ON THE CONNECTION BETWEEN PERSUASION AND DELEGATION

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Abstract. [preliminary] We compare two problems. One is the monotone Bayesian persuasion problem in which messages are monotone functions of state realizations. Another is the delegation problem with an extra assumption that extreme actions are always permitted. (E.g., an employee may need the employer's permission to reduce the job involvement to part time, but no permission is needed to remain at full time or to quit the job entirely.) We prove that these problems are equivalent, in the sense that one problem can be mapped into the other. We use this equivalence to apply known techniques in the persuasion literature to the delegation problem, and vice versa.

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1. Introduction

We compare two problems, the Bayesian persuasion problem of Kamenica and Gentzkow (2011), with interval action and state spaces, and the delegation problem of Alonso and Matouschek (2008) and Amador and Bagwell (2013), with interval action and type spaces and more general payoff functions.

In the persuasion problem, there are a sender (she) and a receiver (he). The receiver must choose an action. His preferred action depends on an unobservable state. The sender is biased and prefers the receiver to choose a different action. Ex-ante, the sender and receiver are uninformed about the state and have a common prior. The sender designs an experiment that generates an informative message about the realized state. The receiver observes the message, forms a posterior belief about the state, and then chooses the optimal action given the posterior. By designing an experiment, the sender manipulates the receiver's beliefs about the state to further her own goals.

We assume that the sender's choice is restricted to monotone experiments. A monotone experiment sends messages that are deterministic and monotone functions of the state. That is, higher states always result in weakly higher messages. A simple example is the experiment that divides of the state space into intervals and informs the receiver about the interval containing the realized state. We refer to the problem with the restriction to monotone experiments as the *monotone persuasion problem*.

In the delegation problem, there are a principal (she) and an agent (he). The principal has the rights to choose an action, but only the agent has access to the relevant information for a decision. The principal delegates the action choice to the agent. As the agent has biased preferences, the principal restricts the set of actions for the agent to minimize the payoff loss from the agent's biased decision.

We assume that the principal, when choosing a delegation set for the agent, cannot exclude the two extreme actions on the opposite ends of the action interval. For example, an employee (agent) wishes to reduce his job engagement to part time. The employer (principal) has the power to permit or prohibit any choice of the employee, except for the two extreme decisions of no action (remaining at full time job) and quitting the job entirely. We refer to the problem with the above restriction on delegation sets as the *constrained delegation problem*.

The common between these two problems is that both have a roughly similar structure and both address the problems where one party designs a mechanism that influences actions of the other. However, the mechanisms of influence are different. In the delegation problem, the principal influences the agent's decisions by directly restricting the set of allowed actions. In the persuasion problem, the sender influences the receiver's decisions by altering his beliefs about a payoff-relevant state.

Despite the difference between these two problems, we show that they are equivalent. Any monotone persuasion problem can be formulated as the constrained delegation problem, and vice versa.

In addition, we consider the subclasses of these problems, with an extra assumption of payoff linearity in the state/type, are also equivalent. In the persuasion problem, we assume that the sender's and receiver's payoffs are linear in the state, as in Gentzkow and Kamenica (2016) and Kolotilin et al. (2017). In the delegation problem, we assume that the principal's and agent's payoffs are linear in the type, as in Alonso and Matouschek (2008) and Amador and Bagwell (2013).[1] We show that these problems with linear payoffs are also equivalent.

To prove the equivalence results, we show that the monotone persuasion problem and the constrained delegation problem are both equivalent to a third problem. This is the problem of persuasion with binary actions and a privately informed receiver, as in Kolotilin (2017) and Kolotilin et al. (2017). In this problem, the receiver chooses one of two actions. His preferred action depends on an unobservable state and his private type. The sender designs a menu of experiments that generate informative messages about the realized state. We restrict attention to menus of *cutoff experiments* that inform the receiver whether the state is above or below a given cutoff. The receiver chooses an experiment from the menu, observes the message, forms the posterior belief about the state, and then chooses the optimal action given the posterior.

The equivalence between these three problems can be used to use the known methodology and solution techniques for one problem to solve other problems. We use this equivalence to connect the interval delegation results for the delegation problem and the interval revelation and the upper/lower-censorship results for the persuasion problems.

[1]In Alonso and Matouschek (2008), the payoffs are quadratic. But they can be written as linear in the type without loss of generality, as the quadratic term of the type does not interact with the action, and thus can be omitted.

2. Two Problems

We present the monotone persuasion problem and the constrained delegation problem side-by-side to highlight their similarities and differences.

Monotone Persuasion Problem

There are a sender (she) and a receiver (he).

