Cheap Talk with Coarse Understanding*

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September 17, 2019

Abstract

We use the analogy-based expectation equilibrium (Jehiel, 2005) to study cheap talk from a sender who does not perfectly understand all the messages available to him. The sender is endowed with a privately known language competence corresponding to the set of messages he understands. For the messages he does not understand, the sender has correct but only coarse expectations about the equilibrium response of the receiver. An analogy-based expectation equilibrium is always a Bayesian solution and usually differs from an equilibrium with language barriers (Blume and Board, 2013). Partial language competence rationalizes information transmission and lies in pure persuasion problems, and can facilitate information transmission from a moderately biased sender.

Keywords: Analogy-based expectations; bounded rationality, cheap talk; language; pure persuasion; strategic information transmission.

JEL Classification: C72; D82

1 Introduction

In cheap talk games, the meaning of messages is not literal but comes from equilibrium strategies. In particular, in equilibrium, a sender is supposed to understand perfectly the receiver's reaction to his messaging strategy. And yet, strategic situations where messages are wrongly interpreted and/or induce unexpected responses are prevalent: the choice of a casual outfit can be perceived as an expression of indifference when it was done to convey confidence; arriving right on time to a social event can be viewed as impolite when it is was supposed to show

*We thank Philippe Jehiel, Hamid Sabourian and seminar participants at Cambridge University and Paris School of Economics for useful comments and suggestions. A previous version of this article was entitled "Partial language competence".

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enthusiasm; the opening of a gift just received can be viewed as indelicate while the movement was meant to express gratitude; a foreign word can be used without a full understanding of its usage and exact meaning to natives. In all these examples, the privately informed sender can send a message despite having an imperfect expectation of the receiver's reaction to it. This paper proposes a way to incorporate such a sender in cheap talk games, and to study how his presence affects information transmission.

Blume and Board (2013) already propose a way to incorporate the fact that a privately informed sender may not understand all the messages available to him. In their sender-receiver framework, every player is endowed with a privately known *language competence*, which is a set of messages that he understands. A key assumption is that a sender cannot emit messages out of his language competence. That is, language competence translates into a restriction on the sender's set of strategies. While we also endow the sender with a privately known set of messages that he understands, we assume that the sender can practically send every available message. However, since it can be particularly complex to learn how the receiver reacts facing any possible message, we consider a sender who has a simplified view of the receiver's strategy. In our model, the sender's language competence relates to his ability to predict finely or not the impact of his messages on the receiver.

More precisely, we incorporate the sender's language competence into the solution concept by adopting the notion of analogy-based expectation equilibrium (ABEE) developed by Jehiel (2005), Jehiel and Koessler (2008), Ettinger and Jehiel (2010). This concept assumes that bounded rational players do not perceive the strategy of other players as finely as rational players do. In our communication context, the sender has bounded cognitive rationality: he does not fully understand how the receiver's strategy maps each message received into actions. Instead, the sender bundles all messages he does not understand into the same analogy class, and perceives only coarsely, but correctly on average, the response of the receiver to such messages.[1] We describe how we incorporate the sender's language types in sender-receiver cheap talk games in Section 2. We define an analogy-based expectation equilibrium (ABEE) for such games in Section 3. When the sender understands all messages available to him, then the definition of an ABEE coincides with Nash equilibrium.

In Section 4, we illustrate and compare ABEE outcomes to other solution concepts and approaches to partial language competence. As in standard cheap talk games, there always exists an equilibrium with no information transmission, that is, the babbling (non-revealing) outcome is always an ABEE outcome. Hence, the set of ABEE outcomes is always non-empty.

[1] We can view the sender's perception of the receiver's strategy as resulting from a learning process: senders and receivers from a large population are randomly matched and play a best response to the feedback they receive about the past behavior in the population, with the property that feedbacks of a sender are aggregated whenever a message outside of his language competence has been used (see, e.g., Ettinger and Jehiel, 2010 or Esponda and Pouzo, 2016 for more details).

We then provide a tight bound on the outcomes that partial language competence permits to reach: an ABEE outcome is always a Bayesian solution (Forges, 1993, 2006) of the basic game without communication. A Bayesian solution corresponds to an outcome that can be achieved with some information system consistent with the prior probability distribution over states. For some language competences, the set of ABEE outcomes is equal to the set of Bayesian solutions. In particular, the two sets coincide if the sender is fully incompetent and the message space is large enough. Next, we illustrate that an ABEE might Pareto dominate all communication equilibria (see Example 2 and Section 5.2). While partial language competence can improve communication, it can also be a barrier to communication. As in Blume and Board (2013), efficiency in common interest games might not be achieved when language competence is privately known, even when the sender's language competence would be sufficient to achieve efficiency if language competence were commonly known (see Example 3). In Section 4.3 we compare more precisely our approach to Blume and Board (2013) and to other strategy restrictions in cheap talk games.

In Section 5 we apply our approach to situations in which the language competence of the sender is perfectly correlated with his payoff type and increasing with this type: a sender with a high payoff type understands more messages than a sender with a lower payoff type. This setting could correspond, for example, to a job interview situation in which the candidate (sender) is characterized by his ability (payoff type) to perform a certain task and his understanding of a technical jargon (message) depends on his ability. To convince the recruiter (receiver) to give him a job, he must decide whether he speaks only about what he correctly understands, or whether he tries to persuade the recruiter by using a technical jargon beyond his own competence. Considering such a language setting, we first study pure persuasion problems in which the sender's preference is state-independent: he only cares about the receiver's estimate of the state and wants to maximize it. Second, we consider a moderately biased sender as in Crawford and Sobel (1982): he wants the receiver's estimate to match the state plus a bias.

In the pure persuasion context, cheap talk involving a fully competent sender cannot be influential. In contrast, we show that there exists a threshold ABEE in which (i) sender types above the threshold reveal that threshold (which is the highest message they commonly understand); (ii) sender types below the threshold lie about their type by sending messages they do not understand. This outcome constitutes an ABEE for every threshold corresponding to a payoff type of the sender. If the sender's utility can only take two values (for example, the sender wants to convince the receiver to choose an alternative action against the status-quo), then there exists a threshold equilibrium that implements the best Bayesian solution for the sender (i.e., the one that maximizes the probability that the alternative action is chosen). When the sender is moderately biased, we show that there exists a unique threshold equilibrium which is such that (i) sender types below the threshold reveal their type truthfully; (ii) sender types above the threshold pool. This ABEE Pareto dominates and is more informative than every Nash equilibrium.

