rho}$.

Proof See the Appendix.

So, when only considering marginal deviations, we obtain that an alternating planning equilibrium exists if ρ is not too negative. The reason is the same as the one explained after Lemma 6. However, the critical ρ is now given by $\check{\rho}$, which is weakly larger than $\hat{\rho}$ as determined in Lemma 6. This is due to the fact that the upper bound of firm $1'$s equilibrium range, $\bar{d}_a^1$, is strictly below firm $2'$s upper bound, $\bar{d}_a^2$. As shown in Lemma 6, if ρ is sufficiently negative, $\underline{d}_a^2$ may become larger than $\bar{d}_a^2$. But the fact that $\bar{d}_a^1 < \bar{d}_a^2$ implies that $\underline{d}_a^2$ is larger than $\bar{d}_a^1$ as well. Thus, the requirement that an alternating planning equilibrium exists if and only if the ranges $[\underline{d}_a^1, \bar{d}_a^1]$ and $[\underline{d}_a^2, \bar{d}_a^2]$ overlap implies a tighter lower bound on ρ compared to the condition that ensures that firm $2'$s range is non-empty.

So far we only considered marginal deviations. Now, we turn to the analysis of non-marginal deviations. Here we consider deviations for firm 1 to period lengths of $d[1 \pm (l/m)]$, for natural numbers $l \leq m$. The reason why we confine the analysis to fractional deviations is the following. With fractions l/m, we can approximate any real number between zero and one arbitrarily closely. In addition, the stream of expected losses that is induced by the fractional deviations allows us to concisely characterize the bounds on the range of equilibrium inattentiveness periods. Similarly, for firm 2, we consider deviations to period lengths of $d[1 \pm (l/m)]$ both in the (S) and the (A) mode.

The following lemma states that for r close to 0 we can without loss of generality concentrate on one particular type of non-marginal deviation: firm 2 deviates to planning mode (S) but keeps $d_a^\star$. In the remainder, we will refer to this type of deviation as the "RTS deviation".[16]

[16]The acronym RTS abbreviates "return to synchronous" planning.

Lemma 8 *Any non-marginal deviation by firm 1 and firm 2 in which firm 2 remains in the (A) planning mode does not exclude potential alternating planning equilibria for r close to 0. Denote by $\mathbb{S}$ the set of potential alternating planning equilibria that survive the RTS deviation and by $\mathbb{S}'$ the set of potential alternating planning equilibria that survive deviations of the following type: Firm 2 deviates to the (S) planning mode and chooses $(1 + l/m)d$. Then, $\mathbb{S} = \mathbb{S}'$ for r close to 0.*

In particular, the RTS deviation does not exclude $d_a^\star$ if

$$rK \leq \frac{d_a^\star}{8e^{3rd_a^\star}(2-\gamma)^2}\Big(e^{5rd_a^\star/2}\left(4\gamma^2(\sigma_\theta^2 + \rho\sigma_\theta\sigma_\xi) + 2\gamma^3(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi) - 4\Lambda_1(\gamma^4 + 4)\right) \tag{50}$$

$$+8\Lambda_1 e^{2rd_a^\star}(2-\gamma)^2 + 4(1 + e^{rd_a^\star/2} - e^{rd_a^\star})\gamma^2\left(\Lambda_1\gamma^2 - (\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\gamma + \sigma_\xi^2 - \sigma_\theta^2\right)\Big).$$

Proof See the Appendix.

The result stated in Lemma 8 allows us, in case r is sufficiently close to 0, to concentrate on the deviation to the sequence $\{d_a^\star, 2d_a^\star, 3d_a^\star, ...\}$ by firm 2 when considering non-marginal deviations. This allows us to characterize the conditions for equilibrium existence in a concise way. It is intuitive that the deviation of firm 2 to the planning mode (S) with the optimal inattentiveness length of firm 1 eliminates many potential equilibria. This is due to the combination of the following two facts: First, firm 2 is per se more prone to deviate from the alternating planning equilibrium as its first inattentiveness period is only half the length of its future ones. Second, the RTS deviation constitutes the only possibility for firm 2 to unilaterally implement a synchronous planning pattern at a potential alternating equilibrium planning frequency.

The result that we can concentrate on the RTS deviation can be shown analytically for $r \to 0$ and, by continuity, this argument extends to r in a neighborhood of zero. However, computations suggest that even for $r > 0$ the RTS deviation excludes the largest set of potential equilibria.

Before we proceed we define ρ^+ as the ρ for which (50) holds with equality. Now, we combine the marginal and non-marginal "no-deviation" conditions in order to state the main result of this subsection:

Proposition 2

(i) For r close to zero, an alternating planning equilibrium does not exist if $\gamma \geq 0$.

(ii) Suppose $\gamma < 0$. For r close to zero, an alternating planning equilibrium exists if $\rho \geq \max[\check{\rho}, \rho^+]$, and the maximum range of equilibrium inattentiveness periods is given by $d_a^\star \in [\underline{d}_a^1, \bar{d}_a^1]$.

(iii) For $r \to 0$, the alternating planning equilibrium is unique, that is $\underline{d}_a^1 = \bar{d}_a^1 = d_a^\star$, and exists if and only if $\gamma < 0$ and

$$\rho \geq \max\left[-\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)}, -\frac{4(4-\gamma)(\sigma_\theta^2+\sigma_\xi^2)-\gamma^2(11\sigma_\xi^2-\sigma_\theta^2)+6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16-4\gamma-5\gamma^2+3\gamma^3)}, -1\right].$$

Proof See the Appendix.

The proposition consists of three parts. First, it states that an alternating planning equilibrium cannot exist if products are strategic complements. This is the case because the non-marginal deviation characterized in Lemma 8 eliminates all potential equilibria with $\gamma \geq 0$. The reason for this closely resembles the strategic effect described above, i.e., that an uninformed firm's optimal price becomes more inaccurate when the rival plans. Thus, for $\gamma > 0$ there is a tendency to synchronize planning decisions. This is particularly pronounced if the degree of strategic complementarity is large. In addition, if γ is relatively small in absolute value, firm 2 prefers a planning sequence with equal period lengths, that is, it prefers the (S) to the (A) mode since this would be its optimal choice if it was a monopolist. In conjunction, these two effects imply that for $\gamma > 0$ there exists no alternating planning equilibrium.

Parts (ii) and (iii) of the proposition state that if products are strategic substitutes, an alternating planning equilibrium exists if the correlation between the shocks is not too negative. The intuition behind this result is a combination of the strategic and the externality effect, similar to the one explained in Section 2.2. First, due to the strategic effect, if $\gamma < 0$ an uninformed firm's price becomes more accurate when the rival plans. Thus, there is an inherent tendency towards alternating planning. Second, by planning, firm j induces an externality on firm i. Suppose for example that $\sigma_\theta > \sigma_\xi$. In this case the informed firm j adjusts its price (in expectation) in the direction of the shock to the intercept. Thus, the covariance between p_j and the realization of the cost shock is negative, which implies, together with $\gamma < 0$, that firm j exerts a negative (expected) externality on firm i. Since the variance of the shocks increases linearly in the time that elapses after a planning date of firm i, this externality is the larger the longer the time distance between firm i's latest planning date and the next planning date of firm j. In order to reduce this externality, firm i is inclined to move closer to the planning date of firm j. Thereby, it destroys the potential equilibrium.

Interestingly, as can be seen from statement (iii), in the limit as r tends to zero, one of the conditions that ρ has to meet so that an alternating planning equilibrium exists is precisely the one for which information acquisition decisions are strategic substitutes in the example presented in Section 2.2. This shows that the interplay between the strategic and the externality effect is present in the dynamic game as well and that its implications resemble the ones given in the example.

Part (iii) of the proposition also states that the alternating planning equilibrium is unique for $r \to 0$. In this case the two marginal "no-deviation" conditions of player 1 are identical. The reason for this is the following: If the future is undiscounted, the difference between marginally extending and shortening the inattentiveness period vanishes since the time period for which this difference matters – the present – becomes negligible relative to the future. Of course, this is not the case if $r > 0$, i.e., in this case there are multiple alternating planning equilibria as stated in Part (ii) and the maximum range is the one that is bounded by the marginal "no-deviation" conditions of player 1.

The proposition also implies that $\gamma < 0$ is only a necessary but not a sufficient condition for the existence of an alternating planning equilibrium. For example, one can check numerically that if γ is close to zero, such an equilibrium does not exist since (50) can never be satisfied. However, our numerical computations confirm that the range of γ for which alternating planning equilibria exist is quite sizeable, even if r is strictly positive, and often starts at relatively small absolute values of γ, such as -0.1.

Having characterized the equilibria of the game and under which conditions they exist we are now in a position to determine how the equilibrium inattentiveness lengths change with the degree of strategic substitutability. This is of interest since it helps to discuss in more detail how strategic interaction shapes the equilibrium outcome given that there is a finite number of players. In the exposition we focus on the equilibria in which the bounds of the equilibrium range are determined by the marginal deviations of player 1. We obtain the following results:

Proposition 3 *Suppose that $\gamma < 0$ and ρ are so that the range of alternating planning equilibria is non-empty and bounded by the marginal "no-deviation" conditions of firm 1.*
(i) For $r \to 0$, $d_a^\star$ is strictly increasing in $|\gamma|$.

(ii) For r positive but small the equilibrium range of inattentiveness periods, $[\underline{d}_a^1, \bar{d}_a^1]$, becomes strictly larger as $|\gamma|$ increases.

Proof See the Appendix.

The first result states that, in the limit as r tends to zero, $d_a^\star$ increases as $|\gamma|$ increases. Put differently, if the degree of strategic substitutability rises, i.e., γ becomes more negative, firms choose longer inattentiveness lengths in equilibrium. Although we can only show this analytically for $r \to 0$, the result that the equilibrium range of inattentiveness periods shifts upward as $|\gamma|$ increases is for $r > 0$ computationally robust for most parameter constellations.

The comparative static effect with respect to the degree of strategic substitutability differs from the one in Hellwig and Veldkamp (2009). They find that an increase in the degree of strategic substitutability decreases equilibrium inattentiveness lengths. This difference stems from the fact that the degree of strategic substitutability has different implications on the impact of uncertainty in the two models. In their model a firm's objective is to minimize the expected distance between its price and a target price. The latter is a weighted average of the shock realization and the average of the other firms' prices where the weights are determined by the substitutability parameter. Now, as the degree of substitutability increases, the target price puts more weight on the shock realization. Ultimately, this implies that in the alternating planning equilibrium inattentiveness periods become shorter. To the contrary, in our model the equilibrium inattentiveness periods become longer as the degree of strategic substitutability increases. The intuition behind our result is the following. At its planning date firm j will adjust its price to the realization of the shocks. Since $\gamma < 0$ the strategic effect is so that firm j's planning decision renders an uninformed firm's price less inaccurate which reduces firm i's incentive to plan. This effect increases in the degree of strategic substitutability. As a consequence, the firms' inattentiveness length in an alternating planning equilibrium increases in the degree of strategic substitutability.

As mentioned above, one can demonstrate computationally that even if r is strictly positive, an increase in $|\gamma|$ increases both $\underline{d}_a^1$ and $\bar{d}_a^1$. The only exception can occur in the case in which both ρ and γ are sufficiently negative and $\sigma_\xi << \sigma_\theta$. In this case $\underline{d}_a^\star$ decreases in $|\gamma|$. This result is driven by the externality effect. By planning firm j exerts a negative (expected) externality on firm i, and firm i can ameliorate this externality by moving closer to the planning date(s) of firm j. Thus, firm i has a stronger incentive to set a marginally shorter inattentiveness length, and this incentive increases in $|\gamma|$. As a consequence, the lower bound falls as $|\gamma|$ increases.

Finally, the second result of Proposition 3 states that the range of equilibria widens as the degree of strategic substitutability increases. Thus, although both $\underline{d}_a^1$ and $\bar{d}_a^1$ usually increase as $|\gamma|$ increases, $\bar{d}_a^1$ increases by a larger extent than $\underline{d}_a^1$. The intuition behind this result can be easily gained from the discussion following Lemma 5. As pointed out there, the difference between the two bounds stems from the fact that firm j reacts in the future in a different way if firm i shortens or extends its inattentiveness interval. Clearly, the extent of this difference depends on the degree of strategic interaction, i.e., on $|\gamma|$. If $|\gamma|$ becomes larger, firm j reacts more strongly to each deviation. Ultimately, this increases the difference between the two bounds.

After having characterized the conditions for existence of an alternating planning equilibrium, we now turn to the analysis of synchronous planning equilibria.

3.3.2 Synchronized Planning

In order to characterize the set of inattentiveness intervals for which synchronous planning is potentially an equilibrium we set out by considering unilateral marginal deviations. More specifically, we analyze whether a firm has an incentive to deviate from a synchronous planning pattern with an inattentiveness interval of length d by either marginally shortening or extending the current inattentiveness period. The set of synchronous inattentiveness periods for which marginal deviations are not profitable is characterized in the following lemma.

Lemma 9 *Suppose that a synchronized planning equilibrium exists and let $d_s^\star$ denote a common synchronous equilibrium inattentiveness period. Then $d_s^\star$ is bounded above by $\bar{d}_s$, where $\bar{d}_s$ solves*

$$\bar{d}_s\Lambda_1 - rK - \left(\frac{e^{r\bar{d}_s} - 1}{re^{r\bar{d}_s}}\right)\Gamma_1 = 0, \tag{51}$$

and $d_s^\star$ is bounded below by $\underline{d}_s$, where $\underline{d}_s$ solves

$$e^{-r\underline{d}_s}\left(\underline{d}_s\Lambda_2 - \frac{\Lambda_1}{r}\right) - \frac{rK}{e^{r\underline{d}_s} - 1} + \frac{\underline{d}_s e^{-r\underline{d}_s}}{e^{r\underline{d}_s} - 1}\Gamma_1 = 0. \tag{52}$$

Proof See the Appendix.

It is evident from Lemma 9 that a synchronous planning equilibrium may exist only if $\bar{d} \geq \underline{d}$. We formulate the condition that has to be met so that a synchronous planning equilibrium potentially exists in terms of a threshold correlation stated in the following lemma.

Lemma 10 *If strategy variables are strategic substitutes and the intercept shock is more volatile than the cost shock, i.e., $\gamma < 0$ and $\sigma_\theta > \sigma_\xi$, then $\bar{d}_s \geq \underline{d}_s$ if $-1 \leq \rho \leq \hat{\rho}$, where $\hat{\rho}$ is bounded above by*

$$\rho' = -\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}. \tag{53}$$

If strategy variables are strategic complements and the intercept shock is less volatile than the cost shock, i.e., $\gamma > 0$ and $\sigma_\theta < \sigma_\xi$, then $\bar{d}_s \geq \underline{d}_s$ if $\rho \geq \max\{-1, \hat{\rho}\}$.
If $\gamma > 0$ and $\sigma_\theta > \sigma_\xi$ then $\bar{d}_s \geq \underline{d}_s$ for all $\rho \in [-1, 1]$.
Finally, if $\gamma < 0$ and $\sigma_\theta < \sigma_\xi$ then $\bar{d}_s \leq \underline{d}_s$ for all $\rho \in [-1, 1]$.

Proof See the Appendix.

Put differently, Lemma 10 states that if strategy variables are strategic complements and the intercept shock is less volatile than the cost shock, the synchronous equilibrium does not exist for a sufficiently negative correlation, that is for $-1 \leq \rho \leq \hat{\rho}$. In addition, if strategy variables are strategic substitutes and the intercept shock is more volatile than the cost shock, simultaneous planning can be an equilibrium if $-1 \leq \rho \leq \hat{\rho}$.

The reason for our result can be most easily seen by alluding to the two-stage game analyzed in Section 2.2: the upper bound of the threshold correlation coincides with the condition that identifies the parameter regions in which the externality effect overcompensates the strategic effect. Thus, if firms compete in strategic complements, the expected negative externality that firms exert on each other when planning jointly may be so large that one firm is, for every inattentiveness interval, better off by acquiring information either shortly before or after the other firm. To the contrary, if strategy variables are strategic substitutes, it may be profitable for a firm to synchronize its planning decision because this reduces the expected negative externality that it has to endure.

Lemma 9 and Lemma 10 identify and characterize the candidates for a stationary synchronized planning equilibrium. As mentioned before, the prerequisite in deriving the bounds on the optimal inattentiveness period was that for a given behavior of one firm the other firm may only deviate marginally from the candidate inattentiveness interval. Thus, it remains to analyze whether the candidate equilibria are robust to non-infinitesimal deviations.

