Information Exchange and Consumer Search

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Abstract

Many benchmarks in over-the-counter markets, such as LIBOR, are formed based on the submission of banks' interbank interest rates. Benchmarks are important, as they help consumers to search and provide firms with a basis for price setting. This paper explores firms' incentives to contribute information about their costs for the purpose of benchmark formation. We show that benchmarks reduce price variance and lead consumers to search less. As a result, firms charge higher prices and consumers end up buying from less efficient firms, which negatively affects total welfare. We find that in order to deter consumer search, firms find it optimal to share their private costs if the search friction is sufficiently high. Moreover, the reduction of search in these markets can dramatically decrease welfare even when consumers are not aware of the content of the information exchange, but observe that it took place.

Keywords: information sharing, consumer search, benchmark, LIBOR.

JEL-Codes: D43, D83, L13

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1 Introduction

In many industries, third parties actively collect market-sensitive information and disseminate the aggregate statistics across market participants. This information is usually collected and shared on behalf of firms themselves and takes the form of industry reports and/or benchmarks.[1] Consumers rely on benchmarks to improve their search behavior and firms take benchmarks into account when they set prices. In this paper, we explore the implications of this form of information sharing in consumer search markets. Consumer search is an important aspect of our study, as information sharing affects firms' behavior not only directly, but also indirectly, through its impact on consumer search: consumers will search differently under the knowledge that the benchmark exists. We explain how information sharing affects search, competition and total welfare depending on the size of the search friction. We also inquire into the incentives of firms to share private information with each other.

Benchmarks are used extensively in Over-the-Counter (OTC) markets. According to a report of the Market Participant Group (MPG), trillions of dollars in loans are tied to the LIBOR and EURIBOR benchmarks. Among them are business loans (1,2$ TNs), commercial and retail mortgages (1,4$ TNs each)[2], syndicated loans (3,4$ TNs) and variable-rate bonds (1,2$ TNs). The volume of the OTC derivatives linked to LIBOR is approximately 100 trillion dollars. LIBOR and EURIBOR are computed as the average of the interbank borrowing rates (the costs of raising funds) of the member banks.[3] In addition to these financial markets, there are more than 7000 trade associations in various industries across the US and approximately 2000 in the UK that publish benchmarks and industry reports based on the private information of firms.[4] Due to recent legal cases concerning the manipulation of a

[1] The European Commission defines a *benchmark* as any commercial index or published figure calculated by the application of a formula to the value of underlying assets or prices, including estimated prices, interest rates or other values.

[2] In 2012, the Federal Reserve Bank of Cleveland found that 45% of prime adjustable-rate mortgages (ARMs, 22% of outstanding volume) and 80% of subprime ARMs used LIBOR as a benchmark.

[3] There are 17 banks that currently contribute to the fixing of the US Dollar LIBOR, such as Bank of America, Barclays, JP Morgan etc. LIBOR is administrated by the Intercontinental Exchange and EURIBOR is published by the European Money Markets Institute.

[4] Average Wholesale Price (AWP) is a benchmark that refers to the average price at which drugs are purchased at the wholesale level. LBMA administrates benchmarks for gold (25$ BNs per month) and silver (3$ BNs per month) based on the daily auctions in which large traders determine the price. The

variety of benchmarks, many authorities have begun to thoroughly consider the problem of benchmark formation based on firms' private information.[5]

As in the case of LIBOR and EURIBOR, we consider a market in which firms have private information about their respective costs. Information revelation improves market transparency both for the competing firms and for consumers, and has an impact on market outcomes in two important ways. First, better informed consumers search more efficiently, which leads to greater consumer surplus. Second, benchmarks may also deter search as increased transparency makes firms use benchmarks as the basis for their pricing. As a result, prices become more aligned and the expected gains yielded by searching diminish, allowing firms to set higher prices. We show the conditions on the market structure under which information revelation transfers welfare from consumers to firms and reduces matching efficiency, as well as total welfare.

We build a model in which in the first-stage firms decide whether or not to truthfully submit their private costs to the trade association (TA), which calculates the average costs of all firms that have submitted information.[6] The TA then disseminates this benchmark across all firms and consumers. In the second stage of the game, consumers search and firms, anticipating the search decision of consumers, simultaneously set their prices based on private costs and the benchmark. Consumers are heterogeneous in their valuation of acquiring the products and have different search costs. We assume that there is a positive fraction of consumers with zero search costs which we refer to as *shoppers*. Shoppers always compare products and prices before buying. On the other hand, *non-shoppers* engage in costly search and decide how many firms to visit based on the size of the search costs and the revealed benchmark. It is also important to note that consumers take into account not only the benchmark itself, but the fact that firms have exchanged information on their private costs in order to create the benchmark.

Aluminium Association collects data on the costs incurred by the main producers and publishes monthly reports: www.aluminum.org

[5] See the reports on *Principles for Financial Benchmarks* by IOSCO, *Regulation of Indices* by EC and *Principles for Benchmark-Setting Processes* by ESMA and EBA.

[6] We assume that the trade association provides the exogenous mechanism of truthful reporting. The LIBOR scandal, in which banks manipulated reporting rates, induced governments and trade associations to redesign the benchmarks to discourage untruthful reports by banks. Benchmarks design has been explored in recent papers by Duffie and Dworczak (2014) and Coulter and Shapiro (2015).

We show that the size of the search friction plays a crucial role in a firm's decision whether or not to share private information. Given that benchmarks affect firms' pricing and consumers' search behavior, the reactions of both sides of the market play important roles. More accurate information on rivals' costs allows firms to choose the prices that best reflect the underlying market conditions and this unambiguously increases profits. On the other hand, the pooling of information leads firms' pricing strategies to be increasingly correlated. Indeed, if the benchmark reveals that competitors' costs are higher than the expected level, then a firm has an incentive to increase its price. Firms with relatively high costs will decrease their prices.[7] Therefore, due to the strategic complementarity of prices, information exchange decreases variance in pricing, which, has two important effects. First, less dispersed prices intensify competition for shoppers. Second, it decreases non-shoppers' incentives to search. We will refer to this second effects as the *variance effect*[8]. In addition to these two main effects, the benchmark provides better information and shrinks the range of prices that consumers might have expected, thereby reducing the incentives to search even further.

Given these different effects, our first result shows that firms find it optimal to reveal their private costs when the search friction is sufficiently high, whereas they prefer to conceal their private information when the search friction is sufficiently low. If the fraction of shoppers is high and many consumers decide to compare prices before buying, then information sharing will lead to intensified competition (on average), putting downward pressure on prices. Thus, firms prefer to conceal their private costs and the benchmark is not formed. This result is in line with the standard literature on information exchange in oligopoly where all consumers are informed about prices.[9] However, when we consider consumers with relatively high search costs, the incentives to share dramatically change. In this case, consumers will search less, allowing firms to increase prices. We show that if the fraction of consumers with high search costs is large enough, then the gains from search deterrence outweigh the losses from

[7] It can also be the case that all firms have relatively low (high) costs in comparison to the expected level and they will all decrease (increase) their prices. What is important in this case is that firms with higher costs will decrease (increase) prices by more (less) than firms with low cost. Therefore, prices become less dispersed. Technically, this is a case of the natural assumption of increasing differences in prices, see Vives (2000).

[8] Our terminology is in line with Benabou and Gertner (1993).

[9] See Gal-Or (1986), Raith (1996), Vives (2000).

intensified competition for shoppers and firms decide to reveal.

Our second result is is a welfare result. We show that if the search friction is sufficiently high, information revelation reduces consumer surplus and total welfare. The increase in correlation of prices, as well as the improved information on the average cost, allows consumers to avoid wasteful search, which is good for total welfare. On the other hand, less-active search increases matching inefficiency. Since more consumers decide not to compare prices before buying, high-cost firms enjoy a greater market share and more consumers end up buying from inefficient firms. This generates lower gains from trade that cannot be recovered by the savings accrued in terms of search costs. Moreover, because of the variance effect, firms increase their prices thereby reducing consumer surplus. As a result, the total welfare loss from information revelation in markets with sufficiently high search costs is paid by the consumers.

Our final result is that, qualitatively, the above results continue to hold for the case of *private* information exchange, in which consumers only know that information exchange took place, but do not know the content of the information that was exchanged. We show that the variance effect by itself can severely deter search such that the losses from matching inefficiency outweigh the benefits from search costs savings.

A recent paper by Duffie, Dworczak and Zhu (2017) also explores how benchmarks affect social welfare. The authors build a model in which firms have identical cost and explore the question of whether or not welfare is greater if consumers know this cost. If there is no benchmark, consumers learn about the underlying costs from prices. Based on the mechanism of learning described by Janssen, Pichler and Weidenholzer (2011), they show that benchmarks improve social welfare. However, they do not study how the benchmarks come about or whether or not firms have an incentive to share information that would result in the creation of a benchmark. In contrast, the focus of our paper is benchmark formation. As in the case of LIBOR, benchmarks are often created on the basis of information provided by firms, where each of the firms may face a different (possibly correlated) cost. To draw a comparison with Duffie, Dworczak and Zhu (2017), two additional points become important: (i) whether or not consumers know that information has been exchanged and (ii) whether or not firms have an incentive to share this information. Thus, consumers have an additional source of learning: if information is exchanged, prices become less dispersed and consumers search less. In order to focus on these two points, we eliminate the learning-from-prices

channel and consider non-sequential search.[10]

More generally, this paper is related to two strands of literature. First, it relates to the literature on *learning in search markets*. Most of this literature considers firms' information structures as exogenously given. In some studies, all firms have the same information on the parameters of the model (Varian (1980), Stahl (1989), Wolinsky (1986), Dana Jr (1994) and Janssen, Pichler and Weidenholzer (2011), Janssen, Parakhonyak and Parakhonyak (2017)). In this case price dispersion arises as a result of firms employing mixed strategies. In other approaches, price dispersion can arise as a result of asymmetry of information between competitors (Reinganum (1979), Benabou and Gertner (1993)). In these models, firms have private information about their own costs. The static modelling in these papers sheds some light on what strategies are employed in a particular asymmetric information game. The question as to why a specific type of asymmetry arises has not yet received any attention in the search literature. In order to fill this gap, we allow for information exchange between firms before the pricing game takes place.

Second, there is a large literature on *information sharing* in markets. In much of this literature, firms reveal their private costs to a trade association that discloses the average cost that we refer to as a benchmark. The literature on information sharing in oligopoly (Gal-Or (1985), Gal-Or (1986), Vives (1990), Raith (1996) and Vives (2010)) underlines that the most important determinant of the incentives to reveal information is the reaction of competitors. Key to this literature is the assumption that consumers are able to perfectly observe prices. Under this assumption, consumers' purchasing strategy does not depend on (i) whether or not firms exchange information and (ii) whether or not this information is also revealed to consumers. Therefore, the problem of information exchange is reduced to the exploration of the reaction of competitors. However, in search markets such as OTC markets, information exchange also affects the behavior of consumers. We show that results from this standard literature do *not* hold when the search friction is sufficiently large.

Information exchange between firms is of primary importance to antitrust authorities and

[10] In their paper, De los Santos, Hortaçsu and Wildenbeest (2012) observe that people may act as if they search non-sequentially. Morgan and Manning (1985) show that depending on the search problem, both sequential and non-sequential protocols can be optimal. The non-sequential search protocol better fits markets with economies of scale in the search trials (searching online, using price aggregators). In some OTC markets, and especially in their interdealer market segments, there are brokers that provide summarized information about prices, resulting in economies of scale.

is regulated by the Guidelines on Horizontal Agreements. Information exchange has never been analyzed in search markets. Therefore, we also contribute to the growing body of work that analyzes the impact of consumer search on traditional competition policy. Mergers in search markets have been analyzed by Moraga-González and Petrikaitė (2013), collusion by Petrikaitė (2016) and vertical restraints by Janssen and Shelegia (2015).

The rest of the paper is organized as follows. Section 2 describes the model. In section 3, we analyze firms' pricing and the search behavior of consumers in each possible information revelation scenario. In section 4, we characterize the equilibrium revelation strategy and provide conditions on the size of the search friction under which benchmarks are formed. In section 5 we explore how information revelation affects social welfare. Section 6 concludes. Technical proofs are presented in the Appendix.

2 Model

Consider an oligopolistic market with $N \geq 2$ firms that produce a differentiated good and compete in prices.[11] Importantly, firms have asymmetric production costs c_i, $i = 1, \ldots, N$ which is their private information. We assume that c_i is independently and identically distributed on the compact support $[\underline{c}, \overline{c}]$ according to the distribution $G(c)$ with a variance $\sigma_c^2 > 0$.[12] Although we do not explicitly model the wholesale market, one may view the asymmetry of costs coming from the fact that firms might work either with different wholesalers or with the same wholesaler offering different contracts. In banking, asymmetry costs is a result of different sources of how banks raise their funds.

