# Optimal Contract Regulation in Selection Markets

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**Abstract**

We develop a tractable model of competitive insurance markets with a continuum of types and exogenous restrictions on the set of allowed contracts. Our model nests, as special cases, the market for lemons of Akerlof [1970] and the unrestricted contracts setting of Rothschild and Stiglitz [1976]. We allow purchase to be mandatory or voluntary. Equilibrium generically exhibits partial pooling and therefore depends non-trivially on the type distribution. An increase in the maximal allowed coverage always increases welfare. Increases in the minimal allowed coverage have ambiguous (and possibly non-monotonic) effects on welfare. In markets for lemons, if the first best is not an equilibrium, then the socially optimal level of mandated coverage is interior (i.e, below full insurance).

**JEL**: C62, G22, I13, D82, L51

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# 1 Introduction

In many insurance markets, and other markets with adverse selection, the set of available contracts is restricted by a regulator. In US health insurance exchanges, purchase is typically seen as mandatory (the option not to purchase is heavily taxed). In many North American and European markets, there are requirements that any insurance contract must provide a minimum level of coverage (see, for instance, Jensen and Gabel [1992]). Also in the US, a controversial "Cadillac Tax" on the most generous health insurance plans effectively imposes a cap on the generosity of plans offered by firms to their employees (Bram et al. [2016], Drake et al. [2017]).

This paper develops a tractable model of competitive insurance markets that allows for a flexible contract space. Our model nests, as special cases, the market for lemons of Akerlof [1970] and the unrestricted contracts setting of Rothschild and Stiglitz [1976]. We use the model's flexibility to show how restrictions on contracts affect equilibrium and welfare in several settings commonly considered in the existing literature.

To be more precise, we consider a model where a continuum of individuals differ in their risk types, but are homogeneous in their risk aversion. As in Rothschild and Stiglitz [1976] and a large ensuing literature, there is a competitive insurance industry which offers contracts with varying levels of generosity. Insurance generosity is parameterized as the share of the individual's wealth shock that is absorbed by the insurer, and denoted by $x$. Generosity ranges from no coverage $(x = 0)$ to full insurance $(x = 1)$. A regulator determines the set of insurance contracts available to individuals and firms, denoted $X \subseteq [0, 1]$. That is, only contracts $x \in X$ are allowed. Risk types determine each individual's demand for insurance, but also the expected cost imposed on insurers, resulting in adverse selection. We assume that moral hazard is absent so the first best would consist of all individuals obtaining full insurance. We consider the notion of competitive equilibrium of Azevedo and Gottlieb [2017], which always exists and has an intuitive and familiar structure, as we describe below.[1]

As a benchmark, we begin by recalling the model studied in Levy and Veiga [2021].[2] There is no minimal coverage restriction (laissez faire), so

$$X = [0, \overline{x}].$$

For any $\overline{x} > 0$, equilibrium is unique and has a structure which generalizes Rothschild and Stiglitz [1976]. There is full separation of types (each type purchases a different contract). Each contract breaks even, given its generosity and the type purchasing it. The generosity chosen by each type is increasing in risk. The highest-cost type purchases full insurance. Given the continuum of types, prices and equilibrium choices are defined by a simple differential equation.

Firstly, we consider a setting where there are minimum and maximum coverage requirement and purchase is mandatory, as in the US healthcare exchanges. In this case,

$$X = [\underline{x}, \overline{x}].$$

Equilibrium is unique. The minimum coverage requirement typically results in a partially-pooling equilibrium. This is illustrated in Figure 1. Risk types are $\mu \in \left[\underline{\mu}, \overline{\mu}\right]$, with higher values of $\mu$ being more costly to insurers. $\sigma(\mu)$ denotes the contract chosen by risk type $\mu$ in equilibrium. There

[1] The Azevedo and Gottlieb [2017] equilibrium, loosely speaking, requires that individuals optimize, each contract breaks even, and a specific robustness notion discussed further in Section 3. It is related to the equilibrium concept used by Dubey and Geanakoplos [2002].

[2] That paper takes $X = [0, 1]$ but, as pointed out there, the main characterization of equilibrium carries through for an arbitrary maximal coverage $\overline{x}$. It is, however, important in that article that the minimal coverage is $0$.

is a threshold type $\mu^*$ who purchases a contract with generosity $x^* = \sigma(\mu^*)$. Among consumers whose risk type $\mu \geq \mu^*$, there is full separation, with each type purchasing a different contract. In fact, these types purchase the same contracts as under laissez faire. All the remaining types purchase the minimum allowed coverage, $\underline{x}$. In equilibrium, there is a "gap": contracts $x \in (\underline{x}, x^*)$ are allowed, but not purchased by any type. If the range of allowed contracts is sufficiently small, all individuals pool at the minimal coverage.

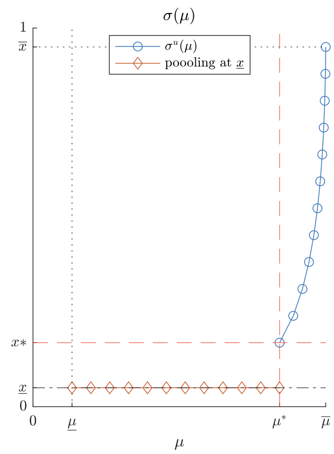


Figure 1: Illustration of equilibrium when purchase is mandatory, so $X = [\underline{x}, \overline{x}]$.

When purchase is mandatory, increasing the maximal coverage increases the mass of types in the separating region and increases welfare. Increasing the minimal coverage results in a trade-off: it reduces the mass of types in the separating region but increases the coverage of those pooling at the lowest level of coverage, so the overall effect on welfare is ambiguous. In simulations, we show that welfare can be non-monotonic in the region of partial pooling.

Secondly, we consider a setting where there is both a minimum and maximum coverage requirement, but individuals are also free not to purchase insurance. That is,

$$X = \{0\} \cup [\underline{x}, \overline{x}].$$

This model nests, as special cases, the unrestricted setting of Rothschild and Stiglitz [1976] ($X = [0, 1]$) and the market for lemons of Akerlof [1970], Einav and Finkelstein [2011] ($X = \{0, \underline{x}\}$). In general, equilibrium need not be unique.[3] However, uniqueness follows if the distribution of types is log-concave. In equilibrium, again, consumers whose riskiness is above a certain cut-off purchase the same contracts as under laissez faire. A further subset of intermediate types pool at the minimal allowed coverage. Those types whose riskiness is yet lower do not purchase at all. Again, there is a "gap" of contracts which are allowed but not purchased by any individual.

When purchase is voluntary, increasing the maximal coverage increases welfare, but increasing the minimal coverage has an ambiguous welfare effect. In particular, simulations show that welfare can be non-monotonic in the region of partial-pooling.

Thirdly, we explicitly consider markets for lemons, as in Akerlof [1970], Einav and Finkelstein [2011], where

$$X = \{0, \underline{x}\}.$$

[3] This was shown in markets for lemons by Scheuer and Smetters [2014].

Again, log-concavity of the type distribution guarantees equilibrium uniqueness. Here, we suppose that the regulator chooses the generosity of the single non-zero contract $\underline{x}$. The welfare maximizing coverage is interior (i.e., less than full insurance), unless full coverage for all individuals (i.e., the first best) is an equilibrium . The reason is that increasing generosity also increases adverse selection, thereby decreasing the mass of individuals who purchase.

Our model is tractable enough to approximate many of the contract restriction typically seen in selection markets. Taken together, these results suggest that restricting the most generous insurance contracts (as in the case of "Cadillac taxes") is likely to reduce consumer welfare. On the other hand, the welfare effects of minimal coverage restrictions are ambiguous a priori and depend on the distribution of types in each market. In these cases, precise policy recommendations would require estimating these cost distributions.

The rest of the paper is organized as follows. Section 2 describes the related literature. Section 3 describes the model and the equilibrium concept. Section 4 describes a benchmark with no contract restrictions. Section 5 discusses a setting with contract restrictions and mandatory purchase. Section 6 allows for purchase to be voluntary. Section 7 considers markets for lemons. Section 8 concludes. Most proofs are given in the appendices following the main text.

## 2 Literature

This article is related to the large literature studying regulation of markets for lemons, when there is a single product available, as in Akerlof [1970]. Einav et al. [2010a] discuss mandates and subsidies. Weyl and Veiga [2016] show how different pricing institutions impact welfare. Einav et al. [2019] contrasts risk adjustment and subsidies. Handel et al. [2015] discuss how community rating can reduce risk due to stochastic changes in type over time, but may increase adverse selection. Veiga [2020] shows that community rating can reduce the static welfare loss from adverse selection, and considers also regulation by means of information design (i.e., "Bayesian persuasion"). None of these articles discuss constraints on the contract space, as we do.

A smaller literature has considered regulation of insurance markets a la Rothschild and Stiglitz [1976, hecenforth RS], where firms endogenously choose the menu of contracts they offer. Azevedo and Gottlieb [2017, henceforth AG] discuss the effect of an increase in the minimum coverage on prices, assuming mandatory purchase. Those authors do not characterize equilibrium as we do, and do not discuss welfare.

A small number of papers based on the RS 2-type framework have obtained (often contradictory) results by making different assumptions on the equilibrium concept. Neudeck and Podczeck [1996] use the Grossman [1979] equilibrium concept (where each firm offers a single contract) to show that imposing a minimum standard can result in decreased welfare with some insurers earning positive profits. Encinosa [2001] comments on Neudeck and Podczeck [1996] showing that, when each firm is allowed to offer multiple contracts, a minimum standard does not result in positive profits and does increase welfare. McFadden et al. [2015] considers Nash equilibrium but assumes each firm offers a single contract. Then, those authors show that a sufficiently high minimum coverage requirement results in desirable a pooling equilibrium where types cross-subsidize, but a weak minimum coverage requirement results in an equilibrium where insurers earn positive profits and all individuals are worse off relative to laissez faire. Relative to these articles, we consider the intuitive equilibrium concept suggested by Azevedo and Gottlieb [2017], which allows us to obtain tractable equilibrium characterizations for any type distribution and for a flexible set of contract spaces, which therefore allows us to more closely approximate contract regulations in real world insurance markets.

A small empirical literature has focused on the effects of regulating contracts. Finkelstein [2004] shows empirically that the introduction of a minimum standard in the US Medigap market decreased the amount of individuals covered, which is consistent with our theoretical predictions. Einav et al. [2010b] shows that a mandate can we welfare improving in the context of UK annuities

Our equilibrium characterization is also related to other studies of insurance markets, that do not necessarily focus on regulation. Our analysis begins by generalizing the equilibrium characterization in Levy and Veiga [2021] (but that articles focuses on how the cost distribution affects equilibrium, not on regulation). Farinha Luz et al. [2021] focus on characterizing equilibrium in a setting where types are 2-dimensional but there is a sufficiently small range of risk aversion. Those authors do not consider regulation of the allowed contracts, as we do. Interestingly, Farinha Luz et al. [2021] focus on "no gap" equilibria where the set of contracts purchased in equilibrium is connected. Our results show that even small regulations of the contract space typically induce gaps (i.e., contracts which are not purchased).

# 3 Model Setup

## 3.1 Ingredients

There is a continuum of individuals with types $\Theta = [\underline{\mu}, \overline{\mu}] \subseteq \mathbb{R}_+$, distributed according to the PDF $f(\mu) > 0$. We denote the induced measure on $\Theta$ by $P$. We assume $\overline{\mu} > \underline{\mu}$. An individual of type $\mu$ has an expected cost of $\mu$.[4]

Individuals can purchase contracts characterized by their level of coverage $x \in X \subseteq [0, 1]$. An economy is a triple $\mathcal{E} = [\Theta, X, P]$. If an individual purchases a contract with coverage $x$, then the insurer covers a share $x$ of the individual's loss, and the individual is left covering a share $1 - x$. Therefore, the firm's cost is

$$c = x\mu.$$

A contract is a pair $(x, p)$, where $p \geq 0$ denotes price. If an individual of type $\mu$ purchases contract $(x, p)$, the profit to the firm from that purchase is

$$\pi(\mu, x, p) = p - x\mu.$$

The utility of type $\mu$ from choosing contract $(x, p)$ is

$$u(\mu, x, p) = x\mu + g(x) \cdot \nu_0 - p. \tag{1}$$

This specification can be derived in a CARA-Gaussian framework and is widely used in the insurance literature.[5] An individual who purchases insurance automatically transfers to the insurer the burden of her expected cost $c = x\mu$. Moreover, individuals are risk averse and therefore insurance results in an additional surplus, captured by the term $g(x) \cdot \nu$, where the parameter $\nu > 0$ captures risk aversion. We assume that $\nu$ is the same for all individuals. The function $g : [0, 1] \to \mathbb{R}_+$ is twice continuously differentiable with $g(0) = 0$, $g'(x) > 0$ in $[0, 1)$, $g'(1) = 0$, $g''(x) < 0$. Intuitively, the marginal surplus from insurance is positive but decreasing in the amount of coverage $x$, and the marginal surplus vanishes at full insurance.

[4]For instance, in the context of health insurance, $\mu$ might refer to the individual's expected yearly medical bill. In the context of auto-insurance, $\mu$ is proportional to the probability of having an accident.

[5]See for instance Farinha Luz et al. [2021], Veiga and Weyl [2016], Levy and Veiga [2021], Weyl and Veiga [2016], Handel et al. [2015], Azevedo and Gottlieb [2017].

**Example 1.** Suppose that an individual has CARA $U = -e^{-aw}$, where $w$ is final consumption is $a$ is the CARA coefficient. The individual is exposed to Gaussian wealth shocks $Z \sim \mathcal{N}(\mu, \sigma^2)$. If initial wealth is $w_0$, final wealth is $w_0 - (1-x)Z - p$ distributed log-normal. Then, the log of expected utility satisfies (1) with $g(x) = \frac{1}{2}\left(1 - (1-x)^2\right)$.

An allocation is a distribution $\alpha$ on $\Theta \times X$ with marginal $P$ on $\Theta$, and marginal $\alpha_X$ on $X$. Intuitively, $\alpha(\{\theta, x\})$ is the mass or density of types $\theta$ purchasing alternative $x$ under allocation $\alpha$.

Since $\nu$ is homogeneous, for a given allocation $\alpha$, social welfare is proportional to

$$W = \int_{\Theta \times X} g(x) d\alpha(\mu, x)$$

Notice that, in this setting, the socially efficient outcome is for all individuals to obtain full coverage ($x = 1$).[6]

## 3.2 Equilibrium

In this article, we consider exclusively the equilibrium concept described in Azevedo and Gottlieb [2017]. Intuitively, an AG equilibrium is a set of prices and an allocation $(p, \alpha)$ such that: 1) individuals optimize; 2) each contract breaks even; and 3) choices and prices of non-traded contracts are robust to small perturbations in fundamentals (which we describe in detail below). AG consider more general economies than the ones described above but, for simplicity, below we express the AG results specialized to the settings considered in this article.

AG define a "weak equilibrium" as a pair $(p, \alpha)$ where 1) individuals optimize and 2) each contract breaks even.

**Definition 1.** $(p, \alpha)$ is a *weak equilibrium* if

1. Individuals maximize:

$$\sup_{x' \in X} u(\mu, x', p(x')) = u(\mu, x, p(x)) \qquad \text{for } \alpha - a.e.\ (\mu, x)$$

2. Each contract breaks even:

$$p(x) = x \cdot \mathbb{E}_\alpha[\mu \mid x] \qquad \text{for } \alpha - a.e.\ x.$$

Typically, there exists many weak equilibria because, if $x$ is not traded, $p(x)$ can take any value. This motivates AG's definition of *equilibrium*: a weak equilibrium that is robust to the introduction of a small mass of zero-cost "behavioral" consumers who purchase every alternative $x \in X$. More precisely, an equilibrium is the limit of a sequence of weak equilibria of perturbed economies $\mathcal{E}_j$ on finite grids in $X$ where the mass of behavioral types vanishes.

**Definition 2.** Consider the economy $\mathcal{E} = [\Theta, X, P]$, and the sequence of perturbed economies $\mathcal{E}_j = [\Theta \cup \overline{X}_j, \overline{X}_j, P + \eta_j]$. Let $(\overline{X}_j)_{j \in N}$ be a sequence of finite subsets of $X$ which converge to

[6] This would not necessarily be the case, for instance, if moral hazard or insurance loads were present. For a discussion, see Einav et al. [2010a]. Handel et al. [2015] estimate the distribution of preferences and costs in the context of US employer-provided health insurance and find that it is socially optimal for all individuals to obtain the maximal available level of insurance.

$X$ in the sense of Haussdorf,[7],[8] Let $(\eta_j)_{j\in N}$ be a sequence of measures, with $\eta_j$ supported on $\overline{X}_j$, strictly positive on $\overline{X}_j$, and $\eta_j(\overline{X}_j) \to 0$. Suppose there exists a sequence of pairs $(p_j, \alpha_j)_{j\in\mathbb{N}}$, satisfying the following conditions. First, $(p_j, \alpha_j)$ is a weak equilibrium of $\mathcal{E}_j$, where the *behavioral type* $x \in \overline{X}_j$ has zero cost and prefers $x$ to any other alternative regardless of price. Second, $\alpha_j \to \alpha$ weakly.[9] Third, whenever $(x_j)_{j\in\mathbb{N}}$ converges to $x \in X$ with $x_j \in \overline{X}_j$, then $p_j(x_j) \to p(x)$. Then, the pair $(p, \alpha)$ is an *equilibrium* of $\mathcal{E} = [\Theta, X, P]$.

AG assume utility $u(\mu, x, p)$ is continuous and strictly decreasing in $p$ and satisfies a form of Lipschitz-ness in $X$ uniformly over types (which is satisfied in the environment we consider). AG also assume that cost is continuous and weakly positive, and the type space $\Theta$ is bounded (so utility and cost are also bounded).[10] Given these assumptions, AG prove the following result.

**Theorem 1.** *Every economy has an equilibrium.*

Notice AG equilibrium need not be unique. AG also show that, for any equilibrium $(p, \alpha)$, $p(\cdot)$ is continuous.

## 4 A benchmark case

We begin by presenting, as a benchmark case, some results from Levy and Veiga [2021]. Recall that the set of types is $\Theta = [\underline{\mu}, \overline{\mu}] \subseteq \mathbb{R}_+$. In this benchmark setting, importantly, there is no minimal coverage. Therefore, the space of allowed contracts is[11]

$$X = [0, \overline{x}].$$

Full coverage ($x = 1$) may or may not be available, since $\overline{x} \leq 1$. Despite the restriction on maximal coverage, equilibrium in setting is very similar to that in Rothschild and Stiglitz [1976], where firms were not restricted in what contracts they can offer, as we discuss below.

Let the set of contracts bought by the consumers in equilibrium (the support of $\alpha_X$)[12] be

$$\mathcal{B} = supp(\alpha_X).$$

The following result summarizes Theorem 2 and Corollary 2 from Levy and Veiga [2021].

**Theorem 2.** *Suppose $X = [0, \overline{x}]$. There is a unique equilibrium $(p, \alpha)$, where:*

1. *There exists a threshold contract $x^* \in [0, \overline{x})$ s.t. $\mathcal{B} = [x^*, \overline{x}]$.*

2. *$\alpha_X$ has the same null sets as the Lebesgue measure in $[x^*, \overline{x}]$.*

3. *Price $p(x)$ is continuous and strictly increasing on $\{x \mid p(x) > 0\}$. Also, $p(x) \leq \overline{\mu} \cdot x$.*

[7] I.e., for each $x \in X$, there is $(x_n)_{n\in\mathbb{N}}$ converging to $x$ with $x_n \in \overline{X}^n$ for each $n \in \mathbb{N}$.

[8] In Levy and Veiga [2021], $X$ is not assumed to be compact but have a compactification $\overline{X}$ which $X$ embeds into, e.g., $X = [0, 1)$ which naturally embeds in $\overline{X} = [0, 1]$.

[9] That is, for each $f : \Theta \times X \to \mathbb{R}$ continuous and bounded, we have $\int f d\alpha^n \to \int f d\alpha$.

[10] Levy and Veiga [2021] discuss AG equilibria when cost is unbounded.

[11] Levy and Veiga [2021] take $\overline{x} = 1$ but, as pointed out in a footnote of their Section 4.1, the results hold for any maximal coverage $\overline{x} \in (0, 1]$.

[12] Recall that the support $supp(\mu)$ is the smallest closed set of $\mu$-measure 1.

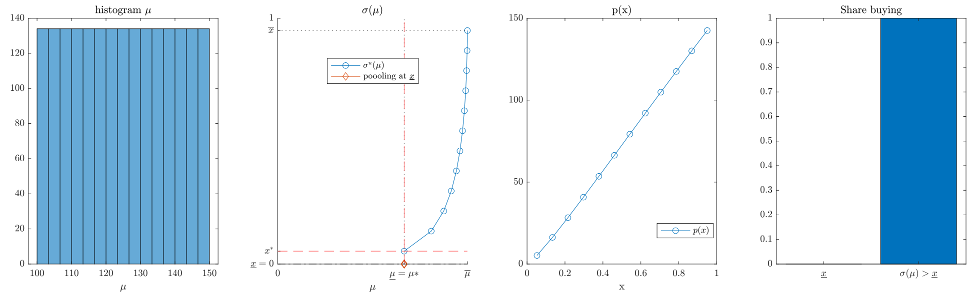


Figure 2: Equilibrium when there is no minimal coverage, so$X = [0, \overline{x}]$. Here, $\mu \sim \mathcal{U}\,[70, 150]$ to more clearly illustrate that $x^* > 0$.

4. *There is a continuous and strictly increasing mapping* $\sigma : \Theta \to X$ *that assigns to each type* $\mu$*, the contract* $\sigma(\mu)$ *that she choses* $\alpha$*-a.s. Formally,* $\alpha\big\{(\mu, x) \mid x = \sigma(\mu)\big\} = 1$*. Moreover, the choice rule* $\sigma$ *satisfies*

$$\overline{\mu} - \mu = \nu \cdot \int_{\sigma(\mu)}^{\overline{x}} \frac{g'(x)}{x} dx, \qquad \forall \mu > \underline{\mu}. \tag{2}$$

   *In particular,* $\sigma > 0$*. Let the type that purchases contract* $x$ *be* $\tau(x) = \sigma^{-1}(x)$*.*

5. *Each contract* $x \in X$ *breaks even:*

$$p(x) = x \cdot \tau(x) \qquad P\text{-a.s.}$$

   *In particular,* $p(0) = 0$*.*

6. $p$ *is Lebesgue-a.e. differentiable. Moreover,*

$$p'(x) = \tau(x) + \nu \cdot g'(\tau(x)), \qquad \text{a.e.} \quad x \in (x^*, \overline{x})$$

In this benchmark environment with no minimal coverage requirement, equilibrium has several properties familiar from Rothschild and Stiglitz [1976]. First, there is full separation of types: each type $\mu$ purchases a different contract $x = \sigma(\mu)$. In the environments we will discuss below, restrictions on the contract space $X$ will often induce "pooling" (multiple types purchasing the same contract).

Second, each contract breaks even via the simple rule $p(x) = x \cdot \tau(x)$, where $\mu = \tau(x)$ is the type that purchases contract $x$.[13]

Third, the mapping between types and contracts $\sigma(\cdot)$ (2) follows from incentive compatibility. In equilibrium, the highest type $\overline{\mu}$ purchases the highest available coverage $\overline{x}$ and each type $\mu$ is indifferent between the contract she purchases under $\sigma(\cdot)$ and the contract immediately "below," as in Rothschild and Stiglitz [1976], Levy and Veiga [2021]. Intuitively, from (5), as risk aversion shrinks ($\nu \to 0$), the coverage purchased by each type $\mu < \overline{\mu}$ shrinks to zero ($\sigma(\mu) \to 0$).

Fourth, all individuals purchase some insurance. More precisely, the mass of individuals choosing $x = 0$ is zero. In the environments we will discuss below, restrictions on the contract space $X$ can induce an atom of individuals to purchase the lowest available level of coverage, or to obtain no coverage at all. Figure 2 illustrates the equilibrium in this benchmark setting.

