Competition in Matching Technologies*

Thomas Trégouët[†]

Preliminary and incomplete version - Comments welcome - Please do not circulate

Abstract

We analyze the trade-off between monopoly and competition in matching markets. Intermediaries use prices to sort agents on one side of the market, while, on the other side, they rely solely on an imperfect screening technology. The socially efficient outcome is characterized by an optimal quality provision and an efficient market coverage: matched agents are matched assortatively and all agents find a partner. Under monopoly, most agents are matched assortatively, but some of them are mismatched. Besides, the market is fully covered only if the population of agents who pay for intermediation service is not too heterogeneous. Under competition, matched agents are matched efficiently, but some of them remain unmatched. The quality provision and market coverage objectives cannot be reached simultaneously. We also show that, under monopoly, if one population of agents is quite heterogenous, then, exempting these agents from payment may have a positive impact on market coverage and total surplus.

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[†]Thema, Université de Cergy-Pontoise. Email: thomas.tregouet@u-cergy.fr

1 Introduction

One of the many implications of the IT (information technology) has been the growing success of matching markets.[1] This renewal of interest for matching markets has also reemphasized one of their major problem, namely, that participants may have incentives to misrepresent their true characteristics.[2] Although people can easily find partners, they cannot be sure of their "qualities". Economic theory suggests price discrimination to be a good way to alleviate this quality provision problem. In a nutshell, the agents (men or women, employers or workers) who benefit more from finding a partner may accept to pay higher subscription fees to get access to more selective meeting places. Then, it seems possible to create several meeting places, ranked by entrance fees, where agents on both sides self select.

Yet, it is then surprising that, in many matching markets, participants on one side used to be exempted from payment. In many dating agencies or single clubs, subscription is free to women. In the labor market, job seekers almost never pay for intermediation service. The International Labor Organization even recommends that "private employment agencies [should] not charge directly or indirectly, in whole or in part, any fees or costs to workers" (Article 7, Convention 181). In these markets, intermediaries have thus find other ways to screen participants: dating agencies use personality testing to pair people up; employment agencies select workers with skills testing.

In this context, the quality provision problem has to be reassessed. In particular, the incentives of a monopoly matchmaker to match agents efficiently may change dramatically. If it happens that a monopoly matchmaker introduces distortion in quality provision, a natural question is whether competition can achieve a better outcome. This question seems particulary relevant in the market for job placement. Indeed, it is worth noticing that, in the past few years, many OECD's countries have promoted competition between private employment agencies, while having little idea about the potential trade-off between monopoly and competition.[3,4]

[1] According to a recent survey, almost 20 percents of European and North American Internet users visited dating web sites in 2006. (Source: ComScore) The Internet has also changed the job search and recruiting strategies of unemployed workers and firms. In August 2000, one in four unemployed US workers reported that they regularly looked for job online. (Source: Source: August 2000, Current Population Survey, Computer and Internet Use Supplements)

[2] For instance, The Wall Street Journal has recently reported several cases of identity thefts on dating websites. "The Cut-and-Paste Personality", Wall Street Journal, Feb. 15, 2008.

[3] In 1996, the Australian authorities ran a massive procurement auction to outsource the former public employment service to private operators. In 2001, a similar auction took place in the Netherlands. In Germany, the Hartz's reforms (2003-2005) authorized private employment agencies to offer intermediation service. In 2005, French authorities abolished the monopoly of the public employment service.

[4] The International Labor Organization has endorsed the view that competition between employment agencies may be beneficial. Indeed, Convention 181 (1997) recognizes "the role which private employment agencies may play in a well-functioning labor market".

In this paper, we shed light on the trade-off between monopoly and competition in matching markets when one side is exempted from payment. We consider a model in which two equal-sized populations of heterogenous agents, namely, employers and workers, can meet in matchmaking agencies. Employers and workers have private information about their one-dimensional type, which can be either low or high, and the match value function exhibits complementarities between types. An agency has three components: subscription fees, a screening technology and a matching function. On the employers' side of the market, agencies use subscription fees to sort employers by type. On the other side of the market, workers are sorted with an imperfect screening technology: after a worker joins an agency, the agency observes a signal correlated with the worker's type. Then, each employer, depending on the fee he paid, and each worker, depending on the signal he obtained, are assigned probabilities for being matched with each type of agents on the other side. These probabilities are given by the matching function.

To understand the differences between the monopoly and competition outcomes, consider as a benchmark the situation where the total surplus from matches is maximal. Because employers' and workers' types are complements, the total surplus from matches is maximal when agents are assortatively matched, i.e. when the highest type employers are matched with the highest type workers, and the second highest type employers are matched with the second highest type workers, and so on. Besides, since the value of a match between an employer and a worker is positive, whatever the agents' types, it is socially efficient to find a partner to each agent. Put differently, the socially efficient outcome is characterized by an efficient *quality provision* (matched agents are matched assortatively) and an efficient *market coverage* (all agents are matched).

A monopoly agency introduces distortions in quality provision and, sometimes, chooses to serve only the high type agents. The distortion in quality provision stems from the fact that a monopoly agency cannot sort workers perfectly. Because workers are exempted from payment, an agency cannot provide incentives for workers to self-select through different subscription fees. It must instead rely on an imperfect screening technology. Therefore, under monopoly, there must be some employers and workers who are not matched assortatively. However, a monopoly agency has the right incentives to match agents assortatively once it has observed workers' signals, i.e. to match the highest type employers with the highest signal workers, and the second highest type employers with the second highest signal workers. Indeed, since the agency can extract part of the employers' surplus through the subscription fees, it has incentives to maximize the total employers' surplus, hence, to match agents assortatively. The only drawback is that the agency may prefer to serve only the high type employers. This arises when the cost of eliciting employers' private information is too high. In order to provide incentives for employers to self-select, the agency must pay an informational rent to high type employers. If this informational rent is too high, which arises when the employers' population is too heterogenous, then, the agency prefers to serve only the high type employers. In this case, the low type employers and the workers who obtain a low signal remain unmatched.

The impact of competition on welfare is ambiguous: matched agents are matched efficiently, but some high type agents remain unmatched. Put differently, under competition, there is no distortion in match quality provision, but, on the other hand, the market is only partially covered. When two agencies compete to attract employers and workers, high type and low type agents do not register with the same agency. The key consequence is that employers and workers are perfectly sorted, so that matched agents are matched assortatively. Yet, this "separating" equilibrium can be sustained only if some high type agents remain unmatched. Intuitively, if all high type agents were matched, then, in particular, it would mean that the agency which serves the high type agents had proposed a partner to each worker, whatever his signal. Therefore, if a low type worker were to deviate and to join the high type agency, he would also be matched with a high type employer, i.e. he would be strictly better off than in the low type agency. This implies that the high type agency must commit not to match some of the low signal workers. Indeed, in this case, a low type worker has a strictly lower probability to be matched in the high type agency than a high type worker.

We also investigate the consequences of exempting one group of agents from payment. This is done by describing the monopoly outcome when agencies can charge workers for intermediation service. This allows us to discuss the impact of exempting workers from payment on market coverage. When both sides pay, the market is partially covered when the cost of eliciting private information on both sides of the market is too high. Therefore, the market is more covered when only employers pay if the cost of eliciting private information is low on the employers' side and high on the workers' side.

The remainder of this paper is organized as follows. In section 2, we lay out the framework of our model. Section 3 derives the monopoly outcome. Section 4 is devoted to the analysis of competition. Section 5 investigates the impact on welfare of exempting one side of payment. Section 6 discusses several robustness checks of our framework. Section 7 concludes.

Related literature.

- Profit-maximizing intermediary in matching markets: Damiano and Li (2007), Damiano and Li (2008), Caillaud and Jullien (2003).

- Institutions or matching mechanisms that may implement an efficient match quality: McAfee (2002), Hoppe, Moldovanu, and Ozdenoren (2007), Hoppe, Moldovanu, and Sela (2008).

- Industrial organization of intermediation markets: literature on multi-sided platforms Armstrong (2006), Caillaud and Jullien (2003), Rochet and Tirole (2003 and 2006) + others.

- "neoclassical" assignment theory, and assignment models with frictions (Shimer and Smith (2000), Atakan (2006)).

- Empirical relevance of sorting. Recent literature in labor economy. See, e.g., Eeckhout and Kircher (2010).

2 The model

Employers and workers. There are two populations of agents: employers and unemployed workers. The two populations have the same size, normalized to one. Employers and workers have heterogeneous one-dimensional characteristics, called types. Half of the employers are of type y_l and the others are of type y_h with $\Delta y = y_h - y_l > 0$. Similarly, half of the workers are of type x_l and the others are of type x_h with $\Delta x = x_h - x_l > 0$.[5] A match between a type x worker and a type y employer produces value xy to both the worker and the employer. Agents are risk neutral and have quasi-linear preferences. They only care about the difference between the expected match value and the entry fee they pay. An unmatched agent gets a payoff of 0, regardless of his type. Thereafter, subscript e (resp. u) will refer to the population of employers (resp. the population of workers). Denote by $\alpha_x = x_l/x_h$ and $\alpha_y = y_l/y_h$.

Employers and workers cannot find a partner by themselves. They can only be matched in agencies.

Agencies. There are three components in an agency: the subscription fees, a screening technology and a matching function.

Subscription fees. Subscription fees are paid only by employers. These can choose between two different fees p_l and p_h, which give them access to two different populations of workers. The vector of subscription fees is denoted $P = (p_l, p_h)$.

Screening technology. On the other side of the market, the population of workers who join an agency is partitioned according to a screening technology. This technology allows an agency to observe an imperfect signal on each worker who subscribes. More precisely, when a type x worker ($x \in \{x_l, x_h\}$) joins an agency, the agency observes a signal $\sigma(x)$. The signal can be either low ($\sigma(x) = x_l$) or high ($\sigma(x) = x_h$), depending on the worker's type.

[5] These assumptions are discussed in Section 6.

We assume that this signal is positively correlated with the worker's type, so that it is more likely to be high when the the worker's type is high, and conversely. Formally, there exists some $\rho \in [1/2, 1)$ such that:

$$\begin{aligned} Pr[\sigma(x) = x_h | x = x_h] &= Pr[\sigma(x) = x_l | x = x_l] = \rho, \\ Pr[\sigma(x) = x_h | x = x_l] &= Pr[\sigma(x) = x_l | x = x_h] = 1 - \rho. \end{aligned}$$

The variable ρ measures the quality of the screening technology. When it is high, an agency observes a partition of the workers which is close to the real one. On the other hand, when ρ is close to $1/2$, the signal is not informative. We assume that an agency can take a costly effort in order to improve the quality of its screening technology. Formally, if an agency wants to obtain the screening technology ρ, it has to pay the sunk cost $C(\rho)$. We assume that function $C(.)$ is twice differentiable, strictly increasing and strictly convex. We also assume that $C(1/2) = 0$, $C'(0) = 0$ and $C'(1) = \infty$. Last, for ease of notation, we take $C(\rho) = -\infty$ for all $\rho < 1/2$.

Matching functions. Let us first say that a precise definition of a matching function is rather complex and not very informative. We thus refer the reader to Appendix A.1 for a formal definition.

A matching function describes how an agency matches employers and workers. Suppose that some employers and workers join an agency. This population can be represented by a four-dimensional vector $\mathcal{N} = (n_i^j)_{i \in \{l,h\}, j \in \{e,u\}}$, where n_i^j is the number of type i agents within the $j-$population. With a slight abuse of notations, we will sometimes refer to n_i^j as a population of agents instead of the size of this population. For instance, n_l^u and n_h^e are respectively the number or workers who obtain the low signal and the number of employers who choose to pay p_h. A vector $\mathcal{N}$ is called a *distribution of agents.* Basically, for any distribution of agents, a matching function gives the probability for a given agent to be matched with each type of agents on the other side of the market.

To grasp the idea, consider a matching function $\mu(.)$ and a distribution of agents $\mathcal{N}$. Denote by $(\mu_{xy}(\mathcal{N}))_{x \in \{x_l, x_l\}, y \in \{y_l, y_l\}}$ the set of matches assigned to $\mathcal{N}$ by $\mu(.)$. Suppose for instance that n_h^e is non-empty, then, the probability for an employer in n_h^e to be matched with a worker in n_h^u is $\mu_{x_h, y_h}(\mathcal{N})/n_h^e$. We denote by $\mathfrak{M}$ the set of matching functions and we assume that an agency chooses its matching function in this set. Notice that we make no restriction on the set of feasible matching functions. In particular, nothing prevents an agency to commit to matching rules under which some agents remain unmatched. This will play a crucial role in the following.

Timing. The chronology of events runs as follows:

1. Agencies choose their subscription fees and their matching technologies.

2. Employers and workers choose which agency (if any) to register with. The agencies observe signals on each workers and the contracts chosen by each employer and, then, match agents according to their matching functions.

We assume that workers join at least one agency in equilibrium. Note that it is a weak assumption since subscription is free to workers. We also assume that employers and workers cannot join multiple agencies. Last, we assume that in equilibrium all agents of the same type play the same pure strategy in stage 2, i.e. we focus on the analysis of "pure" subgame equilibria.