Due to this asymmetry, firms can decide whether or not to make an agreement to share their costs with other market participants. The information exchange is performed by the trade association (TA) that collects all submissions and calculates the aggregate statistics that we refer to as a *benchmark*. If $K \subseteq \{1, \ldots, N\}$ is the non-empty set of firms which

[11] Many financial markets in which benchmarks are extensively used are characterized by substantial product differentiation as it was documented by various studies on credit markets (Matutes and Vives (1996), Kim, Kristiansen and Vale (2005)) and mortgage markets (Allen, Clark and Houde (2012), Del Prete, Demma and Rossi (2017)). The differentiation comes from the variety of complex contracts that might be suitable for some borrowers but not for others.

[12] I separately analyze the model with correlated costs and show that the results qualitatively remain the same. The preliminary version of this extension is available upon request.

decided to join the trade association, then the benchmark b is modelled as the average of submitted costs, so $b = 1/|K| \sum_{j \in K} c_j$.

We will assume that the trade association *publicly* announces the benchmark and all market participants can observe it. Later on, we will distinguish between *public* and *private* information exchange. In contrast, if information is shared privately then consumers cannot see the benchmark but might observe or believe that information is exchanged between firms. We postpone the discussion of how the observability of benchmarks by consumers affects our results.

There are two important assumptions we make on how firms exchange information. First, as in the literature on information exchange mediated by third parties, we assume that firms have to decide whether to reveal costs or not *before* the actual arrival of private information. Therefore, we consider the incentives to reveal costs unconditionally on the information itself. This assumption naturally describes the formation of many benchmarks such as LIBOR or EURIBOR, in which banks commit themselves to submit their costs on a regular basis. Second, we assume that the costs of manipulating the reports are high enough so that the information is revealed *truthfully*. Otherwise, firms would always try to exaggerate their costs to induce their competitors to set higher prices, and the formation of benchmarks would be impossible.

The demand side of the market is represented by an infinite number of consumers, each of whom wishes to buy one unit of at most one of N products. The heterogeneity of consumers' preferences is described by the Chen and Riordan (2007) spokes model. This model captures the nonlocalized oligopolistic competition in which any firm directly competes with each of its $N - 1$ rivals based on the Hotelling framework. For concreteness, we assume that each consumer cares only about two products, and valued all the $N - 2$ remaining products at zero. All consumers assign valuation v to these two products and we assume that v is large enough so the market is fully covered. The preference relation between two products is described by the Hotelling line of unit length, with two firms located at the endpoints. We say that consumer $\ell \in L_{ij}$ (L stands for the Hotelling line) if she cares about products i and j. Consumers are uniformly distributed on the corresponding lines and have linear transportation costs measured by t. The location of consumer ℓ relative to the firm i is denoted by $x_{\ell i}$ ($x_{\ell i} + x_{\ell j} = 1$). There is a mass of $2/(N - 1)$ consumers located between two

randomly selected firms i and j, so the total demand of each firm cannot exceed 2.[13]

Although a random consumer ℓ knows which two products she is interested in, we assume that her location and all prices are ex ante unknown. Consumers have to engage in costly search and are heterogeneous in search costs. We assume that in each Hotelling line there is a fraction of *shoppers* $\lambda \in [0, 1]$ that have zero search cost and visit both firms before buying. Notice, that if $\lambda = 1$ then the model coincides with a generalized version of Gal-Or (1986).[14] The remaining $1 - \lambda$ are *non-shoppers* and have positive search costs which are distributed according to the uniform distribution $s_\ell \sim U[0, \overline{s}]$. It is assumed that consumers search *non-sequentially* and become informed about prices and their locations by paying search costs. Otherwise, consumers choose in which firm to buy and randomize with probability 1/2 if they are indifferent.

In order to avoid the boundary problem in which some firms cannot attract informed consumers because of high prices, we impose the following constraint on the relation between t and the support of the costs distribution:

Assumption 1. $\overline{c} - \underline{c} < 2t$.

This assumption implies that the variance of costs is not too high $\sigma_c^2 \leq (\overline{c} - \underline{c})^2 < 4t^2$. As long as t is high enough, meaning that the product differentiation is sufficiently strong, the informed consumers will not react too drastically on the difference in prices. This will ensure that the location of the indifferent consumer lies in the interior of the respective Hotelling line. Therefore, firms with the highest possible cost $\overline{c}$ will sell to some informed consumers even if they compete with the most efficient firms with costs $\underline{c}$.

The timing goes as follows. In the first stage, firms simultaneously decide whether to join the trade association and reveal their costs. Then the private costs are generated and the TA conducts the transmission of the private information according to the firms' commitments. Then, the TA calculates the average cost b and publicly announces $\{b, k\}$, where k is the number of contributing firms.[15]

[13] The total demand in the market is $C_N^2 2/(N-1) = N$, so 1 per firm.

[14] In Gal-Or(1986), the author considers the information exchange problem in a duopoly, assuming that costs are normally distributed. In contrast, we analyze an oligopolistic market and work with a general distribution of production costs.

[15] We assume that consumers observe k but cannot see the identities of firms that made the submission. In the Appendix we show that this assumption is not restrictive and was made for the model simplicity. In reality, trade associations might reveal the list of contributing firms and consumers might use it.

In the second stage firms simultaneously set prices p_i, $i = \{1, \ldots, N\}$ and consumers search. Consumer ℓ cannot observe prices and make the search decision $a_\ell \in A = \{0, 1\} = \{do\ not\ search, search\}$ simultaneously with other consumers. The resulting fraction of informed consumers interested in product i and j is $\mu_{ij} = \int_{L_{ij}} a_\ell d\ell > \lambda$.

Thus, the profit of firm i that charges p_i and has cost c_i is given by

$$\pi_i(p_i, p_{-i}, a) = (p_i - c_i) \sum_{j \neq i} \frac{2}{N-1} \int_{L_{ij}} \phi_\ell(p_i, p_j, a_\ell) d\ell,$$

where the summation is taken over the competing products j and ϕ_ℓ equals to 1 if consumer ℓ buys the product i and 0 otherwise. The net surplus of consumer ℓ who decided to buy from firm i is given by:

$$u_\ell(p_i, x_{\ell i}) = v \mathbb{I}_{\ell \in \cup_{j \neq i} L_{ij}} - (p_i + t x_{\ell i} + a_\ell s_\ell)$$

The strategy profile of firm i includes any mapping p from the production cost c_i to the actual price: $p = p_i(c_i, b)$.[16] Consumers search strategy is described by $q_\ell(\cdot; b, s_\ell) \in \Delta(A)$, so consumers can randomize their search decisions.

We look for a profile of strategies in which: *1)* firms optimally decide whether or not to contribute to the benchmark formation $e_i \in \Delta E = \Delta\{0, 1\} = \Delta\{reveal, conceal\}$ and *2)* observing this benchmark, consumers search optimally given the pricing strategies of the firms and firms maximize their profits given the number of consumers that become informed. We formally define the equilibrium notion below:

Definition 1. *A Perfect Bayesian Equilibrium (PBE) in the model of benchmark formation in search markets is the profile $(e_i, p_i(c_i, b))$ for all i and $q_\ell(\cdot)$ for all ℓ such that:*

1. *Each firm i chooses e_i to maximize its expected profits given the revelation strategies of other firms e_{-i} and anticipating that consumers will search according to $q_\ell(\cdot; b, s_\ell)$ and firms will set prices according to $p_j(\cdot, b)$;*

2. *Using the Bayes' Rule, consumers update their beliefs about the underlying price dispersion based on the realized benchmark b. Then consumers optimally choose their*

[16] We do not exclude price distributions from the consideration, but since the pricing game is based on the Hotelling framework, it is easy to see that firms will never use mixed strategies. We prove this fact in the next session.

search strategies q_ℓ given their beliefs about prices and the search strategies of the other consumers. The resulting total fraction of informed consumers in each market L_{ij} is $\mu_{ij}(b)$;

3. *Based on the benchmark b, each firm i optimally sets its price according to the function $p_i(c_i, b)$, given the price strategies of other firms $p_j(\cdot, b)$ and the belief about the total fraction of informed consumers $\mu^e_{ij}(b)$ which is correct in equilibrium.*

3 Analysis: The Second Stage

We analyse the model by solving it backwards and start with the second stage of the game in which firms set prices and consumers search. Suppose that $K \subseteq \{1, \ldots, N\}$ is a non-empty set of $k = |K|$ firms which decided to reveal their costs and form a benchmark $b = 1/k \sum_{j \in K} c_j$. If all firms decide to conceal their cost we set $K = \emptyset$ and $k = 0$, so the benchmark is not formed. In this section, we characterize the equilibria in every possible subgame described by the outcome of the preceding benchmark formation stage $\{b, k\}$.

3.1 Firms Pricing

We start from the investigation of the optimal pricing of firms which revealed or concealed their costs for a given search behavior of consumers. Since consumers follow the non-sequential search and decided whether to be informed or not, the outcome of their decision is a fraction of informed consumers. Since consumers do not know the identities of the firms which shared information, we have that the resulting fraction of informed consumers must be the same for all direct markets L_{ij}, so $\mu_{ij}(b) = \mu(b)$. Firms do not know the resulting $\mu(b)$ but have the correct expectation in equilibrium.

In the degenerate case when $\mu(b) = 0$, firms are local monopolists which set the same monopoly prices v. Despite this, there is a positive mass of consumers with very small search costs who will still search in order to learn their locations. Therefore, there is an $\epsilon > 0$, such that all $\mu(b) < \epsilon$ cannot constitute an equilibrium. We focus on the positive $\mu(b)$ and assume that v is high enough that all consumers buy.

Consider how the information of consumers affects firms demand. Suppose an informed consumer who has to choose between firm i and j and is located at the distance x from

product i. This consumer will buy products i if p_i is low or she is located close enough to firm i: $p_i + tx \leq p_j + t(1-x)$ or, equivalently:

$$x \leq \overline{x}_{ij} = \frac{1}{2} + \frac{p_j - p_i}{2t}$$

,where $\overline{x}_{ij}$ is the indifferent consumer in the market L_{ij}. Thereafter, we will assume that the indifferent consumer is located at the interior of the corresponding Hotelling line L_{ij}, so $\overline{x}_{ij} \in (0,1)$, and we will show that this is the case under the assumption (1). Uninformed consumers $1-\mu(b)$ split evenly among the firms and buy there.

The expected demand of firm i setting price p_i is given by:

$$D(p_i) = \sum_{j \neq i} \frac{2}{N-1} \left\{ \mu(b) \left(\frac{1}{2} + \frac{\mathbb{E}[p_j|I_i] - p_i}{2t} \right) + \frac{1-\mu(b)}{2} \right\} \tag{1}$$

The expression in the curly brackets represents the demand of consumers choosing between products i and j. The first term is the expected fraction of informed consumers who are located closer to firm i than the indifferent consumer $\overline{x}_{ij}$. The expectation are taken conditionally on the information I_i if firm i that includes the benchmark $\{b, k\}$ and whether firm i contributed to it. The summation is taken over all $N-1$ Hotelling lines, each of which contains the mass of $2/(N-1)$ consumers.

By simplifying the demand expression, the profit function can be written in the following way:

$$\pi_i(p_i) = \frac{\mu(b)}{t}(p_i - c_i) \left(\frac{t}{\mu(b)} + \frac{1}{N-1} \sum_{j \neq i} \mathbb{E}[p_j|I_i] - p_i \right) \tag{2}$$

There are two important points about the functional form of the profit (2). First, a change in the fraction of informed consumers $\mu(b)$ acts as a change in transportation costs t but with a different magnitude. Precisely, the decrease in $\mu(b)$ by $x\%$ is equivalent to the increase in t by $x/(1-x/100)\% > x\%$. So the market power of firms is higher for lower $\mu(b)$. Second, since the profit function is quadratic we are able to solve for the equilibrium pricing analytically using the similar technique as in Li (1985) and Shapiro (1986) who analyse the information sharing in Cournout oligopoly and abandon the assumption of normality of costs distribution.

The first order condition of firm i's profit maximization problem is:

$$2p_i = t_1 + c_i + \frac{1}{N-1} \sum_{j \neq i} \mathbb{E}[p_j|I_i] \tag{3}$$

The linearity of (3) with respect to c_i hints that there is an equilibrium in linear strategies. Assume that firms follow the symmetric linear strategies with respect to their revelation decisions. Define the candidate strategies as:

$$\begin{cases} p_R(c_i) = \alpha_R + \beta_R b + \gamma_R c_i, \text{ if } i \in K \\ p_C(c_i) = \alpha_C + \beta_C b + \gamma_C c_i, \text{ if } i \in N \setminus K \end{cases} \tag{4}$$

,where indexes R and C stand for "reveal" and "conceal" respectively. The next proposition shows that there is a unique profile of pricing strategies satisfying (3) and it has the linear form given by (4).