Related Literature The most closely related papers in the literature are Blume and Board (2013) and Giovannoni and Xiong (2019). They introduce players' language competence, which restricts the set of possible strategies by directly modifying the extensive form of the cheap talk game (the set of available messages for the sender, and the information sets for the receiver), and use the standard Nash equilibrium as a solution concept. Hence, in their framework, a sender can only use a message that he perfectly understands, so our approaches do not usually lead to the same equilibrium outcomes. This difference in modeling language competence makes a significant difference in our applications to pure persuasion and cheap talk with a biased sender. With their approach, if a higher payoff type of the sender understands messages which are not understood by lower types, then the games in these applications are equivalent to disclosure games with hard information (as in Grossman, 1981; Milgrom, 1981, Seidmann and Winter, 1997) and complete information disclosure would always be an equilibrium.

Blume and Board (2013) show that efficiency may not be attained, even if it would be attained if players' language competences were commonly known. Then, they consider a specific class of common interest sender-receiver games. When only the sender has partial language competence, they show that in the best equilibrium the sender uses all his messages, that messages also transmit information about language types, and that there exists a language type and a message such that this message does not induce the same belief for the receiver as the one that the receiver would have if he knew the sender's language type. When only the receiver has partial language competence, they show that in the best equilibrium the receiver's response to a message depends on his language type, and is therefore stochastic from the point of view of the sender. The role of higher-order uncertainty about language types is investigated further in Blume (2018). Giovannoni and Xiong (2019) show that standard Nash equilibria can be replicated in a game with language barriers (in the sense of Blume and Board, 2013) whenever the set of messages M is extended to multidimensional strings of messages in M^N, for N large enough. Blume and Board (2010) and Giovannoni and Xiong (2019) also show that there exist language barriers that achieve a welfare which is at least as good as (and sometimes stricter better than) the one achievable with mediated or noisy communication.

Another related work is Eliaz, Spiegler, and Thysen (2019). They consider sender-receiver games in a pure persuasion context with binary actions. Contrary to our approach, they assume that it is the receiver (instead of the sender) who partially understand the equilibrium messages. In addition, they allow the sender to choose the analogy partition of the receiver, through the choice of a "dictionary" for the receiver. Finally, they assume that the sender is uninformed and able to commit to a messaging strategy. In this framework, they characterize the optimal messaging strategy for the sender and identify conditions under which the sender is able to achieve his first best.

2 Model

Basic Game. There are two players: the sender (S) and the receiver (R). The set of *payoff types* of the sender is T. The sender is privately informed about his payoff type. Let $p^0 \in \Delta(T)$ be the prior probability distribution over payoff types. The set of actions of the receiver is A. Unless stated otherwise, we assume for expositional simplicity that the sets T and A are finite. In Section 5 we consider simple classes of games in which the setting extends straightforwardly when the type and action sets are intervals of the real lines.

A mixed action of the receiver is denoted by $y \in \Delta(A)$. The utility of the sender (receiver, respectively) is given by $u(a;t)$ ($v(a;t)$, respectively) when the payoff type of the sender is $t \in T$ and the action of the receiver is $a \in A$. With some abuse of notation, the utilities are extended as follows for every $y \in \Delta(A)$ and $p \in \Delta(T)$:

$$u(y;t) = \sum_{a \in A} y(a)u(a;t) \text{ and } v(y;t) = \sum_{a \in A} y(a)v(a;t),$$

$$u(y;p) = \sum_{t \in T} p(t)u(y;t) \text{ and } v(y;p) = \sum_{t \in T} p(t)v(y;t).$$

Messages and Languages. The set of all available messages is M. A *language type* is a subset $\lambda \subseteq M$ of messages, that is interpreted as the set of messages that the sender understands. The sender is privately informed about his language type. The set of all possible language types is a non-empty set $\Lambda \subseteq 2^M$. We allow the language type to be correlated with the payoff type. Let $\pi : T \to \Delta(\Lambda)$ be the conditional probability distribution over the language types. For every payoff type $t \in T$ and language type $\lambda \in \Lambda$, $\pi(\lambda \mid t)$ denotes the probability that the sender's language type is λ when his payoff type is t.

We say that the sender is *fully competent* if $\lambda = M$ and *incompetent* if $\lambda = \emptyset$. It is commonly known that he is fully competent (incompetent, resp.) if $\Lambda = \{M\}$ ($\Lambda = \{\emptyset\}$, resp.). $L = (M, \Lambda, \pi)$ is called a *language system*.

Cheap Talk Extension. We are interested by the following extended cheap talk game (G, L):

1. Nature selects the sender's payoff type, $t \in T$, according to the probability distribution $p^0 \in \Delta(T)$, and the language type $\lambda \in \Lambda$, according to the probability distribution $\pi(\cdot \mid t) \in \Delta(\Lambda)$;

2. The sender is privately informed about $t \in T$ and $\lambda \in \Lambda$;

3. The sender sends a message $m \in M$ to the receiver;

4. The receiver observes the message m and chooses an action $a \in A$.

A strategy for the sender is a mapping $\sigma_S : T \times \Lambda \to \Delta(M)$. A strategy for the receiver is a mapping $\sigma_R : M \to \Delta(A)$. An outcome (distribution of actions induced by players' strategies in each state) of the cheap talk game is denoted by $\mu : T \to \Delta(A)$. That is, for every $a \in A$ and $t \in T$:

$$\mu(a \mid t) = \sum_{\lambda \in \Lambda} \pi(\lambda \mid t) \sum_{m \in M} \sigma_S(m \mid t, \lambda)\sigma_R(a \mid m).$$

3 Analogy-Based Expectation Equilibrium

Given a strategy σ_S for the sender, denote by

$$\Pr(m) = \sum_{t \in T} p^0(t) \sum_{\lambda \in \Lambda} \pi(\lambda \mid t)\sigma_S(m \mid t, \lambda),$$

the total probability that the sender sends message $m \in M$. The corresponding random variable is denoted by $\mathbf{m}$ when there is a possible confusion with the realization of a message.

The next definition describes how an actual strategy profile (σ_S, σ_R) translates into a *strategy of the receiver perceived by the sender* of a given language type. For each message that the sender understands, he correctly perceives the receiver's response. For messages that the sender does not understand (if such messages are sent with strictly positive probability according to σ_S), he perceives the actual average reaction of the receiver to all such messages. In the terminology of Jehiel (2005), this perceived strategy corresponds to the analogy-based expectation of a sender who bundles in the same analogy class the receiver' decisions nodes that follow a message that the sender does not understand.