Agents' payoffs. Precise equations of agents' payoffs are rather complex and not very informative. Again, we thus refer the reader to Appendix A.1 for a more formal definition.

Consider an agency which sets P, ρ and $\mu(.) \in \mathfrak{M}$. The pair $(\rho, \mu(.))$ is called a *matching technology*. If an employer joins this agency and chooses to pay the fee p_i ($i \in \{l, h\}$), we say that he chooses the *contract* $(p_i, \rho, \mu(.))$. Given that a distribution of agents $\mathcal{N}$ join the agency, the expected utility of a type i employer ($i \in \{l, h\}$) who chooses the contract j ($j \in \{l, h\}$) is denoted by $V^e_{ij}(p_j, \rho, \mu(.), \mathcal{N}$. On the other side of the market, the expected utility of a type i worker ($i \in \{l, h\}$) is denoted by $V^u_i(\rho, \mu(.), \mathcal{N})$. Last, the expected profits of the agency is $\Pi(P, \rho, \mu(.), \mathcal{N})$.

Examples of matching functions. We would like to give some examples of matching functions. Suppose that all agents join the same agency and that all type y_i employers ($i \in \{l, h\}$) choose the contract $(p_i, \rho, \mu(.))$. Notice that the agency observes the distribution of agents $\mathcal{N} = (1/2, 1/2, 1/2, 1/2)$. In stage 1, it may have committed to many different matchings in case it would face this distribution in stage 2. Let us give two examples. First, the agency may have decided ex-ante to match employers and workers *assortatively*. In this case, a worker is matched with an employer who chooses the "high type" contract $(p_h, \rho, \mu(.))$ if he gets a high signal. Otherwise, he is matched with an employer who chooses the "low type" contract $(p_l, \rho, \mu(.))$ if he gets a low signal. This is called *positive assortative matching* (PAM): high type agents are matched together and low type agents are matched together. Second, agents can be matched *randomly* with each other. Although these two examples are quite natural, the agency may have committed to very different matching rules. For instance, the agency can choose to match randomly half of the workers who get a high signal, while the other workers remain unmatched. In this case, three-quarters of the employers and workers do not find a partner.

3 Monopoly agency

In this section, we analyze the matching technology decision of a monopoly agency. Since subscription is free to workers, we assume that all workers join the agency. This assumption allow us to abstract from coordination problems.[6]

We prove the following proposition:

Proposition 1. *If $y_h \leq 2y_l$, then, a monopoly agency serves all agents and matches agents assortatively: high (low) type employers are matched with high (low) signal workers. Besides, the agency's screening technology is given by $C'(\rho) = \Delta x \Delta y$.*
If $y_h > 2y_l$, then, a monopoly agency serves only high type employers and these are matched with high signal workers. Besides, the agency's screening technology is given by $C'(\rho) = \frac{1}{2}\Delta x y_h$.

Proof. See Appendix A.2. □

A couple of things are worth noticing here. First, a monopoly agency chooses to match all agents assortatively if $y_h/y_l \leq 2$ and to match only high type agents otherwise. To grasp the intuition, consider that the agency has to choose between these two matching functions and that it has already chosen the quality of its screening technology $\rho > 1/2$. Notice that the total surplus from matches is maximal when employers and workers are matched assortatively. In this case, a high (low) type employer has a probability ρ for being matched with a high (low) type worker and a probability $1-\rho$ for being matched with a low (high) type worker. Besides, the agency's profits would be maximal if it could extract all surplus from employers. Its profits would then be

$$\Pi_{\max} = 1/2 \cdot (\rho x_h + (1-\rho)x_l)y_h + 1/2 \cdot (\rho x_l + (1-\rho)x_h)y_l - C(\rho).^7$$

Let Π_h^{SB} and Π_{h+l}^{SB} denote respectively the agency profits if it chooses to serve only high type agents and if it matches all agents assortatively.

If the agency chooses to match only high type agents, a high type employer has a probability ρ for being matched with a high type worker and a probability $1-\rho$ for being matched with a low type worker. Hence, his expected utility is given by: $V_{hh}^e = (\rho x_h + (1-\rho)x_l)y_h - p_h$. Since the high type contract is the only contract in which an employer can be matched, a high type employer subscribes to this contract if $V_{hh}^e \geq 0$. Notice that if the agency extracts all high type employers' surplus, i.e. if $p_h = (\rho x_h + (1-\rho)x_l)y_h$, then, a low type employer

[6]Indeed, if both subscription fees are positive, there always exists an equilibrium in stage 2 in which no agents join the agency.

[7]Notice that $\Pi_{\max}$ is not the first-best profits since workers' types are private information. The first best-profits would be $1/2.(x_h y_h + x_l y_l) > \Pi_{\max}$.

is not willing to subscribe to the high type contract since this would give him a negative utility: $V_{lh}^e = (\rho x_h + (1-\rho)x_l)y_h - p_h = -(\rho x_h + (1-\rho)x_l)\Delta y < 0$. Therefore, the agency sets $p_h = (\rho x_h + (1-\rho)x_l)y_h$ and makes profits

$$\Pi_h^{SB} = 1/2 \cdot (\rho x_h + (1-\rho)x_l)y_h - C(\rho) = \Pi_{\max} - 1/2 \cdot (\rho x_l + (1-\rho)x_h)y_l. \tag{1}$$

The opportunity cost of matching only high type agents is that the agency extracts no surplus from low type employers.

On the other hand, the agency can choose to serve all agents and to match them assortatively. Suppose that the fee schedule (p_l, p_h) is incentive compatible, so that a high (low) type employer chooses the high (low) type contract. In this case, the expected utilities of a high type and a low type employer are given by:

$$\begin{aligned} V_{hh}^e &= (\rho x_h + (1-\rho)x_l)y_h - p_h, \\ V_{ll}^e &= (\rho x_l + (1-\rho)x_h)y_l - p_l. \end{aligned}$$

A standard argument in contract theory then shows that the incentive compatible pair of fees that maximizes the agency's profits is such that: (i) the low type employers are left with zero surplus, and (ii) the high type employers' incentive constraint is binding. Formally, the agency sets $p_l = (\rho x_l + (1-\rho)x_h)y_l$ and $p_h^e = (\rho x_h + (1-\rho)x_l)y_h - (\rho x_l + (1-\rho)x_h)\Delta y$. Hence, it makes profits:

$$\Pi_{h+l}^{SB} = 1/2(p_h^e + p_l^e) - C(\rho) = \Pi_{\max} - 1/2 \cdot (\rho x_l + (1-\rho)x_h)\Delta y. \tag{2}$$

Put differently, the opportunity cost of serving all agents is the informational rent paid to high type employers.

This analysis shows that the agency chooses to serve all agents if the informational rent paid to high type employers is lower than the total surplus of low type employers. By equations (1) and (2), this arises if $(\rho x_l + (1-\rho)x_h)\Delta y \leq (\rho x_l + (1-\rho)x_h)y_l$, i.e. if $y_h/y_l \leq 2$.

Proposition 1 tells us that the monopoly outcome depends on the degree of heterogeneity between employers. If the population of employers is too heterogenous, namely, if y_h/y_l is too high, then, a monopoly agency has incentives to serve only the high type agents. On the other hand, if the difference between y_l and y_h is not too high, then, the market is fully covered. However, notice that in both cases, matched agents are matched efficiently given the screening technology. Put differently, most of the employers and workers are matched assortatively, but some of them are mismatched, i.e. some high (low) type employers are matched with low (high) type workers.

The number of mismatched agents depends on the screening technology chosen by the agency. For instance, if ρ were maximal ($\rho = 1$), there would be no mismatched agents. The agency chooses ρ to balance two effects: while the agency can extract more surplus from employers when ρ is high, because the total surplus from matches is higher when ρ is high, an efficient screening technology is costly. Suppose that the agency serves all employers. Then, a small increase in ρ raises (lowers) the probability for a high type employer to be matched with a high (low) type worker. In the same way, it raises (lowers) the probability for a low type employer to be matched with a low (high) type worker. Therefore, a small increase in ρ allows the agency to extract more (less) surplus from high (low) type employers. More precisely, following an increase $\delta\rho$ in ρ, the agency can extract an additional surplus $\delta\rho(x_h y_h - x_l y_h)$ from type y_h employers, while it has to lowers low type employers' subscription fees from $\delta\rho(x_h y_l - x_l y_l)$. The overall effect on agency's profits is thus $\delta\rho\Delta x\Delta y$. Therefore, when employers' and workers' populations are highly heterogenous, the agency has strong incentives to chose an efficient screening technology: when Δx or Δy are high, the marginal effect of an increase in ρ on agency's profits is high.

Proposition 1 is reminiscent of a standard price discrimination problem.[8] In a price discrimination problem, a monopoly firm controls the quality or quantity of a goods. In our model, the quality of the "good" traded by the principal (the agency) is the mean workers' type of the pools of workers to which the low and high "quality" contracts give an access.[9] Following standard arguments in the price discrimination literature, the agency should propose the efficient high quality to high type employers and a sub-optimal low quality to low type employers. However, unlike the standard price discrimination problem, the agency cannot choose the mean workers' type of each pool independently: if the agency puts more high signal workers in the high quality pool, then, this necessary lowers the quality of the low quality pool. Therefore, in order to offer a sub-optimal "quality" for low type employers, the agency has to lower the probability for a low type employer to get access to workers when he chooses the low type contract. If λ_l denotes this probability, then, the "quality" of the low type contract is $\lambda_l \hat{x}_l$, where $\hat{x}_l$ is the mean workers' type in the low type pool. Then, the agency chooses either the minimal ($\lambda_l = 0$) or maximal ($\lambda_l = 1$) quality. The reason why it does not choose an intermediate "quality" is that the problem is linear in λ_l.

4 Competing agencies

In this section, we analyze the outcome of competition between two agencies. In stage 1, agencies 1 and 2 set their fee schedules $P^k = (p_l^k, p_h^k)$ and their matching technologies

[8] Classical references in the price discrimination literature are Mussa and Rosen (1978) and Maskin and Riley (1984).

[9] In this interpretation, high type employers have higher willingness to pay for quality because they benefits more than low type employers from being matched with high type workers.

$M^k = (\rho^k, \mu^k(.))$, $k \in \{1, 2\}$. In stage 2, employers and workers choose which agency (if any) to register with. Thereafter, $P = (P^1, P^2)$ and $M = (M^1, M^2)$ denote the agencies' subscription fees and matching technology choices.

4.1 Equilibrium concept

Before defining our equilibrium concept, let us note that a subgame equilibrium is a distribution of agents $\mathcal{N} = (n_i^u(k), n_i^e(k, j))_{(i,j)\in\{l,h\}^2, k\in\{\emptyset,1,2\}}$, where $n_i^u(k)$ is the number of type i workers ($i \in \{l, h\}$) who join agency k ($k \in \{\emptyset, 1, 2\}$) and $n_i^e(k, j)$ is the number of type i employers who register with agency k and choose the contract j ($j \in \{l, h\}$). Here, an agent "joins" agency $\emptyset$ if he joins neither agency 1 nor agency 2. Thereafter, if $\mathcal{N}$ is the distribution of agents across the two agencies, the expected utility of a type i worker who subscribes in agency k is denoted by $V_i^u(k, P, M, \mathcal{N})$. Similarly, the expected utility of a type i employer who joins agency k and chooses the contract j is denoted by $V_{ij}^e(k, P, M, \mathcal{N})$. Last, we say that $\mathcal{N}$ is a *monopoly-like* distribution if all workers join the same agency and a *separating* distribution if type x_l and x_h workers join different agencies.

Definition 1. *A distribution of agents $\mathcal{N}$ is an equilibrium distribution for the system (M, P) if, for all $k \in \{\emptyset, 1, 2\}$ and all $j \in \{l, h\}$,*

$$\begin{aligned} n_i^u(k) > 0 \quad &\Rightarrow \quad V_i^u(k, P, M, \mathcal{N}) = \max\nolimits_{h\in\{\emptyset,1,2\}} V_i^u(h, P, M, \mathcal{N}), \\ n_i^e(k, j) > 0 \quad &\Rightarrow \quad V_{ij}^e(k, P, M, \mathcal{N}) = \max\nolimits_{h\in\{\emptyset,1,2\}, t\in\{l,h\}} V_{it}^e(h, P, M, \mathcal{N}). \end{aligned}$$

A market allocation is a mapping $\mathcal{N}(.)$ that associates to each (M, P) a pure equilibrium distribution of agents.

To grasp the idea, let us analyze the agents' incentives to deviate in an equilibrium distribution. Consider a distribution of agents across agencies $\mathcal{N}$ and suppose that it is an equilibrium distribution. Suppose also that $\mathcal{N}$ is such that some type i workers join agency k. The expected utility of these workers is given by $V_i^u(k, P, M, \mathcal{N})$. If one of them deviates from agency k to agency h, he obtains $V_i^u(h, P, M, \mathcal{N})$. Indeed, since we consider continuum of agents, the distribution of agents across agencies remains unchanged after he deviates. Therefore, a necessary condition for $\mathcal{N}$ to be an equilibrium is that $V_i^u(k, P, M, \mathcal{N}) \geq V_i^u(h, P, M, \mathcal{N})$, for all $h \neq k$. Similar conditions must hold for employers.

