Price and Quality Competition with Boundedly Rational Consumers[*]

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Abstract

We consider a market model with n rational firms (doctors) and a continuum of boundedly rational consumers (patients). Following Spiegler (2006a), we assume that patients are not familiar with the market and rely on anecdotes.

We analyze the price setting game played by doctors with given, different healing qualities. Doctors know their own quality, as well as the qualities of their competitors. We find a unique equilibrium in mixed strategies. All doctors, no matter how bad, make positive profits that are typically considerably higher than their maxmin payoffs.

In order to analyze welfare, we introduce a pre-stage where doctors choose qualities. Even though a better quality comes for free, doctors mainly offer mediocre qualities in all SPNE. If the highest possible quality is high enough, welfare strictly decreases in the number of doctors.

JEL Classification: L13, L15, D44

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1   Introduction

A consumer who turns to an unfamiliar market may rely on the advice of other consumers. Anticipating such search behavior, it may be beneficial for firms to commit on low qualities as a product differentiation strategy, even if raising quality is costless. With low qualities, every single firm is considered by less consumers. Yet the firm has to compete for potential consumers less fiercely, as these take only few other firms into consideration. This is the situation explored in this paper.

There are plenty of markets in which consumers are not fully aware of the different qualities of specialists and rely on anecdotal evidence. Whereas consumers do not know well how the market they are in exactly looks like, specialists know the market perfectly: They know how good they are, how good their competitors are, and how consumers use to search for them. Specialists act rationally, while consumers apply a rule of anecdotal reasoning.

Our running (toy) example is given by a patient thinking about consulting some doctor because a friend got cured at that doctor. Like in Spiegler's "Market for Quacks" (2006a) on which our model is based, patients are assumed to rely on the experiences of others in order to judge the qualities of different doctors. Each patient asks one former client of each doctor whether the doctor cured him or not. A positive report makes the patient believe that that doctor will cure him as well - whereas a negative report makes the patient shy away from that doctor. Among the recommended doctors, patients choose the cheapest one.

Another example is given by the market of piano-teaching: Parents, not knowing how to play the piano themselves, rely on recommendations to find a good piano teacher for their children. Further applications are the markets for financial advice, personal coaching, business consulting, special repair services, or spiritual guidance.

We define the quality of a doctor as the probability with which he can help a patient. If patients ask one client of each doctor while searching for a good treatment, the probability that a doctor is recommended to a patient is given by his success probability. We first assume that qualities are given exogenously and analyze the price setting behavior of the doctors. Later we allow doctors to choose their qualities. Thinking about choosing a quality as a process of specialization, it is natural to assume that doctors first choose their quality, then set their prices.

In the pricing game with exogenous, asymmetric qualities, we find that even the worst doctors survive in the market, and earn equilibrium payoffs much larger than their maxmin payoffs. The reason is that good doctors do not feel threatened enough by bad doctors to set low prices. Instead good doctors mix over rather high prices, leaving room for bad doctors to earn considerable payoffs.[1] No doctor has an incentive to reveal his true quality: If he would state his true quality, his position in competition with other recommended doctors would be weakened.

We then analyze the full game in which doctors first choose their qualities. Considering pure quality-setting strategies, we find that in all SPNE doctors mainly offer low qualities. A low quality attracts fewer patients, but it softens price competition. The latter effect dominates, thus doctors do not set high qualities, even if setting higher qualities is costless. This extended model allows us to analyze how welfare (i.e. the overall proportion of patients cured[2]) is affected by the number of doctors in the market.

We find that heavily restricting the number of doctors improves welfare since it increases the quality of treatments offered. If the maximal quality doctors can offer is high enough, having a monopolistic doctor maximizes welfare (and welfare strictly decreases in the number of doctors).[3]

It may seem cynical to think of a physician (or other specialist) in front of a patient applying a bad technique when he could without problems or direct costs apply a better technique instead. The quality choice should be seen to take place on a more fundamental level: A doctor comes to the market. He can specialize on the best-available treatment. Yet he can as well specialize on an alternative approach of curing the same disease, a method that maybe helps less often. Anticipating that choosing the best available treatment will lead to strong competition, the doctor may specialize on an alternative, weaker method. Our model thus predicts a differentiation of treatments at the expense of overall quality.

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[1]This may explain why approved therapies do not crowd out the myriads of obscure nutritional supplements which were never clinically tested.

[2]In our model, all payments made are just transfers within society. Moreover, neither doctors nor patients face any direct costs. Thus, in our setting, maximizing social welfare is equivalent to maximizing the proportion of patients cured.

[3]It is even more beneficial for welfare not to limit entry but to exogenously prescribe a fixed price. Then doctors cannot not take advantage of the weaker price competition induced by low qualities, and thus have no incentive to choose a lower than maximal quality.

1.1 Related Literature

Our model is based on Ran Spiegler's "Market for Quacks" (2006a). We extend his model of the pricing game to the case of asymmetric qualities. This allows us to introduce a pre-stage of quality-setting which makes the model more suitable for the study of social welfare. The sampling rule patients apply to evaluate doctors is the S(1) rule which was introduced by Osborne and Rubinstein (1998). Spiegler and Rubinstein have utilized the S(1) rule to model consumer behavior in a variety of settings (see Spiegler (2006a, 2006b) and Rubinstein and Spiegler (2008)). We have used the S(1) procedure as well in a companion paper, Szech (2008).[4]

Besides S(1) there are other related approaches for modeling boundedly rational consumer behavior, such as Ellison and Fudenberg's (1995) "word-of-mouth learning" and Rabin's (2002) "law of small numbers". More broadly, our paper contributes to the literature on interactions between rational firms and boundedly rational consumers (see the survey by Ellison (2006)). To our knowledge, this paper is the first to extend a price competition game with boundedly rational consumers via introducing a preceding quality setting stage in order to study how bounded rationality affects welfare.

Technically, our analysis has some parallels to papers on price dispersion like Varian (1980) or to papers on complete information all-pay auctions such as Baye, Kovenock and de Vries (1996).[5] Like in those models, the equilibrium of our pricing stage is in mixed strategies. We first specify equilibrium payoffs for all possible equilibria and then identify sequentially the unique equilibrium candidate - a similar approach has been used, for instance, by Siegel (2008) in the context of generalized all-pay auctions. In light of that literature, an interesting feature of our price setting game is that we obtain a unique equilibrium in non-degenerate mixed strategies in the price setting game both in the symmetric and the asymmetric case. This sets our model apart from the complete information all-pay auction or Varian's model of sales. There, in the asymmetric case, all but two firms play pure strategies. Furthermore,

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[4]In there, we consider a variant of Spiegler's (2006a) model in which the doctors' qualities are privately known random variables. We identify an equilibrium in monotone pricing strategies of that model and show that welfare goes to zero in the number of doctors. This happens because patients always attend the cheapest among the doctors who are recommended - in monotone strategies this is also the worst recommended doctor.

[5]Indeed, our model is a complete-information first-price (procurement) auction with stochastic participation.

in the symmetric case, there is a multiplicity of equilibria.[6] The central reason for these differences is that in our model, for any pair of firms, there is a group of consumers who just decide between these two firms. Notably, in the asymmetric case, Varian's model of sales can only explain sales (i.e. mixing) by the two firms with the smallest "home-base".[7] Our analysis shows that by introducing further groups of consumers (whose "sophistication" lies between that of the two groups considered by Varian) sales by all firms in the market can be explained.[8]

Reinterpreting the model with rational patients, i.e., considering a model where each patient likes or dislikes (notices or does not notice) a firm's product with some probability,[9] one sees the close relation of our model to the advertising model of Butters (1977). A similar reinterpretation with rational agents is possible with the Spiegler model, which is then a special case of the Perloff and Salop (1985) model of product differentiation. Our quality-setting stage parallels Shaked and Sutton (1982): They introduce a quality setting pre-stage to the Gabszewicz and Thisse (1979) pricing model, while we do the same with a Perloff-Salop-type pricing model. In Shaked and Sutton (1982), only a limited number of firms can make positive profits. This is not true in our model. The reason lies in the different modeling of consumers' preferences: In Shaked and Sutton (1982), all consumers share the same ranking of products. In our paper, for each product there is a group of consumers who prefer it to all other products.

1.2 Outline

The paper is structured as follows: Section 2 presents the model and describes the $S(1)$ procedure in detail. In Section 3, we analyze the second stage of decision making in the game (the price setting stage), and identify its unique Nash equilibrium. Pure equilibria of the quality setting stage and their welfare implications are discussed in Section 4. Section 5 presents a number of extensions and variations of our

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[6]See Baye, Kovenock and de Vries (1992, 1996).

[7]By asymmetry we mean that the mass of consumers tied to one firm varies across firms. The asymmetric case is not treated in Varian's paper. It is covered, e.g., by the analysis of Siegel (2008).

[8]In our terminology, in Varian's model patients either get a recommendation for all doctors or for only one doctor. The interpretation is that some consumers are aware of all firms while others consider only the nearest firm. Under this interpretation, we have that in our model for any subset of firms there is a group of consumers who are aware exactly of that subset. See also footnote 21.

[9]This reinterpretation of quality as "mass appeal" or "intensity of advertising" affects the welfare implications of our model, but not the equilibrium analysis.

model. First, we show that the equilibrium analysis is robust to the introduction of convex costs. Then, we point out the differences between our model and a parallel model with rational patients holding incomplete information about the doctors' qualities. Finally, we consider the reinterpretation of our model as a model of product differentiation or advertising with rational consumers. Section 6 concludes. In Appendix A we discuss mixed strategy equilibria of the quality setting stage. All proofs are in Appendix B.

2 The Model

We consider a market with n doctors who are familiar with the market and act rationally. Patients are not familiar with the market and apply a simple sampling rule described below. Patients form a continuum of mass one. We assume that the doctors know each other very well and hence know the qualities of each other when playing the pricing game. Qualities can be anything between zero and some upper bound $0 < \overline{\alpha} < 1$. A doctor's quality is the probability with which he can cure a patient. With the counter probability, the patient remains ill. Before we specify the patients' behavior, we give the exact **timing** of the model:

1. Doctors simultaneously set their qualities $\alpha_i \in [0, \overline{\alpha}]$.

2. Doctors observe each others' qualities.

3. Doctors simultaneously set their prices P_i.

4. Patients decide if they want to attend a doctor and if so, which one.

Patients are initially ill and have a utility of one from getting cured and a utility of zero from staying ill. They decide according to the behavioral rule **S(1)** as introduced by Osborne and Rubinstein (1998), and as utilized in Spiegler (2006a):

- Each patient samples each doctor once.

- With probability α_i, a patient gets a positive signal $S_i = 1$ on doctor i ("**a recommendation**").

