Estimating a War of Attrition: The Case of the US Movie Theater Industry*

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Abstract

This paper estimates the impact of competition and exogenous demand decline on the exit process of movie theaters in the US from 1950-1965. I modify Fudenberg and Tirole (1986)'s model of exit in a duopoly with incomplete information to an oligopoly. I estimate this model with panel data, using variations in TV diffusion across households and other market characteristics to identify the parameters in the theater's payoff function and the distribution of unobservable exit values. Using the estimated model, I show that theaters who are making negative profits may choose to remain in the market if they expect to outlast their competitors, because at that point their profits would increase. This creates a significant delay in the exit process. In addition, by decomposing the across-market variation in the exit process, I find that TV diffusion explains a substantial proportion of the variation in the exit process.

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1 Introduction

As demand decreases in an industry, firms often exit sequentially within a short period of time. One example of this is single-screen theaters in the US, where a drop in demand caused the number of movie theaters to decrease in the 1950s and 1960s. For example, in Illinois, more than 50% of theaters that were open in 1950 had exited the market by 1960. This demand decline was mostly due to exogenous forces, such as the nationwide penetration of televisions (Lev (2003) and Stuart (1960)). Since costs were mostly fixed and capacity adjustments were usually infeasible, theaters had to respond to declining demand by leaving the market. At the same time, since local markets were oligopolies, theaters considered their opponents' behaviors when choosing an optimal exit time. These interactions and demand decline jointly determine the exit process.

There are many examples of when diffusion of new technology creates a nonstationary environment, causing sequential exits from an oligopoly market. For example, today, online movie services are pressuring local video rental stores. Nevertheless, to the best of my knowledge, there are no empirical studies that separate players' interactions from exogenous demand factors in a declining industry. This paper contributes to the literature by providing a tractable framework for analyzing such industries.

This paper estimates the effect of competition among theaters separately from the effect of exogenous demand decline on the devolution of the industry. The starting point of this paper is the assumption that competition in the US movie theater industry in the 1950s and the 1960s resembled a "war of attrition." In a war of attrition, some players will eventually exit. As time passes, competition becomes costly, and each player incurs a loss while competing. If a player outlasts its competitors, it earns a positive profit. Therefore, at each moment, players decide if they should wait longer or not, for if they wait, there is some chance their competitors will exit. I assume there is asymmetric information about each player's profitability. Thus, as players stay longer, they learn more about their competitors. To separate these strategic interactions between theaters (i.e., outlasting motives) from exogenous demand change, I need to infer theaters' expectations and learning process, as well as theaters' payoff functions. This allows me to quantify the delay of exits (and the resulting cost) due to theaters' outlasting motives.

For this purpose, I modify Fudenberg and Tirole (1986)'s model of exit in duopoly with incomplete information to work in an oligopoly. At each instant, theaters choose whether to exit or stay in the market. I assume each theater knows its own value of exit (scrap value) but not that of its competitors. In equilibrium, theaters get discouraged and exit if competitors stay open long enough. As a result, theaters sequentially exit as follows: The theater with a higher exit value leaves earlier.

Using data from the US movie theater industry, I estimate theaters' payoff functions and the distribution of exit values. I use TV penetration rates, which vary across locations and time, to measure changes in demand. By imposing the equilibrium condition, the model predicts the distribution of theaters' exit times for a given set of parameters and unobservables. I estimate the parameters by matching the predicted distribution with the observed distribution of exit times.

In equilibrium, the value of exit for a marginal player (i.e., a player whose cost and benefit of waiting are exactly equal) decreases as time passes. The path of this value is given by a set of differential equations, and it serves as a policy function for theaters. By solving the set of differential equations, I can find equilibrium exit times. I can then repeatedly solve the model in a nested fixed point algorithm (the full solution approach), and this allows me to explicitly take theaters' expectations and unobservable market level heterogeneity into account.

This paper also develops a method to solve the initial condition problem. In particular, I control for the endogeneity generated by the correlation between the initial number of theaters and the unobservable variables of the model. To achieve the goal, I set up a model of entry at the beginning of my sample period. For a given set of unobservables, the entry game predicts the number of entrants in equilibrium. I restrict the support of market heterogeneity and exit values such that, in equilibrium, the entry game predicts the same number of entrants as is observed. Since it is hard to analytically characterize such a restriction, I use an MCMC-like procedure to approximate the joint distribution of unobservable variables, conditional on the observed number of entrants. This simulated distribution can then be used as an input to simulate and solve the dynamic stage of the game. This is another contribution of the paper to the literature of estimation of dynamic games.

The estimated parameters imply that, other things being equal, one additional competitor erodes 4.6% of theaters' mean profit. A change in TV penetration rates accelerates the decay of theaters' profit. In addition, demand decline is severe if population outflow is large.

Using the estimated model, I quantify the significance of a war of attrition. In a war of attrition with learning, players get discouraged and exit if competitors stay open long enough. I show that many theaters delay exiting, even when making negative profits, in hopes that their competitors will exit first.

These delays come from asymmetric information. If exit values are common knowledge, then in equilibrium, a theater exits the game exactly when its profit becomes lower than its exit value. I call this scenario the case of complete information. Cumulative market profits in the war of attrition and under complete information differ by up to 40%-50% of the monthly market profit.

In addition, I find that both TV penetration rates and population growth explain a substantial portion of across-market variation in the exit process. I also simulate the effect of a delayed diffusion process of televisions on theaters' exits. This is a policy relevant exercise because delaying the diffusion of a new technology is an option when the government wants to protect (at least in the short run) the affected sector. The simulation shows that when the nationwide penetration of televisions is delayed by 2 years, the share of surviving theaters increases by 15% in 5 years and by 4% in 15 years.

Related Literature

Klepper and Simons (2000) and Jovanovic and MacDonald (1994) investigate an industry where a large number of firms exited within a relatively short period of time. They analyze a shakeout that happened in the US tire industry, which they assume to be competitive. In their model, innovation opportunities encourage entry in the early stage of the industry. As the price decreases due to the new technology, firms that failed to innovate exit. Competition affects the devolution of the industry through the market price. In comparison, in the movie theater industry competition was local, and hence strategic interactions among theaters should be taken into account using an oligopoly setting. Another important difference is that the shakeout in the US tire industry was not due to declining demand.

Bulow and Klemperer (1999) analyze a general game in which there are $N + K$ players competing for N prizes. My model is different because the values of the prize (operating profits) change over time and are affected by the number of surviving players, which is endogenous. The theoretical difficulties of the N player case have been often discussed in the literature (see Argenziano and Schmidt-Dengler (2008) and Haigh and Cannings (1989)). I follow one of their assumptions to avoid the problem of non-existence of a symmetric equilibrium.

I use a full solution approach to estimate this dynamic game, instead of the 2-step method that has recently been used (e.g., Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007), Pakes, Ostrovsky, and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008)). In the first stage of the 2-step method, a policy function is calculated for every possible state, which is difficult in a non-stationary environment.[1] Moreover, unobservable variables play an important role in my model, so the first stage estimation in the 2-step method would not be consistent. For these reasons, I construct a tractable game, allowing me to repeatedly solve the equilibrium in the estimation algorithm. An additional advantage of the full solution method is the efficiency of the estimator.

Schmidt-Dengler (2006) analyzes the timing of new technology adoption, separately estimating how it is affected by business stealing and preemption, respectively. He imposes a sequential structure of moves of players and their orders of moves in each period of time. I do not need nor impose such a restriction.

The remainder of the paper is organized as follows. Section 2 briefly summarizes the background of the US movie theater industry from 1950-1965. Section 3 modifies the model of Fudenberg and Tirole (1986) to be used in an oligopoly. In Section 4, I describe the data and present simple regression analyses to see how theaters' exits and market characteristics are related. Section 5 discusses my estimation strategy. I present estimation results and simulation analyses in Section 6. Section 7 concludes. All proofs are shown in the Appendices.

2 The US Movie Theater Industry, 1950-1965

After a big boom starting in the 1920s, the US movie theater industry faced a severe decrease in demand in the 1950s and the 1960s, primarily due to the growth of television broadcasting. In 1950, less than one out of ten households in the US owned a TV set. In 1960, however, the share of households with a television reached almost 90%. In response, demand for theaters decreased. Movie attendance declined most quickly in places where televisions became available first, providing evidence that TV penetration caused a decline in demand. According to Stuart (1960), whenever an additional broadcast channel was added in a market, the decline in movie theater attendance accelerated immediately. Figure 1 demonstrates the relationship between TV penetration and decline in demand.

I focus on demand and exit behavior in the classic single-screen movie theater industry. Thus, suburbanization also contributed to the downturn in demand for those movie theaters. Many young people got married and moved to the suburbs, far away from the downtown movie theaters. Suburb growth and motorization facilitated the growth of drive-in theaters, which in turn further decreased demand for classic movie theaters.

[1] One notable exception is Weintraub, Benkard, Jeziorski, and Roy (2008).

Changes in government policy in the end of the 1940s also promoted this downturn in demand.[2] Vertical integrations between producers, distributors, and exhibitors had been wide-spread until the late 1940s. The major movie producers (called "Hollywood majors"[3]) formed an oligopoly in which they colluded to divide the country. The major producers had control over theaters through exclusive contracts and explicit price management. They owned 3,137 of 18,076 movie theaters (70% of the first-run theatres in the 92 largest cities). However, the Paramount decree (1946), put an end to this vertical integration, resulting in the segregation of those producers from their vertical chains of distributors and exhibitors. For example, explicit price management by distributors was prohibited. The government also mandated that the spun-off theater chains would have to further divest themselves of between 25 to 50 percent of their theater holdings.

This breakdown of the dominance of the major producers led movie theaters to a more unstable and risky business environment. For example, movie producers no longer had a strong incentive to produce movies year round. Furthermore, according to Lev (2003), the production companies started to regard television as an important outlet for their movies. In the era of vertical integrations, producers had an incentive to withhold their movies from televisions for the interest of their exhibitor-partners. However, after divestment, movie theaters became just one of the customers for producers, same as the television companies.

Because of all of these factors, demand for incumbent movie theaters shrunk in an arguably exogenous way. Almost all theaters had only a single screen in those days (e.g., the first twin theater in the Chicago area opened in 1964), and their fixed investments were often heavily mortgaged. Therefore, they could not adjust capacity to deal with declining demand. They could only bear the loss and stay open, or exit the market. As a result, during the 1950s and the 1960s, the number of theaters dropped sharply. In Figure 2, I plot the share of theaters that were open in 1950 and remained open in each year. Since a few theaters could still operate profitably, each theater preferred to stay open as long as it expected some competitors to exit early enough. This situation fits nicely into the framework of a war of attrition.

The structure of the US movie theater industry changed significantly in the late 1960s, when multiplex theaters emerged. This arguably changed the nature of competition. Once a theater has multiple screens, it can potentially respond to a change in demand by, for example, closing several screens. The structure of the industry became even more different

[2]The figures and facts in this paragraph are from Chapter 6 of Melnick and Fuchs (2004).

[3]Majors in this era include "Big Five" (Loew's/MGM, Paramount, Fox (which became 20th Century-Fox after a 1935 merger), Warner Bros., and RKO) and "Little Three" (Universal, Columbia, and United Artist).

and complicated after the 1980s because of the advent of home videos/DVD and horizontal integrations by big theater chains. Thus, I focus my analysis on the period between 1950 and 1965.

3 The Model

I assume that theaters play an exit game in each local market. Markets are independent from each other. I suppress the market subscript in this section for notational simplicity. I modify the model of exit in duopoly developed by Fudenberg and Tirole (1986) to an N-player oligopoly setting. I detail the three player case, because this setting is general enough in order to show the essence of this modification. I also focus on a symmetric equilibrium. Throughout this section, I use t to denote time, τ to denote a player's choice of exit time, and T to denote a player's strategy. In addition, I use the terms "player" and "theater" interchangeably in the rest of the paper.

3.1 Setup

Three theaters start the game at $t = 0$.[4] Time is continuous. At each instant, each theater chooses whether to exit the market or to stay open. Once a theater exits the market, it cannot re-enter. At time t, if $n \leq 3$ theaters are operating, each theater receives the common value $\Pi_n(t)$. I assume that this instantaneous profit is deterministic and decreases in the number of surviving theaters; i.e., $\Pi_1(t) > \Pi_2(t) > \Pi_3(t)$ for all $t \in [0, \infty]$.

When a theater exits, it receives its value of exit θ, which is private information. At the start of the game, theater i draws θ_i from the common distribution G, which has a density g on the support $\left[\underline{\theta}, \overline{\theta}\right]$, and $g > 0$ everywhere. θ can also be interpreted as the fixed cost that a theater incurs at each instant. Each theater discounts the future at a common rate r.

The game proceeds as follows. At $t = 0$, each theater decides when to exit, conditional on none of the competitors having exited by then. In Appendix A, I show that this decision is equivalent to choosing to exit or to remain open at every instant. When one theater exits, other surviving theaters revise their exit times, based on the currently available information. A strategy is a mapping from the state space of each subgame to the real line (exiting time),

[4] I use 1950 as the starting year of games, but for ease of exposition, I use $t = 0$ throughout our discussion of the model.

$T : S \to [0, \infty]$, where S denotes the domain of the mapping.[5] The domain of the strategy depends on the current number of survivors. I let T_1, T_2, and T_3 denote a player's strategy when there are one, two, and three surviving theaters in the game, respectively.

The value of being a monopolist is embedded into the two player game as a prize of winning the two player game. The value of playing the two player game is in turn embedded into the three player game. Because of the nested structure of the game, I begin with a monopolist's problem, then solve a two-player game, and finally characterize a three-player game. Throughout this section, let $t_{\mathcal{M}}$ and $t_{\mathcal{D}}$ denote the time when the one-player game and when the two-player game starts, respectively.