[13] This is an assumption of the AG equilibrium concept. For other equilibrium concepts (e.g., Nash equilibrium) it is possible that firms make positive profits (see Levy and Veiga [2020] and the references therein).

To avoid repetition and aid our analysis below, we define the following functions, which apply to the *unconstrained* (hence the superscript "$^u$") markets defined above:

$$p^u, \tau^u : \{(x, \overline{x}) \in \mathbb{R}_{++} \times \mathbb{R}_{++} \mid x \leq \overline{x}\} \to \mathbb{R}_{++}, \tag{3}$$

$$\sigma^u : \{(\mu, \overline{x}) \in \mathbb{R}_{++} \times \mathbb{R}_{++} \mid \mu \leq \overline{\mu}\} \to \mathbb{R}_{++}. \tag{4}$$

The function $p^u$ gives the equilibrium price for an economy with contracts $X = [0, \overline{x}]$ and types $\mu \in [0, \overline{\mu}]$.[14] The function $\sigma^u$ gives the chosen levels of coverage in equilibrium, for that same economy, for each type $\mu \in \Theta$. The function

$$\tau^u(\cdot, \overline{x}) = (\sigma^u)^{-1}(\cdot, \overline{x}, \overline{\mu})$$

gives the type $\mu$ that purchases a given coverage level $x$ in equilibrium. Since each contract breaks even, $p^u(x, \overline{x}) = x \cdot \tau^u(x, \overline{x})$.[15] Notice that $\sigma^u$ satisfies (2).

# 5 Mandatory purchase

In this section, we assume that each individual must purchase a level of coverage of at least $\underline{x}$.[16] Formally, we assume

$$X = [\underline{x}, \overline{x}].$$

This generalizes the benchmark model of Section 4, where $\underline{x} = 0$. The model is only interesting if $\overline{x} > \underline{x} > 0$, which we assume.[17]

## 5.1 Equilibrium characterization

The restrictions on contracts considered in this section often result in "pooling" in equilibrium. More specifically, it is possible that multiple types (or even all types) purchase the minimal coverage contract $\underline{x}$. We therefore introduce the following terminology.

**Definition 3.** Let $(p, \alpha)$ be an equilibrium:

- There is *no pooling* if $\alpha_X(\{x \mid x > \underline{x}\}) = 1$: all individuals purchase strictly more than $\underline{x}$.
- There is *partial pooling* if $\alpha_X(\{x \mid x > \underline{x}\}) \in (0, 1)$: an atom of (but not all) individuals purchase $\underline{x}$.
- There is *full pooling* if $\alpha_X(\{x \mid x > \underline{x}\}) = 0$: all individuals purchase $\underline{x}$.

Proposition 1 below characterizes equilibrium when there is no or partial pooling. If there is no pooling, Proposition 1 reduces to Theorem 2. Then, Proposition 2 discusses necessary and sufficient conditions for partial or full pooling to occur.

Recall that $\mathcal{B} = supp(\alpha_X)$ is the set of contracts bought by the consumers, and $\sigma : \Theta \to X$ is the a mapping that assigns types $\mu$ to their chosen contract $x = \sigma(\mu)$. Recall also the definitions of $\sigma^u, p^u, \tau^u$ from (3) and (4):

[14] Later, we will consider changes in $\overline{x}$ but not in $\overline{\mu}$, so we have these being functions of the former but not the later.

[15] When $\overline{x}, \overline{\mu}$ are implicitly understood, we omit them for notational simplicity. The lower bound $\underline{\mu}$ plays no role in the definition of the functions $p^u, \tau^u, \sigma^u$ and therefore is omitted as a functional argument.

[16] Such minimum coverage regulations are common in many regulated insurance markets, such as health insurance in the US healthcare exchanges and auto insurance in many countries.

[17] If $\overline{x} = \underline{x}$, all individuals mechanically purchase the unique available contract. If $\underline{x} = 0$, equilibrium follows Theorem (2).

**Proposition 1.** *Suppose $X = [\underline{x}, \overline{x}]$. Any equilibrium $(p, \alpha)$ with $\alpha_X(\{x \mid x > \underline{x}\}) > 0$,*

1. There is a *cut-off coverage* $x^* \in [\underline{x}, \overline{x})$ s.t.
   (a) $\mathcal{B} = [x^*, \overline{x}]$ (no pooling) or
   (b) $\mathcal{B} = \{\underline{x}\} \cup [x^*, \overline{x}]$ (partial pooling)
2. $\alpha_X$ has the same null sets as the Lebesgue measure in $[x^*, \overline{x}]$.
3. $p(x)$ is continuous, and is strictly increasing on $\{x \mid p(x) > 0\}$, and $p(x) \leq \overline{\mu} \cdot x$ for all $x \in X$.
4. If all contracts in $[\underline{x}, \overline{x}]$ are bought, then $x^*$ is not an atom of $\alpha_X$. Formally, $x^* = \underline{x} \Leftrightarrow \alpha_X(\{\underline{x}\}) = 0$.
5. $\sigma$ is non-decreasing, and is strictly increasing and continuous for $\mu \in \sigma^{-1}([x^*, \overline{x}])$. Let $\tau = \sigma^{-1}$ be defined on $[x^*, \overline{x}]$. Denoting the type that purchases $x^*$ as the *cut-off type*

$$\mu^* = \tau(x^*),$$

   the choice rule $\sigma$ satisfies

$$\sigma(\mu) \equiv \sigma^u(\mu, \overline{x}), \qquad \forall \mu \in [\mu^*, \overline{\mu}]\,. \tag{5}$$

6. The cut-offs $(x^*, \mu^*)$ and $\sigma$ satisfy:
   (a) $\mathcal{B} = [x^*, \overline{x}] \Leftrightarrow \mu^* = \underline{\mu}$ (no pooling), or
   (b) $\mathcal{B} = \{\underline{x}\} \cup [x^*, \overline{x}] \Leftrightarrow \mu^* \in (\underline{\mu}, \overline{\mu})$ (partial pooling), in which case
      i. $\sigma(\mu) = \underline{x}$ for $\mu \in [\underline{\mu}, \mu^*)$
      ii. $\sigma(\mu) \in [x^*, \overline{x}]$ for $\mu \in [\mu^*, \overline{\mu}]$.
7. All contracts break even (and (6) is vacuous if $\mu^* = \underline{\mu}$):

$$p(x) = x \cdot \tau(x, \overline{x}) \equiv p^u(x, \overline{x}), \qquad \text{a.e.} \quad x \in [x^*, \overline{x}].$$

$$p(\underline{x}) = \underline{p} = \underline{x} \cdot \mathbb{E}[\mu \mid \mu \leq \mu^*] \tag{6}$$

8. $p$ is Lebesgue-a.e. differentiable and

$$p'(x) = \tau(x) + \nu \cdot g'(\tau(x)), \qquad \text{for a.e.} \quad x \in (x^*, \overline{x}) \tag{7}$$

9. For all $x \in [\underline{x}, x^*)$, price $p(x)$ is s.t. the cut-off type $\mu^*$ is indifferent between $(x, p(x))$ and $(x^*, p(x^*))$, i.e.

$$(g(x^*) - g(x)) \cdot \nu = \mu^* \cdot x - p(x), \qquad \forall x \in [\underline{x}, x^*]. \tag{8}$$

10. For $x \in (\underline{x}, x^*)$, $p < p^u(\cdot, \overline{x})$.

*Proof.* See Appendix A. □

If purchase is mandatory ($X = [\underline{x}, \overline{x}]$), equilibrium is characterized by the cut-off coverage and type $x^*, \mu^*$. Types $\mu \in [\mu^*, \overline{\mu}]$ are fully separated. These types purchase contracts $x \in [x^*, \overline{x}]$ and each type $\mu \in [\mu^*, \overline{\mu}]$ purchases the same contract she would purchase if there was no minimal coverage: $\sigma(\mu) \equiv \sigma^u(\mu, \overline{x}), \forall \mu \in [\mu^*, \overline{\mu}]$.

Insurers break even on each contract. In the fully separated region, the price of each contract $x \in [x^*, \overline{x}]$ is actuarily fair given the type that purchases it: $p(x) = x \cdot \tau(x)$. The cut-off type $\mu^*$ purchases the contract $x^* = \sigma(\mu^*)$ and is indifferent between that contract and the minimal coverage contract $(\underline{x}, p(\underline{x}))$.[18] The atom of types $\mu \in [\underline{\mu}, \mu^*)$ purchase the minimal coverage contract $\underline{x}$, which also breaks even given its buyers: $p(\underline{x}) = \underline{x} \cdot \mathbb{E}[\mu \mid \mu < \mu^*]$.

The highest type $\overline{\mu}$ purchases the maximal available insurance coverage at a fair price. Then, down to type $\mu^*$, there is continuous full separation, according to a first-order condition that follows from incentive compatibility. Part 9 follows from Part 2 of Proposition 1 of Azevedo and Gottlieb [2017].[19] Part 10 then follows by comparing Parts 8, 9.

Parts 4 and 6 are the key novelty. It is tempting to think that there would be full separation of types starting from maximal coverage and all the way down to minimal coverage, always at fair prices, and only after all contracts from $\overline{x}$ to $\underline{x}$ had been purchased would those types below $\mu^*$ pool at $\underline{x}$. However, the price per unit of coverage at $\underline{x}$ would then be strictly less than $\mu^*$, while prices per unit just above $\underline{x}$ would be above $\mu^*$. Price $p(x)$ would then be discontinuous. Hence, the pooling produces a gap $(\underline{x}, \overline{x}^*)$ in which no purchasing occurs.

We now discuss conditions under which there is zero, partial or full pooling at $\underline{x}$.

**Proposition 2.** *Suppose $X = [\underline{x}, \overline{x}]$. Let $(p, \alpha)$ be an equilibrium. Then:*

1. *There is an atom at $\underline{x}$ (i.e., there is partial or full pooling) iff*

$$\frac{1}{\nu}(\overline{\mu} - \underline{\mu}) > \int_{\underline{x}}^{\overline{x}} \frac{g'(x)}{x} dx. \tag{9}$$

2. *There is full pooling (all types purchase $\underline{x}$) iff the highest type $\overline{\mu}$ weakly prefers $(\underline{x}, \underline{x} \cdot E[\mu])$ to $(\overline{x}, \overline{\mu} \cdot \overline{x})$, i.e.*

$$\underline{x}\left(\overline{\mu} - \mathbb{E}\left[\mu\right]\right) \geq \left(g(\overline{x}) - g(\underline{x})\right)\nu. \tag{10}$$

*Proof.* See Appendix C for the first part. Appendix C shows that full pooling implies (10), and Appendix C shows the converse. □

To build (9), one follows (5), i.e. the definition of the choice rule $\sigma^u(\cdot, \overline{x})$ in the region of full separation of types. Intuitively, if one "runs out of" contracts $x$ before all types are assigned a contract according to $\sigma^u(\cdot, \overline{x})$, then there is some pooling at $\underline{x}$. This explains why the presence of pooling at $\underline{x}$ is independent of the distribution of types $f(\mu)$.

By inspection of (9), pooling at $\underline{x}$ is more likely when $\nu$ is small. When $\nu$ is large, individuals obtain large surpluses from insurance and therefore are more likely to purchase more than the minimal coverage $\underline{x}$. Also, other things being equal, pooling at $\underline{x}$ is more likely when the range

[18] In fact, prices $p(\hat{x})$ for $\hat{x} \in [\underline{x}, x^*]$ are such that type $\mu^*$ is indifferent between contract $(x^*, p(x^*))$ and any contract $\hat{x} \in [\underline{x}, x^*]$.

[19] Proposition 1 of Azevedo and Gottlieb [2017] informally states "*every contract that is not traded in equilibrium has a low enough price for some consumer to be indifferent between buying it or not, and the cost of this consumer is at least as high as the price.*"

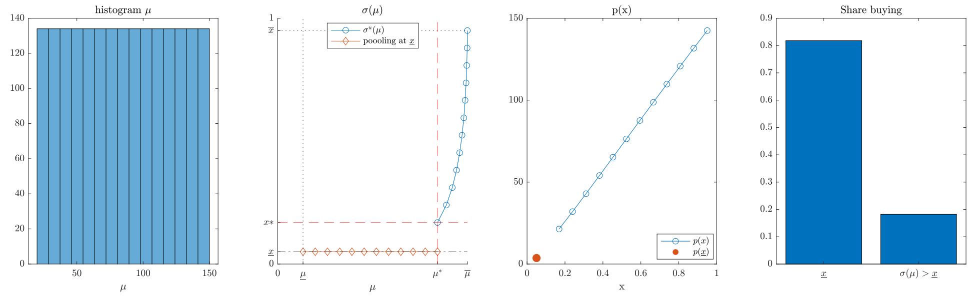


Figure 3: Illustration of Equilibrium when purchase is mandatory, so $X = [\underline{x}, \overline{x}]$.

of types $\overline{\mu} - \underline{\mu}$ is larger. Since $g'(x)/x > 0$, pooling is also more likely if the range of contracts is smaller (low $\overline{x}$ and/or high $\underline{x}$).[20]

To build (10), we show that full pooling occurs if the highest type $\overline{\mu}$ prefers to purchase the minimal coverage at the average cost across all individuals ($\underline{x} \cdot \mathbb{E}[\mu]$) relative to purchasing the most generous contract $\overline{x}$ at the actuarily fair price under $\sigma^u(\cdot, \overline{x})$, $p(\overline{x}) = \overline{\mu} \cdot \overline{x}$. Notice that (10) implies (9): if there is full pooling, the conditions for partial pooling are satisfied.[21] (10) is a necessary (and sufficient) condition for full pooling, as $p(\overline{x}) \leq \overline{\mu} \cdot \overline{x}$. If it did not hold, agents with high enough risk $\mu$ would prefer maximal coverage.

Let us now consider more closely how the minimal coverage $\underline{x}$ determines pooling in equilibrium. If $\underline{x} = 0$ no pooling occurs (Theorem 2). As the minimal coverage $\underline{x}$ increases, the right-hand-side (RHS) of (9) shrinks, so pooling at $\underline{x}$ becomes more likely. If $\overline{x} = \underline{x}$, the RHS of (10) vanishes so there is full pooling. In sum, as $\underline{x}$ increases, the economy transitions from no, to partial, to full pooling. This is further illustrated below by Lemma 3 and Figure 6.

Figure 3 illustrates equilibrium when $X = [\underline{x}, \overline{x}]$.

In this setting we are also able to show, without additional assumptions, equilibrium uniqueness.

**Proposition 3.** *If $X = [\underline{x}, \overline{x}]$, equilibrium is unique.*

*Proof.* See Appendix D.1. □

Uniqueness will follow from the characterizations of equilibrium above by checking various cases. First, we show that any two equilibria with partial (but not full) pooling coincide. Second, we show that any two equilibria which are not fully pooled coincide. That is, there cannot be two different equilibria with no pooling (this follows from (5)) and there cannot be an equilibrium with no pooling alongside an equilibrium with partial pooling. Third, if there is a full pooling equilibrium, it is (by the second part of Proposition 2) the unique equilibrium. These steps are proven by comparing the slope of $p(x)$ in equilibrium (from (7) and (8)) and the price of $\underline{x}$ determined by the break-even condition (6).

## 5.2 Log-Concave Distributions

Before we proceed, we introduce the following definition.

[20] Weyl and Veiga [2016] showed, in a simpler 2 contract setting, that if the two contracts have similar levels of coverage, the market "collapses" to individuals purchasing the lowest level of coverage.

[21] The second inequality implies $\frac{\overline{\mu}-\mathbb{E}[\mu]}{\nu} \geq \frac{g(\overline{x})-g(\underline{x})}{\underline{x}}$. Then, comparing the the LHS of both inequalities, we observe $\frac{1}{\nu}(\overline{\mu}-\underline{\mu}) > \frac{1}{\nu}(\overline{\mu}-\mathbb{E}[\mu])$, while comparing the RHS of both inequalities, we observe $\frac{g(\overline{x})-g(\underline{x})}{\underline{x}} > \int_{\underline{x}}^{\overline{x}} \frac{g'(x)}{x} dx$.

**Definition 4.** A twice differentiable function $f(\cdot) > 0$ is *log-concave* iff $\frac{\partial^2}{\partial \mu^2} \ln [f(\mu)] \leq 0$.

Below we will frequently assume that the PDF $f(\mu)$ is log-concave, which is a feature of many commonly used statistical distributions.[22] We further define[23]

$$\phi(\mu^*) = \mu^* - \mathbb{E}[\mu \mid \mu < \mu^*] \geq 0, \tag{11}$$

$$\psi(\mu_*) = \mathbb{E}[\mu \mid \mu > \mu_*] - \mu^* \geq 0. \tag{12}$$

**Proposition 4.** *(Bagnoli and Bergstrom [2005]) If $f(\mu)$ is log-concave, then $\phi' > 0$ and $\psi' < 0$.*[24]

For instance, if $f(\mu)$ is uniform (and thus log-concave) then $\phi = \psi = \frac{1}{2}(\mu^* - \mu_*)$.

The next sections will examine the effects of changes of $(\overline{x}, \underline{x})$ while holding the distribution of types fixed. Each pair $(\underline{x}, \overline{x})$ defines a unique threshold type $\mu^* = \mu^*(\underline{x}, \overline{x})$ and a unique threshold coverage $x^* = x^*(\underline{x}, \overline{x})$. By convention, if there is full pooling, we define $x^* = \overline{x}$ (to preserve the continuity of $x^*$).

We introduce the following terminology. The pair $(\underline{x}, \overline{x})$ is:

- "in the region of no pooling" if $\mu^*(\underline{x}', \overline{x}') = \underline{\mu}$ for all pairs $(\underline{x}', \overline{x}')$ sufficiently close to $(\underline{x}, \overline{x})$.
- "in the region of partial pooling" if $\underline{\mu} < \mu^*(\underline{x}', \overline{x}') < \overline{\mu}$ for all pairs $(\underline{x}', \overline{x}')$ sufficiently close to $(\underline{x}, \overline{x})$.
- "in the region of full pooling" if $\mu^*(\underline{x}', \overline{x}') = \overline{\mu}$ for all pairs $(\underline{x}', \overline{x}')$ sufficiently close to $(\underline{x}, \overline{x})$.

Inspection of the equations defining the cut-off type and cut-off coverage, implies the following result.

**Corollary 1.** *The thresholds $x^*, \mu^*$ associated with equilibria change continuously with $\underline{x}, \overline{x}$ and the distribution $f(\cdot)$, the latter w.r.t. weak convergence of probability measures.*[25]

### 5.3 Adjusting maximal coverage

We now discuss the effects of changes in the level of the maximal coverage, $\overline{x}$. We theoretically characterize the effect of $\overline{x}$ on $\mu^*, x^*$, and welfare, and then illustrate the theoretical results with numerical simulations.

[22] The following distribution are log-concave: uniform, normal, exponential, logistical, extreme value, Laplace, Maxwell, Reyleigh. The following distribution are log-concave for some parameter values: power, Weibull, Gamma, Chi-Squared, Chi, Beta. See Bagnoli and Bergstrom [2005] for a detailed discussion.

[23] Bagnoli and Bergstrom [2005] refer to $\phi$ as the *mean-advantage-over-inferiors* and $\psi$ as the *mean-residual-lifetime (MRL)*.

[24] Bagnoli and Bergstrom [2005] actually only states $\phi' \geq 0$ and $\psi' \leq 0$, but the proof - see in particular the proofs of Lemma 3 and Lemma 4 in the appendix there - shows that when $f$ is supported on a compact interval, we have strict inequalities.

[25] $f_n \to f$ in this topology on distributions if for each continuous bounded function $\phi : \mathbb{R}_+ \to \mathbb{R}$, $\int f_n \phi \cdot dx \to \int f \phi \cdot dx$

### 5.3.1 Theoretical Results

Let $\mu^* = \mu^*(\underline{x}, \overline{x})$ and $x^* = x^*(\underline{x}, \overline{x})$ define the cut-off coverage and types associated with the economies with contract space $[\underline{x}, \overline{x}]$.

**Lemma 1.** *In each of the regions of no / partial / full pooling, $\mu^*, x^*$ are differentiable in $\overline{x}$. Furthermore, in each such region,*

$$\frac{\partial \mu^*}{\partial \overline{x}} \leq 0$$

*with strict inequality in the region of partial (but not full) pooling.*

*In the region of no pooling,*

$$\frac{\partial x^*}{\partial \overline{x}} > 0$$

*In the region of partial (but not full) pooling, if the PDF $f(\mu)$ is log-concave,*

$$\frac{\partial x^*}{\partial \overline{x}} < 0.$$

*Proof.* For the first and second parts, see Appendix D.2. For the last part, see Appendix D.4.2. □

When there is partial pooling, allowing for more generous insurance contracts (increasing $\overline{x}$) results in fewer individuals purchasing the minimal coverage $\underline{x}$ (reduces $\mu^*$). When there is no pooling, an increase in $\overline{x}$ results in all types purchasing higher coverage (in particular type $\underline{\mu}$, who purchases $x^*$, so $x^*$ increases also). When there is partial pooling, the effect of $\overline{x}$ on the cut-off contract $x^*$ is ambiguous, as shown in Figure 5. However, if $f(\cdot)$ is log-concave, increasing $\overline{x}$ lowers $x^*$.

Recall that $\sigma^u(\mu, \underline{x}, \overline{x})$ is the choice of type $\mu$ when the set of coverages available is $X = [\underline{x}, \overline{x}]$. We now leverage the above results to discuss the effect of contract space regulation on social welfare. Since $\nu$ is homogeneous across individuals, welfare is proportional to the average of $g(\sigma(\mu))$ across all types, or

$$W = W(\underline{x}, \overline{x}) = F(\mu^*)g(\underline{x}) + \int_{\mu^*(\underline{x},\overline{x})}^{\overline{\mu}} g\left(\sigma^u\left(\mu, \underline{x}, \overline{x}\right)\right) f(\mu) d\mu. \tag{13}$$

With zero pooling, $\mu^* = \underline{\mu} \Rightarrow F(\mu^*) = 0$ . With full pooling, $\mu^* = \overline{\mu} \Rightarrow F(\mu^*) = 1$.

We begin by establishing the following preliminary result.

**Lemma 2.** *For $\mu \in (\mu^*, \overline{\mu})$, where $\mu^* = \mu^*(\underline{x}, \overline{x})$, in each of the regions of full / partial / no pooling, $\sigma^u(\mu, \cdot, \cdot)$ is differentiable in $\overline{x}$ and*

$$\frac{\partial \sigma^u(\mu, \overline{x}, \underline{x})}{\partial \overline{x}} = \frac{g'(\overline{x})}{\overline{x}} \frac{\sigma^u(\mu, \overline{x}, \underline{x})}{g'(\sigma^u(\mu, \overline{x}, \underline{x}))} > 0.$$

*Proof.* Follows from implicit differentiation of (2). See Appendix D.2.4 for details. □

Increasing $\overline{x}$ increases the level of coverage chosen by each type in the fully separated region $\mu \in [\mu^*, \overline{\mu}]$. This implies the following result.

**Proposition 5.** *Suppose $X = [\underline{x}, \overline{x}]$. Then $W(\underline{x}, \overline{x})$ is differentiable as a function of $\overline{x}$ in each of the regions of full / partial / no pooling. Moreover:*

- *If* $(\underline{x}, \overline{x})$ *are in the regions of none or partial pooling, then*

$$\frac{\partial W}{\partial \overline{x}} > 0.$$

- *If* $(\underline{x}, \overline{x})$ *are in the regions of full pooling, then*

$$\frac{\partial W}{\partial \overline{x}} = 0.$$

*Proof.* See Appendix D.3. □

Proposition 5 shows that increasing $\overline{x}$ is always (weakly) beneficial. If there is zero pooling, all agents increase their purchased coverage. If there is partial pooling, an increase in $\underline{x}$ affects welfare in two ways. First, it increases the level of coverage chosen by each type in the fully separated region $\mu \in [\mu^*, \overline{\mu}]$. Second, it lowers the threshold $\mu^*$, so there are fewer individuals purchasing the minimal coverage $\underline{x}$. Both effects result in higher welfare. If there is full pooling, increasing $\overline{x}$ does not change any choices, so it does not affect welfare.