- With probability $1 - \alpha_i$, a patient gets a negative signal $S_i = 0$ on doctor i ("**no recommendation**").

- A patient attends the doctor with the highest $S_i - P_i$ ...

6

- unless $\max_i S_i - P_i < 0$. Then the patient stays out of the market and expects a utility of 0 at a price of 0.

Note that the last two points implicitly contain a tie-breaking rule: If a patient has to choose between consulting a recommended doctor charging a price of one and staying at home, the patient opts for the doctor. It can be shown that in the pricing stage no equilibrium exists if we depart from this assumption. For all other possible types of ties, no special breaking rule is needed for our results - ties can be broken arbitrarily.

Note that patients rely far too much on the signal they get - they overinfer from their sample. The idea behind the S(1) rule is to capture a simple way of anecdotal reasoning: Each patient independently asks some "former" client of each doctor.[10] A client of doctor i got cured with probability α_i. Thus, with probability α_i, he recommends doctor i to the patient. The patient trusts in this report - he either thinks the doctor can cure him as well for sure or not at all.

Choosing a higher quality comes at no direct costs for the doctors. The motivation for this assumption is that we want to study how the patients' boundedly rational behavior induces doctors to set a low quality. Our model is to be understood as a benchmark case which ignores costs that give doctors another, separate reason for choosing a low quality. In Section 5.1 we show that our results are robust to the introduction of convex costs.

3   The pricing stage

We search for SPNE using backwards induction, and thus we start with an analysis of the price setting game for given quality levels α_i.[11] We first show that the equilibrium payoffs of the price setting stage are uniquely determined. Then we identify the unique equilibrium of the pricing stage.

Proposition 1 *Fix $\alpha_1,..., \alpha_n$. Then in all equilibria of the price setting game, the payoff of doctor i is given by*

$$\pi_i = \alpha_i \prod_{j \neq j^*} (1 - \alpha_j) \tag{1}$$

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[10]Of course, we are not in a dynamic model here. This is only some motivating story.

[11]The price setting game is essentially a generalization of the game analyzed by Spiegler who assumes that all (or all but one) doctors have the same α.

where $j^* \in argmax_j \, \alpha_j$.

The intuition for this is as follows: Consider doctor j^* who offers the highest quality.[12] This doctor can make a positive profit independent of his competitors' strategies, as there will be patients who only get a positive report on him, but not on the other doctors. (The fraction of these patients is $\alpha_{j^*} \prod_{j \neq j^*}(1 - \alpha_j)$). Our doctor will thus set a positive lowest price in any equilibrium. Consider one equilibrium and assume our doctor makes there a payoff of $\alpha_{j^*} C$ and sets a lowest price of p_L. By charging an only slightly lower price than this, every other doctor i can make, at least, a profit arbitrarily close to $\alpha_i C$. Reasoning vice versa we see that the payoffs indeed have to equal $\alpha_i C$ and cannot be higher. To determine C, we find out that there is one doctor i who has a price of 1 in the support of his equilibrium price setting strategy. This doctor only makes a profit if he is the only recommended doctor, which happens with a probability of $\alpha_i \prod_{j \neq i}(1 - \alpha_j)$. Finally, we find that this doctor has to be the best doctor, as otherwise C would be so low that the best doctor would prefer to deviate. Thus we can specify $C = \prod_{j \neq j^*}(1 - \alpha_j)$.

From Proposition 1, we see that the quality of the best doctor does not appear in the payoff formulas of the other doctors. Hence it does not matter for the competitors' profits whether the best doctor is equally good as the second best doctor or much better. For the second best doctor, this is not true: His quality always affects the competitors' profits.[13]

One important question left open by Proposition 1 is equilibrium existence to which we turn now: We find a unique equilibrium which is in mixed strategies. To get an intuition for the equilibrium distribution functions, assume all doctors use some price $\widetilde{p}$ in their supports. Then, the equilibrium distribution functions G_i must fulfill:

$$\pi_1(\widetilde{p}) \overset{(1)}{=} \alpha_1 \prod_{j \neq j^*}(1 - \alpha_j) = \alpha_1 \widetilde{p} \prod_{j \neq 1}(1 - \alpha_j G_j(\widetilde{p})).$$

The middle part of the equation is the equilibrium payoff specified above. The right hand side is the payoff doctor 1 makes from playing $\widetilde{p}$ given that the other doctors mix according to G_j: $\widetilde{p}$ is the price he earns if he is consulted, and α_1 is the

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[12]If there is more than one, we (arbitrarily) determine as best doctor one of the doctors with highest qualities.

[13]Thus, if more than one quack in Spiegler's market would be changed into some expert, the payoffs of the remaining quacks would be diminished. This result complements Proposition 2 of Spiegler (2006a).

probability that he is recommended. The product term is the probability that all competitors are not recommended, or pricier than i. The same must hold for doctor 2.

$$\pi_2(\widetilde{p}) \overset{(1)}{=} \alpha_2 \prod_{j \neq j^*} (1 - \alpha_j) = \alpha_2 \widetilde{p} \prod_{j \neq 2} (1 - \alpha_j G_j(\widetilde{p})).$$

We thus have

$$\frac{1}{\widetilde{p}} \prod_{j \neq j^*} (1 - \alpha_j) = \prod_{j \neq 1} (1 - \alpha_j G_j(\widetilde{p}))$$

and

$$\frac{1}{\widetilde{p}} \prod_{j \neq j^*} (1 - \alpha_j) = \widetilde{p} \prod_{j \neq 2} (1 - \alpha_j G_j(\widetilde{p}))$$

from which we see that

$$1 - \alpha_1 G_1(\widetilde{p}) = 1 - \alpha_2 G_2(\widetilde{p})$$

or, more generally,

$$1 - \alpha_k G_k(\widetilde{p}) = 1 - \alpha_l G_l(\widetilde{p})$$

for all k, l.

We can thus write

$$\prod_{j \neq j^*} (1 - \alpha_j) = \widetilde{p} \prod_{j \neq 1} (1 - \alpha_j G_j(\widetilde{p})) = \widetilde{p}(1 - \alpha_i G_i(\widetilde{p}))^{n-1}$$

and therefore

$$\frac{1}{\alpha_i} \left(1 - \sqrt[n-1]{\frac{\prod_{j \neq j^*}(1 - \alpha_j)}{\widetilde{p}}} \right) = G_i(\widetilde{p})$$

for all i.

This is the basic reasoning behind Proposition 2, but since the doctors may mix over different price intervals, the equilibrium looks a bit more complicated. Before stating the proposition, we depict the price intervals doctors typically mix on.
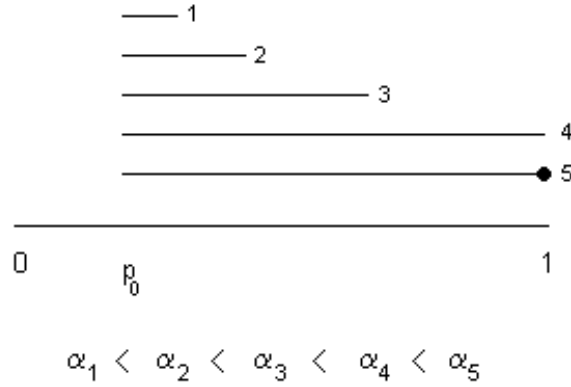
Figure 1: Supports of the doctors' pricing strategies for $n = 5$

The doctors' price supports all start at the same lowest price p_0. Furthermore, we see that the higher the quality of a doctor, the larger the support of his pricing strategy.

Proposition 2 *Consider $0 < \alpha_1 \leq \ldots \leq \alpha_n \leq \bar{\alpha}$. Then the unique equilibrium of the pricing game is given as follows:*

Define a sequence of prices $p_0, \ldots, p_n$ by

$$p_i = \frac{(1 - \alpha_{i+1}) \cdot \ldots \cdot (1 - \alpha_{n-1})}{(1 - \alpha_i)^{n-i-1}}$$

for $1 \leq i \leq n - 2$,

$$p_0 = \prod_{i=1}^{n-1}(1 - \alpha_i) \quad and \quad p_{n-1} = p_n = 1.$$

Each doctor i mixes over the interval $[p_0, p_i]$ using the distribution function G_i defined by

$$G_i(p) = \frac{1}{\alpha_i}\left(1 - \sqrt[n-j]{\frac{(1 - \alpha_j) \cdot \ldots \cdot (1 - \alpha_{n-1})}{p}}\right)$$

for $p \in [p_{j-1}, p_j] \subset [p_0, p_i]$ with $1 \leq j \leq n - 1$. On $[0, p_0]$, define $G_i = 0$ and on $[p_i, 1]$, $G_i = 1$. G_n places an atom of size $1 - \frac{\alpha_{n-1}}{\alpha_n}$ at 1.[14]

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[14]Note that Propositions 1 and 2 still hold in the (excluded) case where exactly one doctor has

10

Note that the upper boundaries p_i coincide if the corresponding α_i coincide. The distribution functions G_i are continuous except for G_n where doctor n puts an atom on $p = 1$ (if $\alpha_{n-1} \neq \alpha_n$).

We first showed equilibrium payoff uniqueness (Proposition 1) and then sequentially constructed a unique equilibrium candidate (Proposition 2). A very similar approach was recently taken by Siegel (2008) to analyze complete information all-pay auctions with general cost functions. Furthermore, our equilibrium somewhat resembles the mixed-strategy equilibrium of a complete information all-pay auction. Of course, the price setting game we consider is not an all-pay auction, but a complete-information first price auction with equal valuations and random participation. What these two auction models have in common is that some players can secure a positive maxmin payoff by making a high bid. Such behavior is, however, not consistent with equilibrium behavior. Thus a mixed strategy equilibrium arises.

Note that in our equilibrium weak players typically earn much more than their maxmin payoffs. As an example, consider the case $n = 2, \alpha_1 = 0.9, \alpha_2 = 0.3$. Then

$$\pi_1 = \text{maxmin}_1 = 0.9(1 - 0.3) = 0.63,$$

whereas

$$\pi_2 = 0.3(1 - 0.3) = 0.21 > \text{maxmin}_2 = 0.3(1 - 0.9) = 0.03.$$

A good doctor can make a high payoff from being a monopolist. Thus when facing a weak competitor he is "unwilling" to play too low prices. Hence there is room left for the weak doctor to make a considerable profit.

The following corollary characterizes the equilibrium further. It shows that while better doctors have larger market shares in equilibrium, differences in market shares are smaller than differences in qualities:

Corollary 1 *Consider the equilibrium of Proposition 2. Consider two doctors i and j with qualities $\alpha_i < \alpha_j$. Denote by m_i and m_j the doctors' market shares, i.e. the*

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a quality of 1. If there are more than one doctor with a quality of 1, payoffs will be zero due to Bertrand competition. Then, many equilibria are possible as long as two "perfect" doctors set prices of 0.

proportions of patients visiting the doctors. Then

$$\frac{\alpha_i}{\alpha_j} < \frac{m_i}{m_j} < 1.$$

Note that α_i/α_j is also the ratio between the equilibrium payoffs of doctor i and j. Thus the corollary implies that better doctors make higher payoffs partly due to higher prices and partly due to attracting more patients.