Definition 1 also introduces the notion of *market allocation*. Intuitively, a market allocation determines the demand of each type of agents for each agency. There can be multiple market allocations.

Definition 2. *An equilibrium is a triple $(M^*, P^*, \mathcal{N}(.))$, such that:*

(i) (M^*, P^*) *is a Nash equilibrium of the reduced-form game induced by* $\mathcal{N}(.)$ *with profits* $\Pi^k(M, P, \mathcal{N}(M, P))$*,*

(ii) $\mathcal{N}(.)$ *is a monotone market allocation,*

An equilibrium consists of a set of subscription fees and matching technologies chosen by agencies and of a system of demands that describes how employers and workers choose among them for all possible fees and matching technologies.

In the following, we study separating equilibria (section 4.2). Then, we show existence of monopoly-like equilibria and we claim that these equilibria can give rise to unrealistic situation. This leads us to propose a more restrictive definition of an equilibrium (section 4.3).

4.2 Separating equilibria

This section is devoted to the analysis of separating equilibria.

We first prove that the unrealistic situation where the high (low) type employers join the same agency as the low (high) type workers cannot arise in equilibrium.

Lemma 1. *There exists no separating equilibrium such that high type workers and employers join a different agency.*

Proof. See Appendix A.3. □

Intuitively, in a separating distribution were high type workers and employers join a different agency, either high type workers or high type employers would be willing to deviate.

It is worth noticing that a thorough characterization of the set of separating equilibria is rather complex. There are several reasons for this. The first one is basic: in our model, an equilibrium is a complex object and it is already a difficult task to describe one of them.

The second reason is more technical. Since we make no assumptions on how agents choose among agencies out of the equilibrium path, it is likely that many different situations can arise in equilibrium. To grasp the intuition, suppose that there exists a separating equilibrium in which all low type agents join the same agency, say agency 2, and each of them finds a partner. Denote by (M, P) agencies' matching technologies and prices in this equilibrium. Notice first this equilibrium can always be sustained by a "sharp" market allocation $\mathcal{N}(.)$ such that:[10]

- $\mathcal{N}(M, P)$ is a separating distribution of agents;

[10] As stated above, there may be other market allocations that sustain the matching technology an price system (M, P) in a separating equilibrium.

- Following any deviation by agency i ($i \in \{1, 2\}$), employers and workers coordinate on the equilibrium that yields zero market share for agency i, whenever possible.

Clearly, with such a market allocation, no matchmaker is willing to deviate since this would leave it with zero profits. Let $\mathcal{N}(.)$ be a "sharp" market allocation that sustains (M, P) in a separating equilibrium. By Lemma 1, since the $(M, P, \mathcal{N}(.))$ is a separating equilibrium, the high type agents are in agency 1. Suppose also that a given employer or worker, either in agency 1 or in agency 2, gets a strictly positive utility and that he is strictly better off in his agency than if he had registered to the other instead. Now, start from this situation and assume that agency 2's matching function is slightly modified: employers and workers in agency 2 now have a small but positive probability to remain unmatched. This defines a new system of subscription fees and matching functions. We claim that it is also an equilibrium of the competition game. Notice first that, since the previous equilibrium distribution is a strict equilibrium, it is also an equilibrium in our new setting. Then, a "sharp" market allocation can sustain the new system as an equilibrium. Although this other equilibrium seems unlikely to arise, it cannot be ruled out without being more specific on what happens out of the equilibrium path.

We now suppose that there exists separating equilibria and we prove that some high type employers and workers must remain unmatched.

Proposition 2. *Some high type employers and workers remain unmatched in a separating equilibrium.*

Proof. Assume the contrary, i.e. there exists a separating equilibrium such that all high type employers and workers are matched to one another. Without loss of generality, suppose that the high type and low type agents register with agency 1 and 2 respectively. Let ρ^i denote the quality of agency i's screening technology. Consider now what happens in agency 1 in equilibrium. This agency matches everyone. Looking into the details, this means that a worker is matched, whatever his signal. More precisely, agency 1 committed in stage 1 to a matching function such that, if it were in a situation in which a mass 0.5 of employers and masses $0.5\rho^1$ and $0.5(1 - \rho^1)$ of respectively high and low type workers subscribe to its services, then, it would propose a partner to everyone. In this case, a low type worker should find it profitable to deviate from agency 2 to agency 1. Indeed, if he joins agency 1, then, he is sure to be matched with a high type employer, even if he is likely to have a low signal. This is a strictly better option than what he can expect in agency 2, namely, being matched with a low type employer. This a contradiction. □

Proposition 2 gives some indications on what could be the matching function of the agency that serves high type agents in a separating equilibrium. Indeed, the previous situation fails to be an equilibrium because agency 1 has committed in stage 1 not to use its screening technology if it were in a situation in which a mass 0.5 of employers and masses $0.5\rho^1$ and $0.5(1-\rho^1)$ of respectively high and low type workers subscribe to its services. Arguably, this is not a reasonable – and realistic – commitment. If, for instance, it had instead committed not to match the workers who receive a low signal, then, this would have been sufficient to sustain a separating distribution of agents in equilibrium. Intuitively, suppose that agency 1 chooses this matching function and a quite efficient screening technology, say $\rho^1 = 3/4$. Then, a low type worker has a probability $1 - \rho^1 = 1/4$ to be matched in agency 1 and, therefore, he is probably better off staying in agency 2. On the other hand, a high type worker is matched with probability 3/4 with a high type employer. This is obviously worse than being matched with probability 1, but this may still be better than subscribing to agency 2 and being matched with a low type employer.

Inertia condition: in equilibrium, if one agency deviates and the original distribution of agents remains consistent with the deviation, then this configuration should be selected.

Proposition 3. *Suppose that the inertia condition applies. Then, in any separating equilibrium:*

- *Each low type worker is matched with a low type employer in an agency which randomly matches agents.*
- *There exists an upper bound $r(\rho) < 1$ for the proportion of matched high type employers and workers, where ρ is the high type agency's screening technology and*

$$r(\rho) = \begin{cases} 1 - \frac{1-\rho}{\rho}(1-\alpha_y) & \text{if } \rho \geq 1-\alpha_y, \\ \frac{\rho}{1-\rho}\alpha_y & \text{if } \rho < 1-\alpha_y. \end{cases}$$

Proof. See Appendix A.4. □

Proposition 3 predicts that, under competition, the equilibrium configuration exhibits endogenous differentiation between agencies: the high type agency chooses a selective screening technology and commits to propose employers candidates who are likely to be good, while the low type agency does not make any screening. The low type agency just matches employers and workers, whatever their preferences over candidates or job offers. Intuitively, it anticipates that it will serve only low type agents. Therefore, it has no incentives to invest in a costly screening technology, since selecting candidates carefully will not allow it to extract more surplus from low type employers.

Proposition 3 also states that competition between agencies create frictions in the matching process between employers and workers, i.e. unemployed workers and job vacancies coexist in equilibrium. The literature in labor economy usually represents labor market frictions by stochastically arriving matching opportunities: because workers and firms lack information, it takes time to find jobs or candidates.[11] More precisely, the number of matches is given by a matching function $M(.)$ which takes into arguments the number of unemployed workers U and vacancies V.[12] Labor market frictions are then captured by assuming that $M(U, V) < \inf\{U, V\}$, i.e. that strictly less matches occur than it is possible. Under this assumption, unemployed workers and job vacancies coexist in a stationary equilibrium because, in each period, jobs are created and destroyed. By contrast, our results show that frictions can arise in a static framework in which matching functions are not assumed ex-ante inefficient.

Proposition 4. *Let* $\rho^* = arg\max_\rho 1/2(r(\rho)x_h y_h - x_l \Delta y) - C(\rho)$.

Suppose the inertia condition applies. Then, a separating equilibrium exists iff:

(i) $1/2\,(r(\rho^*)x_h y_h - x_l\Delta y) - C(\rho^*) > 0$.

(ii) $r(\rho^*) \geq \alpha_x$.

Besides, in the maximum profits equilibrium, agencies' prices are given by:

$$p_l = x_l y_l,\;\; p_h = r(\rho^*)x_h y_h - x_l\Delta y,$$

where p_l *and* p_h *are the prices in the low and high type agencies respectively.*

Proof. See Appendix A.5. □

Intuitively, in a separating equilibrium, the high type agency chooses a quite efficient screening technology and recovers the sunk cost by charging high type employers a sufficiently high registration fee. Then, if the sunk cost $C(\rho)$ is high for low values of ρ, there may not exist separating equilibria. In other words, a necessary condition for existence of separating equilibria is that an agency can obtain a quite efficient screening technology at non-prohibitive cost. This the meaning of condition (i).

Proposition 4 shows that there exists a separating equilibrium in which the high type agency chooses a screening technology ρ *and matches the maximal number of high type agents. The logic at work is the following. High type workers agree on being matched with a strictly lower probability than low type workers in order to signal their type to high type employers. In equilibrium, low type workers are not willing to "mimic" high type workers because they are likely to fail the test in the high type agency and, therefore, to remain unmatched.*

[11] See, for instance, Pissarides (2000).

[12] In each period, a worker and an employer therefore find a partner with probabilities $M(U, V)/U$ and $M(U, V)/V$ respectively.

Propositions 3 and 4 also sheds light on the possible trade-off between monopoly and competition in matching markets. We have shown that, from an efficiency point of view, the outcome of competition may be ambiguous: some employers and workers are not matched in equilibrium, but matched agents are matched efficiently. This has to be compared with the monopoly outcome. As stated in Proposition 1, there are two different outcomes under monopoly, depending on y_h being larger or smaller than $2y_l$. On the one hand, if $y_h \leq 2y_l$, then, there is full market coverage: all agents find a partner and most of them are matched assortatively. The drawback is that, in this case, a monopoly agency matches some high (low) type workers with low (high) type employers because it imperfectly observes workers' types. On the other hand, if $y_h > 2y_l$, then, the market is only partially covered: each high type employer is matched with a worker who receives a high signal, but all other agents remain unmatched. Therefore, the trade-off between monopoly and competition actually depends on which of these two situations we fall into.

4.3 Monopoly-like equilibria

In this section, we prove existence of monopoly-like equilibria and we argue that these equilibria can give rise to unrealistic situation.

Let us first show that the monopoly outcome is always an equilibrium. Suppose that agency 1 chooses the monopoly strategy described in Proposition 1. Suppose also that agency 2 chooses some subscription fees and a matching technology such that $\rho^2 = 1/2$. These strategies can be made part of an equilibrium in which all agents join agency 1, and, thus, in which agency 1 gets the monopoly profits. Indeed, if after any deviation by agency 2, agents coordinate on a subgame equilibrium with zero market share for agency 2, whenever possible, agency 2 cannot prevent agency 1 from implementing its monopoly outcome. Intuitively, in stage 2, employers and workers anticipate that none of them will join agency 2 and, therefore, they all join agency 1.

We have shown that the monopoly outcome is an equilibrium of the competition game. Now, we claim that this can give rise to unrealistic situations in equilibrium. Consider the following situation. Suppose that $y_h > 2y_l$. Then, as pointed out in the previous section, the monopoly outcome is such that the high type employers are matched with the workers who get a high signal, while the other agents remain unmatched. This can also be the outcome of competition. Put differently, there exists an equilibrium of the competition game such that all agents join the same agency and half of them remain unmatched. Besides, this can arise in equilibrium even if the other agency offers to match everyone for free. It does not seem desirable to obtain this situation as the outcome of our model. Indeed, in this situation, it seems likely that the low type agents would join the other agency.

In order to rule out unrealistic equilibria, we propose a refinement of our concept of equilibrium. We then show that, with this refinement, monopoly-like distribution of agents can no longer be part of an equilibrium.

We require that, on the equilibrium path, the distribution of agents satisfies some stability notion.

Definition 3. *An equilibrium distribution $\mathcal{N}$ for the system (M, P) is stable if there exists some $\varepsilon' > 0$ such that, for any $\varepsilon < \varepsilon'$ and any numbers*

$$\begin{aligned} \{\varepsilon_i^u(k)\}_{i\in\{l,h\},k\in\{\emptyset,1,2\}} \quad &\textit{with} \quad 0 < \varepsilon_i^u(k) < \varepsilon, \\ \{\varepsilon_i^e(k,j)\}_{(i,j)\in\{l,h\}^2,k\in\{\emptyset,1,2\}} \quad &\textit{with} \quad 0 < \varepsilon_i^e(k,j) < \varepsilon, \end{aligned}$$

the game where a fraction $\varepsilon_i^u(k)$ of type i workers are constrained to join agency k and where a fraction $\varepsilon_i^e(k,j)$ of type i employers are constrained to choose the contract j in agency k, has an equilibrium distribution $\mathcal{N}^\varepsilon$ for the system (M, P) such that the unconstrained agents make the same participation choice than in $\mathcal{N}$.

Before motivating this definition, we introduce the following technical assumption.

