

Consumer Search and Double Marginalization*

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Abstract

This paper shows that the well-known double marginalization problem underestimates the inefficiencies arising from vertical relations in markets where consumers who are uninformed about the wholesale arrangements between manufacturers and retailers search for the best retail price. Consumer search provides manufacturers with an additional incentive to increase wholesale prices, resulting in higher retail prices. The incentives of firms to reveal the wholesale arrangement are analyzed. We also show that, when the upstream price is unobserved, retail prices decrease, and both industry profits and consumer surplus increase in search cost, whereas the opposite is true when the upstream price is observed.

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1 Introduction

To understand the implications of consumers' search in retail markets, it is important to know whether or not consumers observe retailers' cost. Most of the literature assumes that consumers know these costs and therefore that they can fully understand the distribution of a retailer's (price) offer on the next search (see, e.g., Stahl (1989), Wolinsky (1986) and other papers in the consumer search literature). In most retail markets, however, consumers do not know retailers' cost: as consumers have to search the prices they have to pay, it is unlikely they will know the prices retailers pay to manufacturers. Therefore, upon observing a relatively high price, consumers may be uncertain whether this high price is due to a high margin for the retailer, or whether it is due to a high underlying cost (which may be common to all retailers). This has led Benabou and Gertner (1993), Dana (1994) and Fishman (1996), and more recently, Tappata (2009), Chandra and Tappata (2011) and Janssen et al. (2011) to incorporate cost uncertainty into the search literature by assuming retailers' cost follows some random process.

In many markets, retailer's cost is not random, however, but chosen by an upstream firm. In this paper we introduce vertical relations between a manufacturer and retailers into an otherwise standard model of sequential consumer search. As a reference point, we consider the case where consumers are fully informed about the upstream price retailers have to pay to the manufacturer and show that the familiar double marginalization problem arises in this context. We then consider the often more realistic setting where consumers are *uninformed about retailers' cost*. In this case, consumers cannot condition their search rule (reservation price) on the upstream price. We show that this results in a more inelastic demand curve for the upstream firm, and hence, in even higher upstream and downstream prices.

The phenomenon we discuss arises in all markets with the following three features: (*i*) in the retail market consumers search (and retailers thus have some market power), (*ii*) the upstream market is also characterized by some market power and (*iii*) consumers do not observe the upstream price that is paid by retailers to manufacturer(s). There are many vertical markets that share these three features, e.g. consumer electronics markets (computers, cameras, TVs, refrigerators, etc.), supermarkets, the automobile market, but also the financial industry with financial intermediaries as retailers. In all these markets, there is a limited set of manufacturers and retailers, consumers search for better prices and typically consumers are not aware of the upstream price.¹

These markets differ in important other features, such as, whether products are homogeneous or heterogeneous, the type of contracts that are used between retailers and manufacturers, and the nature of competition at each level of the product channel. In the main body of this paper we focus on a homogeneous products market with one manufacturer who sells its product to two retailers using a linear pricing scheme. This is the context where the quantitative effect of consumers not observing the manufacturer's price is probably the strongest.² Both retailers

¹That consumer search is important in these markets is exemplified by many studies (see, for example, Giulietti et al. (2013) on electricity markets, Lin and Wildenbeest (2013) on Medigap insurance markets, Chandra and Tappata (2011) and Tappata (2009) on gasoline markets) and Baye et al. (2004) on online markets).

²As we argue in the next section, our insights are far more general and apply to any market where the three features listed above are present. Extensions deal with markets where there is an oligopoly at the retail or wholesale level.

buy the manufacturer's product at the same upstream price and sell to consumers in a retail market that has downward sloping demand. As Stahl (1989) is the standard sequential search model for homogeneous goods, we extend this model by incorporating a vertical structure. In Stahl (1989) there are two types of consumers. Some consumers, the shoppers, can search at no cost and simply buy at the lowest price. Other consumers, the non-shoppers, have to pay a positive search cost for each shop they visit after the first one. As explained before, we analyze two versions of this model: one where consumers are informed about the upstream price, and one where they are not.

In the baseline model where retailers' cost is observed by consumers, we show that for a small search cost the upper bound of the retail price distribution is given by the reservation price (which depends on the upstream price); for a larger search cost it is given by the retailers' monopoly price (for given upstream price). This behavior of the downstream market creates a derived expected demand for the upstream monopolist.

In the case where the upstream price is unobserved by consumers, non-shoppers cannot condition their reservation price on the wholesale price. Instead, they form beliefs about the upstream price, and in equilibrium, these beliefs are correct. For a large search cost, the equilibrium is exactly the same as in the case where the upstream price is observed by consumers. The crucial difference is when the search cost is smaller. In case a reservation price equilibrium exists, we show that the upstream price is *larger* than when consumers observe the wholesale price. The reason is as follows. The non-shoppers' reservation price is based on a *conjectured* level of the upstream price. If the manufacturer chooses an upstream price that is higher than this conjectured level, non-shoppers do not adjust their reservation price. As a result, retailers are squeezed: they face a higher cost, but cannot fully adjust their prices upwards as they would do if consumers knew that their marginal cost is higher. The downstream price adjustment to an increase in the upstream price is therefore smaller than in the case where consumers observe the upstream price. For the upstream manufacturer this means that its expected demand is less sensitive to price changes and, therefore, it has an incentive to charge higher prices.³ This effect is strongest when the search cost is small (as in that case the reservation price is always close to the conjectured level of retailers' cost and retailers are maximally squeezed) and when the fraction of non-shoppers is large. In the latter case, the incentives to increase upstream price (for a given conjectured reservation price) can be so strong that a reservation price equilibrium fails to exist.

Since rational consumers correctly anticipate the upstream firm's incentive to charge higher prices when they do not observe the wholesale price, in equilibrium they adjust their belief to an appropriately high level such that the upstream firm is unwilling to increase its price further. This leads to an equilibrium where, as compared to the observed wholesale price case, the wholesale price is higher, the retail prices are higher⁴ and expected consumer surplus is lower. In addition, as prices are higher, expected upstream profits, expected consumer surplus, and under some conditions, expected retail profits are lower.

³In this case where his actions are unobservable, the manufacturer's problem may be regarded as one of a lack of commitment. As Bagwell (1995) has shown, commitment requires that there is a first mover whose actions are observable and the observability in this context is not given.

⁴Since retail price are randomized, by higher retail prices we mean that the distribution of retail prices in the unobserved case first-order stochastically dominates the same distribution in the observed case.

For linear demand, we show that the comparative static results with regard to search cost crucially depend on whether or not consumers observe the upstream price. When the search cost of non-shoppers increases from an initially low level, downstream expected prices are decreasing when consumers do not observe the upstream price, but they are increasing when they do.⁵ Thus, paradoxically, when they do not know the upstream price, consumers are better off with a higher search cost. The underlying reason is as follows. Conditional on a given upstream price, a higher search cost leads to higher retail margins and thus higher prices. In the unobserved case, starting from a high wholesale price at an initially low search cost value, an increase in the search cost decreases demand to even lower levels. This softens the upstream firm's incentive to charge high prices in the unobserved case. For linear demand, this effect is so strong that the resulting retail prices are decreasing in search cost, even though retail margins are increasing. This leads to another interesting effect: in the unobserved case, unlike the observed case, total industry profits are increasing in search cost. This is because by taming the upstream firm's incentive to charge a high price, a higher search cost leads to retail prices closer to those that would be charged by an integrated vertical firm, and thus higher industry profits. The distribution of profits between upstream and downstream firms is, however, affected by the level of search cost - retailers benefit from a higher search cost, while the upstream firm does not.