4   The quality setting stage

As the equilibrium of the price setting game is unique, the doctors' payoffs in every SPNE of the complete game just depend on the qualities chosen by the doctors in the first stage. From now on, we can thus consider the game as a one-stage quality setting game and search for its Nash equilibria.

For intuition, consider first the two doctors case with $\overline{\alpha} > \frac{1}{2}$. By Proposition 1, the payoff of doctor 1, given quality choices α_1 and α_2, is

$$\Pi_1(\alpha_1, \alpha_2) = \begin{cases} \alpha_1(1 - \alpha_2) & \text{if } \alpha_1 \geq \alpha_2 \\ \alpha_1(1 - \alpha_1) & \text{if } \alpha_1 \leq \alpha_2. \end{cases}$$

It is thus not surprising that the best response curve of doctor 1 contains only $\frac{1}{2}$ and $\overline{\alpha}$:

$$BR_1(\alpha_2) = \begin{cases} \overline{\alpha} & \text{if } \alpha_2 \leq 1 - \frac{1}{4\overline{\alpha}} \\ \frac{1}{2} & \text{if } \alpha_2 \geq 1 - \frac{1}{4\overline{\alpha}}. \end{cases}$$

The two equilibria in pure quality setting strategies are, accordingly: $(\overline{\alpha}, \frac{1}{2})$ and $(\frac{1}{2}, \overline{\alpha})$.

With more than two doctors, it remains true that a doctor's best response to any pure strategies of his opponents is either $\overline{\alpha}$ or $\frac{1}{2}$. Furthermore, if $\overline{\alpha} > \frac{1}{2}$, given that one of his opponents plays $\overline{\alpha}$, doctor i will prefer playing $\frac{1}{2}$ to $\overline{\alpha}$. Intuitively, doctor i chooses a low quality to prevent a fierce price competition with the strong competitor. We can now characterize the SPNE as follows:

Observation 1 (*i*) *If* $\overline{\alpha} \in (\frac{1}{2}, 1)$, *all SPNE in pure quality setting strategies are of the following form: One doctor i sets $\alpha_i = \overline{\alpha}$, all other doctors j set $\alpha_j = \frac{1}{2}$.*

Pricing strategies are as calculated in Proposition 2. We will call these equilibria $\frac{1}{2}$-$\overline{\alpha}$-**equilibria**.

(**ii**) *If* $\overline{\alpha} < \frac{1}{2}$*, the unique SPNE is given by all doctors* i *setting* $\alpha_i = \overline{\alpha}$ *and playing prices as in Proposition 2.*

An immediate corollary of Observation 1 is the following:

Corollary 2 *With* $\overline{\alpha} > \frac{1}{2}$*, there is no SPNE where all doctors offer the highest quality.*

There is a positive effect of raising α above $\frac{1}{2}$: With a higher quality, a doctor gets recommended to more patients. Yet, if strong competitors are present, this effect on payoff is dominated by the negative effect of making competition in the market fiercer. This behavior of the doctors already suggests the following conclusion which is made precise in Proposition 3: Having many doctors in the market cannot be good for welfare (i.e. the overall proportion of patients cured) since the average quality offered by the doctors is quite small then (close to $\frac{1}{2}$).

We now determine the market size which maximizes social welfare in the $\frac{1}{2}$-$\overline{\alpha}$-equilibria. We find that monopoly is optimal for $\overline{\alpha} \geq \frac{3}{4}$. For smaller $\overline{\alpha}$ having a few doctors in the market is best.

Proposition 3 *As* n *increases, welfare in the* $\frac{1}{2}$-$\overline{\alpha}$-*equilibria converges to* $\frac{1}{2}$ *from above. For* $\frac{3}{4} < \overline{\alpha} < 1$ *welfare strictly decreases in* $n \geq 1$*. For* $\frac{1}{2} < \overline{\alpha} \leq \frac{3}{4}$ *welfare increases up to some finite optimal market size* n^* *and decreases from there on. Furthermore, for* $\frac{1}{2} < \overline{\alpha}$ *the optimal market size is bounded from above through*

$$ n^* \leq 10.4 + 2.3 \ln \left(1.7 + \frac{0.35}{\alpha - \frac{1}{2}} \right). $$

To see why a market with a small number of doctors may generate more welfare than monopoly, consider as an example $\overline{\alpha} = 0.6$. If there is only one doctor, he offers the best possible quality of 0.6 and sets a price of 1. Thus 60% of the patients get a positive report and attend this doctor. Of this fraction of patients, 60% get cured. Thus, in monopoly, 36% of all patients get cured. With more doctors, more patients get at least one positive report and attend a doctor at all. In our example, with two doctors, we have $1 - (1 - 0.6) \cdot (1 - \frac{1}{2}) = 80\%$ of all patients getting at least one positive report and thus attending a doctor at all. Using formula (10) of

the Appendix it can be seen that in this case 44.25% of patients get cured, and that the optimal number of doctors for $\overline{\alpha} = 0.6$ is 7. This is the positive welfare effect of a larger number of doctors. Yet if the number of doctors increases, more and more doctors offer only low quality treatments ($\alpha = \frac{1}{2}$). This is the downside of a high number of doctors, which dominates if n gets larger: The positive effect of increasing the market size vanishes exponentially in n while the market share of the good doctor decreases much slower (roughly as $\frac{1}{n}$). Thus for n sufficiently large the proportion of patients cured is always above $\frac{1}{2}$.

While the upper bound on the optimal market size given in the proposition is not very sharp, it makes clear that the optimal market size goes to infinity only very slowly as $\overline{\alpha}$ approaches $\frac{1}{2}$. For instance, for $\overline{\alpha} = 0.50001$ we get $n^* \leq 34$:[15] The presence of a doctor with a slightly better quality drastically reduces the optimal market size, as infinitely many doctors would maximize welfare if $\overline{\alpha} = \frac{1}{2}$. The proposition is summed up in Figure 2 which depicts the proportion of patients cured as a function of market size for different values of $\overline{\alpha}$.

Let us now turn to the case where the best possible method of healing only leads to a recovery probability of $\overline{\alpha} \leq \frac{1}{2}$. Then, as noted in Observation 1, the unique SPNE is that all doctors offer the best possible treatment with healing probability $\overline{\alpha}$. But what would happen if there was a new technology that could lead to a higher $\overline{\alpha}$? Observation 2 tells us that a rise of $\overline{\alpha}$ in between $[\frac{1}{n}, \frac{1}{2}]$ would not be welcomed by the doctors:

Observation 2 *The equilibrium profit of each doctor*

$$\overline{\alpha}(1 - \overline{\alpha})^{n-1}$$

is strictly decreasing in $\overline{\alpha}$ on $[\frac{1}{n}, \frac{1}{2}]$.

Hence doctors would try to block or delay the approval of promising new drugs or treatments.[16]

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[15]The exact value (which can be found numerically) is $n^* = 24$.

[16]Note, however, that doctors would, of course, welcome innovations that allowed themselves but not the other doctors to set a higher quality.
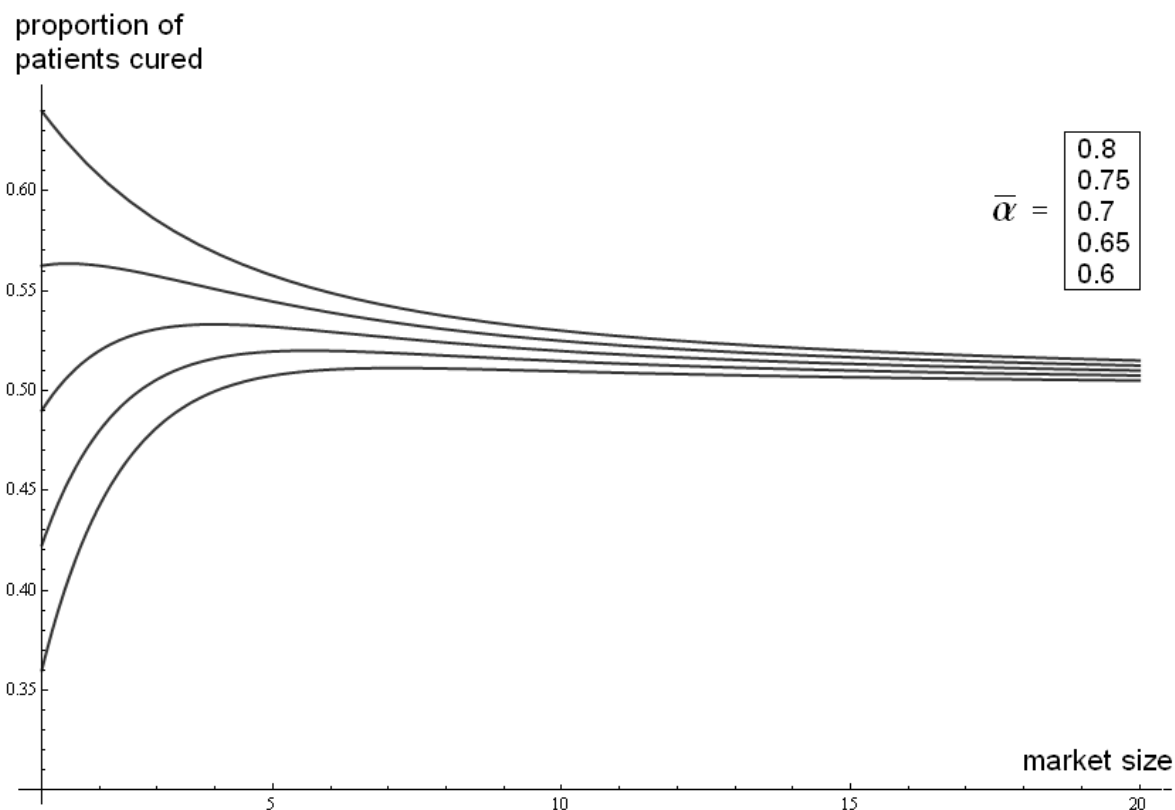
Figure 2: The effect of market size on welfare

For $\overline{\alpha} \leq \frac{1}{2}$, the proportion of patients who are healed increases with the market size. Note, however, that a restriction to a finite market size will not do much harm unless $\overline{\alpha}$ is small since the proportion of patients healed, which is given by

$$\overline{\alpha}\left[1-(1-\overline{\alpha})^n\right],$$

converges to $\overline{\alpha}$ exponentially fast.

