Market Entry and Dynamic Quality Competition

Marc Bourreau
Telecom Paris, IP Paris

Yutec Sun
CREST-ENSAI

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Abstract

This paper examines high-speed mobile network investments in a market challenged by a new entry that induced a breakdown of incumbents' practice of locking in customers with high switching costs. The entry led to both diminished market power and extended rivalry among the incumbents, two forces known to generate opposite incentives for firm's investments in product innovation. We explore which of the two effects prevailed by estimating a dynamic model of innovation competition where the sizes of locked-in demand are determined by the network's pricing and investment strategies. We find evidence that the entry had adverse effect on the incumbents' investment despite the product-market competition softened by consumer switching costs.

Keywords: competition, innovation, investment, dynamic, switching cost

1 Introduction

We examine investments in high-speed networks in the French mobile telecommunications market challenged by a new entry, which was facilitated by the competition authority to stimulate product-market competition in recent years. Before the entry, a predominant share of consumers had long been locked into three established network services that imposed on their customers high barriers to terminating contracts. The limited competition motivated the government's policy to open the market with a spectrum license reserved for the fourth entrant.

The new entry gave rise to the emergence of new products characterized by the absence of contractual switching costs. Contract-free services were introduced not only by the entrant, but also by the incumbents through new product lines that provided them with access to consumers no longer deterred from switching to competing networks.[1] The new products of both entrant and incumbents continued to gain adoption by growing number of consumers, weakening the effectiveness of the existing switching barriers. Hence, it is not surprising that the entrant, along with the fighting brands, has generated significant social surplus at least in the short run (Bourreau, Sun and Verboven, 2021b).

[1] In the literature, it is often called *fighting brand* when the incumbents release such products as a strategic response to the arrival of a new competitive entrant but are unwilling to do so in the absence of the new competition (Johnson and Myatt, 2003; Bourreau, Sun and Verboven, 2021b).

Nevertheless, it is not as straightforward to measure the long run welfare impact of the entry since it produces ambiguous implications for the investment of the incumbent networks. On the one hand, the contract-free services were made available to a broad range of consumers, thus creating new rivalry for consumers who were free to switch. This indicates a possibility that the investment race may have accelerated in the post-entry market (Shapiro, 2011).[2] On the contrary, the intensified competition may have led to reduced demand and market power, diminishing the incentives for investment. This Schumpeterian view is shared by a long line of research including the literature of industrial organization.[3] However, no empirical study has so far analyzed entry in a market operating under high switching costs.

Our goal is to understand how investments respond to competition in a market where the effective barriers to customer's switching behavior are shaped by strategic considerations. We build our analysis on the previous study that analyzes the incumbent's incentives for the fighting brands in the same market (Bourreau et al., 2021b). They find empirical evidence that the extended product lines, which became available only after the entry, came as a result of breakdown in collusion on the restricted supply of contract-free services. Adopting their conclusion, we measure the impact of entry on the innovation of the incumbent high-speed networks under the premise that the entry was an exogenous change of market structure.

The regulated nature of the entry poses at least two important challenges to model-free analysis. First, the entry was a simultaneous nationwide event, leaving no cross-sectional or temporal variation in entry across markets. Thus without directly analyzing the profit incentives, one can only compare investment levels before and after entry, which would be invalid if the entry coincides with other changes omitted in the model. Even if valid instruments were available, they would still be insufficient for a reduced form analysis of our sample since the firms may have already been reacting to the anticipated entry, which was publicly announced in advance.[4]

We develop a dynamic oligopoly model of innovation to measure the impact of entry on the incumbent's incentives for investment in high-speed cellular base stations.[5] The model is characterized by mobile network operators (MNOs) that compete on prices and network qualities under customer switching costs and irreversible investment, both of which generate dynamic implications for investment strategies. To characterize the

[2]This view has its origin from Arrow (1962), who argued for the positive role of competition. In addition to the literature reviewed by Shapiro (2011), Vives (2008) discusses a few oligopoly models where an increase in product substitutability can stimulate product quality innovations. Goettler and Gordon (2011) confirm in a variant of their model that the innovation of technology leaders can increase monotonically with product substitutability.

[3]Some prominent examples include Gilbert and Newbery (1982) and Vives (2008) for theoretical analysis. The empirical literature also finds evidence supporting the same view that innovations are stimulated by increased market concentration (Xu, 2008; Goettler and Gordon, 2011; Hashmi and Van Biesebroeck, 2016).

[4]The announcement was made about two years before the arrival of the entrant in January 10, 2012. It was only two days before the date on which the entrant committed to initiate its service as a part of the 3G spectrum license agreement (ARCEP, 2012, p.63).

[5]We quantify the innovation by the growth of network capitals that determine the quality of communications services. We consider it as a progressive innovation that is continuation of existing innovations. It is analogous to the non-drastic innovation of Tirole (1988), which contrasts with radical or drastic innovation that replaces the legacy technology.

dynamic competition, we adopt the Markov perfect equilibrium framework of Ericson and Pakes (1995) and Doraszelski and Satterthwaite (2010), which has been employed in the relevant empirical literature (Goettler and Gordon, 2011; Ryan, 2012; Collard-Wexler, 2013; Hashmi and Van Biesebroeck, 2016).

We use the two step approach proposed by Bajari, Benkard and Levin (2007) for estimating the sunk cost of investment. The first stage analysis involves estimating the consumer demand and firm's pricing and investment strategies in geographically separated markets. Given the estimated structural parameters, we conduct a counterfactual analysis that excludes the entrant from the market for measuring the impact of entry on the incumbent investments. To solve the large scale dynamic optimization problem, we adopt the Smolyak method of Judd, Maliar, Maliar and Valero (2014), which provides an efficient solution approach that allows us to relax the commonly imposed assumptions such as symmetry, anonymity, and time homogeneity in firm's dynamic strategies.[6] In benchmark analysis, we find that equilibrium network investment increases for all the incumbents in the absence of the entrant. This result confirms the previous findings that the efficiency effect dominates the innovation incentives.

Our analysis adds a new perspective to the broad literature on the role of market structure for innovation, productivity, and economic growth. In the existing literature, extensive evidence has been found for more nuanced effects of competitive pressure (Blundell, Griffith and Van Reenen, 1999; Aghion, Bloom, Blundell, Griffith and Howitt, 2005; Aghion, Blundell, Griffith, Howitt and Prantl, 2009). In particular, Aghion et al. (2005) developed a theory of inverted-U relationship governed by the steady state of innovation race between two rivals. While the nonlinear relationship is identified under varying degrees of market power arising from collusive behavior in their model, the switching costs are not considered in their analysis as a source of market power.

Our work is also closely related to the debate on the role of mergers on innovation. Motta and Tarantino (2017) and Federico et al. (2018) both analyze a oligopoly model of investments under horizontal mergers and find that a merged firm lowers investment since it internalizes the competitive pressure from the innovated product on its other products. Haucap et al. (Forthcoming) provide theoretical and empirical evidence from a pharmaceutical industry that R&D investments decline in the post-merger market. In contrast, Genakos et al. (2018) examine mobile telecom markets across 33 OECD countries and find positive relationship between market concentration and operator's investment. Schmutzler (2013) highlights the asymmetric effects of mergers on the innovation of firms that vary with factors such as cost efficiency. Bourreau, Jullien and Lefouili (2021a) identify various countervailing effects of mergers, which may generate overall ambiguous implications for investments.

In a separate stream, there have been numerous studies in economics and marketing on the relationship between competition and switching costs (for survey of the literature, see Klemperer (1995) and Farrell and Klemperer (2007)). Yet, their analysis has primarily focused on price competition and welfare consequences (e.g., Viard (2007), Dubé, Hitsch

[6] See page 1,894 of Doraszelski and Pakes (2007) for the formal definitions of the first two.

and Rossi (2009), Dubé, Hitsch and Rossi (2010), Biglaiser, Crémer and Dobos (2013), and Shcherbakov (2016)). In contrast, there is still a lack of understanding of how the innovation incentives depend on competition under switching costs. Our empirical study complements this body of research.

Our methodological contribution is concerned with the solution approach for our large scale dynamic model. The firms operate a portfolio of product lines differentiated by production costs, product qualities, and switching costs. In order to account for the persistent variation in the composition of firms and products, we have to relax the assumption of symmetric and anonymous firms that are typically imposed for the sake of tractability. Our model also needs to include multiproduct firms to analyze the innovation incentives generated by extended product portfolios. These considerations require multiple state variables to characterize each firm's competitive position, resulting in the widely known problem of the curse of dimensionality (Aguirregabiria and Nevo, 2013). Furthermore, the anticipated timing of the entry renders the firm's dynamic investment to be nonstationary, i.e., the firm's optimal investment depends on time in addition to the state variables until the arrival of the entrant. Our empirical strategy departs from the empirical IO literature by accommodating these extensions with an efficient empirical strategy.