Combining the marginal and non-marginal "no-deviation" conditions allows us to state the main result of this subsection:

Proposition 4 *A synchronized planning equilibrium exists for r close to zero if and only if $\gamma > 0$ and $\bar{d}_s > \underline{d}_s$. In such an equilibrium, any (common) inattentiveness period is given by $d_s^\star \in [d_s^l, d_s^u]$, where*

$$d_s^l = \frac{2\sqrt{6}(2-\gamma)\sqrt{Km^\star}}{\sqrt{\Theta(m^\star+1)}}, \tag{54}$$

$$d_s^u = \frac{8\sqrt{6}\sqrt{Km^\star}}{\sqrt{\Theta(m^\star+1)}}, \tag{55}$$

with

$$\begin{aligned}\Theta &= (\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)(2+m^\star)2\gamma^3-((7+5m^\star)\sigma_\xi^2+6\rho\sigma_\theta\sigma_\xi(m^\star+1)+\sigma_\theta^2(m^\star-1))\gamma^2\\ &\quad +4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)(m^\star-1)(\gamma(m^\star-1)+3),\\ m^\star &= \left[\sqrt{\frac{4(\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)\gamma^3+(\sigma_\theta^2-6\rho\sigma_\theta\sigma_\xi-7\sigma_\xi^2)+4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)(3-\gamma)}{\gamma(2(\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)\gamma^2-(\sigma_\theta^2+6\rho\sigma_\theta\sigma_\xi+5\sigma_\xi^2)\gamma+4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2))}}\right],\end{aligned} \tag{56}$$

that is, $m^\star$ is the closest integer to the right-hand side of (56).

Proof See the Appendix.

Proposition 4 states that a synchronous planning equilibrium cannot exist if products are strategic substitutes. Here, the non-marginal deviations eliminate all potential synchronous planning equilibria for $\gamma < 0$. Reversely, when products are strategic complements, the proposition implies that for $r \to 0$ the non-marginal deviations do not exclude the existence of a synchronous planning equilibrium whenever it is robust against the marginal deviations. Instead, the non-marginal deviations only tighten the upper and lower bounds of the range of equilibrium inattentiveness periods, that is, the range of synchronous equilibria shrinks from $[\underline{d}_s, \bar{d}_s]$ to $[d_s^l, d_s^u]$. By continuity, this argument extends to discount rates that lie in a neighborhood of zero.

Though we can show existence of the synchronous planning equilibrium analytically only for r in a neighborhood of zero, numerical simulations suggest that the result stated in Proposition 4 is robust for r strictly larger than zero. More specifically, it turns out that even for $r > 0$, d_s^l and d_s^u remain to be the tightest bounds on the range of common inattentiveness periods that constitute synchronous planning equilibria.

To sum up, our analysis shows that after combining marginal and non-marginal deviations synchronous planning equilibria exist only if $\gamma > 0$ while alternating planning equilibria exist only if $\gamma < 0$. Thus, the prediction of this model for the existence of synchronous and alternating planning equilibria is clear-cut. In addition, both types of equilibria only exist if the correlation between the demand and the cost shock is not too negative.

4 Conclusion

In this paper we considered an infinite-horizon dynamic duopoly model in which firms can choose to costly acquire and process information about the realization of a common demand and a common cost shock. We identify the effects that are decisive for the firms' choice to synchronize or stagger their planning decision. These are the strategic effect and the externality effect. The strategic effect determines if a firm's price, when being inattentive, becomes more or less inaccurate when the rival plans at the same instant. The externality effect determines how a firm, via planning, can change the expected externality that its rival induces. This externality effect is inherent in the nature of a model with a discrete number of players and cannot occur with a continuum of players. We show that due to the combination of these two effects, alternating planning equilibria exist only if products are strategic substitutes while synchronous planning equilibria exist only if products are strategic complements. In addition, we find that in both classes multiple equilibria with different inattentiveness lengths exist. Finally, we show that the equilibrium inattentiveness length tends to be shorter, the larger is the degree of strategic complementarity. This is the case because if firm j plans and adjusts its price in accordance with the shocks, firm i's price when staying inattentive becomes more incorrect, and so it has an incentive to plan earlier.

We restricted our attention to the case of common demand and cost shocks. As mentioned above, this is reasonable if firms operate in the same market area and procure inputs from the same suppliers. However, in some markets firms may face idiosyncratic shocks, i.e., because they buy inputs from different suppliers or, due to location differences, their demands are affected in different ways. An interesting direction for future research would be to allow for such idiosyncratic shocks. In this case, when planning, a firm observes its own shock realizations but cannot fully infer the shock realizations of its rival from this information. It is of interest to analyze how this affects the extent of the strategic and the externality effect and, more specifically, if the equilibrium inattentiveness lengths become shorter or longer, and how this is affected by the degree of strategic complementarity.

A second extension of our analysis could be to consider the case in which firms can coordinate their planning decisions. One could imagine for example that firms are still rivals at the product market at each instant but have sourced out their planning decisions to a third party that acts as a consultant and is payed according to profits. In this case, the third party may act as a collusion device in planning dates. It is then possible to determine if and how the structure of planning dates that maximizes joint profits differs from the equilibrium one. This can give new insights into the interplay of the strategic and the externality effect, in particular, if the effects are favorable or detrimental to firms.

As mentioned in the introduction, the analysis presented in this paper is the first to study the implications of rational inattention in a model with a finite number of players. To do so we used the formulation of inattentiveness developed in Reis (2006b). In this formulation firms, when deciding to be attentive, become aware of all relevant information. Although extreme, this feature has the advantage of making the model highly tractable. However, a different theory of rational inattention was proposed by Sims (2003). In his model agents cannot attend to all information because of limited capacity. This model was used e.g., by Moscarini (2004) to study optimal sampling of a decision maker and by Mackowiak and Wiederholt (2009) who analyze a model with an infinite number of firms which face an aggregate and an idiosyncratic shock but cannot fully absorb both shocks due to limited capacity. A similar idea as in Mackowiak and Wiederholt (2009) can also be incorporated in our structure, i.e., firms can only absorb the cost or the demand shock at each instant but not both. It is then possible to analyze to which shock firms pay more attention and how this is affected by the competitiveness of the market. However, such a model does not allow us to draw conclusions about the optimal inattentiveness period. Nevertheless, it is of interest to compare the two approaches and to determine if the results concerning the mode of strategic interaction and the equilibrium structure of planning dates obtained in this chapter are also valid under this alternative formulation of rational inattention.

5 Appendix

Proof of Lemma 5

Let us consider the following candidate equilibrium in which firms plan in an alternating and sequential order: The inattentiveness length of each firm is given by $d \in \mathbb{R}^+$ and firm i plans exactly in the middle of firm j's inattentiveness period, that is the time that elapses between a planning date of firm i and a planning date of firm j is $d/2$. This sequence of planning dates can only be an equilibrium if an infinitesimal deviation is not profitable for firm i. There are two forms of infinitesimal deviations. The first is that firm i chooses a longer inattentiveness period, that is it deviates to $d' = d + \triangle$, with $\triangle > 0$. The second infinitesimal deviation is that firm i chooses a shorter inattentiveness period $d'' = d - \triangle$.

Firm 1

We start with firm 1. In order to derive the "no-deviation" conditions we have to compare firm $1'$s expected loss from following the proposed equilibrium sequence with the expected losses from the sequences $\mathcal{D}' = \{d', 2d', 3d', ...\}$ and $\mathcal{D}'' = \{d'', 2d'', 3d'', ...\}$.

Expected loss for $\mathcal{D}$

When determining the expected loss from following the equilibrium strategy we have to distinguish between the case where firm 1 is the better informed firm since it was the last one that planned, and the case where it is the worse informed firm since firm 2 was the last to plan. In the first case we know from Lemma 4 that firm $1'$s expected instantaneous loss at any instant t, with $nd \leq t < (n + 1/2)d$, for all $n \in \mathbb{N}_0$, is given by

$$E[\mathcal{L}_1^e | I_0] = \Lambda_1 \tau,$$

where τ denotes the time that elapsed since the last planning date, that is $\tau = t - nd$, for all $n \in \mathbb{N}_0$. In the second case, we know from Lemma 4 that firm $1'$s expected instantaneous loss at any instant t, with $(n + 1/2)d \leq t < (n + 1)d$, for all $n \in \mathbb{N}_0$, is given by

$$E[\mathcal{L}_2^e | I_0] = \Lambda_1 \tau + \frac{(2+\gamma)^2}{4} \Lambda_1 \frac{d}{2},$$

where τ denotes the time that elapsed since the last planning date of firm 2, that is $\tau = t-(n+1/2)d$, for all $n \in \mathbb{N}_0$.

Calculating the expected loss implied by the candidate equilibrium inattentiveness interval we obtain

$$E[\mathcal{L}(\mathcal{D})|I_0] = \frac{8e^{rd} + rd\gamma(4+\gamma)(e^{rd/2}-1) - 8(1+rd)}{8r^2(e^{rd}-1)}\Lambda_1 + \frac{K}{e^{rd}-1}. \tag{57}$$

Expected loss for $\mathcal{D}'$

In order to consistently transform expected profits into expected losses we have to scale the expected stream of profits under the infinitesimal deviations by the equilibrium expected full information profits. For that reason we first write out these full information profits. In case firm 1 is the better informed firm, that is, it has planned at nd, $n \in \mathbb{N}$ while firm 2 has planned at $(n-1/2)d$, $n \in \mathbb{N}$, and we look at an instant t with $nd \leq t < (n+1/2)d$ the per instant full information profit can be written as

$$E[\Pi^{FI}(t)\,|\,nd \leq t < (n+1/2)d] = \frac{(\alpha-(1-\gamma)c)}{(2-\gamma)^2} + \Lambda_1(nd+\tau) \tag{58}$$

$$+\gamma\Big(\frac{4(\sigma_\theta^2-\sigma_\xi^2)+2\gamma\rho\sigma_\theta\sigma_\xi+\gamma(3\sigma_\xi^2-\sigma_\theta^2)}{4(2-\gamma)^2}\Big)\frac{nd}{2},$$

where $\tau = t - nd$. Similarly, in case firm 1 has planned at nd, $n \in \mathbb{N}$ while firm 2 has planned at $(n+1/2)d$, $n \in \mathbb{N}$, and we look at an instant t with $(n+1/2)d/2 \leq t < (n+1)d$ the per instant full information profit can be written as

$$E[\Pi^{FI}(t)\,|\,(n+1/2)d/2 \leq t < (n+1)d] = \frac{(\alpha-(1-\gamma)c)}{(2-\gamma)^2} + \Lambda_1((n+1/2)d+\tau) \tag{59}$$

$$+\gamma\Big(\frac{\sigma_\theta^2-\sigma_\xi^2+\Lambda_1}{4}\Big)\frac{(2n+1)d}{2} + \gamma^2\Big(\frac{2(\sigma_\theta^2-\sigma_\xi^2)+2\gamma(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)+\gamma^2\Lambda_1}{4(2-\gamma)^2}\Big)nd,$$

where $\tau = t-(n+1/2)d$.

Before we turn to the different intervals we introduce some notation. Let $\tilde{m} \in \mathbb{N}$ denote a natural number for which $\tilde{m} \leq d/(2\triangle)$ and $(\tilde{m}+1) > d/(2\triangle)$.

In the following we derive the expected loss from the sequence $\mathcal{D}'$. First, we turn to the expected instantaneous loss that firm 1 incurs in the interval $[d, d']$ if it deviates to the sequence $\mathcal{D}'$.[17]

From d to $d' = d + \triangle$

If firm 1 deviates to $\mathcal{D}'$ then its expected profit function at instant $d+\tau$, $\tau \in [0, \triangle)$ is given by

$$\begin{aligned} E[\Pi(d+\tau)|I_0^1] &= (\alpha - p_1(d+\tau) + \gamma E[p_2(d/2+\tau)|I_0^1])(p_i(d+\tau) - c) \\ &- E[\xi(d+\tau)(\theta(d+\tau) + \gamma p_2(d/2+\tau))|I_0^i]. \end{aligned} \tag{60}$$

In this interval firm 1 is the firm that has more outdated information. However, since firm 2 has last planned at $d/2$, it has not yet detected firm 1$'$s deviation. Thus, firm 2 still believes that firm 1 plans after d periods and, therefore, 2 thinks that the information set of firm 1 is I_d. Hence, as shown in Lemma 3 firm $2's$ optimal price at $d+\tau$ is given by

$$p_2^\star(d+\tau) = \frac{\alpha + \theta(d/2) + c + \xi(d/2)}{2-\gamma}. \tag{61}$$

Therefore, since firm 1 has worse information than firm 2 at instant $d+\tau$, $\tau \in [0, \triangle)$, firm 1's optimal price is

$$p_1^\star(d+\tau) = E[p_2^\star(d+\tau)|I_0^1, I_0^2] = \frac{\alpha + c}{2-\gamma}. \tag{62}$$

Using (62) and (61) in (60) yields that firm 1$'$s expected instantaneous profit under $\mathcal{D}'$ in the interval $t \in [d, d']$ is given by

$$E[\Pi^\star(d+\tau)|I_0^1, I_0^2] = -\frac{\gamma(2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)}{2-\gamma} d - \rho\sigma_\theta\sigma_\xi\tau + \frac{(\alpha - (1-\gamma)c)}{(2-\gamma)^2}. \tag{63}$$

Subtracting (63) from (58) with $n = 1$ yields that firm 1$'$s expected instantaneous loss in the period $[d, d+\triangle)$ is

$$E[\mathcal{L}_1'|I_0] = \Lambda_1(d+\tau) + \frac{\gamma(4-\gamma)}{4}\Lambda_2\frac{d}{2}, \tag{64}$$

[17] Note that the expected loss from the candidate equilibrium sequence $\mathcal{D}$ and from the deviation sequence $\mathcal{D}'$ in the interval $t \in [0, d)$ is the same since $\mathcal{D}'$ means that planning is postponed by $\triangle$.

with $\tau \in [0, \triangle)$.

From d' to $3d/2$

In this time interval firm 2 still believes that firm 1 sticks to the proposed equilibrium strategy. Thus, its optimal price is at each instant given by (61). Firm 1 acquires new information at instant $d' = d + \triangle$. Therefore, its expected profit function in this interval is given by

$$\begin{aligned} E[\Pi(d'+\tau)|I^1_{d'}] &= (\alpha - p_1(d'+\tau) + \gamma p_2(d/2+\tau))(p_1(d'+\tau) - c) \\ &- E[\xi(d'+\tau)(\theta(d'+\tau) + \gamma p_2(d/2+\tau))|I^1_{d'}], \end{aligned} \tag{65}$$

with $\tau \in [0, d/2 - \triangle)$. Proceeding in the same way as before, we obtain that

$$p_1^\star(d'+\tau) = \frac{\alpha + \theta(d') + c + \xi(d')}{2} + \frac{\gamma\Big(\alpha + \theta(d) + c + \xi(d)\Big)}{2(2-\gamma)} \tag{66}$$

Using (66) and (61) in (65), taking expectations and subtracting the resulting expression from (58) with $n = 1$ yields for the interval $[d', 3d/2)$ the following instantaneous loss function

$$E[\mathcal{L}_2'|I_0] = \Lambda_1 \tau, \tag{67}$$

with $\tau \in [0, d/2 - \triangle)$.

From $3/2d$ to $2d$

At $3d/2$ firm 2 detects that firm 1 deviated to the sequence $\mathcal{D}'$. This implies that firm 2 knows that firm 1's consecutive planning dates will be at nd', for $n \in \mathbb{N} \setminus \{1\}$. As firm 2 is better informed in the interval $[3/2d, 2d)$ than firm 1, the latter's expected profit function equals

$$\begin{aligned} E[\Pi(3d/2+\tau)|I^1_{d'}] &= (\alpha - p_1(3d/2+\tau) + \gamma E[p_2(3d/2+\tau)|I^1_{d'}](p_1(3d/2+\tau) - c) \\ &- E[\xi(3d/2+\tau)(\theta(3d/2+\tau) + \gamma p_j(3d/2+\tau))|I^1_{d'}], \end{aligned} \tag{68}$$

with $\tau \in [0, d/2)$. As a consequence the firms' optimal prices in this period are given by

$$p_2^\star(3d/2+\tau) = \frac{\alpha+\theta(3d/2)+c+\xi(3d/2)}{2} + \frac{\gamma\Big(\alpha+\theta(d')+c+\xi(d')\Big)}{2(2-\gamma)}, \tag{69}$$

$$p_1^\star(3d/2+\tau) = E[p_2^\star(3d/2+\tau)|I_{d'}^1] = \frac{\alpha+\theta(d')+c+\xi(d')}{2-\gamma}. \tag{70}$$

Using (70) and (69) in (68), taking expectations and subtracting the resulting expression from the corresponding full information profit, which is given by (59) with $n = 1$, yields the instantaneous expected loss in this interval, which is given by

$$E[\mathcal{L}_3'|I_0] = \Lambda_1 \frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1 \triangle + \Lambda_1 \tau, \tag{71}$$

with $\tau \in [0, d/2)$.