Finally, note that for a given vector of (positive) prices, it is a unique best response for doctors to set their qualities to $\overline{\alpha}$. An obvious policy implication of this fact is the following: If a policy maker could prescribe an arbitrary fixed price for the doctors' services, the problem of low qualities would vanish. Doctors would set their qualities as high as possible. As discussed at the end of Section 5.1 however this simple policy of fixing prices at an arbitrary value will not work anymore if we introduce costs of quality setting: Then only a carefully chosen fixed price may have the desired effect.

15

5 Discussion

5.1 Costly Quality Choice

We now discuss the robustness of our results with regard to costly qualities. We find that our results are robust to the introduction of convex costs, i.e., there are SPNE which are structurally similar to the $\frac{1}{2}$-$\overline{\alpha}$-equilibria of the previous section:

Proposition 4 *Assume that setting α is associated with a cost $c(\alpha)$, where c is a continuously differentiable, increasing and convex function with $0 < c'(0) < 1$ and $c(0) = 0$. Then there exists an SPNE where all but one doctors set $0 < \alpha^l < \frac{1}{2}$ and one doctor sets α^h where $\alpha^h > \alpha^l$. α^l solves*

$$c'(\alpha^l) = (1 - 2\alpha^l)(1 - \alpha^l)^{n-2}$$

and α^h solves

$$c'(\alpha^h) = (1 - \alpha^l)^{n-1}$$

if $c'(1) \geq (1 - \alpha^l)^{n-1}$ and $\alpha^h = 1$ otherwise. Furthermore, α^l and α^h are decreasing in n.

Note that all doctors make positive profits in equilibrium no matter how large n is. Of course, if we introduce fixed costs of entry, some doctors may decide to stay out of the market. An implication of the proposition is that the significance of the value $\alpha = \frac{1}{2}$ in the basic model (which may have seemed a bit strange) was caused by the assumption of no costs.

In Section 4 we saw that, in the model without costs, fixing prices at any arbitrary $p \in [0, 1]$ makes doctors set their qualities to the maximum and thus to the social optimum (for each fixed n). In the model with costs this simple policy does not work anymore: as a symmetric Nash equilibrium, doctors play qualities $\alpha(p, n)$ that sensitively depend on the fixed price p. As one would expect, the quality $\alpha(p, n)$ is decreasing in n and increasing in p.[17] For example, if p was fixed at zero, all doctors would choose zero qualities. No patient would get cured. Thus we find that in the model with costs only a careful choice of a fixed price p may induce the social optimum.

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[17]$\alpha(p, n)$ is implicitly given as the solution of $p(1 - (1 - \alpha)^n)/n\alpha = c'(\alpha)$ if a solution exists, otherwise $\alpha(p, n) \in \{0, 1\}$. Since the left hand side is decreasing and the right hand side is strictly increasing in α there is at most one solution. Technical details are available upon request.

5.2 Comparison with a Model of Incomplete Information

It is natural to ask whether our results could be reproduced in a model with incomplete information and complete rationality. Despite some similarities, this is generally not the case. Consider first a price setting game in which patients believe in some prior distribution of doctors' qualities. Patients receive either a good or a bad report on each doctor and update their prior in a Bayesian way. Thus there are two possible expectations α^l and α^h a patient can have about a doctor's quality. This does lead to a mixed strategy equilibrium like in our model. Yet, apart from this, there are considerable differences. For instance, in the incomplete information model, if we change the patients' prior beliefs to more optimistic ones (such that α^l and α^h are replaced by $L\alpha^l$ and $L\alpha^h$, $L > 1$) the patients' willingness to pay increases, and doctors then charge higher prices in equilibrium. This is in contrast to the $S(1)$ model, in which increasing qualities leads to lower equilibrium prices. For a more detailed discussion, see Spiegler (2006a).

A two stage model with strategical patients differs even more drastically from our model. In such a model, in equilibrium patients know the doctors' strategies (even though they may not observe qualities perfectly). It is thus an SPNE for doctors to set the highest possible quality and charge a price of zero and for patients to choose an arbitrary doctor among those offering the lowest price.

Having results such as Milgrom and Roberts (1986) in mind one might expect that if doctors had a chance to disclose their qualities they would do so in equilibrium and most of our results would break down. Yet this is not the case. In our model, doctors have no incentive to disclose their qualities: A doctor who is assumed to have a quality of 1 by a fraction α_i of patients is in a better position when competing with other doctors than a doctor who is known to have quality α_i by all patients. Unlike rational agents, our boundedly rational patients do not draw any conclusions from a doctor's decision not to disclose his quality. For a detailed exposition of this point, see Proposition 3 in Spiegler (2006a).[18]

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[18]Spiegler shows that any strategy involving disclosure is weakly dominated by some strategy involving no disclosure. The equilibrium we have identified in Theorem 2 persists because if some doctor could profitably deviate to a strategy involving disclosure there would also exist a strategy involving no disclosure he could deviate to. But then our equilibrium would not have been an equilibrium in the original game. Furthermore, in the working paper version Spiegler (2003), Spiegler shows that no Nash equilibrium involves disclosure. Hence the equilibrium from our Theorem 2 is still the unique equilibrium of the game where disclosure is allowed.

5.3 Rational Agents

Our model can be seen as a model of product differentiation with fully rational consumers. This is because the Spiegler model, i.e. our price setting stage, can be reinterpreted as a variant of Perloff and Salop's (1985) model of product differentiation (see also Gabaix, Laibson and Li (2005)). In this interpretation, consumers (independently) attribute to a firm i's service a valuation of 1 with probability α_i (and a valuation of 0 otherwise).[19] Thus we reinterpret "quality" as "mass appeal" here. A higher mass appeal is costless to the firms. Our results then show that, under competition, most firms produce niche products, i.e. products with a lower mass appeal. A quality setting stage (or mass appeal setting stage) is not present in other papers based on the Perloff Salop model. Shaked and Sutton (1982), however, consider a quality setting stage extending the Gabszewicz and Thisse (1979) model of product differentiation. What the analysis of Shaked and Sutton has in common with ours is that in both models firms reduce price competition by differentiating their products in a preliminary stage. A notable difference is, however, that in the model of Shaked and Sutton firms offer different combinations of quality and price in order to exploit differences between consumers with regard to their willingness to pay. In contrast, consumers in our model are ex ante symmetric and firms do not differentiate by trying to match different needs of consumers: Instead our firms differentiate their products by modifying them so that some consumers do not want them anymore while the other consumers' valuations of the product remain unchanged.

Our model can also be seen as a model of advertising: Assume that all firms offer services consumers value at one. Consumers need some information (e.g. a flyer) to become aware of a firm. Each firm can specify the proportion α_i of consumers (but not the concrete identities of consumers) that should get informed. Our analysis then suggests that firms do not necessarily inform as many consumers as possible, even though advertising is costless.[20] Butters (1977) is a seminal paper in the advertising literature which considers a related model.

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[19]Perloff and Salop consider continuous distributions instead of this Bernoulli distribution. This leads to pure strategy equilibria of the price setting game.

[20]Note, however, that under these reinterpretations, social welfare is identical no matter which firm is selected. Thus the decrease in social welfare described in Proposition 3 is not present in the reinterpretations.

6 Conclusion

We have seen that if consumers rely on anecdotes, all firms, no matter how bad, survive in the market. Low quality firms typically earn much more than their maxmin payoffs. If firms can choose, they will mostly opt for low qualities: A lower quality makes the firms attract less consumers. Yet it also softens price competition, and thus allows them to set higher prices in equilibrium. The latter, positive effect on payoffs dominates.

Having many firms in the market does not help to cure the problem. Indeed, welfare falls for larger numbers of firms, as the average quality offered decreases. Depending on the maximally possible quality, a monopoly or oligopoly of firms is best for welfare.

Fixing prices exogenously would destroy the incentives of the firms to choose low qualities. Then, all firms would offer the best possible quality. An increase in qualities offered could also be achieved by making the market more transparent to the patients.

A Mixed Strategy Equilibria of the Quality Setting Game

In this section we round out the game-theoretic analysis with a discussion of mixed strategy equilibria of the quality setting game with $\overline{\alpha} \in (\frac{1}{2}, 1)$. While showing existence of a symmetric mixed strategy equilibrium based on abstract existence results is straightforward, we cannot give such equilibria explicitly for $n > 2$. Nevertheless, in the following proposition we show that (symmetric) mixed equilibria are characterized by rather low qualities as well: In expectation, the quality a doctor sets lies in the lower half of $[\frac{1}{2}, \overline{\alpha}]$.

Proposition 5 *For $\overline{\alpha} \in (\frac{1}{2}, 1)$, the quality setting game has a symmetric mixed strategy equilibrium. Fix such an equilibrium given by a distribution function F on $[0, \overline{\alpha}]$. Then supp $F \subseteq [\frac{1}{2}, \overline{\alpha}]$ and $\{\frac{1}{2}, \overline{\alpha}\} \subseteq$ supp F. Denote by μ a doctor's expected equilibrium quality, i.e.*

$$\mu = \int_{\frac{1}{2}}^{\overline{\alpha}} \alpha \, dF(\alpha).$$

Then

$$\mu \leq 1 - \frac{1}{4\overline{\alpha}} \leq \frac{1}{2}\left(\frac{1}{2} + \overline{\alpha}\right).$$

Furthermore, equilibrium payoffs π are bounded from below by

$$\pi \geq \frac{1}{4}\left(\frac{1}{4\overline{\alpha}}\right)^{n-2} \geq \left(\frac{1}{2}\right)^{2n-2}.$$

While our bound on the expected equilibrium quality is not very sharp, note that it does not depend on n. Recall from Corollary 1 that market shares are more evenly distributed than qualities. Thus, although for large n with high probability there will be a considerable number of high quality doctors, this will not prevent a considerable portion of patients from choosing low quality treatments. Hence also in the mixed strategy equilibrium, competition does not force doctors to offer high qualities and social welfare remains bounded away from $\overline{\alpha}$.

For the two player case we can give a symmetric equilibrium of the quality setting game explicitly: It is easy to check that F given by

$$F(\alpha) = \frac{4\alpha^2 - 1}{4\alpha^2} \text{ for } \alpha \in [\frac{1}{2}, \overline{\alpha})$$

and $F(\overline{\alpha}) = 1$ is indeed an equilibrium.

Finally note that there are also asymmetric mixed strategy equilibria. For instance, with two players, an equilibrium is given by one doctor setting a quality of $1 - \frac{1}{4\overline{\alpha}}$ and the other doctor setting a quality of $\overline{\alpha}$ with probability $\frac{1}{3 - \frac{1}{\overline{\alpha}}}$ and of $\frac{1}{2}$ with the counter probability.