Assumption 1. *Agencies choose matching functions $\mu(.)$ such that, for all sequences of distribution of agents $(\mathcal{N}[m])_{m\geq 0}$ such that:*

- *$(\mathcal{N}[m])_{m\geq 0}$ converges to 0;*
- *there exists $i \in \{e, u\}$ such that sequence $(n_l^i, n_h^i)_{m\geq 0}$ converges to 0 infinitely faster than $(n_l^j, n_h^j)_{m\geq 0}$, $j \neq i$,*

then, for m sufficiently large, $\mu(\mathcal{N}[m])$ is such that all agents in population i are matched.

In words, Assumption 1 states that, if dozens of workers join an agency where there is only one employer, then, this agency should propose a worker to the employer. At first sight, this assumption seems innocuous, but it will be important in the following.

Our notion of stability is motivated by the following lemma.

Lemma 2. *Under Assumption 1, monopoly-like equilibrium distributions are unstable, while strict separating equilibrium distributions are stable.*

Proof. See Appendix A.6. □

The immediate consequence of Lemma 2 is that, if we add the following requirement in Definition 2:

$$(S) \quad \mathcal{N}(M^*, p^*) \text{ is a stable distribution of agents,}$$

then, there exists no monopoly-like equilibria.

In order to understand the impact of the stability requirement in our model, we would like to give an intuitive overview of the proof of Lemma 2. Consider first a system of subscription fees and matching functions such that there exists a monopoly-like equilibrium distribution in stage 2. Suppose, for instance, that all agents join agency 1 in this equilibrium distribution. Suppose now that, for some reasons, a small proportion of each type of agents indeed join agency 2. Then, the agents remaining in agency 1 may want to reconsider their decisions to join agency 1 since they can now find a partner in agency 2. Yet, this may not be sufficient to make them leave agency 1. Indeed, if low type employers and workers are over-represented in agency 2, then, the other agents are probably better off staying in agency 1. Therefore, to make the story interesting, suppose instead that high type employers are over-represented in agency 2. Then, a worker in agency 1 is likely to find a better partner in agency 2. Since subscription is also free to workers in agency 2, he will therefore find it profitable to leave agency 1. This shows that the monopoly-like distribution is no longer an equilibrium if a small proportion of agents is constrained to join agency 2. In other words, this distribution is unstable.

Second, Lemma 2 states that strict separating equilibrium distributions are stable. To grasp the intuition, consider, for instance, the incentive of a high type worker to deviate from agency 1 to agency 2 if a small proportion of high type employers join agency 2. On the one hand, in agency 1, this worker is likely to be matched with a high type employer. On the other hand, if he deviates and join agency 2, then, in the best scenario, he could be matched with a high type employer if he obtained a high signal. However, in this case, he would face the competition of low type workers who also obtained a high signal. These workers are numerous since there are many low type workers in agency 2. Therefore, if he deviates, a high type worker has only a small probability to be matched with a high type employer in agency 2. In other words, he is strictly better off staying in agency 1.

We would like to make two comments on our definition of stable equilibrium distribution. First, our notion of stability is inspired by Kohlberg and Mertens (1986)'s (hereafter KM) notion of stable set of equilibria. Actually, our notion of stability is stronger than KM's one. In our framework, an equilibrium distribution $\mathcal{N}$ would be KM-stable if, for any sufficiently small "trembles" ε, there exists an equilibrium distribution $\mathcal{N}^{\varepsilon}$ of the perturbed game that is *close* to $\mathcal{N}$ in the space of strategies. Then, for $\mathcal{N}$ to be KM-stable, it is sufficient that, in the perturbed game, a sufficiently large proportion of unconstrained agents make the same participation choice than in $\mathcal{N}$. This definition does not require that, in $\mathcal{N}^{\varepsilon}$, unconstrained agents makes the same participation choice than in $\mathcal{N}$.

From a theoretical point view, it would be more satisfying to use KM's notion of stability.[13]

[13]Indeed, our notion of stability is not well-founded in the sense that, if we adapt it to a standard normal game, a stable equilibrium may not exist.

Unfortunately, KM's notion is untractable in our framework. The main reason is that, in order to apply KM's notion, we had to be to able to describe the set of equilibria in each perturbed game. In particular, this would require to describe matching functions in a neighborhood of the candidate equilibrium distribution. In general, this is not feasible.

Second, we think that our notion of stability makes sense in the labor market and, more generally, in any matching market. Suppose there are two agencies in the market, say an incumbent and an entrant. In this situation, it seems credible that most employers and workers will join the incumbent agency. However, it seems also credible that some agents will join the entrant agency, either because they lack information about the different matching channels or because they wrongly anticipate the other agents' search strategies. This should be taken into account by the first group of employers and workers – the better-informed agents. When choosing their agency, they should understand that they may find a good partner if they join the entrant agency. The stability requirement captures this idea. In Definition 3, one should think about the $\varepsilon_i^u(k)$ and $\varepsilon_i^e(k,j)$ as small populations of workers and employers who make their participation decisions while being poorly informed about all the options. Therefore, stability means that an equilibrium distribution has to be robust to some agents choosing "irrational" search strategies.

5 Extension to the monopoly case

This section discusses the impact on the monopoly outcome of allowing a monopoly agency to charge subscription on both sides of the market.

If it can charge both the employers and the workers, a monopoly agency is able to extract more surplus from workers. Therefore, it should have better incentives to implement the efficient matching than when subscription is free to workers. We prove that this intuition may be misleading.

The following proposition characterizes the monopoly outcome when both employers and workers pay for intermediation service.

Proposition 5. *If $\frac{x_h}{x_l} + \frac{y_h}{y_l} \leq 4$, then, a monopoly agency matches employers and workers assortatively: high (low) type employers are matched with high (low) type workers.*
If $\frac{x_h}{x_l} + \frac{y_h}{y_l} > 4$, then, a monopoly agency matches high type employers and workers together, while low type employers and workers remain unmatched.
In any case, the agency chooses the lowest quality of screening technology: $\rho = 1/2$.

Proof. See Appendix A.7. □

If the agency matches all agents assortatively, then, the total surplus is maximized. Yet, the agency cannot extract the whole surplus since it has to pay an informational rent both to the high type employers and workers. More precisely, as pointed out in section 3, high type employers and workers are respectively left with positive surpluses $x_l(y_h - x_l)$ and $(x_h - x_l)y_l$. In other words, the agency' profits are given by the total surplus from matches under PAM minus the cost of eliciting private information:

$$\Pi_{h+l} = x_h y_h + x_l y_l - 1/2 \cdot x_l(y_h - x_l) - 1/2 \cdot (x_h - x_l)y_l. \tag{3}$$

On the other hand, if the agency serves only the high type agents, these agents are left with zero surplus and the agency's profits are given by:

$$\Pi_h = x_h y_h. \tag{4}$$

Therefore, by equations (3) and (4), the agency chooses to serve all agents when, under PAM, the surplus of low type agents is greater than the cost of eliciting private information. This arises when $x_l y_l - 1/2 \cdot x_l(y_h - x_l) - 1/2 \cdot (x_h - x_l)y_l \geq 0$, i.e. when $\frac{x_h}{x_l} + \frac{y_h}{y_l} \leq 4$.

Proposition 5 states that a monopoly agency chooses the lowest quality of screening technology. Intuitively, the agency can screens workers through incentive compatible subscription fees without incurring sunk cost $C(\rho)$.

We now compare the two-sides-pay regime (TSP) to the one-side-pay regime (OSP). This comparison is done in two different ways: in terms of market coverage and in terms of total surplus.

As stated in Propositions 1 and 5 either all agents are matched or only half of the employers and workers are matched.[14] Thereafter, we say that there is full market coverage (FMC) when all agents are matched and that there is partial market coverage (PMC) when only half of the employers and workers are matched.

In both regimes, the market is fully covered when the cost of eliciting private information is not too high. Under the TSP regime, the market is fully covered when the sum of the informational rents paid to the high type employers and workers is not too high. This arises when $\frac{x_h}{x_l} + \frac{y_h}{y_l} \leq 4$ (see Proposition 5). On the other hand, under the OSP regime, the cost of eliciting private information is only the informational rent paid to the high type employers. Thus, full market coverage arises when $\frac{y_h}{y_l} \leq 2$ (see Proposition 1). It follows immediately that the market is more covered under OSP than under TSP when the cost of eliciting private information on the workers' side of the market is high, i.e. when the population of workers is quite heterogenous. Then again, if the cost of eliciting private information is high on the employers' side of the market and low on the workers' side, the market is more covered under

[14]Note that when only half of the employers and workers are matched, the unmatched workers under the two regimes. Indeed, under the OSP regime, fractions $1 - \rho$ of high type workers and ρ of low type workers remain unmatched, while, under TSP, only low type workers are left aside.

TSP. Figure 1 summarizes this discussion.

Proposition 6. *When the population of workers is quite heterogenous (homogeneous), the market is more (less) or equally covered when only employers pay than when both sides pay.*

Proof. Immediate. □

Exempting workers from payment may have strong impact on market coverage. This is beneficial when the populations of workers is more heterogenous than the population of employers. However, if the reverse holds, it has a strong negative impact: while the market is fully covered under TSP, an OSP monopoly agency serves only high type agents, therefore leaving all low type agents aside. Last, if the degrees of heterogeneity within the two populations is not too different, exempting workers from payment has no effect on market coverage. In particular, when both populations are too heterogeneous, then, low type agents are left aside under both regimes.

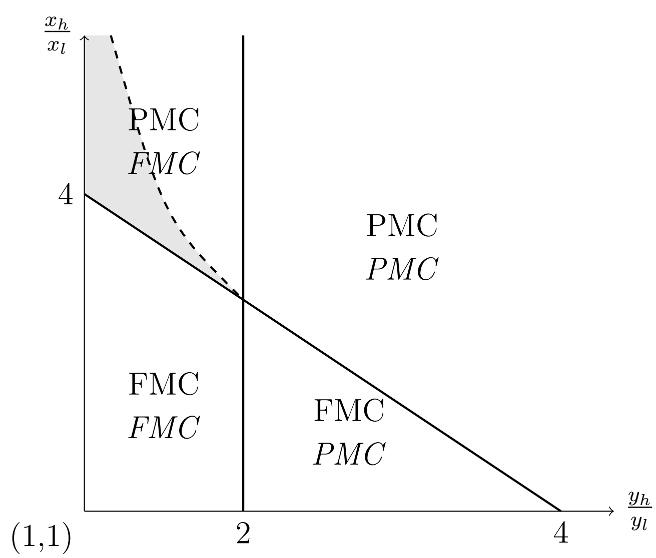

Figure 1: Market coverage when both sides pay (normal characters) and when only employers pay (*italic* characters). In the grey area, the total surplus is higher when only employers pay.

Our results have interesting implications on the design of procurement auctions for labor market services. For instance, when the market for job placement was liberalized in Australia, private employment agencies were asked to bid for providing intermediation services to pools of workers.[15] One objective was to deliver job services to all unemployed workers. The

[15] See OECD (2001).

authorities paid much attention to “sell” homogenous populations of workers.[16] Our results show that this “one-sided” logic may be misleading: the degree of heterogeneity of vacancies should also be taken into account.

We now compare the two regimes in terms of total surplus, which is defined as the sum of the agency's profits and of the employers' and workers' utilities. As pointed out above, the agency should have better incentives to maximize the total surplus when both sides pay. The comparison should therefore turn in favor of the TSP regime, but we find circumstances under which the total surplus is higher under the OSP regime.

Suppose first that the market is fully covered when both sides pay. In this case, employers and workers are matched assortatively in a TSP agency, so that the total surplus is maximized. Put differently, if the market is fully covered when both sides pay, then, the total surplus is lower under the OSP regime than under the TSP regime. Second, assume that the market is partially covered under both regimes. Then, the total surplus is higher under TSP. Indeed, although all high type employers are matched under both regimes, some high type workers remain unmatched under OSP because they get a low signal. In the end, there may be a trade-off between the two regimes only if the market coverage is higher when only employers pay, i.e. if $\frac{x_h}{x_l} + \frac{y_h}{y_l} > 4$ and $\frac{y_h}{y_l} \leq 2$.

Proposition 7. *If the market is more or equally covered under TSP than OSP, then the total surplus is higher under TSP.*
If the market coverage is higher under OSP than under TSP, then the total surplus is higher under OSP if populations of employers and workers are not too heterogeneous.

Proof. See Appendix A.8. □

Proposition 7 states that there exists some parameters such that both the market coverage and the total surplus are higher under the OSP regime. To grasp the intuition, consider a situation where x_h/x_l and y_h/y_l are such that the market is more covered under OSP than TSP. Denote by S_{OSP} and S_{TSP} the total surplus under OSP and TSP respectively. Straightforward calculations yield:

$$S_{OSP} - S_{TSP} = x_l y_l - (1-\rho)\Delta x \Delta y - C(\rho).$$

where ρ is such that $C'(\rho) = \Delta x \Delta y$. Hence, S_{OSP} is greater than S_{TSP} if the cost of mismatching some agents and the screening technology's cost are not too high. This arises when the employers' and workers' populations are not too heterogenous.