Definition 1 Given (σ_S, σ_R), a *strategy of the receiver perceived by the sender* with language type $\lambda \in \Lambda$ is a strategy $\tilde{\sigma}_R^\lambda : M \to \Delta(A)$ such that for every $a \in A$ and $m \in M$:

$$\tilde{\sigma}_R^\lambda(a \mid m) = \begin{cases} \sigma_R(a \mid m) & \text{if } m \in \lambda \\ \sum_{m' \notin \lambda} \Pr(m' \mid \mathbf{m} \notin \lambda)\sigma_R(a \mid m') & \text{if } m \notin \lambda \text{ and } \Pr(\mathbf{m} \notin \lambda) > 0, \end{cases}$$

where for $m' \notin \lambda$,

$$\Pr(m' \mid \mathbf{m} \notin \lambda) = \frac{\Pr(m')}{\Pr(\mathbf{m} \notin \lambda)} = \frac{\sum_{t \in T} p^0(t) \sum_{\lambda' \in \Lambda} \pi(\lambda' \mid t)\sigma_S(m' \mid t, \lambda')}{\sum_{m'' \notin \lambda} \sum_{t \in T} p^0(t) \sum_{\lambda' \in \Lambda} \pi(\lambda' \mid t)\sigma_S(m'' \mid t, \lambda')}.$$

Example 1 Let $T = \{t_1, t_2, t_3\}$, with a uniform prior probability distribution. Let $M = \{m_1, m_2, m_3\}$ and $\Lambda = \{\{m_1\}\}$. The strategy of the receiver perceived by the sender is given by

$$\tilde{\sigma}_R^{\lambda}(m_1) = \sigma_R(m_1),$$

$$\tilde{\sigma}_R^{\lambda}(m_2) = \tilde{\sigma}_R^{\lambda}(m_3) = \Pr(m_2 \mid \{m_2, m_3\})\sigma_R(m_2) + \Pr(m_3 \mid \{m_2, m_3\})\sigma_R(m_3).$$

For example, if the receiver's action matches the message (i.e., $\sigma_R(m_k) = a_k$) and the equilibrium probabilities of messages m_2 and m_3 are given by $\Pr(m_2) = q_2$ and $\Pr(m_3) = q_3$, with $q_2+q_3 > 0$, then the strategy of the receiver perceived by the sender is uniquely defined: for every message $m \in \{m_2, m_3\}$ that the sender does not understand, he believes that the receiver plays actions a_2 and a_3 with probabilities $\tilde{\sigma}_R^{\lambda}(a_2 \mid m) = \frac{q_2}{q_2+q_3}$ and $\tilde{\sigma}_R^{\lambda}(a_3 \mid m) = \frac{q_3}{q_2+q_3}$. $\diamond$

With the sender's coarse perceptions in hands, we are now ready to define the equilibrium concept.

Definition 2 A strategy profile (σ_S, σ_R) is an *analogy-based expectation equilibrium* (ABEE) of the cheap talk extension of G with language system L if there exists a profile of perceived strategies $\{(\tilde{\sigma}_R^{\lambda})\}_{\lambda\in\Lambda}$ such that the following conditions are satisfied:

For the sender: for every $t \in T$, $\lambda \in \Lambda$ and $m^* \in M$ satisfying $\sigma_S(m^* \mid t, \lambda) > 0$,

$$m^* \in \arg\max_{m\in M} \sum_{a\in A} \tilde{\sigma}_R^{\lambda}(a \mid m)u(a;t).$$

For the receiver: for every $m \in M$ satisfying $\Pr(m) > 0$ and every a^* satisfying $\sigma_R(a^* \mid m) > 0$,

$$a^* \in \arg\max_{a\in A} \sum_{t\in T} \Pr(t \mid m)v(a;t).$$

It should be clear that, when it is commonly known that the sender is fully competent ($\Lambda = \{M\}$), the definition of ABEE coincides with the definition of a Nash equilibrium of the cheap talk game.

For every language system L, the set of ABEE outcomes of (G, L) is non-empty because it always includes the set of babbling outcomes. Indeed, it suffices to consider a constant strategy for the receiver, who plays an action in $\arg\max_{a\in A} v(a;p^0)$ for every message, and a constant strategy for the sender, who always sends the same message.

4 Examples and Relationship to other Concepts

In this section we provide examples to illustrate the ABEE concept and compare it to other solution concepts in communication games. We show that ABEE can lead to different outcomes than Nash and communication equilibria. For every language system, an ABEE is a Bayesian solution. When it is commonly known that the sender is incompetent the converse is true as well. We also compare ABEE to Nash equilibrium with restricted strategies for the sender, such as in Blume and Board (2013).

4.1 Full revelation by an incompetent sender

We begin by considering the situation in which it is commonly known that the sender is incompetent, i.e., $\Lambda = \{\emptyset\}$. In this extreme case, full revelation of information is an ABEE even when there is no fully informative Nash equilibrium. This is illustrated in the following example.

Example 2 Consider the following basic game with uniform priors and $\Lambda = \{\emptyset\}$:

	a_1	a_2
t_1	$1, 1$	$0, 0$
t_2	$1, -2$	$0, 0$

Clearly, full revelation cannot be sustained as a Nash equilibrium outcome because type t_2 would have an incentive to mimic type t_1. For the same reason, full revelation is not a communication equilibrium outcome (Forges, 1986; Myerson, 1982, 1986). Nevertheless, the strategy profile $\sigma_S(t_1) = m_1$, $\sigma_S(t_2) = m_2$, $\sigma_R(m_1) = a_1$, $\sigma_R(m_2) = a_2$ is an ABEE. The sender perceives the same receiver's reaction following m_1 and m_2, and therefore has no incentive to deviate from full revelation. $\diamond$

Basically, because an incompetent sender expects the same reaction of the receiver to any of his messages, every game in which it is commonly known that the sender is incompetent has a fully revealing ABEE. For example, a sender may incidentally reveal private information (involvement, motivation etc.) through his outfit while he expects the receiver's reaction to be independent of his clothes. Such an equilibrium seems unnatural in many contexts, but it is similar to what occurs in a Nash equilibrium of a cheap talk game in which every receiver's action leads to the same sender's payoff: the sender fully reveals his payoff type because deviating is never strictly beneficial.