Another interesting comparative statics result is to compare the different ways to converge to a market where search frictions are negligible: one can either let the fraction of fully informed consumers approach one in the limit, or let the search cost approach zero. In the baseline model where consumers observe the upstream price, the equilibrium outcome in both cases converges to the standard outcome where the upstream monopolist charges the monopoly price and there is Bertrand competition downstream. When consumers do not observe the upstream price, however, the situation is very different. When the search cost becomes small, expected upstream price may substantially increase beyond the monopoly price of a vertically integrated firm.

Having discussed many (negative) implications of the unobserved wholesale price to all market participants, and since the phenomenon we discuss may be prevalent in so many different markets, one may wonder why manufacturers and retailers do not reveal information regarding the wholesale contractual arrangement to consumers. We see two reasons. First, in many markets, it may simply be too difficult to announce wholesale arrangements in a credible and understandable way to consumers, as these relationships are usually governed by complex contracts.⁶ Moreover, retailers have an incentive to make consumers believe that their margins are actually lower than they really are, as this makes consumers believe that there is no point in continuing to search. Manufacturers, on the other hand, have an interest in making consumers believe that retail margins are high, that it thus makes sense for consumers to continue to search, as this will lower retail margins and retail prices (for a given upstream price) and increases man-

⁵As with the expected downstream price, the wholesale price is also increasing in the search cost in the unobserved case, but it is not monotone in the observed case. First it is decreasing in search cost, as the upstream firm accommodates higher downstream margins by lowering its price. Afterwards the upstream price starts to increase as downstream margins become inelastic towards the upstream price.

⁶By complex contracts we do not necessarily refer to non-linear pricing arrangements such as two-part tariffs (in the model we consider linear prices). Rather, we refer to provisions for warranties, returns, delivery etc. that affect retailers' marginal cost but are hard to understand for consumers.

ufacturer's demand. Thus, both retailers and manufacturers have an incentive to lie about the details of the wholesale contract, and these incentives go in opposite directions.⁷ Second, as we show in the extension dealing with competition between manufacturers, the incentives for the manufacturer(s) to announce wholesale prices depend on the market power they have. If there is sufficiently strong competition upstream, manufacturers are actually better off in a world where consumers do not observe the upstream price. Thus, they have an incentive to conceal the wholesale arrangement from consumers.

The issues we touch upon in this paper are related to recent literature on recommended retail prices (see, e.g., Lubensky (2011) and Buehler and Gärtner (2012)). Lubensky suggests that by recommending retail prices, manufacturers provide information to consumers on what reasonable retail prices to expect in an environment where the manufacturer's marginal cost is random and only known to the manufacturer. In our framework, when informed about the upstream price, consumers have a better notion of how large retail margins are and this benefits all market participants by reducing the manufacturer's perverse incentive to increase its price. In Buehler and Gärtner (2012), consumers are not strategic and the recommended retail price is used by the manufacturer to communicate demand and cost information to the retailer.

Our paper also provides an alternative perspective on the issue of whether it should be mandatory for intermediaries, for example in the financial sector, to disclose their margins. Recent policy discussions in, for instance, the EU and the USA have lead to legislation mandating intermediaries to reveal this information.⁸ Our paper argues that this legislation may actually benefit most market participants as it provides (i) consumers with a better benchmark on what prices to expect in the market, (ii) upstream firms with a reduced incentive to squeeze retailers, leading to overall price levels and quantities sold to be closer to the efficient outcomes. In a recent paper, Inderst and Ottaviani (2012) come to an opposite conclusion. Their framework focuses on the information an intermediary has on how well a product matches the current state of the economy and how competing manufacturers incentivize the intermediary to recommend their products to consumers. Our paper abstracts from these issues as it deals with a homogeneous product and instead focusses on the effect of observing retail margins on the search behavior of consumers and the incentives of the upstream firm to price its products.

The remainder of this paper is organized as follows. Section 2 provides a general, abstract formulation of the key ingredients of our model and explains when these lead to higher upstream prices. Section 3 discusses the two versions of our consumer search model. In Section 4 we compare both versions for the case of linear demand. The linear demand specification allows us to provide a more detailed discussion on when a reservation price equilibrium exists in the model where the upstream price is unobserved by consumers and perform a comparative statics analysis. We also show numerically the size of the effect stemming from unobserved wholesale prices. Section 5 provides extensions related to the intensity of competition at both the retail and the wholesale level and shows that the qualitative features of our analysis are robust. Here, we also show that manufacturers may not benefit from revealing the upstream price if there is upstream competition. The section also demonstrates that the upstream firm does not benefit

⁷It may also be worthwhile to point out that after the upstream price has been set, consumers do not have an incentive to spend time and resources identifying the price, as in equilibrium they have correct expectations.

⁸See the references given in footnote 4 of Inderst and Ottaviani (2012) for details.

from price discrimination between the two retailers by charging them different prices, or (in the extreme case) by foreclosing one of them. Section 6 concludes. Proofs are provided in the Appendix.

2 A general model

The aim of this section is to illustrate the main argument using simple microeconomic tools. In so doing, we also aim to show that the argument is more general than the search model presented in this paper, and we highlight the main ingredients that are necessary for the argument to hold.

Consider an upstream firm that produces a good. The upstream firm charges a unit price w to retailers who sell the good to consumers. Consumers may or may not observe the upstream price w . As is standard, we assume that the (expected) demand for the upstream firm, denoted by Q , depends on w through some optimal behavior by retailers. We allow Q also to depend on the belief consumers hold about the upstream price, denoted by w^e . In the next Section, we will explicitly model consumer and retail behavior, and show why in search markets Q depends on w^e .⁹

We write (expected) profit of the upstream firm as

$$\pi = w \cdot Q(w, w^e). \quad (1)$$

The upstream firm's profit maximization problem depends on whether or not consumers observe w . If they do, then $w^e = w$, and the upstream firm chooses w , anticipating that consumers observe its choice. If w is not observed, then consumers' belief cannot change with w , but in equilibrium we have to impose that the beliefs w^e are correct.

Assume that the profit function is well-behaved, so that in both the observed and unobserved cases, the profit-maximizing w solves the first-order condition. In the observed case, the upstream firm's price, denoted by w^o , solves

$$w^o = -\frac{Q(w^o, w^o)}{\frac{\partial Q}{\partial w}(w^o, w^o) + \frac{\partial Q}{\partial w^e}(w^o, w^o)}. \quad (2)$$

In the unobserved case, because w^e is not equal to w , when the upstream firm varies its price, the upstream firm's equilibrium price, denoted by w^u , solves

$$w^u = -\frac{Q(w^u, w^u)}{\frac{\partial Q}{\partial w}(w^u, w^u)}, \quad (3)$$

where $w^e = w^u$ is imposed after the derivative is taken.

The only difference between (2) and (3) is the term $\frac{\partial Q}{\partial w^e}$. This is the response of the upstream quantity to the change of the belief about the upstream price if the actual price w is held constant.