Proposition 1. *Given any outcome of the benchmark formation in the first stage $\{b, k\}$, there is a unique Bayesian Nash Best Response of firms to any search behavior described by $\mu(b) > 0$. Depending on whether firm i revealed or concealed its costs, it sets the price according to the linear pricing strategies given by:*

$$\begin{aligned} p_R(c_i) =& \frac{t}{\mu(b)} + \frac{N-k}{2N-1}c^e + \frac{k}{2N-1}b + \frac{N-1}{2N-1}c_i \\ p_C(c_i) =& \frac{t}{\mu(b)} + \left(\frac{1}{2} - \frac{k}{2N-1}\right)c^e + \frac{k}{2N-1}b + \frac{1}{2}c_i \end{aligned} \tag{5}$$

The proof of proposition 1 consists of two steps. First, when we substitute the linear candidate to the first order condition described by (3), we get a linear system of six equations and six unknown coefficients from (4). We show that this system has a unique solution. Second, we show that the FOC (3) connects the pricing strategies in a way that precludes the existence of more than one equilibrium (lemma 5 in the appendix).

In (1) it was assumed that all indifferent consumers are located in the interior of the corresponding Hotelling lines. It is easy to verify that the difference in any two equilibrium prices given by (5) cannot be higher than $(\overline{c} - \underline{c})/2$. Thus, according to the assumption (1), the indifferent consumer is in the interior: $|p_i - p_j| < t$ and $x_{ij} \in (0, 1)$ for all L_{ij}.

There are several important points concerning the equilibrium price strategies (5). First, all firms set higher prices for lower fraction of informed consumers $\mu(b)$ and pass through the benchmark b at the same rate which is more profound if more firms have contributed to the benchmark. Second, the expected prices are the same: $p_R^e = p_C^e$. So, the revelation by itself does not change the relative market power of firms from the ex ante perspective. Third, the revelation changes the price dispersion measured by $\text{Var}(p_1 - p_2)$, that captures

both the variances of p_1 and p_2 and the correlation between them.[17] Notice that the fraction of informed consumers $\mu(b)$ and the benchmark b does not affect the price dispersion and determine only the expected price level relative to which prices vary (terms dependent on $\mu(b)$ and b cancel out in $p_1 - p_2$). Importantly, the price dispersion is determined by the revelation decisions of firms and the following corollary shows that the revelation makes prices less dispersed:

Corollary 1. *Price dispersion of any two given firms i and j is lower if i and/or j reveal their costs.*

Proof. The direct comparison of the variances of price differences establishes the result:

$$\mathbb{E}(p_C(c_i) - p_C(c_j))^2 = \frac{1}{2}\sigma_c^2 > \left(\frac{1}{4} + \left(\frac{N-1}{2N-1}\right)^2\right)\sigma_c^2 = \mathbb{E}(p_R(c_i) - p_C(c_j))^2 >$$
$$> 2\left(\frac{N-1}{2N-1}\right)^2 \sigma_c^2 = \mathbb{E}(p_R(c_i) - p_R(c_j))^2$$

□

We will refer to the decrease of price dispersion due to information sharing as the *variance effect*. The intuition behind the variance effect comes from the fact that holding benchmark b fixed, a revealing firm passes through its cost at a lower rate than a concealing firm. In other words, revealing firms base their pricing on the public information to a greater extent than firms that conceal information which results in more similar prices. For $N = 2$ revealing firms pass through 33% of their private information (c_i) and 66% of the public information (c^e and b). In contrast, concealing firms pass through 50% of their cost. Notice, that the variance effect becomes less severe with an increase in N as the pass through rate of revealing firms approaches the pass through rate of concealing firms: $\lim_{N\to\infty}(N-1)/(2N-1) = 1/2$.

3.2 Consumer Search

On the demand side, consumers cannot observe prices and their locations, so they have to decide if to pay search cost s_ℓ and inspect two products or save it and visit only one firm and

[17] In the standard search literature (Varian(1980) or Stahl(1989)), the dispersion is usually associated with the variance of equilibrium price distribution, since prices are independently and identically distributed. In the general case, price correlation should be taken into account. As an alternative to our approach, one could measure price dispersion in the expected positive price differences $\mathbb{E}\max\{p_1 - p_2, 0\} = \mathbb{E}\,|p_1 - p_2|$.

buy there. They base their decisions on the benchmark $\{b, k\}$ that is publicly announced by TA and the beliefs about firms' pricing. The expected gain of being informed is represented by the difference between the expected effective price and the expected minimum of two effective prices that include the transportation costs of consumers. Consumer ℓ will find it optimal to search if the gain of search compensates her for the search costs incurred:

$$s_\ell \leq \tilde{s}_k(b) = \mathbb{E}[p_i + tx_i|b] - \mathbb{E}[\min\{p_i + tx_i, p_j + t(1 - x_i)\}|b]$$

, where expectation is taken over two prices and consumers' distance x_i to firm i and $\tilde{s}_k(b)$ is the consumer who is indifferent between searching and staying uninformed. Using the identity $-\min\{a, b\} = \max\{-a, -b\}$ we rearrange the expected gain of search in the following form:

$$\tilde{s}_k(b) = \mathbb{E}[\max\{p_i - p_j + t(2x_i - 1), 0\}|b] = 2t\mathbb{E}\left[x_i - \overline{x}_{ij}, 0\}|b\right]$$

, where $\overline{x}_{ij}$ is the location of indifferent consumer in L_{ij}. The gain of search is positive if consumer is located closer to firm j. Next, we take the expectation with respect to x_i:

$$\mathbb{E}_{x_i}[x_i - \overline{x}_{ij}, 0] = \int_{\overline{x}_{ji}}^{1} (x_i - \overline{x}_{ij})dx_i = \frac{1}{2}\left(1 - \overline{x}_{ij}\right)^2 = \frac{1}{2}\overline{x}_{ji}^2.$$

and get that the location of indifferent consumer is given by:

$$\tilde{s}(b) = t\mathbb{E}_p[\overline{x}_{ji}^2|b] = t\mathbb{E}_p\left[\left(\frac{1}{2} + \frac{p_i - p_j}{2t}\right)^2 \middle| b\right] \tag{6}$$

From (6) we see that search becomes more valuable if the variance of the indifferent consumer location $\overline{x}_{ji}$ is high, since in this case there is a high chance that a more desired product is not inspected. The variance of the indifferent consumer location is driven by the price dispersion. Therefore, the price dispersion is the key driver of consumer search.

It is important to notice that price difference $p_i - p_j$ given by (5) does not depend on $\mu(b)$ in equilibrium, thus it does not affect the price dispersion. So, the right side of (6) does not depend on $\mu(b)$ and the equilibrium $\mu(b)$ exists and uniquely determined. Search costs are distributed according to $s_\ell \sim U[0, \overline{s}]$, so the probability that search costs are lower than $\tilde{s}(b)$ is given by $\tilde{s}(b)/\overline{s}$. Since there are $1 - \lambda$ of uninformed consumers, the total fraction if informed consumers is given by:

$$\mu_k(b) = \lambda + (1 - \lambda)\tilde{s}(b)/\overline{s}$$

Next proposition shows that if firms reveal their costs and form the benchmark, then less consumers are expected to to search: the ex ante fraction of informed consumers $\mu_k = \mathbb{E}\mu_k(b)$ (at the stage of benchmark formation) decreases in k.

Proposition 2. *The ex ante equilibrium fraction of informed consumers is lower for a higher number of revealing firms k: $\tilde{s}_k > \tilde{s}_{k+1}$ and $\mu_k > \mu_{k+1}$.*

The intuition of this claim goes as follows. It was shown in corollary 1 that if more firms decide to contribute to the benchmark, then their prices become less dispersed due to the variance effect of information exchange. As a result, the expected gain of search given by 6 becomes lower and consumers are less likely to find a better deal. Diminished necessity of search decreases the fraction of informed consumers μ and pushes firms prices upwards.

4 Information Sharing

We explored the equilibrium pricing and search behavior of consumers for any outcomes of the benchmark formation stage and now we are prepared to inquire into the firms' incentives to reveal their private costs. We consider the benchmark formation stage from the ex ante perspective, so all firms simultaneously decide whether to join trade association before knowing their costs. In this section we show that firms find it optimal to reveal information when search friction is sufficiently strong: i) fraction of informed consumers λ is low and/or ii) the search costs characterized by the upper bound $\overline{s}$ are sufficiently high.

4.1 No search friction

We start the analysis of the first stage by considering the incentives to exchange information in the market without search friction when $\lambda = 1$. Let $\Pi^R(k)$ and $\Pi^C(k)$ be the ex ante profits of revealing and concealing firms respectively, when k firms contribute to the benchmark. These profits are given by the following expressions:

$$\Pi^R(k) = \mathbb{E}(p_R(c_i) - c_i)^2/t \qquad\qquad \Pi^C(k) = \mathbb{E}(p_C(c_i) - c_i)^2/t$$

, where p_R and p_C are the equilibrium pricing strategies given by (5). Information sharing allows firms to choose prices that better correspond to the real market conditions and this unambiguously improves the profits. At the same time, information sharing makes pricing

more correlated and prices become less dispersed by corollary 1. To describe the main tradeoff we consider the profit function in the following general form:

$$\Pi(k) = \mathbb{E}(p_i - c_i)(1 - p_i/t) + \frac{1}{t(N-1)} \sum_{j \neq i} [\underbrace{\mathbb{E}[p_j p_i]}_{+} - \underbrace{\mathbb{E}[p_j c_i]}_{-}] \tag{7}$$

The increased price correlation improves profits since the second term becomes higher due to the strategic complementarity of pricing. On the contrary, higher costs make rivals to set higher prices leading to a higher demand that raises firms' total costs. The following proposition shows that the negative effect always dominates and benchmarks formation is impossible in markets without search friction.

Proposition 3. *It is optimal to conceal private costs in the market without search friction* ($\lambda = 1$)*:* $\Pi^C(k) > \Pi^R(k+1)$ *for all* $k = 0, 1, \ldots N-1$.

This result is a generalization of Gal-Or (1986) to the oligopolistic markets and bounded distributions of production costs. The "no-revelation" equilibrium is in dominant strategies because firms find it optimal to deviate and conceal their costs for any given number of revealing firms: $\Pi^C(k) > \Pi^R(k+1)$. So, in the markets without search friction, price dispersion improves firms' profits.

4.2 Search friction

Consider the problem of benchmark formation in the search markets with positive fraction of non-shoppers $\lambda < 1$.

Denote $\tilde{t}(b) = t/\mu(b)$ as the effective transportation costs that results from the formation of benchmark b. The equilibrium ex ante firms' profit can be written similar to expression (7):

$$\Pi(k) = \mathbb{E}(p_i - c_i)\left(1 + \frac{1}{N-1}\sum_{j \neq i}(p_j - p_i)/\hat{t}(b)\right) =$$
$$= \underbrace{\mathbb{E}(p_i - c_i)(1 - p_i/\tilde{t}(b))}_{+} + \frac{1}{(N-1)}\sum_{j \neq i}\underbrace{(\mathbb{E}[p_j p_i] - \mathbb{E}[p_j c_i])}_{-}\Big/\tilde{t}(b) \tag{8}$$

First, the tradeoff for the markets with $\lambda = 1$ described in (7) is present in the search markets as well(the second term in (8) includes the second term in (7)). According to the proposition 3, firms find it optimal to conceal information when they compete only

for informed consumers. Second, the information exchange decreases price dispersion and consumers search less, which leads to a higher effective transportation cost $\tilde{t}(b)$. From the equation (8), we can see that a higher $\tilde{t}(b)$ increases the first term, since the own-price elasticity of demand becomes lower due to a higher fraction of non-shoppers $1-\mu(b)$. Moreover, a higher $\tilde{t}(b)$ decreases the negative effect of less dispersed prices described by the second term.

Next, we explore the interplay of these two effects and determine the conditions, under which firms find it optimal to contribute to the benchmark. The analysis is heavily based on the comparison between *public* and *private* information sharing. Under *private* information sharing, consumers *can* see the number of firms that revealed their costs k but *cannot* observe the realized benchmark b. In this case, consumers do not learn anything from the benchmark, but rather update their beliefs about prices based on the fact that firms exchange information. We will denote Π_1 and Π_0 as the equilibrium profits in the case of public and private information sharing respectively. If information is shared publicly, then the equilibrium profit of firm i can be written as the expected profit in the second stage given by (2) using the first order condition (3):

$$\Pi_1(k) = \mathbb{E}\left[\frac{\mu(b)}{t}\left(\frac{t}{\mu_k(b)} + \frac{1}{N-1}\sum_{j\neq i}(p_j - p_i)\right)^2\right] = t\mathbb{E}_b\left[\frac{1}{\mu_k(b)} + \mu_k(b)T_k(b)\right] \tag{9}$$

, where $T_k(b)$ describes the term that increases when prices become more dispersed:

$$T_k(b) = \mathbb{E}\left[\left(\frac{1}{N-1}\sum_{j\neq i}(p_j - p_i)/t\right)^2 \middle| b\right] \tag{10}$$

and $p_i = p_C$ or $p_i = p_C$ depending on whether firm i revealed or concealed its cost. By assumption (1) we have that each intermediate consumer x_{ij} is in the interior of the corresponding Hotelling line L_{ij} and therefore we can evaluate $T_k(b)$ from above. By proposition 1 the difference between any two prices cannot be larger than $(\overline{c} - \underline{c})/2)$, thus

$$T_k(b) < \mathbb{E}\left[\left(\frac{1}{N-1}\sum_{j\neq i}(\overline{c} - \underline{c})/2t\right)^2 \middle| b\right] < 1.$$

If information is exchanged privately, then consumers can not condition the expected gains of search on the benchmark b and decide to search if $s < \tilde{s}_k = \mathbb{E}\tilde{s}_k(b)$. The equilibrium profit of firm i in this case is given by:

$$\Pi_0(k) = t\left[\frac{1}{\mu_k} + \mu_k \mathbb{E}T_k(b)\right] \tag{11}$$

, where $\mu_k = \mathbb{E}\mu_k(b)$ is the fraction of searching consumers who cannot observe the benchmark but can see that k firms exchanged information.