3.2 The One Player Case

Define the value of being a monopolist from time $t_{\mathcal{M}}$ onward as

$$V_1(\theta_i, t_{\mathcal{M}}) \equiv \max_{\tau \in [t_{\mathcal{M}}, \infty]} \left[\int_{t_{\mathcal{M}}}^{\tau} \Pi_1(s) e^{-r(s - t_{\mathcal{M}})} ds + \frac{\theta_i}{r} e^{-r(\tau - t_{\mathcal{M}})} \right]. \tag{1}$$

Thus, the strategy is a mapping from the type space and the current time to a real number; i.e., $T_1 : [\underline{\theta}, \overline{\theta}] \times \mathbb{R}_+ \to [t_{\mathcal{M}}, \infty]$. That is,

$$T_1(\theta_i, t_{\mathcal{M}}) \in \arg \max_{\tau \in [t_{\mathcal{M}}, \infty]} \left[\int_{t_{\mathcal{M}}}^{\tau} \Pi_1(s) e^{-r(s - t_{\mathcal{M}})} ds + \frac{\theta_i}{r} e^{-r(\tau - t_{\mathcal{M}})} \right], \tag{2}$$

which is simply a single-agent optimal stopping problem.

3.3 The Two Player Case

Suppose that one theater exited at $t = t_{\mathcal{D}}$. Theaters i and j are the remaining players in this subgame. The only information that each player has about its opponent is the fact that the opponent has survived up until $t_{\mathcal{D}}$. As I will show in Section 3.4, there exists the highest possible value of θ of surviving opponents, denoted by $\widetilde{\theta}$, which is a sufficient statistic for the history of the game up until $t_{\mathcal{D}}$. For ease of exposition, I keep dependence of $\widetilde{\theta}$ on other factors implicit. In the rest of this subsection, I take $(t_{\mathcal{D}}, \widetilde{\theta})$ as given. The domain of the strategy of the two-player subgame is the product of the type space, the current time ($t_{\mathcal{D}}$), and the highest possible value of θ of surviving opponents; i.e., $S = [\underline{\theta}, \overline{\theta}] \times \mathbb{R}_+ \times \mathbb{R}_+$. The strategy of this subgame T_2 is defined as $T_2 : [\underline{\theta}, \overline{\theta}] \times \mathbb{R}_+ \times \mathbb{R}_+ \to [t_{\mathcal{D}}, \infty]$.

[5] I focus on a symmetric equilibrium, so the strategy does not have theater subscript i.

If theater i, with exit value θ_i, chooses stopping time τ, given that theater j follows some strategy T_2, the present discounted value of i's expected profit over time is

$$W_2\left(\tau, T_2(\cdot, t_{\mathcal{D}}, \widetilde{\theta}), t_{\mathcal{D}}, \widetilde{\theta}; \theta_i\right) = \Pr\left(T_2(\theta_j, t_{\mathcal{D}}, \widetilde{\theta}) \geq \tau\right) \left[\int_{t_{\mathcal{D}}}^{\tau} \Pi_2(s) e^{-r(s-t_{\mathcal{D}})} ds + \frac{\theta_i}{r} e^{-r(\tau - t_{\mathcal{D}})}\right]$$
$$+ \int_{\left\{\theta_j | T_2(\theta_j, t_{\mathcal{D}}, \widetilde{\theta}) < \tau\right\}} \left[\int_{t_{\mathcal{D}}}^{T_2(\theta_j, t_{\mathcal{D}}, \widetilde{\theta})} \Pi_2(s) e^{-r(s-t_{\mathcal{D}})} ds\right.$$
$$\left. + e^{-r(T_2(\theta_j, t_{\mathcal{D}}, \widetilde{\theta}) - t_{\mathcal{D}})} V_1\left(\theta_i, T_2(\theta_j, t_{\mathcal{D}}, \widetilde{\theta})\right)\right] g\left(\theta_j | \theta_j \leq \widetilde{\theta}\right) d\theta_j.$$

Notice that the value of being a monopolist $V_1(\theta, t)$ from equation (1) is embedded into this value function.

Now I define the equilibrium in this 2-player subgame.

Definition 1 *For a given pair of* $(t_{\mathcal{D}}, \widetilde{\theta})$, $\left(\widehat{T}_2(\theta_i, t_{\mathcal{D}}, \widetilde{\theta}), \widehat{T}_2(\theta_j, t_{\mathcal{D}}, \widetilde{\theta})\right)$ *is a symmetric Bayesian equilibrium of this two-player subgame if for all* $l \in \{i, j\}$, $\theta_l \in [\underline{\theta}, \widetilde{\theta}]$, *and* $\tau \geq t_{\mathcal{D}}$

$$W_2\left(\widehat{T}_2(\theta_l, t_{\mathcal{D}}, \widetilde{\theta}), \widehat{T}_2(\cdot, t_{\mathcal{D}}, \widetilde{\theta}), t_{\mathcal{D}}, \widetilde{\theta}; \theta_l\right) \geq W_2\left(\tau, \widehat{T}_2(\cdot, t_{\mathcal{D}}, \widetilde{\theta}), t_{\mathcal{D}}, \widetilde{\theta}; \theta_l\right).$$

I make the following assumption throughout this paper, unless otherwise stated.

Assumption 1 (i) $\Pi_n(t)$ *decreases over time for* $n = 1, 2, 3$. (ii) $\lim_{t\to\infty}\Pi_3(t) > \underline{\theta}$. (iii) $\Pi_1(0) < \bar{\theta}$.

Section 3.5 discusses how to apply the argument of Fudenberg and Tirole (1986) to characterize the equilibrium. For the moment, assume that there exists a unique symmetric equilibrium. Let $\left(\widehat{T}_2(\theta_i, t_{\mathcal{D}}, \widetilde{\theta}), \widehat{T}_2(\theta_j, t_{\mathcal{D}}, \widetilde{\theta})\right)$ be the equilibrium strategies. Then, define

$$V_2(t_{\mathcal{D}}, \widetilde{\theta}; \theta_i) \equiv W_2\left(\widehat{T}_2(\theta_i, t_{\mathcal{D}}, \widetilde{\theta}), \widehat{T}_2(\cdot, t_{\mathcal{D}}, \widetilde{\theta}), t_{\mathcal{D}}, \widetilde{\theta}; \theta_i\right).$$

That is, $V_2(t_{\mathcal{D}}, \widetilde{\theta}; \theta_i)$ is i's value of entering the two-player subgame with $\widetilde{\theta}$ at $t = t_{\mathcal{D}}$ when theaters i and j both follow the equilibrium strategy $\widehat{T}_2(\cdot, t_{\mathcal{D}}, \widetilde{\theta})$.

3.4 The Three Player Case

Since the three-player game starts at $t = 0$, no information has been updated at the start of the game. The strategy T_3 only has the type space as its domain; $T_3 : \left[\underline{\theta}, \overline{\theta}\right] \to [0, \infty]$. Suppose theaters i, j, and k start the game at $t = 0$. Let $t^*_{jk}(\theta_j, \theta_k) = \min\{T_3(\theta_j), T_3(\theta_k)\}$

for some strategies $T_3(\cdot)$. Let $T_{-i} = (T_3(\cdot), T_3(\cdot))$ denote a set of strategies followed by i's opponents. Then, for $\tau \geq 0$, let

$$\begin{aligned} W_3\left(\tau, T_{-i}, ; \theta_i\right) = {} & \Pr\left(t^*_{jk}\left(\theta_j, \theta_k\right) \geq \tau\right) \left[\int_0^\tau \Pi_3\left(s\right) e^{-rs} ds + \frac{\theta_i}{r} e^{-r\tau}\right] \\ & + \int_{\left\{\theta_j, \theta_k \mid t^*_{jk}\left(\theta_j, \theta_k\right) < \tau\right\}} \left[\int_0^{t^*_{jk}\left(\theta_j, \theta_k\right)} \Pi_3\left(s\right) e^{-rs} ds \right. \\ & \left. + e^{-rt^*_{jk}\left(\theta_j, \theta_k\right)} V_2(t^*_{jk}\left(\theta_j, \theta_k\right), \widetilde{\theta}; \theta_i)\right] g(\theta_j) g(\theta_k) d\theta_j d\theta_k. \end{aligned} \tag{3}$$

In equation (3), $W_3\left(\tau, T_{-i}, ; \theta_i\right)$ is theater i's value when it exits at τ, given that theaters j and k follow some symmetric strategy $T_3(\cdot)$. Again, note that the value of playing the two-player subgame that a theater obtains in equilibrium, $V_2(t, \widetilde{\theta}; \theta)$, is embedded into the value of the three-player game.

Remember that $\widetilde{\theta}$ represents the highest possible value of θ of surviving opponents. For the sake of argument, suppose for the time being that any equilibrium strategy $T_3(\theta)$ has a differentiable inverse function.[6] Denote the inverse function as $\Phi_3\left(t\right)$. That is, $\Phi_3\left(t\right)$ is the value of exit for a player which in equilibrium exits at t. Then, $\widetilde{\theta}$ in equation (3) is given by

$$\widetilde{\theta} = \Phi_3(t^*_{jk}\left(\theta_j, \theta_k\right)). \tag{4}$$

To see this more clearly, assume that $\theta_k > \theta_l$. Since I am focusing on a symmetric equilibrium, this implies that $t^*_{jk}\left(\theta_j, \theta_k\right) = T_3(\theta_k)$.[7] Substituting this into (4) gives $\widetilde{\theta} = \theta_k$. That is, if player k exits first, the maximum possible value of exit for survivors is simply θ_k. This completes a full characterization of equation (3).

Definition 2 $\left(\widehat{T}_3(\theta_i), \widehat{T}_3(\theta_j), \widehat{T}_3(\theta_k)\right)$ *is a symmetric Bayesian equilibrium of the three-player subgame if for all* $l \in \{i, j, k\}$, $\theta_l \in \left[\underline{\theta}, \overline{\theta}\right]$, *and* $\tau \geq 0$,

$$W_3\left(\widehat{T}_3(\theta_l), T_{-l}; \theta_l\right) \geq W_3\left(\tau, T_{-l}; \theta_l\right).$$

Putting all these subgames together, I define the equilibrium of the entire game.

Definition 3 *A set of symmetric strategies* $(\widehat{T}_1(\theta, t_{\mathcal{M}}), \widehat{T}_2(\theta, t_{\mathcal{D}}, \widetilde{\theta}), \widehat{T}_3(\theta))$ *with posterior beliefs* $g(\theta | \theta \leq \widetilde{\theta})$ *is a symmetric perfect Bayesian equilibrium if*

1. $\widehat{T}_1(\theta, t_{\mathcal{M}})$ *is given by equation (2) for all* $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ *and* $t_{\mathcal{M}} \in [0, \infty)$.

[6] See Lemma 4.

[7] A symmetric equilibrium with a strictly monotonic strategy $T_3\left(\theta\right)$ implies that a player with a higher exit value exits the market earlier.

2. $\widehat{T}_2(\theta, t_{\mathcal{D}}, \widetilde{\theta})$ *constitutes a symmetric Bayesian equilibrium of this two-player subgame as is given by Definition 1.*

3. $\widehat{T}_3(\theta)$ *satisfies the condition given by Definition 2.*

4. *For any opponent* j, $g(\theta_j|\theta_j \leq \widetilde{\theta})$ *is given by*

$$g(\cdot|\theta_j \leq \widetilde{\theta}) = \begin{cases} \frac{g(\theta_j)}{\Pr(\theta_j \in \Theta)}, & \theta_j \in \Theta \\ 0, & \theta_j \notin \Theta \end{cases}$$

where Θ *is the set of types* θ *that can survive on-equilibrium path.*

3.5 Properties of the Symmetric Equilibrium

Since $T_1(\theta_i, t_{\mathcal{M}})$ is simply a solution to a single-agent optimal stopping problem, I start from $\widehat{T}_2(\theta_i, t_{\mathcal{D}}, \widetilde{\theta})$. Fudenberg and Tirole (1986) model a two-player game with $t_{\mathcal{D}} = 0$ and $\widetilde{\theta} = \bar{\theta}$. They show that any equilibrium strategy $\widehat{T}_2(\theta_i, 0, \bar{\theta})$ is strictly decreasing in θ_i and that a differentiable inverse function exists. Denote the inverse function as $\Phi_2(t) \equiv T_2^{-1}\left(t, 0, \bar{\theta}\right)$, where $\Phi_2(t)$ is the exit value of a player which in equilibrium exits at time t.

Given these properties, the first order condition of $W_2\left(\tau, T_2(\cdot, 0, \bar{\theta}), 0, \bar{\theta}; \theta_i\right)$ gives

$$\Phi_2'(t) = -\frac{G(\Phi_2(t))}{g(\Phi_2(t))}\left[\frac{\Phi_2(t) - \Pi_2(t)}{V_1(\Phi_2(t), t) - \Phi_2(t)/r}\right]. \tag{5}$$

Fudenberg and Tirole (1986) demonstrate that the differential equation (5) with the boundary conditions

$$\Phi_2(0) = \Pi_1(0) \tag{6}$$

$$\lim_{t\to\infty} \Phi_2(t) = \lim_{t\to\infty} \Pi_2(t) \tag{7}$$

has a unique solution. They also show that for all $t > 0$ and all i, $0 < \Pi_2(t) < \Phi_2(t) < \Pi_1(t)$. A typical form of $\Phi_2(t)$ is shown in Figure 3. These propositions in Fudenberg and Tirole (1986) still hold when $t_{\mathcal{D}} > 0$ and $\widetilde{\theta} \neq \bar{\theta}$. Before showing this, I construct the equilibrium in the three-player game. All proofs are presented in Appendix B.