4.2 Bayesian solutions

The above observation that an incompetent sender has no strict incentive to deviate from a fully revealing strategy actually extends to any strategy of the sender. Hence, if it is commonly known that the sender is incompetent and if the message space M is large enough, then every Bayesian solution (Forges, 1993, 2006) (in particular, the fully revealing outcome) can be obtained as an ABEE outcome. More precisely, an outcome $\mu : T \to \Delta(A)$ is a Bayesian solution iff it satisfies the following incentive constraints for the receiver:[2]

$$\sum_{t\in T} \mu(a \mid t)p^0(t)v(a;t) \geq \sum_{t\in T} \mu(a \mid t)p^0(t)v(a';t), \quad \text{for every } a \in \text{supp}[\mu] \text{ and } a' \in A. \qquad \text{(RIC)}$$

That is, a Bayesian solution is an outcome that can be obtained when the receiver plays optimally given some information about the sender's payoff type.[3] A fully revealing outcome satisfies $\mu(t) \in \arg\max_{y\in\Delta(A)} v(y;t)$ for every $t \in T$, and is a Bayesian solution (which gives the first best to the receiver). The solution of the Bayesian persuasion problem (Kamenica and Gentzkow, 2011) is also a Bayesian solution (it is the Bayesian solution that maximizes the sender's ex-ante expected payoff).

To see that a Bayesian solution μ is an ABEE outcome when it is commonly known that the sender is incompetent and when the message space is large enough, let $|M| \geq |A|$ and associate to each action $a \in A$ a different message $m_a \in M$. Consider the sender's strategy σ_S such $\sigma_S(m_a \mid t) = \mu(a \mid t)$ for every a and t, and an obedient receiver's strategy satisfying $\sigma_R(m_a) = a$ for every a. By construction, since (RIC) is satisfied, σ_R is a best response to σ_S for the receiver. In addition, since $\lambda = \emptyset$, the strategy of the receiver perceived by the sender is given by

$$\tilde{\sigma}_R^{\lambda}(a \mid m) = \sum_{m'\in M} \Pr(m')\sigma_R(a \mid m') = \sum_{a\in A} \Pr(m_a)\sigma_R(a \mid m_a) = \sum_{a\in A} \Pr_\mu(a),$$

which is independent of m. Hence, the sender's perceived expected payoff is independent of the message he sends, and he thus has no incentive to deviate from σ_S. Therefore, μ is an ABEE outcome.

In the less extreme cases in which the sender has some language competence, not all Bayesian solutions could be achieved as ABEE outcomes. However, the converse is true: for every language system L an ABEE outcome is a Bayesian solution. To see this, consider an ABEE

[2] In contrast, the notion of communication equilibrium additionally requires the following incentive constraint for the sender: $\sum_{a\in A} \mu(a \mid t)u(a;t) \geq \sum_{a\in A} \mu(a \mid t')u(a;t)$ for every $t, t' \in T$.

[3] The formulation above already uses the revelation principle which simplifies the information of the receiver to a recommandation of action, and the optimal strategy of the receiver to an obedient strategy.

(σ_S, σ_R) of (G, L). By definition of an ABEE, we know that the receiver's strategy satisfies

$$\sum_t \Pr(t \mid m) v(a; t) \geq \sum_t \Pr(t \mid m) v(a'; t). \tag{1}$$

Multiplying (1) by $\Pr(m \mid a)$ for every $m \in M$, taking the sum over M and simplifying, we get (RIC).

The following proposition summarizes the link between ABEE and Bayesian solution discussed above.

Proposition 1 *For every language system $L = (M, \Lambda, \pi)$, an ABEE outcome of (G, L) is a Bayesian solution. If $|M| \geq |A|$ and $\Lambda = \{\emptyset\}$, then a Bayesian solution is an ABEE outcome.*

From this result, we conclude that the union of all ABEE outcomes of (G, L) for all possible language systems L coincides with the set of Bayesian solutions of G.

4.3 Language Barriers and Strategy Restrictions

Blume and Board (2013) and Giovannoni and Xiong (2019) also study partial language competence (and private information about language competence) in sender-receiver cheap talk games, but with a different approach.[4] They assume that when the sender's language type is $\lambda \subseteq M$, he is only able to send messages in λ (they therefore assume that $\lambda \neq \emptyset$). That is, the set of strategies of the sender is the set of mappings $\sigma_S : T \times \Lambda \to \Delta(M)$ such that $\sigma_S(m \mid t, \lambda) = 0$ whenever $m \notin \lambda$. An equilibrium is then defined as a standard Nash equilibrium in the cheap talk game with restricted strategy sets.

As in Blume and Board (2013), the next example demonstrates that, even in common interest games, the efficient outcome may not be an ABEE when language competence is privately known even if language competences are sufficient to do so (that is, efficiency would be reached for the language competence considered in the example provided that the language competence were commonly known).

Example 3 Consider the following basic game with uniform priors and common interest:

[4] They also allow for partial language competence of the receiver, assuming that the receiver is not physically able to distinguish the messages he does not understand from each others. In a previous version of this paper (Hagenbach and Koessler, 2019) we show that analogy-based expectations, where the analogy classes refer the groups of messages that the receiver does not understand, leads essentially to the same approach as Blume and Board (2013) and Giovannoni and Xiong (2019) for the receiver.

	a_1	a_2	a_3
t_1	$0,0$	$-2,-2$	$-2,-2$
t_2	$0,0$	$1,1$	$-2,-2$
t_3	$0,0$	$-2,-2$	$1,1$

Consider the following language system: $M = \{m_1, m_2, m_3\}$, the sender's language types are $\lambda^1 = \{m_1\}$, $\lambda^2 = \{m_2\}$, $\lambda^3 = \{m_3\}$ and are uniformly distributed. Therefore, the sender has nine equally likely types in $T \times \Lambda$. The efficient (first best) outcome is $\mu(t_k) = a_k$, $k = 1, 2, 3$. This outcome can be implemented only if the sender uses a separating strategy and the receiver chooses a different action for each message. Consider such a strategy profile. If the receiver chooses a_1 after m_1 then the sender with language type λ^1 and payoff type t_2 or t_3 would like to deviate and send message m_1 since his payoff is 0 by sending m_1 but his perceived payoff is $\frac{1}{2}(1) + \frac{1}{2}(-2) < 0$ if he sends m_2 or m_3. Similarly, if the receiver chooses a_1 after m_2 then the sender with language type λ^2 and payoff type t_2 or t_3 would like to deviate, and if the receiver chooses a_1 after m_3 then the sender with language type λ^3 and payoff type t_2 or t_3 would like to deviate.