B   Proofs

Proof of Proposition 1:
Without loss of generality assume $\overline{\alpha} \geq \alpha_n \geq \alpha_{n-1} \geq \ldots \geq \alpha_1 \geq 0$.
The maxmin payoff of doctor n

$$\alpha_n \prod_{j \neq n}(1 - \alpha_j)$$

is strictly positive. So doctor n must have a strictly positive payoff in any equilibrium. Fix such an equilibrium. p_n^l, the lowest value in the support of doctor n's equilibrium strategy, must be strictly positive. Let π_n be doctor n's expected equilibrium payoff. Define a positive constant C, depending on $\alpha_1, \ldots, \alpha_n$, via $\pi_n = \alpha_n C$.

In this equilibrium, it must then be that each doctor i has an expected payoff π_i of $\alpha_i C$:

If some doctor $j < n$ earned more, say $\alpha_j D$ where $D > C$, doctor n could deviate to a price p^* that was smaller but close to p_j^l. Through this, doctor n could make a profit arbitrarily close to $\alpha_n D$ or even higher. This is because, setting the price p^*, doctor n faces as many or even less cheaper competitors as doctor j did in equilibrium at price p_j^l (as $p^* < p_j^l$). So, given he gets a good report (for which the probability is α_n), doctor n earns at least something close to D: The price is slightly lower, the possibility to be chosen by the patients is equal or higher (due to the slightly lower price).

If some doctor $j < n$ earned less than $\alpha_i C$, he could use an analogous type of deviation (thus deviate to a price slightly below p_n^l) and raise his expected profits.

Hence for any equilibrium there is some constant C such that for all i the equilibrium payoff of doctor i is given by

$$\pi_i = \alpha_i \, C. \tag{2}$$

Next we show that there is some doctor who does not earn more than his maxmin payoff. We start by establishing the following facts:

1) In equilibrium, no doctor places an atom in the interval $[0, 1)$: Placing an atom on $p = 0$ cannot be part of an equilibrium strategy as it leads to zero payoffs. (Recall that we have shown above that equilibrium payoffs are positive.) Placing an atom on $p \in (0, 1)$ cannot be part of an equilibrium strategy either: Assume some doctor i did so. Then consider a small price interval $(p, p+\epsilon)$. If no other doctor sets prices in this interval, doctor i is better off by shifting his atom a bit to the right. If there is some doctor j in this interval, doctor j should better shift the mass he has in the interval to some price slightly left from p. This would give doctor j a slightly lower price in case patients buy his services, but the probability that patients choose him increases substantially. (If ϵ is small enough, the second effect will dominate the first one.)

2) In equilibrium, at most one doctor will place an atom on $p = 1$: If several doctors had an atom on $p = 1$, each of them would have an incentive to deviate and shift his atom to a slightly lower value.[21]

3) In equilibrium, the support of some doctor's pricing strategy must go up to 1: If $p^h < 1$ was the highest price in the supports of the doctors' strategies, some doctor whose support went up to p^h could earn more by shifting probability mass from a neighborhood of p^h to 1. This would give him a substantially higher price while only slightly diminishing his chances of winning. (Recall that no doctor would set an atom on $p^h < 1$.)

We can now show that, in equilibrium, there is some doctor k^* who only earns his maxmin payoff: From 3), we know that there is a doctor whose strategy support

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[21] This argument would not be valid in Varian's (1980) model of sales where, so to say, either all or only one doctor (seller) is recommended. There (as long as other doctors set lower prices with certainty) two or more doctors setting an atom in 1 do not stand in competition because each of them is only attended by patients who are exclusively aware of him. In contrast, in our model for any subset of doctors there is a group of patients who are aware of these doctors and no others. This is the reason for the contrast between our unique equilibrium and the multiplicity of equilibria in the Varian model pointed out by Baye, Kovenock and de Vries (1992).

goes up to 1. This doctor can only earn more than his maxmin payoff in $p = 1$ if he has a competitor who sets an atom on $p = 1$. But then, this competitor cannot earn more than his maxmin payoff (since there cannot be two doctors setting an atom on 1, compare 2)).

Thus, there is some doctor k^* earning his maxmin payoff $\pi_{k^*} = \alpha_{k^*} \prod_{j \neq k^*}(1 - \alpha_j)$ implying $C = \prod_{j \neq k^*}(1 - \alpha_j)$ for all doctors in equilibrium. Thus, in equilibrium,

$$\pi_i = \alpha_i C = \alpha_i \prod_{j \neq k^*}(1 - \alpha_j)$$

for all i. It remains to be shown that $k^* \in \mathrm{argmax}_j \alpha_j$:

Assume $\alpha_{k^*} < \alpha_n$. Then the payoff of doctor n is

$$\pi_n = \alpha_n \prod_{j \neq k^*}(1 - \alpha_j),$$

which is strictly smaller than doctor n's maxmin payoff $\alpha_n \prod_{j \neq n}(1 - \alpha_j)$ since $1 - \alpha_{k^*} > 1 - \alpha_n$. This would give doctor n an incentive to deviate. Thus we get $k^* = n$. $\qquad\square$

Proof of Proposition 2

The proof consists of two parts. In part 1 we show that the vector of strategies defined in the proposition is indeed an equilibrium. In part 2 we show in three steps that the equilibrium is unique.

Part 1

Let $\pi_i(p)$ denote the payoff of doctor i given that he chooses p with certainty and given that the other doctors mix according to the distribution functions G_k described in the proposition. Clearly

$$\pi_i(p) = p\alpha_i \prod_{k \neq i}(1 - \alpha_k G_k(p)). \tag{3}$$

We have to show that, for all i, there exists a set S_i with mass 1 under G_i so that π_i is constant on S_i and π_i is weakly greater on S_i than on S_i^C. We will show this with $S_n = [p_0, p_n]$ and with $S_i = [p_0, p_i)$ for $i < n$.

For $p \in [0, p_0)$ we have $\pi_i(p) \leq \pi_i(p_0)$ as the other doctors do not put any mass on $[0, p_0)$: Deviating below p_0 gives the same chances of attracting a patient as playing p_0 but at a smaller price.

Now we show that $\pi_i(p)$ is constant on $[p_0, p_i] \setminus \{1\}$. The case $p = 1$ will be treated separately afterwards. By (3) and the definition of the G_k

$$\pi_i(p) = p\alpha_i(1 - \alpha_1) \cdot \ldots \cdot (1 - \alpha_{j-1}) \left[\sqrt[n-j]{\frac{(1 - \alpha_j) \cdot \ldots \cdot (1 - \alpha_{n-1})}{p}} \right]^{n-j}$$

for $p \in [p_{j-1}, p_j] \subset [p_0, p_i]$ where we use the fact that $n - j$ of i's opponents have p in the supports of their pricing strategies, while the $j - 1$ remaining doctors put all their probability mass below p. This immediately implies that

$$\pi_i(p) = \alpha_i(1 - \alpha_1) \cdot \ldots \cdot (1 - \alpha_{n-1}). \tag{4}$$

The case $p = 1$ poses a minor technical difficulty due to tie-breaking: If doctor n does not have an atom on 1 (i.e. $\alpha_{n-1} = \alpha_n$), the calculation for $p < 1$ still goes through. If doctor n has an atom on 1, it is easy to check that doctors $i \neq n$ with $p_i = 1$ earn less than the payoff from (4) when playing 1. But this does not give them an incentive to deviate from playing G_i as the probability of playing 1 is zero under G_i. Doctor n plays 1 with positive probability but he does not face opponents who do: He gets his maxmin payoff which corresponds to his payoff from (4) when playing $p = 1$. Thus doctor n does not have an incentive to deviate either.

To conclude the proof, we have to show that doctor i with $p_i < 1$ does not have an incentive to set prices $p \in (p_i, 1]$: Consider $p \in [p_{j-1}, p_j]$ with $j > i$ (so that $p_i \leq p_{j-1}$). By (3) we have

$$\begin{aligned} \pi_i(p) &= p\alpha_i \left[\sqrt[n-j]{\frac{(1 - \alpha_j) \cdot \ldots \cdot (1 - \alpha_{n-1})}{p}} \right]^{n-j+1} \prod_{\substack{1 \leq k \leq j-1 \\ k \neq i}} (1 - \alpha_k) \\ &= \alpha_i \sqrt[n-j]{\frac{(1 - \alpha_j) \cdot \ldots \cdot (1 - \alpha_{n-1})}{p}} \prod_{\substack{k \neq n \\ k \neq i}} (1 - \alpha_k). \end{aligned}$$

The above expression is strictly decreasing in p, so that it is sufficient to consider

deviations to p_{j-1}. Using the definition of p_{j-1} we see that

$$\pi_i(p_{j-1}) = \alpha_i(1 - \alpha_{j-1}) \prod_{\substack{k \neq n \\ k \neq i}} (1 - \alpha_k).$$

This is weakly smaller than the payoff from (4) as $p_i \leq p_{j-1}$ implies $1 - \alpha_i \geq 1 - \alpha_{j-1}$. Thus, doctor i does not have an incentive to deviate to prices above p_i.

Part 2

The uniqueness part of the proof proceeds in a number of steps. We start with some preliminary observations:

First, recall from the proof of Proposition 1 that in any equilibrium doctors will not place atoms except for one doctor possibly putting an atom on 1. From Proposition 1 we also know that, for all i, doctor i's equilibrium payoff is given by

$$\pi_i = \alpha_i C \text{ where } C := \prod_{j=1}^{n-1} (1 - \alpha_i) > 0. \tag{5}$$

Note that the first equality states that all doctors' expected equilibrium payoffs *conditional on being recommended* must be identical. Furthermore, recall from the proof of Proposition 1 that the union of the agents' strategies' supports must go up to 1. Due to the positive equilibrium payoffs this union must be bounded away from zero. Denote by p_L the infimum of the union of equilibrium supports. Now note that this union of supports must be an interval $[p_L, 1]$, i.e. there cannot be any gaps in the union of supports: If there was an interval $[\underline{p}, \overline{p}] \subset [p_L, 1]$ where no doctor was active, a doctor who would be playing prices right below $\underline{p}$ could deviate by moving probability mass from a small interval below $\underline{p}$ to $\overline{p}$ yielding a substantially better price at a marginally lower probability of winning. Thus the union of supports must be an interval $[p_L, 1]$. Note also that there cannot be a subset $[\underline{p}, \overline{p}] \subset [p_L, 1]$ where only one doctor is active: That doctor could profitably deviate by concentrating all probability mass of the interval in an atom at $\overline{p}$ which yields a higher price at the same probability of winning.

Armed with these insights we turn to the first major step in the proof:

1) In any equilibrium, the support of every doctor goes down to the same $p_L > 0$. Furthermore, in any equilibrium, $p_L = C$.