Due to the fact that firm 2 knows firm $1'$s consecutive planning dates, the instantaneous expected loss in each interval $[(n+1/2)d, (n+1)d]$ is given by

$$E[\mathcal{L}_3'|I_0] = \Lambda_1 \frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1 n\triangle + \Lambda_1 \tau,$$

with $\tau \in [0, d/2)$ and $n \in \{2, ..., \tilde{m}\}$.

From $2d$ **to** $2d' = 2(d+\triangle)$

Since firm 2 detects at $3d/2$ that firm 1 deviated and firm 2 does not acquire new information between $2d$ to $2d'$, the expected profit that firm 1 earns between $2d$ to $2d'$ is again given by (68) and the optimal prices are as in (69) and (70). However, when determining the expected loss in this interval, we need to use the full information profit in (58) with $n = 2$. Doing so we obtain

$$E[\mathcal{L}_4'|I_0] = \Gamma_2 d - \Gamma_1 \triangle + \Lambda_1 \tau, \tag{72}$$

with $\tau \in [0, 2\triangle)$.

As above, since firm 2 knows firm $1'$s consecutive planning dates, the instantaneous expected loss in each interval $[nd, nd']$, for $n \in \{2, ..., \tilde{m}\}$, is given by

$$E[\mathcal{L}_4'|I_0] = \Gamma_2 d - \Gamma_1 n\triangle + \Lambda_1 \tau,$$

with $\tau \in [0, n\triangle)$ and $n \in \{2, ..., \tilde{m}\}$.

From $nd' = n(d + \triangle)$ **to** $(n + 1/2)d$**, for** $n \in \{2, ..., \tilde{m}\}$

In these intervals firm $2'$s information set is more outdated. This was also the case in the interval between d' and $3d/2$. But from Lemma 2 and Lemma 3 we know that the firm with the worse information sets the same price independent of the exact planning date of the better informed firm. Therefore, the expected instantaneous loss of firm 1 is in every interval $[nd', (n + 1/2)d)$, for $n \in \{2, ..., \tilde{m}\}$, given by

$$E[\mathcal{L}_5'|I_0] = \Lambda_1 \tau, \tag{73}$$

with $\tau \in [0, d - n\triangle)$ and $n \in \{2, ..., \tilde{m}\}$.

We can now compare the expected loss from the sub-sequence $\mathcal{D}'_{\tilde{m}} = \{d', ..., \tilde{m}d'\}$ with the one of the equilibrium sequence. The expected loss from the sub-sequence $\mathcal{D}'_{\tilde{m}}$ is given by

$$\begin{aligned} E[\mathcal{L}'(\mathcal{D}'_{\tilde{m}})|I_0] &= e^{-rd} \int_{\tau=0}^{\triangle} e^{-r\tau} \Big(\Lambda_1 (d + \tau) + \frac{\gamma(4-\gamma)}{4} \Lambda_2 \frac{d}{2} \Big) \, d\tau + K \sum_{n=1}^{\tilde{m}} e^{-rnd'} \\ &+ \sum_{n=1}^{\tilde{m}} \left(e^{-rnd'} \int_{\tau=0}^{d/2-n\triangle} e^{-r\tau} \Lambda_1 \tau d\tau \right) \\ &+ \sum_{n=1}^{\tilde{m}} \left(e^{-r(n+1/2)d} \int_{\tau=0}^{d/2} e^{-r\tau} \Big(\Lambda_1 \frac{(2+\gamma)^2}{4} \frac{d}{2} - \Gamma_1 n\triangle + \Lambda_1 \tau \Big) d\tau \right) \\ &+ \sum_{n=2}^{\tilde{m}} \left(e^{-rnd} \int_{\tau=0}^{n\triangle} e^{-r\tau} \Big(\Gamma_2 d - \Gamma_1 n\triangle + \Lambda_1 \tau \Big) d\tau \right). \end{aligned} \tag{74}$$

Subtracting the expected loss implied by the equilibrium sub-sequence $\mathcal{D}_{\tilde{m}} = \{d, ..., \tilde{m}d\}$, denoted by $E[\mathcal{L}(\mathcal{D}_{\tilde{m}})|I_0]$, from (74) yields

$$\begin{aligned} &e^{-rd} \int_{\tau=0}^{\triangle} e^{-r\tau} \Big(\Lambda_1 d + \frac{\gamma(4-\gamma)}{4} \Lambda_2 \frac{d}{2} \Big) \, d\tau + K \Big(\sum_{n=1}^{\tilde{m}} e^{-r(nd')} - \sum_{n=1}^{\tilde{m}} e^{-r(nd)} \Big) \\ &- \sum_{n=1}^{\tilde{m}} \left(e^{-rnd'} \int_{\tau=0}^{d/2-n\triangle} e^{-r\tau} \Lambda_1 n\triangle d\tau \right) - \sum_{n=1}^{\tilde{m}} \left(e^{-r(n+1/2)d} \int_{\tau=0}^{d/2} e^{-r\tau} \Big(\Gamma_1 n\triangle \Big) d\tau \right) \\ &+ \sum_{n=2}^{\tilde{m}} \left(e^{-rnd} \int_{\tau=0}^{n\triangle} e^{-r\tau} \Big(\Gamma_2 d - \Gamma_1 n\triangle \Big) d\tau \right) + e^{-r\tilde{m}d} \Upsilon, \end{aligned} \tag{75}$$

where Υ denotes the difference in expected losses beyond date $\tilde{m}d$. This difference in expected losses is at each instant $t > \tilde{m}d$ bounded below by the expected instantaneous loss implied by the sequence $\mathcal{D}$. We get this by assuming the best possible case for the deviation, that is, the expected instantaneous loss that is implied by the sequence $\mathcal{D}'$ is zero from date $\tilde{m}d$ onwards. In addition, we know that the instantaneous loss from the sequence $\mathcal{D}$ is finite since firm i optimally chooses to plan after some time length. Thus, at each instant $t > \tilde{m}d$ the difference in expected losses is bounded below by

$$-e^{-r\tau}\left(1+\frac{(2+\gamma)^2}{4}\right)\Lambda_1\frac{d}{2}$$

for $\tau \in (0, d]$ and by

$$-e^{-rd}K$$

at a planning date d. Both expressions are finite because d is finite.

In order to determine the per instant difference in expected losses in the time period before date $\tilde{m}d$, we divide the first five terms in (75) by $n\triangle$, where n is chosen appropriately for the different intervals. Then, we take the limit $\triangle \to 0$. The last term in (75) $e^{-r\tilde{m}d}\Upsilon$ vanishes as $\triangle \to 0$ implies that $\tilde{m} \to \infty$ since the future is discounted at rate $r > 0$. Therefore, we can concentrate on the first five terms when determining the critical inattentiveness length such that deviating to the sequence $\mathcal{D}'$ is profitable.

We obtain that the loss from deviating to $\mathcal{D}'$ is lower than the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \underline{d}_a^1$, where $\underline{d}_a^1$ solves

$$\frac{\underline{d}_a'\Lambda_2(8-4\gamma+\gamma^2)}{8e^{r\underline{d}_a'}} - \frac{re^{r\underline{d}_a'}K}{(e^{r\underline{d}_a'}-1)^2} - \frac{\Lambda_1 e^{r\underline{d}_a'/2}+\Gamma_1}{r(e^{r\underline{d}_a'}-1)(e^{r\underline{d}_a'/2}+1)} + \frac{\Gamma_2(2e^{r\underline{d}_a'}-1)}{e^{r\underline{d}_a'}(e^{r\underline{d}_a'}-1)^2} = 0. \tag{76}$$

We now turn to existence and uniqueness of $\underline{d}_a^1$. Consider first the case of $\underline{d}_a' \to 0$. In this case the second and the fourth term of the left-hand side of (76) are the dominating terms. Since $rK > 0$, the second term goes to $-\infty$ as $\underline{d}_a' \to 0$ while the fourth term goes to ∞ or $-\infty$ dependent on the sign of Γ_2. So we have to determine which of the two terms tends to the extreme value at a faster rate. To do so we differentiate the two terms with respect to $\underline{d}_a'$. Here we obtain

$$\frac{r^2Ke^{r\underline{d}_a'}(e^{r\underline{d}_a'}+1)}{(e^{r\underline{d}_a'}-1)^3} \tag{77}$$

for the second term and

$$-\frac{\Gamma_2\left(3e^{r\underline{d}_a'}-2e^{2r\underline{d}_a'}-1+r\underline{d}_a'(1+4e^{2r\underline{d}_a'}-3e^{r\underline{d}_a'})\right)}{8e^{r\underline{d}_a'}(e^{r\underline{d}_a'}-1)^3} \tag{78}$$

for the fourth term. Since the numerator of (78) goes to zero as $\underline{d}_a' \to 0$ while this is not the case for (77), we have that the second term tends to $-\infty$ at a faster rate than the fourth term to ∞ or $-\infty$. As a consequence, the left-hand side of (76) goes to $-\infty$ as $\underline{d}_a' \to 0$. Conversely, if $\underline{d}_a' \to \infty$, the left-hand side of (76) goes to 0 from above. This is the case because the last three terms go to zero at a faster rate than the first term, and the first term is strictly positive since $\Lambda_2 > 0$. Thus, there exists a solution to (76) at which $\underline{d}_a' > 0$. It remains to show that this solution is unique. To do so we differentiate (76) with respect to $\underline{d}_a'$ to get

$$-\frac{\underline{d}_a'\Lambda_2(8-4\gamma+\gamma^2)(r\underline{d}_a'-1)}{8e^{r\underline{d}_a'}}+\frac{r^2Ke^{r\underline{d}_a'}(e^{r\underline{d}_a'}+1)}{(e^{r\underline{d}_a'}-1)^3}+\frac{\Lambda_1e^{r\underline{d}_a'/2}\left(e^{r\underline{d}_a'}+e^{3r\underline{d}_a'/2}\right)}{2(e^{r\underline{d}_a'}-1)^2(e^{r\underline{d}_a'/2}+1)^2} \tag{79}$$

$$+\frac{\Gamma_1e^{r\underline{d}_a'/2}\left(3e^{r\underline{d}_a'/2}-1\right)}{2(e^{r\underline{d}_a'}-1)^2(e^{r\underline{d}_a'/2}+1)}-\frac{\Gamma_2\left(3e^{r\underline{d}_a'}-2e^{2r\underline{d}_a'}-1+r\underline{d}_a'(1+4e^{2r\underline{d}_a'}-3e^{r\underline{d}_a'})\right)}{8e^{r\underline{d}_a'}(e^{r\underline{d}_a'}-1)^3}.$$

Now, for $\underline{d}_a' \to 0$, (79) goes to ∞ since the second term dominates the remaining terms, while for $\underline{d}_a' \to \infty$, (79) goes to 0 from below. This is the case because the first term goes to zero at a slower rate than the other terms, and this term is negative. We now look at the five terms of (79) in turn. The first term changes its sign from positive to negative as $\underline{d}_a'$ increases. This, occurs at $\underline{d}_a' = 1/r$. The second and the third are strictly positive and strictly decrease as $\underline{d}_a'$ rises. The fourth and the fifth term are either positive or negative for any $\underline{d}_a'$, depending on the signs of Γ_1 and Γ_2. As the second and the third terms, they become strictly smaller in absolute value as $\underline{d}_a'$ rises, and they do so at a faster rate than the second term. Thus, we have that the first term is the only term in (79) that changes its sign as $\underline{d}_a'$ increases. In addition, this term becomes the dominant one as $\underline{d}_a'$ gets larger and larger. This is the case because the numerator of the first term includes $(\underline{d}_a')^2$ and the denominator is $e^{r\underline{d}_a'}$. This is not the case for any other term. As a consequence, there exists a unique value of $\underline{d}_a'$ at which (79) changes its sign from positive to negative. But this, in combination with the fact that the left-hand side of (79) is negative at $\underline{d}_a' = 0$ and positive at $\underline{d}_a' \to \infty$, implies that there must exist a unique solution to (79).

As a consequence, we have that if $d \geq \underline{d}_a^1$ then the loss from deviating to $\mathcal{D}'$ is larger than the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$.

Expected loss for $\mathcal{D}''$

From $d'' = d - \triangle$ to d

In this interval, firm 2 believes that firm 1 will acquire new information at d. However, firm 1 updates its information set at d'' and is therefore the better informed firm. As a consequence the firm $1'$s expected profit function in this interval is given by

$$\begin{aligned} E[\Pi(d''+\tau)|I_{d''}] &= (\alpha - p_1(d''+\tau) + \gamma p_2(d''+\tau))(p_1(d''+\tau) - c) \\ &- E[\xi(d''+\tau)(\theta(d''+\tau) + \gamma p_2(d''+\tau))]. \end{aligned} \tag{80}$$

Proceeding in a similar way as before, we derive the firms' optimal prices

$$p_1^\star(d''+\tau) = \frac{\alpha + c + \theta(d'') + \xi(d'')}{2-\gamma} + \frac{\gamma((2-\gamma)(\theta(d/2) + \xi(d/2) - 2(\theta(d'') + \xi(d'')))}{4(2-\gamma)}, \tag{81}$$

$$p_2^\star(d''+\tau) = \frac{2(\alpha + c + \theta(d/2) + \xi(d/2)) - \gamma(\tau(d/2) + \xi(d/2))}{2(2-\gamma)}. \tag{82}$$

Using (81) and (82) in (80), taking expectations and subtracting the resulting expression from the corresponding expected full information profit, which is obtained by replacing n by 0 in (59), yields that the expected instantaneous loss in this interval is given by

$$E[\mathcal{L}_1''|I_0] = \Lambda_1\tau. \tag{83}$$

From d to $3/2d$

In this interval firm 2 expects that firm 1 has planned at d—although firm 1 in fact plans at d''—and so it expects to be the worse informed firm. Therefore, the profit of firm 1 is the same as in (74) but the price of firm 2 has changed. Here we get that

$$p_1^\star(d''+\tau) = \frac{\alpha + c + \theta(d'') + \xi(d'')}{2-\gamma} + \frac{\tau(d/2) + \xi(d/2) - \theta(d'') - \xi(d''}{2(2-\gamma)}, \tag{84}$$

$$p_2^\star(d''+\tau) = \frac{\alpha + c + \theta(d/2) + \xi(d/2)}{2-\gamma}. \tag{85}$$

In the same way as above we can calculate the expected instantaneous loss to get

$$E[\mathcal{L}_2''|I_0] = \Lambda_1(\tau + \triangle). \tag{86}$$

From $(n+1/2)d$ to $(n+1)(d-\triangle)$, with $n \in \mathbb{N}$

In this intervals firm 2 realized that firm 1 deviated. The profit function of firm 1 is then given by

$$\begin{aligned} E[\Pi(d''+\tau)|I_{d''}] &= (\alpha - p_1(d''+\tau) + \gamma E[p_2(d''+\tau)|I_{d''}])(p_1(d''+\tau) - c) \\ &- E[\xi(d''+\tau)(\theta(d''+\tau) + \gamma p_2(d''+\tau))|I_{d''}]. \end{aligned} \tag{87}$$

Optimal prices in this case are

$$p_1^\star = \frac{\alpha + c + \theta(n(d-\triangle)) + \xi(n(d-\triangle))}{2-\gamma} \tag{88}$$

and

$$p_2^\star(d''+\tau) = \frac{\alpha+c}{2-\gamma} + \frac{\theta(n(d-\triangle)) + \xi(n(d-\triangle)) + \theta((n+1/2)d) + \xi((n+1/2)d)}{2(2-\gamma)}, \tag{89}$$

which gives an expected loss of

$$E[\mathcal{L}_3''|I_0] = \Gamma_1 n\triangle + \Lambda_1\tau + \frac{(2+\gamma)^2}{4}\Lambda_1\frac{d}{2}. \tag{90}$$

From $(n+1)(d-\triangle)$ to $(n+1)d$, with $n \in \mathbb{N}$

In contrast to the interval from d'' to d, in the intervals from $(n+1)(d-\triangle)$ to $(n+1)d$ firm 2 realized that firm i deviated. The profit function of firm 1 can then be written as

$$\begin{aligned} & E[\Pi((n+1)(d-\triangle)+\tau)|I_{(n+1)(d-\triangle)}] = \\ = \; & (\alpha - p_1((n+1)(d-\triangle)+\tau) + \gamma p_2((n+1)(d-\triangle)+\tau))(p_1((n+1)(d-\triangle)+\tau) - c) \\ - \; & E[\xi((n+1)(d-\triangle)+\tau)(\theta((n+1)(d-\triangle)+\tau) + \gamma p_2((n+1)(d-\triangle)+\tau))]. \end{aligned} \tag{91}$$