### 5.3.2 Numerical simulations

We now present numerical simulations that illustrate these theoretical results. We parameterize the surplus from insurance as

$$g(x) = \frac{1}{2}\left(1 - (1-x)^2\right).$$

This parameterization can be obtained by assuming individuals with CARA preferences exposed to Gaussian wealth shocks.[26] In all simulations, we use $\nu = 25$. For most simulations, we assume types are uniformly distributed as $\mu \sim \mathcal{U}[20, 150]$. However, we will also consider other distributions to illustrate that some effects are not signed in general. We fix $\underline{x} = 0.1$ and consider a grid of $\overline{x} \in [0.15, 1]$.[27] In the event of full pooling ($\mu^* = \overline{\mu}$), we define $x^* = \overline{x}$, to preserve continuity of $x^*$. Welfare is measured as the average of $g(\sigma(\mu))$ across all simulated types. For additional details, see Appendix G.

Consider Figure 4. The first panel shows a histogram of the (uniform) type distribution. The second panel shows $\mu^*$ as a function of $\overline{x}$, with the levels of $\{\overline{\mu}, \underline{\mu}\}$ highlighted by horizontal dashed lines. Full pooling ($\mu^* = \overline{\mu}$) occurs for low values of $\overline{x}$ (when the range of contracts $\overline{x} - \underline{x}$ is small). Under full pooling, $\mu^* = \overline{\mu}$ does not change with $\overline{x}$. Partial pooling ($\mu^* < \overline{\mu}$) occurs for higher values of $\overline{x}$ (when the range of contracts is large). Under partial pooling, $\mu^*$ is decreasing in $\underline{x}$ (Lemma 1). In this range of parameters, zero pooling does not occur.

The third panel shows $x^*$ as a function of $\overline{x}$. As discussed following Lemma 1, $x^*$ can be non-monotonic, even within the region of partial pooling, because $\overline{x}$ affects the threshold $\mu^*$ but also the level of coverage chosen by each type $\mu \in [\mu^*, \overline{\mu}]$. However, since $f(\mu)$ uniform is log-concave, $x^*$ falls with $\overline{x}$ in the region of partial pooling (Lemma 1).[28]

The fourth panel shows the effect on welfare. Under full pooling, welfare does not change with $\overline{x}$. Under partial pooling, welfare is increasing in $\overline{x}$ (Proposition 5).

[26] This specification is used, for instance, by Veiga and Weyl [2016], Levy and Veiga [2021], Weyl and Veiga [2016].
[27] We choose $\underline{x} = 0.1$ because, if $\underline{x} = 0$, pooling never occurs for any $\overline{x}$ (Theorem 2).
[28] Recall that, in the region of full pooling, we set $x^* = \overline{x}$.

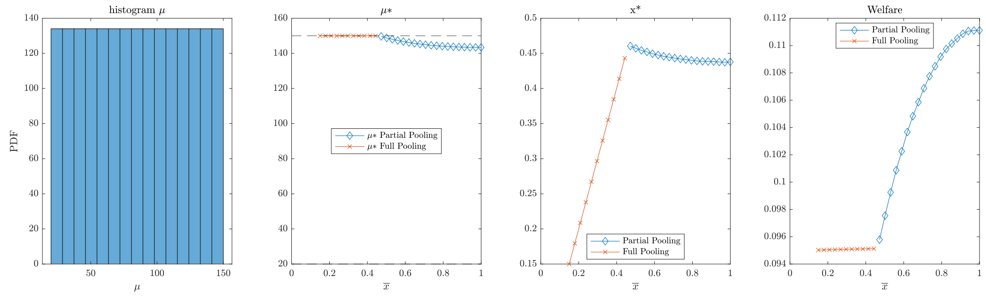


Figure 4: The effect of $\overline{x}$ on $\mu^*, x^*, W$ for a uniform type distribution and mandatory purchase.

Figure 5 illustrates that, in general, the effect of $\overline{x}$ on $x^*$ is not signed, and can even be non-monotonic in the region of partial pooling. For clarity, we show only the range of $\overline{x}$ for which there is partial pooling. The first panel shows the distribution of types, which is not log-concave. The third panel shows that, for this distribution, $x^*$ is non-monotonic. Notice, that $\mu^*$ is still decreasing (Lemma 1) and welfare increasing (Lemma 1), since these results do not depend on the distribution of types.

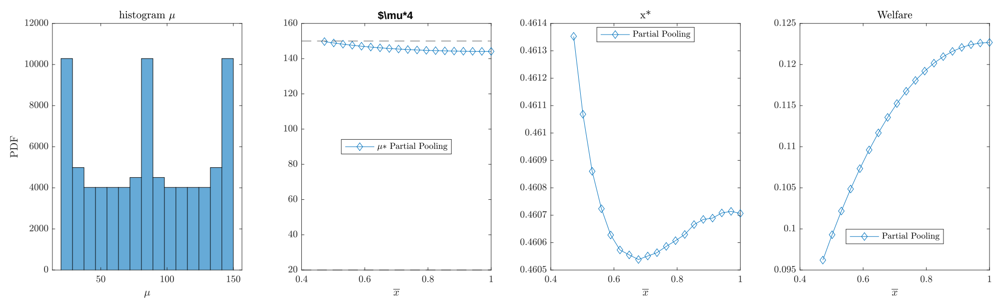


Figure 5: The effect of $\overline{x}$ in a market with mandatory purchase, for a distribution $f(\mu)$ that is not log-concave.

Below we provide a summary of these results.

| Mandatory purchase, effect of $\overline{x}$ | |
|---|---|
| $\frac{\partial \mu^*}{\partial \overline{x}} < 0$ | Lemma 1 |
| $\frac{\partial x^*}{\partial \overline{x}}$ generally ambiguous<br>$\frac{\partial x^*}{\partial \overline{x}} < 0$ if partial pooling & $f$ log-concave<br>$\frac{\partial x^*}{\partial \overline{x}} > 0$ if no pooling | Lemma 1<br>Figure 5 |
| $\frac{\partial W}{\partial \overline{x}} > 0$ if no or partial pooling<br>$\frac{\partial W}{\partial \overline{x}} = 0$ if full pooling | Proposition 5 |

## 5.4 Adjusting minimal coverage

### 5.4.1 Theoretical Results

We now discuss the effects of changes in the level of the minimal coverage, $\underline{x}$. As above, we discuss theoretically the effect of $\underline{x}$ on $\mu^*, x^*$, and welfare. Then, we illustrate the theoretical results with numerical simulations.

**Lemma 3.** *In each of the regions of no / partial / full pooling, $\mu^*, x^*$ are differentiable in $\underline{x}$. Furthermore,*

$$\frac{\partial \mu^*}{\partial \overline{x}} \geq 0, \qquad \frac{\partial x^*}{\partial \overline{x}} \geq 0,$$

*with strict inequality in the region of partial (but not full) pooling.*

*Proof.* See Appendix D.2 (and Appendix D.4.3 for an alternative proof). □

When the minimal level of coverage $\underline{x}$ increases, the cut-off type $\mu^*$ increases. That is, to induce more individuals to purchase coverage in the interval $x \in [x^*, \overline{x}]$, the regulator must make the minimal coverage contract less appealing by reducing $\underline{x}$. Since, $x^* = \sigma^u(\mu^*, \underline{x}, \overline{x})$ and $\sigma^u(\cdot, \underline{x}, \overline{x})$ is increasing in $\mu$ for $\mu \in [\mu^*, \overline{\mu}]$, then $x^*$ also increases with $\underline{x}$.

We proceed by deriving the following preliminary result.

**Lemma 4.** *For $\mu \in (\mu^*, \overline{\mu})$, where $\mu^* = \mu^*(\underline{x}, \overline{x})$, then in each of the regions of full / partial / no pooling, $\sigma^u(\mu, \cdot, \cdot)$ is differentiable in $\underline{x}$ and $\frac{\partial \sigma^u}{\partial \underline{x}}(\mu, \overline{x}, \underline{x}) = 0$.*

*Proof.* Types purchasing $x > \underline{x}$, locally, are not affected by changes in the imposed minimal coverage. See Appendix D.2.4 for details. □

Increasing the minimal coverage $\underline{x}$ does not affect the choice of any type $\mu$ that remains in the fully separated region (but it does raise $\mu^*$, thereby shrinking the set of types that are in this region, by Lemma 3).

Recall that welfare is given by (13), or

$$W = W(\underline{x}, \overline{x}) = F(\mu^*)g(\underline{x}) + \int_{\mu^*(\underline{x}, \overline{x})}^{\overline{\mu}} g\left(\sigma^u\left(\mu, \underline{x}, \overline{x}\right)\right) f(\mu) d\mu.$$

We can now discuss how regulation of $\underline{x}$ affects welfare.

**Proposition 6.** *Suppose $X = [\underline{x}, \overline{x}]$. Then $W(\underline{x}, \overline{x})$ is differentiable as a function of $\underline{x}$ in each of the regions of full / partial / no pooling. Moreover:*

- *If $(\underline{x}, \overline{x})$ are in the region of no pooling, then*

$$\frac{\partial W}{\partial \underline{x}} = 0.$$

- *If $(\underline{x}, \overline{x})$ are in the region of full pooling, then*

$$\frac{\partial W}{\partial \underline{x}} > 0.$$

*Proof.* See Appendix D.4 □

If there is no pooling, a zero measure of individuals purchase $\underline{x}$. Therefore, a change in $\underline{x}$ has no effect on welfare: $\frac{\partial W}{\partial \underline{x}} = 0$.[29] If full pooling occurs, all individuals purchase $\underline{x}$. Then, an increase in $\underline{x}$ has a positive effect on welfare: $\frac{\partial W}{\partial \underline{x}} \geq 0$.

[29] Actually, this can be shown to be true even at the point when pooling begins (i.e., when (9) holds with equality).

In the region of partial pooling, the net effect on welfare is ambiguous; see Figure 7. Indeed, increasing $\underline{x}$ has two effects. Individuals already purchasing $\underline{x}$ obtain better coverage. However, since the minimal coverage $\underline{x}$ becomes more generous, some individuals are induced to switch out of purchasing the higher level of coverage $x = \sigma^u(\mu, \underline{x}, \overline{x})$ into purchasing $\underline{x}$, so the range of fully separated types $\mu \in [\mu^*, \overline{\mu}]$ shrinks.

Formally, the effect of $\underline{x}$ on welfare is characterized by Equation (28) (in Appendix D.4). Intuitively, the magnitude of the second (negative) effect on welfare is proportional to the density of marginal types $f(\mu^*)$. In regions where $f(\mu^*)$ is low, $\frac{\partial W}{\partial \underline{x}} \geq 0$ is more likely, and vice-versa.

### 5.4.2 Numerical Simulations

We now present numerical simulations that illustrate the effects of changes in $\underline{x}$. Unless otherwise stated, the setup for these simulations is the same as the one above. We fix $\overline{x} = 1$ and consider a grid of $\underline{x} \in [0.1, 0.95]$.

Consider Figure 6. The first panel shows the (uniform) distribution of types. The second panel shows $\mu^*$ as a function of $\underline{x}$. Partial pooling ($\mu^* < \overline{\mu}$) occurs for low values of $\underline{x}$ (when the range of contracts $\overline{x} - \underline{x}$ is large). Under partial pooling, $\mu^*$ increases with $\underline{x}$ (Lemma 3). Full pooling ($\mu^* = \overline{\mu}$) occurs for large values of $\underline{x}$ (when the range of contracts is small). Under full pooling, $\underline{x}$ has no effect on $\mu^*$.

The third panel shows $x^*$ as a function of $\underline{x}$. Under partial pooling, both $\mu^*, x^*$ increase in $\underline{x}$ (Lemma 3). Under full pooling, $\underline{x}$ has no effect on $\mu^* = \overline{\mu}$ or $x^* = \overline{x}$.

The fourth panel shows the effect of $\underline{x}$ on welfare. Under full pooling, welfare increases with $\underline{x}$ (Proposition 6). Under partial pooling, and given the uniform type distribution, welfare is increasing in $\underline{x}$. However, in general, the sign of $\frac{\partial W}{\partial \underline{x}}$ depends on the distribution of types, as illustrated below in Figure 7.

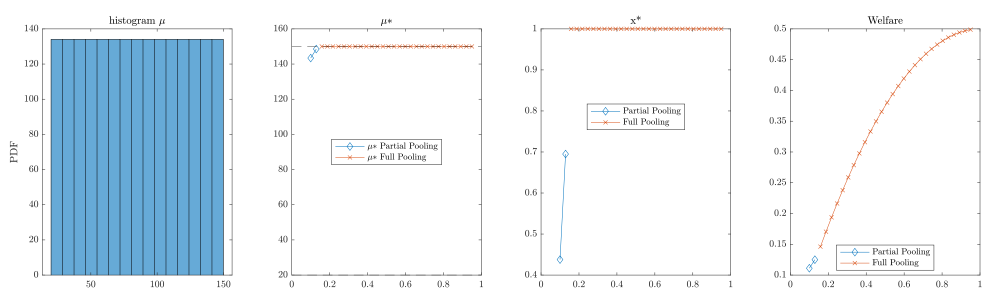


Figure 6: The effect of $\underline{x}$ on $\mu^*, x^*$ and welfare for a uniform distribution and mandatory purchase.

Figure 7 show that, in general, the effect of $\underline{x}$ on welfare is ambiguous. The For clarity, we show only the range of $\underline{x}$ for which there is partial pooling. In each set of graphs, the first panel shows a histogram of the distribution of types. The second and third panels confirm that $\mu^*$ and $x^*$ increase with $\underline{x}$ (Lemma 3). The fourth panel shows that welfare can be non-monotonic in $\underline{x}$, in the region of partial pooling, which illustrates the indeterminacy of the sign of $\frac{\partial W}{\partial \underline{x}}$ (Proposition 6).

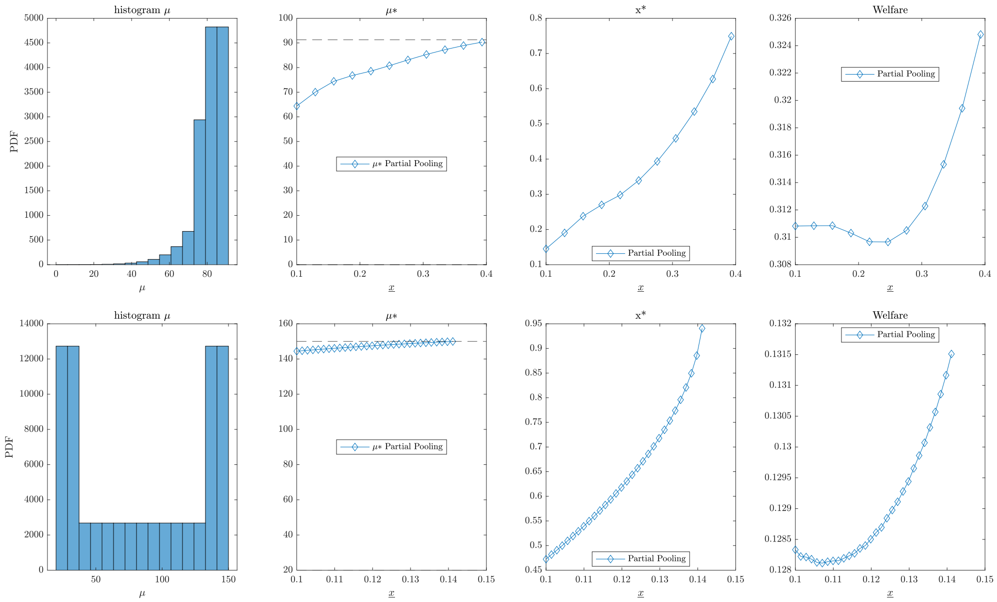


Figure 7: The effect of $\underline{x}$ in a market with mandatory purchase.

Below we provide a summary of these results.

| Mandatory purchase, effect of $\underline{x}$ | |
|---|---|
| $\frac{\partial \mu^*}{\partial \underline{x}} > 0$ | Lemma 3 |
| $\frac{\partial x^*}{\partial \underline{x}} > 0$ | Lemma 3 |
| $\frac{\partial W}{\partial \underline{x}} = 0$ if no pooling<br>$\frac{\partial W}{\partial \underline{x}} > 0$ if full pooling<br>$\frac{\partial W}{\partial \underline{x}}$ ambiguous if partial pooling | Proposition 6<br>Figure 7 |

# 6 Voluntary Purchase

In this section, we assume that individuals are free not to purchase insurance. There exists a contract that offers zero coverage ($x = 0$) and will have a price of $p(0) = 0$ in equilibrium, and this is interpreted as not purchasing insurance. Beyond this, we continue to assume that there is a minimal positive level of coverage $\underline{x}$ that insurers must offer. Formally,

$$X = \{0\} \cup [\underline{x}, \overline{x}].$$

To make the model interesting, we assume $\overline{x} > \underline{x} > 0$.[30]

[30] If $\underline{x} = 0$, then equilibrium is described by Theorem 2. If $\underline{x} = \overline{x}$ then there is a single non-zero insurance contract, as in Akerlof [1970], Einav et al. [2010a], and we discuss this setting in Section 7.

## 6.1 Equilibrium characterization

Unless stated otherwise, we follow the terminology of Section 5.[31] Proposition 7 below shows that full pooling either means that only $\underline{x}$ is purchased, or both $\{0, \underline{x}\}$ are purchased. As we show below, it *cannot* be the case that only $0$ is purchased but $\underline{x}$ is not.

Proposition 7 below characterizes equilibrium. We focus on the case where some individuals purchase more than the minimal positive coverage level $\underline{x}$ (otherwise, equilibrium is described in Section 7). Several properties of equilibrium are the same as those in Proposition 1, so Proposition 7 focuses on the differences between the two settings.

In this context, we define the set of strictly positive coverage ($x > 0$) contracts that are purchased in equilibrium by

$$\mathcal{B}^+ = supp(\alpha_X) \cap [\underline{x}, \overline{x}].$$

That is, individuals purchase contracts in $\mathcal{B}^+$ and, possibly, some individuals also choose not to purchase (i.e., choose $x = 0$).

**Proposition 7.** *Suppose $X = \{0\} \cup [\underline{x}, \overline{x}]$. In any AG equilibrium $(p, \alpha)$ with $\alpha_X(\{x \mid x > \underline{x}\}) > 0$,*

1. *There is a cut-off coverage $x^* \in [\underline{x}, \overline{x})$ s.t.*
   - (a) $\mathcal{B}^+ = [x^*, \overline{x}]$ *or*
   - (b) $\mathcal{B}^+ = \{\underline{x}\} \cup [x^*, \overline{x}]$.
2. *Parts 2, 3, 4, 5, 8, 9, and 10 of Proposition 1 hold. In particular:*
   - (a) *for $\mu \in [\mu^*, \overline{\mu}]$, the choice rule $\sigma(\mu)$ satisfies $\sigma = \sigma^u(\cdot, \overline{x})$ from (5)*
   - (b) *the cut-off type $\mu^*$ satisfies (8)*
3. *The cut-off coverage $x^*$, cut-off type $\mu^*$, and coverage function $\sigma$ satisfy*
   - (a) $\mathcal{B}^+ = [x^*, \overline{x}] \Leftrightarrow \mu^* = \underline{\mu}$ *(no pooling), or*
   - (b) $\mathcal{B}^+ = \{\underline{x}\} \cup [x^*, \overline{x}] \Leftrightarrow \mu^* \in (\underline{\mu}, \overline{\mu})$ *(partial pooling), in which case*
     - i. $\sigma(\mu) \in \{0, \underline{x}\}$ *for* $\mu \in [\underline{\mu}, \mu^*)$,
     - ii. $\sigma(\mu) \in [x^*, \overline{x}]$ *for* $\mu \in [\mu^*, \overline{\mu}]$.
4. *The* cut-off participation type $\mu_*$ *is defined as lowest type that purchases the minimal coverage $\underline{x}$: $\mu_* = \inf\{\mu \mid \sigma(\mu) = \underline{x}\}$. It holds that,*
   - (a) $\mu_* = \underline{\mu}$ *iff all types prefer $(\underline{x}, p(\underline{x}))$ to $(0,0)$, i.e.*

     $$g(0) \cdot \nu \leq \underline{\mu} \cdot \underline{x} + g(\underline{x}) \cdot \nu - p(\underline{x}).$$

     *In this case, all agents purchase insurance; i.e., $x = 0$ is not chosen.*
   - (b) *Otherwise (i.e., if $\mu_* > \underline{\mu}$), $\mu_*$ is the type who is indifferent between these contracts:*

     $$g(0) \cdot \nu = \mu_* \cdot \underline{x} + g(\underline{x}) \cdot \nu - p(\underline{x}). \tag{14}$$

[31] In particular, recall Definition 3. If $\alpha_X(\{x \mid x > \underline{x}\}) = 1$, there is *no pooling*. If $\alpha_X(\{x \mid x > \underline{x}\}) \in (0,1)$, there is *partial pooling*. If $\alpha_X(\{x \mid x > \underline{x}\}) = 0$, there is *full pooling*.

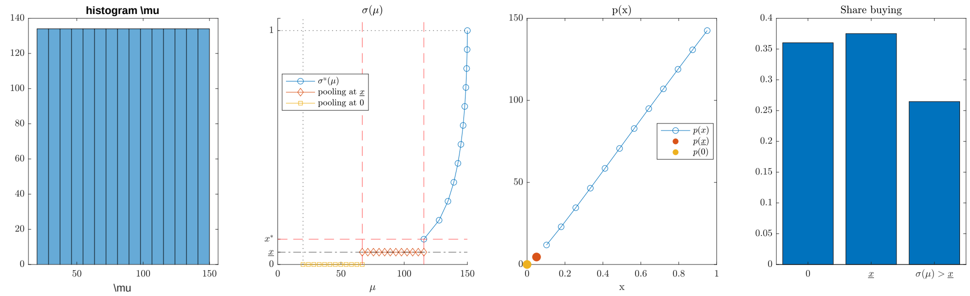


Figure 8: Illustration of Equilibrium when purchase is voluntary, so $X = \{0\} \cup [\underline{x}, \overline{x}]$.

5. *All contracts break even. In particular,* $p(0) = 0$,

$$p(x) = x \cdot \tau(x), \qquad \textit{a.e.} \quad x \in [x^*, \overline{x}]$$

$$\underline{p} = p(\underline{x}) = \underline{x} \cdot \mathbb{E}[\mu \mid \mu \in [\mu_*, \mu^*]], \tag{15}$$

*where (15) is vacuous if* $\mu_* = \mu^*$

6. *If* $x = 0$ *is an atom of* $\alpha_X$*, then* $\underline{x}$ *is as well:* $\alpha_X(\{0\}) > 0 \Rightarrow \alpha_X(\{\underline{x}\}) > 0$.

*Proof.* See Appendix A. □

In an environment with voluntary purchase, equilibrium is defined by two cut-off types. Types $\mu \in [\mu^*, \overline{\mu}]$ are in the fully separated region and purchase the same contracts they would purchase under laissez faire. As in Section 5, type $\mu^*$ is indifferent between the contract she would obtain under laissez faire, and the minimal coverage $\underline{x}$. Types $\mu \in [\mu_*, \mu^*)$ constitute an atom purchasing the minimal coverage $\underline{x}$. The cut-off participation type $\mu_*$ is indifferent between the minimal coverage $\underline{x}$ and no insurance at all ($x = 0$). Types $\mu \in [\underline{\mu}, \mu_*)$ constitute an atom of individuals who do not purchase insurance.

All contracts break even. Not purchasing has a cost of zero and thus a price of zero. In the fully separated region, we have $p(\sigma^u(\mu, \overline{x})) = \mu \cdot \sigma^u(\mu, \overline{x}) = p^u(\sigma^u(\mu, \overline{x}))$. The price of contract $\underline{x}$ is the average cost of all individuals who purchase it: $p(\underline{x}) = \underline{x} \cdot \mathbb{E}[\mu \mid \mu \in [\mu_*, \mu^*]]$. Finally, if some individuals choose not to purchase, then an atom of individuals also chooses to purchase the minimal coverage $\underline{x}$ (there is pooling at $\underline{x}$).[32] Figure 8 illustrates the equilibrium.

The next result describes the conditions under which pooling occurs in this setting.