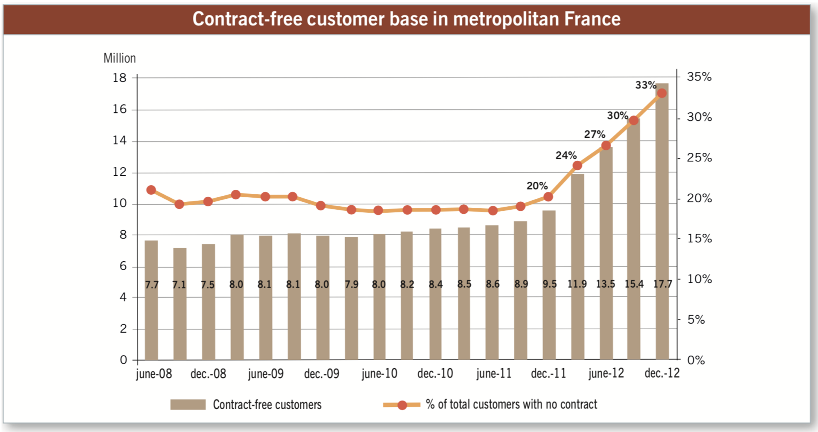
2 Market and data

As commonly observed in industries operating with high customer switching costs, the French mobile market, prior to the latest entry in 2012, was heavily concentrated around three dominant incumbent mobile network operators (MNOs) that had not faced a substantive competition for a long time since 1996.[7] Despite the government's effort to stimulate competition through the entry of mobile virtual network operators (MVNOs), their reliance on the network supply and marketing of the incumbents had limited the ability to aggressively compete with their host operators. As a result, the incumbent MNOs used to control around 88.7% of total subscribers in mainland France in 2011 (ARCEP, 2012, p.64).

In addition, the incumbents had been able to maintain substantial market power by limiting the consumer's mobility between services providers. It was a common practice for the incumbents to lock in consumers with high switching costs from various sources, which include contractual commitment, bundling with other services (e.g., broadband Internet, TV, and fixed telephone, often called as triple or quadruple play) or mobile handset, and the limited duration of prepaid balances that could be extended only through purchasing new credits continuously. The lack of competitive pressure, even in comparison to other OECD countries, had motivated the regulation authority, ARCEP,[8] to set aside a 3G frequency spectrum license for a new entrant (ARCEP, 2012). Finally, Free Mobile obtained the license through a beauty contest and entered the market nationwide in January 2012.

The entry changed the competition through extended rivalry in supplying the contract-free services. Free Mobile began offering postpaid services at competitive prices, most

[7]The last entry was made by the third incumbent in 1996 by Bouygues Telecom.
[8]Autorité de régulation des communications électroniques et des postes.



Source: ARCEP.

Figure 1: Filling the void in the supply of contract-free services

significantly without demanding contractual commitment from the consumers. In response to the new competition, the incumbents immediately followed by introducing similar contract-free services via extended product lines. Interestingly, the extended product line was supplied by each incumbent's new subsidiary that emerged as a *fighting brand* (Bourreau et al., 2021b).

Figure 1, from the ARCEP's annual report, shows that the share of contract-free subscribers had remained stable at 20% until it started to grow rapidly immediately after the entry in early 2012 (ARCEP, 2013, p.85). The continued rise in the contract-free customer base indicates that there may have been a gap in the supply for such products. Our interpretation is supported by an accompanying empirical analysis of Bourreau et al. (2021b), where they find evidence consistent with collusive agreement on the undersupply of products of low switching costs among the incumbents to avoid intense competition in the market without entry.[9] In our analysis, we adopt their view that the expansion of the incumbent's product portfolios was triggered by the change of market structure.

For our analysis, we compile a panel dataset of mobile services in France from January 2011 to December 2014.[10] We obtain average prices and subscriber shares from Kantar consumer survey, which collects the self-reported measures of subscription and consumption from about 7,000 consumers on a monthly basis. The dataset provides information on the demand for the three incumbent MNOs (Orange, SFR, and Bouygues) and the entrant (Free) across 21 geographic divisions called *régions* in mainland France. For the remaining operators, we observe 28 MVNOs, which are then aggregated according to their host networks due to highly sparse observations.

The extensive product variety in the modern mobile market presents a nontrivial challenge to empirical studies. The continuum of price menus available under diverse tariff

[9] Bourreau et al. (2021b) find that the collusion became no longer sustainable due to the competitive pressure from the entrant.

[10] The same data source has also been used by Bourreau et al. (2021b). While they focus on product differentiation by a variety of product lines and subscription types, we consider the market at a more aggregated level of network operators.

structures makes it particularly difficult to construct a price index representing the entire universe of products.[11] As a solution, we adopt a simple approach by defining product as a group of services supplied under each product line brand and considering price as monthly total expenditure on service package, which encompasses subscription and consumption of all component services such as voice call, mobile data, and short messaging services (SMS).[12] Since we study how demand responds to innovation across the product lines, we obtain the price from the average of individual prices for each product line in a given region and time period.[13] We exclude mobile handset payments from the measurement since the competition in the mobile handset industry is not the focus of our analysis.

We measure the network supply by the total number of cellular base stations (often called cell towers) activated by mobile operators in each local region. We obtain information on the location, the date of activation, and the technology generation (2G, 3G, and 4G) from *L'Agence nationale des fréquences* (ANFR), a government agency that authorizes the operation of radio communications facilities. The installed base of the cell towers is an important source of product differentiation for the mobile services since it determines the strength of radio signals and the speed of data transmissions to a large extent. It occupies a major share of capital expenditure for the mobile operators competing to provide a faster and more reliable service. Hence, we use the quantity of cellular base stations as an operating measure of network quality provision and investment.

Prior to the entry, consumer demand appears to have been largely driven by nontrivial switching costs. Table 1 shows that less than 4-5% of the incumbent's subscribers were switching operator from quarter to quarter in the pre-entry periods. After the entry of Free Mobile, we can see in the lower panel new subsidiary brands added to the product lines of the incumbents.[14] The incumbents' main product lines (Orange, SFR, and Bouygues) exhibit a slight drop in the customer retention rates in the post-entry market. But we find that the loss is partly offset by the sales from their subsidiaries, which tends to be relatively high among the customers switching within the same network. Yet, product substitutability is likely to have increased as more consumers migrated to the new contract-free plans supplied by the entrant and the incumbents.

[11] The mobile operators typically offered three types of tariff: prepaid, postpaid, and *forfait bloqué*. Prepaid is a linear tariff that charges only usage fees, and postpaid (called *forfait* in French) is a three-part tariff composed of fixed subscription fee and consumption allowances along with variable fee for usages exceeding the allowances. Forfait bloqué is a flat rate service that comes with fixed allowances only and can thus be viewed as a variation of the postpaid with blocked overusage as its name suggests. Among many factors adding to the complexity of tariff structure are financing for handset device, discounts for within-network call rates or bundling with other services, and duration of contractual commitment.

[12] Only the three incumbent MNOs operated product lines under different brands: Orange, SFR, and Bouygues introduced Sosh, Red, and B&You as fighting brand, respectively.

[13] The individual prices are aggregated across prepaid, postpaid, and forfait bloqué with equal weights to avoid price variation due to fluctuation in demand composition.

[14] Orange, SFR, and Bouygues have introduced Sosh, Red, and B&You, respectively.

	Last period			
	Orange	SFR	Bouygues	Others
Pre entry				
Orange	0.967	0.012	0.009	0.021
SFR	0.010	0.962	0.006	0.017
Bouygues	0.005	0.005	0.957	0.015
Post entry				
Orange	0.940	0.009	0.010	0.014
SFR	0.009	0.940	0.010	0.012
Bouygues	0.004	0.004	0.934	0.008
Free	0.016	0.015	0.016	0.328
Sosh	0.012	0.003	0.002	0.070
Red	0.001	0.013	0.002	0.048
B&You	0.002	0.003	0.011	0.058

The figures display the average share of customer switching on a quarterly basis.
The column heading denotes the network from which consumers switch, and the rows list the network of their destination.

Table 1: Quarterly share of customer switching

3 Model

3.1 Framework

We begin with a model of demand for mobile services of network operators. The list of mobile operators includes all four MNOs (Orange, SFR, Bouygues, and Free) and two MVNO groups (Virgin and others). Their combined market shares are approximately 92.2% on average in our sample. Virgin is the only MVNO that has roaming agreements with all three incumbent operators and occupies the largest share among the MVNOs. The rest of the MVNOs are highly fragmented and thus are grouped by their associated host networks.[15]

Often the MNOs provide multiple product lines through subsidiary brand. Each product line represents a universe of service contracts available to consumers under unique product brands operated by the same operator. For convenience, we use the terms product line and service interchangeably.

For consumer ι subscribing to service k in the preceding time period $t-1$, mobile network operator $i \in \mathcal{N} = \{1, ..., N\}$ offers service $j \in \mathcal{J} = \{1, ..., J\}$, which at present time t yields indirect utility

$$u_{\iota jt|k} = \gamma \log A_{jt} - \alpha p_{jt} + \chi_{jkt} + \xi_{jt} + \epsilon_{jkt} + \epsilon_{\iota jt}, \tag{1}$$

[15]The other choices range from relying on alternative technology such as Wi-Fi to roaming services of foreign operators.

where A_{jt} is a metric of mobile network quality of service j defined as

$$A_{jt} = \delta^{t-1}\tilde{A}_{jt},$$

where $\tilde{A}_{jt}$ is the number of cellular base stations adopting 3G (UMTS) and 4G (LTE) cellular network technology standards. To account for the continual progress in cellular technology, $\tilde{A}_{jt}$ is scaled by spectral efficiency multiplier δ defined as $\delta = \delta^S/\delta^D$, where δ^S represents the rate of spectral capacity made by network equipment suppliers, and δ^D corresponds to the rate of cellular network traffic growth. Therefore, A_{jt} provides an index of quality measured in the effective unit of network traffic normalized to the first quarter of 2011. δ^S is calibrated to be 1.043 based on 18.5% annual growth of spectral capacity reported by Real Wireless Ltd. (2011), and δ^D is set to 1.1 based on the industry estimate of 40% annual mobile traffic growth (Ericsson, 2014, p.12).