To solve for the equilibria in the benchmark formation game we introduce some useful lemmas that significantly help to describe the nature of revelation decisions. First, we determine how consumers' learning from the benchmark affects the equilibrium profit and compare (9) and (11). The first term of (9) is a convex function of $\mu_k(b)$, and by the Jenssen inequality, any uncertainty about consumers' searching behavior improves the profit in comparison to the expected μ_k. Second, since a higher price dispersion described by higher $T_k(b)$ increases the fraction of searching consumers $\mu_k(b)$, we have that $\mu_k(b)$ and the profit from price variability $T_k(b)$ are positively correlated. Therefore, the second term of (9) is higher than the corresponding second term in (11). The next lemma summarizes this discussion and shows that the equilibrium profits are higher if information is exchanged publicly rather than privately.

Lemma 1. *Irrespectively of whether firm $i \in K$ or $i \in N \setminus K$, it is better off if information sharing is public rather than private: $\Pi_1^R(k) > \Pi_0^R(k)$ and $\Pi_1^C(k) > \Pi_0^C(k)$ for all $k > 0$ (equal if $k = 0$).*

Second, we determine how the number of revealing firms in K influences the incentives of concealing firms to deviate and reveal their cost in the case of private information exchange. Next lemma shows that if a firm finds it optimal to reveal its costs when there are k revealing firms in the market, then it will do so when there are more than k revealing firms. In other words, the situation in which many firms join TA and reveal their costs is more attractive for other firms to join TA too.

Lemma 2. *If information sharing is performed privately, then the revelation is more profitable (less unprofitable) for higher number of revealing firms k: $\Pi_0^R(k+1) - \Pi_0^C(k) > \Pi_0^R(k) - \Pi_0^C(k-1)$ for all $k = 0, 1, \ldots N-1$.*

The intuition goes as follows. According to the proposition 2, if more firms decide to reveal information, then prices become less dispersed, so less consumers μ_k decide to search. We show that a change in the total fraction of informed consumers $\mu_k - \mu_{k+1}$ does not depend on k, when one additional firm reveals information. Importantly, the effective transportation cost t/μ increases more rapidly when μ is low. Therefore, the marginal gain from revela-

tion that results in decrease in market competitiveness is higher when more firms reveal information and search is already low.

The next proposition extends this lemma to the situation of public information revelation, in which consumers can observe the benchmark.

Lemma 3. *If information sharing is performed publicly, then the revelation is more profitable (less unprofitable) for higher number of revealing firms k: $\Pi_1^R(k+1) - \Pi_1^C(k) > \Pi_1^R(k) - \Pi_1^C(k-1)$ for all $k = 0, 1, \ldots N-1$.*

Proof. By lemma 1, we have that firms are better off if information is revealed publicly: $\Pi_1^R(k) > \Pi_0^R(k)$ and $\Pi_1^C(k) > \Pi_0^C(k)$ for all k. Using these inequalities, we have that

$$\Pi_1^R(k+1) + \Pi_1^C(k-1) > \Pi_0^R(k+1) + \Pi_0^C(k-1)$$

and

$$\Pi_1^C(k) + \Pi_1^R(k) > \Pi_0^C(k) + \Pi_0^R(k)$$

We subtract the second inequality from the first one and apply lemma 2:

$$\begin{aligned}\Pi_1^R(k+1) - \Pi_1^C(k) - (\Pi_1^R(k) - \Pi_1^C(k-1)) > \\ > \Pi_0^R(k+1) - \Pi_0^C(k) - (\Pi_0^R(k) - \Pi_0^C(k-1)) > 0.\end{aligned}$$

□

This lemma establishes the fact that a higher number of revealing firms makes the revelation strategy more attractive for others. Similarly, it is important to explore how the fraction of informed consumers λ changes the profitability of revelation strategy. Next lemma shows that if the fraction of shoppers is low, then firms have more incentives to reveal their cost.

Lemma 4. $\Pi_1^R(k+1) - \Pi_1^C(k)$ *is decreasing in λ for all $k = 0, 1, \ldots N-1$.*

In fact, the reasoning behind this lemma is quite similar to the intuition of lemma 3. If the fraction of informed consumers is low, then there are less consumers who decide to search. Since the marginal reduction of searching consumers softens the competition more significantly when $\mu_k(b)$ is low, then the revelation becomes more attractive in the markets with a lower fraction of λ.

So far, we showed that everything else being equal, firms have more incentives to contribute to the benchmark if a higher number of firms reveal their costs and the fraction of

informed consumers is low. Denote λ_k as the fraction of informed consumers at which firms in $N \setminus K$ are indifferent between revealing and concealing if k firms decide to contribute to the benchmark. We define λ_k as the solution of the following equation:

$$\Pi_1^R(k+1, \lambda) - \Pi_1^C(k, \lambda) = 0.$$

We don't know yet for which parameters of the model λ_k exists. We will come to this question in the end of this section. Now we continue by assuming that that there exists λ_k for some k. We define $\tilde{k}$ as the maximum number of revealing firms which find it optimal to deviate to concealment for all λ. Therefore, a group consisting of $\tilde{k} + 1 < N$ firms will keep revealing their costs if λ is sufficiently low. We describe $\tilde{k}$ more formally in the next definition.

Definition 2. *Suppose that $\tilde{k} \in K$ is a number of revealing firms such that i) $\tilde{k} + 1$th firm strictly prefers to reveal for some λ, so $\exists \tilde{\lambda}$ $\Pi_1^R(\tilde{k}+1, \tilde{\lambda}) > \Pi_1^C(\tilde{k}, \tilde{\lambda})$ and ii) if $\tilde{k} \geq 1$ each firm in K prefers to conceal for all λ: $\Pi_1^R(\tilde{k}) < \Pi_1^C(\tilde{k}-1)$.*

By proposition 3 firms prefer to conceal their costs if $\lambda = 1$: $\Pi_1^R(\tilde{k}+1, 1) < \Pi_1^C(\tilde{k}, 1)$. If $k = \tilde{k} + 1$ and λ is sufficiently low, then firms in K will not deviate to concealment $\Pi_1^R(\tilde{k}+1, \lambda) > \Pi_1^C(\tilde{k}, \lambda)$. Since the gain from revelation is smaller for higher λ (lemma 4), then by the fixed point argument there exists unique $\lambda_{\tilde{k}}$ such that firms will reveal if $\lambda \leq \lambda_{\tilde{k}}$ and conceal otherwise. Recall, that firms find it more profitable to reveal their costs for higher k. Therefore, if $\lambda < \lambda_{\tilde{k}}$ and it is optimal to join TA consisting of $\tilde{k}$ firms, then it is optimal to do so if TA consists of larger number of firms $k > \tilde{k}$.

Proposition 4. *For all $k \geq \tilde{k}$, $\lambda_k \in (0, 1)$ are uniquely determined and*
i) λ_k increases in k: $\lambda_{\tilde{k}} < \lambda_{\tilde{k}+1} < \ldots < \lambda_N$
ii) λ_k increases in $\overline{s}$: $\partial \lambda_k / \partial \overline{s} > 0$.

Since both larger k and lower λ make revelation decision more attractive, then firms are indifferent between revelation and concealment if there is low fraction of informed consumers and few revealing firms or there is large k and high λ. Therefore λ_k must increase in k. Second, when search costs represented by the upper bound $\overline{s}$ are high and less consumer decide to search, the market becomes more competitive. Thus, to make firms indifferent in their revelation decisions the competition must be intensified, so the fraction of informed consumers has to be higher.

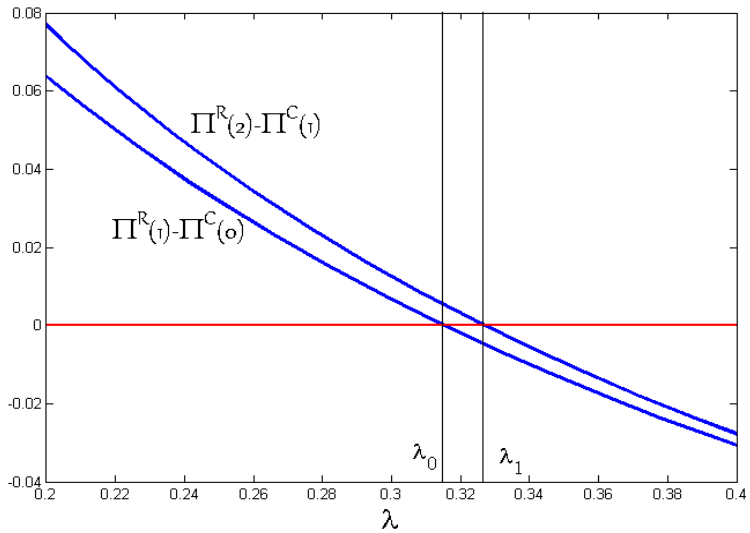


Figure 1: The difference in profits from revelation and concealment: $N = 2$, $\sigma_c^2 = 3$, $t = 2$, $s \sim U[0, 4]$.

Figure 1 illustrates the proposition for $N = 2$. There exist λ_0 and λ_1 such that firms are indifferent between revealing and concealing for $k = 1$ and $k = 2$ consequently. The upper blue line represent the change in profits when $k = 1$ and the second firm reveals it costs: $\Pi_1^R(2) - \Pi_1^R(1)$. The lower blue curve represent the change in profits of a firm that join TA if the other firm conceals. We can see from the picture that it becomes more profitable to reveal information if there is low fraction of informed consumers and more firms decide to join trade association.

Based on the proposition 4 we are ready to characterize the equilibrium in the benchmark formation game. Consider the case in which $\tilde{k} = 0$ and λ_0 exists as in the Figure 1. If $\lambda < \lambda_0$, firms profits are highly dependent on the search strategy of non-shoppers. Therefore, firms will find it optimal to contribute to the benchmark and decrease the search activity of non-shoppers. Less active search allows firms to set higher prices because more consumers buy without comparing the products. Since the majority of consumers are non-shoppers, the losses from shoppers that we considered in the section about no search friction, are compensated by higher profits from non-shoppers. In this case firms will follow the dominant strategy and reveal their costs irrespectively of k.

On the contrary, if the fraction of informed consumers is high and $\lambda > \lambda_N$ then firms will value shoppers more when deciding about revelation strategies. In this case revelation would increase prices not enough to compensate the shrink of price dispersion. Since prices

become less dispersed, firms earn less from shoppers which is the main sources of their profits in this case. Therefore, firms will conceal information if fraction of informed consumers is significantly high.

In the intermediate case when $\lambda_k < \lambda \leq \lambda_{k+1}$ firms play a coordination game in which there are two pure strategy equilibria and one equilibrium is in mixed strategies. Importantly, we show that in this region firms find it more profitable to coordinate on revelation rather than concealment. If $\tilde{k} > 0$ then the intuition remains the same, but in this case, there is no region where revelation is the only equilibrium. The following proposition summarizes the above discussion:

Proposition 5. *Consider the benchmark formation game in which firms decide whether to reveal their costs or not.*

1. If $\tilde{k} = 0$ (high $\overline{s}$), then i) if $\lambda \leq \lambda_0$ then all firms reveal their costs, $k^* = N$. ii) If $\lambda_0 < \lambda \leq \lambda_N$ then there are two pure strategy equilibria, $k_1^* = 0$ and $k_2^* = N$. In this case, firms are better off by coordinating on revelation: $\Pi_1^R(N) > \Pi_1^C(0)$. iii) If $\lambda > \lambda_N$ then all firms conceal their costs, $k^* = 0$.

2. If $\tilde{k} > 0$ (intermediate $\overline{s}$), then j) if $\lambda \leq \lambda_N$ then there are two pure strategy equilibria, $k_1^* = 0$ and $k_2^* = N$. In this case, firms are better off by coordinating on revelation: $\Pi_1^R(N) > \Pi_1^C(0)$. jj) If $\lambda > \lambda_N$ then all firms conceal their costs, $k^* = 0$.

3. If $\nexists \tilde{k}$ (low $\overline{s}$) described by definition 2, then firms will conceal their costs, $k^* = 0$.

It remains to explore how other parameters of the model affect the decision to reveal information. The following proposition establishes the fact that if search costs are sufficiently high, then there exists equilibrium with information revelation for sufficiently low λ.