Proof of 1): Consider two doctors i and j with supports S_i and S_j. Assume $p_L^i < p_L^j$ where $p_L^k = \inf S_k$ for $k = i, j$. Note that with positive probability doctor i plays a price from $[p_L^i, p_L^j]$ and that agent j's payoff from playing p_L^j equals $\alpha_j C > 0$. But this implies that doctor i can earn more than his equilibrium payoff of $\alpha_i C$ by playing p_L^j: Since - unlike doctor j - doctor i does not have himself as a possibly cheaper competitor when playing p_L^j, his expected payoff conditional on being recommended must be higher than that of j. This is, however, a contradiction to (5) and thus the support of every doctor goes down to the same p_L. To see that $p_L = C$, note that, for all j, doctor j's payoff from playing p_L must be $\alpha_j p_L$: The other doctors ask for higher prices with probability 1 and thus doctor j gets all the patients to which he was recommended and they pay him p_L. These payoffs of $\alpha_j p_L$ are, however, only consistent with (5) if $p_L = C$.

The next step further characterizes the functional form of the doctor's equilibrium distribution functions:

2) Let $\mathcal{D} \subset \{1, \ldots, n\}$ denote the set of doctors who are active on some interval $I = (\underline{p}, \overline{p})$ in some arbitrary but fixed equilibrium. Assume all doctors $j \in \mathcal{D}$ are active at any $p \in I$ and let $m = \#\mathcal{D}$. (Note that from our preliminary observations we have $m \neq 1$.) We will show that for all $j \in \mathcal{D}$ the equilibrium distribution functions $G_j(p)$ must satisfy for all $p \in I$

$$G_j(p) = \frac{1}{\alpha_j} \left(1 - {}^{m-1}\!\sqrt{\frac{H}{p}} \right) \tag{6}$$

where $H > 0$ depends on the α_i and on the probability mass placed below $\underline{p}$ by the doctors from $\mathcal{D}^C$, namely,

$$H = \frac{C}{\prod_{i \in \mathcal{D}^C}(1 - \alpha_i G_i(\underline{p}))}.$$

Proof of 2): To see this, note that for all $j \in \mathcal{D}$ and all $p \in I$ the expected payoff of doctor j from playing p must equal the equilibrium payoff of $\alpha_j C$. Using (3) and the fact that distribution functions of the inactive doctors are constant over I this condition reads

$$\alpha_j C = p\alpha_j \left[\prod_{i \in \mathcal{D}^C}(1 - \alpha_i G_i(\underline{p})) \right] \left[\prod_{k \in \mathcal{D} \setminus \{j\}}(1 - \alpha_k G_k(p)) \right].$$

Rearranging and using the definition of H yields for all $p \in I$ and $j \in \mathcal{D}$

$$\prod_{k \in \mathcal{D} \setminus \{j\}} (1 - \alpha_k G_k(p)) = \frac{H}{p}. \tag{7}$$

Now consider (7) for two different doctors $j_1, j_2 \in \mathcal{D}$. Taking the quotient of (7) for j_1 and (7) for j_2 yields that for all $p \in I$

$$1 = \frac{1 - \alpha_{j_2} G_{j_2}(p)}{1 - \alpha_{j_1} G_{j_1}(p)}$$

which implies that there is a function $h(p)$ such that $h(p) = \alpha_k G_k(p)$ for all $k \in \mathcal{D}$. Substituting $h(p)$ for $\alpha_k G_k(p)$ on the left hand side of (7) and then solving for h yields

$$h(p) = 1 - \sqrt[m-1]{\frac{H}{p}}$$

and thus

$$G_j(p) = \frac{1}{\alpha_j} \left(1 - \sqrt[m-1]{\frac{H}{p}} \right)$$

as required.

The last main step before we can conclude the proof shows that all doctors' supports are intervals, i.e. no doctor will be inactive over some range of prices (above p_L) but put positive probability mass on prices above that range:

3) For all j the support of doctor j's strategy is of the form $[p_L, p_H^j]$ for some $p_L < p_H^j \leq 1$.

Proof of 3): The proof is by contradiction. Assume that some price $\bar{p}$ is in the support of the strategy of doctor j but j is inactive on some interval below $\bar{p}$. Choose $\underline{p} < \bar{p}$ such that for all $p \in I = (\underline{p}, \bar{p})$ the set of doctors who are active at p is identical. (This is possible since there are no atoms and thus the G_i are continuous.) Denote the set of doctors active on I by $\mathcal{D}$. Using (3) as in step 2) we can write the payoff of player j from playing some $p \in I \cup \{\bar{p}\}$ as

$$\pi_j(p) = \alpha_j p \left[\prod_{i \in \mathcal{D}^C \setminus \{j\}} (1 - \alpha_i G_i(\underline{p})) \right] \left[\prod_{k \in \mathcal{D}} (1 - \alpha_k G_k(p)) \right]$$

Defining the constant factor from the other inactive doctors as

$$K := \left[\prod_{i \in \mathcal{D}^C \setminus \{j\}} (1 - \alpha_i G_i(\underline{p})) \right].$$

and using (6) from the last step, we can express $\pi_j(p)$ as

$$\pi_j(p) = \alpha_j p K \left(\sqrt[m-1]{\frac{H}{p}} \right)^m = \alpha_j K H^{\frac{m}{m-1}} \sqrt[m-1]{\frac{1}{p}}$$

where the constant H is as defined in Step 2. Note that this implies that $\pi_j(p)$ is strictly decreasing in p over $I \cup \{\overline{p}\}$. By assumption, doctor j is active at $\overline{p}$ and thus earns his equilibrium payoff there:

$$\pi_j(\overline{p}) = \alpha_j C.$$

But since $\pi_j(p)$ is decreasing, this implies that for $p \in I$

$$\pi_j(p) > \alpha_j C.$$

such that doctor j can profitably deviate - which is a contradiction.

To conclude the proof, we still have to show that the vector of strategies defined in the proposition is actually the only candidate for an equilibrium. We have seen that all supports start at $p_L = C$ and since doctors do not set atoms or leave gaps in their supports, all doctors remain active up to the price p_1 where the first doctor(s) j have $G_j(p_1) = 1$. Note that on any interval $[p_L, \overline{p}]$ where all doctors are active, all distribution functions are uniquely pinned down by Step 2. This also uniquely determines p_1 and the set of agents who have $G_j(p_1) = 1$. Above p_1, all doctors who still have probability mass to spend must remain active. By Step 2, distribution functions above p_1 are again uniquely determined, determining in turn uniquely the price $p_2 > p_1$ where the next supports end. Continuing this procedure sequentially until $p = 1$ or until all or all but one distribution functions equal 1 determines a unique candidate for an equilibrium. It is easy to check that this unique candidate is actually the vector of strategies stated in the proposition. $\square$

Proof of Corollary 1

Note that we can write

$$m_i = \int_{p_0}^{p_i} \alpha_i \prod_{k \neq i} (1 - \alpha_k G_k(p)) dG_i(p).$$

Note furthermore that the G_k were chosen such that for $p \in [p_0, p_i]$

$$\prod_{k=1}^{n-1} (1 - \alpha_k) = p \prod_{k \neq i} (1 - \alpha_k G_k(p)).$$

Thus

$$m_i = \alpha_i \prod_{k=1}^{n-1} (1 - \alpha_k) \int_{p_0}^{p_i} \frac{1}{p} dG_i(p).$$

Define $b_i = \int_{p_0}^{p_i} \frac{1}{p} dG_i(p)$. Recall that because of the shape of the G_k and because of $\alpha_i < \alpha_j$ we can see G_j as a mixture of G_i and a probability distribution over $[p_i, p_j]$. Since $\frac{1}{p}$ is decreasing, this implies $b_i > b_j$ and thus

$$\frac{m_i}{m_j} = \frac{\alpha_i b_i}{\alpha_j b_j} > \frac{\alpha_i}{\alpha_j}.$$

It remains to be shown that $m_i < m_j$. For this denote by $g_i(p)$ and $g_j(p)$ the densities of $G_i(p)$ and $G_j(p)$.[22] Note that for $p \in [p_0, p_i]$ we have $\alpha_i G_i(p) = \alpha_j G_j(p)$ and thus $\alpha_i g_i(p) = \alpha_j g_j(p)$. Thus we can deduce:

$$
\begin{aligned}
m_i &= \prod_{k=1}^{n-1} (1 - \alpha_k) \int_{p_0}^{p_i} \frac{1}{p} \alpha_i g_i(p) dp = \prod_{k=1}^{n-1} (1 - \alpha_k) \int_{p_0}^{p_i} \frac{1}{p} \alpha_j g_j(p) dp \\
&< \prod_{k=1}^{n-1} (1 - \alpha_k) \int_{p_0}^{p_j} \frac{1}{p} \alpha_j g_j(p) dp = m_j.
\end{aligned}
$$

$\square$

Proof of Proposition 3

The quantity of interest in this proof is $w(\overline{\alpha}, n)$, the proportion of patients healed in the $\frac{1}{2}$-$\overline{\alpha}$-equilibria with n doctors where $n \geq 1$ and $\frac{1}{2} < \overline{\alpha} \leq 1$. Clearly, $w(\overline{\alpha}, n)$

---

[22]Note that these densities are well-defined but exhibit jumps at the p_k. Without loss of generality we can choose the densities to be left-continuous. In this proof we chose to ignore the atom of G_n since it only complicates notation without adding further insight.