The utility function also includes price p_{jt} defined as average fee for both subscription and usage of service j. As discussed in the previous section, we abstract away from particular details on tariff structure to simplify the analysis. On the other hand, ξ_{jt} denotes unobservable component of product quality exogenous to network innovations. For expositional purpose, we omit the expression for observable product quality in this section. ϵ_{jkt} is an independently and identically distributed (i.i.d.) random shock to the utility of switching from k to j, and $\epsilon_{\iota jt}$ is an i.i.d. random taste shock drawn from a normalized extreme value distribution.

In Equation 1, χ_{jkt} captures the costs of switching to alternatives (including outside option) other than service k at time t. Instead of including χ_{jkt} in the utility of every option other than k, without loss of generality we can normalize the utility of outside good to $u_{\iota 0t|k} = \epsilon_{\iota 0t}$ by parametrizing the switching costs as $\chi_{jkt} = \chi 1\{j = k\}$, where χ_k can be conversely interpreted as savings in opportunity costs of switching from service k. To avoid the curse of dimensionality, we further assume that $\chi \neq 0$ only if k is one of the services of the incumbent MNOs under existing contract-based tariff models. The set of such services are denoted by $\mathcal{K} \subset \mathcal{J}$, from which consumer switching entails nontrivial costs.

Since network is shared by all product lines $\mathcal{J}_i \subset \mathcal{J}$ of each operator i, it follows that $A_j = A_i$ for all $j \in \mathcal{J}_i$. Then, the share of consumers switching from service k to j ex post the demand shock ϵ_{jkt} can be written as

$$S^{\epsilon}_{jt|k} = \frac{e^{\delta_{jt}+\chi_{jkt}+\epsilon_{jkt}}}{1 + \sum_{l\in\mathcal{J}} e^{\delta_{lt}+\chi_{lkt}+\epsilon_{lkt}}} \qquad \text{for} \quad j \in \mathcal{J}, \tag{2}$$

where $\delta_{jt} = \gamma \log A_{jt} - \alpha p_{jt} + \xi_{jt}$. Subsequently, the share of subscribers of service j at time t is determined as

$$S_{jt} = \sum_{k\in\mathcal{J}} S^{\epsilon}_{jt|k} S_{kt-1}.$$

In each period, each operator $i \in \mathcal{N} = \{1, ..., N\}$ sets price and network investment (p_{it}, a_{it}) simultaneously to maximize the net present value of flow profits expected to receive over the infinite time horizon.[16] The one-period profit of firm i has the form

$$\Pi_{it} = \sum_{j \in \mathcal{J}_{it}} (p_{jt} - c_{jt}) D_{jt}(s_t, p_t) - C_i(a_{it}, \nu_{it}),$$

where D_{jt} is an ex ante expected demand for service j at prices $p_t = (p_{1t}, ..., p_{Jt})$; c_{jt} is marginal cost, and s_t is a collection of payoff-relevant states observed by the firms; $C_i(a_{it}, \nu_{it})$ is the cost of investing a_{it} units of network capital, which is assumed to be a convex function of a_{it}. ν_{it} is a cost shock that is private information to firm i.

We assume that firms have imperfect knowledge of demand shocks $\xi_t = (\xi_{1t}, ..., \xi_{Jt})$ and $\epsilon_{kt} = (\epsilon_{1kt}, ..., \epsilon_{Jkt})$. Our assumption is motivated first of all by the need to ensure the existence and stability of price equilibrium. Under high switching costs, the firm's price strategy often responds discontinuously to a purturbation in the price of rivals such that pure-strategy equilibrium does not exist. The problem is solved when uncertainty is introduced in ξ_t and ϵ_{kt}. At time t, the firms are assumed to form rational expectations of ξ_{jt} based on a first-order autoregressive (AR 1) process. The firm's belief about ϵ_{jkt} is represented by a normal distribution $N(\mu^{\epsilon}_{jt}, \sigma^{\epsilon}_{j})$, where its mean μ^{ϵ}_{jt} varies across products, time and markets with product-specific variance σ^{ϵ}_{j}. Then under the belief denoted by $F_{\xi,\epsilon}$ about the shocks, the ex ante demand is $D_{jt} = MS_{jt}$, where M is the population of the corresponding region,[17] and S_{jt} is ex ante market share

$$S_{jt} = \sum_{k \in \mathcal{J}} \int S^{\epsilon}_{jt|k} dF_{\xi,\epsilon}(\xi_t, \epsilon_{kt}) S_{kt-1}.$$

Therefore, each firm's period payoff is characterized by a collection of common knowledge states s_t and a private information cost shock ν_{it}, where s_t will be defined in the next section. After observing s_{it} and ν_{it}, firm i simultaneously sets the price and network investment. We assume that it takes one period for the investment to take effect while price decision becomes effective immediately. After the decisions are made, the firms receive the revenue net of the investment cost, and the next-period states are determined. In each time period t, firm i accumulates network capital by investing a_{it} units of base stations in network A_{it} to obtain

$$A_{it+1} = \delta(A_{it} + a_{it}),$$

where a_{it} is the effective investment in new base stations satisfying $a_{it} = \delta^{t-1}(\tilde{A}_{it+1} - \tilde{A}_{it})$.

We focus on the Markov perfect equilibrium (MPE), where firm's strategy depends only on the payoff-relevant state variables (Ericson and Pakes, 1995; Doraszelski and Satterthwaite, 2010). For given pair of the states (s_t, ν_{it}), firm i's best response $\sigma_i(s_t, \nu_{it}) =$

[16]For a model of collusive investment, see Nocke (2007).

[17]The information on M is obtained from the census provided by the national statistics agency, *L'Institut national de la statistique et des études économiques* (INSEE).

(p_{it}, a_{it}) maximizes the intertemporal profit formally defined by value function

$$V_i(s_t, \nu_{it}) = \max_{\{\sigma_{i\tau}\}_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \mathrm{E}\big[\Pi_i(s_\tau, \nu_\tau, \sigma_{i\tau}, p_{-i\tau})|s_t, \sigma_t\big], \tag{3}$$

where $\sigma_{it} = \sigma_i(s_t, \nu_{it})$, $\sigma_t = (\sigma_{1t}, ..., \sigma_{Nt})$, and E is the expectations operator for (s_τ, ν_τ) conditional on (s_t, σ_t) for $\tau = t, ..., \infty$. The firms are assumed to have rational expectation on the state transition that follows a Markov process $F(s_{t+1}|s_t, \sigma_t)$ and the private shock distribution $F_\nu(\nu)$. The equilibrium strategy profile $\sigma = (\sigma_1, ..., \sigma_N)$ is a collection of the best responses that jointly satisfy the Bellman equation

$$V_i(s_t, \nu_{it}) = \max_{\sigma_{it}} \left[\Pi_i(s_t, \nu_{it}, \sigma_{it}, p_{-it}) + \beta \int V_i(s_{t+1}, \nu_{t+1}) dF(s_{t+1}|s_t, a_t) dF_\nu(\nu_{t+1})\right], \tag{4}$$

for all $i = 1, ..., N$.[18] Under static price competition, p_t does not affect the future state and is thus omitted in the state transition process $F(s_{t+1}|s_t, a_t)$.

Under the assumptions made so far, the MPE is a subgame perfect Nash equilibrium that maps payoff-relevant state space $\mathcal{S} \times \mathcal{V}$ to action space $\mathcal{A}$: $\sigma_i(s, \nu_i) = \big(p_i, a_i\big)$ for all $i = 1, ..., N$. The existence of MPE in dynamic game with continuous actions is established by Escobar (2013) and has been assumed in the empirical literature (e.g., Ryan (2012)). However, the existence and uniqueness of pure-strategy equilibrium is not guaranteed for our oligopoly pricing game, due to the departure from the standard logit demand. Our demand system does not belong to the class of models that satisfy the quasi-concavity in profit function (Caplin and Nalebuff, 1991), or the independence of irrelevant alternatives property in demand (Nocke and Schutz, 2018). Hence, there is *a priori* no guarantee that price equilibrium exists in our model. The problem is solved by our assumption on firm's uncertainty about the demand shocks ξ_{jt}, which recovers the existence of price equilibrium in our empirical analysis.[19]

3.2 Investment incentives and market structure

Before setting up the econometric specifications, we seek to obtain a simpler representation of the investment competition from the theoretical model that will guide our understanding the underlying incentives. To simplify the analysis, we consider the network stock A_t as the sole component of the state vector s_t. The Bellman equation (Equation 4) can then be expressed as

$$V_i(s, a_{-i}) = \max_{a_i} \left[\Pi_i(s, a_i) + \beta \mathrm{E}\Big[V_i(s', a'_{-i})\Big| s, a\Big]\right], \tag{5}$$

where we denote the next period actions and states as $a' = a_{t+1}$ and $s' = s_{t+1}$ and suppress the time index t for $a = a_t$ and $s = s_t$ as well as cost shock ν_{it} to simplify the discussion.