In many search models, including the Stahl (1989) model we use in this paper, the term $\frac{\partial Q}{\partial w^e}(w, w)$ is negative. This is because if consumers believe that the upstream firm has charged a higher price than it actually has, then consumers accept higher prices, and thus retail prices

⁹Guo (2011) presents a behavioral model where buyers care about the seller's cost as they have a preference for a fair division of the surplus. In that case, demand may also depend on retailers' cost.

are higher. This results in lower quantity sold by the upstream firm. Along with the fact that the common term in the denominator of (2) and (3) is also negative, we have that the upstream price is higher in the unobserved case than in the observed case.¹⁰

Note that, regardless of whether w^u or w^o is larger, as long as $w^o \neq w^u$, equilibrium profit is larger in the observed case than in the unobserved case:

$$w^o \cdot Q(w^o, w^o) > w^u \cdot Q(w^u, w^u). \quad (4)$$

This is because in equilibrium $w^e = w$, and so equilibrium profit always equals $wQ(w, w)$. In the observed case the upstream firm maximizes this profit directly, so at w^o this profit is at its highest. If, in the unobserved case, the upstream firm distorts its price, given that in equilibrium consumers' belief is set accordingly, profit has to be lower than in the observed case.

For another way to see that $w^o < w^u$, note that if $w^o > w^u$ we would have

$$w^o \cdot Q(w^o, w^u) > w^o \cdot Q(w^o, w^o) \quad (5)$$

if upstream profits are decreasing in w^e . Combining inequalities (4) and (5) leads to a contradiction (that w^u is not the optimal choice in the unobserved case), and thus to the conclusion that it has to be the case that $w^o < w^u$.

This simple model indicates several requirements that are necessary for the upstream price to be higher in the unobserved than in the observed case. First, demand has to depend on w^e as otherwise $w^u = w^o$. For example, in the standard double marginalization model where demand only depends on retail prices, $\frac{\partial Q}{\partial w^e}(w, w) = 0$ and observability of w is irrelevant. Second, there has to be some market power downstream. If not, retail prices equal w , so that $\frac{\partial Q}{\partial w^e}(w, w) = 0$, and again the two cases coincide. Third, there has to be market power upstream.¹¹

To summarize, our results hold if upstream profits depend not only on upstream price but also on consumers' belief about it, quantities sold decline as the expected upstream price increases, and there is market power upstream and downstream. As we show below, a standard sequential search model with a vertical structure imposed upon it, has all these features.

3 Retail search markets with an upstream monopoly

This section introduces a wholesale level in the search model developed by Stahl (1989). We discuss two versions, depending on whether consumers do or do not observe the upstream price, henceforth referred to as the observed and unobserved case. Using this model, we prove the main result that the wholesale price is larger in the unobserved case than in the observed case, and we relate this result to the discussion of the general mechanism underlying our paper in the previous section.

We modify Stahl's model in two dimensions. First, for analytic tractability we consider duopoly (and relegate the numerical analysis of downstream oligopoly to Section 5), and, second, we explicitly consider retailers' marginal cost, whereas Stahl normalized it to zero. So, we

¹⁰This can be shown using the second-order condition of i 's profit maximization.

¹¹Even though in the model of this section the upstream firm is a monopolist, as long as $\frac{\partial Q}{\partial w}(w, w) > -\infty$, the same qualitative results would hold even if we considered an oligopoly upstream.

consider a homogenous goods market where an upstream firm chooses a price w for each unit it sells to retailers. For simplicity, we abstract away from the issue of how the cost of the upstream firm is determined and set this cost equal to 0.¹² Retailers take this upstream price (their marginal cost) w as given and compete in prices. Each firm's objective is to maximize profits, taking the prices charged by other firms and the consumers' behavior as given.

On the demand side of the market, we have a continuum of risk-neutral consumers with identical preferences. A fraction $\lambda \in (0, 1)$ of consumers, the *shoppers*, have zero search cost. These consumers sample all prices and buy at the lowest price. The remaining fraction of $1 - \lambda$, the *non-shoppers*, have positive search cost $s > 0$ for every second search they make.¹³ These consumers face a non-trivial problem when searching for low prices, as they have to trade off the search cost with the (expected) benefit from search. Consumers can always come back to previously visited firms incurring no additional cost. If a consumer buys at price p she demands $D(p)$. In our general analysis we assume that the demand function satisfies the following properties: (i) $D'(p) < 0$, (ii) for some finite \hat{p} , $D(p) = 0$ for all $p \geq \hat{p}$ and $D(p) > 0$ otherwise, (iii) for every w , $\pi_r(p; w) \equiv (p - w)D(p)$ is increasing in p up to the retail monopoly price $p^m(w)$ and (iv) for all given w , $(p - w)\pi'_r(p)/\pi_r(p)^2$ is decreasing in p for all $p \in (w, p^m(w)]$.¹⁴ We also assume $D(p)$ is such that the relevant upstream profit functions are strictly quasiconcave. We will formulate this assumption more precisely when we have derived these profit functions. In the next section, we consider the special case of linear demand with $D(p) = 1 - p$ so that the retail monopoly price equals $p^m(w) = \frac{1+w}{2}$ and $\hat{p} = 1$.

The timing is as follows. First, the upstream firm chooses w , which is observed by retailers and, (only) in the observed case, also by consumers. Then, given w , each of the retailers i sets price p_i . Finally, consumers engage in optimal sequential search given the equilibrium distribution of retail prices, not knowing the actual prices set by individual retailers.

3.1 Retailers' cost is observed

In the case where consumers observe the price set by the upstream firm, our model simply adds a price setting stage to the sequential search model of Stahl (1989) where the upstream firm chooses the upstream price it charges to the retailers. With the upstream price observed to consumers, there exists a unique symmetric subgame perfect equilibrium where consumer behavior satisfies a *reservation price property*. As consumers know w , consumers' reservation prices are dependent on w and denoted by $\rho(w)$. To characterize this equilibrium, it is useful to first characterize the behavior of retailers and consumers for given w . We use the following

¹²To focus on the new insights one derives from studying the vertical relation between retailers and manufacturers in a search environment, we assume all market participants know the manufacturer's cost equals 0. Alternatively, the manufacturer's cost could be chosen by a third party or could be uncertain. This would create, however, additional complexity that may obscure the results.

¹³Our analysis for low search cost is unaffected when consumers also have to incur a cost for the first search they make as the expected consumer surplus of making one search is larger than the search cost in that case. For higher search cost, the analysis would become more complicated as we then have to consider the participation decision of non-shoppers (see, Janssen et al. (2005) for an analysis of the Stahl model where the first search is not free).

¹⁴These are standard assumptions in this literature. For example, the last condition (accounting for the fact that retailers' marginal cost is w) is used by Stahl (1989) to prove that the reservation price is uniquely defined. Stahl (1989) shows that the condition is satisfied for all concave demand functions and also for the demand function of the form $D(p) = (1 - p)^\beta$, with $\beta \in (0, 1)$.

notation: $F(p)$ for the distribution of retail prices charged by the retailers (with density $f(p)$), and $\underline{p}(w)$ and $\bar{p}(w)$ for the lower- and upper- bound of their support, respectively. This behavior is by now fairly standard and Proposition 1 below is stated without proof.