Calculating optimal prices in the same way as above yields

$$p_1^{\star} = \frac{\alpha + c}{2-\gamma} + \frac{\tau((n+1/2)d) + \xi((n+1/2)d) + \theta((n+1)(d-\triangle)) + \xi((n+1)(d-\triangle))}{2(2-\gamma)} \quad (92)$$

and

$$p_2^{\star}(d'' + \tau) = \frac{\alpha + c + \theta((n+1/2)d) + \xi((n+1/2)d)}{2-\gamma}. \quad (93)$$

We then obtain an expected instantaneous loss of

$$E[\mathcal{L}_4''|I_0] = \Lambda_1\tau + \gamma^2\Big(\gamma^2\frac{\Lambda_2}{16} + \frac{\sigma_\xi^2 - \sigma_\theta^2 - \gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)}{2(2-\gamma)^2}\Big)\frac{d}{2}. \quad (94)$$

From $(n+1)d$ **to** $(n+3/2)d$**, with** $n \in \mathbb{N}$

The difference in these intervals to the interval from d to $3/2d$ is just that firm 2 knows that firm 1 has deviated. However, since firm 2 always expects to be the worse informed firm in these intervals, we know from Lemma 2 and 3 that the price of firm 2 is the same independent of the fact that it is aware of the deviation or not. As a consequence, the expected instantaneous loss is given by

$$E[\mathcal{L}_5''|I_0] = \Lambda_1(\tau + (n+1)\triangle). \quad (95)$$

In the same way as in the case of $\mathcal{D}'$ we can now calculate the expected loss from the sequence $\mathcal{D}''$. Subtracting the expected loss of the equilibrium sequence from the expected loss from the sequence $\mathcal{D}''$, dividing the difference by $n\triangle$ and taking the limit $\triangle \to 0$ yields that the expected loss from deviating to $\mathcal{D}''$ exceeds the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \bar{d}_a^1$, where $\bar{d}_a^1$ solves

$$\frac{\bar{d}_a'\Lambda_1(8+4\gamma+\gamma^2)}{8e^{r\bar{d}_a'}} - \frac{re^{r\bar{d}_a'}K}{(e^{r\bar{d}_a'}-1)^2} - \frac{\Lambda_1 e^{r\bar{d}_a'/2} + \Gamma_1}{r(e^{r\bar{d}_a'}-1)(e^{r\bar{d}_a'/2}+1)} + \frac{\Gamma_2(2e^{r\bar{d}_a'}-1)}{e^{r\bar{d}_a'}(e^{r\bar{d}_a'}-1)^2} = 0. \quad (96)$$

We can show existence and uniqueness in exactly the same way as they were shown for $\underline{d}_a$.

Firm 2

We now turn to firm 2. In an asynchronous equilibrium, firm 2 chooses the sequence $\mathcal{D} = \{d/2, 3d/2, 5d/2, ...\}$. Thus, when considering marginal deviations we need to check that the firm has no incentive to deviate to the sequences $\mathcal{D}' = \{(d+\triangle)/2, 3(d+\triangle)/2, 5(d+\triangle)/2, ...\}$ and $\mathcal{D}'' = \{(d-\triangle)/2, 3(d-\triangle)/2, 5(d-\triangle)/2, ...\}$. We can now proceed in the same way as above, namely first calculating the instantaneous expected loss if firm 2 chooses the sequence $\mathcal{D}$ and compare it with the one of the sequences $\mathcal{D}'$ and $\mathcal{D}''$ and then let $\triangle \to 0$.

Again we start with the equilibrium candidate sequence $\mathcal{D}$. By calculations in the same way as for firm 1 we obtain that the expected instantaneous loss between time 0 and $d/2$ is given by

$$E[\mathcal{L}_1^e|I_0] = \Lambda_1\tau, \tag{97}$$

where τ denotes the time that elapsed since date 0. Similarly, the expected instantaneous loss at t with $(n+1/2)d \leq t < (n+1)d$, $n \in \mathbb{N}_0$ is also given by (97) with $\tau = t-(n+1/2)d$. Finally, the expected instantaneous loss at t with $(n+1)d \leq t < (n+3/2)d$, $n \in \mathbb{N}_0$ for all $n \in \mathbb{N}_0$, is given by

$$E[\mathcal{L}_2^e|I_0] = \Lambda_1\tau + \frac{(2+\gamma)^2}{4}\Lambda_1\frac{d}{2},$$

where τ denotes the time that elapsed since the last planning date of firm 1, that is $\tau = t-(n+1)d$.

Now let us look at the deviation sequence $\mathcal{D}'$. As above in this case the expected instantaneous loss between time 0 and $(d+\triangle)/2$ is given by $E[\mathcal{L}_1'|I_0] = \Lambda_1\tau$ with $\tau = t$ and the expected instantaneous loss at t, with $(d+\triangle)/2 \leq t < d$ is given by $E[\mathcal{L}_2'|I_0] = \Lambda_1\tau$ with $\tau = t - 1/2(d+\triangle)$. Now at $t = d$ firm 1 observes that firm 2 has deviated to $(d+\triangle)/2$. Thus, is it will update its belief that the new sequence of firm 2 is $\mathcal{D}'$ and sets its prices accordingly. As was calculated above the instantaneous expected loss of firm 2 from d to $3d/2$ is given by

$$E[\mathcal{L}_3'|I_{d'}^1, I_{3d/2}^2] = \Lambda_1\frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1\frac{\triangle}{2} + \Lambda_1\tau, \tag{98}$$

with $\tau \in [0,d)$, and, more generally, the instantaneous expected from nd to $(n+1/2)d$, $n \in \mathbb{N}$ can be written as

$$E[\mathcal{L}_3'|I_0] = \Lambda_1\frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1\left(n+\frac{1}{2}\right)\triangle + \Lambda_1\tau. \tag{99}$$

Finally, from $(n+1/2)d$ to $(n+1/2)(d+\triangle)$, $n \in \mathbb{N}$, the instantaneous expected loss of firm 2 is given by

$$E[\mathcal{L}_4'|I_0] = \Gamma_2 d - \Gamma_1\left(n+\frac{1}{2}\right)\triangle + \Lambda_1\tau, \tag{100}$$

with $\tau \in [0,(n+1/2)\triangle)$.

We can now compare the difference between the expected loss from the sequence $\mathcal{D}$ with the one of $\mathcal{D}'$. As above, dividing the respective losses by $(n+1/2)\triangle$, where $n \in \mathbb{N}_0$ is appropriately chosen for the respective intervals, and then taking the limit $\triangle \to 0$, we obtain that the loss from deviating to $\mathcal{D}'$ exceeds the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \geq \underline{d}_a^2$, where $\underline{d}_a^2$ solves

$$\frac{\Lambda_1\left(\underline{d}_a' r(e^{r\underline{d}_a'}-1) - 2e^{r\underline{d}_a'/2}\left(e^{r\underline{d}_a'/2}-1\right)\right)}{2r(e^{r\underline{d}_a'}-1)} - \frac{rKe^{r\underline{d}_a'}}{e^{r\underline{d}_a'}-1} - \frac{\Gamma_1(e^{r\underline{d}_a'}-1)}{r\left(e^{r\underline{d}_a'/2}-1\right)} - \tag{101}$$

$$-\Gamma_2\underline{d}_a'\left(\frac{1}{e^{r\underline{d}_a'}(e^{r\underline{d}_a'}-1)} + \frac{\ln(1-e^{-r\underline{d}_a'/2}) - \ln(1+e^{-r\underline{d}_a'/2})}{e^{-r\underline{d}_a'/2}}\right) = 0.$$

Existence and uniqueness of $\underline{d}_a^2$ follow from similar arguments as in the case of firm 1.

Turning to the comparison of $\mathcal{D}$ with $\mathcal{D}''$ we can proceed exactly in the same way as in above, i.e., first calculating the expected instantaneous losses for the two sequences, then dividing the respective losses by $(n+1/2)\triangle$, where $n \in \mathbb{N}_0$ and then taking the limit $\triangle \to 0$. Here we obtain that the expected loss from deviating to $\mathcal{D}''$ exceeds the expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \bar{d}_a^2$, where $\bar{d}_a^2$ solves

$$\frac{\Lambda_1\bar{d}_a'}{2e^{r\bar{d}_a'}} - \frac{\Lambda_1 e^{r\bar{d}_a'/2}(e^{r\bar{d}_a'/2}-1)}{r(e^{r\bar{d}_a'}-1)} + \frac{\Gamma_1(2+r\bar{d}_a'-2e^{r\bar{d}_a'/2})}{2re^{r\bar{d}_a'/2}(e^{r\bar{d}_a'/2}-1)} - \frac{rKe^{r\bar{d}_a'}}{e^{r\bar{d}_a'}-1} = 0. \tag{102}$$

Existence and uniqueness of $\bar{d}_a^2$ follow from similar arguments as in the case of firm 1. ■

Proof of Lemma 6

Suppose first that $\underline{d}_a^1 = \bar{d}_a^1 = d$. Subtracting the left-hand side of (45) from the left-hand side of (44) we obtain

$$\frac{\gamma^2 de^{dr}(8-\gamma^2)(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)}{32}.$$

Thus, the two sides are equal if and only if $\gamma = 0$ or $\rho = -(\sigma_\theta^2 + \sigma_\xi^2)/(2\sigma_\theta\sigma_\xi)$, where $-(\sigma_\theta^2 + \sigma_\xi^2)/(2\sigma_\theta\sigma_\xi) \leq -1$. But the latter equality can never be fulfilled since $\rho \geq -1$, and for $\rho = -1$ and $\sigma_\theta^2 = \sigma_\xi^2$, i.e., the case in which $-(\sigma_\theta^2 + \sigma_\xi^2)/(2\sigma_\theta\sigma_\xi) = -1$. we have that firms never plan since their profit at each instant is certain due to the perfect negative correlation of the shocks. Since planning only incurs costs, they have no incentive to plan. Hence, $\underline{d}_a^1 = \bar{d}_a^1$ if and only if $\gamma = 0$.

Now let us look at the case in which $\gamma \neq 0$. Here we know that the left-hand side of (45) is lower than the left-hand side of (44) if $\underline{d}_a^1 = \bar{d}_a^1$. From the proof of Proposition 1 we also know that the derivatives of the left-hand sides of (44) and (45) at their respective solutions are strictly positive. But from this it follows that $\bar{d}_a^1 > \underline{d}_a^1$ for all $\gamma \neq 0$.

We can conduct a similar analysis for firm 2. Suppose that $\underline{d}_a^2 = \bar{d}_a^2 = d$. By subtracting the left-hand side of (46) from the left-hand side of (47) and simplifying we obtain that this can only occur if

$$-\frac{\Lambda_1}{2(1-e^{-rd})} - \frac{\Gamma_2\left(\ln(1+e^{-rd/2}) - \ln(1-e^{-rd/2})\right)}{e^{-rd/2}} = 0, \tag{103}$$

The first term on the left-hand side of (103) is negative while the second one is negative if Γ_2 is positive. Solving $\Gamma_2 = 0$ for ρ wo obtain that $\Gamma_2 \geq 0$ if

$$\rho \geq -\frac{4(\sigma_\theta^2 + \sigma_\xi^2)(2-\gamma) + \gamma^2(\sigma_\theta^2 - 3\sigma_\xi^2 + 2\gamma\sigma_\xi^2)}{2\sigma_\theta\sigma_\xi(8 - 4\gamma - \gamma^2 + \gamma^3)}. \tag{104}$$

It is easy to check that the right-hand side of (104) is larger than -1 if and only if $\sigma_\xi > \sigma_\theta$. Thus, for $\sigma_\theta > \sigma_\xi$, (104) is for sure fulfilled which implies that the second term on the left-hand side of (103) is negative. Thus, in this case $\underline{d}_a^2$ can not be equal to $\bar{d}_a^2$. Since the left-hand side of (47) is increasing in $\bar{d}_a^2$ and the left-hand side of (46) is increasing in $\underline{d}_a^2$, we obtain that in this case $\bar{d}_a^2 > \underline{d}_a^2$ because the difference between (47) and (46) at $\bar{d}_a^2 = \underline{d}_a^2$ is negative. If, on the other hand, $\sigma_\xi > \sigma_\theta$, (104) may not fulfilled. Then, the left-hand side of (103) consists of two countervailing terms. In this case $\bar{d}_a^2$ is smaller than $\underline{d}_a^2$ if $\rho < \hat{\rho}$, where $\hat{\rho}$ is the solution to (103). Since ρ is bounded below by -1, the result stated in the lemma follows.

It remains to show that $\hat{\rho}$ is the unique solution to (103). To do so we first use equations (47) and (46) the build the implicit functions $d\bar{d}_a^2/d\rho$ and $d\underline{d}_a^2/d\rho$. Then subtracting $d\underline{d}_a^2/d\rho$

from $d\bar{d}_a^2/d\rho$ and evaluating it at $\underline{d}_a^2 = \bar{d}_a^2 = d$, we obtain

$$\frac{\sigma_\theta\sigma_\xi d(1-e^{-rd})}{\Lambda_1(2e^{-rd}+rd\,(3-e^{-rv}))} \tag{105}$$

$$+\frac{d(1-e^{-rd})(\sigma_\theta\sigma_\xi(8-4\gamma-\gamma^2+\gamma^3))\left(\ln(1+e^{-rd/2})-\ln(1-e^{-rd/2})\right)}{4\Gamma_2(2-\gamma)^2\left((\ln(1+e^{-rd/2})-\ln(1-e^{-rd/2}))\,(1+rd)(e^{-rd}-1)-rde^{-rd/2}\right)}.$$

The first term of (105) is positive while the second term is positive only if $\Gamma_2 < 0$. But we know from above that $\underline{d}_a^2 = \bar{d}_a^2$ can only occur if (104) is not fulfilled which implies that $\Gamma_2 < 0$. Therefore, at $\underline{d}_a^2 = \bar{d}_a^2$ we have that both terms of (105) are positive which implies that $d\bar{d}_a^2/d\rho > d\underline{d}_a^2/d\rho$. It follows that $\hat{\rho}$, i.e., the value of ρ at which $\underline{d}_a^1 = \bar{d}_a^1$, is unique. ■

Proof of Lemma 7

We start with a comparison of $\bar{d}_a^1$ with $\bar{d}_a^2$. To simplify this comparison we first multiply (47) by $(e^{r\bar{d}_a^2}-1)^{-1}$. Clearly, this does not change the optimal solution of this equation. However, it is helpful because it eliminates K when comparing the left-hand sides of (47) and (45). Suppose now that $\bar{d}_a^1 = \bar{d}_a^2 = d$. Then, subtracting the left-hand side of (47) from the left-hand side of (45), we obtain

$$\frac{\Lambda_1 d(8-4\gamma-\gamma^2+e^{rd}(2+\gamma)^2)}{8e^{rd}(e^{rd}-1)} + \frac{\Gamma_2 d(2e^{rd}-1)}{e^{rd}(e^{rd}-1)^2} + \frac{\Gamma_1 d}{(e^{rd}-1)^2}, \tag{106}$$

where the terms involving K are eliminated since (47) was multiplied by $(e^{r\bar{d}_a^2}-1)^{-1}$. One can easily check that (106) is strictly positive. Since we know that the left-hand sides of (47) and (45) are strictly increasing at their respective solutions $\bar{d}_a^1$ and $\bar{d}_a^2$, it follows that $\bar{d}_a^1 < \bar{d}_a^2$.

From Lemma 6 we know that $\bar{d}_a^2 < \underline{d}_a^2$ if ρ is sufficiently negative. Since $\bar{d}_a^1 < \bar{d}_a^2$, there must exist a critical ρ such that for all ρ below this critical ρ we have $\bar{d}_a^1 < \underline{d}_a^2$. To determine this critical ρ we multiply the left-hand side of (46) by $(e^{r\underline{d}_a^2}-1)^{-1}$ and then set it equal to the left-hand side (47). We obtain that the two sides are equal if and only if (49) holds where d_a is the solution to (46) and (47) at the critical ρ. In the same way as in the proof of Lemma 6 we can show that the critical ρ is unique. Finally, denoting the maximum of the critical ρ and -1 by $\check{\rho}$, it follows that $\check{\rho} \leq \hat{\rho}$, since $\bar{d}_a^1 < \bar{d}_a^2$. ■

Proof of Lemma 8

Firm 1

We start with firm 1. We confine our attention to the case of a non-marginal deviation of the form in which firm 1 lowers (or extends) the inattentiveness period by $1/m$. We will show later that $m \to \infty$ yields the tightest bounds in the case in which firms plan in an alternating manner. Thus, even if firm 1 deviates to an inattentiveness period of the form l/m, we obtain the same result. It is therefore without loss of generality to consider deviations of the form $d(1-1/m)$ instead of $d(1-l/m)$, $l > 1$.