The receiver chooses an action a from the real line $\mathbb{R}$. The sender's and receiver's payoffs

$$U_S(a,\omega) \text{ and } U_R(a,\omega)$$

depend on the chosen action a and on a state $\omega \in \Omega = [\underline{\omega}, \overline{\omega}]$. Assume

$$\tfrac{\partial}{\partial a} U_R(a_R(\omega),\omega) = 0 \text{ for some } a_R : \Omega \to \mathbb{R},$$
$$\tfrac{\partial^2}{\partial a^2} U_R(a,\omega) < 0 \text{ and } \tfrac{\partial^2}{\partial a \partial \omega} U_R(a,\omega) > 0.$$

The sender and receiver have a common prior that ω has a distribution F with a strictly positive density f.

The sender designs an experiment that generates an informative message about ω that the receiver observes. An *experiment* π is a function from Ω into $\mathbb{R}$.

We consider *monotone* experiments that generate a monotone partition of Ω. Denote the set of such experiments by

$$\Pi^* = \{\pi \mid \pi(\omega) \text{ is non-decreasing in } \omega\}.$$

Given message m of experiment π, the receiver chooses the optimal action

$$a^*_\pi(m) = \arg\max_{a \in A} \mathbb{E}[U_R(a,\omega) \mid \pi(\omega) = m].$$

The sender chooses a monotone experiment $\pi \in \Pi^*$ to maximize her expected payoff

$$\max_{\pi \in \Pi^*} \int_\Omega U_S(a^*_\pi(\pi(\omega)),\omega) \mathrm{d}F(\omega).$$

Constrained Delegation Problem

There are a principal (she) and an agent (he).

The agent chooses a decision y from the real line $\mathbb{R}$. The principal's and agent's payoffs

$$U_P(\theta, y) \text{ and } U_A(\theta, y)$$

depend on the chosen decision y and on a type $\theta \in \Theta = [\underline{\theta}, \overline{\theta}]$. Assume

$$\tfrac{\partial}{\partial y} U_A(\theta, y_A(\theta)) = 0 \text{ for some } y_A : \Theta \to \mathbb{R},$$
$$\tfrac{\partial^2}{\partial y^2} U_A(\theta, y) < 0 \text{ and } \tfrac{\partial^2}{\partial \theta \partial y} U_A(\theta, y) > 0.$$

The agent privately knows the type θ. The principal has a prior that θ has a distribution G with a strictly positive density g.

The principal chooses a *delegation set* of decisions that the agent is allowed to choose from. A delegation set X is a compact subset of $\mathbb{R}$.

We consider *constrained* delegation sets that contain extreme decisions $y_A(\underline{\theta})$ and $y_A(\overline{\theta})$. Denote the collection of such sets by

$$\mathcal{X}^* = \{X \mid \{y_A(\underline{\theta}), y_A(\overline{\theta})\} \subset X \subset \mathbb{R}\}.$$

Given type θ and delegation set X, the agent chooses the optimal decision

$$y^*_X(\theta) \in \arg\max_{y \in X} U_A(\theta, y).$$

The principal chooses a delegation set $X \in \mathcal{X}^*$ to maximize her expected payoff

$$\max_{X \in \mathcal{X}^*} \int_\Theta U_P(\theta, y^*_X(\theta)) \mathrm{d}G(\theta).$$

The main difference between the above two problems is in the method of control over the receiver's/agent's actions.

In the persuasion problem, the sender manipulates the receiver's beliefs about the state by choosing how the receiver is informed. Kamenica and Gentzkow (2011) allow for experiments that map states to any (stochastic) messages. Without loss of generality, the experiments are *direct*, in the sense that each message $\pi(\omega)$ has the meaning of the recommendation to the receiver about the action to choose. In this paper we restrict attention to direct monotone experiments whose recommendations are nonrandom and nondecreasing in the state.

In the delegation problem, the principal affects the agent's choices by restricting the set of allowed actions. In this paper we make an unusual extra assumption. A delegation set X must contain the end-point actions, $\underline{y}$ and $\overline{y}$. Put differently, the principal can prohibit the agent to take any actions, except the very extremes. One of these extremes can be interpreted as the inaction or status quo, and hence, naturally, cannot be excluded from the delegation set. The justification why the other extreme action cannot be excluded is not as ubiquitous and depends on the context.

We now state our main result. Two problems are *equivalent* if one problem can be identically formulated the other, and vice versa. Our main result is that the monotone persuasion problem and the constrained delegation problem are equivalent.

An *instance* of a problem is a tuple of its primitives. Without loss of generality, assume that the state space and the type space are unit intervals, $\Omega = \Theta = [0, 1]$. Denote by $P = (U_S, U_R, F)$ and $D = (U_P, U_A, G)$ instances of the persuasion and delegation problems. Denote by $\mathcal{P}$ the set of all instances that satisfy the assumptions made above (notice we make identical assumptions about the primitives of the persuasion and delegation problems).