**Proposition 8.** *Suppose* $X = \{0\} \cup [\underline{x}, \overline{x}]$. *Let* $(p, \alpha)$ *be an equilibrium of the economy. Then:*

1. The first part of Proposition 2 holds, i.e., there is an atom at $\underline{x}$ (partial or full pooling) iff

$$\frac{1}{\nu}(\overline{\mu} - \underline{\mu}) > \int_{\underline{x}}^{\overline{x}} \frac{g'(x)}{x} dx. \tag{16}$$

2. If there is full pooling, then the highest type $\overline{\mu}$ weakly prefers the contract $(\underline{x}, \underline{x} \cdot \mathbb{E}[\mu])$ to the contact $(\overline{x}, \overline{\mu} \cdot \overline{x})$, i.e., (10) holds:

$$\underline{x}(\overline{\mu} - \mathbb{E}[\mu]) \geq (g(\overline{x}) - g(\underline{x}))\,\nu. \tag{17}$$

[32]The final point of Proposition 7 has a simple intuition: if only 0, but not $\underline{x}$, is an atom of the distribution, then in particular $\mu_* = \mu^*$, and $\sigma(\mu^*) = x^*$and $p(x^*) = x^* \cdot \mu^*$. By risk aversion, type $\mu^*$ (and hence types just below $\mu^*$, who are purchasing $(0, 0)$) strictly prefer $(x^*, p(x^*))$ to $(0, 0)$, a contradiction.

*Proof.* See Appendix C for the first part. Appendix C shows that full pooling implies (10). The proofs are the same as the case of mandatory purchase. □

Notice that, when purchase is mandatory (as in Section 5), (17) was also a sufficient condition for there to be at full pooling. In this case, it is only necessary (as far as know).[33]

### 6.2 Log-concavity and uniqueness

We now show how, in this setting, log-concavity implies equilibrium uniqueness. We define the average risk within the set of individuals pooling at $\underline{x}$ as

$$E = E\left(\mu^*, \mu_*\right) = \mathbb{E}\left[\mu \mid \mu \in (\mu_*, \mu^*)\right].$$

Notice that $E$ is increasing in both arguments.[34] We introduce the following generalizations of $\phi$ and $\psi$:

$$\phi(\mu_*, \mu^*) = \mu^* - E\left(\mu^*, \mu_*\right) \geq 0 \tag{18}$$

$$\psi(\mu_*, \mu_*) = E\left(\mu^*, \mu_*\right) - \mu_* \geq 0 \tag{19}$$

Using these definitions, and applying Proposition 4 to the log-concave distribution $f(\cdot)$ conditional on sub-intervals, implies

$$\frac{\partial \phi}{\partial \mu^*} > 0, \qquad \frac{\partial \phi}{\partial \mu_*} < 0, \qquad \frac{\partial \psi}{\partial \mu^*} > 0, \qquad \frac{\partial \psi}{\partial \mu_*} < 0. \tag{20}$$

which implies

$$1 > \frac{\partial E}{\partial \mu^*}, \qquad 1 > \frac{\partial E}{\partial \mu_*}.$$

In general, equilibrium need not be unique. Recall that equilibrium is also not unique in general in the classical Akerlof [1970] model, which is a special case of the model being discussed (in that model, $\underline{x} = \overline{x}$).[35] However, a log-concave type distribution guarantees uniqueness. This condition is known to be sufficient for uniqueness in models with a single insurance contract, which we discuss below in Section 7.[36] The novelty of Proposition 9 is to show that the same condition also implies uniqueness in this more general setting with a continuum of contracts.

**Proposition 9.** *If $f(\mu)$ is log-concave, equilibrium is unique.*

*Proof.* See Appendix E.1. □

As in the proof of Proposition (3), uniqueness follows from the characterizations above and by checking the various cases. First, if there are agents not purchasing, then there are also those purchasing $\underline{x}$. Second, if one equilibrium has at least partial pooling, then all equilibria do. Third, if neither of two equilibria has pooling, or if both have pooling with price coinciding at $\underline{x}$, then they are the same equilibrium. Fourth, if each equilibrium has at least partial pooling but

[33] It is not clear if a necessary and sufficient condition for full pooling can be phrased in terms of the model primitives.

[34] Intuitively, an increase in $\mu^*, \mu_*$ results in the expectation being conditional on a set of higher values of $\mu$.

[35] Scheuer and Smetters [2014] discuss multiplicity in the Akerlof [1970] model.

[36] For a proof, see Bagnoli and Bergstrom [2005]

different prices of $\underline{x}$, we deduce a contradiction. It is only in this last step that log-concavity is used.[37]

For the remainder of Section 6, we assume $f(\cdot)$ is log-concave.

Given uniqueness, the following result follows from the characterizations in Proposition 7.

**Corollary 2.** *If $f(\mu)$ is log-concave, the thresholds $x^*, \mu^*, \mu_*$ associated with equilibria change continuously with $\underline{x}, \overline{x}$ and the distribution $f(\cdot)$, the latter w.r.t. weak convergence of probability measures.*

Recall the terminology of being in the regions of full, partial, or no pooling from Section 5.3. In this setting, since $\mu_* = \mu_*(\underline{x}, \overline{x})$ is also well-defined when $f(\mu)$ is log-concave, we say that $(\underline{x}, \overline{x})$ is in the region of zero pooling if $\mu_*(\underline{x}', \overline{x}') = \underline{\mu}$ for all $(\underline{x}', \overline{x}')$ close enough to $\underline{x}, \overline{x}$. Similarly, there is full pooling if $\mu_*(\underline{x}', \overline{x}') = \overline{\mu}$ and partial pooling otherwise.

## 6.3 Adjusting maximal coverage

We now consider the effects of $\overline{x}$ on the thresholds $\mu^*$, $\mu_*$, $x^*$, then its effect on welfare, and finally we illustrate the theoretical results with numerical simulations. We focus on the region of partial pooling because, in the region of zero pooling, equilibrium behaves as in Section 4. In the region of full pooling, equilibrium behaves as in Section 7.

### 6.3.1 Theoretical Results

Let $\mu^* = \mu^*(\underline{x}, \overline{x})$, $x^* = x^*(\underline{x}, \overline{x})$, and $\mu_* = \mu_*(\underline{x}, \overline{x})$ define the cut-off coverage and types associated with the economies with contract space $[\underline{x}, \overline{x}]$. We refer to "full purchasing" if all individuals are purchasing some strictly positive coverage (i.e., $\sigma(\mu) > 0, \forall \mu$)

**Lemma 5.** *Given $f(\cdot)$ log-concave, in the region of partial pooling,*

$$\frac{\partial \mu^*}{\partial \overline{x}} < 0, \qquad \frac{\partial \mu_*}{\partial \overline{x}} < 0 \qquad \textit{if } \mu_* > \underline{\mu},$$

$$\operatorname{sign}\left(\frac{\partial x^*}{\partial \overline{x}}\right) = \operatorname{sign}\left(\frac{\partial E}{\partial \mu_*} + \frac{\partial E}{\partial \mu^*} - 1\right) \qquad \textit{if } \mu_* > \underline{\mu}$$

*while $\frac{\partial x^*}{\partial \overline{x}} < 0$ in the domain of full purchasing and partial pooling.*

*Proof.* See Appendix E.5. Note that, if all individuals are buying (i.e., nobody chooses $x = 0$), the result follows from Lemma (1), since in this region the option $x = 0$ plays no role. □

An increase in the maximal coverage $\overline{x}$ lowers the cut-off type $\mu^*$ thereby increasing the mass of individuals in the fully separated region $\mu \in [\mu^*, \overline{\mu}]$. This is similar to the result in Lemma 1. Moreover, an increase in $\overline{x}$ also lowers the cut-off participation types $\mu_*$ which also reduces the number of individuals not purchasing any insurance.

The effect of $\overline{x}$ on $x^*$ is ambiguous, although in all simulations it seems that $\frac{\partial x^*}{\partial \overline{x}} \leq 0$ always. See Figure 9 and Figure 10 as examples. A change in $\overline{x}$ not only changes the cutoff type $\mu^*$, but also has a direct effect on the shape of the choice function in the fully separated region. For instance, in the knife-edge case where the distribution of types is uniform, the two effects are of equal importance, so $\frac{\partial E}{\partial \mu_*} = \frac{\partial E}{\partial \mu^*} = \frac{1}{2}$ and $\frac{\partial x^*}{\partial \overline{x}} = 0$.

As in Lemma 2, recall that $\sigma^u(\mu, \underline{x}, \overline{x})$ denotes the choice under laissez faire.

[37] As in the proof of Proposition 3, these steps are established largely by comparing the slope of $p(x)$ in equilibrium ((7) and (8)) with $p(\underline{x})$ determined by the break-even condition (15).

**Lemma 6.** *For $\mu \in (\mu^*, \overline{\mu})$, then in each of the regions of full / partial / no pooling, $\sigma^u(\mu,\cdot,\cdot)$ is differentiable and*

$$\frac{\partial \sigma^u}{\partial \overline{x}}(\mu, \underline{x}, \overline{x}) = \frac{g'(\overline{x})}{\overline{x}} \frac{\sigma^u(\mu, \underline{x}, \overline{x})}{g'(\sigma^u(\mu, \underline{x}, \overline{x}))} \geq 0.$$

In this setting, welfare is

$$W = W(\underline{x}, \overline{x}) = [F(\mu^*) - F(\mu_*)]\, g(\underline{x}) + \int_{\mu^*}^{\overline{\mu}} g(\sigma^u(\mu, \underline{x}, \overline{x}))d\mu.$$

The dependence on $(\underline{x}, \overline{x})$ is via both $\mu^* = \mu^*(\underline{x}, \overline{x})$ and $\mu_* = \mu_*(\underline{x}, \overline{x})$. Proposition 10 below focuses on the case where some individuals purchase contracts $x > \underline{x}$ (i.e., there is partial pooling).

**Proposition 10.** *If $f(\cdot)$ is log-concave, then in the region of partial pooling,*

$$\frac{\partial W}{\partial \overline{x}} > 0.$$

*Proof.* See Appendix E.2, in particular Section E.7. □

Given log-concave $f(\cdot)$, raising $\overline{x}$ decreases the amount of pooling at $\underline{x}$. First, some agents who had been purchasing coverage $\underline{x}$ now purchase higher levels of insurance. Second, this reduces the price of $\underline{x}$ , leading to more agents choosing it over no coverage ($x = 0$). The calculation performed in Appendix E.2 essentially guarantees the fall of price at $\underline{x}$ would not cause too many of those agents who had elected for higher levels of coverage to 'fall back down' to $\underline{x}$.

### 6.3.2 Numerical Simulations

We now present numerical simulations that illustrate the results above. The values of $\underline{\mu}, \overline{\mu}, \underline{x}, \overline{x}$ considered are the same as those in the calibrations of Section 5.[38]

Figure 11 describes the effects of changes in $\overline{x}$. The first panel shows a histogram of the (uniform) distribution of $\mu$. The second panel shows $\mu^*$ and $\mu_*$ as a function of $\overline{x}$ (where $\overline{\mu}, \underline{\mu}$ are highlighted with horizontal dashed lines). There is full pooling ($\mu^* = \overline{\mu}$) for low values of $\overline{x}$ (when the range of available contracts $\overline{x} - \underline{x}$ is small). When there is full pooling, there are effectively only two contracts $x \in \{0, \underline{x}\}$ and equilibrium behaves as in Section 7 below.

There is partial pooling ($\mu^* < \overline{\mu}$) for higher values of $\overline{x}$ (when the range of available contracts is sufficiently large). Under partial pooling, and given that $f(\mu)$ is uniform and thus log-concave, increases in $\overline{x}$ decrease both $\mu^*$ and $\mu_*$ (Lemma 5). That is, individuals increase the coverage they purchase (from 0 to $\underline{x}$ and from $\underline{x}$ to some $x > \underline{x}$).

The third panel shows $x^*$ as a function of $\overline{x}$. In the region of partial pooling, since $f(\mu)$ is uniform, $x^*$ is constant in $\overline{x}$ (Lemma 5, specialized to the case of a uniform distribution).[39]

The fourth panel shows welfare as a function of $\overline{x}$. Welfare is increasing in $\overline{x}$ when there is partial pooling (Proposition 10), and flat in the region of full pooling.

[38] As above, in the event of full pooling ($\mu^* = \overline{\mu}$), we define $x^* = \overline{x}$, to preserve continuity in the graphs.

[39] Recall that, in the region of full pooling, we set $x^* = \overline{x}$ as a convention.

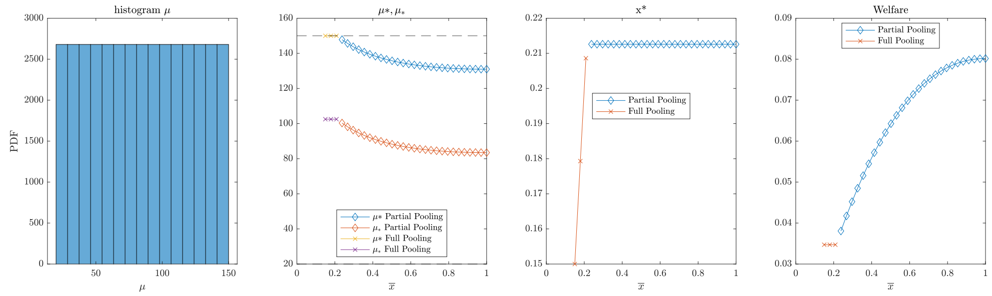


Figure 9: Effects of $\overline{x}$ in a market with voluntary purchase.

Figure 10 shows that $\overline{x}$ can decrease $x^*$. The first panel shows the distribution of types, which is not uniform but still log-concave (so equilibrium is unique). The second panel shows that $(\mu^*, \mu_*)$ are decreasing in $\overline{x}$ in the region of partial pooling (Lemma 5). The third panel shows that $x^*$ is, in this case, decreasing in $\overline{x}$ in the region of partial pooling. In all simulations performed in the preparation for this article, we obtain $\frac{\partial x^*}{\partial \overline{x}} \leq 0$, when the type distribution is log-concave. However, we have not been able to prove this result formally. Welfare is increasing in $\overline{x}$ when there is partial pooling (Proposition 10).

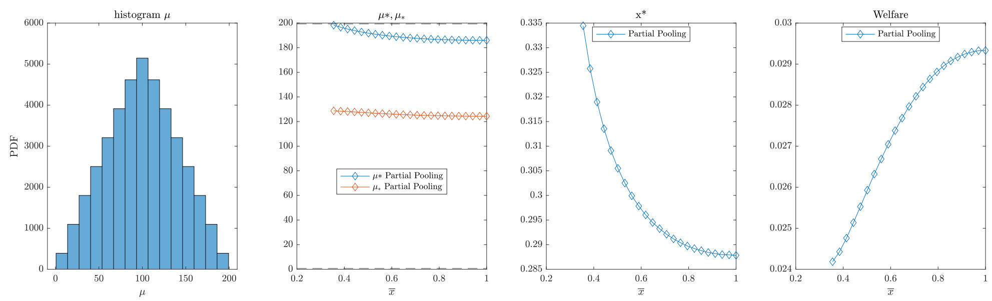


Figure 10: The effect of $\overline{x}$ in a market with voluntary purchase.

Below we provide a summary of these results. Recall that "full purchase" means that all individuals choose $x > 0$.

| Voluntary purchase, effect of $\overline{x}$ | |
|---|---|
| $\frac{\partial \mu^*}{\partial \overline{x}} < 0$ if partial pooling & not full purchasing | Lemma 5 |
| $\frac{\partial \mu_*}{\partial \overline{x}} < 0$ if partial pooling & not full purchasing | Lemma 5 |
| $\frac{\partial x^*}{\partial \overline{x}}$ generally unknown when partial pooling & not full purchasing (but $\leq 0$ in all simulations)<br>$\frac{\partial x^*}{\partial \overline{x}} = 0$ if $\mu$ uniform | Figure 10<br>Lemma 5 |
| $\frac{\partial W}{\partial \overline{x}} \geq 0$ | Proposition 10 |

## 6.4 Adjusting minimal coverage

We now consider the effect of changes in $\underline{x}$ on the thresholds $\mu^*, \mu_*, x^*$, then its effect on welfare, and finally we illustrate the results with numerical simulations. Again, we focus on the case of partial pooling.

### 6.4.1 Theoretical Results

In parallel to Lemma 2, $\sigma^u = \sigma^u(\mu, \underline{x}, \overline{x})$ denotes the allocation under laissez faire. Then, denoting $\mu^* = \mu^*(\underline{x}, \overline{x})$, we have the following result.

**Lemma 7.** *For $\mu \in (\mu^*, \overline{\mu})$, then in each of the regions of full / partial / no pooling, $\sigma^u(\mu, \cdot, \cdot)$ is differentiable and $\frac{\partial \sigma^u}{\sigma \underline{x}}(\mu, \underline{x}, \overline{x}) = 0$.*

We now consider a change in $\underline{x}$ on the thresholds $\mu^*, \mu_*$ and $x^*$. In general, these effects are hard to sign analytically, so below we focus on the case where $f(\mu)$ is uniform.

**Lemma 8.** *In general, $sign(\frac{\partial \mu^*}{\partial \underline{x}}) = sign(\frac{\partial \mu^*}{\partial \underline{x}})$. If $f(\mu)$ is uniform, then in the region of partial pooling,*

$$\frac{\partial \mu^*}{\partial \underline{x}} > 0, \qquad \frac{\partial x^*}{\partial \underline{x}} > 0.$$

*Proof.* See Appendix E.6. □

Since a change in $\underline{x}$ does not affect the shape of $\sigma^u(\cdot, \underline{x}, \overline{x})$, an increase in the cut-off type $\mu^*$ mechanically results in an increase in the cut-off contract $x^*$. When $f(\mu)$ is uniform, an increase in the minimal coverage $\underline{x}$ increases the cut-off type $\mu^*$. Since the minimal coverage $\underline{x}$ becomes more generous, some types come to prefer $\underline{x}$ to the coverage they choose under laissez faire, $\sigma^u(\cdot, \underline{x}, \overline{x})$. This result is similar to that of Lemma 3. The same sign of the change has been observed in a large number of simulations with log-concave distributions, suggesting it may be true more generally. However, we have been unable to show this analytically.

Appendix E.6 shows that an increase in $\underline{x}$ has an ambiguous effect on the cut-off participation type $\mu_*$. Increasing the generosity $\underline{x}$ has the direct effect of making that contract more attractive, both to individuals with low risk (who might previously have not purchased) and high-risk individuals who might otherwise have purchased $\sigma^u(\mu, \underline{x}, \overline{x})$. Depending on the mass of each of these groups, the price $p(\underline{x})$ can increase or fall. If the price increases significantly, this can result in the overall reduction in $\mu_*$ (i.e., in forcing some individuals out of the market) despite the higher generosity of contract $\underline{x}$.[40]

In the region of partial pooling, the effect of $\underline{x}$ on welfare is ambiguous. The reason for this ambiguity is similar to that case of mandatory purchase case: if $\underline{x}$ rises, those purchasing $\underline{x}$ are now better off, but others who had been purchasing higher levels of coverage may now switch their choice to purchase $\underline{x}$. This can raise the price, potentially resulting in individuals no longer purchasing $\underline{x}$. Moreover, those who had not been purchasing at all and now may join.

[40] For instance, even when $f(\mu)$ uniform, $\frac{\partial \mu_*}{\partial \underline{x}}$ is not obviously signed since $\text{sign}\left(\frac{\partial \mu_*}{\partial \underline{x}}\right) = \text{sign}\left(E - \mu_* - g'(\underline{x})\nu\left[1 - \frac{\underline{x}}{x^*}\right]\right)$.

### 6.4.2 Numerical Simulations

Figure 11 shows the effects of changes in $\underline{x}$. The first panel shows the (uniform) type distribution.

The second panel shows $\mu^*, \mu_*$ as functions of $\underline{x}$. There is partial pooling ($\mu^* < \overline{\mu}$) for low values of $\underline{x}$ and full pooling ($\mu^* = \overline{\mu}$) for larger values of $\underline{x}$. Under full pooling, all individuals purchase $x \in \{0, \underline{x}\}$ and equilibrium is as discussed in Section 7 below. Under partial pooling, an increase in $\underline{x}$ increases $\mu^*$, which illustrates Lemma 8. We also find, in these simulations, that a higher value of $\underline{x}$ results in an increase in $\mu_*$ (although we have not been able to prove this result formally). Therefore, in this market, an increase in $\underline{x}$ results in all individuals obtaining a weakly lower coverage.

The third panel of Figure 11 shows $x^*$ as a function of $\underline{x}$. Since $\underline{x}$ raises $\mu^*$, then it also raises $x^*$, as described by Lemma 8.

The fourth panel of Figure 11 shows welfare as a function of $\underline{x}$. Under partial pooling, in these simulations, we find that an increase in $\underline{x}$ results in a rise in welfare. Once $\underline{x}$ is large enough that full pooling occurs, then welfare increases with $\underline{x}$, then decreases (in the region of full pooling, equilibrium behaves as discussed in Section 7 below).

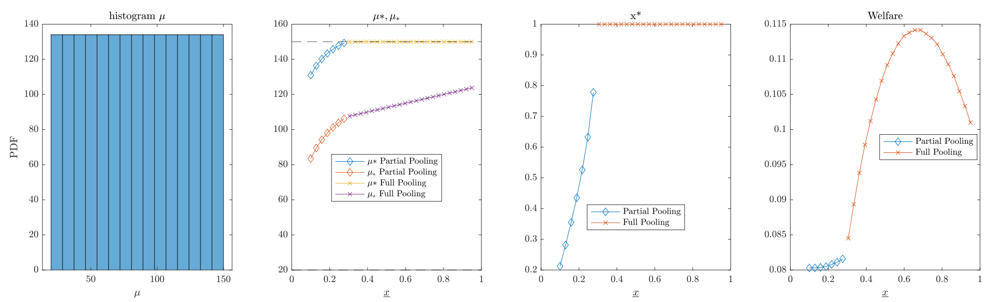


Figure 11: Effects of $\underline{x}$ in a market with voluntary purchase.

Figure 12 illustrates the effect of $\underline{x}$ for another type distribution). The distribution is log-concave, so equilibrium is unique. For clarity, only the region of partial bunching is shown. The fourth panel shows the effect of $\underline{x}$ on welfare, showing that it is non-monotonic in the region of partial bunching. As discussed above, the signs of $\frac{\partial \mu^*}{\partial \underline{x}}, \frac{\partial x^*}{\partial \underline{x}}, \frac{\partial \mu_*}{\partial \underline{x}}$ are unknown in general, although simulations tend to suggest they are always $\geq 0$.

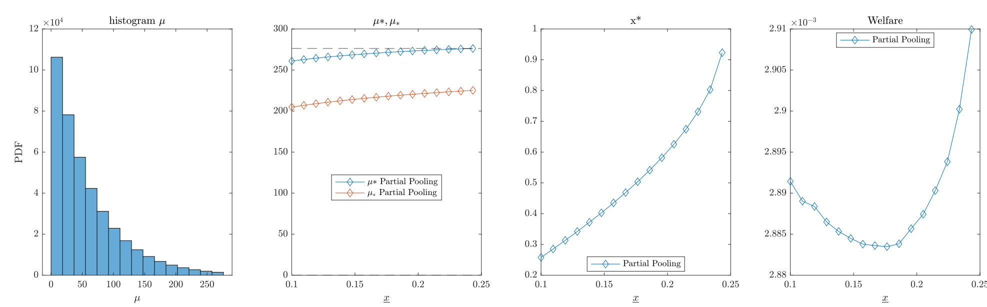


Figure 12: The effect of $\underline{x}$ in a market with voluntary purchase. Types follow a truncated exponential distribution.

