Auctions with Unknown Capacities: Understanding Competition among Renewables*

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Abstract

We analyze a multi-unit auction model in which bidders' capacities are private information. Firms compete by choosing price and quantity offers, and prices are determined through market clearing (uniform price auction). The actual mode of competition is endogenously determined in equilibrium. For large realized capacities, firms behave *à la* Cournot: they bid at marginal costs, and exercise market power by withholding output. Otherwise, firms behave *à la* Bertrand-Edgeworth: they offer all their capacity at prices above marginal costs, with markups decreasing in the firm's capacity. We find that private information serves as an imperfect coordination device, which allows firms to obtain higher profits than if capacities were unknown, but lower than if capacities were publicly known. We also analyze the effects of switching to a discriminatory auction and the effects of fragmenting the market structure. The results shed light on the nature of the strategic interaction between renewable producers in electricity auctions.

Keywords: multi-unit auctions, electricity markets, renewables.

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1 Introduction

Ambitious environmental targets, together with decreasing investment costs, have fostered the rapid deployment of renewable energy around the world. Installed renewable capacity has more than doubled over the last ten years, and it is expected to further increase during the coming decade. In several jurisdictions, the goal is to achieve a carbon-free power sector by 2050, which will require almost all electricity supplies to come from renewable resources.[1] This global trend begs the question: how will electricity markets perform in the future once renewables become the major energy source?

Whereas competition among conventional fossil-fuel generators is by now well understood (e.g. Borenstein (2002); von der Fehr and Harbord (1993) Green and Newbery (1992), among others), much less is known about competition among wind and solar producers (which we broadly refer to as *renewables*).[2] Competition-wise, there are two key differences between these two technologies. First, the marginal costs of conventional plants depend on their efficiency rate as well as on the price at which they buy the fossil fuel. In contrast, the marginal costs of renewable generation equal zero as they produce electricity out of freely available natural resources (e.g. wind or sun). Second, the available capacity of conventional plants is well known as they tend to be available at all times (absent rare outages). In contrast, the availability of renewable plants is uncertain as it depends on weather conditions that are difficult to forecast. Hence, the move from fossil fuel generation towards renewables will imply a change in paradigm. Whereas the previous literature has analyzed environments where marginal costs are private information but production capacities are publicly known, the relevant setting will soon be one in which marginal costs are known (and zero) but firms' capacities become private information. In this paper we build a model that captures this new competitive paradigm in electricity markets.

The auction literature typically assumes that bidders are privately informed about

[1]The International Renewable Energy Agency estimates that compliance with the 2017 Paris Climate Agreement will require overall investments in renewables to increase by 76% in 2030, relative to 2014 levels. Europe expects that over two thirds of its electricity generation will come from renewable resources by 2030, with the goal of achieving a carbon-free power sector by 2050 (European Commission (2012)). Likewise, California has recently mandated that 100% of its electricity will come from clean energy sources by 2045.

[2]Strictly speaking, there are renewable energies other than wind and solar. However, these two are the dominant ones. Not all renewable technologies share the same properties as wind and solar. For instance, hydro electricity is storable (in contrast to wind and solar which are flow technologies); the available capacity of biomass is known very much like a thermal plant, etc.

their costs or valuations, but tends to omit that bidders may also be privately informed about the maximum number of units they are willing to buy or sell.[3] Yet, this source of private information is present in several auctions, beyond electricity markets. To name just a few, in Treasury Bill auctions (Hortasu and McAdams (2010)), banks are privately informed about their hedging needs; in Central Banks' liquidity auctions (Klemperer (2019)), banks are privately informed about their toxic assets; and in emission permits auctions, firms are privately informed about their carbon emissions.[4] Likewise, firms have private information on capacities in a wide range of markets that can be analyzed through the lens of auction theory (Klemperer (2003)), e.g. the markets for hotel bookings or cab services, in which firms are privately informed about the number of empty rooms or available cars. In this paper we show that, in these set-ups, private information on capacities has a key impact on bidding behavior and market outcomes.

In our model, firms' capacity realizations are subject to common and idiosyncratic shocks. Whereas the former are publicly observable, the latter are firms' private information. Firms compete by submitting a price-quantity pair (i.e., an inverted-L supply function), indicating the maximum quantity they are willing to produce (not exceeding their own capacity) and the minimum price they are willing to receive. The auctioneer calls firms to produce in increasing price order until total demand is satisfied, and pays all accepted output at the market-clearing price (uniform-price auction).

In equilibrium, firms behave competitively only when the common shock is so large that each firm would have enough capacity to serve total demand, regardless of their idiosyncratic shocks. In all other cases, firms exercise market power. At the symmetric equilibrium, when realized capacities are large, firms exercise market power *à la* Cournot, i.e., by withholding output. In contrast, when realized capacities are small, they exercise market power *à la* Bertrand-Edgeworth, i.e., by bidding all their capacity above marginal costs. In this case, the equilibrium mark-up is decreasing in firms' realized capacities, i.e. the more capacity a firm has, the lower is the price at which it is willing to supply

[3] Arguably, incomplete information on capacities can be captured through incomplete information on costs or valuations. Simply, a capacity constraint can be modelled as an infinite cost or as a zero valuation beyond a certain number or units. However, existing papers have not allowed for this, either because bidders are not capacity constrained, or because such discontinuities in marginal costs or valuations are not allowed for.

[4] Other examples include spectrum auctions (Milgrom (2004)), procurement auctions for a wide range of goods and services, auctions for electricity generation capacity (Fabra (2018) and Llobet and Padilla (2018)) or auctions for investments in renewables (Cantillon (2014)), among others.

it. This equilibrium property reflects the standard trade-off faced by competing firms: raising the price leads to an output loss (quantity effect), but it also leads to a higher market price if the rival bids below (price effect). Since larger firms lose more from the quantity effect, they are less eager to submit high mark-ups. From this it follows that, in the short-run, expected market prices tend to be low when the common capacity shocks are large. Similarly, in the long-run, an increase in total investment smoothly depresses expected market prices, albeit non-linearly.

We assess the impact of private information on bidding behavior by comparing the above equilibria with the ones that arise when realized capacities are either publicly known, or unknown prior to bidding. We find that private information serves as a imperfect correlation device, which allows firms to make higher profits than in the case of unknown capacities, but lower than in the case of publicly known capacities. This result is reminiscent to the literature on Treasury auctions (LiCalzi and Pavan, 2005) which shows that introducing noise in the demand function rules out the seemingly collusive equilibria that arise otherwise (Back and Zender, 2001).

We extend our main model in several directions. First, we study the effects of firms' entry and changes in market structure. We show that an increase in the number of (symmetric) firms brings about a standard pro-competitive effect: the more firms there are, the bigger is the output loss when a firm outbids the rivals (quantity effect) and the smaller is the market share through which the firm internalizes the market price increase (price effect). Both effects lead to more competitive outcomes. If we leave total capacity fixed and consider splitting it among a larger number of firms, an information-related anti-competitive effect arises as smaller firms face more uncertainty regarding the rivals' capacities. Although this mitigates the former pro-competitive effect, the price-depressing effect of fragmenting the market structure dominates.

We also characterize equilibrium bidding in discriminatory auctions (that pay all winning firms their price offers). Similarly to Holmberg and Wolak (2018), we show that in this format firms submit higher bids than in uniform price auctions: firms are simply discouraged from bidding low prices as they are paid as they bid. Thus, the choice of the auction format introduces a trade-off, as uniform price auctions induce more competitive bidding but pay winning bidders the highest accepted bid. In contrast with Holmberg and Wolak (2018), we show that there is no revenue equivalence across auction formats.[5] The main force driving our different predictions is that equilibrium quantities are affected by private information on capacities, but not by private information on costs. Numerical solutions indicate that the discriminatory auction typically leads to higher prices, for example, when capacity availability arises from a Beta distribution.

Related Literature Holmberg and Wolak (2018) analyze competition in electricity markets in a model in which firms are privately informed about their marginal costs, rather than about their capacities. In their model, private information moves supply functions up or down (when realized costs are either high or low) while firms' quantity offers remain unchanged (given that capacities are assumed to be known and fixed).[6] This is unlike our model in which private information on capacities moves the supply functions up and to the left (when realized capacities are low) or down and to the right (when realized capacities are high). In our model, which allows for capacity withholding, changes in private information also leave supply functions unchanged when firms find it optimal to behave as if their realized capacity was slightly below total demand.

Vives (2011) studies a model of competition where firms offer linear supply functions under imperfect information about marginal costs. Since firms' marginal costs are correlated, the market price aggregates firms' private information. Firms' willingness to learn about their own costs from the market price induces them to submit steeper supply functions, yielding less competitive outcomes. This effect is not present in our model since we are dealing with a private value setup. In this respect, our model is less general than Vives (2011)'s but it allows us to consider constant marginal costs up to capacity, which is suitable for the case of renewable energy. This cost function contains kinks that would make the linear supply-function approach intractable.

Other recent papers have also analyzed competition among renewables (Acemoglu et al. (2017) and Kakhbod et al. (2018)). While they share with us the fact that capacities are random, they differ from our approach in that they assume Cournot competition, thus restricting firms to exerting market power only through capacity withholding. In

[5] They find revenue equivalence across auction formats for the case of independent costs. When marginal costs are positively affiliated, they also find, similarly to us, that the discriminatory auction pays firms more.

[6] More precisely, they assume that each firm might be unaware about its own capacity when it submits its bids. However, expected capacities are uncorrelated to the cost realization. As a result, the equilibrium outcomes are the same as if capacities were fixed and known at the time when firms submit their bids.

this paper we show that if firms are allowed to choose both prices and quantities, they exercise market power by raising prices above marginal costs when they are capacity constrained to serve total demand, and only resort to capacity withholding otherwise.

Acemoglu et al. (2017)'s analysis applies to markets at an earlier stage of renewables deployment, with firms owning a portfolio of conventional and renewable plants. They show that the common ownership of these two technologies mitigates the price depressing effect of renewables as firms withhold more output from their conventional plants when there is more renewable generation. This effect is not present in our model since renewable capacity is often enough to cover total demand. Furthermore, in order to focus on the strategic interaction among renewables, we assume either that conventional power plants are not present, or that they are owned by independent producers, thus constraining renewable producers from raising prices above the conventional plants' marginal costs.[7]

Kakhbod et al. (2018)'s focus is on the impact on market outcomes of the heterogeneity in stochastic renewable availability across locations. They show that firms withhold more output when they are closely located (i.e., when their output is more positively correlated). This implies that the market fails to induce investments at the optimal locations.

Even though we have motivated our model in the context of electricity markets, it is applicable to other relevant contexts. Notably, in the context of treasury auctions it is typically assumed that bidders are privately informed about their valuation up to a fixed number of units (Hortasu and McAdams, 2010). Our model would allow us to extend those analyses to situations in which this limit is private information (e.g. bidders might have different hedging needs that are unknown to their rivals) as long as it is positively related to the bidders' valuations.

The reminder of the paper is structured as follows. In Section 2 we describe the model. In Section 3 we characterize the bidding equilibria when firms' capacities are private information. In Section 4 we assess the impact of private information by comparing equilibrium outcomes with the ones that emerge when capacities are either publicly known or unknown to both firms prior to bidding. Section 5 discusses extensions, including the discriminatory auction and the effect of changing the number of firms. Section 6

[7]If we allowed the independent conventional producers to exercise market power, an increase in renewables would reduce the residual demand faced by the conventional producers, thus reducing their incentives to increase prices.

concludes.