However, observe that if the sender's language type is commonly known, then there is an efficient ABEE. For example, if $\Lambda = \{\{m_1\}\}$, then $\sigma_S(t_1) = m_2$, $\sigma_S(t_2) = m_1$, $\sigma_S(t_3) = m_3$ and $\sigma_R(m_1) = a_2$, $\sigma_R(m_2) = a_1$, $\sigma_R(m_3) = a_3$ is an efficient ABEE. A symmetric argument applies when $\Lambda = \{\{m_2\}\}$ or $\Lambda = \{\{m_3\}\}$. $\diamond$

In general, ABEE outcomes differ from Nash equilibrium outcomes with restricted strategy sets. In particular, while in Blume and Board (2013) the sender never uses a message he misunderstands (by assumption), with our approach the sender might strictly prefer to use such a message. To see this, consider the following example.

Example 4 Let $T = \{t_1, t_2, t_3\}$, with a uniform prior probability distribution. Consider the following language system: $M = \{m_1, m_2, m_3\}$, $\Lambda = \{\{m_1\}, M\}$ and assume that the language type of the sender is independent of his payoff type. Consider the following payoff matrix:

	a_1	a_2	a_3
t_1	$2,1$	$0,0$	$0,0$
t_2	$2,0$	$4,1$	$1,0$
t_3	$2,0$	$1,0$	$4,1$

Let $\sigma_R(m_k) = a_k$, $k = 1, 2, 3$, be the strategy of the receiver. There is a Nash equilibrium with strategy restrictions à la Blume and Board (2013) in which the sender fully reveals when his language type is $\lambda = M$ (i.e., $\sigma_S(t_k, \lambda) = m_k$, $k = 1, 2, 3$) and always sends the message he understands when his language type is $\lambda = \{m_1\}$ (i.e., $\sigma_S(t_k, \lambda) = m_1$, $k = 1, 2, 3$). However, this is not an ABEE. When $t \in \{t_2, t_3\}$, the sender of language type $\lambda = \{m_1\}$ deviates by sending a message that he does not understand because he believes that with such a message he induces actions a_2 and a_3 with probability $\frac{1}{2}$ instead of inducing action a_1 with message m_1. His perceived expected payoff by doing so is $\frac{1}{2}4 + \frac{1}{2}1$ instead of 2. The Nash equilibrium above is however an ABEE if we replace the sender's payoffs 1 by x with $x \leq 0$. ◇

In the next section we will consider an ordered set of payoff types and assume that the set of messages that the sender of payoff type t understands is $\lambda(t) = \{s \in T : s \leq t\}$. That is, the sender's language and payoff types are perfectly correlated, and a sender of higher payoff type understands all the messages that a sender of lower type understands. With the strategy restriction of Blume and Board (2013) and Giovannoni and Xiong (2019), the game is equivalent to a disclosure game with evidence (as in, e.g., Grossman, 1981; Milgrom, 1981, Seidmann and Winter, 1997). As illustrated in the next section, ABEE outcomes differ from equilibrium outcomes in disclosure games.

Another natural approach to partial language understanding would be to restrict the sender to use all the messages he does not understand with the same (possibly zero) probability (that is, $\sigma_S(m \mid t, \lambda) = \sigma_S(m' \mid t, \lambda)$ for every $m, m' \notin \lambda$). This is a weaker strategy restriction than in Blume and Board (2013) and Giovannoni and Xiong (2019). In Example 4, the ABEE in which (i) the strategy of the receiver is $\sigma_R(m_k) = a_k$, $k = 1, 2, 3$, (ii) the sender fully reveals when his language type is $\lambda = M$ or his payoff type is t_1 (i.e., $\sigma_S(t_k, M) = m_k$, $k = 1, 2, 3$ and $\sigma_S(t_1, \{m_1\}) = m_1$) and sends the messages he does not understand (m_2 and m_3) with probability $\frac{1}{2}$ when his language type is $\lambda = \{m_1\}$ and his payoff type is in $\{t_2, t_3\}$, is also an Nash equilibrium of the game with the restricted strategies we just mentioned. However, this is a coincidence coming from the fact that, in this equilibrium, the probabilities of messages m_2 and m_3 are the same. If the prior probability distribution of payoff types is not uniform anymore, then, with the strategy profile above, the equilibrium probabilities of m_2 and m_3 are not the same anymore. The sender of language type $\lambda = \{m_1\}$ and payoff type t_2 would deviate to message m_1 if $\Pr(m_2 \mid \{m_2, m_3\}) < \frac{1}{3}$, and the sender of language type $\lambda = \{m_1\}$ and payoff type t_3 would deviate to message m_1 if $\Pr(m_2 \mid \{m_2, m_3\}) > \frac{2}{3}$.

5 Persuasion with Ordered Understanding

In this section we apply the ABEE concept to situations in which the set T is ordered and the sender's language competence is ordered according to his payoff type: a sender of a given payoff type understands all the messages that lower payoff types understand. Precisely, we assume that $M = T \subseteq [0, 1]$ and that for each payoff type t there is only one possible language type for the sender (the sender's language type is perfectly correlated to his payoff type), given by:

$$\lambda(t) = \{s \in T : s \leq t\}.$$

As an example, consider that the sender's payoff type represents his ability to use or provide a service related to a technical product, such as his knowledge of more or less complex features of a software, a network server or a camera. To communicate about his ability to a receiver, the sender can use a technical jargon related to various features of the product, which he understands better when of greater true ability.[5]

We start by establishing that, with the language system considered, the set of Nash equilibrium outcomes of the cheap talk game in which the sender plays in pure strategies is always included in the set of ABEE outcomes. The following two subsections will show how partial language competence of the sender can lead to equilibrium outcomes which are impossible to implement when the sender is fully competent.