Below we provide a summary of these results.

| Voluntary purchase, effect of $\underline{x}$ | |
|---|---|
| $\text{sign}\left(\frac{\partial\mu^*}{\partial\underline{x}}\right) = \text{sign}\left(\frac{\partial x^*}{\partial\underline{x}}\right)$ $\frac{\partial\mu^*}{\partial\underline{x}}, \frac{\partial x^*}{\partial\underline{x}} > 0$ if $\mu$ uniform (and $\geq 0$ in all simulations) | Lemma 8 |
| $\frac{\partial\mu_*}{\partial\underline{x}}$ unknown (but $\geq 0$ in all simulations) | Figure 12 |
| $\frac{\partial W}{\partial\underline{x}}$ ambiguous | Figure 12 |

# 7 Markets for Lemons

In this Section, we assume that there is a single non-zero contract. Individuals make a binary choice, as in Akerlof [1970], Einav et al. [2010a].[41] Formally, we assume

$$X = \{0, \underline{x}\}.$$

This setting corresponds to a special case of the model with voluntary purchase (Section 6, $X = \{0\} \cup [\underline{x}, \overline{x}]$), where $\underline{x} = \overline{x}$. This setting is not directly comparable to the model of voluntary purchase (Section 5).

## 7.1 Equilibrium characterization

Recall the notation (12),

$$\psi(\mu_*) = \mathbb{E}[\mu \mid \mu > \mu_*] - \mu_*.$$

For instance, if $\mu$ is distributed uniformly, then $\psi(\mu_*) = \frac{1}{2}(\overline{\mu} - \mu_*)$. The following results follows from Theorem 1 and simplified versions of the analysis from earlier sections.

**Proposition 11.** *Suppose $X = \{0, \underline{x}\}$. Then there exists an equilibrium $(p, \alpha)$ such that*

1. There is a cut-off participation type $\mu_* \in (\underline{\mu}, \overline{\mu}]$ s.t. that
   (a) types $\mu > \mu_*$ purchase $\underline{x}$
   (b) types $\mu < \mu_*$ purchase $0$.
2. Type $\mu_*$ is indifferent between contract $(\underline{x}, p(\underline{x}))$ and $(0, 0)$. Formally:
$$\psi(\mu_*) - \frac{g(\underline{x})}{\underline{x}}\nu = 0. \tag{21}$$
3. Each contract breaks even:
$$p(\underline{x}) = \underline{x} \cdot \mathbb{E}[\mu \mid \mu > \mu_*]$$
$$p(0) = 0$$

[41] In Handel et al. [2015] individuals also make a binary choice, but there are two contracts with strictly positive coverage.

In general, there can be multiple equilibria, as discussed by Scheuer and Smetters [2014]. However, here too, a log-concave type distribution ensures that (21) holds for a single type $\mu_*$ so equilibrium is unique.

**Proposition 12.** *When $f$ or $1-F$ are log-concave:*

1. *$\psi(\mu_*)$ is monotone decreasing ($\psi'(\mu_*)<0$) .*
2. *Equilibrium is unique.*

*Proof.* See Appendix E.1. □

The proof is similar to the final steps of the proof of Proposition 9, which shows the uniqueness of equilibrium in the market with voluntary purchase. We show that two equilibria with partial pooling must coincide.[42]

In the rest of this section, we assume $f(\cdot)$ is log-concave.

### 7.2 Optimal mandated coverage

We now discuss a regulator's welfare maximizing choice of coverage for the single insurance contract, $\underline{x}$. Notice that, as in the analysis above, we assume that the regulator can regulate the allowed contracts (i.e., choose the coverage $\underline{x}$) but prices are set by competitive forces in equilibrium.

Before proceeding, we present the following preliminary results.

**Lemma 9.** *$g(x)/x$ is decreasing in $x$.*

*Proof.* Recall $g(0)=0$. The derivative of $\frac{g(x)}{x}$ has the sign of $g'(x)\cdot x-g(x)\leq 0$. This is negative by the concavity of $g(x)$, since $\frac{g(x)}{x}=\frac{g(x)-g(0)}{x-0}\geq g'(x)$.[43] □

The following results describes how regulation of $\underline{x}$ affects the marginal type $\mu_*$.

**Lemma 10.** *If $f(\mu)$ is log-concave and $\mu\in(\underline{\mu},\overline{\mu})$ (i.e., some consumers purchase but others do not), then*

$$\frac{\partial\mu_*}{\partial\underline{x}}=\frac{\nu}{\psi'(\mu_*)}\left[\frac{g(x)}{x}\right]'\geq 0.$$

*Proof.* See Appendix F.1. □

Lemma 10 shows that, when the type distribution is log-concave, an increase in coverage $\underline{x}$ (and a corresponding adjustment in price) causes the set of buyers to shrink. The reason is adverse selection. When mandated coverage increases, the cost associated with covering inframarginal types (and thus the price) increases faster than the willingness to pay of marginal types, leading to a reduction in the overall number of buyers.[44]

[42] Note that here we can use the weaker condition of $1-F$ being log-concave instead of $f$ being log-concave (the latter implies the former by Theorem 3 of Bagnoli and Bergstrom [2005]) since for Proposition 9 we need the relevant log-concavity to pass to distributions that result from conditioning on a sub-interval.

[43] For instance, in a CARA-Gaussian setting, $\frac{g(x)}{x}=1-\frac{x}{2}$.

[44] Notice that this result depends directly on the log-concavity of the type distribution. If $\psi'(\mu_*)>0$, then even if somehow $\mu_*$ is well-defined (despite the a priori non-uniqueness of equilibrium), the result of Lemma 10 would be reversed.

We now characterize the socially optimal level of coverage $\underline{x}$. It will be instructive to proceed by analogy with the classical problem of a monopoly's choice of a profit-maximizing price. Let $q(\underline{x}) = 1 - F(\mu_*(\underline{x}))$ be the demand for insurance when mandated coverage is $\underline{x}$ and price is set competitively. Notice that this demand is somewhat atypical: changes in $\underline{x}$ have both a direct effect on the number of buyers, but also an indirect effect mediated by the endogenous adjustment of price. Since

$$\frac{\partial q}{\partial \underline{x}} = -f(\mu_*) \frac{\partial \mu_*}{\partial \underline{x}} \leq 0,$$

demand for insurance slopes down with mandated coverage $\underline{x}$. This is due to adverse selection, as discussed above. A given change in $\underline{x}$ results in a larger change in $q$ when a) the density of marginal types $f(\mu_*)$ is large; b) individuals are more risk averse ($\nu$ large); and c) the marginal surplus from insurance decreases quickly with coverage ($g(x)/x$ falls steeply with $x$).

Social welfare is

$$W = q(\underline{x}) \cdot g(\underline{x}).$$

We show below that the socially optimal choice of $\underline{x}$ is interior. Therefore, it satisfies the First Order Condition (FOC)

$$\frac{\partial W}{\partial \underline{x}} = 0 \Leftrightarrow g(\underline{x}) + q(x) \cdot g'(\underline{x}) \frac{1}{\frac{\partial q}{\partial \underline{x}}} = 0 \tag{22}$$

Interestingly, the intuition for the social planner's optimal choice of quality $\underline{x}$ is similar to that of a monopoly's profit maximizing choice of price. The regulator considers selling insurance to one additional marginal individual. There are marginal gains but infra-marginal losses. Doing so increases welfare by the surplus generated by the additional marginal individual that is induced to buy insurance, $g(\underline{x})$. However, because of adverse selection, doing so requires lowering coverage $\underline{x}$ for all $q$ infra-marginal individuals, whose surplus therefore falls proportionately to $g'(\underline{x})$ and inversely proportionately to the responsiveness of demand, $\frac{\partial q}{\partial \underline{x}}$.[45]

The first best outcome is for all individuals to obtain full insurance ($x = 1$) because all individuals obtain a positive surplus ($g(x) \geq 0$). However, the social planner cannot directly mandate the first best. Instead, she can only regulate the level of generosity $\underline{x}$. Clearly, mandating $\underline{x} = 0$ is suboptimal. However, mandating $x = 1$ is also suboptimal in this setting. (22) shows that, locally to full insurance, it is welfare enhancing to increase the number of buyers $q$, which requires lowering $\underline{x}$.

In Appendix F we show that log-concavity of demand $q$ implies that welfare is quasi-concave, and show that this is the case, for instance, when $\mu$ is distributed uniformly.

For what follows, it is worthwhile noticing that it is possible that the first best (everyone purchases $\underline{x} = 1$) is an equilibrium.[46] This occurs if type $\underline{\mu}$ strictly prefers $(1, E[\mu])$ to $(0, 0)$: that is, she strictly prefers full coverage at the average market riskiness to no coverage. In this case, we say that "the first best is an equilibrium." This is, of course, unlikely to happen in most markets.

**Proposition 13.** *Suppose $f(\mu)$ is log-concave. Then, the first best is not an equilibrium iff*

$$\frac{dW}{d\underline{x}} \mid_{\underline{x}=1} < 0.$$

[45] Saltzman [2021] discusses a similar trade-off between "underinsurance vs. under-enrollment." His model considers only 2 contracts and focuses on the empirical analysis of other government interventions (namely risk adjustment and mandates).

[46] This was already remarked by Einav and Finkelstein [2011]. In their framework, this occurs if the average cost curve is everywhere below the demand curve.

*In this case, the socially optimal level of mandated coverage is interior ($\underline{x} \in (0,1)$). Otherwise, $\underline{x} = 1$ is optimal.*

It is worth noting that the assumption that the lowest type (weakly) prefers full coverage over weak coverage at the average market riskiness is equivalent to the assumption that all agents have this preference, and Lemma 10 implies that under $f(\cdot)$ log-concave, this is the case iff all agents prefer coverage $\underline{x}$ at price $\underline{x} \cdot E[\mu]$ to no coverage; hence, for any $\underline{x} \in (0,1]$, in the unique equilibrium, all agents purchase insurance.

*Proof.* If the condition does not hold - if $\underline{\mu}$ weakly prefers $(1, E[\mu])$ to $(0,0)$, i.e., $\mu_*(1) = 0$ - then all types purchase $\underline{x}$ for any $\underline{x} \in (0,1]$, since by Lemma 10, $\mu_*(\underline{x}) = 0$ for all $\underline{x} \in (0,1]$. Hence, in that case, welfare increases with $\underline{x}$. Moreover, welfare is minimized when $\underline{x} = 0$. To show the desired conclusion in the case that the condition holds, a direct calculation, based on Lemma 10, is carried out in Appendix F.2. □

### 7.3 Numerical Simulations

We now present numerical simulations that illustrate the theoretical results of Section 7.

Figure 13 illustrates the effects of changing $\underline{x}$ in this setting. The first panel shows a histogram of the (uniform) distribution of types.

The second panel shows that $\mu_*$ is monotonic increasing in the mandated coverage $\underline{x}$, as described by Lemma 10.[47] In particular, for $f(\cdot)$ uniform and $g(x) = \frac{1}{2}\left(1 - (1-x)^2\right)$, if some agents do not purchase coverage, then $\frac{\partial \mu_*}{\partial \underline{x}} = \nu$, which explains why $\mu_*$ changes linearly with $\underline{x}$ in Figure 13. The second panel shows that, in this particular simulation, welfare is quasi-concave (Lemma 26) and has a maximum for an interior level of $\underline{x}$ (Proposition 13).

Notice that the behavior of welfare in this case is the same as that in Figure 11 (Section 6, where $X = \{0\} \cup [\underline{x}, \overline{x}]$), when $\underline{x}$ is sufficiently large and therefore the market exhibits full pooling at $\underline{x}$. That is because, in that case, no individual is purchasing $x > \underline{x}$, so effectively only contracts $x \in \{0, \underline{x}\}$ are relevant.

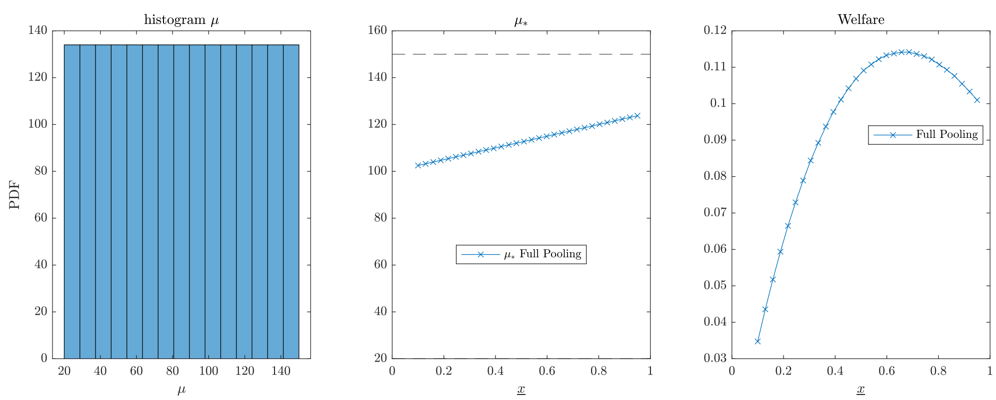


Figure 13: Effects of $\underline{x}$ in a market for lemons with internally maximized welfare.

Figure 14 shows a case of welfare being maximized at full insurance. In this case, the level of $\nu$ is higher, so all agents prefer full insurance at price $p = E[\mu]$ to no insurance.

[47] Since all types purchase $\{0, \underline{x}\}$ we are always in the region of "full bunching" (in the language of Section 6).

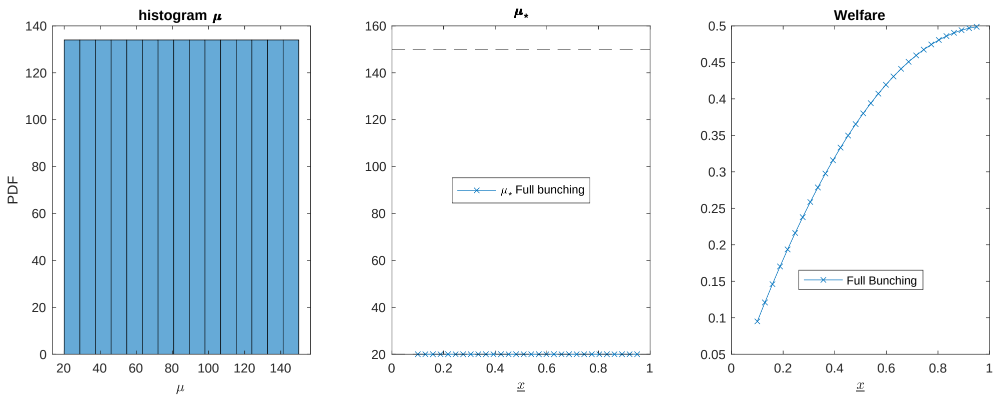


Figure 14: Effects of $\underline{x}$ in a market for lemons with welfare maximized at full insurance.

Below we provide a summary of these results.

| Market for Lemons, effect of $\underline{x}$ | |
|---|---|
| $\frac{\partial \mu_*}{\partial \underline{x}} \geq 0$ | Lemma 10 |
| $\frac{\partial W}{\partial \underline{x}} > 0$ for $\overline{x} \approx 0$<br>$\frac{\partial W}{\partial \underline{x}} < 0$ for for $\underline{x} \approx 1$ if the first best is not an equilibrium | Proposition 13<br>Figure 13 |

## 8 Conclusion

We have developed a tractable model of competitive insurance markets with a continuum of types and exogenous restrictions on the set of allowed contracts. Our model nests, as special cases, the market for lemons of Akerlof [1970] and the unrestricted contracts setting of Rothschild and Stiglitz [1976]. We have considered settings where purchase is mandatory or voluntary, and a regulator imposes minimal and maximal coverage requirements. Equilibrium generically exhibits partial pooling at the lowest available non-zero contract. Therefore, the structure of equilibrium depends non-trivially on the type distribution, unlike in Rothschild and Stiglitz [1976].

We have shown that an increase in the maximal allowed coverage always increases welfare. Increases in the minimal allowed coverage have ambiguous (and possibly non-monotonic) effects on welfare. In markets for lemons, if the first best is not an equilibrium, then there is an interior optimal level of coverage that can be imposed by the regulator to maximize welfare.

How do these results inform our understanding of real world markets with adverse selection? Typically, such markets feature a discrete set of contracts. However, modeling a setting with many discrete contracts can quickly become intractable. We view our model as a tractable way of approximating such real world settings, in a way that allows for relatively simple comparative statics and policy-relevant policy guidelines. Our model allows us to quantity the extent to which changes the set of allowed contracts affect welfare, and how this is related to the distribution of costs in a given market.

Our model also makes several qualitative predictions. For instance, we expect significant numbers of individuals to purchase the lowest available coverage in each market. We also expect very few individuals to purchase levels of coverage which are only marginally above the lowest available coverage.

Our results also have important implications for firm managers in markets with adverse selection. In settings where offering additional contracts is costly, managers can safely refrain from offering contracts that are only marginally more generous than the minimal coverage requirement.

While we make a number of important simplifications, we view these results as a step towards a deeper understanding of such markets. There remain several interesting avenues for future work. We have assumed unidimensional types, although this assumption would be interesting to relax. For instance, we assumed that the insurance surplus $\nu$ was homogeneous across individuals, but it would be interesting to allow for heterogeneity in this parameter. In settings with either mandatory and voluntary purchase, it would also be useful to obtain necessary and sufficient conditions on primitives to sign the effect of $\underline{x}$ on welfare.

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# A Propositions 1 and 7

We now recall some results from Levy and Veiga [2021] (henceforth LV21) used towards proving Theorem 2. We highlight important differences en route to the proofs of Propositions 1 and 7.

## A.1 Following Levy and Veiga [2021]

Parts 1, 2, 3, 5, and 8 of Proposition 1 were established in Levy and Veiga [2021], as generalizations of Rothschild and Stiglitz [1976] to a continuum of types.

Like in Appendix B of LV21, we fix a distribution $\alpha$ on $\Theta \times X$, with $X = [\underline{x}, \overline{x}]$ (e.g., $(p, \alpha)$ may be an equilibrium) and marginal $P := f d\mu$ on $\Theta = [\underline{\mu}, \overline{\mu}]$. LV21 assumed $X = [0, 1]$. Let the marginal of $\alpha$ on $X$ be $\alpha_X$. We define analogues of the maximum and minimum risk $\mu$ purchasing each alternative $x$. To avoid the influence of zero-measure sets of types, we use a variation of the essential supremum and infimum, defined for $x \in supp(\alpha_X)$ as[48]

$$\psi^+(x) = \lim_{\delta \to 0^+} \left[ \sup \left\{ \mu \mid \alpha\big(\{\theta \mid \mu_\theta \geq \mu\} \times (x - \delta, x + \delta)\big) > 0 \right\} \right] \tag{23}$$

$$\psi^-(x) = \lim_{\delta \to 0^+} \left[ \inf \left\{ \mu \mid \alpha\big(\{\theta \mid \mu_\theta \leq \mu\} \times (x - \delta, x + \delta)\big) > 0 \right\} \right]. \tag{24}$$

Intuitively, $\psi^+(x)$ captures the largest value of $\mu$ which purchases $x$ under $\alpha$, and $\psi^-(x)$ as the lowest such value of $\mu$.

Appendix C of LV21 establishes several claims when $(p, \alpha)$ is an equilibrium. Although they are made for the case $X = [0, 1]$, almost all hold when $X = [\underline{x}, \overline{x}]$ with the same proofs, with one exception which we shall point out.

We omit those results which are irrelevant or trivial in the framework of this paper, which is slightly more restrictive than that in LV21 (in particular, in LV21 types may differ in both risk and riskiness, although they are still unidimensional in terms of being linearly ordered w.r.t. willing to pay).

**Lemma 11.** *Let $(\mu, x)$ refer to a type and the contract purchased by that type. It holds $\alpha$-a.s. that for each pair $(\mu_2, x_2), (\mu_1, x_1)$, $x_2 > x_1$ implies $\mu_2 \geq \mu_1$. This is also true if $X' \subseteq X$ is finite and $\alpha'$ is a weak equilibrium of the economy $[\Theta \cup X', P \cup \eta, X']$, where $X'$ refers to behavioral types as well.*[49]

Lemma 11 (which is Lemma 2 in LV21) follows, roughly speaking, as lower types have lower willingness to pay. Hence it follows (Corollary 5 in LV21):

**Corollary 3.** *For $x < y < z$, we have $\psi^+(x) \leq \psi^-(y) \leq \psi^+(y) \leq \psi^-(z)$. This is also true if $X' \subseteq X$ is finite and $\alpha'$ is a weak equilibrium of the perturbed economy $[\Theta \cup X', P \cup \eta, X']$ (as discussed in Section 2).*

One can then obtain the following sequence of lemmas (which are Lemma 3, 4, 6 in LV21, modified to $[\underline{x}, \overline{x}]$):

**Lemma 12.** $\psi^-(x) = \psi^+(x)$ *for* $x \in supp(\alpha_X) \cap (\underline{x}, \overline{x}]$.

Hence, denote $\psi = \psi^- = \psi^+$.

[48] For $\alpha$-a.e. $x \in X$, $\psi^+(x)$ (resp. $\psi^-(x)$) is the supremum (resp. infimum) of the support of $\alpha(\cdot \mid x)$. The limits exist as the terms they are taken over are monotonic.

[49] The conclusion then holds $\alpha'(\cdot \mid \Theta)$-a.s., i.e., holds for types in $\Theta$, not behavioral types for whom $\mu$ is not defined.

**Lemma 13.** $\alpha(\{\theta, x \mid p(x) \neq \mu(\theta) \cdot x\}) = 0$, *and (equivalently),* $p(z) = z \cdot \psi(z)$ *for all* $z \in supp\,(\alpha_X)$ .

*Proof.* From Lemma 12 and the break-even condition for price. □

**Lemma 14.** $\psi\,(x)$ *is strictly increasing for* $x \in supp(\alpha_X) \cap (\underline{x}, \overline{x}]$.

By taking $\sigma = \psi^{-1}$ on a subset $\Phi \subseteq [\underline{\mu}, \overline{\mu}]$ which satisfies $\alpha(\{\mu \in \Phi\}\Delta\{x > \underline{x}\}) = 1$, we have (Corollary 6 in LV21):

**Corollary 4.** *There is a mapping* $\sigma : [\underline{\mu}, \overline{\mu}] \to [\underline{x}, \overline{x}]$*, strictly increasing and continuous on* $[\underline{\mu}, \overline{\mu}] \backslash \sigma^{-1}(\{\underline{x}\})$*, s.t.* $\alpha\big\{(\mu, x) \mid x = \sigma(\mu)\big\} = 1$.

Lemma 7 in LV21 claims that full insurance is in the support of the equilibrium. Although the proof is insensitive to the choice of maximal coverage $\overline{x}$, it does rely on the minimal coverage being $0$, since it relies on the fact that, in equilibrium, the highest type $\overline{\mu}$ must be purchasing some contract $x^{**}$ at price $x^{**} \cdot \overline{\mu}$. While this fact is true there - there is no pooling at positive coverage levels there, and it holds trivially at $0$ - this need not be the case in our model if $x^{**} = \underline{x}$, as there could be pooling which would lead to $p(\underline{x}) < \overline{\mu} \cdot \underline{x}$. However, if pooling is not total (i.e., if $x^{**} > \underline{x}$), the conclusion and proof both hold, so we obtain:

**Lemma 15.** *If* $\alpha_X((\underline{x}, \overline{x}]) > 0$*, the supremum of the support of* $\alpha_X$ *is maximal insurance, i.e.,* $\sigma(\overline{\mu}) = \overline{x}$.

Proposition 13 in Appendix E.1 in LV21 shows (with $\underline{x}$ replacing $0$, and the proofs following otherwise verbatim) the following:

**Proposition 14.** *Suppose the distribution of risk* $\mu$*, conditional on some interval* $(\mu_{**}, \mu^{**})$*, has full support with a.e. strictly positive density w.r.t. the Lebesgue measure, and* $(p, \alpha)$ *is an equilibrium with associated coverage function* $\sigma$ *with* $\sigma(\mu_{**}) > \underline{x}$*, then*

$$\frac{1}{\nu}\left(\mu^{**} - \mu_{**}\right) = \int_{\sigma(\mu_{**})}^{\sigma(\mu^{**})} \frac{g'(x)}{x} dx \tag{25}$$

Recall $\tau = \sigma^{-1}$. The proof also establishes that

$$p'(x) = \tau(x) + x \cdot \tau'(x), \qquad a.e\, x \in [\sigma(\mu_{**}), \sigma(\mu^{**})].$$

### A.2 Completing Proof of Propositions 1, 7

Parts 1, 2, 3, 5, 8 of Proposition 1 follow from piecing together the results recalled in the previous section. Part 7 follows the break-even conditions of equilibrium. Part 9 follows from Part 2 of Proposition 1 of Azevedo and Gottlieb [2017].