2 The Model

Two firms $i = 1, 2$ compete in a market to supply a perfectly price-inelastic demand, denoted as $\theta > 0$. Firms are capacity constrained. The capacity of firm i, denoted $k_i > 0$, is assumed to be random. In particular, $k_i \in \left[\underline{k}, \bar{k}\right]$ for $i = 1, 2$ is distributed according to the function $G(k_i)$ with density $g(k_i) > 0$ in the whole interval. The capacity of both firms is assumed to be independently distributed. For simplicity, we assume that firm i can observe its own capacity but not that of its rival, i.e., available capacities are private information. Each firm can produce at a constant marginal cost $c \geq 0$ up to its capacity.

Firms compete on the basis of the bids submitted to an auctioneer. Each firm simultaneously and independently submits a price quantity pair (b_i, q_i) specifying the minimum price b_i at which it is willing to supply the corresponding quantity q_i. We assume $b_i \in [0, P]$, where P denotes the "market reserve price",[8] and $q_i \in [0, k_i]$, for $i = 1, 2$.

The auctioneer ranks firms according to their price offers, and calls them to produce in increasing rank order. In particular, if firms submit different prices, the low-bidding firm is ranked first while the high-bidding firm is ranked second. If firms submit equal prices, firm i is ranked first with probability $\alpha(q_i, q_j)$ and it is ranked second with probability $1 - \alpha(q_i, q_j)$. We assume a symmetric function $\alpha(q_i, q_j) = \alpha(q_j, q_i) \in (0, 1)$.[9] If firm i is ranked first it produces $\min\{\theta, q_i\}$, while if it is ranked second it produces $\max\{0, \min\{\theta - q_j, q_i\}\}$.

Firms receive a uniform price per unit of output, p, which is set equal to the highest accepted price offer.[10] If the sum of both firms' quantity offers is enough to cover total demand, $p = b_j$ if $b_i \leq b_j$ and $q_i < \theta$; $p = b_i$ otherwise. If the sum of both firms' quantity offers is not enough to cover total demand, $q_i + q_j < \theta$, the market price is set at P.

The profits made by each firm are computed as the product of their per unit profit margin $(p - c)$ and their output. As explained before, both the market price p as well as firms' outputs are a function of demand θ and the prices (b_i, b_j) and quantities (q_i, q_j)

[8] This price can be interpreted as the marginal cost of the next available technology, as long as it is offered competitively, or as an explicit or implicit price cap.

[9] Hence, when firms' quantity offers are equal, $\alpha(q, q) = 1/2$. We do not need to specify $\alpha(q_i, q_j)$ outside of the diagonal as it is inconsequential for equilibrium bidding.

[10] In section 5.1 we characterize the equilibrium under a discriminatory auction, in which firms are paid according to their bid.

offered by both firms. Firms, which are assumed to be risk neutral, bid so as to maximize their individual expected profits, given their realized capacities.

3 Equilibrium Characterization

In this section we characterize the Bayesian Nash Equilibria (BNE) of the game in which capacities are private information. It is simple to characterize equilibrium bidding in two extreme cases. First, when $\underline{k} > \theta$, either firm can cover total demand regardless of their realized capacities. Hence, Bertrand forces drive equilibrium prices down to marginal costs. Second, when $\bar{k} < \theta/2$, aggregate capacity is never enough to cover total demand. Hence, both firms sell all their capacity at P with no need to compete.

We thus turn attention to the remaining cases. We first assume that firms never have enough capacity to cover total demand on their own, $\bar{k} < \theta$. In this case we say that firms are pivotal with certainty. We next analyze the case in which firms are non-pivotal with some probability, $\bar{k} \geq \theta$.

3.1 Certain Pivotality

We start by identifying three key properties that any equilibrium must satisfy when firms are certain to be pivotal, $\bar{k} < \theta$.

Lemma 1. *Under certain pivotality,*

(i) Capacity witholding is never optimal, $q_i^*(k_i) = k_i$.

(ii) All Nash Equilibria must be in pure strategies.

(iii) The optimal price offer of firm i, $b_i^*(k_i)$*, must be non-increasing in* k_i.

The first part of the lemma rules out capacity witholding in equilibrium. The reason is two fold. First, conditionally on having the low bid, the firm maximizes its output by offering to sell at capacity. And second, conditionally on having the high bid, the firm faces the residual demand. Hence, the firm's output would be unchanged unless witholding constrained the firm from serving the residual demand, $q_i < \theta - k_j$.[11] In that

[11] In particular, offers $q_i \in [\theta - \underline{k}, k_i]$ are payoff equivalent to not withholding at all. Those indifference cases are also referred in the rest of the paper as non-withholding strategies.

case, its profits would be $(P - c)q_i$, which are below the ones that it can guarantee by bidding all its capacity at P, $(P - c)(\theta - k_j)$.

The second part of the lemma rules out non-degenerate mixed strategy equilibria. The underlying reason is simple: a firm's profits at a mixed-strategy equilibrium depend on its realized capacity, which is non-observable to the rival. If the competitor randomizes its bids in a way that makes the firm indifferent between two of them for a given capacity realization, the same randomization cannot make the firm indifferent for other capacity realizations as well. It follows that the equilibria must involve pure-strategies.

The last part of the above lemma rules out bids that are increasing in the firm's capacity. When a firm considers whether to reduce its bid marginally, two effects are at play for a given bid of the rival: a profit gain due to the output increase (*quantity effect*), and a profit loss due to the reduction in the market price (*price effect*). On the one hand, the *quantity effect* is increasing in the firm's capacity, as if its bid falls below the rival's, it would sell at capacity rather than just the residual demand. On the other hand, the *price effect* is independent of the firm's capacity as, contingent on bidding higher than the rival, the firm always sells the residual demand. Combining these two effects, the incentives to bid low are (weakly) increasing in the firm's capacity, giving rise to optimal bids that are non-increasing in k_i.[12]

Building on this Lemma, we now turn to characterizing the Bayesian Nash equilibria. We first characterize the pure-strategy asymmetric equilibria, and then move on to characterizing the unique pure-strategy symmetric Bayesian Nash equilibrium of the game.

Proposition 1. *Under certain pivotality, there exists a continuum of outcome equivalent asymmetric pure-strategy equilibria in all of which* $p^* = P$. *In these equilibria,* $q_i^*(k_i) \in [\theta - \underline{k}, k_i]$ *and* $b_i^*(k_i) = P$, *while* $q_j^*(k_j) = k_j$ *and* $b_j^*(k_j) < \underline{b}$, *where* $\underline{b}$ *is low enough so as to make undercutting by firm* i *unprofitable,* $i, j = 1, 2$.

When firms are always pivotal there exist asymmetric equilibria in which one firm bids sufficiently low so as to discourage its rival from undercutting it. This firm is then forced to maximize its profits over the residual demand by bidding at the highest possible price, P. The low bidder makes higher expected profits than the high bidder, as it sells

[12] The incentives to bid low are strictly increasing in the firm's capacity if marginally reducing the bid implies a strictly positive probability of increasing the firm's output, i.e., a strictly positive *quantity effect*. This need not be the case if the equilibrium is asymmetric.

at capacity rather than the residual demand. Unless firms can resort to an external correlation device, they thus face a coordination problem. This might preclude them from playing the asymmetric equilibria, even if these equilibria allow firms to maximize joint profits. For this reason we now focus on characterizing the symmetric equilibrium.

It is useful to distinguish two cases. First, assume $\underline{k} \geq \frac{\theta}{2}$, which guarantees that both firms always have enough joint capacity to serve total demand. In this case, and in contrast to the asymmetric equilibria characterized above, the optimal price offer at a symmetric equilibrium must be *strictly* decreasing in k_i. Following Lemma 1 part (*iii*), we already know that the price offers must be non-increasing in capacity. The reason why they have to be strictly decreasing follows from standard Bertrand arguments: equilibrium bid functions cannot contain flat regions, as firms would otherwise have incentives to slightly undercut those prices. This property allows us to invert $b_j(k_j)$ to write the expected profits of firm i when firm j bids according to a candidate symmetric equilibrium, as follows

$$\pi_i(b_i; k_i, b_j(k_j)) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (b_i - c)(\theta - k_j)g(k_j)dk_j.$$

When $k_j < b_j^{-1}(b_i)$, firm i has the low bid and hence sells up to capacity at the price set by firm j. Otherwise, firm i serves the residual demand and sets the market price at b_i.

In the remaining case, $\underline{k} < \frac{\theta}{2}$, the capacity of both firms might not always be enough to cover total demand. In particular, if $k_i < \theta - \underline{k}$, with probability $G(\theta - k_i)$ the rival has a capacity low enough so that they cannot jointly cover total demand. In this case since both firms produce at capacity regardless of their price offers, the quantity effect does not operate and the optimal price offer is flat as a function of capacity.[13]

Maximizing profits with respect to b_i and applying symmetry we can characterize the optimal bid at a symmetric equilibrium.

Proposition 2. *Under certain pivotality, there exists a unique symmetric Bayesian Nash equilibrium. Each firm $i = 1, 2$ offers all its capacity, $q^*(k_i) = k_i$, at a price given by*

$$b^*(k_i) = c + (P - c)\exp\left(-\omega(k_i)\right), \tag{1}$$

for $k_i \geq \hat{k} \equiv \max\left\{\frac{\theta}{2}, \underline{k}\right\}$ where

$$\omega(k_i) = \int_{\hat{k}}^{k_i} \frac{(2k - \theta)g(k)}{\int_{k}^{\bar{k}} (\theta - k_j)g(k_j)dk_j} dk,$$

[13]The expression for the profits in this case is provided in equation (4) in the Appendix.

and $b^*(k_i) = P$ *otherwise.*

First assume that $\underline{k} \geq \frac{\theta}{2}$, in which case equation (1) characterizes the optimal price offer for all capacity realizations. In this case, the optimal price offer adds a markup above marginal costs that is strictly decreasing in k_i. In order to provide some intuition, it is useful to implicitly re-write the optimal price offer as follows

$$-\frac{b'^*(k_i)}{b^*(k_i) - c} = \omega'(k_i) = \frac{(2k_i - \theta)g(k_i)}{\int_{k_i}^{\bar{k}} (\theta - k_j)g(k_j)dk_j} \tag{2}$$

This equation captures the incentives to marginally reduce the bid, which in turn reflect the trade-off between the *quantity effect* and the *price effect*, as captured by the ratio on the right-hand side of the above equation.

The price effect (on the denominator), or price loss from marginally reducing the bid, is relevant only when the firm is setting the market price, i.e., when the rival firm's capacity is above k_i. In this case, reducing the bid implies that the firm keeps on selling the expected residual demand, $\int_{k_i}^{\bar{k}} (\theta - k_j)g(k_j)dk_j$, but at a lower market price. The quantity effect (on the numerator), or output gain from marginally reducing the firm's bid, is relevant only when the two firms tie in prices, i.e., when both firms have the same capacity k_i, an event that occurs with probability $g(k_i)$. In this case, reducing the bid implies that the firm sells all its capacity rather than just the residual demand, i.e. its output jumps up by $k_i - (\theta - k_i) = 2k_i - \theta$.