can be written as

$$w(\overline{\alpha}, n) = p_g \overline{\alpha} + p_b \frac{1}{2} + p_0 0 \tag{8}$$

where p_g, p_b and p_0 denote the fractions of patients consulting the good doctor (i.e. the doctor offering $\overline{\alpha}$), the fraction consulting the other doctors and the fraction who stays at home, respectively. Note that p_g, p_b and p_0 are unique since the price setting game has a unique equilibrium by Proposition 2 and that they can be calculated from the equilibrium strategies given there: The good doctor mixes over $[2^{-(n-1)}, 1]$ using the distribution function

$$F_g(p) = \frac{1}{\alpha}\left(1 - \sqrt[n-1]{\frac{1}{2^{n-1}p}}\right)$$

and puts an atom of size $1 - \frac{1}{2\alpha}$ on 1. The remaining doctors mix over the same interval with

$$F_b(p) = 2\left(1 - \sqrt[n-1]{\frac{1}{2^{n-1}p}}\right).$$

Note that the pricing strategy of the good doctor can be interpreted in the following way: With probability $1 - \frac{1}{2\alpha}$ he sets a price of 1 and with probability $\frac{1}{2\alpha}$ he uses exactly the same pricing strategy as the other doctors. In the first case the good doctor only gets patients if no other doctor is recommended (which happens with probability $2^{-(n-1)}$). In the second case, the good doctor has exactly the same chances to acquire a patient as the other recommended doctors. Let the random variable r_n denote the number of bad doctors who are recommended to a patient. Then the market share of the good doctor can be written as

$$p_g = \overline{\alpha}\left[\frac{1}{2\alpha}E\left[\frac{1}{1+r_n}\right] + (1 - \frac{1}{2\alpha})2^{-(n-1)}\right] \tag{9}$$

where the leading factor $\overline{\alpha}$ results from the fact that the doctor is only competing for the patients to which he is recommended. Note that r_n is distributed binomially with parameters $n-1$ and $\frac{1}{2}$ and thus

$$
\begin{aligned}
E\left[\frac{1}{1+r_n}\right] &= \sum_{k=0}^{n-1}\binom{n-1}{k}2^{-(n-1)}\frac{1}{1+k} = \frac{1}{n}\sum_{k=0}^{n-1}\binom{n}{k+1}2^{-(n-1)} \\
&= \frac{2}{n}\sum_{k=1}^{n}\binom{n}{k}2^{-n} = \frac{2}{n}\left(-2^{-n} + \sum_{k=0}^{n}\binom{n}{k}2^{-n}\right) \\
&= \frac{2}{n}(1 - 2^{-n})
\end{aligned}
$$

where in the first step we have used that $n\binom{n-1}{k} = (k+1)\binom{n}{k+1}$ and in the final step we used that the sum equals 1 since it simply adds up all probabilities of a Binomial $(n, \frac{1}{2})$ distribution. Putting this into (9) and rearranging terms gives us

$$p_g = \frac{1}{n}(1 - 2^{-n}) + (\overline{\alpha} - \frac{1}{2})2^{-(n-1)}.$$

Clearly $p_0 = (1 - \overline{\alpha})2^{-(n-1)}$. Thus we see that - as claimed in the main text - as n gets large p_g goes to zero like $\frac{1}{n}$ and thus much slower than p_0 which decreases exponentially. Inserting these expressions for p_g and p_0 and $p_b = 1 - p_g - p_0$ into (8) and rearranging we obtain

$$w(\overline{\alpha}, n) = \frac{1}{2} + (\overline{\alpha} - \frac{1}{2})\frac{1}{n}(1 - 2^{-n}) + (2\overline{\alpha}^2 - \overline{\alpha} - \frac{1}{2})2^{-n}. \tag{10}$$

The remainder of the proof consists of an analysis of the function $w(\overline{\alpha}, n)$.[23] We will proceed in the following way: First, we will show that for every α as n gets large w approaches $\frac{1}{2}$ from above. Then we will show that $w(\overline{\alpha}, x)$, where $x \in \mathbb{R}_{\geq 1}$, is monotonically decreasing in x for $\alpha \geq \frac{1}{4}(1 + \sqrt{5}) \approx 0.809$. For $\frac{1}{2} < \overline{\alpha} < \frac{1}{4}(1 + \sqrt{5})$ we will show that w increases in x up to some value $x^* \geq 1$ and decreases from there on. From this we can conclude that $w(\overline{\alpha}, 1) \geq w(\overline{\alpha}, 2)$ is a sufficient condition for $n^* = 1$ being a maximizer of $w(\overline{\alpha}, n)$. Then we will verify that $w(\overline{\alpha}, 1) \geq w(\overline{\alpha}, 2)$ if and only if $\overline{\alpha} \geq \frac{3}{4}$. From these results we can conclude that for $\overline{\alpha} > \frac{3}{4}$ $w(\overline{\alpha}, n)$ is maximized at $n^* = 1$ while for $\overline{\alpha} \geq \frac{3}{4}$ there is a finite $n^* > 1$ which maximizes $w(\overline{\alpha}, n)$. Furthermore it follows that there are at most two maximizers and if there are two, these must be subsequent integers. In the final part of the proof we will show an upper bound on n^* in terms of $\overline{\alpha}$.

It is immediate that $w(\overline{\alpha}, n)$ converges to $\frac{1}{2}$ as n goes to infinity. To show that this convergence is from above we have to show for every fixed $\overline{\alpha}$ that $w(\overline{\alpha}, n) > \frac{1}{2}$ for sufficiently large n. Here and in the following we will substitute n by the real-valued parameter x and view w as the weighted sum of two functions f and g which do not depend on $\overline{\alpha}$:

$$w(\overline{\alpha}, x) = \frac{1}{2} + (\overline{\alpha} - \frac{1}{2})f(x) + (2\overline{\alpha}^2 - \overline{\alpha} - \frac{1}{2})g(x)$$

where

$$f(x) = \frac{1}{x}(1 - 2^{-x}) \text{ and } g(x) = 2^{-x}.$$

Note that the coefficient of f is always positive while the coefficient of g is zero at $\overline{\alpha} = \frac{1}{4}(1 + \sqrt{5})$ and strictly increasing over $[\frac{1}{2}, 1]$. Note furthermore that g and f are strictly decreasing and positive. For g this is clear, for f note that

$$f'(x) = -[2^x - (1 + x\ln(2))]x^{-2}2^{-x}.$$

The factors outside the brackets are clearly positive. The term in squared brackets is positive for all $x \geq 1$ since it is the difference between the convex function 2^x and its first order Taylor approximation in 0. Thus $f'(x) < 0$ for all $x \geq 1$.

Note that in the case $\overline{\alpha} \geq \frac{1}{4}(1 + \sqrt{5})$ where the coefficients of f and g are both non-negative it is immediate that w is strictly decreasing in n, always greater than $\frac{1}{2}$ and maximized by $n^* = 1$. The case where the coefficient of g is negative requires more work however. The key observation driving the argumentation that follows now is that f decreases much slower than g and thus the term with the positive coefficient will eventually dominate - even if for $\overline{\alpha}$ near $\frac{1}{2}$ this coefficient is very small.

Note first that we can rewrite w to

$$w(\overline{\alpha}, x) = \frac{1}{2} + g(x)\left[(\overline{\alpha} - \frac{1}{2})\frac{f(x)}{g(x)} + (2\overline{\alpha}^2 - \overline{\alpha} - \frac{1}{2})\right].$$

In order to show that this is greater than $\frac{1}{2}$ for x sufficiently large (and finite) it is sufficient to show that f/g tends to infinity as x gets large because this guarantees that the first (positive) summand in the squared brackets will eventually dominate the second one making the term in the squared brackets positive. Now f/g is easily calculated to be

$$\frac{f(x)}{g(x)} = \frac{2^x - 1}{x}$$

which obviously tends to infinity as x gets large.

Next we show for fixed $\overline{\alpha}$ that $w(\overline{\alpha}, x)$ has exactly one local maximum in x. This is equivalent to showing that (depending on $\overline{\alpha}$) the x-derivative of $w(\overline{\alpha}, x)$ is either negative for all x or changes signs exactly once on $[1, \infty)$ from positive to negative.

Note that we can write this derivative as

$$\frac{\partial w}{\partial x}(\overline{\alpha}, x) = g'(x)\left[(\overline{\alpha} - \frac{1}{2})\frac{f'(x)}{g'(x)} + (2\overline{\alpha}^2 - \overline{\alpha} - \frac{1}{2})\right]. \tag{11}$$

Recalling that $g' < 0$ it is clear that it is sufficient to prove that f'/g' is monotonically increasing and tends to infinity. It is easily calculated that

$$\frac{f'(x)}{g'(x)} = \frac{-1 + 2^x - x\ln(2)}{x^2\ln(2)} \tag{12}$$

which clearly tends to infinity as x gets large. In order to show monotonicity consider the derivative of f'/g' which can be written as

$$\frac{d}{dx}\frac{f'(x)}{g'(x)} = \frac{2^x(x\ln(2) - 2) - (-x\ln(2) - 2)}{x^3\ln(2)}.$$

To determine the sign of this expression we can concentrate on the numerator. Note that the numerator is exactly the difference between the function $2^x(x\ln(2)-2)$ and its first order Taylor approximation around 0. Since $2^x(x\ln(2)-2)$ is strictly convex on $(0, \infty)$ (its second derivative is $2^x x\ln(2)$) this difference is positive for $x > 0$. Thus f'/g' is monotonically increasing for $x \geq 0$ as desired. Thus we have shown for every $\overline{\alpha} > \frac{1}{2}$ that $w(\overline{\alpha}, x)$ has a unique maximizer $1 \leq x^* < \infty$.

Now we will consider $w(\overline{\alpha}, n)$ with an integer parameter n again. From the previous analysis it is clear that $w(\overline{\alpha}, n)$ is globally maximized by $n = 1$ if and only if

$$w(\overline{\alpha}, 1) - w(\overline{\alpha}, 2) \geq 0$$

(and $n = 1$ is the unique maximizer if the inequality holds strictly). Now we have

$$w(\overline{\alpha}, 1) - w(\overline{\alpha}, 2) = \frac{1}{2}\overline{\alpha}^2 + \frac{1}{8}\overline{\alpha} + \frac{3}{16}.$$

Since this is a quadratic polynomial, it is easily seen that it is increasing over $[\frac{1}{2}, 1]$ and zero for $\overline{\alpha} = \frac{3}{4}$. Thus $n^* = 1$ for $\overline{\alpha} \geq \frac{3}{4}$ and $n^* > 1$ for $\overline{\alpha} < \frac{3}{4}$. That for fixed $\overline{\alpha}$ there are at most two integer maximizers of $w(\overline{\alpha}, n)$ and that, if there are two, those must be subsequent integers follows trivially from the fact that $w(\overline{\alpha}, x)$ has a unique real-valued local (and thus global) maximizer.

In the final part of the proof we show an upper bound on n^* in terms of $\overline{\alpha}$. We will again consider the function $w(\overline{\alpha}, x)$ with real valued argument $x \geq 1$. Note that since $w(\overline{\alpha}, x)$ has a unique local maximum for fixed $\overline{\alpha}$, any point x where the x-derivative of $w(\overline{\alpha}, x)$ is negative must lie to the right of the maximizer x^*. We will construct a function $B(\overline{\alpha})$ with the property that

$$x > B(\overline{\alpha}) \Rightarrow \frac{\partial}{\partial x} w(\overline{\alpha}, x) < 0.$$

This implies $x^* \leq B(\overline{\alpha})$ and thus $n^* \leq B(\overline{\alpha}) + 1$.