Proposition 1. *For $\lambda \in (0, 1)$, the equilibrium price distribution for the subgame starting with w is given by*

$$F(p) = \frac{1 + \lambda}{2\lambda} - \frac{(1 - \lambda)\pi_r(\bar{p}(w))}{2\lambda\pi_r(p)} \quad (6)$$

respectively

$$f(p) = \frac{(1 - \lambda)\pi_r(\bar{p}(w))\pi_r'(p)}{2\lambda\pi_r(p)^2} \quad (7)$$

with support on $[\underline{p}(w), \bar{p}(w)]$ where $\underline{p}(w)$ is the solution to:

$$(1 + \lambda)\pi_r(\underline{p}(w)) = (1 - \lambda)\pi_r(\bar{p}(w)). \quad (8)$$

In order to finalize the description of the equilibrium of the downstream market, we need to find the upper bound (the lower bound is fully defined by the upper bound and w). Clearly, the upper bound $\bar{p}(w)$ may never exceed the monopoly price $p^m(w)$. We define the non-shoppers' reservation price $\rho(w)$, as a solution to

$$ECS(w, \rho(w)) \equiv \int_{\underline{p}(w)}^{\rho(w)} D(p)F(p) dp = s, \quad (9)$$

where $F(p)$ (in (6)) and $\underline{p}(w)$ (from (8)) are taken for $\bar{p}(w) = \rho(w)$.¹⁵ If the solution to (9) does not exist, then we set $\rho(w) = \hat{p}$.¹⁶ For $\rho(w) \leq p^m(w)$, $ECS(w, \rho(w))$ is the expected benefit from searching the second firm when a non-shopper encounters a price equal to $\rho(w)$ at the first firm, and given such a reservation price, all non-shoppers who encounter prices below $\rho(w)$ do not search, whereas those that observe prices above $\rho(w)$ do search.

The upper bound for the equilibrium retail price distribution for a given w is thus the following: $\bar{p}(w) = \min(\rho(w), p^m(w))$. For relatively small search cost, $\rho(w)$ will be the upper bound, and for relatively large search cost $p^m(w)$ will be the upper bound.

This completes the description of the behavior of the downstream market for a given w . Now we turn to the optimal behavior of the manufacturer who determines w . One can easily verify that by increasing w , the upstream firm shifts the retailers' price distribution to the right, regardless of whether $p^m(w)$ or $\rho(w)$ is the upper bound, so that upstream expected demand is decreasing in w . For a given w expected profit of the upstream firm is given by:

¹⁵ Assumption 1, to be made later, guarantees that $ECS(w, \rho(w))$ is monotone in $\rho(w)$, so if the solution to (9) exists, it is unique.

¹⁶ This definition of the reservation price differs somewhat from Stahl's definition. Our and Stahl's definitions coincide when $\rho(w) \leq p^m(w)$. For $\rho(w) > p^m(w)$ Stahl sets $\rho(w) = \hat{p}$ while we still use the root of (9) even though $ECS(w, \rho(w))$ no longer represents the expected benefit from further search (the upper bound $\bar{p}(w)$ may never exceed $p^m(w)$, so the definition of $F(p)$ from (6) for $\bar{p}(w) = \rho(w) > p^m(w)$ results in $F(p) > 1$ for some p). We nevertheless use this definition in order to avoid discontinuity of $\rho(w)$ in w at $\rho(w) = p^m(w)$. We never use $\rho(w)$ as a reservation price, however, if $\rho(w) > p^m(w)$.

$$\pi(w, \bar{p}(w)) = \left((1 - \lambda) \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) f(p) dp + 2\lambda \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) f(p) (1 - F(p)) dp \right) w. \quad (10)$$

The first integral is the expected demand from the non-shoppers; the second integral is expected demand from the shoppers who buy at the lowest retail price. For non-shoppers the density of prices is $f(p)$ as they only sample one firm, whereas for the shoppers the density function is given by $2f(p)(1 - F(p))$.

Henceforth, when we write $\pi(w, x)$ we refer to (10) where x is substituted for $\bar{p}(w)$ everywhere (including in $f(p)$ and $F(p)$), while $\underline{p}(w)$ solves (8) where again x is substituted for $\bar{p}(w)$.¹⁷

In order to simplify the analysis, for the remainder of the paper we assume

Assumption 1. $D(p)$ is such that for any s and λ , $\pi(w, \bar{p}(w))$ and $\pi(w, \rho(w))$ are strictly quasiconcave.¹⁸

As $\rho(w)$ and $p^m(w)$ are continuous functions, so is $\pi(w, \bar{p}(w))$ in $\bar{p}(w)$. It is, however, possible that $\pi(w, \bar{p}(w))$ is not differentiable as the upper bound switches. We show in the appendix (proof of Theorem 1) that at $\rho(w) = p^m(w)$ the derivative of $\pi(w, \bar{p}(w))$ is the same regardless of which upper bound is taken (see also Footnote 17). In addition, the upstream profits equal 0 at $w = 0$ and at $w = \hat{p}$. It follows that there is a unique optimal value of $w \in (0, \hat{p})$, denoted by w^* , where w^* solves

$$\frac{\partial \pi(w, \bar{p}(w))}{\partial w} = 0. \quad (11)$$

Whether $\bar{p}(w) = \rho(w)$ or $\bar{p}(w) = p^m(w)$ in addition to (11) determines the equilibrium upstream price depends on the parameters s and λ and this issue is resolved in Section 3.3. In either case, as $w^* > 0$ and at the retail level $\underline{p}(w^*) > w^*$, both the upstream and downstream firms are able to charge a margin above their respective marginal cost. Thus, although downstream prices follow a mixed strategy, a familiar double-marginalization problem arises in this model with an observed upstream price.

3.2 Retailers' cost is unobserved

We now turn to the analysis of the model where consumers do not observe the upstream price. All other aspects of the model remain the same. At a technical level, this implies that for a given w the downstream market can no longer be analyzed as a separate subgame. As this change in the information structure transforms the interaction into a game with asymmetric information we look for a Perfect Bayesian Equilibrium (PBE) of the game, focusing on equilibria where buyers use reservation price strategies. As non-shoppers are not informed about w the reservation price is now a number ρ that is independent of the upstream price. Non-shoppers buy at a retail price

¹⁷For $x > p^m(w)$ the function $\pi(w, x)$ is well defined, but is no longer the expected profit of the upstream firm (see Footnote 16). We shall nevertheless abuse notation and use $\pi(w, x)$ for such cases, but we ensure that it is used as the profit function only where $x \leq p^m(w)$.

¹⁸In particular, for $s > \hat{p}$ this assumption implies that $\pi(w, p^m(w))$ is quasiconcave.

¹⁹If at w^* we have $\rho(w^*) = p^m(w^*)$, as shown in the Appendix, appropriately chosen right and left derivatives are equal.

$p \leq \rho$ and continue to search otherwise. The reservation price is based on beliefs about w , and in equilibrium, these beliefs are correct.

As retailers know the upstream price, their pricing remains dependent on the upstream price. PBE imposes the requirement that retailers respond optimally to any w , not only the equilibrium upstream price. Therefore, as in the previous subsection, in equilibrium retailers choose retail prices from the range $[\underline{p}(w), \bar{p}(w)]$, where now $\bar{p}(w) = \min(\rho^*, p^m(w))$, where ρ^* is the reservation price when consumers correctly anticipate the equilibrium upstream price. As before, ρ^* is the root of (9) if it exists, and is equal to \hat{p} otherwise.