The quantity effect is weighted by two forces as captured by the left-hand side of equation (2). On the one hand, the quantity effect is more relevant when the rival's price offer is flatter, since a given reduction in the firm's bid makes it more likely that the firm will sell at capacity. On the other hand, the quantity effect is less relevant when the mark-up on the increased sales is smaller.

Consider now the case in which $\underline{k} < \frac{\theta}{2}$. The equilibrium strategy calls firms to bid at P whenever their realized capacities are at or below $\theta/2$. To understand this result, assume that firm i's realized capacity is exactly equal to $\theta/2$. If the rival's capacity was at or below $\theta/2$, reducing the price below P would not allow firm i to increase its profits as both firms would sell all capacity at P regardless of their price offers. In contrast, if the rival's capacity was above $\theta/2$, firm i would serve the residual demand at its own price. Hence, it would be optimal for firm i to raise the market price up to P, as stated in the Proposition. A similar reasoning implies that bidding at P is optimal when the

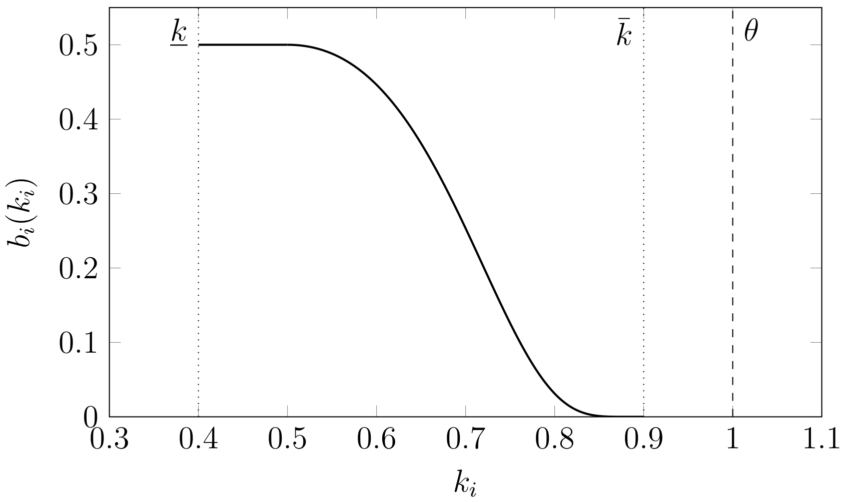

Figure 1: Equilibrium bids when $k_i \sim U[0.4, 0.9]$, with $\theta = 1$, $c = 0$, and $P = 0.5$.

firm's capacity is below $\theta/2$.

Overall, the optimal bid starts at P for the lowest possible capacity realization and ends at c for the largest one. When $k_i = \underline{k}$, firm j is bigger by construction, so firm i faces the residual demand and sets the market price with probability one. A price offer below P could never be part of an equilibrium as firm i could sell the same output at a higher price by bidding at P. When $k_i = \bar{k}$, firm j is smaller by construction, so firm i never sets the market price. Hence, the firm's bid has no impact on the price and only the quantity effect matters. Therefore, a price offer above c could never be part of an equilibrium as firm i could expect to sell more output at the same price by bidding at c.

Figure 1 depicts the equilibrium price offer in the case where $\underline{k} < \frac{\theta}{2}$. When $\underline{k} \geq \frac{\theta}{2}$, the equilibrium does not contain a flat region.

Finally, given the equilibrium bidding behavior, each firm's expected profits are equal to the minmax, $(P - c)(\theta - E(k))$, when $k_i < \hat{\underline{k}}$, and strictly higher otherwise. The reason is simple. A firm can always mimic a smaller capacity firm by withholding output and replicating its bid. The fact that firms prefer to offer all their capacity means that larger firms make higher profits in equilibrium. Since the minmax does not depend on the firm's capacity, in equilibrium a firm's expected profits exceed the minmax. As a result, expected equilibrium market prices are higher than when firms just obtain their minmax.

3.2 Uncertain Pivotality

We now analyze the case in which firms are non-pivotal with some probability, $\overline{k} \geq \theta$. If a firm with $k_i > \theta$ offers to produce up to capacity, it will serve total the demand at its own price when the rival bid's is higher. Interestingly, this makes withholding optimal, in contrast with the case with certain pivotality. Indeed, a firm whose capacity exceeds total demand wants to behave *as if* its capacity was just slightly below θ. This allows the firm, for a given price offer, to sell almost the same quantity while increasing the probability that the rival sets a higher market price. Essentially, this is a *Cournot strategy* that arises endogenously in a game in which firms are allowed to choose both prices and quantities.

Because all firms offer quantities below θ, *in equilibrium*, they are certain to be pivotal. This brings us back to the equilibrium characterization in the previous section.

Proposition 3. *The equilibria characterized in Proposition 1 and 2 continue to exist under uncertain pivotality with* $q_i^*(k_i) = \theta - \varepsilon$*, where* $\varepsilon \to 0$ *for all* $k_i \geq \theta$*, and with* $G(k_i)$ *now adjusted to* $G(q_i^*(k_i))$*,* $i = 1, 2$*.*

When firms play the symmetric equilibrium, as the probability that capacity exceeds θ goes up (e.g. because the distribution function puts more weight on higher capacity values or because the whole capacity interval shifts up), firms are more likely to bid at c. In turn, this implies that the expected market price smoothly converges towards marginal costs. Eventually, as $\underline{k}$ reaches θ, the market becomes competitive at all times as the equilibrium bid function puts all the mass at marginal costs.

4 What is the impact of private information?

In order to understand the impact of private information, we now characterize equilibrium outcomes under two benchmarks in which there is no private information: either because capacities are publicly known or because they are unknown to both firms prior to bidding. For simplicity, we compare outcomes at the symmetric equilibria.[14]

First, suppose that firms observe realized capacities prior to submitting their bids. Accordingly, firms' bids can be conditioned on realized capacities. The following Lemma characterizes the level of profits that can be sustained by symmetric equilibria, either in pure on in mixed strategies.

[14]Comparing the asymmetric equilibria would be uninteresting as the asymmetric equilibria characterized in Proposition 1 can be sustained in all three information treatments.

Lemma 2. *If realized capacities (k_i, k_j) are publicly known prior to bidding:*

(i) All symmetric pure-strategy equilibria result in joint profits $(P - c)\theta$.

(ii) There exists a continuum of symmetric mixed-strategy equilibria resulting in expected joint profits bounded from above by $(P - c)\theta$ and from below by $2(P - c)(\theta - E[k])$.

The game with known capacities allows firms to sustain equilibria in which all their output is sold at P. Just as we described in Section 3, these equilibria are characterized by asymmetric bidding, with one firm bidding at P while the rival bids low enough so as to make undercutting unprofitable. The main difference between this game and the one in which capacities are private information, is that firms can use realized capacities to overcome their coordination problem. For instance, they can now share profits symmetrically by designating the small firm to bid low and the large firm to bid high.[15] Therefore, when realized capacities are publicly known, there exist symmetric equilibria that allow firms to obtain maximum profits.

The game with known capacities also gives rise to a continuum of mixed-strategy equilibria, with firms randomizing their bids between P and c. On the one hand, equilibrium profits are the lowest when firms do not condition their bidding on realized capacities. In this case, the mixed strategy equilibrium involves no firm playing a mass point at P. On the other hand, equilibrium profits are the highest when one of the two firms plays P with probability almost equal to one, thus converging to the pure-strategy equilibrium characterized above. In turn, this shows that all the mixed strategy equilibria are Pareto-dominated by the pure-strategy equilibrium.

Now, suppose that firms do not observe realized capacities prior to bidding. In this case, we assume that the best they can do is to offer to supply their expected capacity $E[k]$ at a price which is endogenously determined in equilibrium.[16] The following Lemma shows that the unique symmetric equilibrium involves mixed strategy pricing.

Lemma 3. *If realized capacities (k_i, k_j) are not known prior to bidding, the unique symmetric equilibrium involves mixed strategies, with firms randomizing their bids in (c, P).*

[15] The only exception is when firms' realized capacities are equal. If firms can condition on an external correlation device, they can still share profits symmetrically. Otherwise, the unique symmetric equilibrium involves mixed-strategy pricing, with firms making (weakly) lower expected profits. However, one can disregard this event as it occurs with probability zero.

[16] The implicit assumption is that firms can buy at the market price the difference between their commited output and their actual realized capacity.

Expected equilibrium joint profits are $2(P - c)(\theta - E[k])$.

Since bids cannot be conditioned on capacities, in a symmetric equilibrium, both firms would have to either charge equal prices or use the same mixed strategy to randomize their prices. Since the former is ruled out by standard Bertrand arguments, the only symmetric equilibrium involves mixed strategies. Since at P the rival firm is bidding below with probability one, and since all the prices in the equilibrium support yield equal expected profits, it follows that at the unique symmetric equilibrium, each firm makes expected profits equal to $(P - c)(\theta - E[k])$.

We are now ready to rank expected prices at the symmetric equilibria across all three information treatments.

Proposition 4. *The comparison of the symmetric equilibria in the games in which capacities are unknown, private information, or publicly known, shows that:*

(i) The lowest expected prices are obtained with unknown capacities.

(ii) The highest expected prices are obtained with publicly known capacities.

The proposition shows that the more information firms have regarding realized capacities, the higher the expected prices they can obtain at a symmetric equilibrium. As already argued, in a uniform auction, firms can achieve maximum profits through asymmetric bidding but this faces them with a coordination problem as they both want to be the low bidder. Observing each others' capacities serves as a perfect coordination device, that allows them to symmetrically share maximum profits. When capacities are private information, each firm's own capacity realization plays the role of a coordination device, albeit weaker than when both capacities are known: large (small) firms find in their own interest to bid low (high), but not as low (high) as if they knew with certainty that the rival firm is bidding above (below). When capacities are unknown to both firms, such a coordination device is absent as all firms face fully symmetric incentives. This explains why private information leads to higher prices than in the case with unknown capacities, which are nevertheless lower than when capacities are publicly known.

5 Extensions

In this section we consider three extensions to the Baseline case. We allow for affiliated capacities, we characterize and compare the equilibria with the one that would arise if

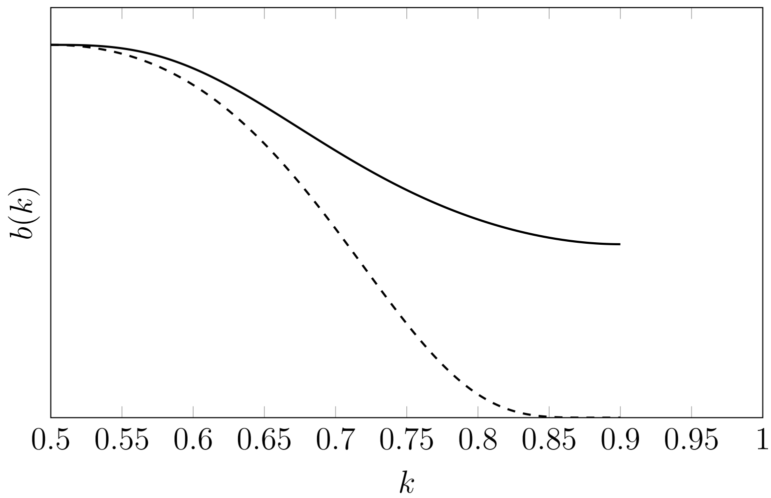

Figure 2: Comparison between the optimal bid function under the uniform auction (dashed line) and the discriminatory auction (solid line). Parameter values: $k_i \sim U[0.5, 0.9]$, $c = 0$, $P = 0.5$ and $\theta = 1$.

firms were paid according to their own bids (discriminatory auction), and we extend the equilibrium to allow for an arbitrary number of firms.