Part 10 follows from parts 5, 8, 9, as follows: observe that (7) implies that $p' > \frac{dp^u}{dx}$ a.e., in $[\underline{x}, x^*]$ since a.e.

$$p'(x) = \mu^* + \nu \cdot g'(x) > \tau^u(x, \overline{x}) + \nu \cdot g'(x, \overline{x}) = \frac{dp^u}{dx}(x, \overline{x})$$

and in $[x^*, \overline{x}]$, $p \equiv p^u$ by comparing (5) and (2).

Finally, to show Part 4 of Proposition 1 (as pointed out earlier, Part 6 is a reformulation of Parts 1 and 4): we show that if $supp(\alpha_X) \cap [\underline{x}, \overline{x}] = [\underline{x}, \overline{x}]$ (i.e., if $x^* = \underline{x}$), then $\underline{x}$ cannot be an atom of $\alpha_X$. Indeed, if $\underline{x}$ is an atom of $\alpha_X$ but $x^* = \underline{x}$, then, since $\underline{x} = \lim_{\mu \to (\mu^*)^+} \sigma(\mu)$, we have

$$p(\underline{x}) = \underline{x} \cdot E_\alpha[\mu] < \underline{x} \cdot \mu^* = \lim_{\mu \to (\mu^*)^+} \mu \cdot \sigma(\mu) = \lim_{x \to \underline{x}^+} p(x)$$

contradicting the continuity of prices.

Proposition 7 follows similarly.

# B Mandatory or voluntary purchase

In this section we present results valid for the case where purchase is mandatory ($X = [\underline{x}, \overline{x}]$) or voluntary ( $X = [\underline{x}, \overline{x}] \cup \{0\}$). These results will then be used towards various proofs below.

**Lemma 16.** *Let $(p_1, \alpha_1), (p_2, \alpha_2)$ be equilibria with zero or partial (but not full) pooling at $\underline{x}$, and cut-off coverages $x_1^*, x_2^*$. If $x_1^* > x_2^*$, then $p_1 \leq p_2$, with strict inequality for $x \in [\underline{x}, x_1^*)$.*

*Proof.* Let $\mu_1^*, \mu_2^*$ be the associated cut-off coverages, and associated coverage functions $\sigma_1, \sigma_2$. By (5), $\mu_1^* > \mu_2^*$. In $[x_1^*, \overline{x}]$, $p_1 = p_2 = p^u(\cdot, \overline{x})$; in particular $p_1(x_1^*) = p_2(x_1^*)$. In $(x_2^*, x_1^*)$, $p_2 \equiv p^u(\cdot, \overline{x})$ and $p_1 < p^u(\cdot, \overline{x})$. For $x \in (\underline{x}, x_2^*)$, $p_2'(x) = \mu_2^* + g'(x) \cdot v < \mu_1^* + g'(x) \cdot \nu = p_1'(x)$. Therefore, $p_1(x_1^*) = p_2(x_1^*)$ and $p_1(x) < p_2(x)$ a.e. in $(\underline{x}, x_1^*)$. $\square$

**Lemma 17.** *If there is one equilibrium with at least partial pooling at $\underline{x}$, then then all equilibria have at least partial pooling at $\underline{x}$*

*Proof.* Suppose $(p_1, \alpha_1)$ has an atom at $\underline{x}$ while $(p_2, \alpha_2)$ does not; $\mu_1^* > \underline{\mu} = \mu_2^*$.

First let's deal with the case $(p_1, \alpha_1)$ has only partial, not full, pooling. Hence, $x_1^* = \sigma_1(\mu_1^*) = \sigma_2(\mu_1^*) > x_2^*$, so by Lemma (16), $p_1 \leq p_2$ with strict inequality in $[\underline{x}, x_1^*)$. In particular, since $p_2(x_2^*) = \underline{\mu} \cdot x_2^*$, we have $p_1(x_2^*) < \underline{\mu} \cdot x_2^*$. Along with $p_1' \geq \underline{\mu}$ (its slope is a.e. the slope of some indifference curve), we have $p_1(\underline{x}) < \underline{x} \cdot \underline{\mu}$. But since $\underline{x}$ is an atom of $(p_1, \alpha_1)$, $p_1(\underline{x}) = \underline{x} \cdot E[\mu \mid \sigma_1(\mu) = \underline{x}] \geq \underline{x} \cdot \underline{\mu}$, a contradiction.

Now we deal with the case $(p_1, \alpha_1)$ displays full pooling. Then $p_1(\overline{x}) \leq \overline{\mu} \cdot \overline{x} = p_2(\overline{x})$, $p_1'(x) = \overline{\mu} + \nu \cdot g'(x) > \sigma^{-1}(x) + \nu \cdot g'(x) = p_2'(x)$ in $(\underline{x}, \overline{x})$, so $p_1 < p_2$ in $[x_2^*, \overline{x})$, and particular $p_1(x_2^*) < p_2(x_2^*) = \underline{\mu} \cdot x_2^*$; and the argument concludes verbatim as in the above case. $\square$

# C pooling

In this section, we prove the results regarding pooling, i.e., Propositions 2 and 8.

## C.1 Mandatory and Voluntary Purchase

The first result holds for settings where purchase is mandatory or voluntary, and provides a necessary and sufficient condition in terms of model primitives for (full or partial) pooling to occur.

**Lemma 18.** *With either mandatory or voluntary purchasing,*

$$\frac{1}{\nu}(\overline{\mu} - \underline{\mu}) > \int_{\underline{x}}^{\overline{x}} \frac{g'(x)}{x} dx$$

*if and only if pooling (full or partial) occurs.*

*Proof.* First we show that, if the condition holds, then pooling occurs. If, by way of contradiction, there was no pooling, then $\sigma^{-1}(x^*) = \underline{\mu}$. (5) shows that

$$\int_{x^*}^{\overline{x}} \frac{g'(x)}{x} dx = \frac{1}{\nu}(\overline{\mu} - \sigma^{-1}(x^*)) = \frac{1}{\nu}(\overline{\mu} - \underline{\mu}) > \int_{\underline{x}}^{\overline{x}} \frac{g'(x)}{x} dx \geq \int_{x^*}^{\overline{x}} \frac{g'(x)}{x} dx,$$

a contradiction.

We now show that if $\frac{1}{\nu}(\overline{\mu} - \underline{\mu}) \leq \int_{\underline{x}}^{\overline{x}} \frac{g'(x)}{x} dx$, then any equilibrium does not have an atom at $\underline{x}$. By Lemma (17), it is enough to present one equilibrium which does not have an atom at $\underline{x}$. Indeed, the equilibrium defined by

$$\frac{1}{\nu}(\overline{\mu} - \mu) = \int_{\sigma(\mu)}^{\overline{x}} \frac{g'(x)}{x} dx$$

gives a well-defined $\sigma(\mu) \in [\underline{x}, \overline{x}]$ for any $\mu \in [\underline{\mu}, \overline{\mu}]$, by the assumed inequality, which we claim defines an an equilibrium allocation rule. Indeed, the economy satisfies the assumptions of AG's Theorem 1 (hence guaranteeing equilibrium existence), and the equilibrium we have presented is the one characterized by Proposition 1 (for mandatory purchase) or Proposition 7 (for voluntary purchase). □

The following result again holds for settings where purchase is mandatory or voluntary. We provide a condition that is necessary, in either setting, for full pooling to occur. Intuitively, it is necessary that the highest type $\overline{\mu}$ prefers to purchase $\underline{x}$ at the average cost in the population $\underline{x}\mathbb{E}[\mu]$, over purchasing the contract typically assigned to type $\overline{\mu}$ when full pooling is not the case, ie $(\overline{x}, \overline{\mu} \cdot \overline{x})$.

**Lemma 19.** *With either mandatory or voluntary purchasing, if there is full pooling, then $\overline{\mu}$ weakly prefers $(\underline{x}, \underline{x} \cdot E[\mu])$ to to $(\overline{x}, \overline{\mu} \cdot \overline{x})$, i.e.,*

$$\underline{x}\left(\overline{\mu} - \mathbb{E}[\mu]\right) \geq \left(g(\overline{x}) - g(\underline{x})\right)\nu. \tag{26}$$

*Proof.* Suppose there is such an equilibrium $(p, \alpha)$. Since agents prefer $(\underline{x}, p(\underline{x}))$ to $(\overline{x}, p(\overline{x}))$, and $p(\overline{x}) \leq \overline{\mu} \cdot \overline{x}$ by Propositions 1 and 7, for mandatory/voluntary purchase,

$$\overline{\mu} \cdot \underline{x} + \nu \cdot g(\underline{x}) - p(\underline{x}) \geq \overline{\mu} \cdot \overline{x} + \nu \cdot g(\overline{x}) - p(\overline{x}) \geq \nu \cdot g(\overline{x}).$$

Since there is full pooling, $p(\underline{x}) = \underline{x} \cdot E[\mu \mid \mu \geq \mu_*] \geq \underline{x} \cdot E[\mu]$ (with equality when $\mu_* = \underline{\mu}$, which is the case with mandatory purchase), we have

$$\overline{\mu} \cdot \underline{x} + \nu \cdot g(\underline{x}) \geq \nu \cdot g(\overline{x}) + \underline{x} \cdot E[\mu]$$

from which the conclusion follows. □

## C.2 Mandatory Purchase

The following result considers settings with mandatory purchase, and derives implications of full pooling.

**Lemma 20.** *In any equilibrium with mandatory purchase and full pooling:*

- $p(\underline{x}) = \underline{x} \cdot E[\mu]$
- *for any $x \in [\underline{x}, \overline{x}]$, $\overline{\mu}$ is indifferent between $(\underline{x}, \underline{x} \cdot E[\mu])$ and $(x, p(x))$.*

*Proof.* $p(\underline{x}) = \underline{x} \cdot E[\mu]$ since all contracts break even and $\underline{x}$ is the only purchased contract; the latter conclusion follows from Part 2 of Proposition 1 in Azevedo and Gottlieb [2017], along with the observation that in $[\underline{x}, \overline{x}]$, in such an equilibrium, the indifference curves of $\overline{\mu}$ through $\underline{x}$ are the highest of all such indifference curves. □

Now we complete the proof of Part 2 of Proposition C, showing the converse direction which applies only with mandatory purchase.

**Lemma 21.** *If $\overline{\mu}$ weakly prefers $(\underline{x}, \underline{x} \cdot E[\mu])$ to $(\overline{x}, \overline{\mu} \cdot \overline{x})$, i.e. (26) holds, then there is an equilibrium with full pooling.*

*Proof.* By Theorem 1, taken from Azevedo and Gottlieb [2017], an equilibrium exists. If there is no equilibrium with full pooling, then there must be an equilibrium $(p, \alpha)$ with $\alpha((\underline{x}, \overline{x}]) > 0$, hence satisfying the characterization given by Proposition 1, and in particular, $p(\overline{x}) = \overline{x} \cdot \overline{\mu}$ and $\tau(\overline{x}) = \overline{\mu}$, i.e., the highest type is purchasing the highest coverage at the break-even price.

By the first part of Proposition 2, there is partial pooling, i.e. $\mu^* < \overline{\mu}$, so $p(\underline{x}) = \underline{x} \cdot E[\mu \mid \mu \leq \mu^*] < \underline{x} \cdot E[\mu]$. But then types sufficiently near $\overline{\mu}$, who are now purchasing close to $(\overline{x}, \overline{\mu} \cdot \overline{x})$, strictly prefer $(\underline{x}, p(\underline{x}))$, as $\overline{\mu}$ at least weakly prefers $(\underline{x}, \underline{x} \cdot E[\mu])$ to the contract $(\overline{x}, \overline{\mu} \cdot \overline{x})$ and $p(\underline{x}) < \underline{x} \cdot E[\mu]$. □

# D Mandatory purchase

This section contains results which pertain to the model with mandatory purchase, so $X = [\underline{x}, \overline{x}]$.

## D.1 Uniqueness

Towards proving equilibrium uniqueness (Proposition 3), we first establish several preliminary results and then complete the proof.

**Lemma 22.** *Suppose $X = [\underline{x}, \overline{x}]$. Let $(p_1, \alpha_1), (p_2, \alpha_2)$ be equilibria with partial (but not full) pooling at $\underline{x}$. Then, $(p_1, \alpha_1) = (p_2, \alpha_2)$.*

*Proof.* If they are different, WLOG, $x_1^* > x_2^*$, so $\mu_1^* > \mu_2^*$. By Lemma 16, $p_1 \leq p_2$ with strict inequality in $[\underline{x}, x_1^*)$; in particular, $p_1(\underline{x}) < p_2(\underline{x})$. But

$$p_1(\underline{x}) = \underline{x} \cdot E[\mu \mid \mu \leq \mu_1^*] > \underline{x} \cdot E[\mu \mid \mu \leq \mu_2^*] = p_2(\underline{x})$$

a contradiction. □

**Lemma 23.** *Suppose $X = [\underline{x}, \overline{x}]$. There can be at most one equilibrium without full pooling; i.e., if two equilibria each have partial or no pooling (meaning $\alpha_X((\underline{x}, \overline{x}]) > 0$), then they coincide.*

*Proof.* Let $(p_1, \alpha_1), (p_2, \alpha_2)$ be two equilibria without full pooling, with associated coverage functions $\sigma_1, \sigma_2$, and points $x_1^*, x_2^*$ as in the proposition above, with associated $\mu_1^*$, $\mu_2^*$. If neither has an atom, then $x_1^* = x_2^* = \sigma_1(\underline{\mu}) = \sigma_2(\underline{\mu})$, as $\sigma_1, \sigma_2$ are then determined by the same integral equation.

Assume then that one equilibrium has an atom at $\underline{x}$; then so does the other by Lemma (17); if they are different equilibria, we can assume w.l.o.g. $x_1^* > x_2^*$; hence $\mu_1^* > \mu_2^*$, so

$$p_1(\underline{x}) = \underline{x} \cdot E[\mu \mid \mu \leq \mu_1^*] > \underline{x} \cdot E[\mu \mid \mu \leq \mu_2^*] = p_2(\underline{x})$$

However, since $x_1^* > x_2^*$, the Lemma 16 implies $p_1(\underline{x}) < p_2(\underline{x})$, a contradiction.[50] □

[50]Note that this argument fails if $0 \in X$. In that case, we

$$p_1(\underline{x}) = \underline{x} \cdot E[\mu \mid \mu_{*1} \leq \mu \leq \mu_1^*] \quad ? \quad \underline{x} \cdot E[\mu \mid \mu_{*2} \leq \mu \leq \mu_2^*] = p_2(\underline{x})$$

and instead of ? a priori we can have either $>$ or $<$.

**Lemma 24.** *Suppose $X = [\underline{x}, \overline{x}]$. If there is an equilibrium $(p, \alpha)$ with full pooling at $\underline{x}$, then it is the unique equilibrium of the economy.*

*Proof.* Using Part 2 of Proposition C, and Lemma 20, in order to show the lemma, we need to show that if $\overline{\mu}$ weakly prefers $(\underline{x}, \underline{x} \cdot E[\mu])$ to $(x, p(x))$ for all $x \in X$, then full pooling is the only equilibrium. Let $(\alpha, p)$ be an equilibrium $\alpha_X(\{\underline{x}\}) = 1$ with price $p(\cdot)$ as described, and $(\beta, q)$ is an equilibrium with $\overline{x} \in supp(\beta)$. We deal with two cases:

If $\beta_X(\{\underline{x}\}) > 0$ (partial pooling), then then $p(\underline{x}) = \underline{x} \cdot E[\mu] > \underline{x} \cdot E[\mu \mid \mu^* \geq \mu] = q(\underline{x})$ , where $\mu^*$ is the cut-off type for $(\beta, q)$. However, we know, letting $I$ be the indifference curve of $\overline{\mu}$ through $(\underline{x}, \underline{x} \cdot E[\mu])$, $p' = I'$ while $q' \leq I'$, and $q(\overline{x}) = \overline{\mu} \cdot \overline{x} \geq I(\overline{x}) = p(\overline{x})$, so $q(\underline{x}) \geq p(\underline{x})$, a contradiction. (In words: $q$ covers greater ground from $\underline{x}$ to $\overline{x}$, yet is flatter as $p$ follows indifference curve of highest type.)

If $\beta_X(\{\underline{x}\}) = 0$ (no pooling), letting $x^*$ be the type purchased under $(\beta, q)$ of $\underline{\mu}$ (i.e, $x^* = \min supp(\beta_X)$). Then since $q' \geq \underline{\mu}$ and $q(x^*) = \underline{\mu} \cdot x^*$, we have $q(\underline{x}) \leq \underline{\mu} \cdot \underline{x} < E[\mu] \cdot \underline{x} = p(\underline{x})$. Since $p'(x) = \overline{\mu} + \nu \cdot g'(x) > \sigma^{-1}(x) + \nu \cdot g'(x) = q'$ in $(\underline{x}, \overline{x})$, where $\sigma$ is the associated coverage function of $(\beta, q)$, and $q(\underline{x}) < p(\underline{x})$, we have $p(\overline{x}) > q(\overline{x}) = \overline{\mu} \cdot \overline{x}$, which is not possible as we must have $p(x) \leq \overline{\mu} \cdot x$ for all $x \in X$. (Similar idea, just explanation of why $q(\underline{x}) \leq p(\underline{x})$ is different.) □

Then, uniqueness of equilibrium when $X = [\underline{x}, \overline{x}]$ (Proposition 3) follows from Lemma 24 (if there is a full pooling equilibrium, it is unique) and Lemma 23 (if there is a non-full-pooling equilibrium, it is unique).

## D.2 Adjusting Maximal and Minimal Coverage

We now prove two preliminary results which concern the effects of changes in $\underline{x}, \overline{x}$ on $\mu^*, x^*$ and $\sigma^u(\mu, \underline{x}, \overline{x})$.

### D.2.1 Lemma 1 (First Part)

We now prove the first part of Lemma 1, which concerns the effects of changes in $\overline{x}$ on $\mu^*$.

Assume $\underline{x} < \underline{x}_1 < \underline{x}_2$. Fix two equilibria $(p_1, \alpha_1)$, $(p_2, \alpha_2)$ respectively for the economies with contract spaces $[\underline{x}, \overline{x}_1]$, $[\underline{x}, \overline{x}_2]$ with cut-offs types $\mu_1^*, \mu_2^*$, coverage functions $\sigma_1, \sigma_2$, and $x_1^* = \sigma_1(\mu_1)$, $x_2^* = \sigma_2(\mu_2)$. Assume both equilibria have partial pooling. If neither have pooling then $\mu_1^* = \mu_2^* = \underline{\mu}$, and if one has pooling, one can insert an intermediate equilibrium in which the condition for pooling holds with equality, and deduce the claim using transitivity and a limit arguing.

Assume by way of contradiction $\mu_1^* < \mu_2^*$. Since there is pooling in both equilibria $\mu_1^* > \underline{\mu}$ and $\mu_2^* > \underline{\mu}$. Then,

$$p_1(\underline{x}) = E[\mu \mid \mu \leq \mu_1^*] < E[\mu \mid \mu \leq \mu_2^*] = p_2(\underline{x})$$

However, since $p_1(\overline{x}_1) = \overline{\mu} \cdot \overline{x}_1 > \sigma_2^{-1}(\overline{x}_1) \cdot \overline{x}_1 = p_2(\overline{x}_1)$, this means that the price curves must intersect somewhere in $(\underline{x}, \overline{x}_1)$; let $z = \max\{x \in [\underline{x}, \underline{x}_1] \mid p_1(x) = p_2(x)\}$. We check the following cases:

- Suppose $x_2^* > x_1^*$ and $z \in (x_1^*, x_2^*)$. In this interval $p_2(x) < \mu_2^* \cdot x$, and hence $p_1(z) = p_2(z) < \mu_2^* \cdot z$, so the point $y := \sigma_1^{-1}(\mu_2^*)$ satisfies $y > z$. If $y \geq x_2^* \geq x_1^*$, then $p_1(y) = \mu_2^* \cdot y = p_2(y)$ since both curves give fair prices in this domain, so $z \geq y$, contradicting $y > z$. So $y < x_2^*$; but then in $[\underline{x}, y]$, and hence in particular in $[\underline{x}, z]$, $p_1' = \sigma_1^{-1} + \nu \cdot g' < \mu_2^* + \nu \cdot g' = p_2'$, which combined with $p_1(\underline{x}) < p_2(\underline{x})$ implies $p_1(z) < p_2(z)$, a contradiction.

- Suppose $x_1^* > x_2^*$ and $z \in (x_2^*, x_1^*)$. Then in $[x_2^*, z]$, $p_1' = \mu_1^* + \nu \cdot g < \mu_2^* + \nu \cdot g \leq \sigma_2^{-1} + \nu \cdot g = p_2'$ and in $[\underline{x}, x_2^*]$, $p_1' = \mu_1^* + \nu \cdot g < \mu_2^* + \nu \cdot g = p_2'$; either way, $p_1' < p_2'$ in $[\underline{x}, z]$, which with $p_1(z) = p_2(z)$ gives $p_1(\underline{x}) > p_2(\underline{x})$, a contradiction.

- Similarly, if $z \leq \min[x_1^*, x_2^*]$, then in $[\underline{x}, z]$, $p_1' = \mu_1^* + \nu \cdot g < \mu_2^* + \nu \cdot g = p_2'$, giving a contradiction.

- Suppose $z \geq \max[x_1^*, x_2^*]$; then $\sigma_2^{-1}(z) = \sigma_1^{-1}(z)$, denote this common type as $\mu_z$. We know that if we look at the restricted economy with $X_z = [\underline{x}, z]$ and $P_z = P(\cdot \mid \mu \in [\underline{\mu}, \mu_z])$, we have a unique equilibrium; but restricting and conditioning $(\alpha_1, p_1)$, $(\alpha_2, p_2)$ to $X_z$ and $P_z$ give precisely such an equilibrium, so they must coincide, and hence $p_1(\underline{x}) = p_2(\underline{x})$, a contradiction.

#### D.2.2 Lemma 1 (Second Part)

We now prove the second part of Lemma 1, which concerns the effects of changes in $\overline{x}$ on $x^*$ when there is no pooling. Indeed, in the region of no pooling,

$$\int_{x^*}^{\overline{x}} \frac{g'(x)}{x} dx = \frac{\overline{\mu} - \underline{\mu}}{\nu}$$

Hence differentiation w.r.t. $\overline{x}$ gives

$$\frac{g'(\overline{x})}{\overline{x}} - \frac{\partial x^*}{\partial \overline{x}} \cdot \frac{g'(x^*)}{x^*} = 0 \to \frac{\partial x^*}{\partial \overline{x}} = \frac{g'(\overline{x})}{g'(x^*)} \cdot \frac{x^*}{\overline{x}} > 0$$

#### D.2.3 Lemma 3

We now prove Lemma 3, which concerns the effects of changes in $\underline{x}$.

Assume $\overline{x} > \underline{x} > \underline{x}_2 > \underline{x}_1$. Fix two equilibria $(p_1, \alpha_1)$, $(p_2, \alpha_2)$ respectively for the economies with contract spaces $[\underline{x}_1, \overline{x}]$, $[\underline{x}_2, \overline{x}]$ with cut-offs types $\mu_1^*, \mu_2^*$, coverage functions $\sigma_1, \sigma_2$, and $x_1^* = \sigma_1(\mu_1)$, $x_2^* = \sigma_2(\mu_2)$. As in the proof of Lemma 1, we may assume both equilibria have partial pooling.