Any price outside the interval $[\underline{p}(w), \bar{p}(w)]$, is an out-of-equilibrium price and consumers have to form beliefs about who has deviated if such a retail price is observed.²⁰ If $\bar{p}(w) = \rho^* < p^m(w)$ any price above ρ^* is interpreted by consumers as a deviation by the retailer, and *not* by the manufacturer, and thus we have that consumers believe the upstream price to be w^* after observing such a price. Such a deviation is thus “punished” by further search. The belief that it is the retailer, and not the manufacturer, that has deviated is necessary to have a reservation price equilibrium. Otherwise, if consumers believe that it is the manufacturer that has deviated to an upstream price $w > \rho^*$ and that therefore all retailers set a price $p = w > \rho^*$, then uninformed consumers will not want to incur the search cost to find out the other retail price. But this would be inconsistent with a reservation price strategy, and defies the notion of equilibrium as retailers would have an incentive to deviate and choose prices above the reservation price. Thus, the belief that prices above the reservation price stem from a deviation by the retailer, and not the manufacturer, is a necessary condition for consumers to have a reservation price strategy. Note that if the manufacturer deviated to an upstream price $w < \rho^*$ and the retailers did *not* deviate from their equilibrium strategy, retail prices would not be larger than the reservation price ρ^* and consumers would not know that the manufacturer had deviated.²¹

If, however, $\bar{p}(w) = p^m(w) < \rho^*$, then the retailers have no incentive to deviate to a price above $p^m(w)$ unless the manufacturer has also deviated. In this case it is natural to assume that consumers believe the manufacturer has deviated. In particular, we assume that consumers believe the manufacturer has chosen w such that the observed price corresponds to the monopoly price given w , i.e., $p = p^m(w)$.²²

If a price below the lower bound $\underline{p}(w)$ is observed, consumers accept it immediately, which, for example, can be supported by a belief that an individual retailer has deviated. This specific belief is not necessary, but whatever their belief, consumers have to accept a lower than expected price.

Definition 1. *A PBE where buyers use reservation price strategies is characterized by a price w^* set by the upstream firm, a distribution of retail prices $F(p)$, one for each w , and non-shoppers’ reservation price ρ^* such that*

²⁰This was not an issue in the previous section because consumers observed w , so if any retailer charged an out-of-equilibrium price this indicated a deviation by that particular retailer.

²¹Thus, if consumers would like to attribute a price observation above the reservation price to the manufacturer only, then they have to assume that the manufacturer has set the wholesale price $w > \rho^*$. Given this, the working paper version of this paper shows that the beliefs we use can be rationalized further and are consistent with *the logic* of the D1 refinement that considers which firm has most incentives to deviate (Cho and Sobel (1990)). The D1 criterion is, however, developed for different types of games than ours.

²²The precise specification of these beliefs is not important. What is important is that when manufacturer adjusts its prices upwards, the upper bound of the retail price distribution remains the retail monopoly price.

1. the manufacturer chooses w to maximize its expected profit (which depends on $F(p)$, ρ^* and non-shoppers' out-of-equilibrium beliefs);
2. each retailer uses a price strategy $F(p)$ with support $[\underline{p}(w), \bar{p}(w)]$, where $\bar{p}(w) = \min(\rho^*, p^m(w))$, that maximizes expected profit, given the actual w chosen by the manufacturer, the competing retailer's price strategies, non-shoppers' reservation price ρ^* , and non-shoppers' out-of-equilibrium beliefs;
3. non-shoppers' reservation price ρ^* is such that they search optimally given their beliefs about w and $F(p)$; shoppers observe all prices and then buy at the lowest retail price.
4. non-shoppers' beliefs are updated using Bayes' Rule when possible. Out-of-equilibrium beliefs are such that
 - i. if $\bar{p}(w) = \rho^* \leq p^m(w)$, then consumers believe that the upstream price equals w^* if a price $p > \rho^*$ is observed;
 - ii. if $\bar{p}(w^*) = p^m(w^*) < \rho^*$, then if a price $p > p^m(w^*)$ is observed, consumers believe that the upstream price w is such that $p = p^m(w)$ and no retailer has deviated.
 - iii. if a price $p < \underline{p}(w)$ is observed, consumers believe that the upstream price equals w^* .

It is not difficult to see that the behavior of downstream retailers in this unobserved case is very similar to what was described in the previous subsection, the main difference being that now the upper bound is given by $\bar{p}(w) = \min(\rho^*, p^m(w))$ instead of $\bar{p}(w) = \min(\rho(w), p^m(w))$. Since we impose $\rho^* = \rho(w^*)$ in equilibrium, the difference between the case where w is observed and where it is unobserved is fairly subtle. In the unobserved case considered here, for every $w \leq \rho^*$ expected profit of the upstream firm is given by $\pi(w, \rho^*)$ while in the observed case it is given by $\pi(w, \rho(w))$. Note also that Assumption 1 does not imply that $\pi(w, \rho^*)$ is quasi-concave, and in fact it may well not be, as we indicate when discussing the possible non-existence of a reservation price equilibria in the next Section. The upstream profit function is also given by (10), with the difference that now $\bar{p}(w) = \min(\rho^*, p^m(w))$.

As with the observed case, depending on parameters, one of two types of equilibria prevails. One type is where $\bar{p}(w^*) = p^m(w^*) < \rho^*$ and the upstream firm chooses w^* such that

$$\frac{\partial \pi(w, p^m(w))}{\partial w} = 0.$$

The other type is where $\rho^* < p^m(w^*)$. In this case, the necessary condition on the upstream price w^* is

$$\frac{\partial \pi(w, \rho^*)}{\partial w} = 0,$$

where the reservation price ρ^* equals $\rho(w^*)$. For this to be an equilibrium, for a fixed ρ^* , and given the optimal reaction by retailers, w^* should maximize profits out of all possible $w \in [0, \hat{p}]$. In Section 4 we will show that for some parameter values this equilibrium may fail to exist.

3.3 A Characterization and Comparison

In order to facilitate the comparison of equilibrium behavior in the two cases, define $w^m = \arg \max \pi(w, p^m(w))$. By Assumption 1, w^m is unique and solves

$$\frac{\partial \pi(w, p^m(w))}{\partial w} = 0. \quad (12)$$

It is clear from this definition that if $p^m(w)$ is the upper bound of the retail price distribution for all w , then there is no difference between the observed and unobserved cases and w^m is the upstream firm's equilibrium choice. The intuition is simple: the only difference between the observed and unobserved cases is in the determination of the upper bound of the retail price distribution and whether this upper bound depends on the expected or the actual upstream price. In case this upper bound is given by the retailers' behavior and not determined by the search behavior of consumers, the upper bound is determined in the same way across the two cases. As this happens when the search cost is high, it is clear that for sufficiently high search cost the two models coincide.

For relatively small search costs in the observed case, define w^o as the solution to

$$\frac{\partial \pi(w, \rho(w))}{\partial w} = 0. \quad (13)$$

By Assumption 1, the solution exists and is unique. As we show later, w^o is the equilibrium choice of the upstream firm in case the upper bound for the price distribution charged by the retailer is the non-shoppers' reservation price $\rho(w)$.

For the unobserved case, let w^u , along with ρ^u , be the solution, if it exists,²⁴ to

$$\frac{\partial \pi(w, \rho^u)}{\partial w} = 0,$$

where $\rho^u = \rho(w^u)$. As with the observed case, and to be shown later, w^u will be the equilibrium upstream price in the unobserved case if the upper bound of the retailers' price distribution is given by the reservation price ρ^u .

Finally, for every λ define \bar{s}_λ as the search cost such that $\rho(w^m) = p^m(w^m)$, i.e., \bar{s}_λ is such that the non-shoppers' reservation price equals the retail monopoly price in case the manufacturer sets its price equal to w^m . As $w^m < p^m(w^m) < \hat{p}$ and the reservation price increases in s up to \hat{p} starting from w^m (when s equals 0), it is clear that \bar{s}_λ is uniquely defined.