5.1 Discriminatory Auctions

In this section we characterize equilibrium bidding under the discriminatory auction in which each firm is paid according to its own bid.

Proposition 5. *In the discriminatory auction, there exists a unique symmetric Pure Strategy Equilibrium: each firm $i = 1, 2$ chooses a bid that is strictly decreasing in k_i according to the function*

$$b_d^*(k_i; \underline{k}, \bar{k}) = c + (P - c)\exp\left(-\omega_d\left(k_i\right)\right),$$

where

$$\omega_d\left(k_i\right) = \int_{\underline{k}}^{k_i} \frac{(2k - \theta)g(k)}{kG(k) + \int_k^{\bar{k}}(\theta - k_j)g(k_j)dk_j} dk,$$

with $b_d^*(\underline{k}) = P$.

Firms now face stronger incentives to increase their bids. Unlike the uniform auction, a higher bid under the discriminatory auction always translates into a higher price, even when the firm bids below the rival. In other words, the price effect is always stronger. In particular, in this case for $k = \overline{k}$ the optimal bid is strictly above marginal cost, unlike in the uniform auction.

Since firms submit higher bids under the discriminatory auction, it follows that the highest accepted bid is higher than under the uniform auction. However, this does not necessarily imply that firms make higher profits under the discriminatory auction. The reason is that they receive their own bid rather than the highest.

When costs rather than capacities are private information Holmberg and Wolak (2018) provide a *revenue equivalence* result between the two auction formats. We now show that this is not the case when there is private information regarding the capacity.

Lemma 4. *The uniform and the discriminatory auction are not revenue equivalent.*

Suppose that the private information is on cost, as in Holmberg and Wolak (2018). In the symmetric equilibrium, small changes in costs affect prices, but due to the Envelope Theorem, this effect on profits is zero. Furthermore, contingent on having either the low or the high cost, the quantity is independent of the private information. Thus, to the extent that the probability that two firms have the same cost is zero, the cost ranking and quantities are not affected. Hence, revenue stays unchanged. The only effect of private information on profits is through changes in the costs of production, but this effect is the same across auction formats.

Matters are different when private information is on capacities, as changes in k affect firms' profits through their profit margin. In turn, since this profit margin is not the same across the two auctions, the impact of private information on revenues differs depending on the auction format in place (Lemma 4). Beyond this, it is difficult to arrive at general conclusions regarding the revenue ranking across auctions. However, numerical results suggest that the less competitive bidding under the discriminatory auction dominates over the fact that the uniform auction pays the highest bid to all units. Thus, at leasts when capacities are distributed according to a Beta distribution (Figure 2), the discriminatoy auction results in higher payments to firms.

5.2 N Firms

In this section we extend our equilibrium analysis to allow for an arbitrary number of firms, $N \geq 2$. This increase in the number of firms allows us to carry out two kinds of exercises: the effects of entry and the effects of changes in the market structure. Regarding the first, an increase in the number of firms brings in new capacity into the

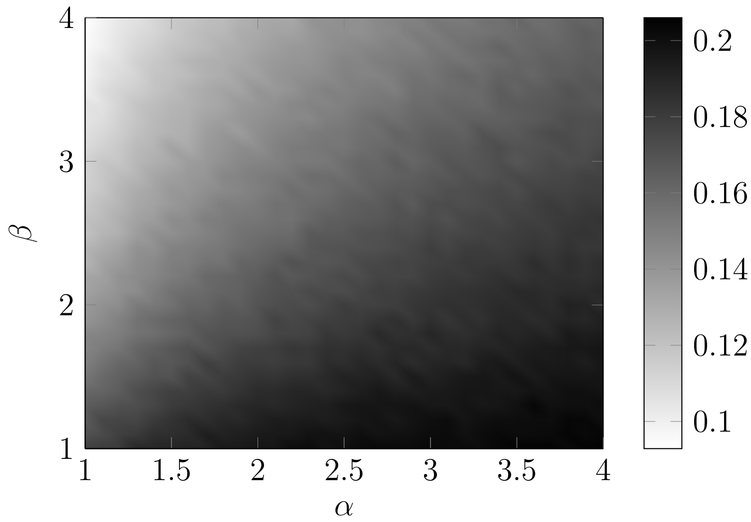

Figure 3: Difference in equilibrium prices between the discriminatory and the uniform auction when $k \sim Beta(\alpha, \beta)$ in the support $[0.5, 0.7]$.

market. In the second case, a fixed distribution of capacity is allocated among a different number of firms.

For this purpose, we need some extra notation. Let k_j be the minimum capacity among those of firm i's rivals, i.e., $k_j = \min\{..., k_{i-1}, k_{i+1}\}$. Its cumulative distribution function and density are

$$\begin{aligned}\Phi(k_j) &= 1 - (1 - G(k_j))^{N-1} \\ \varphi(k_j) &= (N-1)\, g(k_j)\,(1 - G(k_j))^{N-2}\end{aligned}$$

We are going to focus on the case in which firms are pivotal. As a result, all firms but the one with the highest bid (and the smallest capacity) will sell at capacity. This means that from the point of view of firm i the N-firm problem can be reinterpreted as if it was only facing the smallest competitor.

The following result extends Proposition 2 to N firms.

Proposition 6. *Assume $\frac{\theta}{N} < \underline{k} < \bar{k} < \frac{\theta}{N-1}$. There exists a unique symmetric Pure Strategy Equilibrium: each firm $i = 1, ..., N$ chooses a bid that is strictly decreasing in k_i according to the function*

$$b^*(k_i) = c + (P - c)\exp\left(-\omega\left(k_i; \underline{k}, \bar{k}, N\right)\right),$$

where

$$\omega\left(k_i; \underline{k}, \bar{k}, N\right) = \int_{\underline{k}}^{k_i} \frac{\left(2k + \int_{k}^{\bar{k}} (N-2)\, kg(k)dk - \theta\right)\varphi(k)}{\int_{k}^{\bar{k}}\left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k)dk\right)\varphi(k_j)dk_j} dk,$$

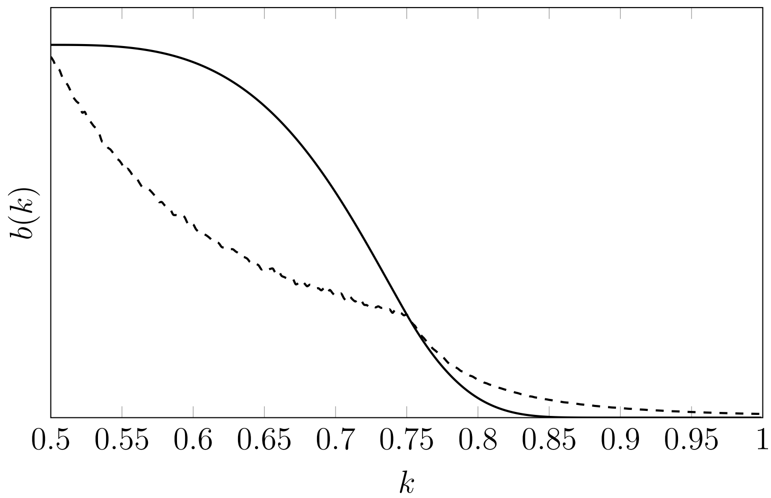

Figure 4: Comparison between $N = 2$ and $N = 4$. The dashed line corresponds to the maximum of the bids of two firms when $N = 4$ and their capacities are uniformly distributed, $k_i \sim U[0.25, 0.5]$. The solid line is the equilibrium bid of a firm when $N = 2$ and each firm has two plants that have the previous capacity distribution. The rest of parameters are $c = 0$, $P = 0.5$ and $\theta = 1$.

with $b^*(\underline{k}) = P$ *and* $b^*(\bar{k}) = c$.

As compared to the solution in the duopoly case, N enhances the quantity effect because the loss in production from marginally increasing the bid is higher the more competitors there are in the market. At the same time the price effect is reduced because the firm only benefits from increasing the bid through the residual demand, which is now smaller. Both effects imply that the optimal bid goes down with N and so does the equilibrium price.

The model also allows us to understand what is the effect of changing market structure for a given total capacity distribution. Suppose that N is the number of plants and consider a situation in which we move from single-plant firms to a fewer number of firms owning multiple plants. In this case there are two different effects. As the number of firms decreases and each becomes bigger, they tend to behave less competitively and increase their bids. However, a second effect arises, as a decrease in the number of firms changes the distribution of the rivals' capacity. The capacity of a firm owning multiple plants tends to take an intermediate value since smaller realizations of one plant are compensated with larger realizations of another. Instead, in the single plant case what matters is the realization of the smallest capacity which is smaller than the average capacity. Firms face multiple plant competitors need to behave more aggressively as they expect them to submit lower bids.

Figure 4 provides an example of how the previous two forces shape the equilibrium with $N = 2$ as compared to the case with $N = 4$. The dashed line corresponds to the case with $N = 4$ and capacities are uniformly distributed between $U[0.25, 0.5]$, as described at the beginning of this section. The dashed line displays, $\hat{b}(k)$, the average highest bid of the two of these firms given a total capacity k defined as

$$\hat{b}(k) = \int_{2\underline{k}}^{2\overline{k}} \max[b(x), b(k-x)] g(x) g(k-x) dx.$$

The solid line is the equilibrium bid of a firm that owns two of the plants and competes with another firm that owns the other two. Hence, the difference between both cases is only due the ownership of the plants and not the distribution of the capacity available. This example illustrates how an increase in the number of firms translates in lower bids only when capacity is low, whereas the reverse is true when firms have a large aggregate capacity. The equilibrium price is lower when N increases.

5.3 Capacity Withholding is not Allowed

In Section 3.2 we showed that the symmetric equilibrium involves capacity withholding whenever $k_i \geq \theta$. However, in certain contexts, either because of market design, regulatory intervention, or technical constraints, it is plausible that firms might be constrained from withholding capacity. Would the equilibrium characterize in Proposition xx remain the same if capacity withholding was not possible?

In this section, we impose $q_i = k_i$ and characterize the corresponding pricing equilibrium. Clearly, this constraint is not binding in the baseline case, as firms never find it optimal to withhold capacity even if allowed (Lema 1). In contrast, banning capacity withholding in cases with $\bar{k} \geq \theta$ has a dramatic impact on bidding incentives. First, conditionally on having the high bid, a firm produces nothing if its rival's capacity is at or above θ. Second, the bid of a firm whose capacity exceeds θ is payoff relevant even if it has the low bid, as in this case the firm serves total demand at its own bid. The former effect intensifies competition, whereas the second induces firms to charge higher prices.