[16]Unemployed workers passed a series of tests, which aim was to rank them into four different groups. The results of these tests and others criterions (age, location, race, etc.) were then used to form homogenous pools of unemployed workers.

6 Discussion

We now discuss some extensions and robustness check.

More workers than employers. Let n^u denote the size of workers' population. We assume that low and high type agents are still equally represented in each population.

If there are more workers than employers, a monopoly agency has the same incentives to match agents efficiently since it tries to extract maximum surplus from employers. Using the same technique than in Proposition 1, we prove the following proposition.[17]

Proposition 8. *Assume that $n_u > 1$.*
If $y_h > 2y_l$, then, a monopoly agency serves only high type employers and these are matched with high signal workers. Besides, the agency's screening technology is given by $C'(\rho) = \frac{1}{2}\Delta x y_h$.
If $y_h \leq 2y_l$, then, a monopoly agency serves all agents and matches agents efficiently: high type employers are matched with high signal workers; low type employers are matched with high (low) signal workers with probability $\inf\{1, n^u - 1\}$ ($\max\{0, 2 - n^u\}$). Besides, the agency's screening technology is given by $C'(\rho) = \Delta x \Delta y + \Delta x(2y_l - y_h)\inf\{1, n^u - 1\} > \Delta x \Delta y$.

When the employers' population is too heterogeneous ($y_h > 2y_l$), the agency serves only the high type employers and chooses the same screening technology than in Proposition 1. When the employers' population is sufficiently homogeneous ($y_h \leq 2y_l$), the agency serves all agents and some low type employers are matched with high signal workers. More precisely, the high signal workers who are not matched with high type employers are matched with low type employers. The key consequence is that the agency has incentives to choose a more efficient screening technology than when employers' and workers' population had the same size. This stems from the fact that, in the case considered here, the agency can extract more surplus from low type employers when the screening technology is efficient, since low type employers are sometimes matched with high signal workers.[18] Notice also that, in this case, only workers with low signals remain unmatched.[19]

Under competition, there may exist separating equilibria in which all employers are matched. To grasp the idea, consider a situation where high type agents join agency 1, while low type agents join agency 2. Suppose that agency 1 chooses a screening technology ρ

[17] Proof available upon request.

[18] Notice that, if $n_u \geq 2$ and $y_h \leq 2y_l$, then, all employers are matched with high signal workers. In this case, low and high type employers pay the same subscription fee.

[19] When there are less workers than employers, we can prove the following results: if $y_h > 2y_l$, then, the agency serves only high type employers, all high signal workers are matched with high type employers and some low signal workers are matched with high type employers; if $y_h < 2y_l$, then, the agency serves all employers and agents are matched efficiently.

such that $\rho n^u > 1$. In words, in agency 1, the number of high type workers who successfully pass the test is higher than the number of high type employers. Besides, assume that agency 1 chooses to match only workers who obtain a high signal and that agency 2 matches all low type employers. Therefore, in agency 2, a low type worker is matched with probability $1/n^u$. If he deviates and join agency 1, then, he is matched with probability $(1-\rho)/n^u$ with a high type employer. Therefore, if $\rho > \Delta y/y_h$, low type workers are strictly better off in agency 2.[20] The above separating distribution of agents is thus an equilibrium if ρ is sufficiently high.

When there were as much employers and workers, a situation in which all high type employers were matched failed to be an equilibrium because some high type workers who obtained a low signal had to remain unmatched. But when there are more workers than employers, some workers must remain unmatched anyway. Therefore, the high type agency's commitment to leave unmatched some high type workers is innocuous. This is why there may exist separating equilibria in which all employers are matched.

In section 4, we argue that separating equilibria are the most credible outcome of competition. Yet, we think that it may no longer be the case when n^u is high. Suppose for instance that $n^u \geq 2$, so that in a separating equilibrium, at least half of the high type workers remain unmatched. In such a situation, it seems plausible that high type workers would search a job in both agencies. Indeed, since low type employers benefit from being matched with high type workers, the low type agency would probably propose to match them, since this would allow the agency to extract more surplus from low type employers. In the end, if n^u is high, we think that a credible outcome of competition could be a monopoly-like equilibrium if the population of employers is not too heterogenous, or an equilibrium in which low and high type employers join different agencies and workers join both agencies.[21]

More low type than high type agents. Consider now a situation where there are more low type agents than high type agents in each population. More precisely, let n_l and n_h denote the proportions of low and high type agents respectively and assume that $n_h < 1/2 < n_l$.[22] Denote by $\Delta n = n_l - n_h > 0$.

Again, the fact that there are more low type than high type agents should not change the incentives of a monopoly agency. We prove the following proposition.[23]

Proposition 9. *Assume that $n_h < 1/2 < n_l$.*
If $n_h y_h > y_l$, then, a monopoly agency serves only high type employers and these are matched

[20] A high type worker is not willing to deviate if $\rho/n^u y_h > 1/n^u y_l$, i.e. if $\rho > y_l/y_h$.

[21] Notice that, in order to describe such a situation, we would need a model to describe what happens when a worker finds a job in both agencies.

[22] We assume that the proportions of low and high type agents are the same in each population.

[23] Proof available upon request.

with high signal workers. Besides, the agency's screening technology is given by $C'(\rho) = n_h \Delta x y_h$.
If $n_h y_h \leq y_l$, then, a monopoly agency serves all agents and matches agents efficiently: high type employers are matched with high signal workers; low type employers are matched with high (low) signal workers with probability $(1-\rho)\frac{\Delta n}{n_l}$ $(1-(1-\rho)\frac{\Delta n}{n_l})$. Besides, the agency's screening technology is given by $C'(\rho)\frac{(\rho n_h+(1-\rho)n_l)^2}{n_h^2} = \Delta x \Delta y$.

Proposition 9 states that a monopoly agency is less likely to serve only high type employers when there are more low type than high type agents. Intuitively, in this case, this is less profitable to serve only high type employers since there are fewer of them.

When it serves all employers, some low type employers are matched with high signal workers. This stems from the fact that there are more workers who obtain a high signal than high type employers: $\rho n_h + (1-\rho)n_l > n_h$. Besides, in this case, the agency chooses a screening technology which is less efficient than when low and high type agents were equally represented.[24] Intuitively, the agency sets ρ in order to extract more surplus from high type employers. Yet, when $n_h < n_l$, some low type employers are matched with high signal workers. This raises the cost of eliciting employers' private information and, therefore, provides the agency with incentives to choose a lower quality of screening technology.

Under competition, we can derive similar results as in section 4: in a separating equilibrium, some high type agents must remain unmatched. However, existence of separating equilibria may require stronger conditions on function $C(.)$. Indeed, in a separating equilibrium, the "high type" agency must choose an efficient screening technology, i.e. it must pay a high fixed cost. But since there are fewer high type employers, the "high type" agency makes less profits, so that it may not be able to recover this fixed cost.

More than two types. So far we have assumed that there are only two types of agents in each population. A key implication of this assumption is that, under competition, matched agents are matched efficiently. Suppose instead that there are three types of agents: low, medium and high type agents.[25] In this case, we claim that there would be two types of separating equilibria: first, separating equilibria in which low and medium type agents join the same agency, while high type agents join the other; second, separating equilibria in which medium and high type agents join the same agency, while low type agents join the other.

[24]When $n_h = n_l$, the optimal screening technology solves $C'(\rho) = \Delta x \Delta y$. Since $C'(\rho)\frac{(\rho n_h+(1-\rho)n_l)^2}{n_h^2} > C'(\rho)$ when $n_h < n_l$, then, the agency chooses a less efficient screening technology when there are more low type than high type agents.

[25]Notice that the screening technology must be modified to account for more than two types. Suppose that there are three types of workers: $x_l < x_m < x_h$. For instance, we could assume that, when a type x worker joins an agency, the intermediary observes a signal $\sigma(x)$ such that: $Pr[\sigma(x) = x_i | x = x_i] = \rho$ and $Pr[\sigma(x) = x_j | x = x_i] = (1-\rho)/2$, where $(i,j) \in \{l, m, h\}^2$ and $j \neq i$.

Besides, in these equilibria, for the same reasons than in Lemma 2, some agents in the "high type" agency must remain unmatched. In the agency which serves two different types of agents, there is necessary some mismatches since screening technologies are imperfect. However, there are only mismatches between two types of agents, while, under monopoly, there would be mismatches between the three types.

7 Concluding remarks

In this paper, we build a model to shed light on the trade-off between monopoly and competition in matching markets in which one side is exempted from payment. We have shown than this trade-off can be formulated as follows: under monopoly, all agents are matched but some of them are mismatched; under competition, matched agents are matched efficiently, but some of them remain unmatched.

One of the main insight of the paper is that, under competition, some agents takes the risk of remaining unmatched in order to signal their type. It would be interesting to investigate whether this effect is also present in a dynamic version of our model. If there were a second spell of matchings, unmatched agents had another opportunity to find a partner. On the other hand, if "new" employers and workers arrive after the first spell of matchings, the "high type" agency may still have to commit to leave some agents unmatched in order to sustain a separating equilibrium. Analyzing a dynamic version of our framework is a challenging topic and is left for further research.

A Appendix

A.1 Definition of a matching function and agents' payoffs

Last, a matching function describes how an agency matches employers and workers. Suppose that some employers and workers join an agency. This population can be represented by a four-dimensional vector $\mathcal{N} = (n_i^j)_{i\in\{l,h\},j\in\{e,u\}}$, where n_i^j is the number of type i agents within the $j-$population. A vector $\mathcal{N}$ is called a *distribution of agents.* The agency does not observe $\mathcal{N}$. On the employers' side, it observes two populations of employers: those who pay the fee p_l and those who pay the fee p_h. Let $\hat{n}_l^e$ and $\hat{n}_h^e$ denote the size of the population of employers who choose to pay p_l and p_h respectively. On the workers' side, the screening technology splits workers into two groups, according to the signals observed by the agency. Let $\hat{n}_l^u$ and $\hat{n}_h^u$ denote the size of the population of workers who get the low and high signal respectively. For instance, $\hat{n}_l^u$ is the sum of the population of low type workers who get the low signal and of the population of high type workers who get the low signal. Formally:

$$\hat{n}_l^u = \rho n_l^u + (1-\rho) n_h^u,$$
$$\hat{n}_h^u = \rho n_h^u + (1-\rho) n_l^u.$$

In the end, the agency observes the vector $\hat{\mathcal{N}} = (\hat{n}_i^j)_{i\in\{l,h\},j\in\{e,u\}}$ and it knows in which of these four sub-populations each agent is. With a slight abuse of notations, we will sometimes refer to $\hat{n}_i^j$ as a population of agents instead of the size of this population. The agency can make four different types of matches: a worker in the population $\hat{n}_i^u$ ($i \in \{l,h\}$) can be matched either with an employer in $\hat{n}_l^e$ or in $\hat{n}_h^e$. Let $\hat{x}_i$ and $\hat{y}_i$ ($i \in \{l,h\}$) denote the mean "type" of a worker in $\hat{n}_i^u$ and of employer in $\hat{n}_i^e$ respectively.[26] A match between a worker in $\hat{n}_i^u$ and an employer in $\hat{n}_j^e$ is called a $(\hat{x}_i, \hat{y}_j)$-match. For any given $\hat{\mathcal{N}}$, we can define a *feasible set of matches*. Let $\psi(\hat{\mathcal{N}})$ denote the feasible set of matches for a distribution of agents $\hat{\mathcal{N}}$. An element of $\psi(\hat{\mathcal{N}})$ is a four-dimensional vector $(\eta(\hat{x}_i,\hat{y}_j))_{(i,j)\in\{l,h\}^2}$, where $\eta(\hat{x}_i,\hat{y}_j)$ is the number of $(\hat{x}_i,\hat{y}_j)$-matches. Notice that there cannot be more matches that involve workers $\hat{x}_i$ or employers $\hat{y}_j$ than the number of agents in their respective populations. Put differently, we must have $\eta(\hat{x}_i,\hat{y}_l) + \eta(\hat{x}_i,\hat{y}_h) \leq \hat{n}_i^u$ and $\eta(\hat{x}_l,\hat{y}_j) + \eta(\hat{x}_h,\hat{y}_j) \leq \hat{n}_j^e$, for all i and j. Therefore, $\psi(\hat{\mathcal{N}})$ can be formally defined by:

$$\psi(\hat{\mathcal{N}}) = \Big\{ (\eta(\hat{x}_i,\hat{y}_j))_{(i,j)\in\{l,h\}^2} \in [0,1/2]^4, \text{ such that, for all } (i,j) \in \{l,h\}^2 : \\ \eta(\hat{x}_i,\hat{y}_l) + \eta(\hat{x}_i,\hat{y}_h) \leq \hat{n}_i^u \text{ and } \eta(\hat{x}_l,\hat{y}_j) + \eta(\hat{x}_h,\hat{y}_j) \leq \hat{n}_j^e \Big\}$$

A matching function maps any distribution of agents $\hat{\mathcal{N}}$ into an element of $\psi(\hat{\mathcal{N}})$.