Proposition 2 *Assume that $M = T \subseteq [0, 1]$ and $\lambda(t) = \{s \in T : s \leq t\}$, where $\lambda(t)$ is the set of messages that the sender of payoff type t understands. Then, every outcome of a Nash equilibrium in which the sender plays in pure strategies is an ABEE outcome.*

Proof. Consider a Nash equilibrium (σ_S, σ_R) in which every payoff type t sends a message $m(t) \in M$ with probability one, i.e., $\sigma_S(m(t) \mid t) = 1$ for every t. We construct an ABEE (σ_S^*, σ_R^*) in which every sender type t sends a message $m^*(t) \in \lambda(t)$ that he understands with probability one, i.e., $\sigma_S^*(m^*(t) \mid t) = 1$ for every t, and which implements the same outcome as (σ_S, σ_R). For every $t \in T$, let $m^*(t) = \min\{s \in T : m(s) = m(t)\}$ be the smallest payoff type who sends the same message as type t according to the strategy σ_S. Let $\sigma_R^*(m^*(t)) = \sigma_R(m(t))$. By construction, σ_R^* is a best response to σ_S^* because σ_R is a best response to σ_S and σ_S^* induces the same beliefs for the receiver as σ_S:

$$\Pr_{\sigma_S}(s \mid m(t)) = \Pr_{\sigma_S^*}(s \mid m^*(t)), \text{ for every } s, t \in T.$$

For every $t \in T$ we have $m^*(t) \in \lambda(t)$. Thus, along the equilibrium path of (σ_S^*, σ_R^*), the perceived expected utility of the sender is the same as the expected utility of the sender in the Nash equilibrium (σ_S, σ_R). A sender of type t with partial language competence perceives correctly the strategy of the receiver that follows any message that he understands. Hence, a sender of type t with partial language competence does not deviate from $m^*(t)$ to another

[5] These examples suggest that the receiver could also have an active communication role, for example by asking some precisions to the sender when he uses more technical messages. The analysis of such bilateral communication is beyond the objective of the current paper.

message $m' \in \lambda(t)$. If he deviates to a message he does not understand, $m' \notin \lambda(t)$, then his perceived expected utility is a convex combination of expected utilities he would get by deviating from the Nash equilibrium (σ_S, σ_R), and is therefore not profitable. ■

It is clear from the proof of this proposition that the result can be extended to more general language systems and to mixed strategies for the sender as long as a strategy σ_S^* with the following properties could be constructed: (i) it induces the same beliefs as σ_S for the receiver, and (ii) every sender type t only sends messages that he understands to the receiver. These properties do not hold in Example 3 in which we cannot implement the Nash equilibrium outcome as an ABEE outcome.

In the two following subsections, we assume that the receiver's optimal action is the expected value of the state given his belief. This assumption is standard in the economic literature on cheap talk and strategic information disclosure. It can be represented by the set of actions $A = [0, 1]$, and the receiver's utility function $v(a; t) = -(a - t)^2$. Hence, the receiver's strategy is always a pure strategy, that we denote by $\sigma_R : M \to A$. The two applications differ with respect to the sender's preferences.

5.1 Pure Persuasion

In the pure persuasion case, the sender's utility is independent of the state and can be written as $u(a; t) = u(a)$.[6] We further assume that $u(a)$ is non-decreasing, meaning that the sender always wants the receiver's action to be as high as possible. Hence, every Nash equilibrium outcome of the cheap talk game is the babbling outcome.

For simplicity, let $T = \{t_1, \ldots, t_n\}$.[7] The optimal action of the receiver is uniquely defined along the equilibrium path as the expected value of the type given the message and the sender's strategy:

$$\sigma_R(m) = \sum_{t \in T} \Pr(t \mid m)\, t, \text{for every } m \in \text{supp}[\sigma_S], \tag{2}$$

where $\Pr(t \mid m) = \frac{p^0(t)\sigma_S(m|t)}{\sum_s p^0(s)\sigma_S(m|s)}$. We also fix $\sigma_R(m) = 0$ for $m \notin \text{supp}[\sigma_S]$.; this is w.l.o.g. since action 0 is the most severe punishment for every sender type t.

First observe that, contrary to disclosure games with evidence (Milgrom, 1981), there is no fully revealing ABEE. In particular, there is no ABEE in which the two highest types t_{n-1} and t_n get different payoffs (and hence send distinct messages when $u(a)$ is strictly increasing in a). By assumption about the sender's language competences, there is a unique message that the sender of type t_{n-1} does not understand, namely t_n, so he perceives correctly the receiver's

[6]This setting is sometimes called "cheap talk with transparent motives" (Lipnowski and Ravid, 2018)

[7]The same analysis and insight can be obtained with $T = [0, 1]$.

reaction to this message. It follows that, if these two sender's types get distinct equilibrium payoffs, one of them has an interest in mimicking the other one.

One-Threshold ABEE. The following proposition establishes that, for every threshold type $t^* \in T$, there exists an ABEE in which (i) each sender type t below t^* over-reports his type by sending every message $m > t$ with the same probability; (ii) each sender type t above t^* truthfully reports that his type is above t^* by reporting $m = t^*$. In such an equilibrium, low types use all messages they do no understand while high types use the highest message they commonly understand. The intuition of this equilibrium is simple: if sender types below the threshold uniformly over-report their types, then the receiver's expectation about the state (and therefore his action) is increasing with the message he gets, except when he gets the message t^* corresponding to the threshold, for which his expectation is the highest (see Equation (8) in the proof below). Hence, sender types above the threshold report the threshold because they commonly understand this message and it induces the highest equilibrium action of the receiver. On the other hand, types below the threshold understand that reporting their type or under-reporting induces a lower action than over-reporting. However, since they do not understand the meaning of the over-reporting messages, they are unable to identify the best message (the message which consists in reporting the threshold t^*) and it is a best response for them to over-report uniformly. Notice that it is not crucial that types below the threshold precisely use a uniform over-reporting strategy. The uniform strategy is simple and natural, and it guarantees that the ordering of the receiver's strategy (8) is satisfied whatever the prior distribution of types.

Proposition 3 *For every threshold $t^* \in T$, the following strategy for the sender constitutes an ABEE:*

$$\sigma_S(m \mid t_k) = \begin{cases} \frac{1}{n-k} & \textit{if } m > t_k \\ 0 & \textit{if } m \leq t_k, \end{cases} \quad \textit{for every } t_k < t^*, \tag{3}$$

$$\sigma_S(m \mid t_k) = \begin{cases} 1 & \textit{if } m = t^* \\ 0 & \textit{if } m \neq t^*, \end{cases} \quad \textit{for every } t_k \geq t^*. \tag{4}$$

Proof. If $t^* = t_1$ we have a trivial pooling equilibrium, so let $k^* \in \{2, \ldots, n\}$ be such that $t_{k^*} = t^*$. From the sender strategy σ_S and the priors, we can compute the total probabilities of the different messages sent by the sender, and then get the conditional probabilities (beliefs of the receiver) for every message sent with positive probability. Simplifying, we get the following optimal actions for the receiver as a function of the message m he receives:

- For $t_1 < m = t_{k'} < t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1}p^0(t_1)t_1 + \cdots + \frac{1}{n-(k'-1)}p^0(t_{k'-1})t_{k'-1}}{\frac{1}{n-1}p^0(t_1) + \cdots + \frac{1}{n-(k'-1)}p^0(t_{k'-1})}. \tag{5}$$