The first order condition of equation (3) is given by

$$\Phi_3'(t) = -\frac{G(\Phi_3(t))}{2g(\Phi_3(t))}\left[\frac{\Phi_3(t) - \Pi_3(t)}{V_2(t, \Phi_3(t); \Phi_3(t)) - \Phi_3(t)/r}\right]. \tag{8}$$

This serves as a policy function that I use in estimation.

Lemma 4 *Given* $\{T_3(\theta_i), T_3(\theta_j), T_3(\theta_k)\}$ *is the equilibrium strategy, (i) For all* $\theta \in [\Pi_2(0), \overline{\theta}]$, $T_3(\theta) = c > 0$ *cannot be true. (ii)* $T_3(\theta)$ *is continuous and strictly decreasing on* $(\underline{\theta}, \Pi_2(0))$. *(iii)* $\Phi_3(t)$ *is differentiable on* $(0, \infty)$. *Its derivative is given by (8).*

Next, I construct the boundary conditions. For this purpose, I need the following assumption.

Assumption 2 *If more than one firm has the same optimal exit time, the firm with the highest value of* θ *exits first. Then, every other firm reappraises its optimal stopping time, taking into account the fact that there is now one less firm.*

Since $T_3(\theta)$ is strictly decreasing on $(\underline{\theta}, \Pi_2(0))$, the event where more than one firm has the same optimal exit time has measure zero when t is strictly positive. However, it is possible that exiting immediately is optimal for more than one firm. In this case, the symmetric equilibrium does not exit. Assumption 2 avoids this problem. See Haigh and Cannings (1989) and Argenziano and Schmidt-Dengler (2008) for discussion.

Lemma 5 *Suppose Assumption 2 holds. For one of the solutions to equation (8) to be an equilibrium, the boundary conditions should be given by*

$$\Phi_3(0) = \Pi_2(0) \tag{9}$$

$$\lim_{t\to\infty} \Phi_3(t) = \lim_{t\to\infty} \Pi_3(t). \tag{10}$$

The following proposition guarantees that one set of parameters in the model generates a unique solution.[8]

Proposition 6 *The symmetric equilibrium, if it exists, is unique.*

Proof. See Appendix. ■

The next lemma establishes the shape of $\Phi_3(t)$.

Lemma 7 $0 < \Pi_3(t) < \Phi_3(t) < \Pi_2(t)$.

Proof. See Appendix. ■

Now I discuss how players move from the three player game to two player game when one player exits at time $t_{\mathcal{D}}$. The case with $t_{\mathcal{D}} = 0$ was already given by equations (5)-(7).

[8] I do not have a formal proof for existence. In estimation, however, for any set of parameters, I numerically find $\Phi_3(t)$ that satisfies equations (8), (9), and (10). Moreover, as Proposition 5 shows, if I find a symmetric equilibrium, it is the unique symmetric equilibrium.

Lemma 8 *Suppose* $\max\{\theta_i, \theta_j, \theta_k\} < \Pi_2(t)$. *Assume k is the first to exit and $T_3(\theta_k) > 0$. Players i and j do not exit at any $t \in [T_3(\theta_k), \Pi_2^{-1}(\theta_k)]$. That is, $\Phi_2(t) = \theta_k$ for $t \in [T_3(\theta_k), \Pi_2^{-1}(\theta_k)]$. For $t \in [\Pi_2^{-1}(\theta_k), \infty]$, the policy function $\Phi_2(t)$ follows (5) with the boundary conditions*

$$\Phi_2\left(\Pi_2^{-1}(\theta_k)\right) = \theta_k \tag{11}$$

$$\lim_{t\to\infty} \Phi_2(t) = \lim_{t\to\infty} \Pi_2(t). \tag{12}$$

Proof. See Appendix. ■

After one player exits at some positive time, there will be no selection until the duopoly payoff becomes low enough. Once the selection restarts, the game becomes the two-player game with $t_{\mathcal{D}} = \Pi_2^{-1}(\theta_k)$ and $\widetilde{\theta} = \theta_k$. Finally, I summarize the results.

Theorem 9 *Equations (2) and (5)-(10) constitute a symmetric Bayesian equilibrium of the entire game.*

Based on all the claims I have made so far, a typical path of $\Phi_3(t)$ is shown in Figure 4. Suppose that none of the players exits immediately and that player k is the first to exit, i.e., $\max\{\theta_i, \theta_j, \theta_k\} = \theta_k < \Pi_2(0)$. Then, a typical shape of $\Phi_2(t)$ is also shown in Figure 4.

3.6 Computing the Equilibrium of the Model

For a given payoff function and exit values of theaters, I can calculate the equilibrium exit times. First, the policy function $\Phi_3(t)$ is obtained by solving the differential equation given by equations (8)-(10). Assuming $\theta_k = \max\{\theta_i, \theta_j, \theta_k\}$, theater k's exit time, t_k, is such that $\theta_k = \Phi_3(t_k)$. Then, set $\Phi_2(t) = \theta_k$ for $t \in [t_k, \Pi_2^{-1}(\theta_k)]$. That is, $\Phi_2(t)$ stays constant until it equals $\Pi_2(t)$. From this time on, $\Phi_2(t)$ is given by (5) with the boundary conditions (11) and (12). Assuming $\theta_j = \max\{\theta_i, \theta_j\}$, theater j's exit time, t_j, is given by $\theta_j = \Phi_2(t_j)$. Finally, find $T_1(\theta_i, t_j)$ from (2). As I argue in later sections, this process is nested in the estimation algorithm. Note also that this procedure works in the same way for an N-player game.

3.7 Implications of the Model

As time goes by, two factors, which move in opposite directions, determine the observed probability that a theater exits. As a theater survives longer, the econometrician expects the theater to have a lower value of exit (selection). On the other hand, as time passes without competitors having exited the market, the theater infers that its competitors are strong and becomes discouraged to remain active (pessimism). In equilibrium, these two forces jointly determine the observed probability of exit at each point in time, and as a result, the empirical distribution of exit times. Estimating the model parameters allows me to infer the effect of competition on the devolution of the industry.

Another important implication of the model concerns the game's history. Consider two cases that differ only in terms of the histories up to t'. In both cases, player k exits first and players i and j have survived until t'. In the first case, player k exits at t' so that players i and j have just started two-player subgame. Let $\Phi_2(t')$ be the value of exit of the marginal type in this case. In the second case, player k dropped out right after the game started, and players i and j have remained active until t'. Let $\widetilde{\Phi}_2(t')$ be the value of exit of the marginal type in this case. The example in Figure 4 implies that $\Phi_2(t') < \widetilde{\Phi}_2(t')$. The intuition is that, in the first case (when k exits at t'), players were exposed to fierce competition in the past, so the value of exit for the marginal player is lower than in the second case.

For the marginal player i who is indifferent between dropping out at t and staying until time $(t + dt)$ and then dropping out [9], it follows that

$$\Pr\left(\begin{array}{c}\text{conditional probability}\\ \text{theater } j \text{ drops in } [t, t+dt]\end{array}\right)\underbrace{\left[V_1\left(\Phi_2(t), t\right) - \frac{\Phi_2(t)}{r}\right]}_{i\text{'s payoff when } j \text{ drops}} dt = \underbrace{\left[\Phi_2(t) - \Pi_2(t)\right] dt}_{i\text{'s cost of staying in}}. \quad (13)$$

The right-hand side of this equation is increasing in $\Phi_2(t)$. On the other hand, $\left[V_1(a,t) - \frac{a}{r}\right]$ is a decreasing function of a.[10] Thus, if I substitute $\widetilde{\Phi}_2(t')$ for $\Phi_2(t')$ in equation (13), the conditional probability that player j drops in $[t, t+dt]$ should increase. Put differently, as competition was severe in the past, the conditional probability that player j drops in the near future is lower. To conclude, the current number of players is not sufficient to infer the players' beliefs. This is another reason that I use the full solution method rather than the two-step method, which requires choice probabilities as a function observable covariates.

[9] The conditional probability that player j drops in $[t, t+dt]$ is given by $-\frac{g(\Phi_2(t))}{G(\Phi_2(t))}\Phi_2'(t)$. Using this and rearranging (13) give the differential equation (5).

[10] $V_1(a,t) - \frac{a}{r} = \max_{\tau\in[t,\infty]}\left[\int_t^\tau \Pi_1(s)e^{-rs}ds + \frac{a}{r}\left(e^{-r(\tau-t)} - 1\right)\right]$. Hence, $V_1(a,t) - \frac{a}{r}$ is decreasing in a for all $\tau > t$. Lemma 8 implies that $\tau \neq t$.

4 Data

4.1 Data Description

The main data for this study comes from *The Film Daily Yearbook of Motion Pictures* (1950 and 1955) and *The International Motion Picture Almanac* (1961 and 1964), which contain information on every theater that has existed in the US.[11] The dataset includes the name, location, entry and exit years, number of screens, type of the theater (indoor, drive-in, live show, and etc.), and ownership (individually owned or company-owned) of each theater.[12]

I assume that wars of attrition started in 1950, when demand started to shrink rapidly in an exogenous way. I define all the non drive-in theaters that were open in 1950 as players. Theaters that entered after 1950 are treated as exogenous demand shifters. While the focus of this analysis is on single screen theaters, theaters that entered after 1950 (henceforth I call them "latecomers") were brand-new, and sometimes equipped with concession stands and nicer seats. There was certainly competition between classic single screen theaters and these new theaters. It is not unreasonable, however, to assume that the game I developed was played among old theaters and the exit of new theaters was exogenous from the viewpoint of the old theaters. Drive-in theaters are also treated as exogenous demand shifters, regardless of when they entered.[13]

I define a market as a county. There are two major advantages of doing this. This definition of a market is less arbitrary than other possible definitions. Also, data on demand shifters, such as TV penetration and demographics, are mostly at the county-level. One of the major drawbacks of this market definition is that some counties extend over many cities and contain hundreds of theaters (e.g., San Francisco county). To avoid this problem, I focus on markets (counties) with less than or equal to 12 theaters in 1950.[14]

I focus on 1950 to 1964 for the reasons that I mentioned in Section 2 (seemingly different natures of competition before 1950 and after 1965). I divide this sample period into three

[11] The data set is compiled by Mike Rivest and available at http://movie-theatre.org/theatre.html.

[12] For location variables, the exact address is often missing. However, we know the name of the city where the theater is/was located.

[13] The average number of latecomers per market was 0.24 from 1950-1955, 0.30 from 1955-1961, and 0.15 from 1961-1964, respectively. The average number of drive-in theaters per market was 1.39 from 1950-1955, 1.53 from 1955-1961, and 1.39 from 1961-1964, respectively.

[14] Another possible problem is that some counties are sparsely populated so that cities in one county are far away from each other. To deal with this problem, I pick counties with a small number of cities inside and re-estimate the model as one of the robustness checks (in progress).

sub-intervals: 1950-1955, 1955-1961, and 1961-1964. For entry and exit years, sometimes the exact year is not known; in such cases, only intervals are available. For example, I know whether theaters were open or not in 1950, 1955, 1961, and 1964.[15] Therefore, I transform all entry/exit information into intervals. For example, if a theater was observed in 1950 and 1955 but not in 1961, I assume that the theater exited sometime between 1955 and 1961.

For estimation, I use 454 markets, which have a total of 1881 theaters.[16] Table 1 shows the frequency of markets by the initial number of competitors. Figure 2 shows how quickly theaters exited during the period 1950-1965. One fifth of all theaters that were open in 1950 exited in the first five years, and another 50% (of the original set of theaters) exited in the next 6 years. The pace of the decline slowed down in the early 1960s. Overall, about 70% of all the theaters went out of business during the sample period.

I assume that the diffusion of televisions was the main driving force in the decline in demand for classic movie theaters. The *Television Factbook* and the *Television & Cable Factbook* provide TV penetration rates by county and year.[17] The TV penetration rate is defined as the share of households which have at least one TV set. The diffusion process of televisions differs greatly across counties. I calculate the number of years needed (since 1950) for each county to reach the TV penetration rate of 50%. Then, I define the 25th, 50th, and 75th percentiles of the sample counties based on this number.

Figure 5 plots changes of TV penetration rates for three counties. Jim Wells county (in

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[15] This is because *The Film Daily Yearbook of Motion Pictures* and *The International Motion Picture Almanac* are available electronically once in every several years (in 1950, 1955 for the former one and in 1961 and 1964 for the latter one during my sample period).

[16] There are over 3000 counties in the US, but for the time being I only use counties for which an electronic spreadsheet is available. These counties mostly come from the states of Alabama, Arkansas, California, Georgia, Illinois, South Dakota, Texas, Utah, Vermont, and Wyoming. Thus, I have 573 counties. Since I focus on markets with less than or equal to 12 theaters in 1950, I am left with 489 counties. Finally, I exclude counties where changes in TV penetration rates are not monotonic and/or some of market covariates are missing. As a result, I end up with 454 counties. For data selection issues, see Footnote 18.

[17] TV penetration rates by county are available for 1953, 1956, 1958, 1959, and 1967. I arbitrarily set the TV penetration rate in 1950 at 0.01, and specify

$$\log\left(\frac{TV_{tm}}{1-TV_{tm}}\right) = \beta_{0m} + \beta_{1m}t + \beta_{2m}t^2 + \varepsilon_{tm}$$

where TV_{tm} is the TV penetration rate at time $t = \{1950, 1953, 1956, 1958, 1959, 1967\}$ in market m. I obtain $\{\beta_{0m}, \beta_{1m}, \beta_{2m}\}$ for each market by linear projection. Once I get these estimates, I use the fomula above to interpolate TV penetration rates for all $t \in [1950, 1964]$. I dropped markets which experienced a decrease in the TV penetration. I will try other interpolation methods for a robustness check.