Agents' payoffs. Consider an agency which sets P, ρ and $\mu(.) \in \mathfrak{M}$. The pair $(\rho, \mu(.))$ is called a *matching technology*. If an employer joins this agency and chooses to pay the fee p_i ($i \in \{l,h\}$), we say that he chooses the *contract* $(p_i, \rho, \mu(.))$. Given that a distribution of agents $\mathcal{N}$ join the agency, the expected utility of a type i employer ($i \in \{l,h\}$) who chooses the contract j ($j \in \{l,h\}$) is given by:

$$V_{ij}^e(p_j,\rho,\mu(.),\mathcal{N}) = v_{ij}^e(\rho,\mu(.),\hat{\mathcal{N}}) - p_j = \left(\frac{\mu_{\hat{x}_l\hat{y}_j}(\hat{\mathcal{N}})}{\hat{n}_j^e}\hat{x}_l + \frac{\mu_{\hat{x}_h\hat{y}_j}(\hat{\mathcal{N}})}{\hat{n}_j^e}\hat{x}_h\right)y_i - p_j,$$

where $\hat{\mathcal{N}}$ is the distribution of agents observed by the agency and $v_{ij}^e(\rho,\mu(.),\hat{\mathcal{N}})$ is the expected surplus from match. On the other side of the market, the expected utility of a type i worker ($i \in \{l,h\}$) is given by:

$$V_i^u(\rho,\mu(.),\mathcal{N}) = \left\{ \rho\left(\frac{\mu_{\hat{x}_i\hat{y}_l}(\hat{\mathcal{N}})}{\hat{n}_i^u}\hat{y}_l + \frac{\mu_{\hat{x}_i\hat{y}_h}(\hat{\mathcal{N}})}{\hat{n}_i^u}\hat{y}_h\right) + (1-\rho)\left(\frac{\mu_{\hat{x}_{-i}\hat{y}_l}(\hat{\mathcal{N}})}{\hat{n}_{-i}^u}\hat{y}_l + \frac{\mu_{\hat{x}_{-i}\hat{y}_h}(\hat{\mathcal{N}})}{\hat{n}_{-i}^u}\hat{y}_h\right) \right\} x_i.$$

[26] For instance, $\hat{x}_h = \frac{\rho n_h^u}{\hat{n}_h^u} x_h + \frac{(1-\rho) n_l^u}{\hat{n}_h^u} x_l$.

Last, the expected profits of the agency writes:

$$\Pi(P,\rho,\mu(.),\mathcal{N}) = \hat{n}_l^e p_l + \hat{n}_h^e p_h - C(\rho).$$

A.2 Proof of Proposition 1

Proof. When all agents join the agency, the matching is completely described by a 4-uple $\Lambda = (\lambda_i^j)_{(i,j)\in\{l,h\}^2}$, where λ_i^j is the probability for an employer who chooses the contract i ($i \in \{l,h\}$) to be matched with a worker with signal j ($j \in \{l,h\}$). Denote by $P = (p_l, p_h)$ the vector of subscription fees. The agency offers incentive compatible contracts. To be coherent with our notations, we assume that the agency sets P, ρ and $\mu(.)$ such that type y_i employers ($i \in \{l,h\}$) choose the contract $(p_i, \rho, \mu(.))$. Therefore, the expected utilities of a high type and a low type employer are given by:

$$\begin{aligned} V_h^e &= (\lambda_h^h \hat{x}_h + \lambda_h^l \hat{x}_l) y_h - p_h, \\ V_l^e &= (\lambda_l^h \hat{x}_h + \lambda_l^l \hat{x}_l) y_l - p_l, \end{aligned}$$

where $\hat{x}_h = \rho x_h + (1-\rho)x_l$ and $\hat{x}_l = \rho x_l + (1-\rho)x_h$. Employers' participation constraints thus write $V_h^e \geq 0$ and $V_l^e \geq 0$. Let $(IR)_h^e$ and $(IR)_l^e$ denote these two constraints. Besides, employers' incentives constraints are given by:

$$\begin{aligned} (\lambda_h^h \hat{x}_h + \lambda_h^l \hat{x}_l) y_h - p_h^e &\geq (\lambda_l^h \hat{x}_h + \lambda_l^l \hat{x}_l) y_h - p_l^e, \quad (IC_h^e) \\ (\lambda_l^h \hat{x}_h + \lambda_l^l \hat{x}_l) y_l - p_l^e &\geq (\lambda_h^h \hat{x}_h + \lambda_h^l \hat{x}_l) y_l - p_h^e, \quad (IC_l^e) \end{aligned}$$

where (IC_i^e), $i = l,h$, denote the type y_i employers' incentive constraint. The last set of constraints is the feasibility constraints. For instance, the probability for a high type employer to be matched with a worker must lie between 0 and 1, i.e. $0 \leq \lambda_h^h + \lambda_h^l \leq 1$. Denote by $(FC)_h^e$ this constraint and define similarly the feasibility constraints $(FC)_l^e$, $(FC)_h^u$ and $(FC)_l^u$. The problem of the agency thus writes:

$$\begin{aligned} \max_{\Lambda, p_h, p_l, \rho} \Pi &= \tfrac{1}{2}(p_h + p_l) - C(\rho) \\ \text{s.t.} &\quad (IR_i^e), (IC_i^e), (FC)_i^e, (FC)_i^u,\ i \in \{l,h\}, \\ &\quad 0 \leq \lambda_i^j \leq 1,\ (i,j) \in \{l,h\}^2. \end{aligned}$$

The last four constraints state that the variables λ_i^j are probabilities and, then, must lie between 0 and 1.

In order to solve this program, we apply the standard methodology in contract theory. First, we solve the relaxed program without high type employers' participation constraint and low type employers' incentive constraint. Second, we show that a solution to this program also satisfies the omitted constraints.

In the relaxed program, the subscription fees are set so that constraints $(IR)_l^e$ and $(IC)_h^e$ are binding:

$$\begin{aligned} p_l &= (\lambda_l^h \hat{x}_h + \lambda_l^l \hat{x}_l) y_l, \\ p_h &= (\lambda_h^h \hat{x}_h + \lambda_h^l \hat{x}_l) y_h - (\lambda_l^h \hat{x}_h + \lambda_l^l \hat{x}_l) y_h + (\lambda_l^h \hat{x}_h + \lambda_l^l \hat{x}_l) y_l. \end{aligned}$$

Plugging these fees into the program of the agency, we obtain:[27]

$$\begin{aligned} \max_{\Lambda,\rho} \Pi &= \lambda_h^h \hat{x}_h y_h + \lambda_h^l \hat{x}_l y_l + \lambda_l^h \hat{x}_h (2y_l - y_h) \\ &\quad + \lambda_l^l \hat{x}_l (2y_l - y_h) - 2C(\rho) \\ \text{s.t.} &\quad \Lambda \text{ feasible.} \end{aligned}$$

For a given ρ, we solve this linear program with the simplex algorithm.[28] The algorithm is initialized at the vertex $\lambda_h^h = \lambda_h^l = \lambda_l^h = \lambda_l^l = 0$. Notice that the objective function is increasing in λ_h^h and that $\hat{x}_h y_h$ is greater than $\hat{x}_l y_l$, $\hat{x}_h(2y_l - y_h)$ and $\hat{x}_l(2y_l - y_h)$. Therefore, the best move is to raise λ_h^h up to 1. Put differently, we move from the starting vertex to the vertex $\lambda_h^h = 1$ and $\lambda_h^l = \lambda_l^h = \lambda_l^l = 0$. Notice that, once the algorithm has reached a vertex where $\lambda_h^h = 1$, the feasibility constraints imply that $\lambda_h^l = \lambda_l^h = 0$. In words, starting from a vertex where $\lambda_h^h = 1$, any move along the edges of the polyhedron of constraints such that either λ_h^l or λ_l^h increases strictly reduces the agency's profits. Therefore, we only have to consider moves such that λ_l^l increases from 0 to 1. Notice now that this move can raise agency's profits only if $2y_l - y_h \geq 0$. In the final analysis, the solution of the agency's problem is: $\lambda_h^h = 1$, $\lambda_h^l = \lambda_l^h = 0$ and, $\lambda_l^l = 1$ if $y_h \leq 2y_l$ and $\lambda_l^l = 0$ otherwise. Straightforward calculations then show that this solution also satisfies $(IR)_h^e$ and $(IC)_l^e$.

Last, we calculate the optimal screening technology. If $2y_l - y_h \geq 0$ the agency chooses ρ to maximize $\Pi = \hat{x}_h y_h + \hat{x}_l(2y_l - y_h) - 2C(\rho)$, i.e. ρ such that $C'(\rho) = (x_h - x_l)(y_h - y_l)$. On the other hand, if $2y_l - y_h < 0$, the agency chooses ρ to maximize $\Pi = \hat{x}_h y_h + -2C(\rho)$, i.e. ρ such that $C'(\rho) = \frac{1}{2}(x_h - x_l) y_h$.

□

A.3 Proof of Lemma 1

Proof. Assume there exists a separating equilibrium such that low (high) type workers and high (low) type employers join agency 1 (agency 2).

Let ρ^i denote agency i's screening technology, $i = 1, 2$. Let p^i denote the employers' subscription fee in agency i. Last, denote by λ_k^i the probability for a worker with signal x_k ($k \in \{l, h\}$) to be matched in agency i ($i \in \{1, 2\}$). Then, the expected utilities of a type x_l worker, a type y_h employer, a type x_h worker and a type y_l employer are given by:

$$\begin{aligned} V_l^u &= (\lambda_l^1 \rho^1 + \lambda_h^1 (1 - \rho^1)) \cdot x_l y_h, \\ V_h^e &= (\lambda_l^1 \rho^1 + \lambda_h^1 (1 - \rho^1)) \cdot x_l y_h - p^1, \\ V_h^u &= (\lambda_h^2 \rho^2 + \lambda_l^2 (1 - \rho^2)) \cdot x_h y_l, \\ V_l^u &= (\lambda_h^2 \rho^2 + \lambda_l^2 (1 - \rho^2)) \cdot x_h y_l - p^2. \end{aligned}$$

[27] Notice here that we have multiplied the objective of the agency by 2.

[28] Notice here that, the linear problem has not been converted into its augmented form. Strictly speaking, we should introduce stack variables to replace inequalities by equalities and, then, solve the new linear problem with the simplex algorithm. This would introduce dozens of new variables (one per inequality). Therefore, for expositional simplicity, we make a loose use of the simplex algorithm, keeping in mind that it is not completely rigorous.

Let $(IC)_i^j$ denote the incentive constraint of a type i ($i \in \{l,h\}$) agent in population j ($j \in \{e,u\}$). These constraints write:

$$\begin{array}{rcll}
(\lambda_l^1\rho^1 + \lambda_h^1(1-\rho^1))x_l y_h & \geq & (\lambda_h^2(1-\rho^2) + \lambda_l^2\rho^2)x_l y_l, & (IC)_l^u \\
(\lambda_l^1\rho^1 + \lambda_h^1(1-\rho^1))x_l y_h - p^1 & \geq & (\lambda_h^2\rho^2 + \lambda_l^2(1-\rho^2))x_h y_h - p^2, & (IC)_h^e \\
(\lambda_h^2\rho^2 + \lambda_l^2(1-\rho^2))x_h y_l & \geq & (\lambda_h^1\rho^1 + \lambda_l^1(1-\rho^1))x_h y_h, & (IC)_h^u \\
(\lambda_h^2\rho^2 + \lambda_l^2(1-\rho^2))x_h y_l - p^2 & \geq & (\lambda_l^1\rho^1 + \lambda_h^1(1-\rho^1))x_l y_l - p^1. & (IC)_l^e
\end{array}$$

The two employers' incentive constraints imply:

$$\begin{array}{rcl}
p^2 - p^1 & \geq & \left\{(\lambda_h^2\rho^2 + \lambda_l^2(1-\rho^2))x_h - (\lambda_l^1\rho^1 + \lambda_h^1(1-\rho^1))x_l\right\} y_h, \\
p^2 - p^1 & \leq & \left\{(\lambda_h^2\rho^2 + \lambda_l^2(1-\rho^2))x_h - (\lambda_l^1\rho^1 + \lambda_h^1(1-\rho^1))x_l\right\} y_l.
\end{array}$$

Let $\gamma = (\lambda_h^2\rho^2 + \lambda_l^2(1-\rho^2))x_h - (\lambda_l^1\rho^1 + \lambda_h^1(1-\rho^1))x_l$. Then, the two previous inequalities imply:

$$\gamma y_l \geq \gamma y_h.$$

Since $y_l < y_h$, we must have $\gamma < 0$, i.e.:

$$\frac{\lambda_h^2\rho^2 + \lambda_l^2(1-\rho^2)}{\lambda_l^1\rho^1 + \lambda_h^1(1-\rho^1)} < \frac{x_l}{x_h} < 1.$$

Notice now that $(IC)_h^u$ can be rewritten as:

$$\frac{\lambda_h^2\rho^2 + \lambda_l^2(1-\rho^2)}{\lambda_l^1\rho^1 + \lambda_h^1(1-\rho^1)} > \frac{y_h}{y_l} > 1.$$

This is a contradiction. □

A.4 Proof of Proposition 3

Proof. ******* proof to be completed *******

Assume there exists separating equilibria.