[18]Given any opponent strategy (p_{-it}, a_{-it}), the value function is a unique solution to the Bellman equation by Bellman's principle of optimality under certain regularity conditions (Rust, 1988). However, there could exist multiple sets of equilibrium strategy profile satisfying the above Bellman equation.

[19]Without the assumption, the price equilibrium does not exist almost in all cases. A small change in the rival's prices induces a discontinuous jump in the best response price function, such that no configuration of prices satisfies the optimality condition for all firms.

From this, we can derive an Euler equation (the details of derivation are available in the appendix)

$$\mathrm{E}\Big[C'(a_i) - \beta\delta C'(a_i')\Big| s, a_i\Big] = \beta\delta \mathrm{E}\left[\frac{\partial \Pi_i(s', a_i')}{\partial s_i'} + \sum_{j\neq i} \frac{\partial V_i(s', a_{-i}')}{\partial a_j'}\frac{\partial a_j'}{\partial s_i'}\middle| s, a\right]. \tag{6}$$

This Euler equation demonstrates the tradeoff of accelerating the investment schedule in equilibrium. The difference in the left hand side of Equation 6 represents an increase in the expected cost of shifting one unit of future investment a_i' to the present a_i. The right hand side determines the marginal increase in the profit flow as a result of the accelerated investment.

Inside the bracket on the right hand side, the first term reflects the profit incentives of the investment, which can be further decomposed as

$$\frac{\partial \Pi_i(s)}{\partial s_i} = (p_i - c_i)\left[\frac{\partial D_i(s)}{\partial s_i} + \sum_{j\neq i}\frac{\partial D_i(s)}{\partial p_j}\frac{\partial p_j}{\partial s_i}\right] \tag{7}$$

by the envelope theorem applied to the equilibrium flow profit $\Pi_i(s)$, where firm i has a single product, and $\{p_j\}_{\forall j}$ are the Nash equilibrium prices. Within the above bracket, the first term determines the direct impact of investment on demand, and the second characterizes the indirect effect on demand through intensified price competition. In general, the increase in s_i imposes downward pressure on the rival's prices, counteracting the direct impact $\partial D_i/\partial s_i$ to some extent. Hence, the price competition effect generates an incentive to delay the investment to avoid excessive competition (Vives, 2008; Motta and Tarantino, 2017).[20]

Going back to Equation 6, the second term in the bracket defines the strategic effect of investment.[21] It takes into account the impact of rivals' best response a_j' to a marginal improvement in s_i' on the discounted sum of profit flows V_i. Its direction may not be straightforward to determine a priori since the derivative $\partial a_j'/\partial s_i'$ depends on the model primitives as well as the market states. The theoretical literature has considered general cases where the competitive reaction $\partial a_j/\partial s_i$ is simply assumed to be either positive (Aghion et al., 2005) or neutral (Segal and Whinston, 2007), or instead analyzed a specific framework where the rival's equilibrium response can be decreasing in s_i (Nocke, 2007). Similalry, empirical studies have identified negative or non-monotonic relationship between investment and quality of the rivals (Goettler and Gordon, 2011; Hashmi and Van Biesebroeck, 2016; Igami, 2017). Provided that $\partial V_i/\partial a_j < 0$, the downward-sloping strategy in rival's quality motivates firms to accelerate the investment in order to deter the rival's investment.

It can be challenging to characterize the overall impact of market structure on the investment incentives in theoretical analysis. Yet, the direct demand effect $\partial D_i/\partial s_i$

[20] It is also identified as the main source of negative investment impact of mergers by Motta and Tarantino (2017) and Federico et al. (2018).

[21] Without competition, the second term would disappear, and Equation 6 would be reduced to the simplified Euler equations of Pakes (1996) and Aguirregabiria and Magesan (2013) derived for single agent dynamic models.

generally goes down if the incumbent's demand moves to more inelastic area of the curve after the entry. This is analogous to the S-shaped value function of Pakes and McGuire (1994) as described by Whinston (2011). In their model, the R&D rate tends to be high when the firm is located at intermediate states where the value function curve is steepest while it becomes lower as the firm moves away from the steep area of the curve. Our model may also generate similar effect, considering that our framework employs a variation of their discrete choice demand specification.

Nevertheless, there exists one important difference with the standard setting: i.e., the expansion of product lines into the space of contract-free products. The incumbents, by making contract-free services available, began to face different parts of the demand curve depending on the products. Hence, it is not obvious how the overall incentives have responded to the changes in the product-market competition. The analysis is further complicated by the persistent firm heterogeneity in multiple dimensions. Hence, it motivates our empirical analysis to determine which incentive had a prevailing effect.

3.3 Empirical specification

Dynamic games can easily become computationally intractable due to the curse of dimensionality (Doraszelski and Judd, 2012; Aguirregabiria and Nevo, 2013). To mitigate the burden, simplifying assumptions are needed to reduce the state space. We begin by reformulating the profit as a function of margins $m_t = (m_{1t}, ..., m_{Nt})$ instead of prices as

$$\begin{aligned}\Pi_{it} &= \sum_{j \in \mathcal{J}_{it}} (p_{jt} - c_{jt}) D_j(A_t, \xi_t, S_{t-1}, p_t) - C_{it}(a_{it}, \nu_{it}) \\ &= M \sum_{j \in \mathcal{J}_{it}} m_{jt} \sum_{k \in \mathcal{J}} \int \frac{e^{\delta_{jt} + \chi_{jkt} + \epsilon_{jkt}}}{1 + \sum_{l \in \mathcal{J}} e^{\delta_{lt} + \chi_{lkt} + \epsilon_{lkt}}} dF_{\xi,\epsilon}(\xi_t, \epsilon_{kt}) S_{kt-1} - C_{it}(a_{it}, \nu_{it}),\end{aligned}$$

where the common knowledge states s_t are constituted by $\{A_{it}\}_{i \in \mathcal{N}}$, $\{\xi_{jt} - \alpha c_{jt}\}_{j \in \mathcal{J}}$, and $\{S_{kt-1}\}_{k \in \mathcal{K}}$. With abuse of notation, ξ_{jt} denotes the observed part of the exogenous product quality not integrated out above. From this alternative representation, we can see that both product and process innovations, which shift ξ_{jt} and c_{jt} respectively, would have the same implications for the firm's innovation incentives (Motta and Tarantino, 2017).

We abstract away from the pricing decision of the fringe firms by assuming the marginal-cost pricing, i.e., $p_{jt} = c_{jt}$ or equivalently $m_{jt} = 0$ for the MVNO products. This assumption allows us to treat the quality of the MVNO products as exogenous and thus focus on the strategic interactions among the MNOs. This allows us to normalize the product qualities with respect to a composite quality $\omega_t = \log\left(1 + \sum_{j \notin \mathcal{J}} e^{\delta_{jt}}\right)$, where $\mathcal{J}$ is modified to denote the products of the MNOs only. The inclusive value ω_t effectively summarizes the quality of the products of those who are outside the investment competition.[22] Then we can use a new measure $\mu_{jt} = \xi_{jt} - \alpha c_{jt} - \omega_t$ to characterize the exogenous component of the quality of service j relative to the non-investment goods. Under these notations, we

[22] Since we normalize the switching costs to the new products and MVNOs, $\chi_{jk} = 0$ for all $j \notin \mathcal{J}$.

can simplify the profit function as

$$\Pi_{it} = M \sum_{j \in \mathcal{J}_{it}} m_{jt} \sum_{k \in \mathcal{J}} \frac{e^{\gamma \log A_{jt} + \mu_{jt} + \chi_{jk} - \alpha m_{jt}}}{1 + \sum_{l \in \mathcal{J}} e^{\gamma \log A_{lt} + \mu_{lt} + \chi_{lk} - \alpha m_{lt}}} S_{kt-1} - C_{it}(a_{it}, \nu_{it}),$$

where $\mathcal{N}$ now denotes the set of the MNO firms without loss of generality.

Hence, the final set of state variables for firm i is $(s_t, \nu_{it}) = (A_t, \mu_t, S_{t-1}, \nu_{it}) \in \mathcal{S} \times \mathcal{V}$, where $\mathcal{S} = [0, \bar{A}]^N \times \mathcal{R}^J$ for $\bar{A} > 0$, and $\mathcal{V} = \mathcal{R}^1$.[23] The exogenous state μ_{jt} is assumed to follow a stationary first-order Markov process $F_j(\mu_{jt+1} | \mu_{jt})$ parametrized as

$$\mu_{jt+1} = \rho_{j0} + \rho_1 \mu_{jt} + \zeta_{jt} \qquad \forall j \in \mathcal{J}, \tag{8}$$

where ζ_{jt} is an i.i.d. normally distributed random shock. Hence, the dimension of the state space is $N + J + K + 1 = 4 + (2 \times 3 + 1) + 3 + 1 = 15$.