Suppose that firm 1 deviates to a sequence $d - d/m$, where m is an odd number, whereas firm 2 sticks to the equilibrium sequence $\{d/2, 3/2d, 5/2d, ...\}$. Before we proceed, we introduce some notation. We define

$$\Gamma_3 := \frac{\gamma^2(\gamma^2(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - 8\gamma(\sigma_\theta^2 + \rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) + 8(\sigma_\xi^2 - \sigma_\theta^2))}{32(2-\gamma)^2},$$

and $k \in \mathbb{N}_0$.

The deviation of type $d(1-1/m)$ where m is an odd number induces a stream of expected losses that is composed of seven different instantaneous expected losses:

$$\mu_1 := \Lambda_1\left(\tau + \frac{d(2+\gamma)^2}{8}\right), \tag{107}$$

$$\mu_2 := \Lambda_1\tau, \tag{108}$$

$$\mu_3 := \Lambda_1\tau + \Gamma_3 d, \tag{109}$$

$$\mu_4 := \Lambda_1\left(\tau + \frac{(m-1)d}{2m}\right), \tag{110}$$

$$\mu_5 := \Lambda_1\left(\tau + \frac{d(2+\gamma)^2}{8}\right) + \Gamma_1\frac{dk}{2m}, \tag{111}$$

$$\mu_6 := \Lambda_1\tau + \Gamma_1\frac{dk}{2m} - d\left(\frac{\Lambda_2}{32}(16-\gamma^4)\right) \tag{112}$$
$$-d\left(\frac{\gamma^2\left(3\sigma_\xi^2 + 2\rho\sigma_\theta\sigma_\xi - \sigma_\theta^2 + 8\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\right)}{4(2-\gamma)^2}\right),$$

$$\mu_7 := \Lambda_1\tau + \Gamma_1\frac{dk}{2m} - \gamma d\left(\frac{\Lambda_2}{2} - \gamma\frac{\sigma_\theta^2 + 6\rho\sigma_\theta\sigma_\xi - 4\sigma_\xi^2 - 2\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)}{8(2-\gamma)^2}\right). \tag{113}$$

Here μ_1 is the expected instantaneous loss in periods in which firm 2 was the last to plan, e.g., at $d/2$, up to the next planning date of firm 1 under deviation, that is, at $d(1-1/m)$. The component μ_2 captures the instantaneous expected loss in periods in which firm 1 is better informed under the deviation than on the equilibrium path, e.g., at $d(1-1/m)$, up to the next planning date under non-deviation, e.g., at d. However, firm 2 has not yet detected the deviation of firm 1. Similarly, μ_3 captures the same instantaneous expected loss but in periods in which firm 2 has detected the deviation. μ_4 displays the instantaneous expected loss in which firm 1 is better informed than firm 2 on the equilibrium and also under deviation but its latest planning date after deviation was before the one on equilibrium. This occurs between the periods $((m-1)/2-1)d$ and $((m-1)/2)(1-1/m)d$.

The instantaneous expected loss μ_5 occurs in periods in which firm 1 is worse informed after a deviation but was also worse informed on the equilibrium path. However, the firm's latest planning date after a deviation was before the one on the equilibrium path. This occurs for example in periods between $3/2d$ and $2(1-1/m)d$, in which firm 1 on the equilibrium path last planned at d but after deviation at $(1-1/m)d$. The instantaneous expected loss μ_6 is similar to μ_5, i.e., firm 1 is worse informed both on the equilibrium and after a deviation, but in this case firm 1 latest planning date in the deviation case was after the one on equilibrium. This occurs in periods from $((m-1)/k-1/2+k)d$ to $((m-1)/2+k)d$. Finally, μ_7 captures the loss in periods in which firm 1 is worse informed after deviation but better informed on the equilibrium path. This occurs in periods from $((m-1)/2+k)d$ up to $((m-1)/2+k)(1-1/m)d$.

We can now turn to the expected stream of losses. Given the expected instantaneous losses (107) to (113) a deviant's expected stream of losses is given by

$$
\begin{aligned}
E[\mathcal{D}_1|\theta_0,\xi_0] \;=\;& e^{-rd/2}\int_{\tau=0}^{\left(\frac{1}{2}-\frac{1}{m}\right)d} e^{-r\tau}\mu_1 d\tau + e^{-r\left(1-\frac{1}{m}\right)d}\int_{\tau=0}^{d/m} e^{-r\tau}\mu_2 d\tau \\
&+\sum_{k=\frac{m-1}{2}}^{m-2}\left(e^{-r(k+1)\left(1-\frac{1}{m}\right)d}\int_{\tau=0}^{\left(k+\frac{1}{2}-(k+1)\left(1-\frac{1}{m}\right)\right)d} e^{-r\tau}\mu_2 d\tau\right) \\
&+\sum_{k=1}^{\frac{m-1}{2}-1}\left(e^{-r(k+1)\left(1-\frac{1}{m}\right)d}\int_{\tau=0}^{\left(\frac{k+1}{m}\right)d} e^{-r\tau}\mu_3 d\tau\right) \\
&+\sum_{k=1}^{\frac{m-1}{2}-1}\left(e^{-r\left(k+\frac{1}{2}\right)d}\int_{\tau=0}^{(k+1)\left(1-\frac{1}{m}-k-\frac{1}{2}\right)d} e^{-r\tau}\mu_5 d\tau\right)+\sum_{k=\frac{m+1}{2}}^{m-2}\left(e^{-r\left(k-\frac{1}{2}\right)d}\int_{\tau=0}^{d/2} e^{-r\tau}\mu_6 d\tau\right) \\
&+\sum_{n=1}^{\infty}\left(e^{-r\left(n(m-1)+\frac{1}{2}\right)d}\left(\int_{\tau=0}^{\left(\frac{1}{2}-\frac{1}{m}\right)d} e^{-r\tau}\mu_1 d\tau\right)\right)+\sum_{n=1}^{\infty}\left(e^{-rn(k-1)d}\left(\int_{\tau=0}^{d/2} e^{-r\tau}\mu_2 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=\frac{m-1}{2}+1}^{m-1}\left(e^{-r\left(n(m-1)+\left(1-\frac{1}{m}\right)k\right)d}\left(\int_{\tau=0}^{\left(\frac{1}{2}+k-1-k\left(1-\frac{1}{m}\right)\right)d} e^{-r\tau}\mu_2 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-3}{2}}\left(e^{-r\left(n(m-1)+\left(1-\frac{1}{m}\right)(k+1)\right)d}\left(\int_{\tau=0}^{\left(\frac{k+1}{m}\right)d} e^{-r\tau}\mu_3 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\left(e^{-r(m-1)(1/2+n)d}\left(\int_{\tau=0}^{\left(1-\frac{1}{m}-\frac{m-1}{2m}\right)d} e^{-r\tau}\mu_4 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-1}{2}-1}\left(e^{-r\left(n(m-1)+\frac{k}{2}\right)d}\left(\int_{\tau=0}^{\left(\frac{1}{2}-\frac{k+1}{m}\right)d} e^{-r\tau}\mu_5 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=\frac{m+1}{2}}^{m-2}\left(e^{-r\left(n(m-1)+k-\frac{1}{2}\right)d}\left(\int_{\tau=0}^{d/2} e^{-r\tau}\mu_6 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}-2}\left(e^{-r\left((m-1)(n+\frac{1}{2})+1+k\right)d}\left(\int_{\tau=0}^{\left(\left(1-\frac{1}{m}\right)\left(m-\frac{1}{2}+k+2\right)-\left(m-\frac{1}{2}+k+1\right)\right)d} e^{-r\tau}\mu_7 d\tau\right)\right) \\
&+K\sum_{n=1}^{\infty}\left(e^{-rn\left(1-\frac{1}{m}\right)d}\right). \qquad (114)
\end{aligned}
$$

Now, we subtract from this fractional-deviation induced expected stream of losses the equilibrium expected stream of losses which is appropriately adapted to the considered time horizon. This yields

$$E[\mathcal{D}_1|\theta_0,\xi_0]-E[\mathcal{S}_1|\theta_0,\xi_0], \qquad (115)$$

where

$$E[\mathcal{S}_1|\theta_0,\xi_0] = \Lambda_1 \frac{e^{rd/2}(8+rd(2+\gamma)^2) - 8 - rd(8+4\gamma+\gamma^2)}{16r^2(e^{rd}-1)} + \Lambda_1 \frac{2e^{rd/2} - 2 - rd}{4e^{rd/2}r^2(e^{rd}-1)} + \frac{K}{e^{rd}-1}.$$

Thus, it is profitable for a firm to deviate from a proposed alternating equilibrium inattentiveness period d in a non-marginal way if (115) is negative for at least one $m \in \mathbb{N}$.

Now building the difference quotient by dividing the respective terms of (115) by x/m, where x is chosen appropriately for the different intervals, and then letting $r \to 0$, we obtain that (115) is negative if

$$K < d^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2 \left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32} \right) \times \tag{116}$$

$$\left(\frac{8\Lambda_1(40+12\gamma)m^3 + \Lambda_1(4-\gamma)m^2\Gamma_1(36-24\gamma)m}{8\Lambda_1(40+12\gamma)m^3 + \Lambda_2(8-4\gamma+\gamma^2)m^2 + \Gamma_1(132-42\gamma)m + \Lambda_2 + \gamma^2(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)} \right).$$

Differentiating the second term of the right-hand side of (116) with respect to m, we obtain

$$\begin{gathered}24\Lambda_1(40+12\gamma)\Lambda_2(4-3\gamma+\gamma^2)m^4 + 16\Lambda_1(40+12\gamma)\Gamma_1(32+6\gamma-11\gamma^2+2\gamma^3)m^3 \\ + \left(24\Lambda_1(40+12\gamma)(\Lambda_2+\gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)) + 6\Gamma_1(152-122\gamma+46\gamma^2-7\gamma^3)\right)m^2 \\ + (2\Lambda_1(4-\gamma)m + \Gamma_1(36-24\gamma))\left(\Lambda_2 + \gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right).\end{gathered}$$

It is easy to see that this term is strictly positive. Thus, (116) is strictly increasing in m. Therefore, letting $m \to \infty$ is the most profitable deviation for firm 1. We can then write (116) as

$$K < d^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2 \left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32} \right). \tag{117}$$

Now letting $r \to 0$ in (45), i.e., in the condition that gives the upper bound for any potential equilibrium from the marginal deviations of firm 1, $\bar{d}_a^1$, we obtain that $\bar{d}_a^1$ is (implicitly) given by

$$K = (\bar{d}_a^1)^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2 \left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32} \right). \tag{118}$$

But since only $d \leq \bar{d}_a^1$ are potential equilibria, combining (117) and (118), we obtain that (117) can never be satisfied for any $d \leq \bar{d}_a^1$. This implies that the non-marginal deviation of firm 1 to a period length of $d - d/m$ with m being an odd number does not eliminate potential equilibria.

We can proceed in the same way for non-marginal deviations in which firm 1 chooses a period length of $d - d/m$ with m being an even number, and also for non-marginal deviations in which firm 1 sets a period length of $d + d/m$ with m being an odd or even number. In all cases we obtain that in the limit as $r \to 0$, the deviation to $m \to \infty$ excludes the largest set of inattentiveness periods, and that this set is equivalent to the one that is excluded by the marginal deviations. Thus, non-marginal deviations of firm 1 do not exclude potential equilibria in the limit as $r \to 0$. By continuity, if r is positive but sufficiently close to 0, the result applies as well.

Firm 2

Now, we turn to firm 2. Here we derive the expected stream of losses for the case in which firm $2'$s deviates to an inattentiveness interval of length $d + d/m$, where m is an even number, but sticks to the (A) mode. Before we proceed, we introduce some notation. We define $k \in \{0, .., (m-2)/2\}$.

The considered deviation of firm 2 induces a stream of expected losses that is composed of nine different expected instantaneous losses:

$$\vartheta_1 := \Lambda_1 \left(\tau + \frac{d}{2} \right), \tag{119}$$

$$\vartheta_2 := \Lambda_1 \tau, \tag{120}$$

$$\vartheta_3 := \Lambda_1 \tau + \Gamma_3 d, \tag{121}$$

$$\vartheta_4 := \Lambda_1 \left(\tau + \left(\frac{1}{2} - \frac{1+2k}{2m} \right) \right), \tag{122}$$

$$\vartheta_5 := \Lambda_1 \left(\tau + \frac{(2+\gamma)^2}{4} \frac{d}{2} \right) - \Gamma_1 \left(\frac{1+k}{2m} \right) \frac{d}{2}, \tag{123}$$

$$\vartheta_6 := \Lambda_1\tau - \Gamma_1\left(\frac{1+2k}{m}\right)\frac{d}{2} + \Gamma_2 d, \tag{124}$$

$$\vartheta_7 := \Lambda_1\tau + \Gamma_1\left(\frac{1-2k}{m}\right)\frac{d}{2} + \Gamma_2 d, \tag{125}$$

$$\vartheta_8 := \Lambda_1\left(\tau + \frac{(2+\gamma)^2}{4}\frac{d}{2}\right) + \Gamma_1\left(\frac{d}{2m}\right), \tag{126}$$

$$\vartheta_9 := \Lambda_1\tau + \Gamma_1\left(\frac{d}{2m}\right) + \Gamma_2 d, \tag{127}$$

The first four expected instantaneous losses—(119) to (122)—capture firm $2'$s losses when it is the better informed firm. The expected instantaneous loss in the period that elapses between the first planning date on the equilibrium path and the first planning date under the deviation is given by (119). In this period, firm $2'$s deviation is undetected by firm 1. The component (120) captures firm $2'$s expected instantaneous losses in the time period in which it is better informed, irrespective of whether the deviation was detected or not.

If firm 2 deviates to a frequency of $(1 + 1/m)\, d$ then it happens that firm 1 plans twice between two consecutive planning dates of firm 2. This takes place for the first time between the $m/2$th and the $(m+2)/2$th planning date of firm 2 and occurs thereafter between every $m(1/2 + (n+1))$th and $((m+2)/2 + (n+1)m)$th, $n \in \mathbb{N}_0$, planning date of firm 2. At this point, firm 1 has already detected firm $2'$s deviation. Moreover, the $((m+2)/2)$th planning date of firm 2 happens at an instant at which it would have been the worse informed firm if it followed the equilibrium planning horizon. The corresponding expected instantaneous loss is captured by (121). After $(1/2 - (1+2k)/(2m))$ periods firm 2 is still the better informed firm but now in a time period in which it would have also been the better informed firm on the equilibrium path. The corresponding loss is captured by (122).

Note that after each mth repetition of firm $2'$s planning horizon the relative time distance to firm $1'$s planning dates is the same. Thus, in the following we refer to this as a cycle. The components (123) and (124) capture firm $2'$s losses from being worse informed in the first half of the cycle, i.e. the $(1/2 + k + nm)$th to the $((m-1)/2 + nm)$th repetition of the planning horizon $(1 + 1/m)d$. The difference between (123) and (124) is that the former captures the losses in a time period in which firm 2 was the worse informed and the latter the losses in which it was the better informed firm on the equilibrium path. In this part of the cycle, firm 2 is relative to the equilibrium path worse informed for a shorter time.

After firm 1 repeated its planning horizon d for the $((m-1)+k+n(m+1))$th time, firm 2 is the worse informed firm in a time period in which it was already worse informed on the equilibrium path until it repeated its planning horizon for the $((m+1)/2+k+nm)$th time. The expected instantaneous losses in this periods are caught by (125). The cycle is completed after firm 2 repeated its inattentiveness interval for the $(m+1/2+n(m+1))$th time. In this last repetition the components (126) and (127) pick up firm $2'$ expected instantaneous losses from being the worse informed firm. Again, the former component captures the losses in a time period in which firm 2 was the worse informed and the latter the losses in which it was the better informed firm on the equilibrium path.