Let $(M, P, \mathcal{N}(.))$ a separating equilibrium in which, for instance, high (low) type agents join agency 1 (agency 2). Let λ_h^i and λ_l^i denote respectively the probability for a worker to be matched in agency i ($i \in \{1,2\}$) if it has a high and low signal. Let p_h and p_l denote the prices paid by employers in agency 1 and 2 respectively. Last, ρ^1 and ρ^2 are agency 1 and 2's screening technologies.

Since $(M, P, \mathcal{N}(.))$ is a separating equilibrium, we have:

$$\begin{aligned}
&(\rho^1\lambda_h^1 + (1-\rho^1)\lambda_l^1)x_h y_h - p_h \geq (\rho^2\lambda_l^2 + (1-\rho^2)\lambda_h^2)x_l y_h - p_l, && (IC)_h^e \\
&(\rho^2\lambda_l^2 + (1-\rho^2)\lambda_h^2)x_l y_l - p_l \geq (\rho^1\lambda_h^1 + (1-\rho^1)\lambda_l^1)x_h y_l - p_h, && (IC)_l^e \\
&(\rho^1\lambda_h^1 + (1-\rho^1)\lambda_l^1)x_h y_h \geq (\rho^2\lambda_h^2 + (1-\rho^2)\lambda_l^2)x_h y_l, && (IC)_h^u \\
&(\rho^2\lambda_l^2 + (1-\rho^2)\lambda_h^2)x_l y_l \geq (\rho^1\lambda_l^1 + (1-\rho^1)\lambda_h^1)x_l y_h, && (IC)_l^u \\
&(\rho^1\lambda_h^1 + (1-\rho^1)\lambda_l^1)x_h y_h - p_h \geq 0, && (IR)_h^e \\
&(\rho^2\lambda_l^2 + (1-\rho^2)\lambda_h^2)x_l y_l - p_l \geq 0, && (IR)_l^e
\end{aligned}$$

+ feasibility constraints: $0 \leq \lambda_j^i \leq 1,$ for all $(i,j) \in \{1,2\} \times \{l,h\}$.

Step 1: change of variables. Let

$$\mu_l^i = \lambda_l^i\rho^i + \lambda_h^i(1-\rho^i),\ \mu_h^i = \lambda_h^i\rho^i + \lambda_l^i(1-\rho^i).$$

The feasibility constraints then rewrites:

$$\begin{aligned}
0 \leq \rho^i\mu_h^i - (1-\rho^i)\mu_l^i \leq 2\rho^i - 1, \\
0 \leq \rho^i\mu_l^i - (1-\rho^i)\mu_h^i \leq 2\rho^i - 1.
\end{aligned}$$

Let $\mu^i = (\mu_l^i, \mu_h^i)$ and denote by $\mathcal{E}(\rho^i)$ the set of feasible μ^i.

******* Insert picture here *******

Step 2: Prove that $\mu^i \in Fr(\mathcal{E}(\rho^i))$. Proof by contradiction. If not, then agency i could profitably decrease ρ^i to $\hat{\rho}^i$ so that $\mu^i \in Fr(\mathcal{E}(\hat{\rho}^i))$. Deviation makes sense under the inertia condition.

Step 3: Prove that $\mu^2 = \hat{\mu}^2 = (1,1)$ and $\mu^1 = \hat{\mu}^1 = (\alpha_y, \min\{\frac{2\rho^1-1}{\rho^1}\alpha_y, \frac{1-\rho^1}{\rho^1}\alpha_y\})$. Proof by contradiction.

- If $\mu^1 \neq \hat{\mu}^1$, prove that, under inertia condition, agency 1 has a profitable deviation.
- If $\mu^1 = \hat{\mu}^1$ and $\mu^2 \neq \hat{\mu}^2$. Prove that, under inertia condition, agency 2 has a profitable deviation along the frontier of $\mathcal{E}(\rho^2)$.

Step 4: Since $\mu^2 = (1,1)$, $\mathcal{E}(\rho^2) = \mathcal{E}(1/2)$ for all ρ^2. Therefore, under inertia condition, agency is better off choosing $\rho^2 = 1/2$.

□

A.5 Proof of Proposition 4

Proof. ******* proof to be completed *******

Step 1: Characterize agency 1's screening technology and prices in the maximum profits equilibrium.

Step 2: Define out-of-equilibrium distribution of users in the candidate separating equilibrium.

Step 3: Check that this is an equilibrium and that agencies' profits are positive under condition (i) and (ii).

□

A.6 Proof of Lemma 2

Proof. Let (M, P) be agencies' strategies such that, in stage 2, there exists a monopoly-like equilibrium distribution, denoted by $\mathcal{N}$. Without loss of generality, suppose $\mathcal{N}$ is such that all agents join agency 1. Let V_i^u denote the expected utility of a type i worker ($i \in \{l, h\}$) in agency 1. If agents were matched assortatively in agency 1, then, the expected utility of a high type worker would be $(\rho^1 y_h + (1-\rho^1)y_l)x_h$. Since this is the best situation for this worker, we thus have $V_h^u \leq (\rho^1 y_h + (1-\rho^1)y_l)x_h$.

Consider now a sequence of trembles

$$\varepsilon = \left\{(\varepsilon_i^u(k)[m])_{i\in\{l,h\},k\in\{\emptyset,1,2\}}, (\varepsilon_i^e(k,j)[m])_{(i,j)\in\{l,h\}^2,k\in\{\emptyset,1,2\}}\right\}_{m\geq 0}.$$

Assume that ε converges to 0. Suppose also that the two sub-sequences $\{\varepsilon_h^e(2,j)[m]\}_{m\geq 0}$, $j \in \{l,h\}$, converges to 0 infinitely slower than each other sub-sequences. Besides, suppose that the two sub-sequences $\{\varepsilon_l^e(2,j)[m]\}_{m\geq 0}$, $j \in \{l,h\}$, converges to 0 infinitely faster than each other sub-sequences. Let Γ^ε denote the sequence of perturbed game associated to the sequence of trembles ε. By construction, for m sufficiently large, an unconstrained high type worker can profitably deviate from agency 1 to agency 2. Indeed, under Assumption 1, if he deviates, then, he is almost certain to be matched with a high type employer in agency 2, in which case he gets an expected utility $x_h y_h > (\rho^1 y_h + (1-\rho^1)y_l)x_h \geq V_h^u$. Therefore, for m sufficiently large, the game $\Gamma^\varepsilon[m]$ has no equilibrium distribution for the system (M, P) such that all unconstrained agents make the same choice than in $\mathcal{N}$. Put differently, the monopoly-like distribution $\mathcal{N}$ is unstable.

Let us now prove that a strict separating equilibrium distribution of agents is stable. Let $\mathcal{N}$ denotes such a distribution for the matching technology and price system (M, P). Assume that high and low type agents join agency 1 and 2 respectively. Let λ_i^j denote the probability that a type i worker who obtains signal x_j is matched. Thus, in equilibrium, agents' payoffs are given by:

$$\begin{aligned}
V_l^u &= (\rho^2\lambda_l^l + (1-\rho^2)\lambda_l^h)x_l y_l, \\
V_l^e &= (\rho^2\lambda_l^l + (1-\rho^2)\lambda_l^h)x_l y_l - p_l^2, \\
V_h^u &= (\rho^1\lambda_h^l + (1-\rho^1)\lambda_h^h)x_h y_h, \\
V_h^e &= (\rho^1\lambda_h^l + (1-\rho^1)\lambda_h^h)x_h y_h - p_h^1.
\end{aligned}$$

Consider now a sequence of trembles ε that converges to 0. Let Γ^{ε} denote the sequence of perturbed game associated to the sequence of trembles ε. For all integers m, denote by $\mathcal{N}[m]$ the distribution of agents such that the unconstrained agents make the same participation choice than in $\mathcal{N}$.

Let us prove that, if m is sufficiently large, $\mathcal{N}[m]$ is an equilibrium of the game $\Gamma^{\varepsilon}[m]$. Let m be an integer such that $|\varepsilon|$ is close to 0. Assume that unconstrained agents make the same participation choice than in $\mathcal{N}$. Let $\eta_l^1[m]$ denote the ratio between the population sizes of low type employers and high signal workers in agency 1:

$$\eta_l^1[m] = \frac{\varepsilon_l^e(1,h)[m] + \varepsilon_l^e(1,l)[m]}{\rho^1(1/2 - \varepsilon_h^u(2)[m] - \varepsilon_h^u(\emptyset)[m]) + (1-\rho^1)\varepsilon_l^u(1)[m]}$$

Then, a lower bound for an unconstrained high type worker's utilities is given by:

$$(1-\eta_l^1[m])(\rho^1\lambda_h^l + (1-\rho^1)\lambda_h^h)x_h y_h + \eta_l^1[m]x_h y_l.$$

Now, let $\eta_h^1[m]$ denote the ratio between the population sizes of high type employers and high signal workers in agency 2:

$$\eta_h^2[m] = \frac{\varepsilon_h^e(2,l)[m] + \varepsilon_h^e(2,h)[m]}{(1/2-\rho^2)(1-\varepsilon_l^u(1)[m] - \varepsilon_l^u(\emptyset)[m]) + \rho^1\varepsilon_h^u(2)[m]}.$$

Then, an upper bound for an unconstrained high type worker's utilities if he deviates from agency 1 to 2 is given by:

$$(1-\eta_h^2[m])(\rho^2\lambda_h^l + (1-\rho^2)\lambda_l^l)x_h y_l + \eta_h^2[m]x_h y_h.$$

Therefore, a sufficient condition for an unconstrained high type worker to be strictly better off staying in agency 1 is:

$$(1-\eta_l^1[m])(\rho^1\lambda_h^l+(1-\rho^1)\lambda_h^h)x_h y_h+\eta_l^1[m]x_h y_l \geq (1-\eta_h^2[m])(\rho^2\lambda_h^l+(1-\rho^2)\lambda_l^l)x_h y_l+\eta_h^2[m]x_h y_h. \quad (5)$$

Now, notice that $(\rho^1\lambda_h^l + (1-\rho^1)\lambda_h^h)x_h y_h > (\rho^2\lambda_h^l + (1-\rho^2)\lambda_l^l)x_h y_l$ since $\mathcal{N}$ is a strict separating equilibrium distribution. Then, notice than both $\eta_l^1[m]$ and $\eta_h^1[m]$ converges to 0. These two observations together show that there exists an integer m_h^u such that, for all $m \geq m_h^u$, inequality (5) is satisfied, i.e. unconstrained high type workers have no incentives to deviate in $\mathcal{N}[m]$.

By the same reasoning, define integers m_l^u, m_h^e and m_l^e. Denote by $M = \sup\{m_i^k | i \in \{l,h\}, k \in \{e,u\}\}$. Then, by construction, for all $m \geq M$, $\mathcal{N}[m]$ is an equilibrium distribution of the game $\Gamma^{\varepsilon}[m]$. □

A.7 Proof of Proposition 5

Proof. When all agents join the agency, the match makes is fully described by a 8-uple $\Lambda = (\lambda_{ij}^k)_{(i,j,k)\in\{l,h\}^3}$, where λ_{ij}^k denotes the probability for a worker who chooses the contract $(p_i^u, \mu(.))$ and gets the signal j to be matched with an employer who chooses the contract k. Denote by P

the vector of prices $(p_i^j)_{(i,j)\in\{l,h\}\times\{e,u\}}$. The agency offers incentive compatible contracts. In order to be coherent with our notations, we assume that the agency sets the menu of contracts so that high type and low type agents choose the contract with the lower subscript h and l respectively. Let $v_i^j(\Lambda,\rho)$ the expected surplus from match of an agent j ($j\in\{e,u\}$) of type i ($\in\{l,h\}$):

$$\begin{aligned} v_h^u(\Lambda,\rho) &= \left\{\rho\left(\lambda_{hh}^h y_h+\lambda_{hh}^l y_l\right)+(1-\rho)\left(\lambda_{hl}^h y_h+\lambda_{hl}^l y_l\right)\right\}x_h, &\text{(6a)}\\ v_l^u(\Lambda,\rho) &= \left\{\rho\left(\lambda_{ll}^h y_h+\lambda_{ll}^l y_l\right)+(1-\rho)\left(\lambda_{lh}^h y_h+\lambda_{lh}^l y_l\right)\right\}x_l, &\text{(6b)}\\ v_h^e(\Lambda,\rho) &= \left\{\left(\rho\lambda_{hh}^h+(1-\rho)\lambda_{hl}^h\right)x_h+\left(\rho\lambda_{ll}^h+(1-\rho)\lambda_{lh}^h\right)x_l\right\}y_h, &\text{(6c)}\\ v_l^e(\Lambda,\rho) &= \left\{\left(\rho\lambda_{hh}^l+(1-\rho)\lambda_{hl}^l\right)x_h+\left(\rho\lambda_{ll}^l+(1-\rho)\lambda_{lh}^l\right)x_l\right\}y_l. &\text{(6d)}\end{aligned}$$