4 Estimation

While it is straightforward to recover the static parameters, the dynamic parameters in the investment cost function are prohibitively costly to estimate since we have to repeatedly solve the dynamic game for many times, which is infeasible even with the simplifying assumptions introduced so far. Instead, we adopt the two-step estimator of Bajari et al. (2007) (BBL), which provides a computationally efficient alternative.

In the first stage, the BBL algorithm estimates the state transition distribution $F(s'|s)$ and the equilibrium investment policy function $\sigma(s)$. This step involves the estimation of the demand in Equation 1 and the marginal cost, by which the state variables are characterized. The equilibrium investment strategy is estimated using a reduced form model, which will be discussed shortly. Given the estimates, the second stage is to form inequality moments from the following necessary and sufficient condition for MPE:

$$V_i(s; \sigma_i, \sigma_{-i}; \theta) \geq V_i(s; \sigma_i', \sigma_{-i}; \theta), \tag{9}$$

where V_i is the value function that jointly satisfies Equation 4 for all $i \in I$, σ is a MPE strategy profile, and σ_i' is an alternative Markov strategy. The BBL algorithm aims to find the parameter θ_0 that least likely violates this equilibrium condition at the observed data. That is,

$$\theta_0 = \arg\min_{\theta \in \Theta} \int_{\chi} \Big(V_i(s; \sigma_i, \sigma_{-i}; \theta) - V_i(s; \sigma_i', \sigma_{-i}; \theta) \Big)^2 dH(s, \sigma_i'), \tag{10}$$

where H is a distribution over the product of the state and action spaces $\mathcal{S} \times \mathcal{A}$, and $\chi = \{(s, \sigma_i') \in \mathcal{S} \times \mathcal{A} : V_i(s; \sigma_i, \sigma_{-i}; \theta) < V_i(s; \sigma_i', \sigma_{-i}; \theta)\}$. Since computing the exact value function is not feasible in practice, Bajari et al. (2007) propose a forward simulation method that numerically computes the value function by simulating the data-generating process. The state transition distribution and equilibrium policy function estimated in the

[23] For the upper bound $\bar{A}$, we count the total number of antenna sites authorized during the sample period and then multiply it by the number of frequency bands available for the network operators.

first stage are used to calculate the next-period states given the strategy profile σ_i'.[24] Since the estimation proceeds in multiple stages, an analytical form of asymptotic standard error is difficult to evaluate. Hence, we compute the bootstrap standard errors following Bajari et al. (2007) and Ryan (2012).

Table 7

Table 7 presents the results for the demand estimation. Column Logit shows the estimates without controlling for price endogeneity. In Column IV, we account for the price endogeneity by using the IV approach of Berry et al. (1995): we consider the network supply as exogenous to pricing decisions and use them as basis for constructing instruments. Specifically, they are the lagged variables of 2G and 3G/4G networks as well as their BLP-type approximations of the theoretical optimal instruments. The IV approach produces increased price impact and more significant estimates for other parameters. This implies that even after controlling for the most important quality determinants such as network outlays, the products are still differentiated in other dimensions.

Column IV-SWC further includes the opportunity costs of switching, which is significantly high. We find a downward shifts in the product fixed effects for the incumbent operators (i.e., Orange, SFR, and Bouygues) that carry high switching costs as a result of the change. Nonetheless, the coefficients for price and network characteristics remain unchanged in size but their efficiency improves slightly.

For more efficient estimation, we account for heteroscedasticity and autocorrelation in the covariance of unobservables by aggregating the moments across time for given product and region. We also include linear and log time trends to account for the progress in the outside technology. We use time elapsed since entry for the new products to control for the remaining intertemporal dependence in the unobservables that may arise from the word of mouth and learning effects in a parsimonious way.

Service provider	Product group	Price (€)	M.C. (€)	Margin (p-c)
Orange	Orange	25.00	10.23	14.78
	Sosh	16.64	11.55	5.09
SFR	SFR	20.76	8.41	12.35
	Red	15.42	10.76	4.67
Bouygues	Bouygues	23.23	13.53	9.70
	B&You	15.86	11.87	3.99
Free	Free	11.54	7.50	4.04

Bootstrap means across 21 regions for 2011Q2–2014Q4 based on 100 replications

Table 2: Markup estimates

[24] In practice, it may be possible that the moment inequality estimator permits only set identification. In that case, Bajari et al. (2007) provide an alternative estimator with appropriate change of notations.

Table 2 summarizes the average marginal cost and markup estimates generated by 100 bootstrap samples drawn from the asymptotic distribution of the parameter estimates in Column IV-SWC of Table 7. The overall results exhibit quite a departure from the standard logit demand, where markups tend to be homogeneous in the absence of consumer inertia. All the subsidiary lines of the incumbents generate substantially lower markups than their original product lines. Yet, they still charge higher margins than the entrant.

Interestingly, the marginal costs tend to be higher for the supposedly low cost subsidiary lines than those main product lines for the incumbents. For example, Orange's marginal cost is €10.71, which is lower than Sosh's €11.84. While negligible, the reversal may be due to the fact that the Orange product group encompasses a wide range of heterogeneous services ranging from prepaid, postpaid, and *forfait bloqué*, which could have led to the lower marginal costs. Moreover, the new service lines (Sosh, Red, and B&You) all provide larger consumption allowances than their older products, which may have contributed to the increased marginal costs.

Given the demand and marginal cost estimates, we estimate the law of motion for the exogenous quality μ_{it} (Equation 8). The bootstrap estimation results are reported in Table 3. The high estimate of ρ_1 (ρ_1=0.913) indicates that there is a relatively strong state dependence in the product quality process. On the other hand, the product fixed effects show a relatively weak heterogeneity in the quality generating process.

Parameter	Estimate	Std. err
ρ_0	0.638	0.278
ρ_1	0.711	0.039
SFR	0.027	0.136
Bouygues	-0.410	0.106
Free	-0.108	0.104
Sosh	-0.736	0.137
B&You	-0.777	0.122
Red	-0.666	0.125
Observations	1,822	
R^2	0.805	

Based on 100 Bootstrap estimations. Orange's fixed effect normalized to 0.

Table 3: AR 1 process of exogenous quality

For estimating a reduced form of investment policy, we encounter a challenge due to the complexity in the firm's strategic considerations. While various flexible models have been employed in the literature to obtain close approximated equilibrium policy functions,[25] it would be difficult to obtain similarly reliable estimates for a market characterized by a large state space and a high degree of firm heterogeneity. Instead, we adopt random forests

[25]Among the examples are cubic b-splines (Ryan, 2012), local linear regressions (Bajari et al., 2007), or higher order polynomials (Hashmi and Van Biesebroeck, 2016).

of Breiman (2001), which is a flexible yet robust machine learning method that requires minimal tuning efforts by the researcher. Random forests has been found to produce superior out-of-sample predictions in a test of various demand estimation methods (Bajari et al., 2015). While a carefully selected parametric function may outperform random forests in fitting the data, our machine learning method has the advantage of accurate and reliable predictions without the need for user interventions due to model sensitivity.

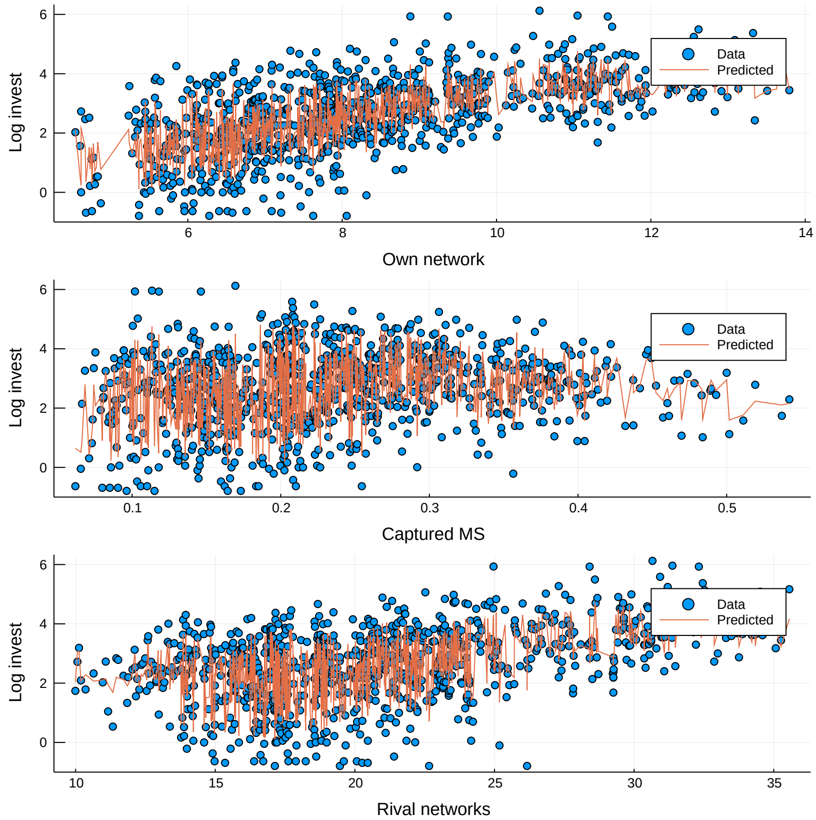


Figure 2: Impact of entry on the incumbent networks

In random forests, we regress the log investment of each firm on the states $s_t = (A_t, \mu_t, S_{t-1})$, population, market fixed effects and time trend.[26] We use 500 trees to obtain function estimates

$$\hat{f}_{it} = \frac{1}{B}\sum_{b=1}^{B} T_b(s_t),$$

where T_b is a single regression tree generated by random selection of training samples and regressors. For each node splitting, the square root number of regressors are selected at random.