Texas) is a county that had a relatively slow diffusion of televisions (25th percentile). Around 40% of the households in this county didn't have a television set in 1960. On the other hand, Jackson county (in Illinois) experienced a quick change in TV penetration (75th percentile). It reached over 50% in 1954. Carter county (in Tennessee) is a median county (50th percentile). Figure 6 draws the whole distribution of years needed to reach the TV penetration rate of 50%. Some counties reached 50% in less than 2 years, while other counties hadn't reached 50% by the end of the sample period. This rich cross-section and over-time variation of the TV penetration rate is the main source of identification of theaters' payoff function. In estimation, I specify the demand decay as a flexible function of the TV penetration rates.

Other data sources also provide across-market variations that will help me to identify the theaters' payoff functions. Basic demographic variables such as population, income, and age distributions are obtained for every county from the US Census. Population is the most important among these variables. Three panels in Figure 7 plot changes in population (percentage change) between 1950-1955, 1955-1961, and 1961-1964, respectively. There were quite a few people moving across counties in each of three time periods. For the variables in the base demand that characterize counties (and determine theaters' profit) in 1950, I use total population, the median income (categorical), the median age (categorical), and the share of population living in urban areas. Table 2 presents summary statistics for these variables.[18]

4.2 Regression Analysis

To summarize the relationship between theaters' exits and market characteristics, I run several regressions. The dependent variable in the first regression is the share of theaters that exited between 1950 and 1964. I regress this variable on the change in population, change in TV penetration, the number of theaters in 1950, market size (measured in log of population), the median age, median family income, and the share of population living in the city area. The first column of Table 3 shows the results. As expected, an increase in TV penetration leads to an increase in exits. Growth of population is negatively correlated with theaters' exits, although

[18] To see if using only 454 markets causes any serious data selection problem, I calculate the same set of statistics as in Table 2, using all US counties (around 3000). The mean values of the median income, the median age of population, and the share of population living in urban areas are 5.06, 4.12, and 0.07, respectively. These values are somewhat close to those of my sample. For total population, my sample does not include big counties (in terms of population), which significantly affect the mean value. Thus, I calculate the median population for all US counties and my sample, and obtain 18,957 and 18,960, respectively. To conclude, I cannot find evidence for a serious data selection problem, based on observable covariates.

the coefficient is statistically insignificant. The coefficient on the number of competitors in 1950 is positive and significant, which implies that severe competition is positively associated with exits.

If there is market level heterogeneity which affects theaters' profits, the coefficient for number of competitors will be biased. To account for such heterogeneity, I exploit the panel aspect of the data. Let $p \in \{1, 2, 3\}$ denote an interval, where each p in $\{1, 2, 3\}$ corresponds to intervals 1950-1955, 1955-1961, and 1961-1964, respectively. Assuming that the share of theaters that exit during each interval is explained by variables at the beginning of the period, let

$$y_{mp} = x_{mp}\beta + c_m + \varepsilon_{mp}, \quad p = 1, 2, 3,$$

where y_{mp} is the share of theaters that exit in market m during time period p, x_{mp} is the set of covariates at the beginning of period p, including the number of competitors, c_m represents unobservable market level heterogeneity, and ε_{mp} is the random shock. Let $x_{mp} = (z_{mp}, n_{mp})$, where z_{mp} is the set of exogenous variables (population growth, change in TV penetration, number of latecomers), and n_{mp} is the number of competitors.[19] Taking the first difference, I eliminate market heterogeneity:

$$\Delta y_{mp} = \Delta x_{mp}\beta + \Delta\varepsilon_{mp}, \quad p = 2, 3. \tag{14}$$

One of the key conditions for OLS to consistently estimate β is

$$\mathbb{E}\left(\Delta x' \Delta \varepsilon\right) = 0.$$

However, in this context, ε_{m2} and n_{m3} are most likely correlated.[20] Thus, β for Δn_{m3} would be inconsistent. To get around the problem, I estimate (14) only with the third time period, $p = 3$, using n_{m1} as an instrument for Δn_{m3}. This is essentially the same as estimating the 2SLS on a cross section.

The second column of Table 3 reports these results. A decrease in population is associated with a lower exit rate, although the coefficient is insignificant. An increase in TV penetration is positively correlated with theaters' exits. The coefficient on the number of competitors is positive and statistically significant. To conclude, the competition variable and

[19] I use only population growth, change in TV penetration, numbers of latecomers for this regression, since I will assume that other variables affect only initial profits in later sections.

[20] A high value of ε_{m2} increases exits in the second time period, which in turn decreases the number of competitors in the third period.

TV penetration have the expected signs, even after controlling for the possible endogeneity and unobservable heterogeneity.

Finally, I explore the relationship between the initial number of theaters and market covariates. The third column of Table 3 shows these results. The explanatory variables are the market size (measured in log of population), the median age, median family income, and the share of population living in the city area. Every covariate has a positive coefficient.

5 Estimation Strategy

For a given set of parameters in the payoff function and in the distributions of unobservable variables, the model predicts the distribution of theaters' exit times. I estimate the parameters by matching the predicted distribution with the observed distribution of exiting times. Because of the model's analytically intractable outcomes and the presence of high dimensional unobservable variables, I use simulation methods to calculate the predicted distribution. Three features are worthy of note.

First, I adopt a full solution approach. The Bayesian equilibrium of the game yields a set of differential equations which is easy to compute.[21] For a given set of parameters and random draws of unobservable variables, I fully solve for the equilibrium of the model to calculate the equilibrium exit times of theaters. By averaging over simulation draws, I obtain the predicted distribution. This method is used instead of a two-step estimation strategy, which is generally less efficient.

Second, I control for the endogeneity generated by the correlation between the initial number of theaters and unobservable market level heterogeneity (i.e., the initial condition problem as formulated by Heckman (1981)). I assume that wars of attrition started in 1950. There is a wide variation in the number of competitors in 1950, even across markets with similar observable characteristics. Therefore, it is highly likely that, before 1950, unobservable market heterogeneity affected theaters' profit. Consequently, because of dynamic selection, as time passes the number of theaters and market heterogeneity will be correlated even if ex ante they were not. In such a case, I cannot simply integrate out unobservable market heterogeneity, even if the ex ante distribution of heterogeneity is known.

To deal with the initial condition problem, I set up a model of entry in 1950 as a prestage of the dynamic game. Each theater draws its value of exit, and based on its value and

[21] Other models such as Ericson and Pakes (1995) are hard to compute, and hence are difficult to use for a full solution approach.

market level heterogeneity, it chooses whether or not to enter the market. Once it enters, the theater carries over its value of exit into the dynamic game of exit. For a given value of market heterogeneity and set of exit values of all potential entrants, the entry game predicts the number of entrants in equilibrium. Since the initial number of theaters is observed, I use the entry game to restrict the support of market heterogeneity and exit values so that, in equilibrium, the entry game predicts the same number of entrants as is observed. Since it is hard to analytically characterize such a restriction, I use a simulation to approximate the joint distribution of market heterogeneity and exit values, conditional on the observed number of entrants. This simulated distribution can then be used as an input to simulate and solve the dynamic stage of the game.

Third, I use indirect inference for estimation. I choose several moments of the data which seemingly capture the relevant features of the data. Then, for a given set of structural parameters, I calculate the equilibrium outcome of the model and obtain the corresponding set of moments from the created data. I choose the set of structural parameters so that these two sets of moments are as close as possible. The major advantage of this approach over likelihood is its ease of implementation. Above all, the structure of my data (only intervals are available for entry/exit years in many cases) makes calculating the likelihood a daunting task, and hence indirect inference is preferable. I jointly use moments from the dynamic game and from the entry game. In the following subsections, I discuss the details of my estimation strategy.

5.1 Specification

To numerically solve the model, I parameterize the payoff function. From this section on, I use a market subscript $m \in \{1, ..., M\}$ for all variables that differ across markets. Let $\Pi_n(t, m)$ be a theater's instantaneous profit in market m at time t, when there are n surviving theaters. I assume that

$$\Pi_n(t, m) = \pi_n(m) \cdot d(t, m),$$

where $\pi_n(m)$ is the base demand and $d(t, m)$ is a decay function. I assume that $d(t, m)$ decreases over time to satisfy Assumption 1 in Section 3. I assume that the decay function is given by

$$d(t, m) = \left\{ \frac{1}{1 + \exp\left(\lambda_0 + \lambda_1 TV_{tm} + \lambda_2 TV_{tm}^2\right)} \right\}^{\exp(\lambda_3 + \lambda_4 \Delta POP_m)},$$

where TV_{tm} is the TV penetration rate in market m at time t and ΔPOP_m is the growth rate of population in market m throughout the sample period. I restrict λ_1 and λ_2 to be positive. Note that $d(t,m)$ is between zero and one, and decreases over time as long as TV_{tm} is increasing in time. This specification is flexible and potentially captures various types of decay.

The base demand is specified as additively separable:

$$\pi_n(m) = \alpha_m + X_m\beta + \delta(n_m - 1) \tag{15}$$

where α_m is unobservable (to the econometrician) market level heterogeneity, X_m is a vector of observable variables, and δ is a parameter that captures the effect of competition. I assume δ is negative to guarantee that $\Pi_n(t,m)$ is decreasing in n for all t. The set of observable variables X_m includes a constant, population size, the median age of population, the median income, and the share of population living in urban areas.

5.2 The Initial Condition Problem

As mentioned earlier, if unobservable market heterogeneity (α_m in equation (15)) affected theaters' profit before 1950, the number of theaters in 1950 and market heterogeneity would be correlated through selection. In fact, there is wide variation in the initial number of theaters among similarly sized markets. Table 4 shows the average, standard deviation, maximum, and minimum of numbers of theaters in 1950 by population size. The variation in the initial number of theaters in each group implies the presence of unobservable market-level heterogeneity, which is correlated with the initial number of theaters. Thus, even if the ex-ante distribution of market heterogeneity were known and independent of the number of theaters, I could not use it to integrate out market heterogeneity. Letting $f_Y(y|z)$ be the joint density of $Y \in \mathbb{R}^K$, conditional on $Z = z$, the above argument suggests that

$$f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|n,X) \neq f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|X),$$

where n is the observed number of theaters in 1950 and X is a set of observable market covariates. For later use, let $F_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|n,X)$ be its CDF.

I approximate $f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta},\alpha|n,X)$ by simulation as follows. I assume that $\bar{N}$ potential players play an entry game at time zero, and entrants play the exit game afterwards. I make the following assumption:

Assumption 3 *All potential players make an entry decision simultaneously at time zero without knowing a war of attrition will immediately follow the entry game.*

I first solve an entry game for each market repeatedly to simulate the joint distribution of $(\boldsymbol{\theta}, \alpha)$ given n. Then I randomly draw $(\boldsymbol{\theta}, \alpha)$ from the simulated distribution to solve the dynamic exit game (i.e., the war of attrition).

5.2.1 Entry Model

The model is similar to Seim (2006). $\bar{N}$ potential players decide whether they enter each market or not in 1950. If n_m players enter the market as a result of their decisions, the payoff for each entrant is given by

$$\Pi_n(0, m) = \alpha_m + X_m\beta + \delta(n_m - 1), \tag{16}$$

where (β, δ) is the set of parameters to be estimated.[22] On the other hand, if player i does not enter, it earns θ_{mi}, which is private information. The game proceeds as follows. First, each player draws its own value of θ. Then, every player simultaneously makes an entry decision. I assume α_m and θ_{mi} follow the distributions $N(\mu_\alpha, \sigma_\alpha^2)$ and $TN(\mu_\theta, \sigma_\theta^2; l_\theta, h_\theta)$, respectively, where $TN(\mu, \sigma^2; l, h)$ represents the truncated normal distribution with mean μ, variance σ^2, and the lower and higher truncation points l and h. Note that these unobservables are carried over to the dynamic stage of the game.

Let $P(X_m, \alpha_m)$ denote a player's subjective probability that a competitor enters the market in a symmetric equilibrium. Player i's choice $D_i = \{0, 1\}$ is then given by

$$D_i = \mathbb{1}\{\alpha_m + X_m\beta + \delta\left(\bar{N} - 1\right) P(X_m, \alpha_m) \geq \theta_i\}. \tag{17}$$

where $\left(\bar{N} - 1\right)$ is the maximum number of potential competitors. The equilibrium belief $P^*(X_m, \alpha_m)$ is thus given by

$$P^*(X_m, \alpha_m) = G(\alpha_m + X_m\beta + \delta\left(\bar{N} - 1\right) P^*(X_m, \alpha_m)), \tag{18}$$

where G is the CDF of θ. In the actual estimation, I assume $\alpha \sim N(0, \sigma_\alpha^2)$ and $\theta \sim TN(0, \sigma_\theta^2; 0, 15)$, where $(\sigma_\alpha^2, \sigma_\theta^2)$ are parameters to be estimated.[23]

[22] Rigorously speaking, $\Pi_n(0, m)$ should be given by

$$\Pi_n(0, m) = (\alpha_m + X_m\beta + \delta(n_m - 1))\, d(0, m).$$

However, $d(0, m)$ is constant so I omit it from this argument for notational simplicity.

[23] I arbitrarily choose $h_\theta = 15$. However, my estimation results are not sensitive to this choice. I re-estimate the model with $h_\theta = 12$ for example, and obtained very similar results.

The number of entrants predicted by this entry model equals the number of $\theta_i s$ in $\{\theta_1, ..., \theta_{\bar{N}}\}$ that satisfy

$$\begin{aligned} & \alpha_m + X_m\beta + \delta\left(\bar{N}-1\right)P^*\left(X_m,\alpha_m\right) - \theta_i \\ & \equiv \widetilde{\pi}\left(0,m\right) - \theta_i \\ & \geq 0. \end{aligned} \tag{19}$$

In estimation, for the benchmark case, I set $\bar{N} = 13$.[24]

5.2.2 Getting the joint distribution of α and θs

Based on the entry model, I simulate the joint distribution of θ and α conditional on n_m in the following way:

Step 1: Draw α^0 from $N\left(0,\sigma_\alpha^2\right)$.