Proposition 6. $\lambda_0 > 0$ *exists and firms reveal their costs for $\lambda < \lambda_0$ if search costs are high enough: $\overline{s}/t > \omega(N, \sigma_c^2/4t^2)$, where*

$$\omega^2(N, x) = \left(\frac{x}{2} + N\left(\frac{1}{4} + z(N)x\right)\right)\left(\frac{1}{2} + x\right)\left(\frac{1}{4} + z(N)x\right)$$

and

$$z(N) = \frac{1}{2}\left(1 - \left(1 - \left(\frac{2N-2}{2N-1}\right)^2 / N\right)\right).$$

The figure 2 illustrate proposition 6. Firms find it optimal to deviate from no benchmark case and reveal their costs if the relative search costs $\overline{s}/t$ are high and/or the relative variance

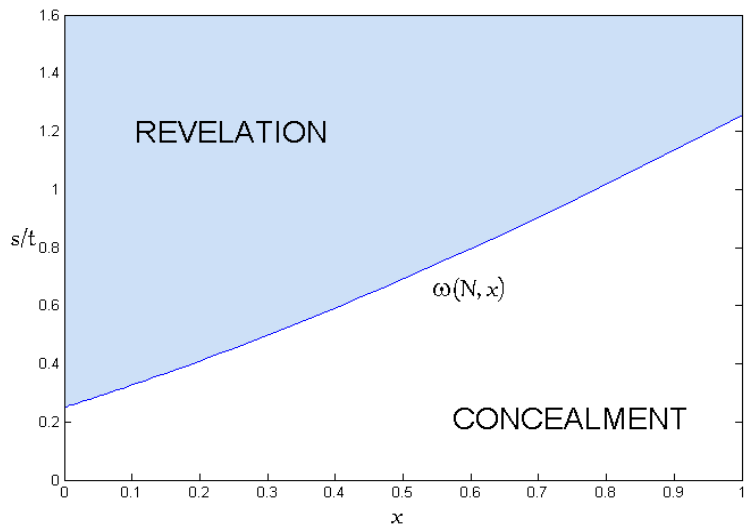


Figure 2: Incentives to deviate from no sharing for small λ: $N = 2$, $s \sim U[0, \overline{s}]$.

of production costs $x = \sigma_c^2/4t^2$ is low. Intuitively, if the variance of costs is high and consumers search more, then the revelation can significantly deter search if and only if search costs are high. On the contrary, if variance of production cost is low and consumer search less, firms find it optimal to reveal even if search is cheap. In the shaded region of parameters firms prefer to reveal information for sufficiently low λ. We can see that everything else equal, a higher search friction is more attractive for benchmark creation.

5 Welfare Analysis

In this section we explore the impact of information exchange on the total welfare. We show that if search costs are high, then the information sharing about private costs leads to the reduction of the total welfare. Since the revelation decision are made before the realization of costs, it is more natural to consider the welfare from the ex ante perspective.

Consider a consumer ℓ who decided to buy product i and has to pay p_i. Clearly, the price does not affect the total surplus and simply divides the gains from trade between firm i and consumer ℓ. The gains from trade can be written in the following form:

$$W_{ij} = v - [a_\ell s_\ell + t x_{\ell i} + c_i] \tag{12}$$

, where a_ℓ equals 1 if consumer decided to search and 0 otherwise. From 12, there three sources of inefficiency. First, search friction does not allow consumers to buy the most

preferred product and they have to incur search costs. Second, inefficiency comes from the preference mismatch. Consumers with higher search costs find it optimal not to search and end up buying a less preferred product incurring higher transportation costs. Third, consumers who do not search might buy products that are produced by less efficient firms. We will refer to it as a cost inefficiency.

Notice, that the market without search friction has the maximal possible total welfare for a given production costs distribution. In this case, consumers will always compare all products and minimize the effective price $tx_{\ell i} + p_i$. Since less efficient firms set higher prices, then consumers' purchasing strategies minimize the matching and cost inefficiencies $tx_{\ell i} + c_i$ and therefore total welfare is maximized.[18] Consequently, there is no reduction of the gains from trade on the side of informed consumers. Therefore, we have to characterize the search strategy of *non-shoppers* that maximizes the social welfare. As it was pointed out earlier, the social maximizer cares about firms' costs rather than prices. The ex ante gain of search of social maximizer that incurs search cost s_ℓ is given by:

$$s_\ell \leq \tilde{s}^* = \mathbb{E}[c_i + tx_i] - \mathbb{E}[\min\{c_i + tx_i, c_j + t(1 - x_i)\}]$$

which can be rearranged as in the case of consumers' optimal search strategy (6)

$$\tilde{s}_k^* = t\mathbb{E}_c[\overline{x}_{ji}^2] = t\mathbb{E}_c\left(\frac{1}{2} + \frac{c_i - c_j}{2t}\right)^2 = \frac{t}{4} + \frac{1}{4t}\mathbb{E}(c_i - c_j)^2 \tag{13}$$

The total welfare attains its maximum when the incurred search costs are counterbalanced with the mismatch and costs inefficiency reduction. If consumers are pushed to search too much, then the generated gains from better information will not recover the search costs. Otherwise, if consumer are not active enough, their search costs saving do not compensate for the foregone reduction in mismatch and cost inefficiencies. According to proposition 1, firms pass through their costs at the rate lower than 1. Suppose k firms reveal their costs then:

$$\tilde{s}^* > \frac{t}{4} + \frac{1}{4t}\mathbb{E}_k(p_i - p_j)^2 = \tilde{s}_k$$

This inequality establishes the fact that in equilibrium consumers search less than needed to maximize the total welfare. The variance effect of information exchange diminishes the

[18] In Duffie et al(2017), benchmarks encourage entry by consumers and increase realized gains from trade. In our welfare analysis we do not consider entry decision, rather than focusing on how benchmarks change the welfare through the impact on the search behaviour of consumers.

search activity even further away from the optimal level $\tilde{s}^*$, reducing the total welfare: $\tilde{s}^* > \tilde{s}_0 > \tilde{s}_N$. The previous section established the fact the information is exchanged if the fraction of shoppers is sufficiently low: $\lambda < \lambda_N$. Therefore, the welfare is the lowest in our model, when the search friction is high because i) a higher fraction of consumers incurs search costs and ii) firms exchange information making consumers search too little.

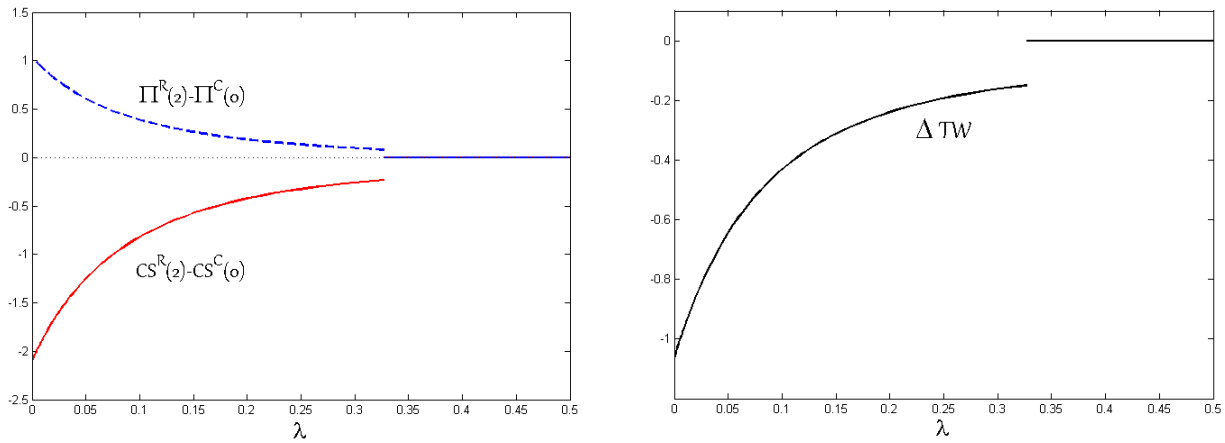
It is important to notice that the reduction of welfare is paid by consumers. If k firms reveal their costs, then the ex ante consumer surplus is given by:

$$CS_k = v - \mathbb{E}[p + tx] + \left\{ \lambda \tilde{s}_k + (1 - \lambda) \int_0^{\tilde{s}_k} (\tilde{s}_k - s) ds \right\} \tag{14}$$

The informed consumers λ buy at the lowest price which is equivalent to saying that they buy at the expected price corrected for the expected gain of search: $\mathbb{E}[p + tx] - \tilde{s}_k$. Uninformed consumers with sufficiently low search costs $s_\ell < \tilde{s}_k$ decide to search and save less than shoppers: $\tilde{s}_k - s_\ell$. Those who decided not to search pay the expected price. It is easy to show that (14) is an increasing function in $\tilde{s}_k$ since the savings of search costs are higher. Firms set higher prices and consumers search less when the benchmark is formed, so we can deduce that consumer surplus is higher if firms conceal information: $CS_0 > CS_N$. We showed in the previous section that prices and profits are higher when firms exchange information $\Pi_1(N) > \Pi_1(0)$ for $\lambda < \lambda_N$, therefore the reduction of total welfare is paid by consumers. When information is exchanged, the welfare is transferred from consumers to firms with subsequent dead weight losses. The following proposition summarizes the discussion:

Proposition 7. *The equilibrium in which firms form the benchmark yields higher profits of firms but reduces consumer surplus as well as total welfare.*

This propositions is illustrated by figures 3(a) and 3(b). We compare the equilibrium profits and consumer surplus to the case where the formation of benchmarks is not possible and firms always conceal information. According to the proposition 5, there is $\tilde{\lambda}$ such that for all $\lambda < \tilde{\lambda}$ firms will reveal information. Therefore, the difference in welfare can be observed only for small fraction of informed consumers. From figure 3(a) we can see that revelation increases profits of firms and decreases the consumer surplus of consumers. Importantly, the consumers surplus decrease is not compensated by higher profits of firms and this is depicted by figure 3(b): $\Delta TW < 0$.



(a) The impact of costs sharing on profits and CS

(b) The impact of costs sharing on welfare

Figure 3: Welfare loss from Information Sharing: $N = 2$, $\sigma_c^2 = 3$, $t = 2$, $s \sim U[0, 4]$

6 Conclusions

Many benchmarks are used as the basis for price setting in various financial and nonfinancial markets, such as OTC markets, in which consumer search is of great importance. Benchmarks and/or industry reports are often formed based on the private information of firms that is collected and disseminated by third parties. This form of information sharing affects the level of uncertainty in markets, which, in turn, has an impact on firms' pricing and consumers' search behavior. In this paper we explore the impact of benchmarks on competition and total welfare and inquire into the incentives to contribute to benchmark formation. Information sharing improves the firms' information about the costs of each other and price become more dependent on benchmarks. We show that this leads to price alignment and consumers have less incentives to search because they believe that a significantly lower price is less likely to be found. Firms find it optimal to contribute to benchmark when search friction is significantly strong, so that information sharing maximally deters search of consumers. Although firms' profits increase, we find that benchmarks reduce total welfare. As a result, consumers search less and buy less desired products from inefficient firms.

In recent paper, Duffie, Dworczak and Zhu (2017) show that benchmarks improve welfare as they encourage entry by consumers and intensify competition. They don't consider information sharing of firms and their result is based on consumers' learning about costs. If consumers know firms' costs, firms cannot manipulate consumers' beliefs and cannot set a

high price because consumers will attribute it to a higher margin and continue to search. In our paper, we uncover another important impact of benchmarks. Precisely, benchmarks facilitate price alignment due to improved information on the firms' side. This unambiguously deters search and reduces total welfare. It is important to investigate the interplay of these two impacts of benchmarks and consider a general model that would incorporate consumers' learning from prices as well as information sharing between firms.

From a more general perspective, in this paper we show that the presence of players who are not involved in communication plays an important role. In their paper, Okuno-Fujiwara, Postlewaite and Suzumura (1990) explore strategic information revelation, and provide sufficient conditions for information revelation. They consider a Bayesian game where players can publicly reveal their type and then take actions. However, there are often some players who are not involved in communication, but their actions influence the payoffs of everybody else. Therefore, when a player reveals her information, she must take into account both the reactions of those players who receive it and of those who do not, but believe or observe that information transmission takes place. Thus, it would be interesting to examine a general model of strategic information revelation in the presence of players who do not communicate.

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Appendix

Lemma 5. *Consider the vectors of random variables X_i and functions h_i which satisfy the following equation for all $i = 1, \ldots, N$:*

$$2h_i(X_i) = \frac{1}{N-1} \sum_{j \neq i} \mathbb{E}[h_j(X_j)|X_i] \tag{15}$$

Then $h_i(X_i) = 0$ almost surely for all i.