Assume, by way of contradiction, that $\mu_1^* \geq \mu_2^*$, or equivalently, $x_1^* = \sigma_1(\mu_1^*) \geq \sigma_2(\mu_2^*) = x_2^*(\geq \underline{x}_2)$. We know

$$p_1(\underline{x}_1) = \underline{x}_1 \cdot E[\mu \mid \mu \leq \mu_1^*] \qquad \text{and} \qquad p_2(\underline{x}_2) = \underline{x}_2 \cdot E[\mu \mid \mu \leq \mu_2^*].$$

In $[\underline{x}_1, \underline{x}_2]$, $p_1' = \mu_1^* > E[\mu \mid \mu \leq \mu_1^*]$, so

$$p_1(\underline{x}_2) = p_1(\underline{x}_1) + \int_{\underline{x}_1}^{\underline{x}_2} p'(t) dt > \underline{x}_2 \cdot E[\mu \mid \mu \leq \mu_1^*]$$

Hence $p_1(\underline{x}_2) > p_2(\underline{x}_2)$. But, by Lemma 16, $p_1(\underline{x}_2) \leq p_2(\underline{x}_2)$, a contradiction.

#### D.2.4 Lemma 2

We now prove Lemma 2, which concerns the effects of $\underline{x}, \overline{x}$ on $\sigma^u(\cdot, \underline{x}, \overline{x})$.

Recall that, in the region of full separation ($\mu \in [\mu^*, \overline{\mu}]$), $\sigma^u(\cdot, \underline{x}, \overline{x})$ satisfies (5), i.e.

$$\overline{\mu} - \mu - \nu \int_{\sigma^u(\mu, \underline{x}, \overline{x})}^{\overline{x}} \frac{g'(x)}{x} dx = 0, \qquad \forall \mu \in [\mu^*, \overline{\mu}].$$

The function $\sigma^u(\cdot, \underline{x}, \overline{x})$ does depend on the exogenous parameters $\overline{x}, \underline{x}$, but is locally independent of $\underline{x}$ - that is, if $\mu > \mu^*(\underline{x}, \overline{x})$, then the above equation shows that $\sigma^u(\mu, \underline{x}, \overline{x}) = \sigma^u(\mu, \underline{x}', \overline{x})$ for $\underline{x}'$ close to $\underline{x}$. By implicit differentiation,

$$\frac{\partial \sigma^u}{\partial \overline{x}} = -\frac{-\nu \frac{g'(\overline{x})}{\overline{x}}}{-\nu\left(-\frac{g'(\sigma^u(\mu,\underline{x},\overline{x}))}{\sigma^u(\mu)}\right)} = \frac{g'(\overline{x})}{\overline{x}} \frac{\sigma^u(\mu, \underline{x}, \overline{x})}{g'(\sigma^u(\mu, \underline{x}, \overline{x}))} \geq 0, \qquad \forall \mu \in [\mu^*, \overline{\mu}].$$

## D.3 Proposition 5

We now prove Proposition 5, which concerns the effect on welfare of changes in $\overline{x}$.

Since $\nu$ is common to all individuals, and firms break even, social welfare (up to scale) is given by:

$$W(\underline{x}, \overline{x}) = F(\mu^*)g(\underline{x}) + \int_{\mu^*}^{\overline{\mu}} g(\sigma^u(\mu, \underline{x}, \overline{x}))f(\mu)d\mu,$$

With full pooling $\mu^* = \overline{\mu}$, so $W(\underline{x}, \overline{x}) = g(\underline{x})$. Then, welfare increases with $\underline{x}$ and is unchanged by $\overline{x}$. With no pooling $\mu^* = \underline{\mu}$ so

$$W(\underline{x}, \overline{x}) = \int_{\underline{\mu}}^{\overline{\mu}} g(\sigma^u(\mu, \underline{x}, \overline{x}))f(\mu)d\mu.$$

When there is no pooling, $\frac{\partial \sigma^u}{\partial \overline{x}} \geq 0, \forall \mu \in [\underline{\mu}, \overline{\mu}]$ (Lemma 2), so an increase in $\overline{x}$ increases welfare and an increase in $\underline{x}$ has no effect on welfare.

We now discuss partial pooling. From Lemma 1, an increase in $\overline{x}$ lowers the cutoff type $\mu^*$, which we can write as $\frac{d\mu^*}{d\overline{x}} \leq 0$.[51] Then, the derivative of welfare is

$$\frac{\partial W}{\partial \overline{x}} = -\underbrace{\frac{d\mu^*}{d\overline{x}}}_{-}\left[g(x^*) - g(\underline{x})\right] f(\mu^*) + \int_{\mu^*}^{\overline{\mu}} g'(\sigma^u(\mu, \underline{x}, \overline{x})) \underbrace{\frac{\partial \sigma^u}{\partial \overline{x}}}_{+} f(\mu)d\mu \geq 0$$

This term is signed since $g'(x) \geq 0$, and $\frac{\partial \sigma^u}{\partial \overline{x}} \geq 0$ for all $\mu > \mu^* = \mu^*(\underline{x}, \overline{x})$, by Lemma 2.

## D.4 Proposition 6

We now prove Proposition 6, which concerns the effects on welfare of changes in $\underline{x}$.

If there is no pooling for at $\underline{x}, \overline{x}$ and also no pooling if $\underline{x}$ is perturbed by a small amount, then changes in $\underline{x}$ have no effect on the equilibrium. If there is full pooling at $\underline{x}, \overline{x}$, and also full pooling when $\underline{x}$ is perturbed by a small amount, then welfare is strictly increasing in $\underline{x}$, as $W(\underline{x}, \overline{x}) = g(\underline{x})$.[52]

Below, we discuss the case of partial pooling. First, we write the system of equations that define equilibrium in matrix form. In the interest of generality, we implicitly differentiate this matrix equation with respect to a generic parameter (which can later be taken to be $\underline{x}$ or $\overline{x}$). We then use these results to characterize $\frac{\partial \mu^*}{\partial \underline{x}}$ and then discuss $\frac{\partial W}{\partial \underline{x}}$.

[51] This is illustrated in Figure 4.

[52] With full pooling or no pooling, $x^*$ is irrelevant for welfare.

### D.4.1 Setup

Recall that

$$\phi(\mu^*) = \mu^* - \mathbb{E}\left[\mu \mid \mu < \mu^*\right] > 0.$$

Notice that $\frac{d}{d\mu^*}\mathbb{E}\left[\mu \mid \mu < \mu^*\right] \geq 0$ since increasing $\mu^*$ results in an average conditional on a set containing larger values of $\mu$. Therefore

$$\frac{d\phi}{d\mu^*} = 1 - \frac{\partial \mathbb{E}\left[\mu \mid \mu < \mu^*\right]}{\partial \mu^*} < 1.$$

If $f(\mu)$ is uniform, $\phi = \frac{1}{2}\left(\mu^* - \underline{\mu}\right)$ and $\frac{d\phi}{d\mu^*} = \frac{1}{2}$.

Let $\eta$ be a generic exogenous parameter (which can later be taken to be $\underline{x}, \overline{x}$). There are two equations which govern equilibrium when $X = [\underline{x}, \overline{x}]$ and there is partial pooling. Each equation contains the endogenous variables $\mu^*, x^*$ and the exogenous variable $\eta$. The first equilibrium condition is the relationship between $\mu^*$ and $x^*$ described by (5), or

$$H\left(\mu^*, x^*, \eta\right) = \overline{\mu} - \mu^* - \nu \int_{x^*}^{\overline{x}} \frac{g'(x)}{x} dx = 0.$$

Recall that $\underline{p} = p\left(\underline{x}\right) = \underline{x} \cdot \mathbb{E}\left[\mu \mid \mu < \mu^*\right]$. The second equilibrium condition is type $\mu^*$'s indifference condition:

$$G\left(\mu^*, x^*, \eta\right) = \left(g(x^*) - g(\underline{x})\right) \cdot \nu - \underline{x}\phi(\mu^*) = 0.$$

These two conditions can be written in matrix form as

$$\begin{bmatrix} H\left(\mu^*, x^*, \eta\right) \\ G\left(\mu^*, x^*, \eta\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Implicit differentiation with respect to $\eta$ implies

$$\begin{bmatrix} \frac{\partial H}{\partial \mu^*} & \frac{\partial H}{\partial x^*} \\ \frac{\partial G}{\partial \mu^*} & \frac{\partial G}{\partial x^*} \end{bmatrix} \begin{bmatrix} \frac{\partial \mu^*}{\partial \eta} \\ \frac{\partial x^*}{\partial \eta} \end{bmatrix} = \begin{bmatrix} -\frac{\partial H}{\partial \eta} \\ -\frac{\partial G}{\partial \eta} \end{bmatrix}. \tag{27}$$

The terms in the square matrix are

$$\frac{\partial H}{\partial \mu^*} = -1, \qquad \frac{\partial H}{\partial x^*} = \nu \frac{g'(x^*)}{x^*},$$

$$\frac{\partial G}{\partial \mu^*} = -\underline{x}\frac{d\phi}{d\mu^*}, \qquad \frac{\partial G}{\partial x^*} = \nu g'(x^*).$$

The determinant of the square matrix is

$$D = \nu g'(x^*) \left[\frac{\underline{x}}{x^*}\frac{d\phi}{d\mu^*} - 1\right] < 0.$$

We have $D < 0$ because $\frac{\underline{x}}{x^*} < 1$ in the presence of partial pooling, and $\frac{d\phi}{d\mu^*} \leq 1$ as discussed above.

#### D.4.2 Lemma 1 (Log-concave Distribution)

We now prove the final part of Lemma 1, which says that if $f(\cdot)$ is log-concave and both equilibria display partial (but not full) pooling, then $\frac{\partial x^*}{\partial \overline{x}} < 0$.

$$\frac{\partial G}{\partial \overline{x}} = 0, \frac{\partial H}{\partial \overline{x}} = -\frac{g(\overline{x})}{\overline{x}}$$

so by applying Cramer's rule to (27),

$$\begin{aligned}
\frac{\partial x^*}{\partial \overline{x}} &= \frac{1}{D}\left[\frac{\partial H}{\partial \mu^*}\left(-\frac{\partial G}{\partial \overline{x}}\right) - \frac{\partial G}{\partial \mu^*}\left(-\frac{\partial H}{\partial \overline{x}}\right)\right] \\
&= \frac{1}{D}\left[\frac{\partial G}{\partial \mu^*}\cdot\frac{\partial H}{\partial \overline{x}}\right] = \frac{1}{D}[(-\underline{x}\frac{d\phi}{d\mu^*})\cdot(-\frac{g(\overline{x})}{\overline{x}})] = \frac{1}{D}\frac{\underline{x}}{\overline{x}}\cdot\frac{g(\overline{x})}{\overline{x}}\cdot\frac{d\phi}{d\mu^*}
\end{aligned}$$

Since $D < 0$, and when $f$ is log-concave, $\frac{d\phi}{d\mu^*} > 0$, we derive $\frac{\partial x^*}{\partial \overline{x}} < 0$.

#### D.4.3 Lemma 3

We now provide an alternative proof of Lemma 3, which shows $\frac{\partial x^*}{\partial \underline{x}} \geq 0$.

Lemma 3 implies that, for fixed $\underline{x}$, the domain of $\overline{x}$ for which partial pooling occurs is connected, and $x^*$ depends continuously on $\underline{x}, \overline{x}$. Therefore, it suffices to show that $\frac{\partial x^*}{\partial \underline{x}}$ is non-negative a.e.. We now take $\eta$ to be $\underline{x}$. We compute the terms on the RHS of (27) as

$$\frac{\partial H}{\partial \underline{x}} = 0, \qquad \frac{\partial G}{\partial \underline{x}} = -g'(\underline{x})\nu - \phi(\mu^*) \leq 0.$$

From the analysis above, Cramer's rule gives

$$\begin{aligned}
\frac{\partial x^*}{\partial \underline{x}} &= \frac{1}{D}\left[\frac{\partial H}{\partial \mu^*}\left(-\frac{\partial G}{\partial \underline{x}}\right) - \frac{\partial G}{\partial \mu^*}\left(-\frac{\partial H}{\partial \underline{x}}\right)\right] \\
&= -\frac{1}{D}\left[g'\left(x\right)\nu + \phi(\mu^*)\right]
\end{aligned}$$

We have $g'\left(x\right) > 0$ in $(0,1)$, $\phi(\mu^*) \geq 0$ and $D < 0$ from the preceding analysis. Therefore, $\frac{\partial x^*}{\partial \underline{x}} \geq 0$.

#### D.4.4 Affect of $\underline{x}$ on Welfare

We now use the results above to complete the proof of Proposition 6 (effect of $\underline{x}$ on welfare).

We take $\eta$ to refer to $\underline{x}$. By Cramer's Rule and using the results above,

$$\begin{aligned}
\frac{\partial \mu^*}{\partial \underline{x}} &= \frac{1}{D}\left[\left(-\frac{\partial H}{\partial \underline{x}}\right)\frac{\partial G}{\partial x^*} - \frac{\partial H}{\partial x^*}\left(-\frac{\partial G}{\partial \underline{x}}\right)\right] \\
&= \frac{1}{x^* - \underline{x}\frac{d\phi}{d\mu^*}}\left[g'(\underline{x})\nu + \phi(\mu^*)\right] \geq 0
\end{aligned}$$

as claimed in Proposition 3. We have $\frac{d\phi}{d\mu^*} \leq 1$ as discussed above, so $x^* - \underline{x}\frac{d\phi}{d\mu^*} \geq 0$. This is consistent with Lemma 3 and is illustrated in Figure 6.

Recall that, with partial pooling, social welfare is

$$W(\underline{x}, \overline{x}) = F(\mu^*)g(\underline{x}) + \int_{\mu^*}^{\overline{\mu}} g(\sigma(\mu; \overline{x}))f(\mu)d\mu.$$

Then, using the results above, the effect of $\underline{x}$ on welfare is

$$\frac{\partial W}{\partial \underline{x}}(\underline{x}, \overline{x}) = -\frac{\partial \mu^*}{\partial \underline{x}} f(\mu^*)\left[g(x^*) - g(\underline{x})\right] + F(\mu^*)g'(\underline{x}).$$

Recall the function $\phi$ defined in (11) Then, using the expression for $\frac{\partial \mu^*}{\partial \underline{x}}$ above yields the following result

$$\frac{\partial W}{\partial \underline{x}} = -\frac{f(\mu^*)\left[g'(\underline{x})\nu + \phi(\mu^*)\right]\left[g(x^*) - g(\underline{x})\right]}{x^* - \underline{x}\frac{d\phi}{d\mu^*}} + F(\mu^*)g'(\underline{x}). \tag{28}$$

# E Voluntary Purchase

In this section we consider the model of voluntary purchase of Section 6, so $X = [\underline{x}, \overline{x}] \cup \{0\}$.

## E.1 Uniqueness under log-concavity

We now show that, if the distribution of types is log-concave, equilibrium is unique in a market with voluntary purchase. Towards this result, we begin by showing (without using of log-concavity) that, if some agents choose not to buy, then some must also choose the minimal coverage.

**Lemma 25.** *In any AG equilibrium $(p, \alpha)$, if $x = 0$ is an atom of $\alpha_X$, then $x = \underline{x}$ is an atom of $\alpha_X$ as well.*

*Proof.* By way of contradiction, assume $\alpha_X(\{0\}) > 0$ but $\alpha_X(\{\underline{x}\}) = 0$. First we deal with the case $\alpha_X((\underline{x}, \overline{x}]) > 0$. Let $\sigma$ be the associated coverage function. By definition $\sigma(\mu^*) = x^*$, $p(x^*) = x^* \cdot \mu^*$, and $\sigma(\mu) = 0$ iff $\mu < \mu^*$. Since agents are strictly risk average, $\mu^*$ strictly prefers $(x^*, p(x^*))$ to $(0, 0)$, and hence so do all types $\mu \in (\mu^* - \delta, \mu^*)$ for some $\delta > 0$, contradicting the fact that types $< \mu^*$ purchase $0$.

If $\alpha_X((\underline{x}, \overline{x}]) = 0$, then $\alpha_X(\{0\}) = 1$. However $p(\underline{x}) \leq \overline{\mu} \cdot \underline{x}$ and hence all types $\mu \in (\overline{\mu} - \delta, \overline{\mu})$ for some $\delta > 0$ strictly prefer $(\underline{x}, p(\underline{x}))$ to $(0, 0)$, and hence $\alpha_X(\underline{x}) > 0$ as well. ☐

We now use Lemma 25 to prove Proposition 9 (uniqueness under log-concavity).

*Proof.* First, we observe that if the distribution is log-concave, then recall the function, the MRL, defined in (19)

$$\psi(\mu_*, \mu^*) = E[\mu \mid \mu^* > \mu > \mu_*] - \mu_*$$

and recall, as mentioned there, that when $f(\cdot)$ is log-concave, then $\psi$ is strictly decreasing in $\mu_*$; Clearly also, $\psi$ is strictly increasing in $\mu^*$.

We show uniqueness of equilibrium by checking several cases. Let $(p, \alpha)$ and $(q, \beta)$ be two different equilibria.

- Lemma 25 shows that if there is pooling at $0$, then there must also be pooling at $\underline{x}$.

- By Lemma (17) one of the equilibria has at least partial pooling at $\underline{x}$, then they all do.
- If neither has pooling, both satisfy $\mu^* = \mu_* = \underline{\mu}$, they coincide by (5).
- If both have at least partial pooling and $p(\underline{x}) = q(\underline{x})$, then Lemma 16 shows both must have the same cut-off type, and clearly they must have the same cut-off participation type, and hence they coincide.

The only remaining case is they both have either partial or full pooling (possibly one of each), but differ with their price at $\underline{x}$. WLOG, $q(\underline{x}) > p(\underline{x})$. Associate with $(\alpha, p)$ the usual cut-offs $\mu^*, \mu_*$, and denote the cut-offs $(\beta, q)$ by $\omega^*, \omega_*$. Under $(\beta, q)$ is less willing to prefer $(\underline{x}, q(\underline{x}))$ over $(0, 0)$, so $\omega_* > \mu_*$. We claim we must have $\omega^* \leq \mu^*$:

- If $(\alpha, q)$ (and possibly also $(\beta, q)$) has full pooling, $\mu^* = \overline{\mu} \geq \omega^*$.
- If only $(\beta, q)$ has full pooling, then since $q(\overline{x}) \leq \overline{\mu} \cdot \overline{x} = p(\overline{x})$ and $q' = \overline{\mu} + \nu \cdot g' > p'$ in $(\underline{x}, \overline{x})$, we have we must have $q(\underline{x}) < p(\underline{x})$, a contradiction.
- If they both have only partial pooling, we also know by Lemma (16), that if $\omega^* > \mu^*$, then we would have $q \leq p$ with strictly inequality below the intersection of supports of $\alpha_X, \beta_X$; hence, $q(\underline{x}) < p(\underline{x})$, a contradiction.

The combination of $\omega^* \leq \mu^*$ with $\omega_* > \mu_*$ shows[53]

$$E[\mu \mid \omega_* < \mu < \omega^*] - \omega_* = \psi(\omega_*, \omega^*) < \psi(\mu_*, \mu^*) = E[\mu \mid \mu_* < \mu < \mu^*] - \mu_*$$

and hence, multiplying both sides by $\underline{x}$, we have $q(\underline{x}) - \omega_* \underline{x} < p(\underline{x}) - \mu_* \underline{x}$. However, since these are equilibria, and the agents $\mu_*$ (resp. $\omega_*$) are indifferent between $(0, 0)$ and $(\underline{x}, p(\underline{x}))$ (resp. $(\underline{x}, q(\underline{x}))$), we know that

$$\nu \cdot g(0) = \mu_* \underline{x} + \nu \cdot g(\underline{x}) - p(\underline{x}) = \omega_* \underline{x} + \nu \cdot g(\underline{x}) - q(\underline{x})$$

a contradiction. □

### E.2 Effect of $\underline{x}, \overline{x}$ on $\sigma^u$

The effects of $\underline{x}, \overline{x}$ on $\sigma^u(\cdot, \underline{x}, \overline{x})$ in the region where types are fully separated ($\mu \in [\mu^*, \overline{\mu}]$) are the same as those described in Lemma 6, namely:

$$g'(\sigma^u(\cdot, \overline{x}, \underline{x})) \frac{\partial \sigma^u}{\partial \overline{x}} = \frac{g'(\overline{x})}{\overline{x}} \frac{\sigma^u(\cdot, \overline{x}, \underline{x})}{g'(\sigma^u(\cdot, \overline{x}, \underline{x}))}, \qquad \frac{\partial \sigma^u}{\partial \underline{x}} = 0.$$

### E.3 Setup

To discuss the effect of $\underline{x}, \overline{x}$ on equilibrium welfare, we proceed in a way similar to that of Appendix D; we will deal with the region of partial pooling but non-full purchase, i.e., $\underline{\mu} < \mu_* < \mu^* < \overline{\mu}$, as the conclusions for region of full purchase arise from the results of Appendix D. First, we write the system of equations that defines equilibrium in matrix form. Then, we implicitly differentiate this matrix equation with respect to an exogenous parameter $\eta$ (which can later be taken to be $\underline{x}$ or $\overline{x}$). We then use these results to discuss $\frac{\partial W}{\partial \overline{x}}, \frac{\partial W}{\partial \underline{x}}$.