Figure 2 shows how the investments vary with some of its state variables. In the top panel, the log investment is displayed as a function of own firm's innovation stock, while

[26]Time trend is included to test for the stationarity of the Markov strategy.

in the middle and bottom panels it is related to share of locked-in customers and sum of rival networks. The dots represent the observed investments, and the lines are the predicted values from one of the random forests estimations. The fitted curves exhibit high fluctuation due to variation in the other states. Still, the overall prediction appears to match the observations reasonably well.[27] While the figure suggests that there might be a positive relationship between investment and own firm's network, a caution is needed for such interpretation since both are endogenously determined.

	1		2	
	Estimate	Std. error	Estimate	Std. error
Invest	50,589	13,803		
Invest^2			435.98	219.95
Log σ_ν	-14.68	13.18	-3.82	13.70
Bootstrap	100		100	
Simulation paths	100		100	

Discount factor β=0.925; each simulation with one shot perturbation above & below the observed equilibrium investments. Investment median=15.4; mean=28.0.

Table 4: Estimation of investment cost

Table 4 displays estimates for the cost function under two different specifications. The first uses a linear specification, and the following column for the quadratic function. In each column, we simulate 100 history paths for future states with a fixed time horizon of 100 quarters, so as to maintain the computation cost within feasible range. Each column simulates two counterfactuals from one-period perturbation of the equilibrium investment policy by one antenna unit upward and downward, respectively.[28]

We find the estimates to be mostly significant. However, the dispersion of cost shock distribution is excessively small. This appears to be an artifact in a computational problem that will be addressed in the future draft. We present Table **??** in the appendix, where the cost function is estimated for a benchmark model that does not incorporate the customer switching costs. The magnitude of the investment costs is in line with those estimates from the benchmark, suggesting that our estimates may not be too far from the correct values.

5 Counterfactual analysis

5.1 Impact of entry on network investment

Given the estimates of the benchmark model, we now analyze the equilibrium investments in counterfactual market that excludes the entrant. It is typically infeasible to numerically solve dynamic games without imposing restrictive assumptions to reduce the state space (Aguirregabiria and Nevo, 2013). To alleviate the problem, the industrial organization

[27]The average R^2 was 0.80 (Orange), 0.79 (SFR), 0.74 (Bouygues), and 0.83 (Free).

[28]We find this approach to be more informative than random or permanent perturbations. The same approach has been used by Hashmi and Van Biesebroeck (2016).

literature usually assumes strategies to be invariant with respect to the identity of own and rival firms (Doraszelski and Pakes, 2007). Often researchers limit the maximum number of active firms or restrict the state space into a small number of discrete values Ryan (2012); Collard-Wexler (2013); Hashmi and Van Biesebroeck (2016). However, these assumptions are overly restrictive in our context since the regulated market accommodates fixed set of firms that operate under heterogeneous quality-generating processes.

Moreover, our model is not time stationary in the pre-entry periods since the optimal investment strategy would adjust to the time remaining until the anticipated entry. Hence, it is necessary to solve the stationary dynamic game in the post-entry periods and then solve backwards the nonstationary games played before the entry recursively. This approach can easily become intractable even in models of relatively small scale.

We overcome the challenge by adopting a sparse-grid algorithm developed by Smolyak (1963). The Smolyak method implemented by Judd et al. (2014) provides an efficient approximation for the value functions in Equation 4. The Smolyak method constructs a sparse set of grid points in the state space on which the value functions are approximated by an efficiently chosen set of basis functions. It has also been used in large scale applications in economics (Winschel and Krätzig, 2010; Brumm and Scheidegger, 2017). Additional implementation details are available in the appendix.

We quantify the changes in the growth of the incumbent networks when the entrant's and incumbent's new products are excluded from the market. This exercise allows us to examine how the innovation incentives respond to an unexpected change in market structure in the long run. To simplify the analysis, we focus on the region of *Île-de-France*, which is the most populated province that includes Paris as a part of the region. We simulate 200 paths of the network growth for the incumbents starting from the second quarter of 2012 for 15 years. By starting from 2012 Q2, our exercise allows us to abstract away without loss of generality from the transitory periods preceding the arrival of the new products from the entrant and incumbents.[29]

Figure 3 displays the growth of the networks starting from 2012 Q2 until the market approximately reaches a steady state. Label "No entry" indicates the effective network supply A_{it} in the market where both Free Mobile and fighting brands are excluded. Label "+Free only" represents the market entered by the entrant exclusively while still excluding the incumbent's new products. Finally, the curve for "+Free/FB" includes all contract-free services from both the entrant and the incumbents.

First of all, the investments in the "No entry" market are highest in all cases across the incumbents. The gap between the curves "No entry" and "+Free/FB" indicates that the investments have gone down for all the incumbents with increased competition in the post-entry market. This result is consistent with the theoretical prediction of Vives (2008) and the empirical findings of Goettler and Gordon (2011) and Hashmi and Van Biesebroeck (2016).

Yet, we find the entry effects to be asymmetric across the firms. In Table 5, the impacts

[29]By the ergodicity of our Markov framework, the long run equilibria remain unaffected by the timing of the counterfactual change.

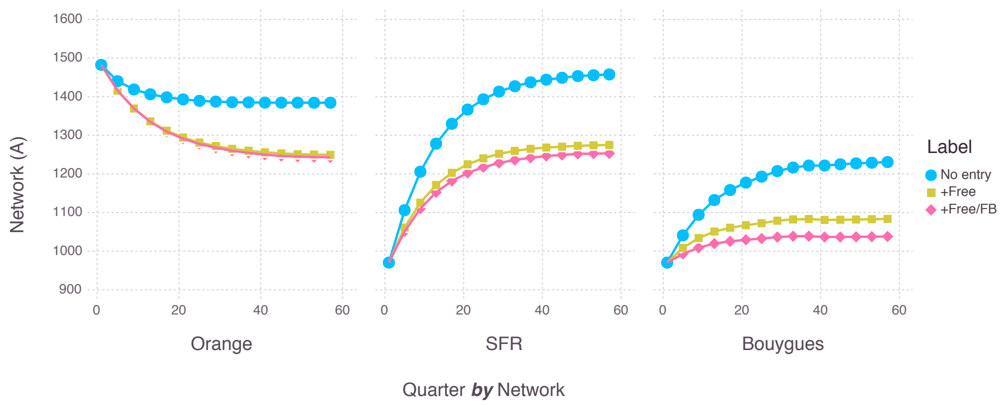


Figure 3: Impact of new competition on the growth of incumbent networks

of the counterfactuals are displayed as changes of network supply with respect to the baseline no entry case. Not surprisingly, we find in Column $\Delta_{\text{Free/FB}}$ that the competitive effect is largest for SFR because it becomes the investment leader in the long run. In terms of relative changes, however, the follower firm Bouygues is predicted to scale down the investment the most by 16% in comparison to Orange and SFR, which had 10% and 14% reductions, respectively. It appears that Bouygues was most vulnerable to the new rivalry since it had the least share of locked-in customers among the incumbents as shown in Figure 6 in the appendix. This may have exposed Bouygues the most to the competitive pressure from the new contract-free products despite its relatively small market share.

Network	No entry	Δ_{Free}	Δ_{Total}
Orange	1,384	-135	-142
SFR	1,458	-183	-205
Bouygues	1,230	-147	-193
Free	0	868	818
Total	4,072	403	279

Long run equilibrium network supplies displayed in the market without Free & fighting brands (No entry), with Free only (Free), and with both Free & the three fighting brands (Total), respectively.

Table 5: Steady state network supply

We find that the new products of the incumbents had moderately negative impact on the investments. Column ΔFree/FB reports larger losses than Column ΔFree, implying that the positive demand curvature effect of the new product introductions is outweighed by the negative competition effect perhaps due to the collective introductions. Indeed, we find significant drop in the incumbent's markups when their new products are taken into account, as shown in Figure 5 in the appendix. Hence, it appears that the rival's new products had prevailing effect on the investment incentives, which was partly offset by the own firm's new customers.

In summary, we confirm the dominant effects of demand and market power in determining the relationship between competition and innovation, despite the substantial increase in the product-market rivalry. In contrast to the new products released by the incumbents that had only moderate or negligible effect, the new entrant imposed intense competitive pressure on the investment incentives of all the incumbents.