Proof. By taking the expectation of (15) conditional on $h_i(X_i)$ and using the tower property of conditional expectation, we get:

$$2h_i(X_i) = \frac{1}{N-1} \sum_{j \neq i} \mathbb{E}[\mathbb{E}[h_j(X_j)|X_i]|h_i(X_i)] = \frac{1}{N-1} \sum_{j \neq i} \mathbb{E}[h_j(X_j)|h_i(X_i)]$$

Notice that now the equation (15) depends only on the functions $h_i(X_i)$, so we can denote $h_i(X_i)$ as Z_i and rewrite:

$$2Z_i = \frac{1}{N-1} \sum_{j \neq i} \mathbb{E}[Z_j|Z_i] \tag{16}$$

Next, we multiply both sides of (16) by Z_i, take the expectation using the law of iterated expectations and add $\mathbb{E}[Z_i^2]/(N-1)$ to both sides:

$$\frac{2N-1}{N-1}\mathbb{E}[Z_i^2] = \frac{1}{N-1} \sum_{j \neq i} \Big(\mathbb{E}[Z_i \mathbb{E}[Z_j|Z_i]] + \mathbb{E}[Z_i^2]\Big) = \frac{1}{N-1} \sum_{j=1}^{N} \mathbb{E}[Z_i Z_j]$$

Applying the Cauchy-Schwarz inequality to the right-hand side of the above equation and subtracting the left hand side from it, we get that the following inequality holds for all i:

$$\sqrt{\mathbb{E}[Z_i^2]} \left(\frac{1}{N-1} \sum_{j=1}^{N} \sqrt{\mathbb{E}[Z_j^2]} - \frac{2N-1}{N-1} \sqrt{\mathbb{E}[Z_i^2]} \right) \geq 0 \tag{17}$$

Without loss of generality, let's assume that $\mathbb{E}[Z_i^2] > 0$, so the second multiplier of (17) is non-negative for all i. Summing up for all i we get that

$$\frac{N}{N-1} \sum_{j=1}^{N} \sqrt{\mathbb{E}[Z_j^2]} - \frac{2N-1}{N-1} \sum_{i=1}^{N} \sqrt{\mathbb{E}[Z_i^2]} = -\sum_{i=1}^{N} \sqrt{\mathbb{E}[Z_i^2]} \geq 0$$

which holds if and only if $\mathbb{E}[(h_i(X_i))^2] = 0$. Therefore, $h_i(X_i) = 0$ almost surely for all $i = 1, \ldots, N$. □

Proposition 1. *Given any outcome of the benchmark formation in the first stage $\{b, k\}$, there is a unique Bayesian Nash Best Response of firms to any search behavior described by $\mu(b)$. Depending on whether firm i revealed or concealed its costs, it sets the price according to the linear pricing strategies given by:*

$$p_R(c_i) = \frac{t}{\mu(b)} + \frac{N-k}{2N-1}c^e + \frac{k}{2N-1}b + \frac{N-1}{2N-1}c_i$$
$$p_C(c_i) = \frac{t}{\mu(b)} + \left(\frac{1}{2} - \frac{k}{2N-1}\right)c^e + \frac{k}{2N-1}b + \frac{1}{2}c_i$$

Proof. Denote the resulting transportation costs as $t_1 = t/\mu(b)$. The profit function of firm i, conditional on the information it has $I_i = \{b, k\} \cup c_i$, is

$$\mathbb{E}_{c_j}[\pi_i(p_i)|I_i] = \frac{1}{t_1}(p_i - c_i)\left(t_1 + \frac{1}{N-1}\sum_{j\neq i}\mathbb{E}[p_j|I_i] - p_i\right) \tag{18}$$

By taking the FOC of (18) we get that optimal p_i satisfies:

$$2p_i = t_1 + c_i + \frac{1}{N-1}\sum_{j\neq i}\mathbb{E}[p_j|I_i]$$

Suppose for now that firms in K follow the symmetric pricing strategy $p_R(c)$ and firms $N \setminus K$ follow symmetric pricing strategy $p_C(c)$. It is clear, that if a firm j didn't reveal its cost, $j \in N \setminus K$, then the cost of firm i, does not contain anything about j's price, so $\mathbb{E}[p_j|I_i] = \mathbb{E}[p_C|b]$. Moreover, since I_i does not allow to distinguish between any two firms in K, we must have that any two expected prices of firms $j_1, j_2 \in K$ are the same: $\mathbb{E}[p_{j_1}|I_i] = \mathbb{E}[p_{j_2}|I_i] = \mathbb{E}[p_R|I_i]$. So, for the symmetric equilibrium we can rewrite FOC in the following way:

$$2p_i = t_1 + c_i + \frac{1}{N-1}\sum_{j\in N\setminus K, j\neq i}\mathbb{E}[p_C|b] + \frac{1}{N-1}\sum_{j\in K, j\neq i}\mathbb{E}[p_R|I_i] \tag{19}$$

The equilibrium candidate is in linear strategies and given by (4):

$$\begin{cases} p_R(c_i) = \alpha_R + \beta_R b + \gamma_R c_i, \text{ if } i \in K \\ p_C(c_i) = \alpha_C + \beta_C b + \gamma_C c_i, \text{ if } i \in N \setminus K \end{cases}$$

We will verify that there are uniquely determined coefficients of p_R and p_C constituting the symmetric equilibrium in linear strategies. Later on, we show that this problem cannot have more than one equilibrium.

So, to compute FOC, we substitute the suggested linear strategies given by (4) into (19). First, suppose firm $i \in K$. The expected price of concealing and revealing firms are given by $\mathbb{E}[p_C|b] = \alpha_C + \beta_C b + \gamma_C c^e$ and $\mathbb{E}[p_R|I_i] = \alpha_R + \beta_R b + \gamma_R \mathbb{E}[c_j|I_i]$ respectively. Let we compute the expected costs of revealing firm j given I_i. Take the sum of $\mathbb{E}[c_j|I_i]$ for all $j \in K$:

$$\sum_{j \in K} \mathbb{E}[c_j|I_i] = \mathbb{E}[kb|I_i] = kb$$

On the other hand, since the strategies are symmetric for any $j_1, j_2 \in K$ we have that $\mathbb{E}[c_{j_1}|I_i] = \mathbb{E}[c_{j_2}|I_i]$ and $\sum_{j \in K} \mathbb{E}[c_j|I_i] = c_i + (k-1)\mathbb{E}[c_j|I_i]$. Therefore, it is natural that the expected costs of competitors is simply the average of $k-1$ costs without taken into account the cost of firm i:

$$\mathbb{E}[c_j|I_i] = \frac{kb - c_i}{k-1}$$

The FOC takes the following form:

$$2p_R(c_i) = t_1 + c_i + \frac{N-k}{N-1}(\alpha_C + \beta_C + \gamma_C c^e) + \frac{k-1}{N-1}\left(\alpha_R + \beta b + \gamma_R \frac{kb - c_i}{k-1}\right)$$

Rearranging to the linear form given by (4), we get first three equation for the coefficients:

$$p_R(c_i) = \frac{1}{2}\left(t_1 + \frac{N-k}{N-1}(\alpha_C + \gamma_C c^e) + \frac{k-1}{N-1}\alpha_R\right) + \frac{1}{2}\left(\frac{N-k}{N-1}\beta_C + \frac{k-1}{N-1}\beta_R + \frac{k}{N-1}\gamma_R\right) b + \\ + \frac{1}{2}\left(1 - \frac{\gamma_R}{N-1}\right) c_i = \alpha_R + \beta_R b + \gamma_R c_i$$

Second, consider a firm $j \in N \setminus K$. This firms has the same expectation about the price of concealing firms $\mathbb{E}[p_C|b] = \alpha_C + \beta_C b + \gamma_C c^e$. Since j didn't contribute to b, its estimation of p_R will be less precise: $\mathbb{E}[c_{i \in K}|I_j] = b$ and so $\mathbb{E}[p_R|I_i] = \alpha_R + (\beta_R + \gamma_R)b$. The FOC of firm j is:

$$2p_C(c_j) = t_1 + c_j + \frac{N-k-1}{N-1}(\alpha_C + \beta_C b + \gamma_C c^e) + \frac{k}{N-1}(\alpha_R + (\beta_R + \gamma_R)b)$$

Rearranging to the linear form given by (4), we obtain

$$p_C(c_i) = \frac{1}{2}\left(t_1 + \frac{N-k-1}{N-1}(\alpha_C + \gamma_C c^e) + \frac{k}{N-1}\alpha_R\right) + \frac{1}{2}\left(\frac{N-k-1}{N-1}\beta_C + \frac{k}{N-1}(\beta_R + \gamma_R)\right) b + \\ + \frac{1}{2} c_j = \alpha_C + \beta_C b + \gamma_C c_j$$

The two FOCs for firms $i \in K$ and $j \in N \setminus K$ result in the system of linear equation with 6

unknowns. The solution of this system is:

$$\alpha_R = t_1 + \frac{N-k}{2N-1}c^e \qquad \beta_R = \frac{k}{2N-1} \qquad \gamma_R = \frac{N-1}{2N-1}$$
$$\alpha_C = t_1 + \left(\frac{1}{2} - \frac{k}{2N-1}\right)c^e \qquad \beta_C = \frac{k}{2N-1} \qquad \gamma_C = \frac{1}{2}$$

Uniqueness. Denote $p^* = (p_R, p_C)$ as an equilibrium pricing strategy. We want to show that there does not exist any other Bayesian equilibrium for any given I_i and μ. Suppose that there is another one equilibrium profile of pricing strategies $\tilde{p}$, possible asymmetric. Then according to the FOC it must hold that for any firm i:

$$2\tilde{p}_i = t_1 + c_i + \frac{1}{N-1}\sum_{j\neq i}\mathbb{E}[\tilde{p}_j|I_i]$$

Subtracting the FOC of p^* from this equation we get:

$$2(\tilde{p}_i(I_i) - p_i^*(I_i)) = \frac{1}{N-1}\sum_{j\neq i}\mathbb{E}[\tilde{p}_j(I_j) - p_j^*(I_j)|I_i]$$

We apply *lemma 5* to this equation by setting $h_i(X_i) = \tilde{p}_i(X_i) - p_i^*(X_i)$, where $X_i = I_i$ and get that $h_i(X_i) = 0$ for all $i = 1, \ldots, N$:

$$\tilde{p}_i = p_i^*$$

Therefore, this problem has the unique equilibrium for every μ that is characterized by the linear pricing strategies p_R and p_C. □

Proposition 2. *The ex ante equilibrium fraction of informed consumers is lower for a higher number of revealing consumers k: $\tilde{s}_k > \tilde{s}_{k+1}$ and $\mu_k > \mu_{k+1}$.*

Proof. The expected fraction of informed consumers μ_k depends on the expected search cost of the indifferent consumer $\tilde{s}_k = \mathbb{E}\tilde{s}_k(b)$:

$$\tilde{s}_k = t\mathbb{E}_p\left(\frac{1}{2} + \frac{p_i - p_j}{2t}\right)^2 = \frac{t}{4} + \frac{1}{4t}\mathbb{E}_p(p_i - p_j)^2 \tag{20}$$

Since consumers do not know which firms revealed information they consider possible three situation. Consider a consumer who is interested in products i and j. Denote the probability that two (both i and j), one (either i or j) or none firms revealed their cost as $q_2(k)$, $q_1(k)$ and $q_0(k)$ correspondingly. Denote the variance of the price difference as $z_m = E_m(p_i - p_j)^2$

if $m = \{0, 1, 2\}$ firms revealed costs. According to the corollary 1, z_m decreases in m since prices become less dispersed. Consider the last term in (20):

$$\mathbb{E}_p(p_i - p_j)^2 = \sum_{m=0}^{2} q_m(k) z_m$$

such that $\sum_{m=0}^{2} q_m(k) = 1$ for any k. Comparing the difference we get:

$$\tilde{s}_{k+1} - \tilde{s}_k = \sum_{m=0}^{2} (q_m(k+1) - q_m(k)) z_m < (q_0(k+1) - q_0(k)) z_0 +$$
$$+ z_1 \sum_{m=1}^{2} (q_m(k+1) - q_m(k)) = \underbrace{(z_0 - z_1)}_{>0} \underbrace{(q_0(k+1) - q_0(k))}_{<0} < 0$$

The first inequality comes from the fact that the probability of dealing with two revealing firms $q_2(k) = C_k^2 / C_N^2$ is higher for higher k and $z_1 > z_2$. Then we use the fact the sum of probabilities $q_0 + q_1 + q_2 = 1$ for all k and since the chance to deal with two concealing firms $q_0(k) = C_{N-k}^2 / C_N^2$ gets lower for higher k, we get the last inequality.

Since $\mu_{k+1} - \mu_k = (1 - \lambda)(\tilde{s}_{k+1} - \tilde{s}_k)/\overline{s}$ we have that μ_k decreases in k as well. □

Proposition 3. *It is optimal to conceal private costs in the market without search friction* ($\lambda = 1$)*:* $\Pi_1^C(k) > \Pi_1^R(k+1)$ *for all* $k = 0, 1, \ldots N-1$.

Proof. Since $\lambda = 1$, all consumers have zero search costs and buy after inspecting two firms, $\mu = 1$. Importantly, the learning of consumers does not take place, so private and public information exchange regimes coincide and will proceed without indexing the equilibrium profit functions: $\Pi_1^R(k) = \Pi_0^R(k) = \Pi^R(k)$. We start from the situation in which k firms decided to reveal their costs and form the revealing group K.