The next theorem states our main result. For search cost values smaller than the critical threshold value \bar{s}_λ , if in the unobserved case a PBE where buyers use reservation price strategies exists, then the manufacturer chooses a higher upstream price than in the equilibrium of the observed case. As in equilibrium consumers correctly anticipate the manufacturer's behavior, reservation prices will be higher in the unobserved case implying that the distribution of retail prices in the unobserved case first-order stochastically dominates the distribution of retail prices

²³Note that w^o may be such that $\rho(w^o) > p^m(w^o)$, in which case $f(p)$ as defined in (7) is negative. Nevertheless, the function $\pi(w, \rho(w))$ is well defined, even if it does not represent the expected profit of the upstream firm. This is not an issue, because we use w^o only where $\rho(w^o) \leq p^m(w^o)$.

²⁴This system may have no solution, as discussed later. Also, as with w^o , w^u may be such that $\rho(w^u) > p^m(w^u)$, in which case $f(p)$ is negative for the range of w where $\rho(w) > p^m(w)$.

in the observed case. For search cost values larger than \bar{s}_λ the manufacturer's behavior is independent of the search behavior of consumers and the equilibria in the two cases coincide.

Theorem 1. (i) If $s \geq \bar{s}_\lambda$, then equilibrium always exists and the upstream price is identical in both cases and is equal to w^m ; (ii) If $s < \bar{s}_\lambda$, then the equilibrium upstream price in the observed case equals w^o and, if equilibrium exists, the upstream price in the unobserved case equals w^u with $w^o < w^u$.

The technical part of the proof, dealing with the threshold value \bar{s}_λ , is in the Appendix. For the substantive part arguing that $w^o < w^u$ when $s < \bar{s}_\lambda$, note that in the previous section, we have argued that the main difference between the two cases is the term $\frac{\partial Q}{\partial w^e}$. In our model, w^e affects demand through ρ . The next Lemma shows that ρ positively depends on w^e . The reason is that if w^e increases, consumers expect to get a worse deal if they continue to search the next retailer and are therefore more willing to buy now.

Lemma 1. *The reservation price is increasing in the non-shoppers' belief about the upstream price: $\rho'(w^e) > 0$.*

For the effect of the reservation price ρ on demand, the following lemma states that upstream demand decreases in ρ and in the proof we argue that the retail price distributions first-order stochastically dominate each other as ρ increases.

Lemma 2. *For a given w , the manufacturer's demand and profit are decreasing in ρ for $w \leq \rho < p^m(w)$.*

Taken together, the two lemmas imply that upstream demand decreases as w^e increases and thus, in terms of the previous section, $\frac{\partial Q}{\partial w^e} < 0$.

Figure 1 illustrates the difference between the two cases for the same increase in w . In the observed case the manufacturer demand falls more because $\rho(w)$ also increases, whereas in the unobserved case ρ is constant, depending on consumers' expectations about w , but not on w itself. Thus, with an increase in w the retail price distribution shifts more to the right if w is observed than when it is not observed.

Since w^o maximizes the upstream firm's profits for correct beliefs, and in the unobserved case beliefs are correct in equilibrium, the upstream firm prefers to be in the observed than in the unobserved case. Since retail prices increase as w increases, consumers are also worse off in the unobserved case. As for the retailers, their profits change in a complex way with w . When w , their marginal cost, increases, their profits fall because the marginal cost is higher, but the reservation price also increases, which given that the reservation price is below p^m , increases their profits. One can show however, that for sufficiently large λ , or s sufficiently close to, but smaller than, \bar{s}_λ , retailers are also worse off in the unobserved case.²⁵

Proposition 2. *The manufacturer and consumers are worse off in the unobserved case compared to the observed case. There exist $\tilde{\lambda}$ and \tilde{s} such that this is also true for retailers for $\tilde{s} < s < \hat{s}_\lambda$ or $\tilde{\lambda} < \lambda < 1$.*

²⁵These conditions are sufficient, but not necessary. We have been unable to find examples where retailers are better off in the unobserved case and believe, but were unable to show, that retailers are always worse off in the unobserved case regardless of λ or s .

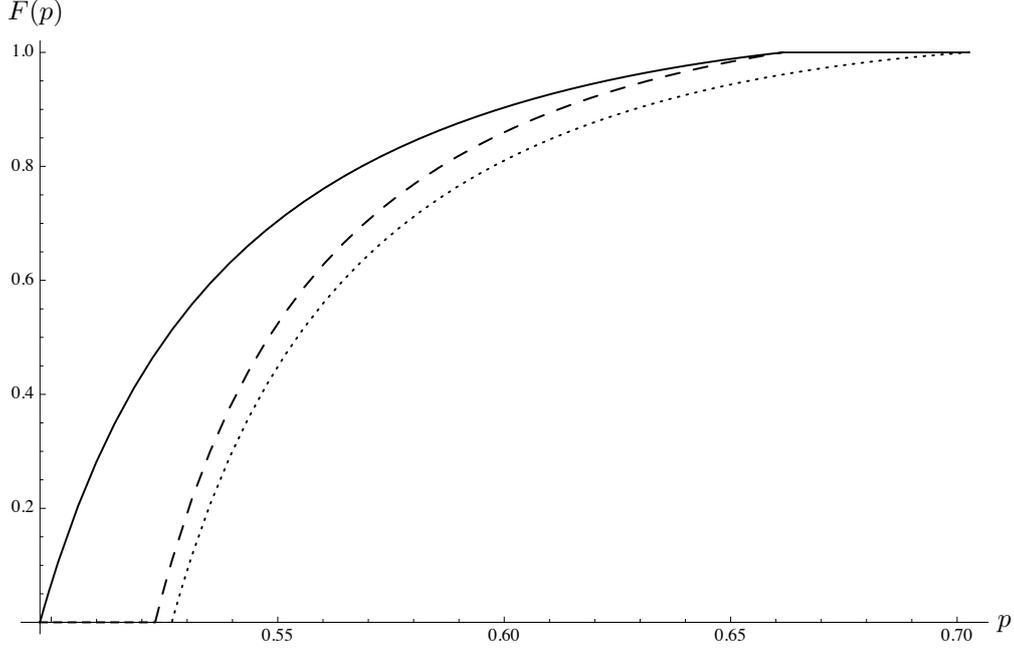


Figure 1: Cumulative distribution of downstream prices for $w = 0.45$ and $\rho = \rho(0.45) = 0.6615$ (solid), $w = 0.48$ and $\rho = \rho(0.48) = 0.7026$ (dotted), and $w = 0.48$ and $\rho = \rho(0.45) = 0.6615$ (dashed).

4 Linear Demand

For general demand functions, it is not technically tractable to obtain an explicit expression for the optimal upstream price w^* . It is therefore impossible to get an impression of how large the effect due to the unobservability of the upstream price can be. Comparative statics are also hard to derive. Moreover, we have assumed so far that a reservation price equilibrium always exists. In the case where upstream prices are observed by consumers, existence is not an issue. In the more important case where they are not observed, existence is a non-trivial issue, however. In this section, we focus on linear demand to be able to discuss these issues in more detail.

4.1 High search cost

From the previous section we know that when search costs are relatively high, the two models coincide. In case of linear demand, we can calculate the equilibrium upstream price and it turns out that it is equal to the monopoly price of a vertically integrated monopolist.

Proposition 3. *Suppose $D(p) = 1 - p$. For all λ , if $s \geq \bar{s}_\lambda$, $w^m = 1/2$.*

For high search cost the upstream firm chooses the monopoly price when demand is linear. This follows from the special relationship between the upper and lower bound of the price distribution where the upper bound is the retailers' monopoly price in this case.