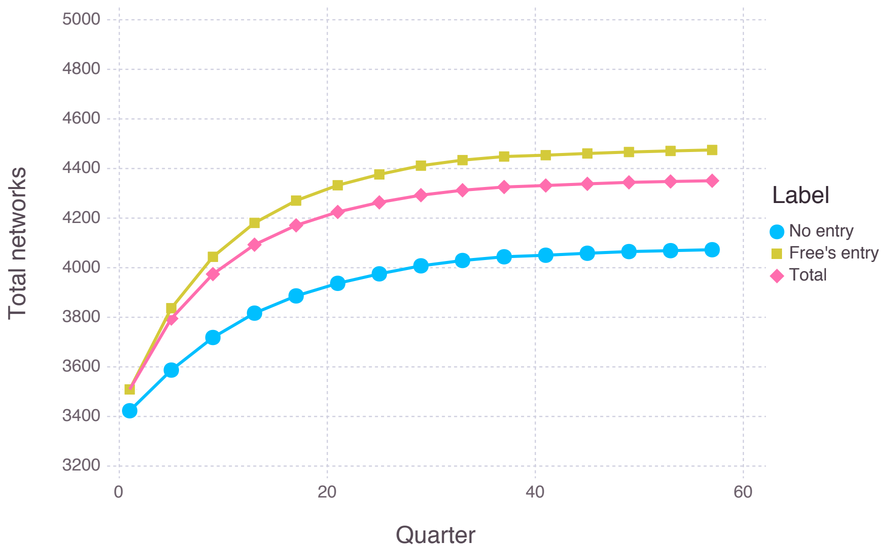


Figure 4: The growth of total networks

In Figure 4, we examine the total investments of all the networks including the entrant. The three curves are the industry-level network supplies in the market without entry (No entry), with Free's entry (Free's entry), and with the new products from both the entrant and the incumbents (Total), respectively.

We find that the entrant drives up the total investments substantially. The figure shows that the overall investment under Free's entry reaches above what is obtained without any entry. This implies that the reduced investments of the incumbents are more than compensated by the entrant's incentive to build its own network. Table 5 reports that the entrant adds 868 units of network stations while the incumbents dropping 465 stations collectively, resulting in the net increase of 403 stations or 10% in proportion. This result can be explained by considering that the entrant does not offer long term contracts to lock in customers. Since the main part of the entrant's demand consists of the competitive market segment not locked into the incumbents, it faces a larger incentive to invest in quality than the incumbents.

5.2 Welfare impacts of entry

While we have found an increase in total network investments with the new competition, it is unclear which market structure generates the optimal outcome in the social planner's perspective. Hence, we measure the average welfare of consumers and producers during the last 15th year of the simulated market estimated for the same *Île-de-France* region in Table 6. As in the previous table, figures in each column correspond to the markets excluding the entrant and the incumbent's new products (No entry), including the entrant only (Free's entry), and all new products from both the entrant and the incumbents (Total), respectively.

	No entry	Free's entry	Total
Consumer	331.2	378.5	404.4
Producer	312.4	271.6	237.9
Total	643.6	650.0	642.3

Quarterly welfare estimates at steady state network supply in region *Île-de-France* based on 200 simulation paths. The unit is in million €.

Table 6: Long run welfare impacts of new competition

As expected, we find that the consumer surplus increases monotonically as more products become available. The entrant alone contributes to about 14% increase in consumer welfare while the incumbent's new products generate additional 7.8% gain. With Free's entry, consumers benefit from intensified price competition as well as the new contract-free service of the entrant. While the remaining customers of the incumbents may suffer moderate deterioration of network quality, the entrant heavily invests in its own network, which likely compensates the loss of welfare. On the other hand, while the incumbent's new products offset the gain in investment incentives generated by the entry, they still produce additional consumer surplus by creating further competitive pressure through their own contract-free services.

In contrast, the producer profits as a whole suffer from the increasing level of competition. Judging by net impacts, Free's entry inflicts the largest loss upon the industry profit since the incumbents stand to lose the customer base to the new competitor particularly when they have no similar products with which to fight back the challenge. With the shift of the demand toward the entrant holding the lowest market power, the industry profits are diminished. The new product lines of the incumbents also cannibalize the customer base with high profit margins, so as to further reducing the total producer surplus.

Overall, the social surplus turns out to be little affected by the market structure changes because of the opposite welfare effects largely cancelling each other. The net impact of Free's entry amounts to only about 1% increase in social surplus, or €6 million per quarter. The welfare estimate becomes even slightly lower than the no entry case when the incumbent's new product lines are included. Hence, it is obvious that even though the concentration and market power have greatly diminished while industry-level investments went up significantly, they merely played a role of redistributing the producer surplus to the consumers without raising the total surplus. If we take into account the cost of entry and new product launches, the estimate may become even lower.

6 Conclusion

In this paper, we examined how an exogenous entry changed firm's investments in product quality in a mobile telecom market where the majority of consumers were already locked into the incumbent networks bearing high switching costs. The entry gave rise to two opposing impacts on the incumbent's investment incentives: reduced profit gains from locked-in customers of the main product line and a new secondary revenue source to extract innovation rents from. To decompose the incentives, we develop a dynamic oligopoly model of investment and estimate the steady state network supplies under counterfactual market structure. We find that the diminished profits from the existing product lines bearing high switching costs dominated the entry's impact on the investment incentives for all the incumbent networks. Nevertheless, the aggregate investments are estimated to rise in the long run due to the intensive investment of the entrant. However, the welfare analysis suggests its limited impact on social surplus because of the opposite effects on consumers and producers cancelling each other. If we take into account the cost of entry and new product introductions, it is possible to obtain even net welfare loss from entry.

Our conclusion contrasts with the conventional view of long-term contracts as anticompetitive strategy. While they undoubtedly insulate the firms from product-market competition, they help protect the appropriability of investments. As the locked-in customer base becomes diminished with competitive pressure from both entrant and incumbent's contract-free services, the individual firm's investment fall unambiguously.

Our analysis provides evidence on the adverse effect of competition. While it may appear to contradict many predictions from the literature, it can be reconciled by the fact that the market structure is exogenously determined in our analyzed market. As Vives (2008) points out, the presence of potential entry threat determines the incumbent's investment and thus their response to actual entry. In the absence of such threat, our finding confirms the prediction of Vives (2008) albeit through different mechanism.

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A Tables

	Logit		IV		IV-SWC	
	Estimate	Std. err	Estimate	Std. err	Estimate	Std. err
Price	0.002	(0.011)	-0.542**	(0.236)	-0.543***	(0.199)
SW cost					6.625***	(0.112)
Log (2G)	-0.203	(0.864)	0.958	(0.919)	0.984*	(0.567)
Log (3G&4G)	0.101	(0.112)	0.415**	(0.190)	0.421***	(0.133)
Orange	1.129	(0.953)	0.376	(1.009)	-1.322**	(0.572)
SFR	1.019	(0.823)	-1.281	(1.388)	-2.980***	(0.986)
Bouygues	0.638	(0.814)	-0.283	(0.957)	-1.976***	(0.560)
Free	1.578*	(0.861)	-5.442*	(3.213)	-5.493**	(2.624)
Sosh	0.154	(0.982)	-4.520*	(2.320)	-4.582**	(1.807)
B&You	-0.050	(0.798)	-4.374**	(2.172)	-4.417***	(1.674)
Red	-0.123	(0.815)	-4.684**	(2.275)	-4.727***	(1.760)
Virgin	0.537	(0.459)	-2.828*	(1.459)	-2.842**	(1.237)
MVNO:O&S	0.127	(0.439)	-1.105*	(0.633)	-1.118**	(0.489)
MVNO:Orange	0.316	(0.460)	-3.192**	(1.518)	-3.207**	(1.287)
MVNO:SFR	0.451***	(0.136)	-4.048**	(1.954)	-4.053**	(1.644)
Time trend	0.108**	(0.046)	0.011	(0.057)	0.010	(0.041)
Log time	-0.336***	(0.104)	-1.892***	(0.687)	-1.900***	(0.589)
Time since entry	-0.856***	(0.187)	-1.274***	(0.238)	-1.267***	(0.222)
Constant	1.240	(4.701)	10.075	(6.556)	9.793**	(4.014)
Observations	12,863		12,863		12,863	
J test (p value)			0.190		0.687	

$*$: p<0.1; $**$: p<0.05; $***$: p<0.01.
Market fixed effects omitted from the table.

Table 7: Estimation of mobile service demand

		No entry				Entry			
Network	Product	Margin	M.S.(%)	$\frac{\partial D_i}{\partial A_i}$	$\sum_{j\neq i} \frac{\partial D_i}{\partial p_j}$	Margin	M.S.(%)	$\frac{\partial D_i}{\partial A_i}$	$\sum_{j\neq i} \frac{\partial Di}{\partial p_j}$
Orange	Orange	11.78	28.10	455.7	730.6	9.28	22.90	559.1	801.0
	Sosh					3.80	5.20	382.2	521.8
SFR	SFR	11.90	30.10	467.8	787.0	9.48	23.40	563.7	789.5
	Red					4.08	6.20	450.4	608.4
Bouygues	Bouygues	9.81	19.30	424.6	604.4	7.34	13.30	482.8	563.9
	B&You					3.33	5.50	524.5	616.6
Free	Free					3.73	16.80	1,572.0	1,614.0

Markup, market share, and derivative of own demand evaluated at steady states in region *Île-de-France*. Margins denote monthly marginal revenue $(p - c)$ in euros. $\partial D/\partial A$ measures change of product demand wrt one additional stock of own firm's network outlay, which corresponds to $\partial D_i/\partial s_i$ in Equation 7. The derivative $\sum_{j\neq i} \frac{\partial Di}{\partial p_j}$ is in the scale of 1,000.