Given the expected instantaneous expected losses (119) to (127) firm $2'$s expected stream of losses from a deviation to $(1+1/m)d$ can be written as

$$\begin{aligned}
E[\mathcal{D}_2|\theta_0,\xi_0] \;=\;& e^{-r\frac{d}{2}}\int_0^{\frac{d}{2m}} e^{-r\tau}\vartheta_1 d\tau + \sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd\left(n(m+1)+\frac{(1+2k)(1+m)}{2m}\right)}\left(\int_{\tau=0}^{\left(\frac{m-(1+2k)}{2m}\right)d} e^{-r\tau}\vartheta_2 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd\left(n(m+1)+\frac{(1+m+2k)(1+m)}{2m}\right)}\left(\int_{\tau=0}^{\left(\frac{m-(1+2k)}{2m}\right)d} e^{-r\tau}\vartheta_3 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd\left(n(m+1)+1+\frac{1+m+2k}{2}\right)}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_4 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd(n(m+1)+k+1)}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_5 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-4}{2}} e^{-rd\left(n(m+1)+k+\frac{3}{2}\right)}\left(\int_{\tau=0}^{\frac{d}{2}\frac{3+k(2-m)}{m}} e^{-r\tau}\vartheta_6 d\tau\right)\\
&+\sum_{n=0}^{\infty} e^{-rd\left(n(m+1)+\frac{m-1}{2}\right)}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_6 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd\left(n(m+1)+\frac{m+2}{2}+k\right)}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_7 d\tau\right)\\
&+\sum_{n=1}^{\infty} e^{-rd(n(m+1))}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_8 d\tau + e^{-r\frac{d}{2}}\int_{\tau=0}^{\frac{d}{2m}} e^{-r\tau}\vartheta_9 d\tau\right)\\
&+K\sum_{n=1}^{\infty}\left(e^{-rd\left(\frac{1}{2}+n\left(1+\frac{1}{m}\right)\right)}\right). \qquad (128)
\end{aligned}$$

In the same way as for player 1 we can now subtract from this fractional-deviation induced expected stream of losses the equilibrium expected stream of losses which is appropriately adapted to the considered time horizon. This yields

$$E[\mathcal{D}_2|\theta_0,\xi_0]-E[\mathcal{S}_2|\theta_0,\xi_0], \tag{129}$$

where

$$E[\mathcal{S}_2|\theta_0,\xi_0]=\Lambda_1\frac{2e^{rd/2}-2-rd}{e^{rd}r^2(e^{rd}-1)}+\Lambda_1\frac{e^{rd/2}(8+rd(2+\gamma)^2)-8-rd(8+4\gamma+\gamma^2)}{4e^{rd/2}r^2(e^{rd}-1)}+\frac{Ke^{rd/2}}{e^{rd}-1}.$$

Thus, it is profitable for firm 2 to deviate from a proposed synchronous inattentiveness period d in a non-marginal way if (129) is negative for at least one $m\in\mathbb{N}$.

Now building the difference quotient by dividing the respective terms of (129) by x/m, where again x is chosen apprppriately for the different intervals, and then letting $r\to 0$, we obtain that (129) is negative if

$$K>d^2\left(\Lambda_2(8-4\gamma)-\frac{\gamma^2\left(3\sigma_\xi^2+2\rho\sigma_\theta\sigma_\xi-\sigma_\theta^2-2\gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right)}{(2-\gamma)^2}\right)\times \tag{130}$$

$$\left(\frac{\Lambda_2\left(\frac{(2+\gamma)^2}{16}\right)m^3+\left(\Lambda_1\frac{1}{2}+\Gamma_2\frac{(2-\gamma)^2}{32}\right)m^2+\left(\Lambda_2\frac{(2-\gamma)^2}{4}+\Gamma_1\frac{(2+\gamma)^2}{16}\right)m+\Lambda_2\left(\frac{12+2\gamma}{(2-\gamma)^2}\right)+\Gamma_3\left(\frac{8+4\gamma+\gamma^2}{32}\right)}{\Lambda_2\left(\frac{(2+\gamma)^2}{16}\right)m^3+\Lambda_2\left(\frac{4+4\gamma+\gamma^2}{16}\right)m^2+\Gamma_2\left(\frac{(2+\gamma)^2(2-\gamma)}{32}\right)m}\right).$$

As above, via differentiating the second term on the right-hand side of (116), one can check that it is strictly decreasing in m. Therefore, letting $m\to\infty$ is the most profitable deviation for firm 2. We can then write (130) as

$$K>d^2\left(\Lambda_2(8-4\gamma)-\frac{\gamma^2\left(3\sigma_\xi^2+2\rho\sigma_\theta\sigma_\xi-\sigma_\theta^2-2\gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right)}{(2-\gamma)^2}\right). \tag{131}$$

Now letting $r\to 0$ in (46) stated in Lemma 5, i.e., in the condition that gives the lower bound from the marginal deviation of player, $\underline{d}_a^2$, we obtain that $\underline{d}_a^2$ is (implicitly) given by

$$K=(\underline{d}_a^2)^2\left(\Lambda_2(8-4\gamma)-\frac{\gamma^2\left(3\sigma_\xi^2+2\rho\sigma_\theta\sigma_\xi-\sigma_\theta^2-2\gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right)}{(2-\gamma)^2}\right). \tag{132}$$

But since only $d \geq \underline{d}_a^2$ can be potential equilibria, combining (131) and (132), we obtain that (131) can never be satisfied for any $d \geq \underline{d}_a^2$. This implies that the non-marginal deviation of firm 2 to a period length of $d + d/m$ with m being an even number does not eliminate any potential equilibria. By the same argument, even if we consider deviations of firm 2 to period lengths of $d(1 + l/m)$, $l \leq m$, this does not eliminate any potential equilibria since $m \to \infty$ is the most profitable deviation in this case either.

We can proceed in the same way for non-marginal deviations in which firm 2 chooses a period length of $d + d/m$ with m being an odd number, and sticks to the (A) mode and also for non-marginal deviations in which firm 2 sets a period length of $d - d/m$ with m being an odd or even number, and sticks to the (A) mode. In all cases we obtain that in the limit as $r \to 0$, the deviation to $m \to \infty$ excludes the largest set of inattentiveness period length, and that this set is equivalent to the one that is excluded by the marginal deviations. Thus, non-marginal deviations in which firm 2 chooses the (A) mode do not exclude any potential equilibria in the limit as $r \to 0$. By continuity, if r is positive but sufficiently close to 0, the result applies as well.

We now turn to the case in which firm 2 deviates to the (S) mode, that is, to sequence of planning dates

$$\left\{ d\left(1 \pm \frac{1}{m}\right), 2d\left(1 \pm \frac{1}{m}\right), ... \right\}.$$

Proceeding in the same way as above, i.e., building the differential quotient between the expected stream of losses and the equilibrium loss and letting r go to zero, we obtain that independent of the inattentiveness length being $d(1-1/m)$ or $d(1+1/m)$ the most profitable deviation is the one in which $m \to \infty$, i.e., the sequence $\{d, 2d, ...\}$.

Now, we determine the condition under which it is not profitable for firm 2 to deviate to the RTS deviation. We do so by determining the differential quotient between the RTS and the equilibrium sequence for r in a neighborhood of zero. This yields that deviating to the RTS sequence is not profitable if and only if

$$rK \leq \frac{d}{8e^{3rd}(2-\gamma)^2}\Big(e^{5rd/2}\left(4\gamma^2(\sigma_\theta^2 + \rho\sigma_\theta\sigma_\xi) + 2\gamma^3(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi) - 4\Lambda_1(\gamma^4 + 4)\right) \tag{133}$$

$$+8\Lambda_1 e^{2rd}(2-\gamma)^2 + 4(1 + e^{rd/2} - e^{rd})\gamma^2\left(\Lambda_1\gamma^2 - (\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\gamma + \sigma_\xi^2 - \sigma_\theta^2\right)\Big).$$

Thus, only those $d_a^\star$ that satisfy (133) are not eliminated by the RTS deviation. Since no potential equilibrium inattentiveness period is eliminated by the non-marginal deviations of firm 1 and the other non-marginal deviations of firm 2, the result of the lemma follows. ■

Proof of Proposition 2

On (i): Taking the limit $r \to 0$ of (133) this inequality can be written as

$$0 \leq -\frac{\gamma d_a^\star \left(4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - \gamma(\sigma_\theta^2 + 6\rho\sigma_\theta\sigma_\xi + 5\sigma_\xi^2) + 2\gamma^2(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\right)}{8(2-\gamma)^2}. \tag{134}$$

Therefore, in the limit as $r \to 0$ an alternating planning equilibrium can only exist for $\gamma > 0$ if and only if

$$\rho < \max\left[-\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}, -1\right]. \tag{135}$$

From the proof of Lemma 8 we also know that, as $r \to 0$, marginal deviations are not profitable if

$$K \leq (d_a^\star)^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2\left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32}\right).$$

Suppose for the moment that $K \to 0$. Then solving the last inequality for ρ we obtain that marginal deviations are not profitable if

$$\rho \geq -\frac{4(4-\gamma)(\sigma_\theta^2 + \sigma_\xi^2) - \gamma^2(11\sigma_\xi^2 - \sigma_\theta^2) + 6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16 - 4\gamma - 5\gamma^2 + 3\gamma^3)}. \tag{136}$$

We can now check if conditions (135) and (136) can be jointly satisfied. To do so we calculate

$$-\frac{4(4-\gamma)(\sigma_\theta^2 + \sigma_\xi^2) - \gamma^2(11\sigma_\xi^2 - \sigma_\theta^2) + 6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16 - 4\gamma - 5\gamma^2 + 3\gamma^3)} - \left(-\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}\right),$$

which yields

$$-\frac{\gamma(2-\gamma)^2(\sigma_\xi - \sigma_\theta)(\sigma_\xi + \sigma_\theta)}{\sigma_\theta\sigma_\xi(16 - 4\gamma - 5\gamma^2 + 3\gamma^3)(4 - 3\gamma + \gamma^2)}.$$

For both conditions to be jointly satisfied at $\gamma > 0$ we must have that the latter term is strictly negative which can only hold true if $\sigma_\xi > \sigma_\theta$. But we have that at $\sigma_\xi > \sigma_\theta$

$$-\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)} < -1,$$

thereby ruling out an alternating planning equilibrium. For $K > 0$ condition (136) becomes even tighter. Thus, an alternating planning equilibrium does not exists for this case either. We have shown the result for $r \to 0$. However, by continuity, we must have that even if r is positive but close to 0, an alternating planning equilibrium can not exist for $\gamma > 0$.

On (ii): From part (i) of this proposition it follows that for r close to zero an alternating planning equilibrium can only exist for $\gamma < 0$. Lemmas 7 and 8 imply that even for $\gamma < 0$ the equilibrium only exists if $\rho \geq \max[\check{\rho}, \rho^+]$. Finally, it follows from Lemmas 5 and 6 that the maximum range of this equilibrium is given by the marginal deviations of player 1, that is $d_a^\star \in [\underline{d}_a^1, \bar{d}_a^1]$.

On (iii): The equilibrium inattentiveness lengths $\underline{d}_a^\star$ and $\bar{d}_a^\star$ are characterized by (44) and (45). Solving each of the two equations for K and then taking the limit $r \to 0$, we obtain that $\underline{d}_a^\star = \bar{d}_a^\star = d_a^\star$, where $d_a^\star$ is characterized by

$$K = (d_a^\star)^2\left(\Lambda_2\frac{4-\gamma}{8} + \frac{\gamma^2\left(6\gamma(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi^2)+\sigma_\theta^2-10\rho\sigma_\theta\sigma_\xi-11\sigma_\xi\right)}{32}\right). \tag{137}$$

The right-hand side of (137) can only be positive if

$$\rho \geq -\frac{4(4-\gamma)(\sigma_\theta^2+\sigma_\xi^2)-\gamma^2(11\sigma_\xi^2-\sigma_\theta^2)+6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16-4\gamma-5\gamma^2+3\gamma^3)}. \tag{138}$$

From the proof of part (i) of this proposition we know that this is also the tightest bound on ρ such that marginal deviations are not profitable. In addition, an alternating equilibrium can only exist if (134) is satisfied. The proof of part (i) of this proposition implies that (134) and (138) can jointly only be satisfied if $\gamma < 0$ and

$$\rho \geq -\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)}.$$

As a consequence, the alternating planning equilibrium exists if and only if $\gamma < 0$ and

$$\rho \geq \max\left[-\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)}, -\frac{4(4-\gamma)(\sigma_\theta^2+\sigma_\xi^2)-\gamma^2(11\sigma_\xi^2-\sigma_\theta^2)+6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16-4\gamma-5\gamma^2+3\gamma^3)}, -1\right].$$

■

Proof of Proposition 3

On (i): We know that for $r \to 0$, $\underline{d}_a^\star = \bar{d}_a^\star = d_a^\star$, where $d_a^\star$ is defined by (137). The left-hand side of (137) is constant in $d_a^\star$ and γ while the right-hand side of (137) is strictly increasing in $d_a^\star$. Now differentiating the right-hand side with respect to γ we obtain

$$\frac{3d_a^\star\left(4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi\sigma_\xi^2)-\gamma(8-6\gamma+\gamma^2)(\sigma_\xi+\rho\sigma_\theta\sigma_\xi)\right)}{16(2-\gamma)^3} > 0,$$

where the inequality sign stems from the fact that at the equilibrium $\gamma < 0$. It therefore follows that after an increase in γ, (137) can only be fulfilled if $d_a^\star$ falls, yielding that $dd_a^\star/d\gamma < 0$.

On (ii): Now consider the case where $r > 0$ but small such that the difference between $\underline{d}_a^\star$ and $\bar{d}_a^\star$ is small. We know that $\underline{d}_a^\star$ and $\bar{d}_a^\star$ are determined by (44) and (45). As is easy to see the last three terms on the left-hand side of both equations have the same structure and only differ because $\underline{d}_a^\star$ and $\bar{d}_a^\star$ differ while the first term is different even if $\underline{d}_a^\star$ and $\bar{d}_a^\star$ were the same. Since we consider the case in which $\underline{d}_a^\star$ and $\bar{d}_a^\star$ are very close to each other, we can concentrate on the difference in the respective first terms in determining if $\underline{d}_a^\star$ and $\bar{d}_a^\star$ change differently with γ.

Totally differentiating the first term of (44) we obtain

$$\frac{d\underline{d}_a^\star}{d\gamma} = -\frac{8\underline{d}_a^\star}{(2-\gamma)(8-4\gamma+\gamma^2)(1-r\underline{d}_a^\star)} < 0,$$

while totally differentiating the first term of (45) we obtain

$$\frac{d\bar{d}_a^\star}{d\gamma} = -\frac{\bar{d}_a^\star(8+4\gamma+\gamma^2)}{2(2+\gamma)(1-r\bar{d}_a^\star)} < 0.$$

Now subtracting $d\underline{d}_a^\star/d\gamma$ from $d\bar{d}_a^\star/d\gamma$ and taking into account that $\underline{d}_a^\star$ is close to $\bar{d}_a^\star$ we obtain

$$\frac{d\bar{d}_a^\star}{d\gamma} - \frac{d\underline{d}_a^\star}{d\gamma} \approx -\frac{\bar{d}_a^\star(96 - 80\gamma + 2\gamma^4 - \gamma^5)}{2(2+\gamma)(2-\gamma)(8-4\gamma+\gamma^2)(1 - r\bar{d}_a^\star)}.$$

Since r is small, the denominator is positive and so the whole expression is negative. But, since both $d\underline{d}_a^\star/d\gamma$ and $d\bar{d}_a^\star/d\gamma$ are strictly decreasing in γ, this implies that $\bar{d}_a^\star$ decreases by more than $\underline{d}_a^\star$ if γ rises. Thus, the equilibrium range of inattentiveness periods shrinks as γ rises. ■

Proof of Lemma 9

Suppose that firm i expects that firm j chooses an inattentiveness period of length $d \in \mathbb{R}^+$ at $D_i(0) = D_j(0) = 0$. The sequence of planning dates induced by d can only be an equilibrium if neither of the following infinitesimal deviations are profitable for firm i: choose a longer inattentiveness period, that is deviate to $d' = d + \triangle$, with $\triangle > 0$, or a shorter period, i.e. $d'' = d - \triangle$. In order to simplify the exposition we assume that the shock realizations at 0 are given by $\theta(0) = 0$ and $\xi(0) = 0$. This assumption is without loss of generality because the shock realizations do not influence a firms' deviation incentive.

In order to derive the "no-deviation" conditions we have to compare firm i's expected loss from following the proposed equilibrium sequence with the expected losses from the sequences $\mathcal{D}' = \{d', 2d', 3d', ...\}$ and $\mathcal{D}'' = \{d'', 2d'', 3d'', ...\}$.

Lifetime expected loss for $\mathcal{D}$

We know from Lemma 5 that firm i's expected instantaneous loss from following the proposed equilibrium sequence at any instant t, with $nd \leq t < (n+1)d$, for all $n \in \mathbb{N}_0$, is given by

$$E[\mathcal{L}^e | I_0] = \Lambda_1 \tau,$$

where τ denotes the time that elapsed since the last planning date, that is $\tau = t - nd$, for all $n \in \mathbb{N}_0$.