The equilibrium profit (multiplied by t) of firm $i \in N \setminus K$ that reveals its costs, is given by:

$$t\Pi^R(k+1) = \mathbb{E}(p_R(c_i) - c_i)^2 = t^{-1}\mathbb{E}\left(t + \frac{N-k-1}{2N-1}(c^e - c_i) + \frac{k+1}{2N-1}(b_{k+1} - c_i)\right)^2 =$$

We rearrange the last expression, by taking into account that $\mathbb{E}[c^e - c_i] = \mathbb{E}[b_{k+1} - c_i] = 0$:

$$= t^2 + \left(\frac{N-k-1}{2N-1}\right)^2 \sigma_c^2 + 2\frac{(N-k-1)(k+1)}{(2N-1)^2}\mathbb{E}(c^e - c_i)(b_{k+1} - c_i) +$$
$$+ \left(\frac{k+1}{2N-1}\right)^2 \mathbb{E}(b_{k+1} - c_i)^2 \qquad (21)$$

To compute (21) we have to separately consider the second and the third terms. In the derivation of the second term we use the fact that the costs are distributed independently:

$$\mathbb{E}(c^e - c_i)(b_{k+1} - c_i) = \frac{1}{k+1}\sum_{j\neq i}\mathbb{E}(c^e - c_i)(c_j - c^e) + \frac{1}{k+1}\sum_{j\neq i}\mathbb{E}(c^e - c_i)^2 = \frac{k}{k+1}\sigma_c^2$$

Next we compute the third term of (21) by substituting the expression for b_{k+1} and simplifying of the square expression:

$$\mathbb{E}(b_{k+1}-c_i)^2 = \frac{1}{(k+1)^2}\mathbb{E}\left(\sum_{j\in K}(c_j - c_i)\right)^2 = \frac{1}{(k+1)^2}\left(\sum_{j\in K}\mathbb{E}(c_j - c_i)^2 + 2\sum_{\substack{j,z\in K\\ j\neq z}}\mathbb{E}(c_j - c_i)(c_z - c_i)\right)$$

It is easy to check that $\mathbb{E}(c_j - c_i)^2 = 2\sigma_c^2$ and $\mathbb{E}(c_j - c_i)(c_z - c_i) = \sigma_c^2$. Since there are k firms in K and C_k^2 combination of two randomly taken firms from K, we get the expression for the third term:

$$\mathbb{E}(b_{k+1} - c_i)^2 = \frac{1}{(k+1)^2}(2\sigma_c^2 k + 2C_k^2\sigma_c^2) = \frac{k}{k+1}\sigma_c^2$$

We notice that the second and the third terms coincide and substitute them back to (21) and get a simple expression for the equilibrium profit:

$$t\Pi^R(k{+}1) = t^2 + \frac{(N-k-1)^2 + 2(N-k-1)k + (k+1)k}{(2N-1)^2}\sigma_c^2 = t^2 + \left(\left(\frac{N-1}{2N-1}\right)^2 + \frac{k}{(2n-1)^2}\right)\sigma_c^2$$

Now consider the equilibrium profit (multiplied by t) of concealing firm $j \in N \setminus K$

$$t\Pi^C(k) = \mathbb{E}(p_C(c_j) - c_j)^2 = \mathbb{E}\left(t + \frac{k}{2N-1}(b_k - c_j) + \frac{1}{2}(c^e - c_j)\right)^2$$

Notice that firm j does not reveal its cost, therefore $\mathbb{E}(b_k - c_j)^2 = \sigma_c^2/k$. By simplifying we get:

$$t\Pi^C(k) = t^2 + \left(\frac{1}{4} + \frac{k}{(2N-1)^2}\right)\sigma_c^2 \tag{22}$$

We prove the proposition by comparing the equilibrium profits:

$$\Pi^C(k) - \Pi^R(k+1) = \left(1 - \left(\frac{2N-2}{2N-1}\right)^2\right)\frac{\sigma_c^2}{4t} > 0$$

□

Lemma 6. *For all $k < N$, the difference $\Delta\mu = \mu_k - \mu_{k+1}$ does not depend on k.*

Proof. To show that $\Delta\mu$ does not depend on k, it suffice to show that the difference in the search costs of indifferent consumers does not depend on k: $\Delta\mu = (1-\lambda)(\tilde{s}_k - \tilde{s}_{k+1})/\overline{s}$. Denote q_m as the probability that $m \in \{0,1,2\}$ firms revealed their costs consequently. Expected gain of search is the expected effective price difference given this weights:

$$\tilde{s}_k/t = q_2\mathbb{E}\left(\frac{1}{2} + \frac{N-1}{2N-1}(c_i - c_j)/2t\right)^2 + q_1\mathbb{E}\left(\frac{1}{2} + \frac{1}{2}(c_i - c^e)/2t + +\frac{N-1}{2N-1}(c^e - c_j)/2t\right)^2 + \\ +q_0\mathbb{E}\left(\frac{1}{2} + \frac{1}{2}(c_i - c_j)/2t\right)^2$$

Denote $\psi = \left(\frac{2N-2}{2N-1}\right)^2 < 1$. By computing the probabilities of these three events and taking the expectations, we get

$$\tilde{s}_k/t = \frac{1}{4} + \frac{1}{2N(N-1)}[k(k-1)\psi + k(N-k)(1+\psi) + (N-k)(N-k-1)]\frac{\sigma^2}{4t^2}$$

which can be simplified further and the final expression is as follows:

$$\tilde{s}_k = \frac{t}{4} + \frac{1}{2N}\left(N - k(1-\psi)\right)\frac{\sigma_c^2}{4t}$$

Next, we compute the difference between the search costs of indifferent consumers in case of k and $k+1$ numbers of revealing firms and see that it does not depend on k

$$\tilde{s}_k - \tilde{s}_{k+1} = \frac{1}{2N}(1-\psi)\frac{\sigma_c^2}{4t} > 0.$$

Therefore, $\Delta\mu$ does not depend on k either. □

Lemma 2. *If information sharing is performed privately, then the revelation is more profitable (less unprofitable) for higher number of revealing firms k: $\Pi_0^R(k+1) - \Pi_0^C(k) > \Pi_0^R(k) - \Pi_0^C(k-1)$ for all $k = 0, 1, \ldots N-1$.*

Proof. The effect of information revelation on the profits can be split in two parts. First, information revelation increases the price level (square brackets) at which firms base their prices. Second, it changes the price variability which negatively affects profits (curly brackets). The profit difference can be written in the following form:

$$\Delta\Pi_0(k) = \Pi_0^R(k+1) - \Pi_0^C(k) = t\left[\frac{1}{\mu_{k+1}} - \frac{1}{\mu_k}\right] + \{\mu_{k+1}(\psi + \phi(k)) - \mu_k(1+\phi(k))\}\frac{\sigma_c^2}{4t} \quad (23)$$

, where $\psi = \left(\frac{2N-2}{2N-1}\right)^2 < 1$ and $\phi(k) = \frac{4k}{(2N-1)^2}$. We want to show that the increase (decrease) in profits is larger (smaller) for higher k: $\Delta\Pi_0(k) > \Delta\Pi_0(k-1)$. We will prove separately

that i) the positive effect of higher price basis described by the curly brackets is larger for higher k and ii) the negative effect of price variability is lower for higher k.

i)Using lemma (6) and the fact that $\mu_{k+1} < \mu_k < \mu_{k-1}$ we have that:

$$\left[\frac{1}{\mu_{k+1}} - \frac{1}{\mu_k}\right] - \left[\frac{1}{\mu_k} - \frac{1}{\mu_{k-1}}\right] = \frac{\Delta\mu}{\mu_{k+1}\mu_k\mu_{k-1}}(\mu_{k-1} - \mu_{k+1}) > 0$$

ii) Denote $V(k)$ as the size of the negative effect of price variability described by the curly brackets in (23). We rearrange the terms in $V(k)$ and use lemma (6):

$$V(k) = \mu_k(1 + \phi(k)) - \mu_{k+1}(\psi + \phi(k)) = \Delta\mu(\psi + \phi(k)) + \mu_k(1 - \psi)$$

To show that the negative effect is stronger for smaller k we have to show that $V(k)$ is decreasing in k. By comparing the difference we prove the proposition:

$$V(k-1) - V(k) = \Delta\mu(\phi(k-1) - \phi(k) + (1 - \psi)) > 0$$

since for any $N \geq 2$ we have that two multipliers are positive:

$$\phi(k-1) - \phi(k) + (1 - \psi) = -\frac{4}{(2N-1)^2} + 1 - \left(\frac{2N-2}{2N-1}\right)^2 = \frac{4N-7}{(2N-1)^2} > 0$$

□

Lemma 4. $\Pi_1^R(k+1) - \Pi_1^C(k)$ *is decreasing in* λ *for all* $k = 0, 1, \ldots N-1$.

Proof. To prove this lemma let's first show that the expected total fraction of consumers μ_k is increasing in λ faster for higher number of revealing firms k. Notice, that we can represent the derivative of μ'_k as the function of μ_k:

$$\mu'_k = 1 - \tilde{s}_k/\overline{s} = \frac{1 - \mu_k}{1 - \lambda}$$

Therefore, the sign of the difference between the derivatives is the opposite sign of the difference between the functions: $sign(\mu'_{k+1} - \mu'_k) = sign(\mu_k - \mu_{k+1}) > 0$, since μ_k is decreasing in k. The equilibrium profit of firm i can be written in the following general form:

$$\Pi_1(k) = t\mathbb{E}_b\left[\frac{1}{\mu_k(b)} + \mu_k(b)T_k(b)\right]$$

, where $T_k(b) < 1$. Next, we show that the derivative of the profits difference is decreasing. Denote $z_1 = \min_b\{\mu_k^2(b), \mu_{k+1}^2(b)\}$ and $z_2 = \max\{T_{k+1}^R(b), T_k^C(b)\}$. It is clear that $z_1 < 1$ as

long as $\lambda < 1$ and $z_2 < 1$ given that $T_k(b) < 1$ for all b and k.

$$\frac{\partial}{\partial\lambda}\left(\Pi_1^R(k+1) - \Pi_1^C(k)\right) = t\mathbb{E}\left[\frac{\mu_k'(b)}{\mu_k^2(b)} - \frac{\mu_{k+1}'(b)}{\mu_{k+1}^2(b)} + \mu_{k+1}' T_{k+1}^R(b) - \mu_k' T_k^C(b)\right] \leq$$
$$\leq t\mathbb{E}\left[\frac{\mu_k'(b) - \mu_{k+1}'(b)}{z_1} + (\mu_k'(b) - \mu_{k+1}'(b))z_2\right] <$$
$$< t\mathbb{E}(\mu_k'(b) - \mu_{k+1}'(b))(1 - 1/z_1) = t\underbrace{(\mu_k' - \mu_{k+1}')}_{>0}\underbrace{(1 - 1/z_1)}_{<0} < 0$$

□

Lemma 1. *Irrespectively of whether firm $i \in K$ or $i \in N \setminus K$, it is better off if information sharing is public rather than private: $\Pi_1^R(k) > \Pi_0^R(k)$ and $\Pi_1^C(k) > \Pi_0^C(k)$ for all $k > 0$ (equal if $k = 0$).*

Proof. The equilibrium profit function in case of public information revelation can be represented as

$$\Pi_1(k) = t\mathbb{E}_b\left[\frac{1}{\mu_k(b)} + \mu_k(b)T_k(b)\right] \tag{24}$$

,where $T_k(b) < 1$ for all b. The equilibrium profit in the case of private revelation is given by:

$$\Pi_0(k) = t\left[\frac{1}{\mu_k} + \mu_k T_k\right] \tag{25}$$

,where $\mu_k = \mathbb{E}_b\mu_k(b)$ and $T_k = \mathbb{E}_b T_k(b)$ are the expectation taken before the benchmark is formed. By Jenssen inequality we have that: $\mathbb{E}\mu_k^{-1}(b) > (\mathbb{E}\mu_k(b))^{-1} = \mu_k^{-1}$. Second, since $\mathrm{Cov}(\mu(b), T_k(b)) > 0$, we have that $\mathbb{E}_b[\mu_k(b)T_k(b)] > \mu_k T_k$. Since each of the term in (24) is higher than in (25) we get that $\Pi_1(k) > \Pi_0(k)$ for all $k > 0$.

□

Proposition 4. *For all $k \geq \tilde{k}$, $\lambda_k \in (0, 1)$ are uniquely determined and increase in k:*

$$\lambda_{\tilde{k}} < \lambda_{\tilde{k}+1} < \ldots < \lambda_N$$

Proof. Consider the situation under which $k > \tilde{k}$ firms decide to reveal their costs and the benchmark b_k is formed.