Let

[53] This is the only point at which the log-concavity is used in en route to proving Proposition 9.

$$\phi\left(\mu^*, \mu_*\right) = \mu^* - \mathbb{E}\left[\mu \mid \mu \in [\mu_*, \mu^*)\right],$$

$$\psi\left(\mu^*, \mu_*\right) = \mathbb{E}\left[\mu \mid \mu \in [\mu_*, \mu^*)\right] - \mu_*.$$

Recall that we assume $f$ is log-concave (so equilibrium is unique), and in this case, recall from Section 5.2,

$$\frac{\partial \phi}{\partial \mu^*} \geq 0, \qquad \frac{\partial \phi}{\partial \mu_*} \leq 0, \qquad \frac{\partial \psi}{\partial \mu^*} \geq 0, \qquad \frac{\partial \psi}{\partial \mu_*} \leq 0 \tag{29}$$

In this case, equilibrium is characterized by 3 equations. Each of them is potentially a function of the three endogenous parameters $(x^*, \mu^*, \mu_*)$ and the generic exogenous parameter $\eta$ (which we will later take to be either $\underline{x}$ or $\overline{x}$). First, as in Section 5, the relationship between $x^*$ and $\mu^*$ satisfies

$$H\left(x^*, \mu^*, \eta\right) = \overline{\mu} - \mu^* - \nu \int_{x^*}^{\overline{x}} \frac{g'(x)}{x} dx = 0.$$

Second, the indifference condition of type $\mu^*$ is:

$$G\left(x^*, \mu^*, \mu_*, \eta\right) = \left(g(x^*) - g(\underline{x})\right)\nu - \underline{x}\phi\left(\mu^*, \mu_*\right) = 0.$$

Third, the indifference condition of type $\mu_*$ (recalling that $g(0) = 0$) is:

$$K\left(\mu^*, x^*, \eta\right) = g(\underline{x}) \cdot \nu - \underline{x}\psi\left(\mu^*, \mu_*\right) = 0$$

This can be written in matrix form as

$$\begin{bmatrix} H\left(x^*, \mu^*, \eta\right) \\ G\left(x^*, \mu^*, \mu_*, \eta\right) \\ K\left(\mu^*, x^*, \eta\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Total differentiation with respect to a generic exogenous parameter $\eta$ and writing in matrix form implies

$$\begin{bmatrix} \frac{\partial H}{\partial \mu^*} & \frac{\partial H}{\partial x^*} & \frac{\partial H}{\partial \mu_*} \\ \frac{\partial G}{\partial \mu^*} & \frac{\partial G}{\partial x^*} & \frac{\partial G}{\partial \mu_*} \\ \frac{\partial K}{\partial \mu^*} & \frac{\partial K}{\partial x^*} & \frac{\partial K}{\partial \mu_*} \end{bmatrix} \begin{bmatrix} \frac{\partial \mu^*}{\partial \eta} \\ \frac{\partial x^*}{\partial \eta} \\ \frac{\partial \mu_*}{\partial \eta} \end{bmatrix} = \begin{bmatrix} -\frac{\partial H}{\partial \eta} \\ -\frac{\partial G}{\partial \eta} \\ -\frac{\partial K}{\partial \eta} \end{bmatrix}.$$

We now compute the terms in the square matrix above and the vector on the RHS. First, we have

$$\frac{\partial H}{\partial \mu^*} = -1 \leq 0, \qquad \frac{\partial H}{\partial x^*} = \nu \frac{g'(x^*)}{x^*}, \qquad \frac{\partial H}{\partial \mu_*} = 0$$

$$\frac{\partial H}{\partial \underline{x}} = 0 \qquad \frac{\partial H}{\partial \overline{x}} = -\nu \frac{g'(\overline{x})}{\underline{x}}$$

Second, we have

$$\frac{\partial G}{\partial \mu^*} = -\underline{x}\frac{d\phi}{d\mu^*}, \qquad \frac{\partial G}{\partial x^*} = \nu g'(x^*), \qquad \frac{\partial G}{\partial \mu_*} = -\underline{x}\frac{\partial \phi}{\partial \mu_*}$$

$$\frac{\partial G}{\partial \underline{x}} = -g'(\underline{x})\nu - \phi \qquad \frac{\partial G}{\partial \overline{x}} = 0$$

Third, we have

$$\frac{\partial K}{\partial x^*} = 0, \qquad \frac{\partial K}{\partial \mu^*} = -\underline{x}\frac{\partial \psi}{\partial \mu^*}, \qquad \frac{\partial K}{\partial \mu_*} = -\underline{x}\frac{\partial \psi}{\partial \mu_*} \geq 0$$

$$\frac{\partial K}{\partial \underline{x}} = -\psi + g'(\underline{x})\nu \qquad \frac{\partial K}{\partial \overline{x}} = 0$$

The determinant of the square matrix is

$$D \;\; = \;\; \nu g'(x^*)\underline{x}\left[\frac{\partial \psi}{\partial \mu_*} - \frac{\underline{x}}{x^*}\left(\frac{\partial \phi}{\partial \mu^*}\frac{\partial \psi}{\partial \mu_*} - \frac{\partial \phi}{\partial \mu_*}\frac{\partial \psi}{\partial \mu^*}\right)\right]. \tag{30}$$

Recall that

$$\frac{\partial H}{\partial \mu_*} = \frac{\partial H}{\partial \underline{x}} = \frac{\partial G}{\partial \overline{x}} = \frac{\partial K}{\partial x^*} = \frac{\partial K}{\partial \overline{x}} = 0.$$

Then, for a generic parameter $\eta$, by Cramer's Rule

$$\frac{\partial \mu^*}{\partial \eta}D \;\; = \;\; -\frac{\partial H}{\partial \eta}\frac{\partial G}{\partial x^*}\frac{\partial K}{\partial \mu_*} + \frac{\partial H}{\partial x^*}\left(\frac{\partial G}{\partial \eta}\frac{\partial K}{\partial \mu_*} - \frac{\partial G}{\partial \mu_*}\frac{\partial K}{\partial \eta}\right).$$

Also, for a generic parameter $\eta$, by Cramer's Rule

$$\begin{aligned}\frac{\partial \mu_*}{\partial \eta}D \;\; = \;\; & \frac{\partial H}{\partial \mu^*}\left(\frac{\partial G}{\partial \eta}\frac{\partial K}{\partial x^*} - \frac{\partial G}{\partial x^*}\frac{\partial K}{\partial \eta}\right) \\ & + \frac{\partial H}{\partial x^*}\left(\frac{\partial G}{\partial \mu^*}\frac{\partial K}{\partial \eta} - \frac{\partial G}{\partial \eta}\frac{\partial K}{\partial \mu^*}\right) \\ & + \frac{\partial H}{\partial \eta}\left(\frac{\partial G}{\partial x^*}\frac{\partial K}{\partial \mu^*} - \frac{\partial G}{\partial \mu^*}\frac{\partial K}{\partial x^*}\right)\end{aligned}$$

Finally, for a generic parameter $\eta$ and again by Cramer's Rule

$$\begin{aligned}\frac{\partial x^*}{\partial \eta}D \;\; = \;\; & -\frac{\partial H}{\partial \mu^*}\left(\frac{\partial G}{\partial \eta}\frac{\partial K}{\partial \mu_*} - \frac{\partial K}{\partial \eta}\frac{\partial G}{\partial \mu_*}\right) \\ & - \frac{\partial H}{\partial \eta}\left(\frac{\partial G}{\partial \mu^*}\frac{\partial K}{\partial \mu_*} - \frac{\partial K}{\partial \mu^*}\frac{\partial G}{\partial \mu_*}\right)\end{aligned}$$

## E.4 Sign of $D$

Let $E = \mathbb{E}\left[\mu \mid \mu \in [\mu_*, \mu^*)\right]$. Notice that $\frac{\partial E}{\partial \mu^*} \geq 0$ and $\frac{\partial E}{\partial \mu_*} \geq 0$ (in both cases, the change involves conditioning on larger values of $\mu$).

Recall that $\phi = \mu^* - E$ and $\psi = E - \mu_*$. Then the determinant $D$ from (30) can be written as

$$\begin{aligned} D &= \nu g'(x^*)\underline{x}\left[\frac{\partial E}{\partial \mu_*} - 1 - \frac{\underline{x}}{x^*}\left(\left[1 - \frac{\partial E}{\partial \mu^*}\right]\left[\frac{\partial E}{\partial \mu_*} - 1\right] - \left[-\frac{\partial E}{\partial \mu_*}\right]\left[\frac{\partial E}{\partial \mu^*}\right]\right)\right] \\ &= \nu g'(x^*)\frac{\underline{x}}{x^*}\left[\left(\frac{\partial E}{\partial \mu_*} - 1\right)(x^* - \underline{x}) - \underline{x}\frac{\partial E}{\partial \mu^*}\right]. \end{aligned}$$

Recall $\frac{\partial E}{\partial \mu^*} \geq 0$ and $x^* - \underline{x} \geq 0$. If $f(\mu)$ log-concave, then for instance $\frac{\partial \psi}{\partial \mu_*} = \frac{\partial E}{\partial \mu_*} - 1 \leq 0$. Therefore, $f(\mu)$ log-concave is a sufficient (but not necessary) condition for $D < 0$.

For instance, if $\mu$ is uniform (and thus log-concave), then $D = -\frac{1}{2}\nu g'(x^*)\underline{x} < 0$.

### E.5 Lemma 5

We now prove Lemma 5, which concerns the effect of $\overline{x}$ on $\mu^*, \mu_*$.

Recall that Appendix E.4 shows that, if $f(\mu)$ log-concave (as assumed), then $D < 0$. Recall that

$$\frac{\partial H}{\partial \mu_*} = \frac{\partial H}{\partial \underline{x}} = \frac{\partial G}{\partial \overline{x}} = \frac{\partial K}{\partial x^*} = \frac{\partial K}{\partial \overline{x}} = 0.$$

Recall that $\frac{\partial \psi}{\partial \mu^*} > 0$ and $\frac{\partial \psi}{\partial \mu_*} < 0$. We take $\eta$ to refer to $\overline{x}$.

First, we compute $\frac{\partial \mu^*}{\partial \overline{x}}$. Using the results above, we obtain

$$\begin{aligned} \frac{\partial \mu^*}{\partial \overline{x}} &= -\frac{1}{D}\frac{\partial H}{\partial \overline{x}}\frac{\partial G}{\partial x^*}\frac{\partial K}{\partial \mu_*} \\ &= -\frac{1}{D}[\nu]^2 g'(\overline{x})g'(x^*)\frac{\partial \psi}{\partial \mu_*} < 0 \end{aligned}$$

This matches the simulations of Figure 9.

We now compute $\frac{\partial \mu_*}{\partial \overline{x}}$. Using the results above, we obtain

$$\begin{aligned} \frac{\partial \mu_*}{\partial \overline{x}} &= \frac{1}{D}\frac{\partial H}{\partial \overline{x}}\frac{\partial G}{\partial x^*}\frac{\partial K}{\partial \mu^*} \\ &= \frac{1}{D}[\nu]^2 g'(\overline{x})g'(x^*)\frac{\partial \psi}{\partial \mu^*} < 0. \end{aligned}$$

This matches the simulations of Figure 9.

We can also obtain

$$\frac{\partial x^*}{\partial \overline{x}}D = -\nu g'(\overline{x})\underline{x}\left(\frac{d\phi}{d\mu^*}\frac{\partial \psi}{\partial \mu_*} - \frac{\partial \psi}{\partial \mu^*}\frac{\partial \phi}{\partial \mu_*}\right)$$

Recall that $\phi = \mu^* - E$ and $\psi = E - \mu_*$. Then this can be written as

$$\frac{\partial x^*}{\partial \overline{x}}D = -\nu g'(\overline{x})\underline{x}\left(\frac{\partial E}{\partial \mu_*} + \frac{\partial E}{\partial \mu^*} - 1\right).$$

If $f(\mu)$ is log-concave, then $D < 0$, so $\frac{\partial x^*}{\partial \overline{x}}$ has the same sign as $\frac{\partial E}{\partial \mu_*} + \frac{\partial E}{\partial \mu^*} - 1$. If $f(\mu)$ uniform, then $\frac{\partial E}{\partial \mu_*} = \frac{\partial E}{\partial \mu^*} = \frac{1}{2}$ so $\frac{\partial x^*}{\partial \overline{x}} = 0$. This matches the simulations in Figure 9 (up to numerical approximation errors).

### E.6 Lemma 8

We now prove Lemma 8 which concerns the effect of $\underline{x}$ on $\mu^*, \mu_*$. Recall that

$$\frac{\partial H}{\partial \mu_*} = \frac{\partial H}{\partial \underline{x}} = \frac{\partial G}{\partial \overline{x}} = \frac{\partial K}{\partial x^*} = \frac{\partial K}{\partial \overline{x}} = 0.$$

Given $f(\mu)$ is log-concave, recall (20),

$$\frac{\partial \phi}{\partial \mu^*} \geq 0, \qquad \frac{\partial \phi}{\partial \mu_*} \leq 0, \qquad \frac{\partial \psi}{\partial \mu^*} \geq 0, \qquad \frac{\partial \psi}{\partial \mu_*} \leq 0.$$

We now take the generic parameter $\eta$ to be $\underline{x}$.

First, we compute $\frac{\partial \mu^*}{\partial \underline{x}}$ . Using the results above,

$$\begin{aligned}
\frac{\partial \mu^*}{\partial \underline{x}} D &= \frac{1}{D} \frac{\partial H}{\partial x^*} \left( \frac{\partial G}{\partial \underline{x}} \frac{\partial K}{\partial \mu_*} - \frac{\partial G}{\partial \mu_*} \frac{\partial K}{\partial \underline{x}} \right) \\
&= \nu \frac{g'(x^*)}{x^*} \underline{x} \left( \left[ g'(\underline{x})\nu + \phi \right] \frac{\partial \psi}{\partial \mu_*} - \left[ \psi - g'(\underline{x})\nu \right] \frac{\partial \phi}{\partial \mu_*} \right)
\end{aligned}$$

This can we written as

$$\frac{\partial \mu^*}{\partial \underline{x}} D = \nu \frac{g'(x^*)}{x^*} \underline{x} \left( -g'(\underline{x})\nu - \phi + [\mu^* - \mu_*] \frac{\partial E}{\partial \mu_*} \right)$$

If $f(\mu)$ is uniform, then $\phi = \psi = \frac{1}{2}(\mu^* - \mu_*)$ and $\frac{\partial E}{\partial \mu_*} = \frac{1}{2}$, so

$$\frac{\partial \mu^*}{\partial \underline{x}} = -\frac{1}{D} [\nu]^2 g'(x^*) g'(\underline{x}) \frac{\underline{x}}{x^*} > 0.$$

This matches the simulations of Figure 11 (where $f(\mu)$ is uniform). $\frac{\partial x^*}{\partial \underline{x}} > 0$ follows similarly, or by using $\frac{\partial \mu^*}{\partial \underline{x}} > 0$ and the fact that (2) implies that $-\frac{\partial \mu^*}{\partial \underline{x}} = -\frac{\partial x^*}{\partial \underline{x}} \nu \frac{g'(x^*)}{x^*}$.

We now compute $\frac{\partial \mu_*}{\partial \underline{x}}$. We obtain

$$\begin{aligned}
\frac{\partial \mu_*}{\partial \underline{x}} D &= \frac{\partial G}{\partial x^*} \frac{\partial K}{\partial \underline{x}} + \frac{\partial H}{\partial x^*} \left( \frac{\partial G}{\partial \mu^*} \frac{\partial K}{\partial \underline{x}} - \frac{\partial G}{\partial \underline{x}} \frac{\partial K}{\partial \mu^*} \right) \\
&= \frac{1}{D} \left[ \nu g'(x^*) \right] \left[ \left[ -\psi + g'(\underline{x})\nu \right] + \frac{\underline{x}}{x^*} \left( \frac{d\phi}{d\mu^*} \left[ \psi - g'(\underline{x})\nu \right] - \left[ g'(\underline{x})\nu + \phi \right] \frac{\partial \psi}{\partial \mu^*} \right) \right]
\end{aligned}$$

In particular, if $f(\mu)$ is uniform, then

$$\frac{\partial \mu_*}{\partial \underline{x}} = \frac{1}{D} \nu g'(x^*) \left[ -\psi + g'(\underline{x})\nu \left[ 1 - \frac{\underline{x}}{x^*} \right] \right].$$

In the simulations of Figure 11 (where $f(\mu)$ is uniform), we see $\mu_*$ increasing with $\underline{x}$ in the region of partial pooling.

### E.7 Proposition 10

We now prove Proposition 10, which uses the results above to describe the effect of $\underline{x}, \overline{x}$ on welfare.

When purchase is voluntary ($X = [\underline{x}, \overline{x}] \cup \{0\}$), social welfare is proportional to

$$W = [F(\mu^*) - F(\mu_*)]\, g(\underline{x}) + \int_{\mu^*}^{\overline{\mu}} g(\sigma^u(\mu; \overline{x})) f(\mu) d\mu$$

The derivative of welfare with respect to $\overline{x}$ is

$$\frac{\partial W}{\partial \overline{x}} = -f(\mu^*) \underbrace{\frac{\partial \mu^*}{\partial \overline{x}}}_{-} [g(x^*) - g(\underline{x})] - f(\mu_*) \underbrace{\frac{\partial \mu_*}{\partial \overline{x}}}_{-} g(\underline{x}) + \int_{\mu^*}^{\overline{\mu}} g'(\sigma^u(\mu)) \underbrace{\frac{\partial \sigma^u}{\partial \overline{x}}}_{+} f(\mu) d\mu \geq 0$$

Recall that $\frac{\partial \sigma^u}{\partial \overline{x}} \geq 0$. Given $D \leq 0$, the overall effect is signed since $\frac{\partial \mu^*}{\partial \overline{x}} \leq 0$, $\frac{\partial \mu_*}{\partial \overline{x}} \leq 0$ and $\frac{\partial \sigma^u}{\partial \overline{x}} \geq 0$. This matches the simulations of Figure 9.

# F Markets for Lemons

We now prove that log-concavity of demand implies welfare is quasi-concave.

**Lemma 26.** *If demand $q(\underline{x}) = 1 - F(\mu_*(\underline{x}))$ is log-concave, then welfare is quasi-concave in $\underline{x}$.*

*Proof.* If $q(\underline{x})$ is log concave, then $[\ln q(\underline{x})]'' = [q'(\underline{x})/q]' \leq 0$. Also, $g'(x)/g(\underline{x})$ is decreasing because $g(\underline{x})$ is increasing concave. Then (22) can be written as $g'/g = -q'/q$. The RHS is increasing and the LHS is decreasing so the FOC holds at a unique point. $\square$

One limitation of Lemma 26 is that it places conditions on endogenous objects, namely $\mu_*$. The following result provides conditions on primitives such that $q(\underline{x})$ is log-concave.

**Lemma 27.** *If $\mu \sim \mathcal{U}[\underline{\mu}, \overline{\mu}]$ and $g(x) = \frac{1}{2}\left(1 - (1-x)^2\right)$, then $q(\underline{x})$ is log-concave.*

*Proof.* We can write

$$\frac{\partial q}{\partial \underline{x}} \frac{1}{q} = \nu \frac{f(\mu_*)}{1 - F(\mu^*)} \left[\frac{g(x)}{x}\right]' \frac{1}{[-\psi'(\mu_*)]}$$

Given the assumptions, then $\psi = \frac{1}{2}(\overline{\mu} - \mu_*)$ and $\frac{f(\mu_*)}{1-F(\mu^*)} = \frac{1}{\mu^*}$ and $g(x)/x = 1 - \frac{x}{2}$. Therefore,

$$\frac{\partial q}{\partial \underline{x}} \frac{1}{q} = -\nu \frac{1}{\mu_*}$$

Then, since $\frac{\partial \mu_*}{\partial \underline{x}} \geq 0$, this implies $\frac{\partial}{\partial \underline{x}}\left[\frac{\partial q}{\partial \underline{x}} \frac{1}{q}\right] \leq 0$, so $q(\underline{x})$ is log-concave. $\square$

### F.1 Lemma 10

We now prove Lemma 10, which concerns the effects of changes in coverage $\underline{x}$ on the mass of buyers. Applying the IFT to (21) implies

$$\frac{\partial \mu_*}{\partial \underline{x}} = \frac{\nu}{\psi'(\mu_*)}\left[\frac{g(x)}{x}\right]' \geq 0.$$

Recall that $\left[\frac{g(x)}{x}\right]' \leq 0$ by Lemma 9. Also, if $f$ is log-concave, $\psi'(\mu_*) < 0$. Notice that $\frac{\partial q}{\partial \mu_*} = -f(\mu^*)$. Then, this implies

$$\frac{\partial \underline{x}}{\partial q} = \frac{1}{\frac{\partial q}{\partial \mu_*}\frac{\partial \mu_*}{\partial \underline{x}}} = \frac{[-\psi'(\mu_*)]}{\nu f(\mu_*)\left[\frac{g(x)}{x}\right]'} \leq 0.$$

### F.2 Lemma 13

We now prove Lemma 13, which shows that the socially optimal level of coverage $\underline{x}$ is interior, assuming $\underline{\mu}$strictly prefers no purchase to purchase $(1, E[\mu])$.

Recall that, in this setting, welfare is $W(x) = q(x) \cdot g(x)$, where $q(x) = 1 - F(\mu_*(x))$. Then, using Lemma 10,

$$\begin{aligned} W'(x) =& q'(x)g(x) + q(x) \cdot g'(x) = -f(\mu_*)g(x)\frac{\partial \mu_*}{\partial \underline{x}} + q(x) \cdot g'(x) \\ =& \frac{\nu}{\psi'(\mu_*)}\left(-\frac{g'(x)}{x} + \frac{g(x)}{x^2}\right) f(\mu_*)g(x) + (1 - F(\mu_*))g'(x) \\ =& \frac{\nu \cdot f(\mu_*)}{\psi'(\mu_*)}\left(\frac{g(x)}{x}\right)^2 + g'(x) \cdot \left((1 - F(\mu^*)) - \frac{\nu \cdot f(\mu_*)}{\psi'(\mu_*)} \cdot \frac{g(x)}{x}\right) \end{aligned}$$

We note that $\mu_*(1) < \overline{\mu}$, since risk aversion implies that even if $\underline{x} = 1$ is given the highest price $\overline{\mu}$, a positive mass of agents will want to buy it; so $\psi'(\mu_*(1)) > 0$. Also $\mu_*(1) > \underline{\mu}$, since by assumption, not all types are purchasing. Since $g'(1) = 0$, $g(1) \neq 0$, and $\psi'(\mu) < 0$, $f(\mu) > 0$ for any $\mu \in (\underline{\mu}, \overline{\mu})$, the result follows.

## G Numerical simulations

In all our simulations, we take $g(x) = \frac{1}{2}(1 - (1 - x)^2)$ implies $g'(x) = 1 - x$ and $g(x)/x = 1 - \frac{x}{2}$. Then, the cut-off type $\mu^*$ satisfies, according to (2) and (5)

$$\mu^* = \overline{\mu} - \nu\left[\ln(\overline{x}) - \ln(x^*) - (\overline{x} - x^*)\right] \tag{31}$$

where $x^* = \sigma(\mu^*)$ is the cut-off coverage. For most cases, we simulated approximately 2000 individuals.[54]

To solve for equilibria, we first focus on finding $x^*$. Once this is done, $\mu^*$ follows from (31). In the case of voluntary purchase, the cut-off purchase type $\mu_*$ then follows from incentive compatibility, i.e., (14). The prices are then computed via the break-even conditions in Propositions (1) and (7), respectively. We describe this process in more detail below.

[54]In same cases, we simulate more individuals to reduce the noise in the graphs.

### G.1 Mandatory purchase

To find the equilibrium values of $(\mu^*, x^*)$ we first find the equilibrium value of $x^*$ and then use it to compute $\mu^*$. We use the following procedure:

1. For each point in a fine grid of $\hat{x} \in [\underline{x}, \overline{x}]$, we compute the value $\hat{\mu^*} = \tau(\hat{x})$ that would be the cut-off type if $\hat{x}$ were the cut-off coverage, using (31).

2. For each such pair $(\hat{x}, \hat{\mu})$ we compute $\underline{p}^1(\hat{x})$ as the price of the minimal coverage contract $\underline{x}$ that satisfies (8), i.e., where type $\hat{\mu^*} = \tau(\hat{x})$ is indifferent between contract $(\hat{x}, \hat{x} \cdot \hat{\mu})$ and contract $\left(\underline{x}, \underline{p}^1(\hat{x})\right)$.[55]

3. For each such pair $(\hat{x}, \hat{\mu})$ , we also compute $\underline{p}^2(\hat{x})$ as the price of the minimal coverage contract $\underline{x}$ implied by the break-even condition (6): $\underline{p}^2(\hat{x}) = \underline{x} \cdot E[\mu \mid \mu \leq \hat{\mu^*}]$.

The equilibrium value of $x^*$ is the value of $\hat{x}$ that solves $\left|\underline{p}^1(\hat{x}) - \underline{p}^2(\hat{x})\right| = 0$. In practice, we minimize this quantity over $\hat{x}$. If this distance does not vanish, then the (unique) equilibrium must be one which satisfies full pooling at $\underline{x}$. Once we have found the equilibrium value of $x^*$, we compute $\mu^* = \tau(x^*)$ - as mentioned, this uses (31).

### G.2 Voluntary purchase

To find the equilibrium values of $(\mu^*, \mu_*, x^*)$ we first find the equilibrium value of $x^*$, find $\mu^*$ as above, and then find $\mu_*$ using (14), or

$$\left([g(0) - g(\underline{x})] \cdot \nu + \underline{p}^1(x)\right) \frac{1}{\underline{x}} = \mu_* \tag{32}$$

If no such $\mu_*$ exists in $[\underline{\mu}, \mu^*]$, then there is full purchasing, and $\mu_* = \underline{\mu}$. The first two steps - leading to the computation of $\underline{p}^1(\hat{x})$ for each point in a fine grid of $\hat{x} \in [\underline{x}, \overline{x}]$, is the same as above. Then, to compute $\underline{p}^2(x)$:

1. For each $\hat{x}$ in a fine grid of $x \in [\underline{x}, \overline{x}]$, we compute the cut-off participation type $\hat{\mu_*}$ that would result if $\hat{x}$ was the cut-off coverage, using (32). We set $\hat{\mu_*} = \underline{\mu}$ if there is no such cut-off participation type.

2. For each such $\hat{x}$, Compute $\underline{p}^2(\hat{x})$ as the price of contract $\underline{x}$ implied by the break even condition:
$$\underline{p}^2(x) = \underline{x} \cdot \mathbb{E}\left[\mu \mid \mu \in \left[\hat{\mu_*}, \hat{\mu^*}\right]\right].$$

The equilibrium value of $x^*$ is the value of $\hat{x}$ that solves $\left|\underline{p}^1(\hat{x}) - \underline{p}^2(\hat{x})\right| = 0$. In practice, we minimize this quantity over $\hat{x}$. If this distance does not vanish, then there is full pooling.

Finally, once we've found the equilibrium value of $x^*$, we compute we compute $\mu^* = \tau(x^*)$ and $\mu_*$ - as mentioned, this uses (31) and (32).

[55] Given $\mu^* = \tau(x)$, price $\underline{p}^1(x)$ satisfies (8), or $(g(x) - g(\underline{x}))\,\nu = \tau(x) \cdot \underline{x} - \underline{p}^1(x))$