To describe more clearly how the equilibrium bids change when capacity withholding is not allowed when $\bar{k} \geq \theta$, we split the equilibrium characterization in two cases, depending on whether the firm's realized capacity is above or below θ.

Lemma 5. *Assume $\bar{k} \geq \theta$. There does not exist a Bayesian Nash Equilibrium in pure strategies. Furthermore, in any equilibrium, for $k_i \geq \theta$, firm i randomizes its bid in a support $\left[\underline{b}, \bar{b}\right]$ independently of k_i, where $\underline{b} > c$ and $\bar{b} < P$.*

If $k_i \geq \theta$, firm i is never capacity constrained. Since its expected profits do not depend on its realized capacity, its optimal bid at a candidate pure-strategy equilibrium is the same for all capacity realizations above θ. However, this would give rise to ties with positive probability and it is thus ruled out by standard Bertrand-Edgeworth arguments. More specifically, ties cannot be part of an equilibrium as firms would be better off by slightly undercutting any price above marginal costs in order to sell more output with only (if any) a slight reduction in the price. Furthermore, tying at marginal cost is ruled out as firms could make positive profits by selling the expected residual demand at P. Thus, at a symmetric equilibrium, firms must randomize their bids for all capacity realizations above θ.

The previous argument implies, of course, that the symmetric Bayesian Nash Equilibrium of the game must be in mixed strategies, as at least for $k_i \geq \theta$ firms randomize their bids. One distinctive feature of this equilibrium is that the upper bound of the price support does not go all the way up to P. The reason is that when firms have a capacity $k_i \geq \theta$, they face a downward sloping residual demand, induced by the downward sloping bid function of the rival when its capacity realization is below θ. We now turn to characterizing the equilibrium in this case. The next proposition describes the optimal bid, and Figure 5 illustrates it.

Proposition 7. *Assume $\bar{k} \geq \theta$. In the unique symmetric Bayesian Nash Equilibrium, if $k_i < \theta$, the optimal bid for firm i is*

(i) $b^*(k_i; \underline{k}, \theta)$ *for* $k_i \in \left[\underline{k}, \hat{k}\right]$ *and* $k_i \in \left[\tilde{k}, \theta\right)$*, as defined in (1).*

(ii) $\hat{b}(k_i; \underline{k}, \theta)$ *for* $k_i \in \left(\hat{k}, \tilde{k}\right)$*, strictly decreasing in* k_i *and strictly lower than* $b^*(k_i; \underline{k}, \theta)$*.*

(iii) $b_i \sim F(b_i)$ *with density* $f(b_i)$ *in a support* $[\underline{b}, \bar{b}]$*.*

The thresholds $\hat{k}$ and $\tilde{k}$ are implicitly defined as $b^*(\hat{k}; \underline{k}, \theta) = \bar{b}$ and $b^*(\tilde{k}; \underline{k}, \theta) = \underline{b}$, where $\underline{b}$ and $\bar{b}$ are defined in Lemma 5.

The optimal bid when k_i belongs to either $\left[\underline{k}, \hat{k}\right]$ or $\left[\tilde{k}, \theta\right)$ is similar to the one in the Baseline case. The sole difference is that, from firm i's point of view, firm j's relevant

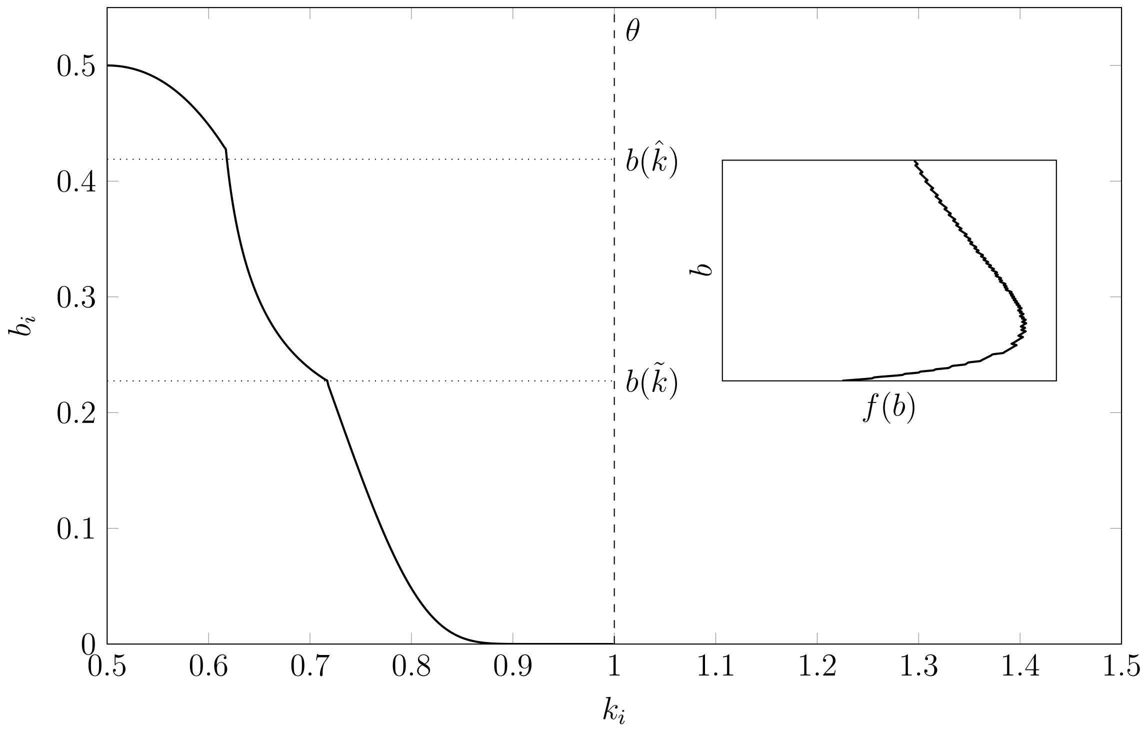

Figure 5: Equilibrium bids and probability density when $k_i \sim U[0.5, 1.1]$, with $\theta = 1$, $c = 0$, and $P = 0.5$.

capacities now range from $\underline{k}$ to θ given that firm i's profits are constant when $k_j \geq \theta$. In particular, for such capacities, firm j randomizes its bid in the support $(\underline{b}, \bar{b})$ and, thus, prices are limited above by $b^*(\hat{k}; \underline{k}, \theta) = \bar{b}$ and below by $b^*(\tilde{k}, \underline{k}, \theta) = \underline{b}$. Hence, if $k_j \geq \theta$, firm i does not produce anything if k_i belongs to $\left[\underline{k}, \hat{k}\right]$, while firm i sells at capacity at the price set by firm j if k_i belongs to $\left[\tilde{k}, \theta\right)$. It follows that firm i's marginal profits are zero whenever $k_j \geq \theta$, and hence its bidding incentives are equal to those in the base line model with $\bar{k}$ arbitrarily close to θ.

This result is in contrast to the case where $k_i \in \left[\hat{k}, \tilde{k}\right)$. For these realizations, firm i might have the low or the high bid depending on the bid chosen by firm j when playing its mixed strategy. In particular, firm i's incentives to bid low are now stronger as compared to the Baseline case, given that by reducing the b it can outbid the rival for a larger range of capacity realizations, including $k_j \geq \theta$.

The previous equilibrium bidding function is not monotonic in k_i, particularly around θ. The optimal bid converges to c to the left of θ as the firm is certain to be selling at capacity at the price set by the rival. In contrast, the bid jumps above c when $k_i > \theta$, as

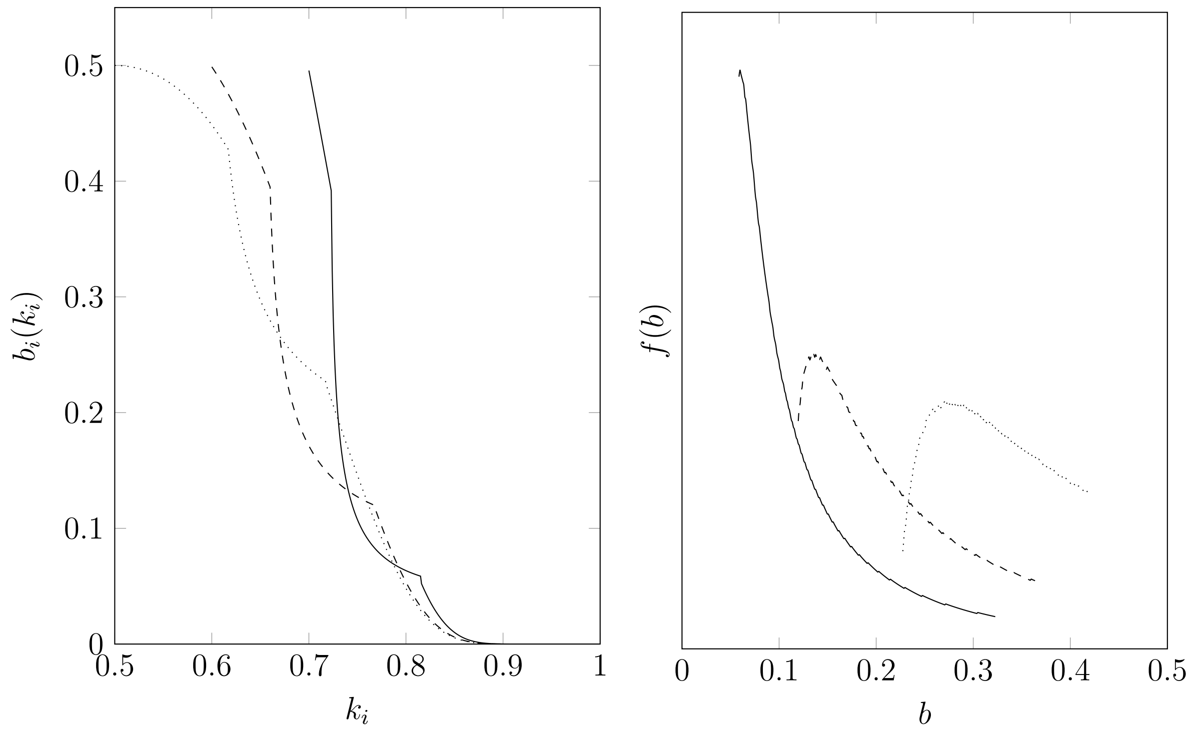

Figure 6: Equilibrium bids and probability density when $k_i \sim U[0.5, 1.1]$ (dotted), $k_i \sim U[0.6, 1.2]$ (dashed) and $k_i \sim U[0.7, 1.3]$ (solid), with $\theta = 1$, $c = 0$, and $P = 0.5$.

the firm is aware that its bid is always payoff relevant.[17]

Allowing $\bar{k}$ to increase above θ shows how the equilibrium bid schedules approach the competitive outcome. Suppose that capacities are uniformly distributed in $\left[\underline{k}, \bar{k}\right]$, and consider moving the whole capacity support to the right. For capacity realizations above θ, the equilibrium mixed strategy puts increasingly more weight on the lower bound of the price support, which converges towards c. In turn, the range $\left(\hat{k}, \tilde{k}\right)$ widens up. This process continues until $\underline{k}$ reaches θ, in which case the equilibrium bid functions become flat at marginal costs. Figure 6 depicts this process of convergence towards the competitive outcome.