First, we prove that λ_k exists and unique. From the definition of $\tilde{k}$ there exists $\tilde{\lambda}$ such that $\tilde{k}+1$ firm find it optimal to contribute to the benchmark formation: $\Pi_1^R(\tilde{k}+1, \tilde{\lambda}) - \Pi_1^C(\tilde{k}, \tilde{\lambda}) > 0$. Since firms gain form revealing costs under the coalition of $\tilde{k}$ firms, we have that by

proposition 3, it will be optimal to reveal costs for all coalitions including more than $\tilde{k}$ firms at $\lambda = \tilde{\lambda}$:

$$\Pi_1^R(k+1, \tilde{\lambda}) - \Pi_1^C(k, \tilde{\lambda}) > \Pi_1^R(\tilde{k}+1, \tilde{\lambda}) - \Pi_1^C(\tilde{k}, \tilde{\lambda}) > 0$$

Since $\Pi_1^R(k+1) - \Pi_1^C(k)$ is decreasing in λ by lemma 4, and firms conceal their costs when $\lambda = 1$ (by proposition 3) and decide to reveal when $\lambda = \tilde{\lambda} < 1$, then due to the fixed point argument we have that there exists unique $\lambda_k \in (\tilde{\lambda}, 1)$ such that firms are indifferent between revelation and concealment of their costs: $\Pi_1^R(k+1, \lambda_k) - \Pi_1^C(k, \lambda_k) = 0$.

Second, we show that λ_k is increasing in k. From the definition of λ_k and proposition 3 we have that it is optimal to reveal at $\lambda = \lambda_k$ if the coalition consists of $k+1$ firms:

$$\Pi_1^R(k+2, \lambda_k) - \Pi_1^C(k+1, \lambda_k) > \Pi_1^R(k+1, \lambda_k) - \Pi_1^C(k, \lambda_k) = 0$$

Applying the fixed point argument, we find that $\lambda_{k+1} \in (\lambda_k, 1)$ and therefore λ_k is an increasing function in k.

Second, we show that λ_k is an increasing function in $\overline{s}$. From the implicit function theorem we have that:

$$\frac{\partial \lambda_k}{\partial \overline{s}} = -\frac{\partial}{\partial \overline{s}} \left(\Pi_1^R(k+1, \lambda_k) - \Pi_1^C(k, \lambda_k) \right) \Big/ \frac{\partial}{\partial \lambda} \left(\Pi_1^R(k+1, \lambda_k) - \Pi_1^C(k, \lambda_k) \right)$$

By lemma 4, we have that the denominator is positive, so it remains to explore the sign of the numerator. The derivative of $\mu_k(b)$ with respect to $\overline{s}$ is given by:

$$\mu_k'(b) = \frac{\partial \lambda_k}{\partial \overline{s}} \mu_k = -\frac{(1-\lambda)\tilde{s}_k(b)}{\overline{s}^2}$$

Denote $\mu_k' = \mathbb{E}_b \mu_k'(b)$, then $\mu_k' > \mu_{k+1}'$ since $\tilde{s}_k < \tilde{s}_{k+1}$. Denote $z_1 = \max_b\{\mu_k^2(b)\} < 1$, $z_2 = \max\{T_{k+1}^R(b), T_k^C(b)\} < 1$.

$$\frac{\partial}{\partial \overline{s}} \left(\Pi_1^R(k+1, \lambda_k) - \Pi_1^C(k, \lambda_k) \right) = t\mathbb{E} \left[\frac{\mu_k'(b)}{\mu_k^2(b)} - \frac{\mu_{k+1}'(b)}{\mu_{k+1}^2(b)} + \mu_{k+1}' T_{k+1}^R(b) - \mu_k' T_k^C(b) \right] >$$
$$> t\mathbb{E} \left[\frac{\mu_k'(b) - \mu_{k+1}'(b)}{z_1} - (\mu_k'(b) - \mu_{k+1}'(b)) z_2 \right] = t(\mu_k' - \mu_{k+1}')(1/z_1 - z_2) > 0$$

Numerator is positive, therefore $\partial \lambda_k / \partial \overline{s} > 0$.

□

Proposition 5. *Consider the benchmark formation game in which firms decide whether to reveal their costs or not.*

1. If $\tilde{k} = 0$ (high $\overline{s}$), then i) if $\lambda \leq \lambda_0$ then all firms reveal their costs, $k^* = N$. ii) If $\lambda_0 < \lambda \leq \lambda_N$ then there are two pure strategy equilibria, $k_1^* = 0$ and $k_2^* = N$. In this case, firms are better off by coordinating on revelation: $\Pi_1^R(N) > \Pi_1^C(0)$. iii) If $\lambda > \lambda_N$ then all firms conceal their costs, $k^* = 0$.

2. If $\tilde{k} > 0$ (intermediate $\overline{s}$), then j) if $\lambda \leq \lambda_N$ then there are two pure strategy equilibria, $k_1^* = 0$ and $k_2^* = N$. In this case, firms are better off by coordinating on revelation: $\Pi_1^R(N) > \Pi_1^C(0)$. jj) If $\lambda > \lambda_N$ then all firms conceal their costs, $k^* = 0$.

3. If $\nexists \tilde{k}$ (low $\overline{s}$) described by definition 2, then firms will conceal their costs, $k^* = 0$.

Proof. 1. First, consider the case in which $\tilde{k} = 0$. i) By proposition 4, there is a positive fraction of informed consumers λ_0, such that it would be optimal to reveal if all firms conceal information. By proposition 3 we find that for all $\lambda \leq \lambda_0$ there is no stable coalition of $m < N$ revealing firms, because each time a concealing firm would deviate to revelation:

$$\Pi_1^R(N, \lambda) - \Pi_1^C(N-1, \lambda) > \ldots > \Pi_1^R(m+1, \lambda) - \Pi_1^C(m, \lambda) > \ldots > \Pi_1^R(1, \lambda) - \Pi_1^C(0, \lambda) > 0$$

ii) Consider an intermediate $\lambda_0 < \lambda \leq \lambda_N$. By proposition 4 there exists λ_k such that $\lambda_{k-1} < \lambda \leq \lambda_k$. Since $\lambda_{k-1} < \lambda$ we have that each coalition consisting of k or lower number of revealing firms $1 \leq m \leq k$ is not an equilibrium, because each firm will deviate to concealment:

$$0 = \Pi_1^R(k, \lambda_{k-1}) - \Pi_1^C(k-1, \lambda_{k-1}) \geq \Pi_1^R(k, \lambda) - \Pi_1^C(k-1, \lambda) > \ldots \Pi_1^R(m, \lambda) - \Pi_1^C(m-1, \lambda)$$

Moreover, since $\lambda \leq \lambda_k$ we have that all revealing coalitions that consists of $k < m < N$ firms cannot constitute an equilibrium because firms from $N \setminus K$ will prefer to reveal:

$$0 = \Pi_1^R(k+1, \lambda_k) - \Pi_1^C(k, \lambda_k) \leq \Pi_1^R(k+1, \lambda) - \Pi_1^C(k, \lambda) < \ldots < \Pi_1^R(m, \lambda) - \Pi_1^C(m-1, \lambda)$$

Therefore, in the intermediate range of $\lambda_0 < \lambda \leq \lambda_N$, there are two equilibria in pure strategies, in which firms either conceal or reveal their costs, $k^* = 0$ or $k^* = N$.

There are also mixed strategy equilibria in each interval $\lambda_{k-1} < \lambda < \lambda_k$ which we do not characterize. Instead, we show that if firms reveal their costs, the industry profit attains its maximum over all other equilibria including the ones in mixed strategies.

Let's check that the profit of concealing firm $\Pi_1^C(k)$ is higher for higher k. The equilibrium profit can be represented in the following form (see the proof of lemma 4):

$$\Pi_1(k) = t\mathbb{E}_b \left[\frac{1}{\mu_k(b)} + \mu_k(b) T_k(b) \right]$$

,where $T_k(b) < 1$ for all b. Denote $z_1 = \min_b \mu_{k-1}(b) < 1$ and $z_2 = \max_b\{T_k(b), T_{k-1}(b)\} < 1$ and compare the difference in profits:

$$\Pi_1^C(k) - \Pi_1^C(k-1) = t\mathbb{E}_b\left[\frac{1}{\mu_k(b)} - \frac{1}{\mu_{k-1}(b)} + \mu_k(b)T_k(b) - \mu_{k-1}(b)T_{k-1}(b)\right] >$$
$$> t\mathbb{E}_b\left[\frac{\mu_{k-1}(b) - \mu_k(b)}{z_1} - (\mu_{k-1}(b) - \mu_k(b))z_2\right] = \underbrace{(\mu_{k-1}(b) - \mu_k(b))}_{>0}\underbrace{(1/z_1 - z_2)}_{>0} > 0$$

Therefore, since $\Pi_1^C(k)$ is increasing in k we have that $\Pi_1^C(N-1) > \Pi_1^C(0)$. We showed above that for all $\lambda_0 < \lambda < \lambda_N$ we have that it is optimal to reveal costs of other $N-1$ firms reveal. Combining these two facts we get that the equilibrium profit under revelation is higher than under concealment:

$$\Pi_1^R(N) - \Pi_1^C(0) > \Pi_1^R(N) - \Pi_1^C(N-1) > 0$$

iii) Consider the case where there is a big fraction of informed consumers $\lambda > \lambda_N$. In this case firms will always deviate to concealing their private costs:

$$0 = \Pi_1^R(N, \lambda_N) - \Pi_1^C(N-1, \lambda_N) > \Pi_1^R(N, \lambda) - \Pi_1^C(N-1, \lambda) > \ldots > \Pi_1^R(m, \lambda) - \Pi_1^C(m, \lambda)$$

2. If $\tilde{k} > 0$ then $\Pi_1^C(m-1) > \Pi_1^R(m)$ for all λ if $m \leq \tilde{k}$. Therefore, similar to the proof of case 2, one can show that there are two equilibria if $\lambda \leq \lambda_N$ and unique equilibrium $k^* = 0$ if $\lambda > \lambda_N$.

3. Third, suppose that there is no $\tilde{k}$ defined in 2, then all revelation coalitions including $k = 1, \ldots, N$ firms are not stable because firms will deviate to concealment: for all λ we have that $\Pi_1^C(k, \lambda) > \Pi_1^R(k, \lambda)$ (from definition). Therefore, firms conceal costs in equilibrium and $k^* = 0$.

□

Proposition 6. $\lambda_0 > 0$ *exists and firms reveal their costs for* $\lambda < \lambda_0$ *if search costs are high enough:* $\overline{s}/t > \omega(N, \sigma_c^2/4t^2)$, *where*

$$\omega^2(N, x) = \left(\frac{x}{2} + N\left(\frac{1}{4} + z(N)x\right)\right)\left(\frac{1}{2} + x\right)\left(\frac{1}{4} + z(N)x\right)$$

and

$$z(N) = \frac{1}{2}\left(1 - \left(1 - \left(\frac{2N-2}{2N-1}\right)^2 / N\right)\right).$$

Proof. Firms find it optimal to deviate from the no sharing regime and reveal their private costs if $\Pi_1^R(1,\lambda) > \Pi_1^C(0,\lambda)$. By lemma 4, λ_0 exists if it is optimal to reveal in the market without shoppers: $\Pi_1^R(1,\lambda=0) > \Pi_1^C(0,\lambda=0)$. Applying lemma 1 we can show that if firms want to reveal under private information sharing, they will do so if information is hared publicly:

$$\Pi_1^R(1,\lambda=0) - \Pi_1^C(0,\lambda=0) > \Pi_0^R(1,\lambda=0) - \Pi_0^C(0,\lambda=0)$$

Explore the sign of the difference in profits $\Delta\Pi_0(1)$ in the private case.

$$t^{-1}\Delta\Pi_0(1) = \left(\frac{1}{\mu_1} - \frac{1}{\mu_0}\right) - (\mu_0 - \psi\mu_1)x = (\mu_0 - \mu_1)\left(\frac{1}{\mu_1\mu_0} - x\right) - (1-\psi)\mu_1 x \tag{26}$$

, where $\psi = (2N-2)^2/(2N-1)^2 < 1$ and $x = \sigma_c^2/4t^2$. By lemma 6, we can find the analytical expression for equilibrium fraction of informed consumers when $\lambda = 0$:

$$\mu_k = \frac{t}{\overline{s}}\left(\frac{1}{4} + \frac{1}{2N}(N-k+\psi k)x\right) \tag{27}$$

To explore the sign of profit difference, we denote $z(N) = \frac{1}{2}\left(1 - \frac{1-\psi}{N}\right)$ and substitute the expressions for μ_0 and μ_1 given by (27) to (26)

$$t^{-1}\Delta\Pi_0(1) = \left(\frac{1}{2} - z(N)\right)\left[\frac{(\overline{s}/t)^2}{\left(\frac{1}{4}+\frac{1}{2}x\right)\left(\frac{1}{4}+zx\right)} - x\right] - x(1-\psi)\left(\frac{1}{4}+zx\right)$$

By simplifying, we get that it is optimal to reveal and $\Delta\Pi_0(1) > 0$ if and only if the search costs are high enough and the following condition holds:

$$\left(\frac{\overline{s}}{t}\right)^2 > \left(\frac{x}{2} + N\left(\frac{1}{4} + z(N)x\right)\right)\left(\frac{1}{2}+x\right)\left(\frac{1}{4}+z(N)x\right) = \omega^2(N,x).$$

□