4.2 Low search cost

A more interesting comparison arises when search costs are low. It is still difficult to solve the maximization problem of the upstream firm for general s , as for s values smaller than \bar{s}_λ

the upper bound of the retailers' price distribution equals the reservation price and this price depends in a non-trivial way on w . Nevertheless, as the next Proposition illustrates, it is possible to obtain an interesting comparison between the limiting results of the two models when the search cost s vanishes.

Proposition 4. *Suppose demand is given by $D(p) = 1 - p$. If s approaches 0, then*

- (a) w^o approaches $1/2$,
- (b) w^u approaches $1/(1 + \lambda) > 1/2$ if a reservation price equilibrium exists.

In the observed case, the limiting result for search cost approaching zero can be easily understood. As the reservation price has to converge to the retailers' cost there is (almost) Bertrand competition at the retail level with retail prices almost equal to marginal cost. As this cost is known to consumers, they effectively demand $1 - w$ and therefore, the upstream profit function is simply $w(1 - w)$, which is maximized at $1/2$.

In the unobserved case, the limiting result is very different. Here, the model exhibits a discontinuity at $s = 0$ that is similar to the Diamond paradox (see Diamond (1971) and Rhodes (2012) for a recent contribution). When $s = 0$, all consumers observe both prices before buying, creating Bertrand competition at the downstream level. However, when s is small, non-shoppers have a reservation price based on an expected upstream price and given this expectation, the manufacturer pushes the upstream price up to the expected reservation price. In equilibrium, it has to be the case that the manufacturer does not have an incentive to raise prices above the level that is expected by consumers. For low search cost, this happens, however, at a much higher upstream price where a further (unobserved) increase in w is ultimately unprofitable.

Theorem 1 and Proposition 4 allude to the possibility that when consumers do not observe the upstream price, a reservation price equilibrium may not exist. The reason for this potential non-existence can be understood intuitively by looking at the profit maximization problem of the upstream firm. As the demand of non-shoppers is relatively inelastic, the manufacturer may have an incentive for a given reservation price to choose higher price levels, lose some demand from shoppers but squeeze retailers. This is especially true when the fraction of non-shoppers is relatively large and so the upstream price is high. It may be so high, that if consumers were to expect it to be charged by the upstream firm, the upstream firm would find it profitable to deviate to a lower price (at or close to $w = 1/2$), but if consumers expected such a price, the upstream firm would again want to deviate to a higher price. Thus, no pure strategy equilibrium would exist.

The next proposition establishes that there is a region of (s, λ) values where a reservation price equilibrium does not exist.²⁶

Proposition 5. *In the unobserved case where demand is given by $D(p) = 1 - p$, there exists a critical value of λ , denoted by λ^* , such that for all $\lambda < \lambda^*$ there exists a $\tilde{s}(\lambda)$ such that for all $s < \tilde{s}(\lambda)$ a reservation price equilibrium does not exist, where $\lambda^* \approx 0.47$ solves $\frac{2(1-\lambda(1+\lambda))}{\lambda} - \frac{(1-\lambda)^2}{\lambda^2} \log\left(\frac{1+\lambda}{1-\lambda}\right) = 0$.*

²⁶If in an equilibrium w is fixed, then optimal stopping rule for nonshoppers is characterized by a reservation price strategy. It follows that if a reservation price equilibrium does not exist, then in any equilibrium the upstream firm has to randomize.

At a technical level, Figure 2 provides more detail on why for these values a reservation price equilibrium does not exist. In each panel of the figure two curves are drawn. The consumers' reservation price is drawn for a given (correctly anticipated) upstream price, and the resulting retail price distribution. The second curve gives the manufacturer's optimal price for a given reservation price. A reservation price equilibrium is represented by the intersection of these two curves inside the shaded area. In panel (a) we see for $\lambda = 0.32$ and $s = 0.02$ that the "best response" curve of the upstream firm is discontinuous. For reservation prices smaller than 0.5 the upstream firm simply sets the monopoly price of the vertically integrated monopolist, inducing all consumers to obtain two price quotes and fully squeezing retailers. For reservation prices larger than 0.75, the manufacturer sets w^m such that the upper bound equals the retail monopoly price. For intermediate levels of the reservation price, the manufacturer again fully squeezes the retailers by setting $w = \rho$ for reservation prices somewhat larger than 0.5, but at higher reservation prices this becomes too costly (as it lowers demand) and he jumps to a lower price level. Non-existence arises if the reservation price curve crosses this hole in the best response curve of the upstream firm. In panel (b), we see that for a larger value of λ the two curves intersect. This is because with more informed consumers a larger fraction of consumers buys at prices at the lower end of the price distribution at prices closer (but larger) than 0.5, where the retailer does not make a high margin. By jumping to a wholesale price equal to the reservation price, the upstream firm loses profits over these consumers and as these consumers have a higher weight in the overall profit considerations of the upstream firm, the incentive to set the upstream price equal to the (expected) reservation price is reduced (and for high enough λ eliminated).

At a more intuitive level, Proposition 5 can be understood by relating it to the Diamond paradox. The Diamond Paradox, where a pure strategy equilibrium does not exist, can arise in search models because for a given level of the (equilibrium) price that is expected by consumers, a firm has an incentive to increase the price. The Diamond paradox relies on the fact that in a search environment, before they engage in search, consumers are uninformed about the price at which they may buy, and after they find out the price the search cost is sunk. This effect is also present in our model, but it is softened by the fact that a fraction of consumers incurs no search cost - the Stahl (1989) "solution" to the Diamond paradox - and the existence of an intermediate retail level. If the search cost is low, the intermediate retail level does not provide much of a buffer, however, as retail prices are close to the upstream price. If the fraction of shoppers is small, the Stahl "solution" is also ineffective. Therefore, when the search cost is low and the fraction of shoppers is small the Diamond paradox enters our model in full force, eliminating any pure strategy equilibrium.

To establish the region where a reservation price equilibrium exists is more complicated. The non-existence result of Proposition 5 is proved by considering when the second-order condition for profit maximization of the manufacturer is violated in the limit as the search cost approaches zero. For existence, this local second-order condition is necessary, but not sufficient. Figure 3 shows the result of a numerical analysis checking, for each parameter combination, whether the manufacturer's choice of upstream price is a global maximum. The Figure provides a complete