6 Concluding Remarks

In this paper we have analyzed equilibrium bidding in multi-unit auctions when bidders' production capacities are private information. Furthermore, we have allowed

[17]We do not allow firms to withhold capacity, which might become a concern, particularly, when $k_i > \theta$. It is easy to see that firms are indifferent between offering any capacity above or at θ. With capacity withholding over this range, firms would have a distribution with a mass point at θ but it would not affect the results in any way. Matters might be different if firms have incentives to withhold capacity and offer a capacity slightly below θ at marginal cost. In this case, the firm would sell more (it would produce almost total demand with probability one) but possibly at a lower price.

changes in capacity to shape the bidding functions, both through changes in the prices and the quantities offered by firms. This is unlike other papers in the literature which typically assume that the private information is on costs (or bidders' valuations) and which, with few exceptions, do not allow bidders to act on both the price and quantity dimensions. We have shown that the nature of private information and the strategies available to firms have a key impact on equilibrium behaviour.

We have motivated the model in the context of electricity markets, in which renewable technologies have constant and zero marginal cost but their capacity is subject to fluctuations that are difficult to forecast by competitors. However, its approach and insights are also applicable to other multi-unit auction settings in which the quantity dimension (e.g. bidders' maximum willigness to buy or sell) need not be common knowledge, as typically assumed. Treasury auctions and the auctions for emissions permits are two examples, among others.

Understanding competition among renewable producers is key to predict the performance of future electricity markets as conventional technologies are being substituted by renewables. The profitably of current investments critically depends on the future prices at which firms expect to sell their renewable generation. Whereas in the past firms were often paid according to fixed prices (i.e., the so-called feed-in tariffs), the European Commission (in line with other jurisdictions) is now advocating to expose renewables to the time-varying prices set in wholesale electricity markets. Thus, to assess whether market revenues will be enough to induce the desired investments, regulators need to understand firms' price setting incentives if such investments indeed take place. Concerns over the profitability of renewable investments, have led regulators in several countries to pay them some sort of market premia that are set through auctions. Likewise, firms need to understand the future market performance in order to determine how to bid in the auctions for the new investments.

The standard approach to assess these issues has been to assume that renewable producers offer their output in the wholesale market at marginal costs. However, this approach tends to overestimate the price-depressing effect of renewables, therefore underestimating firms' investment incentives. As we have shown in this paper, since strategic producers add a markup to their marginal costs, the price impact of renewables is smoothed out, thus making investments more profitable than would otherwise be assumed under perfect competition. Likewise, the assumption of either competitive or strategic bidding also has implications for the expected price volatility, ultimately affecting investment incentives of risk averse firms. While in this paper we do not model investment decisions, analyzing equilibrium market outcomes for given capacities is a necessary first step to characterize equilibrium investment in the future.

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A Proofs

Proof of Lemma 1: For part (i) of the lemma, suppose that firm j chooses a bid according to a distribution $F_j(b_j, q_j|k_j)$. Profits for firm i can be written as

$$\pi_i(b_i, q_i, F_j|k_i) = \int_{k_j}\int_{(b,q)} [(b-c)q_i \Pr(b_i \leq b)+(b_i-c)\min\{q_i, \theta-q\} \Pr(b_i > b)]dF_j(b, q|k_j)g(k_j)dk_j.$$

The above equation is increasing in q_i, indicating that the firm maximizes profits by choosing $q_i^*(k_i) = k_i$. In what follows we simplify the notation and we consider the randomization on prices only, $F_j(b_j|k_j)$.

For part (ii), towards a contradiction, consider two bids b_i and $b_i' > b_i$ for which firm i randomizes. Then, it must be that firm i is indifferent and, thus,

$$\pi_i(b_i', F_j|k_i) - \pi_i(b_i, F_j|k_i) = \int_{k_j}\int_b \{(b-c)k_i [\Pr(b_i' < b) - \Pr(b_i < b)] \tag{3}$$
$$+ \min\{k_i, \theta - k_j\} [(b_i' - c) \Pr(b_i' > b) - (b_i - c) \Pr(b_i > b)]\} dF_j(b)g(k_j)dk_j = 0.$$

Since j cannot condition its strategy on k_i, then $F_j(b_j|k_j)$ must be such that the previous expression holds for all k_i. Hence, either $b_j(k_j) = c$ for all k_j, in which case $F_j(b_j|k_j)$ would be a degenerate mixed strategy, or

$$\int_{k_j}\int_b [\Pr(b_i' < b) - \Pr(b_i < b)] dF_j(b|k_j)g(k_j)dk_j = 0.$$

By Bertrand arguments, F_j cannot contain gaps in the support and therefore this cannot occur. Given that the second part of equation (3) does not depend on k_i this leads to a contradiction.

Regarding part (iii) of the lemma, using the previous result we can focus on firm j choosing a pure strategy. As a result, it is enough to show that the function $\pi_i(b_i, b_j(k_j), k_i)$ has non-increasing differences in b_i and k_j. Using the previous expression and taking the derivative with respect to k_i we have

$$\frac{\partial [\pi_i(b_i', F_j|k_i) - \pi_i(b_i, F_j|k_i)]}{\partial k_i} = \int_{k_j}\int_b (b-c)k_i [\Pr(b_i' < b) - \Pr(b_i < b)] dF_j(b)g(k_j)dk_j \leq 0.$$

In words, larger firms gain (weakly) less from increasing their bids. Hence, the optimal bid function is non-increasing in k_i. □

Proof of Proposition 1: See Fabra et al. (2006).

Proof of Proposition 2: Consider equilibria with the following shape:

$$\tilde{b}_j(k_j) = \begin{cases} P & \text{if} \quad k_j < \underline{\hat{k}}, \\ b_j(k_j) & \text{if} \quad k_j \geq \underline{\hat{k}}. \end{cases}$$

Notice that $b_j(k_j)$ is strictly decreasing k_j in the symmetric equilibrium. Towards a contradiction, suppose that this is not the case. From Lemma 1, this implies that there is a region $[k_a, k_b]$ such that both firms choose the same bid, $b_i = b_j$. At least one of the firms would not sell all its capacity for some capacity realizations. The standard Bertrand argument implies that this cannot be part of an equilibrium, as the firm that sells below capacity could increase its profits by slightly undercutting the competitor in order to sell at capacity.

We start by showing that $\hat{\underline{k}} = \max\left\{\frac{\theta}{2}, \underline{k}\right\}$. First, it cannot be lower than $\theta/2$. Argue by contradiction and suppose $\hat{\underline{k}} < \theta/2$ or, rearranging, $\hat{\underline{k}} < \theta - \hat{\underline{k}}$. This cannot part of a symmetric equilibrium since for $k_i \in \left(\hat{\underline{k}}, \theta - \hat{\underline{k}}\right)$ aggregate capacity is not enough to cover total demand, implying that the best response of firm i includes P, a contradiction.

We now show that $\hat{\underline{k}}$ cannot be greater than $\theta/2$ if $\theta/2 > \underline{k}$. Argue by contradiction and suppose $\tilde{k} > \theta/2$. This cannot be part of a symmetric equilibrium since for $k_i \in \left(\theta/2, \tilde{k}\right)$ firm i would be better off undercutting P. If the rival firm's capacity falls in the interval $k_j \in \left(\theta - k_i, \tilde{k}\right)$, the two firms would tie at P and each one would sell below capacity. By slightly undercutting P, expected profits would increase by $(P - c)\left(G\left(\tilde{k}\right) - G(\theta - k_i)\right)\left(k_i - \frac{\theta}{2}\right)$. It follows that we must have $\tilde{k} = \theta/2$.

In turn, note that this implies that we only have ties at P when both firms have capacity below $\theta/2$ so that aggregate capacity is not enough to cover total demand. Otherwise, when at least one firm is selling below capacity, ties at P can never occur.

Expected profits are

$$\pi_i(b_i, b_j, k_i) = (P - c)\, k_i G(\theta - k_i) + \int_{\max(\theta - k_i, \underline{k})}^{b_j^{-1}(b_i)} (b_j(k_j) - c) k_i g(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (b_i - c)(\theta - k_j) g(k_j) dk_j, \tag{4}$$

and the first-order condition that characterizes the optimal bid of firm i can be written as

$$\frac{\partial \pi_i}{\partial b_i} = b_j^{-1\prime}(b_i) g(b_j^{-1}(b_i))(b_i - c)(k_i + b_j^{-1}(b_i) - \theta) + \int_{b_j^{-1}(b_i)}^{\bar{k}} (\theta - k_j) g(k_j) dk_j = 0. \tag{5}$$

Furthermore, around the candidate equilibrium, the profit function is strictly concave.

Under symmetry, $b_j(k) = b_i(k)$. Accordingly, we can rewrite the expression as

$$\frac{1}{b_i'(k_i)} g(k_i)(b_i(k_i) - c)(2k_i - \theta) + \int_{k_i}^{\bar{k}} (\theta - k_j) g(k_j) dk_j = 0. \tag{6}$$

If $k_i \leq \theta/2$, the first term in the first order condition (6) is always positive, hence it is optimal to bid at P. If $k_i > \theta/2$, the first term of the first order condition (6) is negative and the second term is positive, so an interior solution exists. Indeed, the first order condition takes the form

$$b_i'(k_i) + a(k_i)b_i(k_i) = ca(k_i),$$

where

$$a(k) = \frac{(2k-\theta)g(k)}{\int_k^{\bar{k}}(\theta - k_j)g(k_j)dk_j}\cdot \tag{7}$$

If we multiply both sides by $e^{\int_{\underline{\hat{k}}}^{k} a(s)ds}$ and integrate from $\underline{\hat{k}}$ to k_i we obtain

$$\int_{\underline{\hat{k}}}^{k_i}\left(e^{\int_{\underline{\hat{k}}}^{k} a(s)ds}b_i'(k) + a(k)e^{\int_{\underline{\hat{k}}}^{k} a(s)ds}b_i(k)\right)dk_i = c\int_{\underline{\hat{k}}}^{k_i} a(k_i)e^{\int_{\underline{\hat{k}}}^{k} a(s)ds}dk_i.$$

We can now evaluate the integral as

$$e^{\int_{\underline{\hat{k}}}^{k} a(k)dk}b_i(k)\Big]_{\underline{\hat{k}}}^{k_i} = ce^{\int_{\underline{\hat{k}}}^{k} a(s)ds}\Big]_{\underline{\hat{k}}}^{k_i}.$$

This results in

$$e^{\int_{\underline{\hat{k}}}^{k_i} a(k)dk}b_i(k_i) - b_i(\underline{\hat{k}}) = ce^{\int_{\underline{\hat{k}}}^{k_i} a(k)dk} - c.$$

Solving for $b_i(k_i)$ we obtain

$$b_i(k_i) = c + Ae^{-\int_{\underline{\hat{k}}}^{k_i} a(k)dk} = c + Ae^{-\omega(k_i)},$$

where $A \equiv b_i(\underline{k}) - c$ and $\omega(k_i) \equiv \int_{\underline{\hat{k}}}^{k_i} a(k)dk$.