Step 2: Calculate $P^*\left(X_m,\alpha^0\right)$ using (18). Then, define θ^* as

$$\theta^* = \alpha^0 + X_m\beta + \delta\left(\bar{N}-1\right)P^*\left(X_m,\alpha^0\right).$$

That is, θ^* is the threshold of exit values above which a theater finds it profitable to enter the game.

Step 3: Draw a value of exit n_m times from $TN\left(0,\sigma_\theta^2;0,\theta^*\right)$ and $\bar{N}-n_m$ times from $TN\left(0,\sigma_\theta^2;\theta^*,15\right)$. Sort these values in an ascending order. Call them $\boldsymbol{\theta}^0 = \left(\theta_1^0,...,\theta_{n_m}^0,\theta_{n_m+1}^0,...,\theta_{\bar{N}}^0\right)$.

Step 4: Define α_l and α_h such that

$$\begin{aligned} \theta_{n_m}^0 &= \alpha_l + X_m\beta + \delta\left(\bar{N}-1\right)P^*\left(X_m,\alpha_l\right) \\ \theta_{n_m+1}^0 &= \alpha_h + X_m\beta + \delta\left(\bar{N}-1\right)P^*\left(X_m,\alpha_h\right). \end{aligned}$$

Step 5: Draw α^1 from $TN\left(0,\sigma_\alpha^2;\alpha_l,\alpha_h\right)$.

That is, α^1 is large enough to support n_m entries but not enough to support n_m+1 entries.

Step 6: Return to Step 2 and repeat these steps J times to get $\left(\boldsymbol{\theta}^j,\alpha^j\right)_{j=1}^J$.[25] Call this $\hat{F}_{\theta,\alpha|n_m}$. Note that this procedure is done for each market.

[24] I arbitrarily set $\bar{N}$ at 13, since the number of actual entrants is 12 in my sample. I also change $\bar{N}$ according to population size as a robustness check (in progress).

[25] I discard the first 100 sets of $\left(\boldsymbol{\theta}^j,\alpha^j\right)$ before storing.

5.3 Indirect Inference

I simulate moments from the model and minimize the distance between the simulated moments and data moments. I use p to denote the three time periods:

$$p = \begin{cases} 1 & \text{if } t \in [1950, 1955) \\ 2 & \text{if } t \in [1955, 1961) \\ 3 & \text{if } t \in [1961, 1964]\,. \end{cases}$$

Let $\boldsymbol{\gamma}$ be a set of structural parameters and $\boldsymbol{\rho}$ be a set of auxiliary parameters that summarize certain features of the data. Let $\hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})$ be the set of auxiliary parameters estimated from the simulated data. Note that I keep the dependence of $\hat{\boldsymbol{\rho}}$ on $\boldsymbol{\gamma}$ explicit. s_{mp} and $\hat{s}_{mp}$ denote the empirical and theoretical share of surviving theaters in market m at the end of time period p, respectively. The elements in $\boldsymbol{\rho}$ are

$$M^{-1}\sum_{m=1}^{M} s_{mp} \qquad \text{for } p = 1, 2, 3 \tag{20}$$

$$M^{-1}\sum_{m=1}^{M} s_{mp}\Delta POP_{mp} \qquad \text{for } p = 1, 2, 3 \tag{21}$$

$$M^{-1}\sum_{m=1}^{M} s_{mp}\Delta TV_{mp} \qquad \text{for } p = 1, 2, 3 \tag{22}$$

$$M^{-1}\sum_{m=1}^{M} n_m X_{mq} \qquad \text{for } q = 1, ..., 5. \tag{23}$$

The elements in $\hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})$ are the same as (20)-(23) with s_{mp} and n_m being replaced by their simulated counterparts, $\hat{s}_{mp}$ and $\hat{n}_m$. I use the entry game not only to approximate $F_{\theta,\alpha|n_m}$ but also to form the additional moments in equation (23).

The indirect inference estimator $\hat{\boldsymbol{\gamma}}$ is given by

$$\hat{\boldsymbol{\gamma}} = \arg\min_{\boldsymbol{\gamma}} \left(\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})\right)' W \left(\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})\right) \tag{24}$$

where W is a positive definite weighting matrix. For implementation, I bootstrap the data 1000 times to get $\{\boldsymbol{\rho}_b\}_{b=1}^{1000}$, and then calculate its variance-covariance matrix Ω. Then I set $W_M = \Omega^{-1}$ and use this asymptotic weighting matrix.

5.4 Simulating Moments

Using $\hat{F}_{\theta,\alpha|n}$ obtained as described in Section 5.2.2, I can simulate the moments from the dynamic game of exit. The procedure to calculate the value of the objective function is as follows:

First, I simulate $\hat{n}_m$. The following two steps describe this process.

Step 1: Choose the set of structural parameters $\boldsymbol{\gamma}$.

Step 2: Draw $\{\boldsymbol{\theta}^{ns}\}_{ns=1}^{NS}$ and $\{\alpha^{ns}\}_{ns=1}^{NS}$ independently from their ex ante distributions. Use (18) and (19) to solve the entry game to calculate $\hat{n}_m^{ns}$ for $ns = 1, ..., NS$ and form $\hat{n}_m = \frac{1}{NS}\sum_{ns=1}^{NS} \hat{n}_m^{ns}$.

Next I simulate $\hat{s}_{mp}$, using steps 3 to 5.

Step 3: For X_m and n_m, simulate $\hat{F}_{\theta,\alpha|n_m}$, following the process of Section 5.2.2.

Step 4: Draw $(\boldsymbol{\theta}^{ns}, \alpha^{ns})_{ns=1}^{NS}$ randomly from $\hat{F}_{\theta,\alpha|n_m}$. For each simulation draw, calculate the equilibrium of the dynamic game of exit: $\left\{\left(t_1^{ns}, ..., t_{n_m}^{ns}\right)\right\}_{ns=1}^{NS}$.

Step 5: Calculate the share of surviving theaters for each time period $\hat{s}_{mp}^{ns}$ and form $\hat{s}_{mp} = \frac{1}{NS}\sum_{ns=1}^{NS} \hat{s}_{mp}^{ns}$. Note that these steps are done for each market.

Finally, the objective function is evaluated in step 6.

Step 6: Calculate moments based on (20) to (23) and obtain the value of the criterion function

$$J(\boldsymbol{\gamma}) = (\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma}))' W_M (\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})).$$

Then, repeat Steps 1-6 to minimize $J(\boldsymbol{\gamma})$.

The vector of parameters to be estimated is

$$\boldsymbol{\gamma} = \left(\delta, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \sigma_\alpha^2, \sigma_\theta^2\right).$$

The asymptotic distribution is given by

$$\sqrt{M}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \xrightarrow{d} N(0, \Sigma)$$

where Σ is substituted by its sample analogue

$$\Sigma_M = \left(1 + \frac{1}{NS}\right)\left[H_M' W_M H_M\right]^{-1} H_M' H_M \left[H_M' W_M H_M\right]^{-1}$$

with $H_M = \partial\hat{\boldsymbol{\rho}}(\boldsymbol{\gamma}) / \partial\boldsymbol{\gamma}'$.

6 Estimation Results

This section first presents the model fit and parameter estimates. Using these estimated parameters, I then perform several counter-factual analyses.

6.1 Model Fit

Table 5 shows the moments from the data as well as the moments from the model. The model overestimates the speed of exits in the first interval, and it underestimates the speed of exits in the second interval. To further investigate the model fit, I focus on the first three moments and calculate them conditional on the initial number of competitors. Figure 8 shows three graphs. For markets with one, two, or three theaters in 1950, the model slightly underestimates the number of exits during the period 1955-1961. The model fits very well for markets with four, five, or six theaters in 1950. For markets with more competitors, the model overestimates the number of exits in the first 5 years. As Table 1 shows, about a half of the markets have either one, two, or three theaters in 1950. Thus, the model fit is good in the top panel of Figure 8. Because of theaters' exits, however, the frequency of these markets (one, two, or three theaters in 1950) decreases afterwards. Therefore, the model fit becomes worse in 1961. Instead, the frequency of markets with four, five, and six theaters in 1950 is still high in the 1960s, and hence the model shows a good fit for those markets (the middle panel of Figure 8).

6.2 Parameter Estimates

6.2.1 Base demand

Table 6 presents estimates of the structural parameters. The coefficient on population (β_2) is positive and statistically significant. This implies that theaters are more likely to operate profitably in bigger markets. It is worth noting that the coefficient on median income (β_4) is negative. One possible interpretation is that once I control for other observable and unobservable (to the econometrician) market characteristics, richer people tend to choose other entertainment besides movies. The parameter that captures competition (δ) is negative and significantly different from zero, implying that a theater's profit is eroded by competition. To see the relative sizes of these estimates, I evaluate the value of base demand (16) at the sample mean of X, using the estimated parameters. Then, $\bar{X}\hat{\beta} = 1.74$. Comparing with the parameter of competition, this means that, other things being equal, one additional competitor erodes 4.6% of theaters' profit.

6.2.2 Decay function

Most of the parameters in the decay function are precisely estimated. The fact that the quadratic term for TV rate (λ_2) is significantly different from zero means that the decay of a theater's profit accelerates as televisions are diffused. The coefficient of population growth (λ_4) is negative and significant. Since the term in the large bracket lies between zero and one, the negative estimate of λ_4 means that, if population grows faster, then the demand decay is less severe. Put differently, in a county with outflow of people, decay in demand is faster, which is consistent with a suburbanization story.

6.2.3 Estimates of standard deviations

Estimates of σ_θ and σ_α are also reported in Table 6. The standard deviation of exit values is 2.065 and is statistically significant. This implies that the mean of exit values is 1.65, and 95% of theaters has an exit value below 4.05.[26] Meanwhile, the standard deviation of market level heterogeneity is 0.376, which means that 95% of the value of heterogeneity ranges from -0.75 to 0.75. Compared with the value of base demand (16) evaluated at the sample mean of X and the estimated parameters (i.e., $\bar{X}\hat{\beta} = 1.74$), this variation explains a substantial proportion of the variation in initial numbers of competitors among similarly sized markets.

The variance of exit values can be interpreted as the extent of asymmetric information. If the variance is small, for example, a theater's assessment about its competitors' exit values is more precise. Hence, if a theater's value of exit is significantly higher than the mean when the variance is small, the theater would give up and exit relatively earlier. As the previous paragraph suggests, the value of exit ranges widely (the 95 percentile is 4.05), so I expect that theaters do stay in the market in hopes of outlasting their competitors.

6.3 Simulation Analysis

6.3.1 Delay and cost of asymmetric information

First, I quantify delays in exit generated by war of attrition. As discussed in Section 3, theaters expect a higher profit if they outlast their competitors and thus stay until the expected

[26] Remember that μ_θ is normalized to zero. The mean value is calculated by

$$\mathbb{E}(\theta|0 < \theta < 15) = \mu_\theta + \frac{\phi\left(-\frac{\mu_\theta}{\sigma_\theta}\right) - \phi\left(-\frac{15-\mu_\theta}{\sigma_\theta}\right)}{\Phi\left(\frac{15-\mu_\theta}{\sigma_\theta}\right) - \Phi\left(-\frac{\mu_\theta}{\sigma_\theta}\right)}\sigma_\theta = 1.65$$

and the value of 4.05 is calculated by

$$0.95 = \frac{\Phi\left(\frac{4.05}{\sigma_\theta}\right) - \Phi(0)}{\Phi\left(\frac{15-\mu_\theta}{\sigma_\theta}\right) - \Phi\left(-\frac{\mu_\theta}{\sigma_\theta}\right)}.$$

benefit of waiting becomes lower than the expected cost of waiting. As time goes by, theaters get discouraged and exit if their competitors remain in the market. Notice that this dynamic selection may occur even if demand is not declining.[27] In order to quantify delays in exit generated by war of attrition, I fix the TV penetration rate at its initial level in each market so that the decay function stays constant over time. There are three types of theaters in equilibrium. The first set of theaters does not exit. Since demand is constant, their instantaneous profits are higher than their values of exit forever. The second set of theaters exits as soon as a war of attrition starts. They choose to enter the market in the static entry game. However, it turns out that playing an exit game is not profitable at all, and so they exit right away. The third set of theaters stays in the market for a while, in hopes that they will outlast their competitors. Notice that this is due to asymmetric information. If they did not have this hope, they would exit the market right away. I am interested in such theaters.

Holding demand constant, 94 theaters exited in the sample period. Therefore, 1787 theaters remained in the market (the first set of theaters described above). Out of the 94 exits, 60 exits occurred right after the game started (the second set of theaters). Thus, the remaining 34 theaters stay in the market, hoping that they will outlast their competitors. Some of them exit the market relatively early, while others wait for longer. Figure 9 plots the distribution of years delayed. Most of the delays are distributed below 5 years. However, there are a small number of theaters that wait more than 5 years because of outlasting motives. This can possibly accumulate a big negative profit.

These delays come from asymmetric information. If exit values are common knowledge, in equilibrium, any theater exits the game exactly when the profit becomes lower than its exit value. Thus, no theater incurs a loss. I call this the complete information case. The difference in cumulative industry profits under war of attrition and complete information can be regarded as the cost of asymmetric information. Let $\{n_s\}_{s=1950}^{1964}$ and $\{n_s^*\}_{s=1950}^{1964}$ represent the number of theaters in each year in the war of attrition and in complete information, respectively.