A *solution* of a problem is an algorithm that, for each instance, finds all objects that maximizes the problem's objective function. A solution of the persuasion problem is a correspondence φ that associates with each problem instance $P \in \mathcal{P}$ the set $\varphi(P)$ of all elements of Π^* that maximize the problem's objective function. Similarly, a solution of the delegation problem is a correspondence ψ that associates with each problem instance $D \in \mathcal{P}$ the set $\psi(D)$ of all elements of $\mathcal{X}^*$ that maximizes the problem's objective function.

We say that the two problems are *equivalent* if a solution to one problem translates into a solution to the other problem. Formally, the persuasion and delegation problems are equivalent if there exists one-to-one mappings $\nu : \mathcal{P} \to \mathcal{P}$ and $\eta : \Pi^* \to \mathcal{X}^*$ such that $D = \nu(P)$ if and only if $\psi(D) = \eta(\varphi(P))$.

Theorem 1. *The monotone persuasion problem and the constrained delegation problem are equivalent.*

Proof. Using our assumptions on the payoff functions, without loss of generality, we can assume that the receiver's optimal action at each state is the state itself,

$$a_R(\omega) = \arg\max_{a\in\mathbb{R}} U_R(a,\omega) = \omega,$$

and that the agent's optimal action for each type is the type itself,

$$y_A(\theta) = \arg\max_{y\in\mathbb{R}} U_A(\theta,y) = \theta.$$

Part 1. Constrained delegation problem. Since U_A and U_P are differentiable, we can write

$$U_A(\theta,y) = \int_y^1 u(\theta,\omega)\mathrm{d}F(\omega) + U_A(\theta,1)$$

where $u(\theta,\omega)$ is strictly decreasing in θ and strictly increasing in ω, and $u(\theta,\theta)=0$, and F is any distribution with an almost everywhere positive density. Also,

$$U_P(\theta,y) = \int_y^1 v(\theta,\omega)\mathrm{d}F(\omega) + U_P(\theta,1).$$

For a given delegation set $X \in \mathcal{X}^*$, define

$$\underline{x}(\theta) = \sup\{x\in X : x\le\theta\} \quad \text{and} \quad \overline{x}(\theta) = \inf\{x\in X : x\ge\theta\}.$$

Note that $\underline{x}(\theta)=\overline{x}(\theta)=\theta$ if and only if $\theta\in X$.

The agent's optimization problem is

$$\max_{y\in X}\int_y^1 u(\theta,\omega)\mathrm{d}F(\omega).$$

The solution is

$$y_X^*(\theta) = \begin{cases} \theta, & \text{if } \theta\in X,\\ \overline{x}(\theta), & \text{if } \int_{\underline{x}(\theta)}^{\overline{x}(\theta)} u(\theta,\omega)\mathrm{d}F(\omega) < 0,\\ \underline{x}(\theta), & \text{if } \int_{\underline{x}(\theta)}^{\overline{x}(\theta)} u(\theta,\omega)\mathrm{d}F(\omega) \ge 0. \end{cases}$$

Let

$$I_X(\theta,\omega) = \begin{cases} 1, & \text{if } \underline{x}(\theta)<\overline{x}(\theta) \text{ and } \int_{\underline{x}(\theta)}^{\overline{x}(\theta)} u(\theta,\omega)\mathrm{d}F(\omega)\ge 0, \text{ or } \underline{x}(\theta)=\overline{x}(\theta),\\ 0, & \text{otherwise.} \end{cases}$$

Then, under the agent's optimal action conditional on type θ, the agent's payoff is $\int_\Omega u(\theta,\omega)I_X(\theta,\omega)\mathrm{d}F(\omega)$ and the principal's payoff is $\int_\Omega v(\theta,\omega)I_X(\theta,\omega)\mathrm{d}F(\omega)$. The principal maximizes the expected payoff

$$\max_{X\in\mathcal{X}^*}\int_0^1\int_0^1 v(\theta,\omega)I_X(\theta,\omega)\mathrm{d}F(\omega)\mathrm{d}G(\theta).$$