Thus, the lifetime expected loss implied by any candidate equilibrium inattentiveness interval is

$$E[\mathcal{L}(\mathcal{D})|I_0] = \frac{e^{rd} - 1 - rd}{e^{rd} r^2 (e^{rd} - 1)} \Lambda_1 + \frac{K}{e^{rd} - 1}. \tag{139}$$

In order to consistently transform expected profits into expected losses we scale the expected stream of profits under the infinitesimal deviations by the equilibrium expected full information profits. In the synchronous planning equilibrium the expected full information profit a firm earns

$$\begin{aligned} E[\Pi^{FI}(t)|I_0] \;&=\; \Big(\frac{\sigma_\theta^2 - 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2}{4}\Big)\tau + \Big(\frac{\sigma_\theta^2 - 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2 - \gamma\sigma_\xi^2(2-\gamma) + 2\gamma\rho\sigma_\theta\sigma_\xi}{(2-\gamma)^2}\Big) n d^\star \\ &\quad + \frac{(\alpha - (1-\gamma)c)}{(2-\gamma)^2}. \end{aligned} \tag{140}$$

Lifetime expected loss for $\mathcal{D}'$

We set out by introducing some notation. Let $\tilde{m} \in \mathbb{N}$ denote a natural number for which $\tilde{m}\triangle \leq d$ and $(\tilde{m}+1)\triangle > d$.

In the following we derive the lifetime expected loss from the sequences $\mathcal{D}'$. First, we turn to the expected instantaneous loss that firm i incurs in the interval $[d, d']$ if it deviates to the sequence $\mathcal{D}'$.

From d to $d' = d + \triangle$

If firm i deviates to $\mathcal{D}'$ then its expected profit function at instant $d+\tau$, $\tau \in [0, \triangle)$ is given by

$$\begin{aligned} E[\Pi(d+\tau)|I_0^i, I_0^j] \;&=\; (\alpha - p_i(d+\tau) + \gamma E[p_j(d+\tau)|I_0^i, I_0^j])(p_i(d+\tau) - c) \\ &\quad - \; E[\xi(d+\tau)(\theta(d+\tau) + \gamma p_j(d+\tau))|I_0^i, I_0^j]. \end{aligned} \tag{141}$$

In this interval firm i is the firm with the more outdated information est. However, firm j has not detected firm i's deviation. Thus, firm j still believes that both firms share the same information set I_d. Hence, it follows that firm $j's$ optimal price at $d+\tau$ is given by

$$p_j^\star(d+\tau) = \frac{\alpha + \theta(d) + c + \xi(d)}{2-\gamma}. \tag{142}$$

Therefore,

$$p_i^\star(d+\tau) = E[p_j^\star(d+\tau)|I_0^i, I_0^j] = \frac{\alpha+c}{2-\gamma}. \tag{143}$$

Using (143) and (142) in (141) yields that firm i's expected instantaneous profit under $\mathcal{D}'$ in the interval $t \in [d, d']$ is given by

$$E[\Pi^\star(d+\tau)|I_0^i, I_0^j] = -\frac{2\rho\sigma_\theta\sigma_\xi + \gamma\sigma_\xi^2}{2-\gamma}d - \rho\sigma_\theta\sigma_\xi\tau + \frac{(\alpha-(1-\gamma)c)}{(2-\gamma)^2}. \tag{144}$$

Subtracting (144) from (140) yields that firm i's expected instantaneous loss in the period $[d, d+\triangle)$ is

$$E[\mathcal{L}_1'|I_0] = \Lambda_2 d + \Lambda_1\tau, \tag{145}$$

with $\tau \in [0, \triangle)$.

From d' to $2d$

In this time interval firm j still believes that firm i sticks to the proposed equilibrium strategy. Thus, its optimal price is at each instant given by (142). Firm i acquires new information at instant $d' = d + \triangle$. Therefore, its expected profit function is in this interval given by

$$\begin{aligned} E[\Pi(d'+\tau)|I_{d'}^i, I_d^j] &= (\alpha - p_i(d'+\tau) + \gamma p_j(d'+\tau))(p_i(d'+\tau) - c) \\ &- E[\xi(d'+\tau)(\theta(d'+\tau) + \gamma p_j(d'+\tau))|I_{d'}^i, I_d^j], \end{aligned} \tag{146}$$

with $\tau \in [0, d-\triangle)$. Proceeding in the same way as before, we obtain that

$$p_i^\star(d'+\tau) = \frac{\alpha + \theta(d') + c + \xi(d')}{2} + \frac{\gamma\Big(\alpha + \theta(d) + c + \xi(d)\Big)}{2(2-\gamma)}. \tag{147}$$

Using (147) and (142) in (146), taking expectations and subtracting the resulting expression from (140) yields for the interval $[d', 2d)$ the following instantaneous loss function

$$E[\mathcal{L}_2'|I_0] = \Lambda_1\tau, \tag{148}$$

with $\tau \in [0, d-\triangle)$.

From $2d$ to $2d'$

At $2d$ firm j detects, that firm i deviated to the sequence $\mathcal{D}'$. This implies that firm j knows that firm i's consecutive planning dates will be at nd', for $n \in \mathbb{N} \setminus \{1\}$. As firm j is in the interval $[2d, 2d')$ better informed than firm i, the latter's expected profit function equals

$$\begin{aligned} E[\Pi(2d+\tau)|I^i_{d'}, I^j_{d'}] &= (\alpha - p_i(2d+\tau) + \gamma E[p_j(2d+\tau)|I^i_{d'}, I^j_{d'}](p_i(2d+\tau) - c) \\ &- E[\xi(2d+\tau)(\theta(2d+\tau) + \gamma p_j(2d+\tau))|I^i_{d'}, I^j_{d'}], \end{aligned} \tag{149}$$

with $\tau \in [0, 2\triangle)$. As a consequence the firms' optimal prices in this period are given by

$$p^\star_j(2d+\tau) = \frac{\alpha + \theta(2d) + c + \xi(2d)}{2} + \frac{\gamma\Big(\alpha + \theta(d') + c + \xi(d')\Big)}{2(2-\gamma)}, \tag{150}$$

$$p^\star_i(2d+\tau) = E[p^\star_j(2d+\tau)|I^i_{d'}, I^j_{d'}] = \frac{\alpha + \theta(d') + c + \xi(d')}{2-\gamma}. \tag{151}$$

Using (151) and (150) in (149), taking expectations and subtracting the resulting expression from the corresponding full information profit, which is obtained by replacing d by $2d$ in (140), yields the instantaneous expected loss in this interval, which is given by

$$E[\mathcal{L}'_3|I^i_{d'}, I^j_{d'}] = \Gamma_1(d - \triangle) + \Lambda_1\tau, \tag{152}$$

with $\tau \in [0, 2\triangle)$. Due to the fact that firm j knows firm i's consecutive planning dates, the instantaneous expected loss is in each interval in which firm i is the firm with worse information, that is in $[nd, nd']$, for $n \in \{2, ..., \tilde{m}\}$, given by

$$E[\mathcal{L}'_3|I_0] = \Gamma_1(d - n\triangle) + \Lambda_1\tau,$$

with $\tau \in [0, d - n\triangle)$ and $n \in \{2, ..., \tilde{m}\}$.

From nd' to $(n+1)d$, for $n \in \{2, ..., \tilde{m}\}$

These intervals share the feature that firm j's information set is more outdated. It is easy to see that the firm with the worse information in an alternating planning pattern sets the same optimal price as a firm in the synchronous planning pattern. Thus, it follows immediately, that the expected instantaneous loss is in every interval $[nd', (n+1)d)$, for $n \in \{2, ..., \tilde{m}\}$, given by

$$E[\mathcal{L}_2'|I_0] = \Lambda_1\tau, \tag{153}$$

with $\tau \in [0, d - n\triangle)$ and $n \in \{2, ..., \tilde{m}\}$.

The above analysis implies, that the expected loss from the sub-sequence $\mathcal{D}'_{\tilde{m}} = \{d', ..., \tilde{m}d'\}$ is given by

$$\begin{aligned} E[\mathcal{L}'(\mathcal{D}'_{\tilde{m}})|I_0] &= e^{-rd}\int_{\tau=0}^{\triangle} e^{-r\tau}(\Lambda_2 d + \Lambda_1\tau)\, d\tau \\ &+ \sum_{n=1}^{\tilde{m}}\left(e^{-rnd'}\int_{\tau=0}^{d-n\triangle} e^{-r\tau}\Lambda_1\tau d\tau\right) + K\sum_{n=1}^{\tilde{m}} e^{-rnd'} \\ &+ \sum_{n=2}^{\tilde{m}}\left(e^{-rnd}\int_{\tau=0}^{n\triangle} e^{-r\tau}\Big(\Gamma_1(d-n\triangle)+\Lambda_1\tau\Big)d\tau\right). \end{aligned} \tag{154}$$

Subtracting the expected loss implied by the equilibrium sub-sequence $\mathcal{D}_{\tilde{m}} = \{d, ..., \tilde{m}d\}$, denoted by $E[\mathcal{L}(\mathcal{D}_{\tilde{m}})|I_0]$, from (154) yields

$$\begin{aligned} & e^{-rd}\int_{\tau=0}^{\triangle} e^{-r\tau}(\Lambda_2 d)\, d\tau - \sum_{n=1}^{\tilde{m}}\left(e^{-rnd'}\int_{\tau=0}^{d-n\triangle} e^{-r\tau}\Lambda_1 n\triangle d\tau\right) + K\Big(\sum_{n=1}^{\tilde{m}} e^{-r(nd')} - \sum_{n=1}^{\tilde{m}} e^{-r(nd)}\Big) \\ &+ \sum_{n=2}^{\tilde{m}}\left(e^{-rnd}\int_{\tau=0}^{n\triangle} e^{-r\tau}\Gamma_1(d-n\triangle)d\tau\right) + e^{-r\tilde{m}d}\Upsilon. \end{aligned} \tag{155}$$

where Υ denotes the difference in expected losses beyond date $\tilde{m}d$. This difference in expected losses is at each instant $t > \tilde{m}d$ bounded below by the expected instantaneous loss implied by the sequence $\mathcal{D}$. Again we get this by assuming the best possible case for the deviation, that is, the expected instantaneous loss that is implied by the sequence $\mathcal{D}'$ is zero from date $\tilde{m}d$ onwards. In addition, we know that the instantaneous loss from the sequence $\mathcal{D}$ is finite since firm i optimally chooses to plan after some time length. Thus, at each instant $t > \tilde{m}d$ the difference in expected losses is bounded below by

$$-e^{-r\tau}\Lambda_1 d,$$

for $\tau \in (0, d]$ and

$$-e^{-rd}K,$$

at a planning date d. Both expressions are finite because d is finite.

In order to determine the per instant difference in expected losses in the time period before date $\tilde{m}d$, we divide the first four terms in (155) by $n\triangle$, where n is chosen appropriately for the different intervals. Then, we take the limit $\triangle \to 0$. The last term in (155) $e^{-r\tilde{m}d}\Upsilon$ vanishes as $\triangle \to 0$ implies that $\tilde{m} \to \infty$ and the future is discounted at rate $r > 0$. Therefore, we can concentrate on the first four terms when determining the critical inattentiveness length such that deviating to the sequence $\mathcal{D}'$ is profitable.

We get that the lifetime expected loss from deviating to $\mathcal{D}'$ is lower than the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \underline{d}_s$, where $\underline{d}_s$ solves

$$e^{-r\underline{d}_s}\underline{d}_s\Lambda_2 - e^{-r\underline{d}_s}\frac{\Lambda_1}{r} - \frac{rK}{e^{r\underline{d}_s}-1} + \frac{\underline{d}_s e^{-r\underline{d}_s}}{e^{r\underline{d}_s}-1}\Gamma_1 = 0. \tag{156}$$

If $\underline{d}_s \to 0$, the left-hand side of (156) goes to $-\infty$ because the term involving $-rK < 0$ is dominating term. Conversely, if $\underline{d}_s \to \infty$, the left-hand side of (156) goes to 0 from above. This is the case because the last three terms go to zero at a faster rate than the first term and the first term is strictly positive since $\Lambda_2 > 0$. Thus, there exists a solution to (156) at which $\underline{d}_s > 0$. It remains to show that this solution is unique. To do so we differentiate (156) with respect to $\underline{d}_s$ to get

$$e^{-r\underline{d}_s}\Lambda_1 + \frac{r^2 e^{r\underline{d}_s}K}{(e^{r\underline{d}_s}-1)^2} - \Gamma_1\left(\frac{e^{-r\underline{d}_s} + r\underline{d}_s(2 - e^{-r\underline{d}_s} - 1}{(e^{r\underline{d}_s}-1)^2}\right) + \Lambda_2 e^{-r\underline{d}_s}(1 - r\underline{d}_s). \tag{157}$$

It is easy to check that for $\underline{d}_s \to 0$, (157) goes to ∞ since $K > 0$, while for $\underline{d}_s \to \infty$, (157) goes to 0 from below. This is the case because the term $-\Lambda_2 e^{-r\underline{d}_s} r\underline{d}_s < 0$ goes to zero at a slower rate than the other terms. We now look at the four terms of (157) in turn. The first two terms are strictly positive and strictly decrease as $\underline{d}_s$ rises. The third term is either positive or negative for any $\underline{d}_s$, dependent on Γ_1 being positive or negative. As the frist two terms, it becomes strictly smaller in absolute value as $\underline{d}_s$ increases but it does so at a higher rate than the first two terms. Finally, the fourth term is the only term in (157) that changes its sign from positive to negative as $\underline{d}_s$ increases which occurs at $\underline{d}_s = 1/r$. In addition, this fourth term becomes the dominant term as $\underline{d}_s$ gets larger and larger. As a consequence, there exist a unique value of $\underline{d}_s$ at which (157) changes its sign from positive to negative. But this, in combination with the fact that the left-hand side of (156) is negative at $\underline{d}_s = 0$ and positive at $\underline{d}_s \to \infty$, implies that there must exist a unique solution to (156). As a consequence, we have that if $d \geq \underline{d}_s$ then the lifetime expected loss from deviating to $\mathcal{D}'$ exceeds the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$.

Using a similar derivation we obtain that the lifetime expected loss from deviating to $\mathcal{D}''$ exceeds the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \bar{d}_s$, where $\bar{d}_s$ solves

$$\bar{d}_s\Lambda_1 - rK - \left(\frac{e^{r\bar{d}_s} - 1}{re^{r\bar{d}_s}}\right)\Gamma_1 = 0. \tag{158}$$

Existence and uniqueness of $\bar{d}_s$ are implied by arguments similar to the ones developed for $\underline{d}_s$. ■

Proof of Lemma 10

It follows from (51) and (52) that $\bar{d}_s = \underline{d}_s = \hat{d}_s$ if $\rho = \hat{\rho}$, where

$$\hat{\rho} = \frac{\delta_1 e^{r\hat{d}_s} + \delta_2}{2\sigma_\theta\sigma_\xi\Big((4(1 + r\hat{d}_s) + \gamma^2 - \gamma(3 + r\hat{d}_s))e^{r\hat{d}_s} + \delta_3\Big)}, \tag{159}$$

with

$$\begin{aligned}
\delta_1 &:= (\gamma(r\hat{d}_s + 5)\sigma_\xi^2 + (r\hat{d}_s + 1)\sigma_\theta^2) - 2\gamma^2\sigma_\xi^2 - 4(\sigma_\theta^2 + \sigma_\xi^2)(r\hat{d}_s + 1), \\
\delta_2 &:= 4(\sigma_\theta^2 + \sigma_\xi^2) + \gamma((4r\hat{d}_s - 5)\sigma_\xi^2 - \sigma_\theta^2) - 2\gamma^2\sigma_\xi^2(r\hat{d}_s - 1), \\
\delta_3 &:= -4 - \gamma(2r\hat{d}_s - 3) - \gamma^2(1 - r\hat{d}_s).
\end{aligned}$$

Differentiating $\hat{\rho}$ with respect to r yields

$$\frac{\hat{d}_s\gamma(\sigma_\theta^2 - \sigma_\xi^2)(2-\gamma)(4-\gamma)(1 + e^{2r\hat{d}_s} + (\hat{d}_s r)^2 e^{r\hat{d}_s} - 2e^{r\hat{d}_s})}{2\sigma_\theta\sigma_\xi\Big((4(1 + r\hat{d}_s) + \gamma^2 - \gamma(3 + r\hat{d}_s))e^{r\hat{d}_s} + \delta_3\Big)^2}. \tag{160}$$

Thus,

$$\text{sign}\Big\{\frac{\partial\hat{\rho}}{\partial r}\Big\} = \text{sign}\Big\{\gamma(\sigma_\theta^2 - \sigma_\xi^2)\Big\}.$$

Therefore, $\hat{\rho}$ is decreasing in r if either $\sigma_\theta > \sigma_\xi$ and $\gamma < 0$ or $\sigma_\theta < \sigma_\xi$ and $\gamma > 0$. If these conditions are not met, then $\hat{\rho}$ is increasing in r.