Define Q_m and Q_m^* as

$$\begin{aligned} Q_m &= \int_{1950}^{1964} \sum_{k=1}^{n_s} \left[\Pi_{n_s}(s,m) - \theta_k\right] e^{-rs} ds \\ Q_m^* &= \int_{1950}^{1964} \sum_{k=1}^{n_s^*} \left[\Pi_{n_s^*}(s,m) - \theta_k\right] e^{-rs} ds. \end{aligned}$$

[27] For example, the original game in Fudenberg and Tirole (1986) is mainly for the case of a growing industry. The case of constant demand may be simply thought of as a special case of either a declining or growing market.

Then, calculate the percentage difference:

$$\frac{\sum_{m=1}^{454} Q_m^* - \sum_{m=1}^{454} Q_m}{\sum_{m=1}^{454} Q_m} = 0.13\%.$$

Thus, the industry profit will increase by 0.13% under complete information. This may appear small, but this number understates the cost of asymmetric information, especially since two cases (war of attrition and the case of complete information) become close to each other as time passes.

Define $R_m = \int_{1950}^{1950+dt} \sum_{k=1}^{n_s} [\Pi_{n_s}(s,m) - \theta_k] e^{-rs} ds$. I choose dt such that R_m represents the total profit less exit values earned in one month by all initial entrants in market m. Then, calculate

$$\frac{Q_m^* - Q_m}{R_m}$$

for each market m. In words, this shows the cumulative cost of asymmetric information in market m relative to the monthly market profit. This captures the effect of long delays more appropriately. Table 7 summarizes this statistic by the number of initial competitors. The cost of asymmetric information differs greatly across markets. The cumulative cost can sometimes equal as much as 40%-50% of the total market profit earned in one month.

6.3.2 Source of variation in exit process

I also quantify sources of across-market variation in theaters' exits. I focus on variables in the decay function; TV penetration rates and population growth. Since these two variables are related to each other in the decay function, it is difficult to fully decompose their effects. For every county, I set the value of each variable at certain common levels to see how the equilibrium exit process will change. For each variable, I use the 25th and 75th percentiles. The 25th and 75th percentiles of population growth are -9.1% and 26.5%, respectively. For TV penetration, I use the diffusion processes of the two counties I mentioned in Section 4 (Jim Wells county and Jackson county for the 25th and 75th percentile, respectively, in terms of years needed to reach the TV penetration rate of 50%).

Table 8 summarizes the results. When I use the TV penetration rates of Jim Wells county for every county, the exit process is significantly delayed. The number of surviving theaters in 1961 increases by 35.6% as compared to the equilibrium outcome. When I use the TV penetration rates of Jackson county instead, the number of surviving theaters decreases by 15.9% in 1955, by 18.4% in 1961, and by 8.8% in 1964. Meanwhile, population growth also explains a substantial portion of the variation of the exit process. The demand decline is severe in a county with a large population outflow (a county of at the 25th percentile). In particular, the number of surviving theaters in 1964 is 27% less than in equilibrium.

6.3.3 Role of diffusion process of television

Finally, I simulate the effect of a delayed diffusion process of television on the theaters' exits. This is a policy relevant exercise because delaying the diffusion of a new technology is an important option when the government wants to protect (at least in the short run) the affected sector. Think of the example of video rental stores. The government may potentially want to delay the diffusion process of online movie services to mitigate the severity of demand decline for video rental stores. In a counterfactual scenario, I let the TV penetration rate of each county stay constant for two years from 1950 to 1952, and let it start to change afterwards. Figure 10 shows the result. The simulation shows that when the nationwide penetration of televisions is delayed by 2 years, the share of surviving theaters increases by 15% in 5-6 years and the difference decreased to 4% at the end of my sample (15 years later).

7 Conclusion

This paper estimates the effect of competition among theaters separately from the effect of exogenous demand decline on the devolution of the industry. I modify Fudenberg and Tirole (1986)'s model of exit in duopoly with incomplete information to work in an oligopoly. I use data on the US movie theater industry and rich cross-section and over-time variations of TV penetration rates to estimate theaters' payoff functions and the distribution of exit values. By imposing the equilibrium condition, the model predicts the distribution of theaters' exit times for a given set of parameters and unobservables. I use indirect inference and estimate parameters by matching the predicted distribution with the observed distribution of exit times.

I control for the endogeneity generated by the correlation between the initial number of theaters and the unobservable variables of the model. To achieve this goal, I set up a model of entry at the beginning of my sample period and use MCMC-like procedure to approximate the joint distribution of unobservable variables, consistent with the observed number of entrants. This simulated distribution can be used to simulate the theoretical distribution of exit times.

Using the estimated model, I show that many theaters delay their exits, even when making negative profits, in hopes that their competitors will exit first. The difference between the total net profit in war of attrition and in the case where no theater makes a negative profit can be regarded as the cost of asymmetric information. In some markets, it equals 40%-50% of the total market profit that is earned in one month.

A war of attrition is costly for all theaters. If they are allowed, theaters may want to negotiate with their competitors on exit times. For example, the theater that exits early could be compensated by another theater that stays in the market. Such an arrangement can potentially improve every theater's payoff. However, this is illegal under anti-trust laws. Policymakers could consider an exception to the anti-trust laws in this case because sometimes a war of attrition is very costly. For example, think of the following mechanism. Each theater would "bid" the amount of compensation with which it is willing to exit immediately. Based on each theater's "bid," the mechanism assigns an outcome so that every theater is weakly better off. I plan to calculate the increase in profits under such a mechanism in future work.

I also quantify sources of across-market variation in theaters' exits, finding that both TV penetration rates and population growth explain a substantial portion of the variation in the exit process. I also perform a policy relevant exercise of delaying a diffusion process of televisions. I show that when the nationwide penetration of televisions is delayed by 2 years, the share of surviving theaters increases by 15% in 5 years, but the effect will die off as time passes.

The framework in this paper can be applied to analyze other industries where exogenous demand decline creates a nonstationary environment in an oligopoly. The industry of local video rental stores pressured by online movie services is a good example.

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8 Appendices

8.1 Appendix A

Claim *Consider a two player game. Pick θ arbitrarily such that $\theta > \lim_{t\to\infty} \Pi_2(t)$. Let $T_2\left(\cdot, 0, \overline{\theta}\right)$ be the equilibrium strategy of the game if the two player subgame starts at $t = 0$. Let $\Phi_2\left(t, 0, \overline{\theta}\right)$ be its inverse. For any t_0 such that $0 < t_0 < T_2\left(\theta, 0, \overline{\theta}\right)$, let $T_2(\theta, t_0, \widetilde{\theta})$ denote the equilibrium strategy of the game starting from t_0. Let also $\Phi_2(t, t_0, \widetilde{\theta})$ be its inverse. Then, $\widetilde{\theta} = \Phi_2\left(t_0, 0, \overline{\theta}\right)$ and $T_2\left(\theta, 0, \overline{\theta}\right) = T_2(\theta, t_0, \widetilde{\theta})$.*

In words, the optimal exit time that is planned at time zero equals to the one that is planned at some later time, conditional on nobody having exited until then.

Proof of Claim Since both players follow the same strategy, player i knows that θ_j is equal to or smaller than $\Phi_2\left(t_0, 0, \overline{\theta}\right)$. If not, player j would be better off by exiting at $t_0 - \varepsilon$, which contradicts the construction of $\Phi_2\left(t_0, 0, \overline{\theta}\right)$. Thus, $\widetilde{\theta} = \Phi_2\left(t_0, 0, \overline{\theta}\right)$. At time t_0, player i knows that θ_j is equal to or smaller than $\Phi_2\left(t_0, 0, \overline{\theta}\right)$. If player i has exit value θ and chooses stopping time $\tau \geq t_0$, the present discounted value of its expected profit over time at t_0 is

$$\begin{aligned}
&W_2(\tau, T_2(\cdot, t_0, \widetilde{\theta}), t_0, \widetilde{\theta}; \theta_i) \\
&= \Pr(T_2(\theta_j, t_0, \widetilde{\theta}) \geq \tau | \theta_j \leq \widetilde{\theta}) \left[\int_{t_0}^{\tau} \Pi_2(s)\, e^{-r(s-t_0)} ds + \frac{\theta}{r} e^{-r(\tau - t_0)}\right] \\
&+ \int_{\left\{\theta_j | T_2(\theta_j, t_0, \widetilde{\theta}) < \tau\right\}} \left(\int_{t_0}^{T_2(\theta_j, t_0, \widetilde{\theta})} \Pi_2(s)\, e^{-rs} ds\right. \\
&\left. + e^{-r\left(T_2(\theta_j, t_0, \widetilde{\theta}) - t_0\right)} V_1(\theta_i, T_2(\theta_j, t_0, \widetilde{\theta}))\right) g(\theta_j | \theta_j \leq \widetilde{\theta}) d\theta_j.
\end{aligned}$$

Taking the first-order condition and rearranging give

$$\begin{aligned}
\Phi_2'(t, t_0, \widetilde{\theta}) &= -\frac{G(\Phi_2(t, t_0, \widetilde{\theta}) | \theta_j \leq \widetilde{\theta})}{g(\Phi_2(t, t_0, \widetilde{\theta}) | \theta_j \leq \widetilde{\theta})} \left[\frac{\Phi_2(t, t_0, \widetilde{\theta}) - \Pi_2(t)}{V_1(\Phi_2(t, t_0, \widetilde{\theta}), t) - \Phi_2(t, t_0, \widetilde{\theta})/r}\right] \\
&= -\frac{G(\Phi_2(t, t_0, \widetilde{\theta}))}{g(\Phi_2(t, t_0, \widetilde{\theta}))} \left[\frac{\Phi_2(t, t_0, \widetilde{\theta}) - \Pi_2(t)}{V_1(\Phi_2(t, t_0, \widetilde{\theta}), t) - \Phi_2(t, t_0, \widetilde{\theta})/r}\right]. \qquad (25)
\end{aligned}$$

where the second equality follows from $\frac{G(\Phi_2(t,t_0,\widetilde{\theta})|\theta_j \leq \widetilde{\theta})}{g(\Phi_2(t,t_0,\widetilde{\theta})|\theta_j \leq \widetilde{\theta})} = \frac{G(\Phi_2(t,t_0,\widetilde{\theta}))}{g(\Phi_2(t,t_0,\widetilde{\theta}))}$. The boundary condition is

$$\Phi_2(t_0, t_0, \widetilde{\theta}) = \Phi_2(t_0, 0, \overline{\theta}). \qquad (26)$$

Since (5) and (25) are the same, (26) implies

$$\Phi_2(t, t_0, \widetilde{\theta}) = \Phi_2(t, 0, \overline{\theta}) \ \forall t \geq t_0.$$

Equivalently, $T_2\left(\theta, 0, \overline{\theta}\right) = T_2(\theta, t_0, \widetilde{\theta})$.■

8.2 Appendix B

Proof of Lemma 4 (i) Arbitrarily pick $\theta \in \left[\Pi_2(0), \overline{\theta}\right]$. Suppose $T_3(\theta) > 0$. Then, θ is the marginal type at $t = T_3(\theta)$ that is indifferent between exiting at t and staying until $(t + dt)$ and exiting then. The cost of waiting until $(t + dt)$ is $\{(\theta - \Pi_3(t))\, dt\} > 0$ because $\theta \geq \Pi_2(0) > \Pi_3(0)$. The value of waiting until $(t + dt)$ is the probability that either player j or player k drops out in $[t, t + dt]$ conditional on its having survived until t, times the value of entering the two-player subgame. If player j drops out in the interval, the value of entering the subgame depends on θ_k. Since θ is the marginal type at $T_3(\theta)$, it follows that $\Pr(\theta < \theta_k) = 0$. Otherwise, player θ_k should have dropped out earlier. Thus, player θ drops out immediately after the two-player subgame starts. Therefore, the value of staying should be zero, and player θ cannot be the marginal type. A contradiction.

(ii), (iii) By the same argument as Lemma 1 of Fudenberg and Tirole (1986).■

Proof of Lemma 5 In the previous lemma, I proved that any strategy $T_3(\cdot)$ such that $T_3(\theta) = c > 0$ cannot be an equilibrium. Without Assumption 2, any strategy $T_3(\cdot)$ such that $T_3(\theta) = 0$ for $\theta \in \left[\Pi_2(0), \overline{\theta}\right]$ cannot be an equilibrium either. To show this, assume $T_3(\theta) = 0$ for $\theta \in \left[\Pi_2(0), \overline{\theta}\right]$. Suppose $\theta_i \in \left[\Pi_2(0), \overline{\theta}\right]$. Then there exists $\varepsilon > 0$ such that

$$\Pr(T_3(\theta_j) = T_3(\theta_k) = 0)\, V_1(\theta_i, 0) > [\theta_i - \Pi_2(0)]\, \varepsilon.$$

Thus, player i becomes better off by waiting for ε. This contradicts $T_3(\theta) = 0$. Therefore, Assumption 2 is necessary. With this assumption, the first boundary condition is given by (9). The logic behind the second boundary condition is the same as that of (7)■

Proof of Proposition 6 In the symmetric case, a slight modification of the proof for Lemma 3 in Fudenberg and Tirole (1986) suffices. In particular, letting Φ_3 and $\widetilde{\Phi}_3$ be distinct solutions of (8), (9), and (10), I can obtain a contradiction.