Part 2. Monotone persuasion problem. Since U_R and U_S are differentiable, we can write

$$U_R(a,\omega) = \int_0^a u(\theta,\omega)\mathrm{d}G(\theta) + U_R(0,\omega)$$

where $u(\theta,\omega)$ is strictly decreasing in θ and strictly increasing in ω, and $u(\omega,\omega)=0$, and G is any distribution with an almost everywhere positive density. Also,

$$U_S(a,\omega)=\int_0^a v(\theta,\omega)\mathrm{d}G(\theta)+U_S(0,\omega).$$

Any monotone experiment $\pi\in\Pi^*$ can be equivalently described by a compact set $X\subset[0,1]$ defined as follows. Let S be the set of all states where $\pi(\omega)$ is locally constant,

$$S=\left\{\omega\in(0,1):\lim_{\varepsilon\to 0}\frac{\pi(\omega+\varepsilon)-\pi(\omega-\varepsilon)}{2\varepsilon}=0\right\},$$

and define

$$X=[0,1]\backslash S.$$

That is, X consists of the intervals where π continuously increases, the discontinuity points of π, as well as the endpoints 0 and 1. Notice that set $X\in\mathcal{X}^*$.

Conversely, each $X\in\mathcal{X}^*$ we construct $\pi\in\Pi^*$ by $\pi(\omega)=\underline{x}(\omega)$, where $\underline{x}(\omega)$ is as defined above.

We use notations $\underline{x}(\cdot)$, $\overline{x}(\cdot)$, and $I_X(\cdot,\cdot)$ defined before. Note that after observing a message $\pi(\omega)$, the receiver knows the interval $[\underline{x}(\omega),\overline{x}(\omega)]$ that ω belongs to and maximizes

$$\max_{a\in A}\int_0^a\int_{\underline{x}(\omega)}^{\overline{x}(\omega)}u(\theta,t)\mathrm{d}F(t)\mathrm{d}G(\theta).$$

The solution must satisfy

$$\int_{\underline{x}(\omega)}^{\overline{x}(\omega)}u(a,t)\mathrm{d}F(t)=0.$$

Since $u(a,t)$ is strictly decreasing in a, there is a unique solution. Using the notation I_X, the above is satisfied if, for a given state ω, a is largest value of θ such that $I_X(\theta,\omega)=1$. Thus, for a given state ω, under the optimal action of the receiver, his payoff is

$$\int_0^1\left(\int_{\underline{x}(\omega)}^{\overline{x}(\omega)}u(\theta,t)\mathrm{d}F(t)\right)I_X(\theta,\omega)\mathrm{d}G(\theta)$$

and the sender's payoff is

$$\int_0^1\left(\int_{\underline{x}(\omega)}^{\overline{x}(\omega)}v(\theta,t)\mathrm{d}F(t)\right)I_X(\theta,\omega)\mathrm{d}G(\theta)$$

The sender maximizes

$$\max_{X\in\mathcal{X}^*}\int_0^1\int_0^1\left(\int_{\underline{x}(\omega)}^{\overline{x}(\omega)}v(\theta,t)\mathrm{d}F(t)\right)I_X(\theta,\omega)\mathrm{d}G(\theta)\mathrm{d}F(\omega)$$
$$=\max_{X\in\mathcal{X}^*}\int_0^1\int_0^1 v(\theta,\omega)I_X(\theta,\omega)\mathrm{d}G(\theta)\mathrm{d}F(\omega).$$

□

3. Discussion of Assumptions

We made a number of assumptions for the clarity of exposition. These assumptions can be relaxed without affecting the equivalence result.

The strict concavity and supermodularity of the agent's payoff function in the delegation problem, $\frac{\partial^2}{\partial y^2}U_A(y,\theta) < 0$ and $\frac{\partial^2}{\partial y \partial \theta}U_A(y,\theta) > 0$, can be relaxed the quasiconcavity of $U_A(y,\theta)$ in y, and the condition of the single crossing of differences,

$$U_A(y',\theta) - U_A(y,\theta) \geq (>)\, 0 \implies U_A(y',\theta') - U_A(y,\theta') \geq (>)\, 0$$

whenever $y < y'$ and $\theta < \theta'$. In this case, the agent's optimal action $y^*_X(\theta)$ is defined as a monotone selection from the arg max set.

Similarly, the strict concavity and supermodularity of the receiver's payoff function in the persuasion problem, $\frac{\partial^2}{\partial a^2}U_R(a,\omega) < 0$ and $\frac{\partial^2}{\partial a \partial \omega}U_R(a,\omega) > 0$, can be relaxed to the condition

$$\int_\Omega U(a,\omega)\mathrm{d}H(\omega) \text{ is quasiconcave in } a \text{ for each distribution } H \text{ of } \omega,$$

and the condition of the single crossing of differences,

$$U_R(a',\omega) - U_R(a,\omega) \geq (>)\, 0 \implies U_R(a',\omega') - U_R(a,\omega') \geq (>)\, 0$$

whenever $a < a'$ and $\omega < \omega'$. In this case, the receiver's optimal action $a^*(H_{\pi(\omega)})$ is defined as a monotone selection from the arg max set.

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