Table 8: Impact of entry on product-market competition

Before analyzing the investments, we first examine how the product market competition is affected by the entry. Table 8 presents the product margins, market shares, and derivatives of demand with respect to network supply $\partial D_i/\partial A_i$, i.e., the direct demand effect of investment as discussed in Equation 7. They are evaluated at the steady states after simulating the market in the long run. The columns under the heading *No entry* are obtained from the market without Free Mobile and the subsidiaries of the incumbents. The columns under *Entry*, in contrast, are generated from the market entered by both the entrant and the subsidiaries.

B Figures

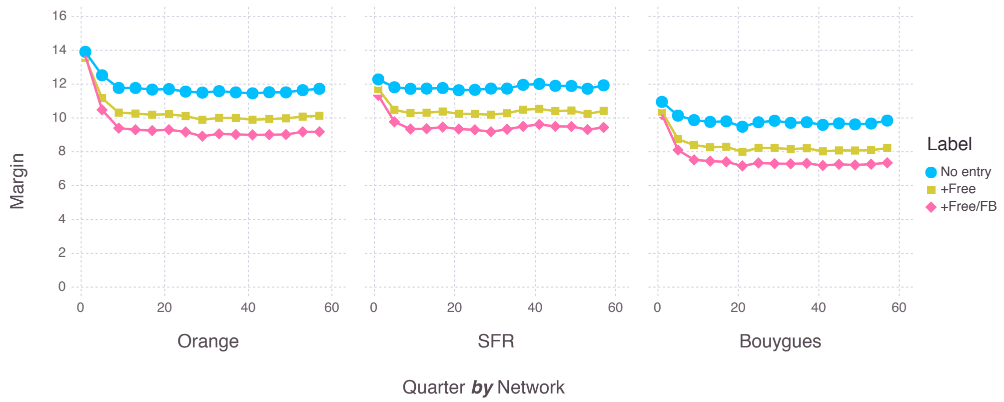

Figure 5: Impact of new entry on margins

Figure 6: Impact of new entry on the share of locked-in customers

C Derivation of Euler equation

From Equation 5, we obtain the first order condition

$$
\begin{aligned}
0 &= \frac{\partial \Pi_i(s,a_i)}{\partial a_i} + \beta \frac{\partial}{\partial a_i} \mathrm{E}\Big[V_i(s', a'_{-i}) \Big| s, a\Big] \\
&= \frac{\partial \Pi_i(s,a_i)}{\partial a_i} + \beta\delta \mathrm{E}\Big[\frac{d}{ds'_i} V_i(s', a'_{-i}) \Big| s, a\Big] \quad \text{since } s'_i = \delta(s_i + a_i).
\end{aligned}
$$

On the other hand, we apply the envelope theorem to Equation 5 to obtain

$$
\begin{aligned}
\frac{dV_i(s,a_{-i})}{ds_i} &= \frac{\partial \Pi_i(s,a_i)}{\partial s_i} + \beta\delta \mathrm{E}\Big[\frac{dV_i(s', a'_{-i})}{ds'_i} + \sum_{j\neq i} \frac{dV_i(s', a'_{-i})}{ds'_j}\frac{\partial a_j}{\partial s_i} \Big| s, a\Big] \\
&= \frac{\partial \Pi_i(s,a_i)}{\partial s_i} - \frac{\partial \Pi_i(s,a_i)}{\partial a_i} + \beta\delta \sum_{j\neq i} \frac{\partial a_j}{\partial s_i} \mathrm{E}\Big[\frac{dV_i(s', a'_{-i})}{ds'_j} \Big| s, a\Big] \\
&= \frac{\partial \Pi_i(s,a_i)}{\partial s_i} - \frac{\partial \Pi_i(s,a_i)}{\partial a_i} + \sum_{j\neq i} \frac{\partial V_i(s,a_{-i})}{\partial a_j}\frac{\partial a_j}{\partial s_i},
\end{aligned}
$$

where the second equality follows from the FOC, and the third equality holds since

$$
\frac{\partial V_i(s,a_{-i})}{\partial a_j} = \beta\delta \mathrm{E}\Big[\frac{dV_i(s', a'_{-i})}{ds'_j} \Big| s, a\Big]
$$

by the envelope theorem. Plugging the above equation into Equation 5, we have

$$
0 = \frac{\partial \Pi_i(s,a_i)}{\partial a_i} + \beta\delta \mathrm{E}\Big[\frac{\partial \Pi_i(s',a'_i)}{\partial s'_i} - \frac{\partial \Pi_i(s',a'_i)}{\partial a'_i} + \sum_{j\neq i} \frac{\partial V_i(s', a'_{-i})}{\partial a'_j}\frac{\partial a'_j}{\partial s'_i} \Big| s, a\Big].
$$

Rearranging and evaluating the terms produces the Euler equation

$$
\mathrm{E}\Big[C'(a_i) - \beta\delta C'(a'_i) \Big| s, a_i\Big] = \beta\delta \mathrm{E}\Big[\frac{\partial \Pi_i(s',a'_i)}{\partial s'_i} + \sum_{j\neq i} \frac{\partial V_i(s', a'_{-i})}{\partial a'_j}\frac{\partial a'_j}{\partial s'_i} \Big| s, a\Big].
$$

D Solution procedure for the stationary equilibrium

1. Set up Smolyak grid $\mathcal{X} = \{x_g\}_{g=1}^G$ and basis functions $\{\psi_g(x)\}_{g=1}^G$, where $x_g \in \mathcal{S} \times \mathcal{V}$, and $\psi(x)$ is a $G \times G$ matrix with $\psi_g(x)$ as a row vector.

2. Set $a_i^0(x) = \hat{a}_i$ obtained from the first-stage estimation for all $x \in \mathcal{X}$.

3. Set $V_i^0(x_t) = \Pi_{it}(x_t, a_{it}^0) + \sum_{\tau=1}^T \beta^\tau \Pi(x_{t+\tau}, a_{it}^0)$ for a given $T < \infty$, where $a_{it}^0 = a_i^0(x_t)$ for all $x_t \in \mathcal{X}$.

4. Solve for $b_i^0 = [\psi(x)]^{-1} V_i^0(x) \in \mathbb{R}^G$.

5. At kth iteration with b_i^{k-1}, solve for $a^k = (a_1^k, ..., a_N^k)$ such that for each $i \in \mathcal{N}$,

$$
a_i^k = \arg\max_{a_i} \Pi_{it}(x_t, a_i) + \beta \mathrm{E}\big[V_i^{k-1}(x_{t+1}) \big| x_t, a_i, a_{-i}^k\big]
$$

where $x_t = (A_t, \mu_t, S_{t-1})$, $x_{t+1} = (A_t + a^k, \mu_{t+1}, S_t)$, and $V_i^{k-1}(x) = \psi(x)b_i^{k-1}$.

6. Given a^k, obtain V^k or b^k by iterating the Bellman equation:

$$\begin{aligned}
V_i^k(x_t) &= \Pi_{it}(x_t, a_i^k) + \beta \mathrm{E}[V_i^{k-1}(x_{t+1})|x_t, a^k] \\
&= R_i(x_t) - C_i(a_i^k) + \beta \sum_{l=1}^{L} V_i^{k-1}(A_t + a^k, \mu_l')P(\mu_l'|\mu)
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \psi(x)b_i^k = R_i(x_t) - C_i(a_i^k) + \beta \left[\sum_{l=1}^{L} \psi(A + a^k, \mu_l')P(\mu_l'|\mu) \right] b_i^{k-1} \\
&\Rightarrow b_i^k = \psi(x)^{-1} \left[R_i(x) - \lambda(a_i^k)^2 + \beta \left[\sum_{l=1}^{L} \psi(A + a^k, \mu_l')P(\mu_l'|\mu) \right] b_i^{k-1} \right]
\end{aligned}$$

7. Repeat steps 5 & 6 until $\max\left\{\|a^k - a^{k-1}\|, \|b^k - b^{k-1}\|\right\} < \epsilon$ for a given threshold $\epsilon > 0$.

E Definition of $\tilde{a}$

In the counterfactual analysis, we solve for a transformed investment $\tilde{a}$ specified as

$$\log A' = \log \delta + \log A + \tilde{a},$$

where $\tilde{a} = \log(A + a) - \log A$. Then

$$a = e^{\tilde{a} + \log A} - A = A(e^{\tilde{a}} - 1).$$

Therefore,

$$\frac{\partial a}{\partial \tilde{a}} = Ae^{\tilde{a}}, \qquad \frac{\partial^2 a}{\partial \tilde{a}^2} = Ae^{\tilde{a}},$$

and

$$\frac{\partial^2 C(a)}{\partial \tilde{a}^2} = \frac{\partial}{\partial \tilde{a}}\left(2\lambda a \frac{\partial a}{\partial \tilde{a}}\right) = 2\lambda\left[\left(\frac{\partial a}{\partial \tilde{a}}\right)^2 + a\frac{\partial^2 a}{\partial \tilde{a}^2}\right] = 2\lambda A \frac{\partial a}{\partial \tilde{a}}\left[2e^{\tilde{a}} - 1\right].$$