Proof of Lemma 7 Firm $\Phi_3(t)$ does not drop out if its triopoly profit is strictly larger than its exit value. If $\Pi_3(t) = \Phi_3(t)$, the firm does not drop out either, because there is a positive probability that some of its competitors exits in the next instant; i.e., $P(\Phi_3(t) < \theta_j < \Phi_3(t+s)) > 0$ for all t and s. Since $\Pr(\theta_j < \lim_{t\to\infty} \Pi_2(t)) > 0$, there is always a positive probability that firm j stays in. Therefore, $\Phi_3(t) < \Pi_2(t)$.$\blacksquare$

Proof of Lemma 8 When $t \in [T_3(\theta_k), \Pi_2^{-1}(\theta_k))$, θ_i is strictly lower than $\Pi_2(t)$. Thus, regardless of what firm j does, firm i does not exit the game. When $t = \Pi_2^{-1}(\theta_k)$, only type $\theta_i = \theta_k$ is indifferent between staying in and exiting. But the event has measure zero. The same logic applies to firm j's strategy. For $t \in [\Pi_2^{-1}(\theta_k), \infty]$, the policy function follows (5). Since no type has exited since $T_3(\theta_k)$, the maximum possible value of opponent's exit value is still θ_k. Thus (11) obtains. The second boundary condition is the same as (7).$\blacksquare$

Tables and figures

Table 1: Number of competitors

Number of competitors in 1950	Frequency	Percent
1	73	16.08
2	78	17.18
3	74	16.30
4	62	13.66
5	56	12.33
6	30	6.61
7	21	4.63
8	15	3.30
9	17	3.74
10	13	2.86
11	8	1.76
12	7	1.54
Total	454	100

Table 2 : Summary statistics of demographic variables in 1950

Variable	Obs	Mean	Std. Dev.	Min	Max
Population	454	27,240	29,032	2,205	209,628
Median age	454	4.06	0.93	2.00	7.00
Median family income	454	5.48	1.46	2.00	9.00
Share of population living in city areas	454	0.05	0.16	0.00	0.92

Note: Median age and median family income are categorical variables.
Source: Haines, Michael R., and the Inter-university Consortium for Political and Social Research. HISTORICAL, DEMOGRAPHIC, ECONOMIC, AND SOCIAL DATA: THE UNITED STATES, 1790-2000.

Table 3: Effect of market structure and competition on exit

Variables	Dependent variable† $S_{1950-64}$	Dependent variable† $S_{1961-64}-S_{1955-61}$	Dependent variable: The initial number of comeptitors
change in population (log)	-0.0235	-0.2498	
	(0.0435)	(0.2724)	
change in TV penetration	0.0054 **	0.0050 ***	
	(0.0023)	(0.0013)	
# of new entrants	0.0013	-0.0179	
	(0.0114)	(0.0229)	
# of competitors	0.0215 ***	0.1052 ***	
	(0.0071)	(0.0132)	
market size (population)	-0.1376 ***	--	1.9825 ***
	(0.0262)		(0.1206)
median age	-0.0037	--	0.2433 ***
	(0.0141)		(0.1026)
median family income	-0.0456 ***	--	0.1361 ***
	(0.0104)		(0.0570)
population share living in city areas	-0.0032	--	1.0213
	(0.0642)		(0.7616)
Constant	1.8186 ***	--	-17.0542 ***
	(0.2401)		(1.1399)
Estimator	OLS	2SLS	OLS
Instrument for # of competitors	--	# of competitors in 1950	--
R^2 (overall)	0.1404	--	0.5367
No. of Market	454	285	454

Note:

Robust standard errors in parentheses.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

† Let $S_{t-t'}$ denote the share of theaters that exited during the period t and t'

For the 2SLS regression, I take the first difference for both dependent and independent variables to eliminate unobserved effects. Then I use the number of competitors in 1950 as an instrument for the change in number of competitors between 1961-1964.

Table 4: Summary statistics of number of theaters in 1950 by population size

Population of counties	Observations	Mean	Std. Dev.	Minimum	Maximum
0-10000	128	2.03	1.17	1	6
10000-20000	114	3.32	1.42	1	7
20000-30000	77	4.23	1.96	1	11
30000-40000	38	5.66	2.53	2	11
40000-50000	38	6.42	2.96	1	12
50000-75000	28	6.86	2.84	2	12
75000-100000	16	7.63	2.28	3	11
100000-150000	9	8.67	1.58	6	11
150000-	6	10.83	1.94	7	12

Source: Haines, Michael R., and the Inter-university Consortium for Political and Social Research. HISTORICAL, DEMOGRAPHIC, ECONOMIC, AND SOCIAL DATA: THE UNITED STATES, 1790-2000.

Table 5: Model fit

Variables	Data moments	Moments from the model
share of surviving theaters in 1955	0.7876	0.7397
share of surviving theaters in 1961	0.2891	0.3810
share of surviving theaters in 1964	0.2571	0.2941
share of surviving theaters in 1955 times pop. growth	0.0384	0.0379
share of surviving theaters in 1961 times pop. growth	0.0202	0.0260
share of surviving theaters in 1964 times pop. growth	0.0093	0.0114
share of surviving theaters in 1955 times change in TV penet. Rate	0.3328	0.2839
share of surviving theaters in 1961 times change in TV penet. Rate	0.0960	0.1327
share of surviving theaters in 1964 times change in TV penet. Rate	0.0167	0.0211
initial # of competitors	4.1432	3.9365
covariance b/w initial # and pop.	0.1801	0.1049
covariance b/w initial # and age	0.0553	0.0060
covariance b/w initial # and income	0.0828	0.0424
covariance b/w initial # and share of population living in city	1.7308	3.4510

Table 6: Estimates of structural parameters

Variables	
δ (competition)	-0.0794 ***
	(0.0068)
β_1 (constant)	1.0150 ***
	(0.0369)
β_2 (population)	0.6328 ***
	(0.0656)
β_3 (median age)	0.3911
	(0.2420)
β_4 (median income)	-0.7766 ***
	(0.2402)
β_5 (city population)	0.7474 ***
	(0.2085)
λ_0 (constant)	-0.2327 ***
	(0.0738)
λ_1 (TV rate)	1.8458 ***
	(0.2201)
λ_2 (TV rate squared)	1.6144 ***
	(0.5288)
λ_3 (constant)	-0.1861
	(0.1782)
λ_4 (change in population)	-0.6967 ***
	(0.0785)
σ_θ (std. of exit value)	2.0645 ***
	(0.4712)
σ_α (std. of demand shifter)	0.3759 ***
	(0.1428)
No. of Markets	454

Note:

Standard errors in parentheses.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table7: Cost of asymmetric information, by number of initial competitors

# of initial competitors	# of markets	(1) Without discount				(2) With discount			
		mean	10 percentile	50 percentile	90 percentile	mean	10 percentile	50 percentile	90 percentile
1	73	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
2	78	1.4%	0.6%	1.2%	2.7%	0.6%	0.3%	0.5%	1.2%
3	74	5.2%	2.3%	4.1%	10.2%	2.5%	1.2%	1.9%	5.2%
4	62	11.2%	5.8%	8.8%	20.7%	5.7%	3.0%	4.5%	9.9%
5	56	15.2%	7.4%	13.3%	28.4%	7.9%	3.8%	7.1%	14.2%
6	30	17.6%	0.7%	16.4%	29.4%	9.4%	0.4%	9.0%	15.7%
7	21	20.0%	0.3%	21.8%	37.8%	10.5%	0.2%	11.9%	18.6%
8	15	19.5%	0.5%	19.0%	42.4%	10.5%	0.2%	10.7%	22.9%
9	17	15.6%	0.6%	19.9%	36.2%	8.6%	0.3%	11.0%	20.6%
10	13	25.3%	0.8%	23.8%	44.1%	13.5%	0.4%	13.4%	22.7%
11	8	19.0%	0.9%	15.1%	40.5%	10.3%	0.5%	8.2%	22.1%
12	7	23.6%	0.4%	11.6%	57.8%	12.6%	0.2%	6.5%	29.6%

Note: Let Q_m and Q'_m be the total cumulative profit (less value of exit) earned by all theaters in market m in war of attrition and in the case of complete information, respectively. Let R_m be the total profit (less value of exit) earned by all theaters in market m in the first month of the sample period. Then, I calculate $(Q'_m - Q_m) / R_m$ and define this as a cost of asymmetric information. The table above summarizes this statistic by the number of initial competitors. For the columns (1), I simply add profits of each time period to calculate Q'_m and Q_m, while for the columns (2), I use the discounted sum of profits.

Table 8: Sources of cross-market variation in exits

	Number of theaters			
	1950	1955	1961	1964
In equilibrium	1881.0	1272.2	654.3	528.6
Different Scenarios				
TV diffusion rate fixed at:				
25th percentile	1881.0	1580.6	887.5	663.6
	(0.0%)	(24.2%)	(35.6%)	(25.6%)
75th percentile	1881.0	1070.2	534.1	482.2
	(0.0%)	(-15.9%)	(-18.4%)	(-8.8%)
Population change fixed at:				
25th percentile	1881.0	1154.5	515.2	385.8
	(0.0%)	(-9.3%)	(-21.3%)	(-27.0%)
75th percentile	1881.0	1295.0	700.6	566.4
	(0.0%)	(1.8%)	(7.1%)	(7.2%)

Note: To identify the source of variations in exit times across markets, I fix the TV diffusion rate of each county at the 25th and 75th percentiles of the sample, and calculate the equilibrium path. Similarly, I fix the growth rate of population of each county at the 25th and 75th percentile of the sample, and calculate the equilibrium path. Numbers in parentheses are deviations (in percentage) from the original equilibrium.

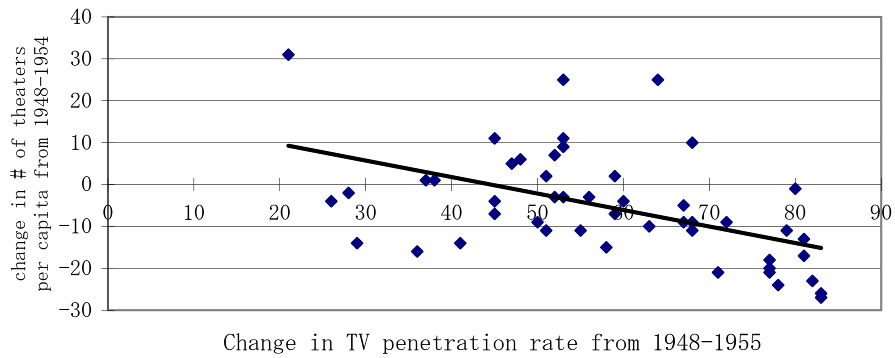

Figure1 : TV penetration and # of theaters per capita across states Source: Stuart (1960)

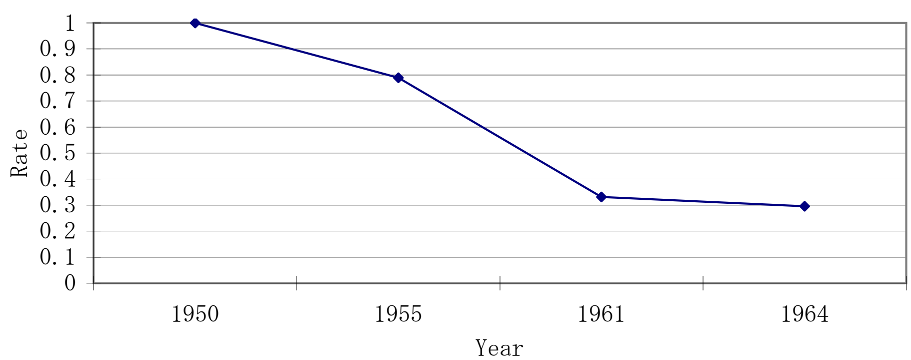

Figure 2: Share of surviving theaters

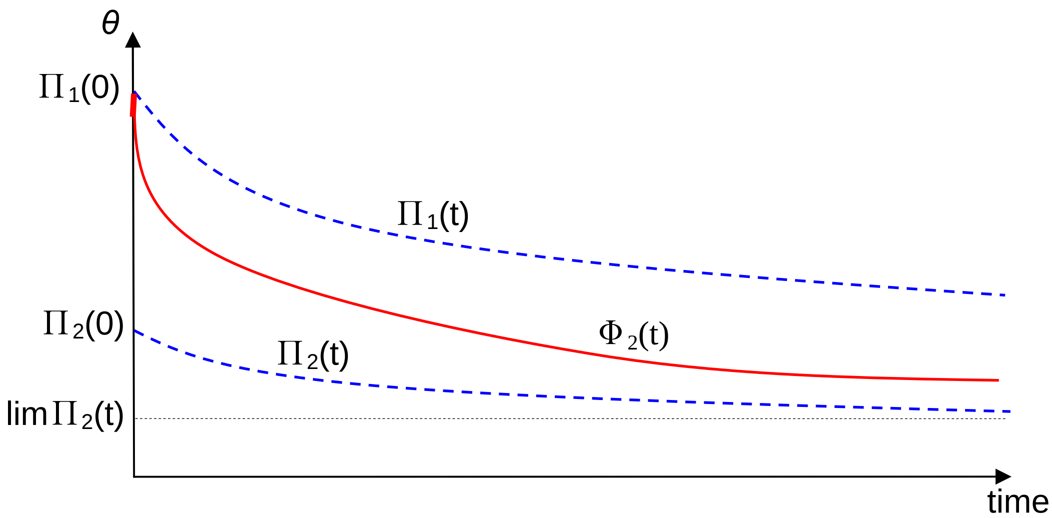

Figure 3: Policy function in the case of duopoly

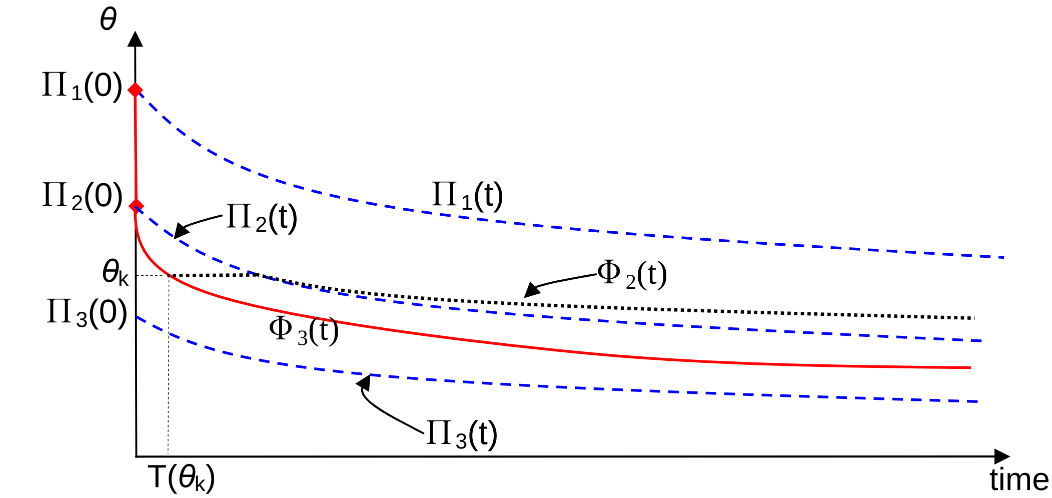

Figure 4: Policy function in the case of oligopoly

Player *k* exits at $T(\theta_k)$ and surviving players form a new policy function, $\Phi_2(t)$.