- For $m > t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1}p^0(t_1)t_1 + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})t_{k^*-1}}{\frac{1}{n-1}p^0(t_1) + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})}. \tag{6}$$

- For $m = t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1}p^0(t_1)t_1 + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})t_{k^*-1} + p^0(t_{k^*})t_{k^*} + \cdots + p^0(t_n)t_n}{\frac{1}{n-1}p^0(t_1) + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1}) + p^0(t_{k^*}) + \cdots + p^0(t_n)}. \tag{7}$$

Hence we have:

$$0 = \sigma_R(t_2) < \sigma_R(t_3) < \cdots < \sigma_R(t_{k^*-1}) < \sigma_R(t_{k^*+1}) = \sigma_R(t_{k^*+2}) = \cdots = \sigma_R(t_n) < \sigma_R(t_{k^*}) \tag{8}$$

From this ordering we can observe that the sender has no profitable deviation. Indeed, for every type $t_k < t^*$, the equilibrium distribution of actions he perceives by sending a message he does not understand is a distribution over $\{\sigma_R(m) : m > t_k\}$, which is always higher than $\sigma_R(t_{k+1})$; if he deviates to a message that he understands he gets at most $u(\sigma_R(t_k)) \leq u(\sigma_R(t_{k+1}))$, so he does not deviate. Any type $t_k \geq t^*$ gets a payoff equal to $u(\sigma_R(t_{k^*}))$, which is the maximum payoff the sender can get given the strategy of the receiver. ■

A particular case of the one-threshold ABEE we just described is one in which the threshold is $t^* = t_1$, which is equivalent to pooling. For all other possible thresholds, some information is transmitted. We can note that $\sigma_R(t^*)$ is increasing in t^*, so when t^* increases sender types $t \geq t^*$ are better off.

Status Quo vs Alternative. Consider now a particular case in which the sender cares about whether the action taken by the receiver is above a threshold or not: $u(a) = 0$ if $a < \bar{a}$ and $u(a) = 1$ if $a \geq \bar{a}$, where $\bar{a} \in (0, 1)$. An interpretation is that the receiver has two actions and chooses the *alternative* instead of the *status quo* if his estimate of the expected value of the state is higher than $\bar{a}$. The sender always wants the alternative to be chosen. We assume that $E(t) < \bar{a}$, implying that the alternative is never chosen in the absence of information transmission.

As demonstrated above, there exists a t^*-threshold equilibrium for every $t^* \in T$. We can therefore characterize the best threshold for the sender, that is, the threshold that maximizes the probability that the alternative is chosen. This optimal threshold is the smallest type in the set $\{\tilde{t} \in T : E(t \mid m = \tilde{t}) \geq \bar{a}\}$ if this set is non-empty (otherwise, the alternative is never chosen whatever the threshold).

Notice that in the limit case in which the set of payoff types is $T = [0,1]$ we have $E(t \mid m = t^*) = E(t \mid t \geq t^*)$, and therefore t^* is the unique solution of $E(t \mid t \geq t^*) = \bar{a}$. For example, if payoff types are uniformly distributed, then $t^* = 2\bar{a} - 1$. It is interesting to observe that this ABEE coincides with the ex-ante optimal Bayesian solution of the sender. Indeed, in this example, the optimal Bayesian solution for the sender is given by a threshold information structure in which the receiver learns that $t \geq t^{\#}$ and is indifferent between the status quo and the alternative whenever $t \geq t^{\#}$. Hence, $t^{\#}$ solves $E(t \mid t \geq t^{\#}) = \bar{a}$, so $t^{\#} = t^*$.

Equilibrium with Multiple Thresholds. The previous proposition shows that there always exist equilibria with a single threshold. Equilibria with multiple thresholds could exist, but the existence of such equilibria depend on the specific parameters of the game. As an illustration, assume that $T = \{t_1, t_2, t_3, t_4\}$, $p^0(t_1) = p^0(t_2) = p^0(t_3) = 1/3$ and $p^0(t_4) = 0$ (the example is robust to any small perturbation of the prior probability distribution). Consider the following 2-threshold strategy for the sender, where the thresholds are $t^* = t_2$ and $t^{**} = t_3$:

$$\sigma_S(m \mid t_1) = \begin{cases} \frac{1}{3} & \text{if } m > t_1 \\ 0 & \text{if } m = t_1, \end{cases}$$

$\sigma_S(t_2 \mid t_2) = 1$, $\sigma_S(t_3 \mid t_3) = 1$, and $\sigma_S(t_3 \mid t_4) = 1$. Using Bayes rule, the following strategy is a best response for the receiver: $\sigma_R(t_1) = \sigma_R(t_4) = 0$, $\sigma_R(t_2) = \frac{3}{4}t_2$, and $\sigma_R(t_3) = \frac{3}{4}t_3$. Clearly, the sender types t_1, t_3 and t_4 have no incentive to deviate. Consider the sender type t_2. By sending message $m = t_2$, his perceived (and correct) expected utility is $\sigma_R(t_2) = \frac{3}{4}t_2$. If he deviates to a message he does not understand ($m = t_3$ or $m = t_4$), then his perceived expected utility is

$$\frac{\Pr(m = t_3)}{\Pr(m = t_3) + \Pr(m = t_4)}\sigma_R(t_3) + \frac{\Pr(m = t_4)}{\Pr(m = t_3) + \Pr(m = t_4)}\sigma_R(t_4) = \frac{3}{5}t_3.$$

Hence, the 2-threshold strategy above constitues an ABEE whenever $\frac{3}{4}t_2 \geq \frac{3}{5}t_3$.

5.2 Biased Sender

In this subsection we assume that the sender's preference over the estimates of the receiver depends on his payoff type and is given by

$$u(a;t) = -(a - (t+b))^2,$$

where $b > 0$ is a bias parameter. To compare our findings with existing results, we assume that the set of payoff types is $T = [0,1]$, with a uniform prior probability distribution.[8]

Partitional ABEE. As shown in Crawford and Sobel (1982), this cheap talk game has informative Nash equilibrium outcomes if and only if $b < 1/4$. Such equilibria are described by a partition of T into K intervals, $[t^*_{k-1}, t^*_k]$, $k = 1, \ldots, K$, such that the sender reveals to the receiver to which of these intervals his payoff type belongs. There is a unique K-partitional equilibrium outcome for every $K \geq 2$ satisfying $b \leq \frac{1}{2K(K-1)}$. In addition, every K-partitional equilibrium satisfies $t^*_K \leq 1 - 4b$, that is, there exists a pooling interval of size at least $4b$ at the top of the type space. From an ex-ante point of view, the best equilibrium is the same for the sender and the receiver and is given by the K^*-partitional equilibrium where K^* is the highest integer satisfying $b \leq \frac{1}{2K^*(K^*-1)}$.