Note that from (11) and (12) $\frac{\partial}{\partial x} w(\overline{\alpha}, x) < 0$ is equivalent to

$$\frac{-1 + 2^x - x \ln(2)}{x^2 \ln(2)} > \frac{\overline{\alpha} + \frac{1}{2} - 2\overline{\alpha}^2}{\overline{\alpha} - \frac{1}{2}}.$$

We will now try to find a sufficient condition for this which is of the desired form. Note that the numerator of the right hand side only fluctuates between $\frac{1}{2}$ and $-\frac{1}{2}$ and thus a sufficient condition is

$$\frac{2^x}{x^2 \ln(2)} > \frac{1 + x \ln(2)}{x^2 \ln(2)} + \frac{1}{2(\overline{\alpha} - \frac{1}{2})}.$$

Now note that the first term on the right hand side is at most $(1 + \ln(2))/\ln(2)$ and thus a sufficient condition is

$$\frac{2^x}{x^2} > 1 + \ln(2) + \frac{\ln(2)}{2(\overline{\alpha} - \frac{1}{2})}.$$

Taking logarithms on both sides yields

$$x \ln(2) - 2 \ln(x) > \ln\left(1 + \ln(2) + \frac{\ln(2)}{2(\overline{\alpha} - \frac{1}{2})}\right).$$

Since $\ln(x)$ is concave we can bound it from above by its first order Taylor approximation in 8 which is $\ln(8) + (1/8)x$.[24] Thus a sufficient condition is

$$x > \frac{2 \ln(8)}{\ln(2) - \frac{1}{4}} + \frac{1}{\ln(2) - \frac{1}{4}} \ln\left(1 + \ln(2) + \frac{\ln(2)}{2(\overline{\alpha} - \frac{1}{2})}\right).$$

---

[24]The choice of 8 as the expansion point is essentially arbitrary, it only matters that the value is large enough so that the left hand side of the equation remains increasing in x.

Since this bound is not very sharp anyway we can afford to improve readability by bounding the logarithms by real numbers in a way that the condition remains sufficient and get

$$x > 9.4 + 2.3 \ln \left(1.7 + \frac{0.35}{\overline{\alpha} - \frac{1}{2}} \right) =: B(\overline{\alpha}).$$

As argued above this implies

$$n^* \leq 10.4 + 2.3 \ln \left(1.7 + \frac{0.35}{\overline{\alpha} - \frac{1}{2}} \right).$$

$\square$

Proof of Observation 2

This observation follows immediately from the fact that

$$\frac{\partial}{\partial \overline{\alpha}} \, \overline{\alpha}(1 - \overline{\alpha})^{n-1} = (1 - n\overline{\alpha})(1 - \overline{\alpha})^{n-2}$$

is negative for $\overline{\alpha} \in (\frac{1}{n}, 1)$. $\square$

Proof of Proposition 4

We first search for a symmetric Nash equilibrium of the quality setting game played by the doctors $1, ..., n-1$ given that doctor n chooses a higher α (and thus does not affect the payoffs of doctors $1, ..., n-1$). Then we show that doctor n will indeed respond to this behavior by setting a higher quality than the other doctors.

Given that doctor n chooses a high α_n, doctors $i < n$ maximize

$$\alpha_i(1 - \alpha_i) \left[\prod_{j \neq i, n} (1 - \alpha_j) \right] - c(\alpha_i)$$

which yields a first order condition of

$$(1 - 2\alpha_i) \left[\prod_{j \neq i, n} (1 - \alpha_j) \right] - c'(\alpha_i) = 0.$$

35

Assuming symmetry for doctors $1, ..., n - 1$ this becomes

$$(1 - 2\alpha)(1 - \alpha)^{n-2} = c'(\alpha).$$

There is a unique $\alpha^l \in (0, \frac{1}{2})$ which solves this equation: the left hand side decreases strictly on $(0, \frac{1}{2})$ taking all values between 1 and 0 while the right hand side is strictly increasing starting with $0 < c'(0) < 1$ as c is assumed to be convex.

We now determine doctor n's best response to the other doctors playing α_l. Note that doctor n will not play qualities below α_l: On $[0, \alpha^l]$ doctor n has the same optimization problem as the other doctors in the previous step,

$$\max_{\alpha} \alpha(1 - \alpha)(1 - \alpha^l)^{n-2} - c(\alpha)$$

which is solved by $\alpha = \alpha^l$. On $[\alpha^l, 1]$ doctor n maximizes

$$\alpha(1 - \alpha^l)^{n-1} - c(\alpha)$$

which gives a first order condition of

$$(1 - \alpha^l)^{n-1} = c'(\alpha).$$

Note that the other doctors' first order condition implies that

$$(1 - \alpha^l)^{n-1} > c'(\alpha^l).$$

Thus we have to distinguish two cases: If

$$(1 - \alpha^l)^{n-1} < c'(1),$$

by the monotonicity of c', there is a unique best response $\alpha^h \in (\alpha^l, 1)$ which solves

$$(1 - \alpha^l)^{n-1} = c'(\alpha^h).$$

If

$$(1 - \alpha^l)^{n-1} \geq c'(1),$$

it is optimal for doctor n to play $\alpha^h = 1$. Finally, note that doctors $1, ..., n - 1$

do not want to deviate to qualities above α^h: If this were a profitable deviation for one of them, α^h would not have been optimal for doctor n (who faces weaker competitors than the other doctors).

To conclude the proof we have to show that α^l and α^h are decreasing in n: To make the dependence on n clearer we write $\alpha^l(n)$ and $\alpha^h(n)$. Recall that from the bad doctors' first order condition $\alpha^l(n)$ was the value of α where $c'(\alpha)$ and $(1-2\alpha)(1-\alpha)^{n-2}$ intersect. Increasing n to $n+1$ shifts the function $(1-2\alpha)(1-\alpha)^{n-2}$ downwards, implying that it intersects c' at a smaller value of α since c' is increasing. Thus $\alpha^l(n)$ is decreasing in n.

We now show that $\alpha^h(n)$ is weakly decreasing: As a preliminary observation, note that $c'(\alpha^l(n))$ is decreasing in n and thus by the bad doctors' first order condition

$$(1 - 2\alpha^l(n))(1 - \alpha^l(n))^{n-2} \tag{13}$$

is decreasing as well. If for some n the good doctor's first order condition is not binding (which implies $\alpha^h(n) = 1$) we obviously have $\alpha^h(n) \geq \alpha^h(n + 1)$. Now consider some n where the good doctor's first order condition is binding so that $\alpha^h(n)$ solves

$$(1 - \alpha^l(n))(1 - \alpha^l(n))^{n-2} = c'(\alpha^h(n)). \tag{14}$$

Clearly, to show that $\alpha^h(n) \geq \alpha^h(n + 1)$ we need to prove that the left hand side of (14) is decreasing: Recall that (13) is decreasing and note that the leading factor $(1 - 2\alpha^l(n))$ is increasing. Thus the second factor $(1 - \alpha^l(n))^{n-2}$ must be decreasing strongly enough to make the product (13) decreasing. The left hand side of (14) has the same second factor as (13). The first factor $(1 - \alpha^l(n))$ is however increasing less strongly than the first factor of (13). Thus the left hand side of (14) is dominated by its second, decreasing factor. $\qquad\square$

Proof of Proposition 5

Note that payoffs in the quality setting game are continuous, bounded and symmetric. Thus, for instance, the main result of Becker and Damianov (2006) shows existence of a symmetric equilibrium. Recall that a doctor's best response to any set of pure strategies by his opponents is playing either $\frac{1}{2}$ or $\overline{\alpha}$. Thus $(\frac{1}{2}, \ldots, \frac{1}{2})$ and $(\overline{\alpha}, \ldots, \overline{\alpha})$ are the only candidates for symmetric pure equilibria. Since neither of these is an equilibrium, we can conclude the existence of a symmetric equilibrium

in (non-degenerate) mixed strategies. Fix such an equilibrium and denote the equilibrium strategy by a distribution function F over $[0, \overline{\alpha}]$. Define $S = \text{supp } F$. Since qualities in $[0, \frac{1}{2})$ are strictly dominated by playing $\frac{1}{2}$ we can conclude $S \subseteq [\frac{1}{2}, \overline{\alpha}]$. Define $\underline{s} = \inf S$ and $\overline{s} = \sup S$. We must have $\overline{s} = \overline{\alpha}$ because otherwise a doctor could profitably deviate by playing $\overline{\alpha}$ instead of qualities near $\overline{s}$: For a doctor who is the (weakly) best doctor with high probability it is optimal to choose $\overline{\alpha}$. Analogously we must have $\underline{s} = \frac{1}{2}$ because otherwise a doctor would want to move probability mass from near $\underline{s}$ down to $\frac{1}{2}$.

What remains to be shown is the lower bound on expected equilibrium qualities and payoffs. Without loss of generality consider the payoff of doctor 1. For doctor 1, $\alpha_2, \ldots, \alpha_n$ are independent random variables with distribution function F. The main idea is to compare payoffs from playing $\frac{1}{2}$ and $\overline{\alpha}$. Denote these payoffs by $\pi_{\frac{1}{2}}$ and $\pi_{\overline{\alpha}}$. Note that

$$\pi_{\overline{\alpha}} = \overline{\alpha} E[\prod_{j=2}^{n}(1 - \alpha_j)] = \overline{\alpha}(1 - \mu)^{n-1}$$

since a doctor playing $\overline{\alpha}$ is the (weakly) best doctor with certainty. Denote by $\alpha_{j:n-1}$ the jth largest of $\alpha_2, \ldots, \alpha_n$. We can then write doctor 1's payoff from setting a quality of $\frac{1}{2}$ as

$$\pi_{\frac{1}{2}} = \frac{1}{4} E\left[\prod_{j=2}^{n-1}(1 - \alpha_{j:n-1})\right] \geq \frac{1}{4} E\left[\prod_{j=2}^{n-1}(1 - \alpha_j)\right] = \frac{1}{4}(1 - \mu)^{n-2}$$

where in the middle step we used that we can bound the product over the $n - 2$ smallest α_j by a product over an arbitrary collection of $n - 2$ of the α_j.[25] In equilibrium we must have that $\pi_{\frac{1}{2}} = \pi_{\overline{\alpha}}$. We can thus conclude that

$$\overline{\alpha}(1 - \mu)^{n-1} \geq \frac{1}{4}(1 - \mu)^{n-2}.$$

Solving for μ we get the bound

$$\mu \leq 1 - \frac{1}{4\overline{\alpha}}.$$

Since the right hand side is a convex function in $\overline{\alpha}$ we can bound it from above by the straight line connecting its values at the boundaries $\frac{1}{2}$ and $\overline{\alpha}$. This yields the

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[25]Note that how much we are giving away in this bound depends on n: Thus while our upper bound is independent of n, the same is not true for μ itself.

easier to read bound

$$\mu \leq \frac{1}{2}\left(\frac{1}{2} + \overline{\alpha}\right).$$

Plugging our upper bound on μ into the expression for $\pi_{\overline{\alpha}}$ we finally get the desired lower bound on equilibrium payoffs. $\qquad\square$

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