A necessary condition for this to constitute an equilibrium is that equilibrium profits are at or above the minmax, which the firm can obtain by bidding at P. Hence, a necessary and sufficient condition for equilibrium existence is that

$$\pi_i(b_i, b_j, k_i) \geq (P-c)\,k_iG(\theta - k_i) + \int_{\max(\theta-k_i,\underline{k})}^{\bar{k}}(P-c)(\theta-k_j)g(k_j)dk_j. \tag{8}$$

Hence, to rule out deviations to P, we now need to prove that minmax profits increase less in k_i as compared to equilibrium profits. First, suppose $k_i \leq \underline{\hat{k}}$. To satisfy condition (8), we must have $b_i(\underline{\hat{k}}) = P$, implying $A = P - C$. In this case, equilibrium profits are exactly equal to the minmax.

Second, suppose $\underline{\hat{k}} < \theta - \underline{k}$. The derivative of the minmax with respect to k_i is

$$(P-c)G(\theta - k_i),$$

whereas, from (4), the derivative of the profit function, using the Envelope Theorem, can be computed as

$$(P-c)G(\theta-k_i)+\int_{\theta-k_i}^{b_j^{-1}(b_i)}(b_j(k_j)-c)k_i g(k_j)dk_j.$$

Clearly, this derivative is greater than that of the minmax.

Last, suppose $k_i > \theta - \underline{k}$. The derivative of the minmax is

$$(P-c)\left(G\left(\theta-k_i\right)-g\left(\theta-k_i\right)k_i\right).$$

The derivative of profits is

$$(P-c)\left(G\left(\theta-k_i\right)-g\left(\theta-k_i\right)k_i\right)+\int_{\underline{k}}^{b_j^{-1}(b_i)}(b_j(k_j)-c)k_i g(k_j)dk_j.$$

Again, this derivative is greater than that of the minmax.

It follows that deviations to P are not profitable since equilibrium profits are always strictly greater than the minmax, except for $k_i \leq \theta/2$ when equilibrium profits are exactly equal to the minmax.

Finally, we need to verify that the candidate equilibrium, indeed, maximizes profits for each of the firms. From the first order condition in (5) we can compute the second derivative of the profit function of firm i, when firm j uses a bidding function $b_j(k_j)$ as

$$\frac{g(b_j^{-1}(b_i))}{b_j'\left(k_j\right)}\left(-\frac{b_j''\left(k_j\right)}{\left(b_j'\left(k_j\right)\right)^2}(b_i-c)(k_i+b_j^{-1}(b_i)-\theta)+\frac{1}{b_j'\left(k_j\right)}\frac{g'(b_j^{-1}(b_i))}{g(b_j^{-1}(b_i))}(b_i-c)(k_i+b_j^{-1}(b_i)-\theta)\right.$$
$$\left.+(k_i+b_j^{-1}(b_i)-\theta)+\frac{1}{b_j'\left(k_j\right)}(b_i-c)-(\theta-b_j^{-1}(b_i))\right).$$

Once we substitute the candidate equilibrium $b_i(k)=b_j(k)$ the previous expression becomes

$$\frac{\partial^2\pi_i}{\partial^2 b_i(k_i)}=\frac{g(k_i)}{b^{*\prime}\left(k_i\right)}\frac{1}{a\left(k_i\right)}<0.$$

□

Proof of Proposition 3: We want to show that there exists an equilibrium in which firm i behaves as in Proposition 2 for $k_i < \theta$, while it bids at marginal cost and offers $q_i = \lim_{\varepsilon\to 0}\left(\theta-\varepsilon\right)$ for all $k_i \geq \theta$. We only need to rule our deviations for $k_i \geq \theta$, as Proposition 2 already shows that deviations for $k_i < \theta$ are unprofitable. Suppose $k_i \geq \theta$, $q_i = \lim_{\varepsilon\to 0}\left(\theta-\varepsilon\right)$ and $b_i(k_i)=b_i \geq c$. At this candidate equilibrium, profits are

$$\pi_i(k_i>\theta;b_i,b_j(k_j))=\int_{\underline{k}}^{b_j^{-1}(b_i)}(b_j(k_j)-c)\theta g(k_j)dk_j+\int_{b_j^{-1}(b_i)}^{\overline{k}}(b_i-c)(\theta-k_j)g(k_j)dk_j. \quad (9)$$

Since profits are increasing in q_i for $k_i < \theta$ (Lemma 1) we do not need to check deviations to $q_i < \theta$. If the firm deviates to $q_i > \theta$, for any candidate price offer $b_i \geq c$, profits are

$$\pi_i(k_i > \theta; b_i, b_j(k_j)) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_i - c)\theta g(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}} (b_i - c)(\theta - k_j) g(k_j) dk_j.$$

Comparing the above equation with (9), shows that the deviation is unprofitable: the second term is the same as in equation (9), while the first term is now smaller since, over this range, $b_j(k_j) > b_i$.

Thus, it only remains to show that the firm does not want to choose a bid above c. We show this by proving that profits are decreasing in $b_i > c$, as this implies that profits are are maximum at c. Taking derivatives,

$$\frac{\partial \pi_i}{\partial b_i} = b_j^{-1\prime}(b_i) g(b_j^{-1}(b_i))(b_i - c) b_j^{-1}(b_i) + \int_{b_j^{-1}(b_i)}^{\overline{k}} (\theta - k_j) g(k_j) dk_j. \tag{10}$$

From Proposition 2, using the equilibrium price offer for firm j,

$$b^*(k_i) = c + (P - c)\exp\left(-\omega(k_i)\right),$$

we can write

$$\exp\left(-\omega(k_j)\right) = \exp\left(-\omega(b_j^{-1}(b_i))\right) = \frac{b_i - c}{P - c}$$

Using the implicit function Theorem,

$$\frac{dk_j}{db_i} = -\frac{\int_{k_j}^{\overline{k}} (\theta - k) g(k) dk}{(b_i - c)\,(2k_j - \theta) g(k_j)}$$

Plugging this into the first-order condition 10, using $\frac{dk_j}{db_i} = b_j^{-1\prime}(b_i)$ and $b_j^{-1}(b_i) = k_j$, and simplifying

$$\begin{aligned} \frac{\partial \pi_i}{\partial b_i} &= \left(-\frac{k_j}{(2k_j - \theta)} + 1\right)\left(\int_{k_j}^{\overline{k}} (\theta - k_j) g(k_j) dk_j\right) \\ &= \left(\frac{k_j - \theta}{2k_j - \theta}\right)\int_{k_j}^{\overline{k}} (\theta - k_j) g(k_j) dk_j < 0, \end{aligned}$$

as desired. □

Proof of Lemma 2: See Fabra et al. (2006). Unlike their paper, the fact that capacities are random and observable allows to symmetrize the equilibrium through perfect correlation between the two asymmetric pure strategy equilibria. □

Proof of Lemma 3: See Fabra et al. (2006). The proof is analogous to the case in which demand is uncertain (long-lived bids). □

Proof of Proposition 4: It follows from the proofs of Lemmas 2 and 3.

Proof of Proposition 5: Expected profits under the discriminatory auction are given by:

$$\pi_i(k_i; b_i, b_j(k_j)) = (b_i - c)\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (\theta - k_j)g(k_j)dk_j\right). \qquad (11)$$

Maximization with respect to b_i implies,

$$\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (\theta - k_j)g(k_j)dk_j\right) + (b_i - c)b_j^{-1\prime}(b_i)\left(g(b_j^{-1}(b_i))(k_i + b_j^{-1}(b_i) - \theta)\right) = 0.$$

Under symmetry, $b_j(k) = b_i(k)$. Accordingly, we can rewrite the expression as

$$k_i G(k_i) + \int_{k_i}^{\bar{k}} (\theta - k_j)g(k_j)dk_j + (b_i - c)\frac{1}{b_i'(k_i)}g(k_i)(2k_i - \theta) = 0$$

This expression is the similar as equation (6) for the uniform auction, but is has an additional term, $k_i G(k_i)$, reflecting the fact that the firm is always paid according to its bid, also when it is the large firm and hence has the low bid. The rest of the proof follows the same steps as the proof of Proposition 2. □

Proof of Lemma 4: Suppose that profits under the uniform and the discriminatory auction, determined by equations (4) and (11), are equal for all k_i. Since the bid of the discriminatory auction is higher than the uniform one, $b_u^*(k_i) < b_d^*(k_i)$, where we denote the latter as b_u^*, it then follows that the first term of the two profit equations must be such that

$$\int_{\underline{k}}^{k_i} (b^u(k_j) - c)k_i g(k_j)dk_j > \int_{\underline{k}}^{k_i} (b^d(k_i) - c)k_i g(k_j)dk_j$$

which implies

$$\int_{\underline{k}}^{k_i} \left(b^u(k_j) - b^d(k_i)\right) g(k_j)dk_j > 0.$$

It can be verified that the previous expression is equal to $\frac{\partial \pi_u(k)}{\partial k} - \frac{\partial \pi_d(k)}{\partial k}$. Using the Fundamental Theorem of Calculus and the fact that $\pi_u(\underline{k}) = \pi_d(\underline{k})$ it follows that $\pi_u(k) > \pi_d(k)$ which contradicts that profits are equal and, hence, there is no revenue equivalence. □

Proof of Proposition 6: Profits for firm i are:

$$\pi_i(k_i) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c) k_i \varphi(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (b_i - c) \left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k) dk \right) \varphi(k_j) dk_j$$

The first-order condition that characterizes the optimal bid of firm i can be written as

$$\frac{\partial \pi_i}{\partial b_i(k_i)} = b_j^{-1\prime}(b_i) \varphi(b_j^{-1}(b_i))(b_i - c) \left(k_j + \int_{k_j}^{\bar{k}} (N-2)\, kg(k) dk + b_j^{-1}(b_i) - \theta \right)$$
$$+ \int_{b_j^{-1}(b_i)}^{\bar{k}} \left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k) dk \right) \varphi(k_j) dk_j = 0.$$

Under symmetry, $b_j(k) = b_i(k)$, we can rewrite the expression as

$$\frac{\partial \pi_i}{\partial b_i(k_i)} = \frac{1}{b_i'(k_i)} \varphi(k_i)(b_i(k_i) - c) \left(2k_i + \int_{k_i}^{\bar{k}} (N-2)\, kg(k) dk - \theta \right)$$
$$+ \int_{k_i}^{\bar{k}} \left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k) dk \right) \varphi(k_j) dk_j = 0$$

Reorganizing it,

$$b_i'(k_i) + b_i(k_i) a(k_i) = c a(k_i)$$

where $a(k_i)$ does not depend on b_i,

$$a(k_i) = \frac{\left(2k_i + \int_{k_i}^{\bar{k}} (N-2)\, kg(k) dk - \theta \right) \varphi(k_i)}{\int_{k_i}^{\bar{k}} \left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k) dk \right) \varphi(k_j) dk_j}.$$

Hence, the solution is the same as above:

$$b_i^*(k_i) = c + (P - c)\, e^{-\omega(k_i)}$$

where $\omega(k_i) \equiv \int_{\underline{k}}^{k_i} a(k) dk$. □

Proof of Lemma 5: We first prove the non-existence of a pure-strategy equilibrium when $k_i \geq \theta$. By way of contradiction, assume that there exists one. Following the same steps as in the proof of Lemma 1, it is easy to show that it must be non-increasing in k_i. Suppose, therefore, that b_j is non-increasing in k_j. As a result, the optimal bid for firm i can be characterized as $b_i \in \arg\max_{b_i} \pi_i(k_i; b_i, b_j(k_j))$.