Agents' participation constraints thus write $v_i^j(\Lambda,\rho)\geq p_i^j$. Let us now write the agents' incentive constraints. For instance, the high type workers' incentive constraint is given by:

$$\begin{aligned} &\left\{\rho\left(\lambda_{hh}^h y_h+\lambda_{hh}^l y_l\right)+(1-\rho)\left(\lambda_{hl}^h y_h+\lambda_{hl}^l y_l\right)\right\}x_h-p_h^u\\ &\geq\left\{(1-\rho)\left(\lambda_{ll}^h y_h+\lambda_{ll}^l y_l\right)+\rho\left(\lambda_{lh}^h y_h+\lambda_{lh}^l y_l\right)\right\}x_h-p_l^u.\end{aligned} \tag{7}$$

The incentive constraints of the other type of agents write similarly. Let $(IR)_i^j$ and $(IC)_i^j$ denote respectively the participation and incentive constraints of of an agent j ($j\in\{e,u\}$) of type i ($\in\{l,h\}$). The last set of constraints are the feasibility constraints. For instance, the probability for a high type worker to be matched with an employer has to lie between 0 and 1, i.e. $0\leq\rho\left(\lambda_{hh}^h+\lambda_{hh}^l\right)+(1-\rho)\left(\lambda_{hl}^h+\lambda_{hl}^l\right)\leq 1$ by equation (6a). Denote by $(FC)_h^u$ this constraint and define similarly the feasibility constraints $(FC)_l^u$, $(FC)_h^e$ and $(FC)_l^e$. The program of the agency then writes:

$$\begin{aligned} \max_{\Lambda,P,\rho}\Pi &= \tfrac{1}{2}\textstyle\sum_{(i,j)\in\{l,h\}\times\{e,u\}}p_i^j-C(\rho)\\ \text{s.t.}&\ \ (IR)_i^j,\ (IC)_i^j,\ (FC)_i^j,\ (i,j)\in\{l,h\}\times\{e,u\},\\ &\ \ 0\leq\lambda_{ij}^k\leq 1,\ (i,j,k)\in\{l,h\}^3.\end{aligned}$$

The last set of eight constraints state that the variables λ_{ij}^k ($(i,j,k)\in\{l,h\}^3$) are probabilities. Thus, they should lie between 0 and 1.

Let $\mu_i^j=\rho\lambda_{ii}^j+(1-\rho)\lambda_{i(-i)}^j$ denote the probability for a worker who chooses the contract i ($i\in\{l,h\}$) to be matched with an employer of type j ($j\in\{l,h\}$). Notice that the agents' participation and incentive constraints and the feasibility constraints can be simply rewritten with respect to this new variables. For instance, the high type workers' incentive constraint (see equation (7)) writes:

$$\left(\mu_h^h y_h+\mu_h^l y_l\right)x_h-p_h^u\geq\left(\mu_l^h y_h+\mu_l^l y_l\right)x_h-p_l^u.$$

Let Λ' denote the 8 dimensional vector $(\mu_i^j,\lambda_{ii}^j)_{(i,j)\in\{l,h\}^2}$. We make the following change of variables in the program of the agency:[29]

$$(\Lambda,P,\rho)\to(\Lambda',P,\rho).$$

[29]This is without loss of generality. Once Λ' is chosen, the variable $\lambda_{i(-i)}^j$ in Λ is given by $\frac{1}{1-\rho}\left(\mu_i^j-\rho\lambda_{ii}^j\right)$.

Given this change of variables, the problem of the agency rewrites:

$$\begin{array}{rcl} \max_{\Lambda', P, \rho} \Pi & = & \frac{1}{2}\sum_{(i,j)\in\{l,h\}\times\{e,u\}} p_i^j - C(\rho) \\ & \text{s.t.} & (IR)_i^j,\ (IC)_i^j,\ (FC)_i^j,\ (i,j)\in\{l,h\}\times\{e,u\}, \\ & & 0 \le \lambda_{ii}^j \le 1,\ (i,j)\in\{l,h\}^2, \\ & & 0 \le \frac{1}{1-\rho}\left(\mu_i^j - \rho\lambda_{ii}^j\right) \le 1,\ (i,j)\in\{l,h\}^2. \end{array}$$

In order to solve this program, we apply the standard methodology in contract theory. First, we solve the relaxed program without the participation constraints of high type agents and without the incentive constraints of low type agents. Second, we show that this solution satisfies the constraints omitted in the relaxed program.

In the relaxed program, the fees are set so that the constraints (IR_l^u), (IR_l^e), (IC_h^u) and (IC_h^e) are binding. Formally, the vector P is given by:

$$\begin{array}{rcl} p_l^u & = & (\mu_l^h y_h + \mu_l^l y_l)x_l, \\ p_l^e & = & (\mu_h^l x_h + \mu_l^l x_l)y_l, \\ p_h^u & = & (\mu_h^h y_h + \mu_h^l y_l)x_h - (\mu_l^h y_h + \mu_l^l y_l)x_h + (\mu_l^h y_h + \mu_l^l y_l)x_l, \\ p_h^e & = & (\mu_h^h x_h + \mu_l^h x_l)y_h - (\mu_h^l x_h + \mu_l^l x_l)y_h + (\mu_h^l x_h + \mu_l^l x_l)y_l. \end{array}$$

Plugging these fees into the program of the agency and rearranging terms, we obtain:[30]

$$\begin{array}{rcl} \max_{\Lambda',\rho} & & \mu_h^h \cdot 2x_h y_h \\ & + & \mu_h^l \cdot (3x_h y_l - x_h y_h) \\ & + & \mu_l^h \cdot (3x_l y_h - x_h y_h) \\ & + & \mu_l^l \cdot (4x_l y_l - x_h y_l - x_l y_h) - 2C(\rho) \\ & \text{s.t.} & (FC)_i^j,\ (i,j)\in\{l,h\}\times\{e,u\}, \\ & & 0 \le \lambda_{ii}^j \le 1,\ (i,j)\in\{l,h\}^2, \\ & & 0 \le \frac{1}{1-\rho}\left(\mu_i^j - \rho\lambda_{ii}^j\right) \le 0,\ (i,j)\in\{l,h\}^2. \end{array} \tag{8}$$

For a given ρ, we solve this linear program with the simplex algorithm.[31] First, we initialize the algorithm at the vertex $\mu_h^h = \mu_h^l = \mu_l^h = \mu_l^l = 0$. Notice that the objective is increasing in μ_h^h and that $2x_h y_h$ is greater than $3x_h y_l - x_h y_h$, $3x_l y_h - x_h y_h$ and $4x_l y_l - x_h y_l - x_l y_h$. Therefore, the best move is to increase μ_h^h up to 1. Put differently, we move from the starting vertex to the vertex $\mu_h^h = 1$, $\mu_h^l = \mu_l^h = \mu_l^l = 0$. Notice now that, once the algorithm has reached a vertex where $\mu_h^h = 1$, the feasibility constraints imply that $\mu_h^l = \mu_l^h = 0$. In words, starting from a vertex where $\mu_h^h = 1$, any move along the edges of the polyhedron of constraints such that either μ_h^l or μ_l^h increases strictly reduces the agency's profits. Therefore, we only have to consider moves such that μ_l^l increases from 0 to 1. Notice that this move can raise agency's profits only if $4x_l y_l - x_h y_l - x_l y_h \ge 0$. In the final analysis, the solution of the agency's problem is: $\mu_h^h = 1$, $\mu_h^l = \mu_l^h = 0$, and $\mu_l^l = 1$ if

[30] Notice here that we have multiplied the objective of the agency by 2.

[31] Same remark as in footnote 28.

$4x_ly_l - x_hy_l - x_ly_h \geq 0$, and $\mu_l^l = 0$ otherwise.

Now that we have characterized the optimal μ_i^j, we deduce the optimal λ_{ij}^k. If $4x_ly_l - x_hy_l - x_ly_h \geq 0$, then, as stated above, $\mu_h^h = \mu_l^l = 1$ and $\mu_h^l = \mu_l^h = 0$. Feasibility constraints implies that $0 \leq \frac{1}{1-\rho}\left(\mu_i^j - \rho\lambda_{ii}^j\right) \leq 1$, for all i and j. Therefore, it is immediate that $\lambda_{ii}^j = 1$ if $\mu_i^j = 1$, and that $\lambda_{ii}^j = 0$ if $\mu_i^j = 0$. The change of variables $\Lambda' \rightarrow \Lambda$ then immediately yields the parameters $\lambda_{i(-i)}^j$: $\lambda_{i(-i)}^j = 1$ if $\mu_i^j = 1$, and $\lambda_{i(-i)}^j = 0$ if $\mu_i^j = 0$.

We now prove that this solution alos satisfies the omitted constraints. Consider for instance the low type workers' incentive constraint. If $4x_ly_l - x_hy_l - x_ly_h \geq 0$, a low type worker get 0 if he chooses the contract $(p_l^u, \mu(.))$ and $x_ly_h - p_h^u = x_ly_h - x_hy_h + y_l(x_h - x_u) = -\Delta x \Delta y < 0$ if he deviates. This proves that $(IC)_l^u$ is satisfied. Similarly, the other constraints are also satisfied. If $4x_ly_l - x_hy_l - x_ly_h < 0$, the omitted constraints are trivially satisfied.

Last, we calculate the optimal screening technology. Notice that, once the optimal λ_{ij}^k are determined, the variable ρ only appears in the sunk cost $C(\rho)$ in the agency's profits. Indeed, suppose for instance that $4x_ly_l - x_hy_l - x_ly_h \geq 0$. Then, the optimal screening technology solves $\max_\rho x_hy_h + 1/2 \cdot (4x_ly_l - x_hy_l - x_ly_h) - C(\rho)$. Therefore, the agency chooses the lowest screening technology's quality: $\rho = 1/2$.

□

A.8 Proof of Proposition 7

Proof. Let $\alpha^u = \frac{x_h}{x_l}$ and $\alpha^e = \frac{y_h}{y_l}$. Let S_{TSP} and S_{OSP} denote respectively the total surplus when both sides pay and when subscription is free to workers.

Suppose that $\alpha^u + \alpha^e > 4$ and $\alpha^e \leq 2$. Then S^{e+u} and S^e are given by:

$$\begin{aligned} S_{TSP} &= x_hy_h, \\ S_{OSP} &= x_hy_l + x_ly_h + \rho^*\Delta x\Delta y - C(\rho^*), \end{aligned}$$

where ρ^* is such that $C'(\rho^*) = \Delta x \Delta y$. The difference between S_{TSP} and S_{OSP} is therefore given by:

$$S^e - S^{e+u} = x_hy_l + x_ly_h - x_hy_h + \rho^*\Delta x\Delta y - C(\rho^*).$$

Notice first that $\rho^*\Delta x\Delta y - C(\rho^*)$ is positive since $\rho^* = \arg\max_\rho \rho\Delta x\Delta y - C(\rho)$ and $\rho\Delta x\Delta y - C(\rho)$ is positive for $\rho = \frac{1}{2}$. A sufficient condition for $S_{OSP} - S_{TSP} > 0$ is therefore $x_hy_l + x_ly_h - x_hy_h > 0$, or equivalently $\frac{1}{\alpha^u} + \frac{1}{\alpha^e} > 1$. □

References

Armstrong, M. (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37(3), 668–691.

Atakan, A. E. (2006): "Assortative Matching with Explicit Search Costs," *Econometrica*, 74(3), 667–680.

Caillaud, B., and B. Jullien (2003): "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 34(2), 309–28.

Damiano, E., and H. Li (2007): "Price discrimination and efficient matching," *Economic Theory*, 30(2), 243–263.

——— (2008): "Competing Matchmaking," *Journal of the European Economic Association*, 6(4), 789–818.

Hoppe, H. C., B. Moldovanu, and E. Ozdenoren (2007): "Coarse Matching and Price Discrimination," CEPR Discussion Papers 6041, C.E.P.R. Discussion Papers.

Hoppe, H. C., B. Moldovanu, and A. Sela (2008): "The Theory of Assortative Matching Based on Costly Signals," *Review of Economic Studies*.

Kohlberg, E., and J.-F. Mertens (1986): "On the Strategic Stability of Equilibria," *Econometrica*, 54(5), 1003–37.

Maskin, E., and J. Riley (1984): "Monopoly with Incomplete Information," *RAND Journal of Economics*, 15(2), 171–196.

McAfee, R. P. (2002): "Coarse Matching," *Econometrica*, 70(5), 2025–2034.

Mussa, M., and S. Rosen (1978): "Monopoly and product quality," *Journal of Economic Theory*, 18(2), 301–317.

OECD (2001): *Innovations in Labour Market Policies: The Australian Way*. OECD Publishing, Paris.

Pissarides, C. (2000): *Equilibrium unemployment theory*. MIT Press, Cambridge, MA, 2nd edn.

Rochet, J.-C., and J. Tirole (2003): "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1(4), 990–1029.

——— (2006): "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, 37(3), 645–667.

Shimer, R., and L. Smith (2000): "Assortative Matching and Search," *Econometrica*, 68(2), 343–370.