From Proposition 2, these partitional Nash equilibrium outcomes are also ABEE outcomes for the language system assumed in this section. An ABEE equilibrium implementing a partitional Nash equilibrium outcome is constructed as follows: for every $k = 1, \ldots, K$, each type $t \in [t^*_{k-1}, t^*_k]$ sends message t^*_{k-1}, and the receiver responds optimally (i.e., chooses action $\frac{t^*_{k-1}+t^*_k}{2}$ after the message t^*_{k-1}), and plays action 0 off the equilibrium path. Consider a sender type $t \in [t^*_{k-1}, t^*_k]$ for some $k = 1, \ldots K$. In a Nash equilibrium, this type prefers to reveal that his type belongs to the interval $[t^*_{k-1}, t^*_k]$ than to any other interval. Hence, a sender with partial language competence does not under-report because he perceives the strategy of the receiver correctly by under-reporting, and he does not over-report because his perceived expected utility by over-reporting is a convex combinaison of the utilities he would get by over-reporting in a Nash equilibrium.

One-Threshold ABEE. First note that the informative threshold ABEE identified in Proposition 3 are never ABEE in the current framework for $b \leq 1/4$ because type $t = 0$ strictly prefers action 0 (what he induces by sending the message $m = 0$) to a distribution of actions whose expectation is 1/2 (what he perceives to induce by sending a message $m > 0$). Also observe that there is no fully revealing ABEE for $b > 0$: a high enough type $t < 1$ would deviate and

[8] The results are similar with a finite grid of types, but the analysis is less tractable due to integer problems.

over-report because, by over-reporting, he would perceive to induce a distribution of actions slightly above t, and hence closer to his ideal action $t + b$.

The next proposition shows that the following strategy for the sender always constitutes an ABEE for some $t^* \in (0, 1)$. If the sender's type t is strictly below t^*, then he fully reveals the state by sending the highest message he understands, i.e., by sending message $m = t$. Otherwise, if the sender's type is above t^*, then he sends message $m = t^*$.

Proposition 4 *The following pure strategy for the sender constitutes an ABEE if and only if* $b \leq \frac{1}{4}$ *and* $t^* = 1 - 4b$*:*

$$\sigma_S(t) = \begin{cases} t & \text{if } t < t^* \\ t^* & \text{if } t \geq t^*. \end{cases} \tag{9}$$

Proof. Consider a strategy for the sender as stated in the proposition. The best response of the receiver is:

$$\sigma_R(m) = \begin{cases} m & \text{if } m < t^* \\ E(t \mid t > t^*) = \frac{1+t^*}{2} & \text{if } m \geq t^*. \end{cases}$$

To constitute an ABEE, the sender type t^* must be indifferent between sending message $m = t^*$ and sending a message $m = t^* - \varepsilon$ when $\varepsilon \to 0$, i.e.,

$$-(t^* - (t^* + b))^2 = -(\frac{1 + t^*}{2} - (t^* + b))^2 \iff b^2 = (\frac{1 - t^*}{2} - b)^2 \iff t^* = 1 - 4b.$$

A sender of type $t > t^*$ prefers higher actions than the sender of type t^*, so the former prefers to pool on $m = t^*$ rather than send a message $m < t^*$. A sender of type $t < t^*$ does not deviate iff

$$-(t - (t + b))^2 \geq -\frac{1}{1 - t}\left(\int_t^{t^*} (m - t - b)^2 dm + (1 - t^*)(\frac{1 + t^*}{2} - t - b)^2\right),$$

where the LHS is the utility of type t if he sends message $m = t$ and the RHS is the perceived expected utility of type t if he over-reports, i.e., if he sends a message he does not understand. A sufficient condition for this inequality to be satisfied is

$$b^2 \leq \frac{1}{1 - t}\left((1 - t^*)(\frac{1 + t^*}{2} - t - b)^2\right).$$

Replacing t^* by $1 - 4b$ and simplifying we get

$$4(1 - t)^2 - 25b(1 - t) + 36b^2 \geq 0.$$

The LHS is equal to zero at $t = 1 - 4b$ and is decreasing in t for $t \leq 1 - \frac{25}{8}b$. Since $t < t^* = 1 - 4b < 1 - \frac{25}{8}b$, we conclude that the LHS is positive for every $t < t^*$. We conclude that the sender has no profitable deviation and therefore the strategy profile described above for $t^* = 1 - 4b$ and $b \leq 1/4$ is an ABEE. ∎

In the ABEE of the previous proposition, the strategy of the sender becomes more informative when the conflict of interest b decreases, and converges to full revelation when b tends to 0. The threshold $t^* = 1-4b$ is higher than the highest possible threshold t_K^* of a Nash equilibrium. Hence, this ABEE is more informative and therefore Pareto superior (at the ex-ante stage) to *all* Nash equilibria of the standard model of Crawford and Sobel (1982).[9]

To conclude, the analysis in Sections 5.1 and 5.2 shows that in two well-known communication settings (pure persuasion and cheap talk with a moderately biased sender) partial language competence can improve communication compared to the case of cheap talk from a fully competent sender. The qualitative properties of the equilibria in the two settings are however quite different: in the pure persuasion case, low types lie about their types by systematically over-reporting (because they have nothing to lose by doing so), and high types pool by truthfully reporting that they are higher than some threshold. This threshold could be any payoff type on which players coordinate on, and can be interpreted as a norm used by high type senders to distinguish themselves from lower types. When the sender is only moderately biased, low types truthfully and perfectly reveal their types (because over-reporting is too risky), and high types pool. In the latter case, the threshold level above which high types pool is not arbitrary and decreases with the conflict of interest between the sender and the receiver.

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[9]For small biases, this ABEE also ex-ante Pareto dominates all (mediated) communication equilibria. This assertion can be verified easily by using the upper bounds of the ex-ante expected communication equilibrium payoffs in Goltsman, Hörner, Pavlov, and Squintani (2009, Lemma 2) and checking that, when b is small enough, the ex-ante expected payoffs at the ABEE constructed above are strictly higher than these upper bounds.

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