If $b_i > b_j(\theta)$, expected profits are

$$\pi_i(k_i; b_i, b_j(k_j)) = (b_i - c) \left(\int_{\underline{k}}^{b_j^{-1}(b_i)} \theta g(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (\theta - k_j) g(k_j) dk_j \right).$$

Instead, if $b_i \leq b_j(\theta)$,

$$\pi_i(k_i; b_i, b_j(k_j)) = (b_i - c)\int_{\underline{k}}^{b_j^{-1}(b_i)} \theta g(k_j) dk_j.$$

In both cases, profit functions do not depend on k_i. Therefore, the optimal bid is the same for all $k_i \geq \theta$. Thus, at the candidate pure strategy equilibrium, $b^*(k_i) = b^*(\theta)$ for all $k_i \geq \theta$.

However, this is ruled out by standard Bertrand-Edgeworth arguments. First, if $b^*(\theta) > c$, firm i would have incentives to slightly undercut $b^*(\theta)$. If $k_j \geq \theta$, this would allow firm i to serve total demand, rather than a share of it, at only a slightly lower price, with almost no effect on firm i's profits if $k_j < \theta$. Second, if $b^*(\theta) = c$, the market price would always be c. Hence, firm i would make zero profits regardless of k_j and would rather deviate to P in order to make positive profits over the expected residual demand. It follows that the equilibrium must involve mixed strategies. Standard arguments imply that firms choose prices in a compact support $\left[\underline{b}, \bar{b}\right]$. □

Proof of Proposition 7: A symmetric Bayesian Nash Equilibrium must have the following properties. First, using the same arguments in Proposition 2, the optimal bid must be strictly decreasing in k_i for $k_i < \theta$. Second, from Lemma 5 it must involve a mixed strategy when $k_i \geq \theta$.

To make things simpler, we first assume $\bar{b} \leq b(\underline{k})$ and $\underline{b} \geq b(\theta)$. At the end of the proof we will show that this assumption must hold in equilibrium. We define $\tilde{k} = b_j^{-1}(\bar{b})$ and $\hat{k} = b_j^{-1}(\underline{b})$. Since $b_j(k_j)$ is decreasing, it follows that $\left[\tilde{k}, \hat{k}\right] \subseteq [\underline{k}, \theta]$. We consider four capacity regions:

Region I. If $k_i \in \left[\underline{k}, \tilde{k}\right]$, expected profits are

$$\pi_i(b_i; k_i, b_j(k_j)) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c) k_i g(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (b_i - c)(\theta - k_j) g(k_j) dk_j$$

Firm i has the low bid when $k_j < b_j^{-1}(b_i)$ and, hence sells up to capacity at the price set by firm j. Otherwise, either it sells the residual demand and sets the price or, if $k_j > theta$ the rival will serve all the market.

Taking derivatives, we obtain a similar First Order Condition as in equation (6), with the only difference that $\bar{k}$ is replaced by θ. Hence, the solution is the same as in Proposition 2, with the only difference that $\bar{k}$ is replaced by θ in equation (7). Hence, the optimal bid in this region is

$$b^*(k_i) = c + (P - c)\, e^{-\omega(k_i)}, \tag{12}$$

where $\omega(k_i) \equiv \int_{\underline{k}}^{k_i} a(k)dk$, and

$$a(k) = \frac{(2k-\theta)g(k)}{\int_k^\theta (\theta - k_j)g(k_j)dk_j}. \tag{13}$$

Using the optimal bid in (12), for given $\bar{b}$, $\tilde{k}$ is implicitly defined by

$$b^*\left(\tilde{k}\right) = \bar{b}.$$

Region II. If $k_i \in \left[\tilde{k}, \hat{k}\right]$, expected profits are

$$\begin{aligned}\pi_i(b_i; k_i, b_j(k_j)) &= \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (b_i - c)(\theta - k_j)g(k_j)dk_j \\ &\quad + (1 - G(\theta)) \int_{b_i}^{\bar{b}} (b_j - c)k_i f_j(b_j)\, db_j.\end{aligned}$$

The profit expression now adds a third term as the firm will serve all its capacity at the price set by the rival whenever $k_j \geq \theta$ and $b_i < b_j$.

The first-order condition that characterizes the optimal bid of firm i can be written as

$$\frac{1}{b_j'(k_j)} g(b_j^{-1}(b_i))(b_i - c)(k_i + b_j^{-1}(b_i) - \theta) + \int_{b_j^{-1}(b_i)}^{\theta} (\theta - k_j)g(k_j)dk_j - (1 - G(\theta))(b_i - c)k_i f_j(b_i) = 0.$$

This expression is similar to equation (6), where $\bar{k}$ replaces θ, plus an additional third term, which is negative. If follows that the optimal bid that solves the above equation is lower than the optimal bid in the baseline case.

Using symmetry, the optimal bid is the solution to

$$\left(1 - \frac{(1 - G(\theta))(b(k) - c)kf(b(k))}{(2k - \theta)g(k)} a(k)\right) b'(k) + a(k)b(k) = ca(k),$$

where $a(k)$ is defined as in equation (13). Note that if $G(\theta) = 1$ we would obtain the same solution as in the Baseline case. Since we now have $G(\theta) < 1$, the solution is lower.

Region III. If $k_i \in \left[\hat{k}, \theta\right]$, expected profits are

$$\begin{aligned}\pi_i(b_i; k_i, b_j(k_j)) &= \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (b_i - c)(\theta - k_j)g(k_j)dk_j \\ &\quad + (1 - G(\theta)) \int_{\underline{b}}^{\bar{b}} (b_j - c)k_i f_j(b_j)\, db_j.\end{aligned}$$

The first-order condition that characterizes the optimal bid of firm i is the same as in Region I as the last term does not depend on b_i. Hence, the solution is also given by expressions (12) and (13). Hence, (12), for given $\underline{b}$, $\hat{k}$ is implicitly defined by $b^*(\hat{k}, \underline{k}, \theta) = \underline{b}$.

Region IV. Last, consider $k_i \in \left[\theta, \bar{k}\right]$. Expected profits are given by,

$$\pi_i(b_i; k_i, b_j(k_j)) = (b_i - c)\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} \theta g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (\theta - k_j)g(k_j)dk_j + (1 - F_j(b_i))(1 - G(\theta))\theta\right) \tag{14}$$

As argued above, this profit function does not depend on k_i, so the optimal bid must be constant in k_i.

At the upper bound of the support, $F_j\left(\bar{b}\right) = 1$. Hence, $\bar{b}$ maximizes

$$\begin{aligned}\pi_i(\bar{b}, k_i) &= (\bar{b} - c)\left(\int_{\underline{k}}^{b_j^{-1}(\bar{b})} \theta g(k_j)dk_j + \int_{b_j^{-1}(\bar{b})}^{\theta} (\theta - k_j)g(k_j)dk_j\right) \\ &= \left(b^*\left(\tilde{k}\right) - c\right)\left(\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j)dk_j\right)\end{aligned}$$

Taking derivatives with respect to $\bar{b}$,

$$\theta G(\theta) - \int_{b_j^{-1}(\bar{b})}^{\theta} k_j g(k_j)dk_j + (\bar{b} - c)\frac{1}{b_j'(k_j)} g(b_j^{-1}(\bar{b}))b_j^{-1}(\bar{b}) = 0$$

Using the definition of $\tilde{k}$ above, it can be re-written as

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j)dk_j + (\bar{b} - c)\frac{1}{b_j^{*\prime}(\tilde{k})} g(\tilde{k})\tilde{k} = 0.$$

From the analysis of the case with certain pivotality we know that

$$b_i'(k_i) + a(k_i)b_i(k_i) = ca(k_i)$$

so that

$$b_j'(k_j) = -\left(b_i(k_i) - c\right)a(k_i)$$

Hence,

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j)dk_j - (\bar{b} - c)\frac{1}{\left(b^*(\tilde{k}) - c\right)a(\tilde{k})} g(\tilde{k})\tilde{k}$$

Since $b^*(\tilde{k}) = \bar{b}$,

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j)dk_j - \frac{g(\tilde{k})\tilde{k}}{a(\tilde{k})} = 0$$

Using the expression for $a(k)$ in equation (13),

$$\theta G\left(\tilde{k}\right) - \frac{\theta - \tilde{k}}{2\tilde{k} - \theta}\int_{\tilde{k}}^{\theta} (\theta - k_j)\, g(k_j)dk_j = 0,$$

which defines $\tilde{k}$. Note that we must have an interior solution, $\tilde{k} \in (\underline{k}, \theta)$. For $\tilde{k} = \underline{k}$, the first term is zero so the left hand side would be negative; whereas for $\tilde{k} = \theta$, the second term is zero so the left hand side would be positive.

At the lower bound of the support, $F_j\left(\underline{b}\right) = 1$. Expected profits are

$$\begin{aligned} \pi_i(\underline{b}, k_i) &= \left(\underline{b} - c\right)\left(\theta - \int_{b_j^{-1}(\underline{b})}^{\theta} k_j g(k_j) dk_j\right) \\ &= \left(b\left(\hat{k}\right) - c\right)\left(\theta - \int_{\hat{k}}^{\theta} k_j g(k_j) dk_j\right) \end{aligned}$$

Since the firm must be indifferent between all the prices in the support, profits at the lower and upper bounds must be equal,

$$\left(\bar{b} - c\right)\left(\theta G\left(\theta\right) - \int_{b_j^{-1}(\bar{b})}^{\theta} k_j g(k_j) dk_j\right) = \left(\underline{b} - c\right)\left(\theta - \int_{b_j^{-1}(\underline{b})}^{\theta} k_j g(k_j) dk_j\right) = \pi^*$$

Using the definitions for $\bar{b}$ and $\underline{b}$,

$$\left(b^*\left(\tilde{k}\right) - c\right)\left(\theta G\left(\theta\right) - \int_{\tilde{k}}^{\theta} k_j g(k_j) dk_j\right) = \left(b^*\left(\hat{k}\right) - c\right)\left(\theta - \int_{\hat{k}}^{\theta} k_j g(k_j) dk_j\right) = \pi^*$$

which defines $\hat{k}$. Hence, equilibrium profits are well defined $\underline{\pi}$ and we can treat them like a constant.

By the above equality, when $\bar{k}$ is just above θ, $\tilde{k}$ is arbitrarily close to $\hat{k}$. Instead, when $\underline{k}$ is so large that $G\left(\theta\right) = 0$, then $b\left(\hat{k}\right) = c$.

Now, we can use the above expression for equilibrium profits to solve for $F\left(b\right)$ in equation (14),

$$F\left(b\right) = \frac{1}{\left(1 - G\left(\theta\right)\right)\theta}\left(\theta - \int_{b^{*-1}(b)}^{\theta} k g(k) dk - \frac{\pi^*}{(b - c)}\right)$$

where $b^*(k)$ is defined above by expressions (12) and (13).

Computing the density,

$$f\left(b\right) = \frac{1}{\left(1 - G\left(\theta\right)\right)\theta}\left(\frac{\pi^*}{(b - c)^2} + \frac{k}{b^{*\prime}(k)} g(k)\right).$$

□