If $\hat{\rho}$ is decreasing in r, then we obtain the upper bound of $\hat{\rho}$, which we denote as $\bar{\rho}$, by taking the limit $r \to 0$. We get that

$$\bar{\rho} = \lim_{r \to 0} \hat{\rho} = -\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}.$$

Thus, $\bar{\rho} = \rho'$.

If $\hat{\rho}$ is increasing in r, we obtain the upper bound of $\hat{\rho}$, which we denote as $\tilde{\rho}$, by taking the limit $r \to \infty$. We get

$$\tilde{\rho} = \lim_{r \to \infty} \hat{\rho} = -\frac{\sigma_\theta^2 + \sigma_\xi^2}{2\sigma_\theta\sigma_\xi}.$$

It is easy to show that $\tilde{\rho} < -1$. ■

Proof of Proposition 4

We constructed the range of candidate equilibrium inattentiveness intervals by assuming that a firm may only deviate marginally from the proposed equilibrium planning horizon. Thus, we have to complement the analysis by considering non-infinitesimal deviations.

Suppose for the remainder of the proof that $\rho > \hat{\rho}$ so that $\underline{d} \leq \bar{d}$. As in the alternating planning scenario we restrict the exposition to fractional deviations: firm j sticks to the proposed inattentiveness period $d \in [\underline{d}, \bar{d}]$ and firm i chooses to remain inattentive for a period of $d\,(1 \pm l/m)$, $m \in \mathbb{N}$ and $l \in \{l \in \mathbb{N} : l \leq m\}$.

The proof proceeds as follows. First, we show that the fractional deviations of the type $d\,(1 \pm 1/m)$ induce the tightest bounds on the range of potential equilibria. Second, we derive a firm's expected lifetime loss from deviating non-infinitesimally from any candidate equilibrium inattentiveness interval. Third, we characterize the tightest bounds that this type of non-marginal deviation imposes on the range of inattentiveness periods that are robust against marginal deviations.

Comparison of the $d(1 \pm l/m)$, $l > 1$, and $d(1 \pm 1/m)$ deviation

Generally, a deviation to a sequence $d(1 + l/m)$, $l > 1$ induces higher expected losses net of planning cost than a deviation to the sequence $d(1 + 1/m)$. This is due to the fact that with the former type of deviation firm i is the worse informed firm for a longer time. The advantage of the former over the latter lies in the planning cost reduction. Thus, firm i prefers the $d(1 + 1/m)$ to the $d(1 + l/m)$, $l > 1$, deviation if

$$E\left[\mathcal{S}\left(\left(1+\frac{l}{m}\right)d\right)\right]-E\left[\mathcal{S}\left(\left(1+\frac{1}{m}\right)d\right)\right]-K\frac{e^{\frac{rd(m+l)}{m}}-e^{\frac{rd(m+1)}{m}}}{\left(e^{\frac{rd(m+1)}{m}}-1\right)\left(e^{\frac{rd(m+l)}{m}}-1\right)}\geq 0, \quad (161)$$

where $E\left[\mathcal{S}((1 + l/m)\, d)\right]$ denotes the expected losses net of planning cost induced by the sequence $(1 + l/m)\, d$ and $E\left[\mathcal{S}((1 + 1/m)\, d)\right]$ denotes the expected losses net of planning cost induced by the sequence $(1 + 1/m)\, d$.

For $d \rightarrow 0$ and $d \rightarrow \infty$ the net expected losses induced by both sequences are identical. In the first case both firms plan at each instant whereas in the case both firms never plan. Thus, in both cases the difference between the net expected losses is zero. Thus, for $d \rightarrow 0$ the left-hand side of (161) converges to $-\infty$. For $d \rightarrow \infty$ the left-hand side of (161) goes to zero. We know from the arguments developed in Lemma 9 concerning existence and uniqueness that the left-hand side of (161) is first increasing and then decreasing in d. Thus, there exists a unique d, denoted by $\hat{d}$, so that the inequality in (161) holds for all $d > \hat{d}$. Thus, for given m the fractional deviation of type $d(1 + 1/m)$ induces the largest lower bound on the set of potential equilibrium inattentiveness periods as compared to fractional deviations of the type $d(1 + l/m)$, $l > 1$.

Similar arguments imply that for given m the fractional deviation of type $d(1 - 1/m)$ induces the lowest upper bound on the set of potential equilibrium inattentiveness periods as compared to fractional deviations of the type $d(1 - l/m)$, $l > 1$.

Now, we derive firm i's expected stream of losses if it deviates to an inattentiveness interval of length $d(1 + 1/m)$, $m \in \mathbb{N}$.

$d(1+1/m)$: **Expected instantaneous losses**

Suppose that firm i deviated to $d + d/m$, $m \in \mathbb{N}$, whereas firm j sticks to the equilibrium planning horizon d. This deviation induces a stream of expected losses that is composed of three different instantaneous expected losses:

$$\nu_1 \quad := \quad \Lambda_1\tau + \Lambda_2 d, \tag{162}$$

$$\nu_2 \quad := \quad \Lambda_1\tau, \tag{163}$$

$$\nu_3 \quad := \quad \Lambda_1\tau + \Gamma_1\Big(\frac{m-k}{m}\Big)d,\ k \in \mathbb{N},\ k < m. \tag{164}$$

The expected instantaneous loss in the period that elapses between the first planning date on the equilibrium path and the first planning date under the deviation are given by (162). In this period, firm i's deviation is undetected by firm j. The component (163) captures firm i's expected instantaneous losses in the time period in which it is the better informed firm, irrespective of whether the deviation was detected or not. (164) captures firm i's expected instantaneous losses in the time period in which it is the worse informed firm after its deviation has been detected.

The counter $((m-k)/m)d$, $k \in \{0, ..., m-1\}$ in (164) measures the time that elapses between a planning date of firm i and the consecutive planning date of firm j. If, e.g., $m = 5$ and $k = 4$, then the time that elapses between the kth planning date of firm i and the $(k+1)$th planning date of firm j is given by $(k+1)d - k(1+1/m)d = 5d - 4(1+1/5)d = 1/5d$. It also becomes evident from this example that the firms plan simultaneously every $(m+1)d$ periods: firm i repeated its planning horizon exactly m times and firm j planned for the $(m+1)$th time at this date. This is due to the fact that we consider deviations in which the deviant expands its inattentiveness period by a fraction.

$d(1+1/m)$: **Expected stream of losses**

Given the expected instantaneous expected losses (162) to (164) a deviant's expected stream of losses is given by

$$\begin{aligned} E[\underline{S}|\theta_0,\xi_0] &= e^{-rd}\int_0^{\frac{d}{m}} e^{-r\tau}\nu_1 d\tau + \sum_{k=1}^{m-1}\left(e^{-r\left(1+\frac{1}{m}\right)kd}\left(\int_{\tau=0}^{\left(\frac{m-k}{m}\right)d} e^{-r\tau}\nu_2 d\tau\right)\right) \\ &+ \sum_{n=1}^{\infty}\sum_{k=0}^{m-1}\left(e^{-r\left(n(m+1)+\left(1+\frac{1}{m}\right)k\right)d}\left(\int_{\tau=0}^{\left(\frac{m-k}{m}\right)d} e^{-r\tau}\nu_2 d\tau\right)\right) \\ &+ \sum_{k=1}^{m-1}\left(e^{-r(1+k)d}\left(\int_{\tau=0}^{\left(\frac{k+1}{m}\right)d} e^{-r\tau}\nu_3 d\tau\right)\right) \\ &+ \sum_{n=1}^{\infty}\sum_{k=0}^{m-1}\left(e^{-r(n(m+1)+1+k)d}\left(\int_{\tau=0}^{\left(\frac{k+1}{m}\right)d} e^{-r\tau}\nu_3 d\tau\right)\right) \\ &+ K\sum_{n=1}^{\infty}\left(e^{-rn\left(1+\frac{1}{m}\right)d}\right). \end{aligned} \tag{165}$$

Now, we derive the deviant's expected stream of losses if it chooses an inattentiveness interval of length $d - d/m$, $m \in \mathbb{N}$ instead of d.

$d(1 - 1/m)$**: Expected instantaneous losses**

Suppose that firm i deviated to $d - d/m$, $m \in \mathbb{N}$, whereas firm j sticks to the equilibrium planning horizon d. This deviation induces a stream of expected losses that is composed of two different instantaneous expected losses:

$$\phi_1 := \Lambda_1\tau, \tag{166}$$

$$\phi_2 := \Lambda_1\tau + \Gamma_1\left(\frac{k}{m}\right)d,\ k \in \mathbb{N},\ k < m. \tag{167}$$

In this scenario, the deviation is detected by firm j at its first planning date. The component (166) captures firm i's expected instantaneous losses in the time period in which it is the better informed firm, irrespective of whether the deviation was detected or not. Correspondingly, (167) captures firm i's expected instantaneous losses in the time period in which it is the worse informed firm.

The counter $(k/m)d$, $k \in \mathbb{N}$, $k < m$ in (167) measures the time that elapses between a planning date of firm i and the consecutive planning date of firm j. In order to illustrate this, suppose again that $m = 5$ and $k = 3$. Then the time that elapses between the kth planning date of firm i and the kth planning date of firm j is given by $(k)d - k(1-1/m)d =$

$3d - 3(1-1/5)d = 3/5d$. It also becomes evident from this example that the firms plan simultaneously every $(m-1)d$ periods: firm i repeated its planning horizon exactly $m-1$ times and firm j planned for the mth time at this date. Again, this is due to the fact that we consider deviations in which the deviant expands its inattentiveness period by a fraction.

$d(1-1/m)$**: Expected stream of losses**

Given the expected instantaneous expected losses (166) and (167) a deviant's expected stream of losses is given by

$$\begin{aligned} E[\bar{\mathcal{S}}|\theta_0,\xi_0] \;=\; & \sum_{n=0}^{\infty}\sum_{k=1}^{m-1}\left(e^{-r\left(k-\frac{k}{m}+n(m-1)\right)d}\left(\int_{0}^{\left(\frac{k}{m}\right)d} e^{-r\tau}\phi_1 d\tau\right)\right)+\sum_{n=0}^{\infty}\left(e^{-r(n(m-1)d)}\int_{\tau=0}^{\left(1-\frac{1}{m}\right)d}\phi_1 d\tau\right) \\ & +\sum_{n=0}^{\infty}\sum_{k=1}^{m-2}\left(e^{-r(n(m-1)+1+k)d}\left(\int_{\tau=0}^{\left(1-\frac{k+1}{m}\right)d} e^{-r\tau}\phi_2 d\tau\right)\right) \\ & +K\sum_{n=1}^{\infty}\left(e^{-rn\left(1-\frac{1}{m}\right)d}\right). \end{aligned} \tag{168}$$

Now, we turn to the third part of the proof and characterize the equilibrium range of synchronous inattentiveness intervals.

To do so, we set out by deriving a firm's incentive to deviate for each type of fractional deviation. Subtracting from each fractional-deviation induced expected stream of losses – (165) to (168) – the equilibrium expected stream of losses which is appropriately adapted to the considered time horizon yields

$$E[\underline{\mathcal{S}}|\theta_0,\xi_0] \quad - \quad E[\mathcal{S}_1|\theta_0,\xi_0], \tag{169}$$

$$E[\bar{\mathcal{S}}|\theta_0,\xi_0] \quad - \quad E[\mathcal{S}_2|\theta_0,\xi_0], \tag{170}$$

where

$$\begin{aligned} E[\mathcal{S}_1|\theta_0,\xi_0] &= \frac{e^{rd}-1-rd}{e^{rd}r^2(e^{rd}-1)}\Lambda_1+\frac{K}{e^{rd}-1}, \\ E[\mathcal{S}_2|\theta_0,\xi_0] &= e^{-r\left(1-\frac{1}{m}\right)d}\int_{\tau=0}^{\frac{d}{m}} e^{-r\tau}\left(\Lambda_1\left(\left(1-\frac{1}{m}\right)d+\tau\right)\right)d\tau+E[\mathcal{S}_1|\theta_0,\xi_0]. \end{aligned}$$

Thus, it is profitable for a firm to deviate from a proposed synchronous inattentiveness period d in a non-marginal way if either (169) or (170) are negative for at least one $m \in \mathbb{N}$.

Using (169) and (170) we can show that synchronous planning cannot be an equilibrium for $\gamma < 0$ and $-1 \leq \rho \leq \hat{\rho}$. Thus, according to Lemma 10 a synchronous planning equilibrium can only exist for $\gamma > 0$ and $\rho \geq \max\{-1, \hat{\rho}\}$.

In order to derive the tightest bounds that the non-marginal "no-deviation" conditions impose on the range of potential equilibria we proceed as follows: we treat, for the sake of simplicity, m as a real non-negative number and minimize (169) and (170) with respect to m. Denote the real numbers that solve the minimization problem of (169) and (170) by $\underline{m}$ and $\bar{m}$. It can be shown that $\underline{m} = \bar{m}$. Accordingly, denote by $m^\star = [\bar{m}]$ the integer which is closest to $\bar{m}$. This yields for each direction the fractional deviation, i.e., $\left(1 - \frac{1}{m^\star}\right) d$ and $\left(1 + \frac{1}{m^\star}\right) d$, that induces the smallest expected stream of losses. As a consequence, the equilibrium inattentiveness intervals for which (169) and (170) are zero for $m = m^\star$ constitute the tightest bounds that characterize the equilibrium range.

For the sake of brevity, we limit the presentation to the derivation of $\underline{m}$. Solving (169) with respect to K and taking $r \rightarrow 0$ in the solution delivers

$$\begin{aligned} K \;=\; & \frac{d^2}{24(2-\gamma)^2 m}\Big(\big(2(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\gamma^2 - (\sigma_\theta^2 + 6\rho\sigma_\theta\sigma_\xi + 5\sigma_\xi^2)\gamma + 4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)\big)\gamma m^2 \\ & +(2(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - (2-\gamma)(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\gamma^2)6m \\ & +4(3-\gamma)(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) + (\sigma_\theta^2 - 6\rho\sigma_\theta\sigma_\xi - 7\sigma_\xi^2)\gamma^2\Big). \end{aligned} \tag{171}$$

Differentiating (171) with respect to m yields

$$\underline{m} = \sqrt{\frac{4(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)\gamma^3 + (\sigma_\theta^2 - 6\rho\sigma_\theta\sigma_\xi - 7\sigma_\xi^2) + 4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)(3-\gamma)}{\gamma(2(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)\gamma^2 - (\sigma_\theta^2 + 6\rho\sigma_\theta\sigma_\xi + 5\sigma_\xi^2)\gamma + 4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2))}}. \tag{172}$$

Solving the second derivative of (169) with respect to m for K, taking $r \rightarrow 0$ in the solution and evaluating the resulting expression at $\underline{m}$ yields that this is indeed a minimum. Thus, it follows from (171) and (169) that a firm has no incentive to deviate to an inattentiveness period of length $d + \frac{d}{m} \,\forall m \in \mathbb{N}$, where

$$d_s^l = \frac{2\sqrt{6}(2-\gamma)\sqrt{Km^\star}}{\sqrt{\Theta(m^\star + 1)}},$$

and

$$\begin{aligned}\Theta &= (\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)(2+m^\star)2\gamma^3 - ((7+5m^\star)\sigma_\xi^2 + 6\rho\sigma_\theta\sigma_\xi(m^\star+1) + \sigma_\theta^2(m^\star-1))\gamma^2 \\ &\quad +4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)(m^\star-1)(\gamma(m^\star-1)+3), \\ m^\star &= \lfloor \underline{m} \rfloor .\end{aligned}$$

Similar arguments imply that a firm has no incentive to deviate to an inattentiveness period of length $d - \frac{d}{m} \, \forall m \in \mathbb{N}$ if $d \leq d_s^u$, where

$$d_s^u = \frac{8\sqrt{6}\sqrt{Km^\star}}{\sqrt{\Theta(m^\star+1)}}.$$

Obviously, $d_s^u > d_s^l$ for $\gamma > 0$.

A similar argument yields that the bounds that are implied by player 2 choosing planning mode (A) and an fractional deviation of the type $(1 + l/m)\, d$ are not tighter than d_s^l and d_s^u. Hence, every common $d \in [d_s^l, d_s^u]$ is a synchronous equilibrium inattentiveness period. ■

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