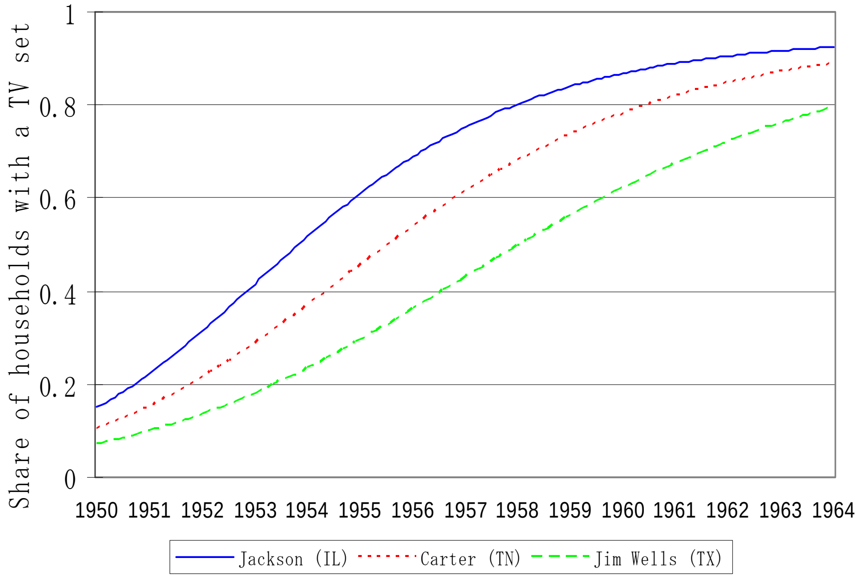

Figure 5: TV penetration rate for selected counties

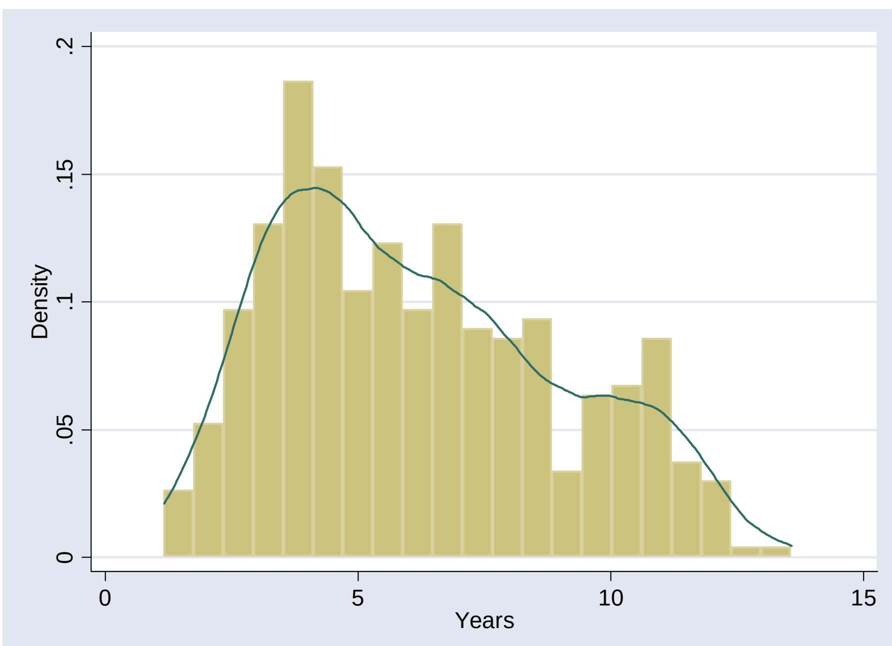

Figure 6: Distribution of years needed to reach TV penetration of 50%

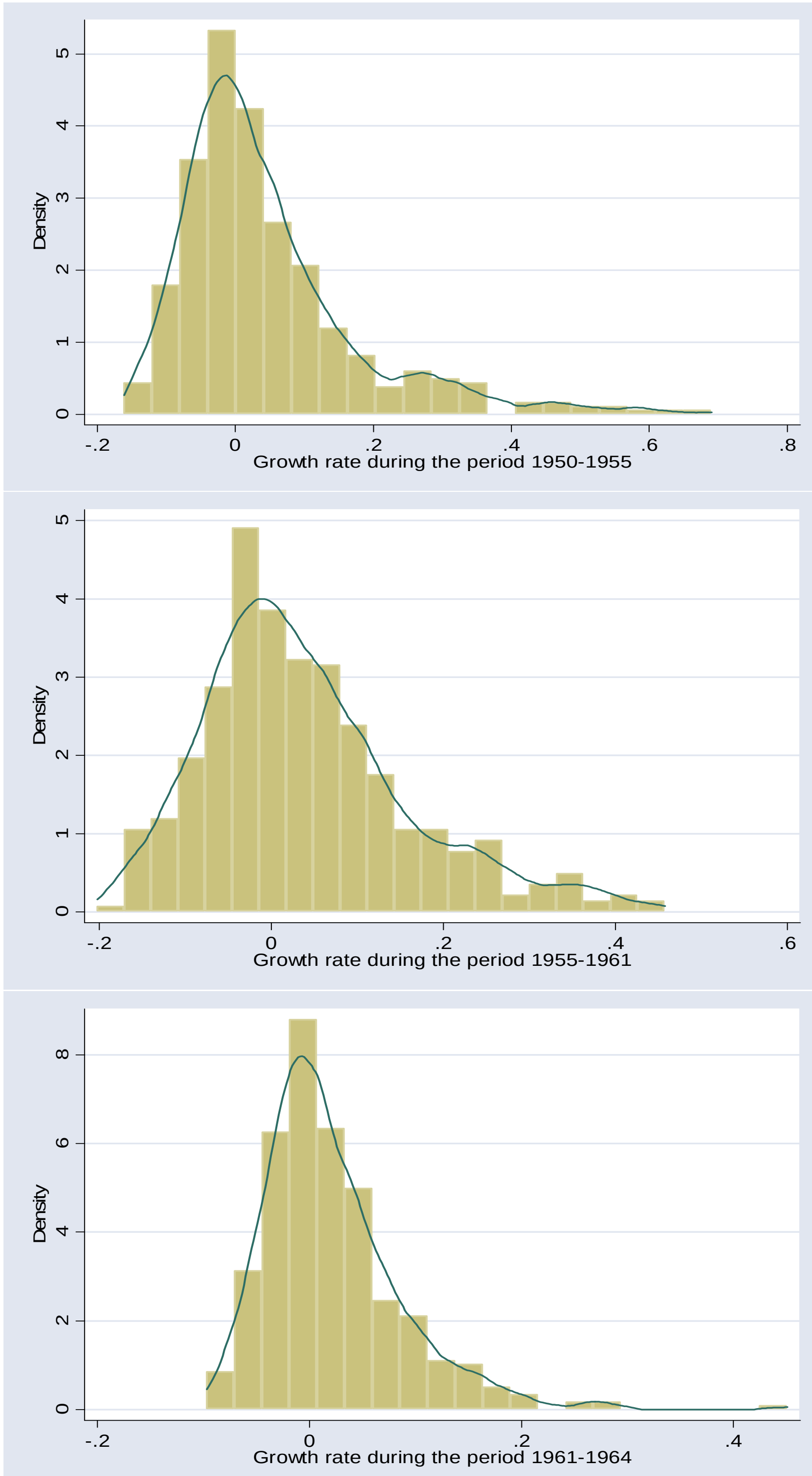

Figure 7: Distributions of percentage changes in population from 1950-55 (top panel), 1955-1961 (middle panel), and 1961-1964 (bottom panel).

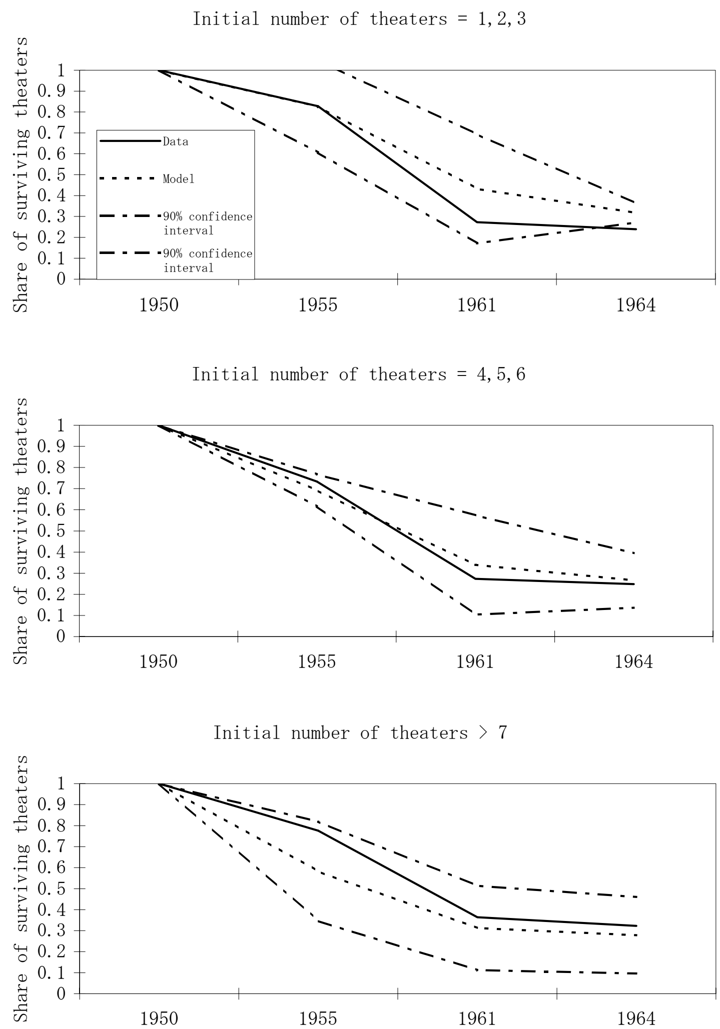

Figure 8: Share of surviving theaters by initial number of competitors

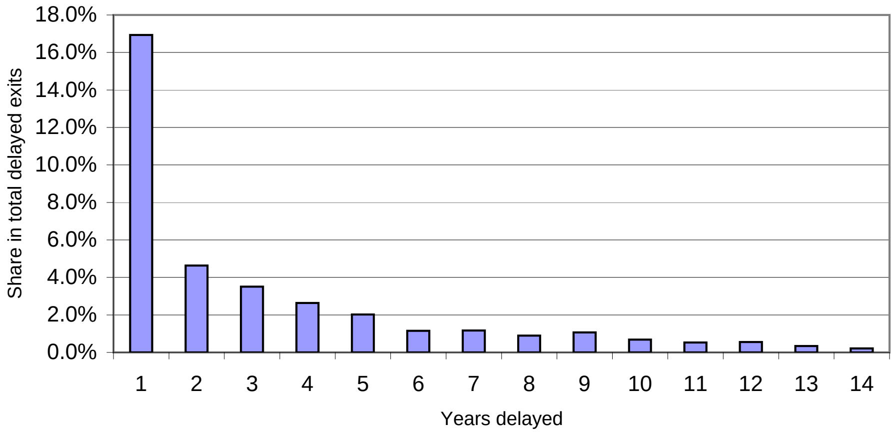

Figure 9: Distribution of years delayed
Out of 94 total exits in a constant demand environment, 34 exits are delayed.

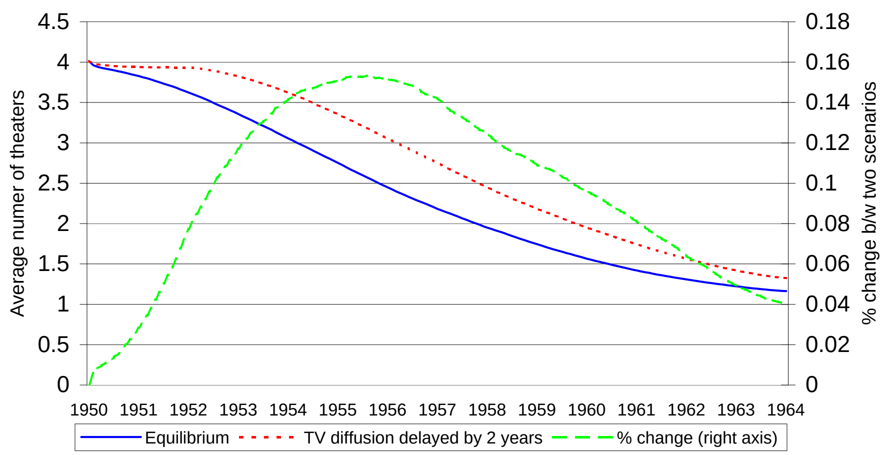

Figure 10: Effect of 2-Year Delay in TV penetration