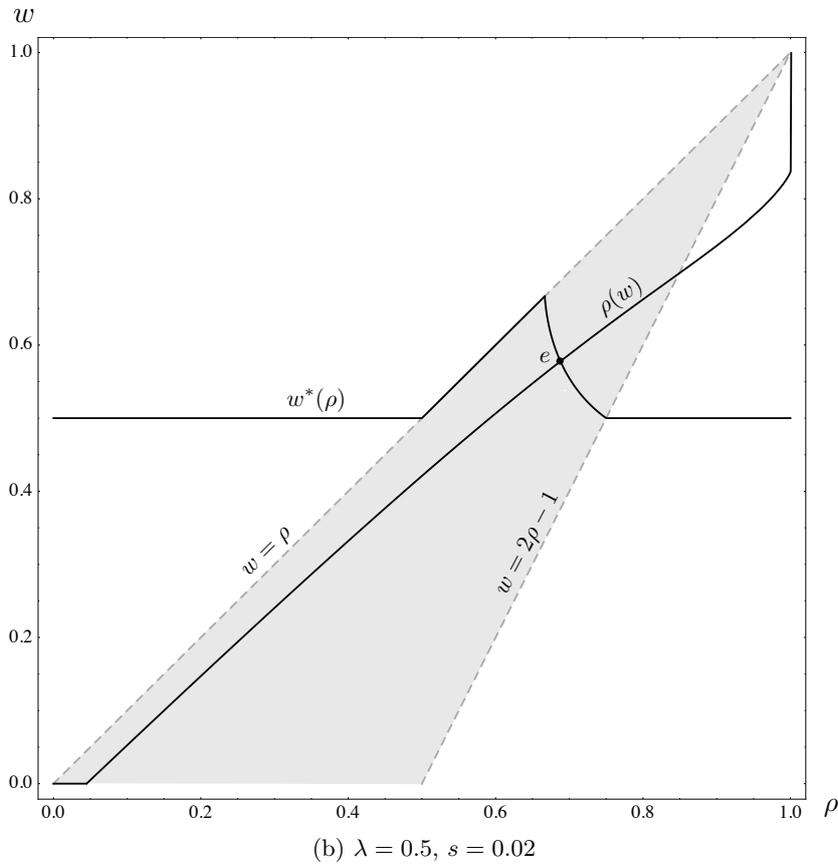
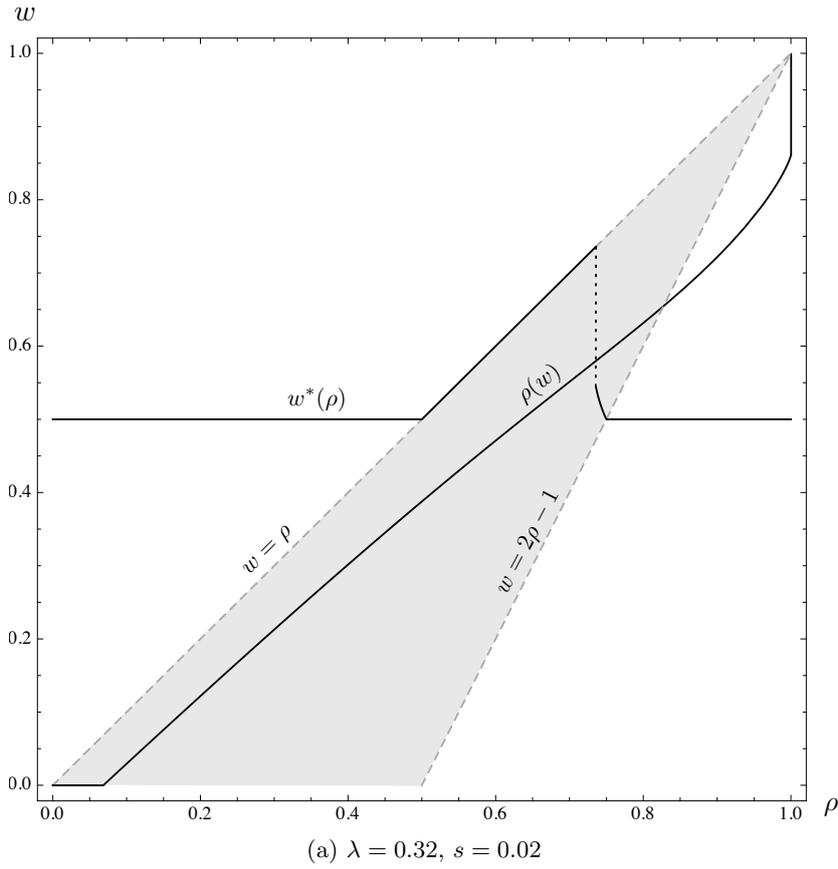


Figure 2: Equilibrium nonexistence for $D(p) = 1 - p$. For the reservation price equilibrium to exist, the two solid curves have to cross in the shaded area. In the first panel the optimal upstream price given a reservation price exhibits discontinuity (dotted line) that leads to equilibrium nonexistence.

characterization when a reservation price equilibrium does and does not exist.²⁷

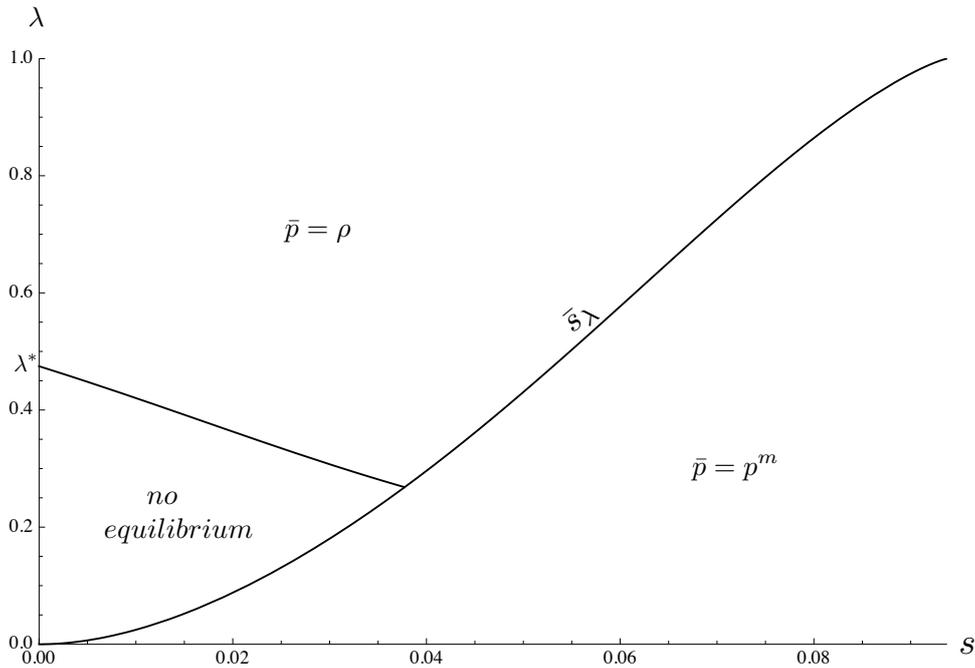


Figure 3: Different equilibrium cases depending on s and λ .

4.3 Comparative statics

We next explore the possible size of the price raising effect due to the non-observability of the upstream price and inquire into the comparative static analysis of changes in s and λ . Figures 4 and 5 illustrate how much the upstream price is higher in the unobserved case for different parameter values. When $\lambda = 1/2$, Figure 4 confirms that for high search cost s , the upstream price w equals $1/2$, while for smaller values of s the upstream price is larger in the observed case. It also confirms the limiting results when s becomes very small (Proposition 4). For these parameter values, the Figure shows that the price raising effect due to unobservability can be quite substantial with the upstream price being up to 33% larger in the unobserved case. Figure 5 shows a similar picture where we fix s at 0.03 and we vary λ . For small λ , $s > \bar{s}_\lambda$ and the upstream price w equals the monopoly price of a vertically integrated monopolist. When λ approaches 1, we also have that the upstream price w approaches this vertically integrated monopoly price. In between $w^u > 0.5 > w^o$.

The two figures also depict $\tilde{p} = \lambda E \min(p_1, p_2) + (1 - \lambda)Ep$ in both cases. These are the prices consumers expect to pay before they “know” whether they are shoppers or non-shoppers. Figure 4 clearly shows the different effect of search cost s on \tilde{p} in both cases. When the upstream price is observed, we have the standard effect in the consumer search literature that expected prices are increasing in the search cost. However, in the unobserved case, \tilde{p} is decreasing in s . The reason for this counterintuitive result is that there are two countervailing forces here. First, when the search cost increases, retailers have more market power and can increase their

²⁷One may wonder what type of equilibria do exist if a reservation price equilibrium does not exist. This is, however, an inherently difficult question to answer as one has to look for mixed strategy (upstream) equilibria where consumers follow a search strategy that is not characterized by a reservation price.

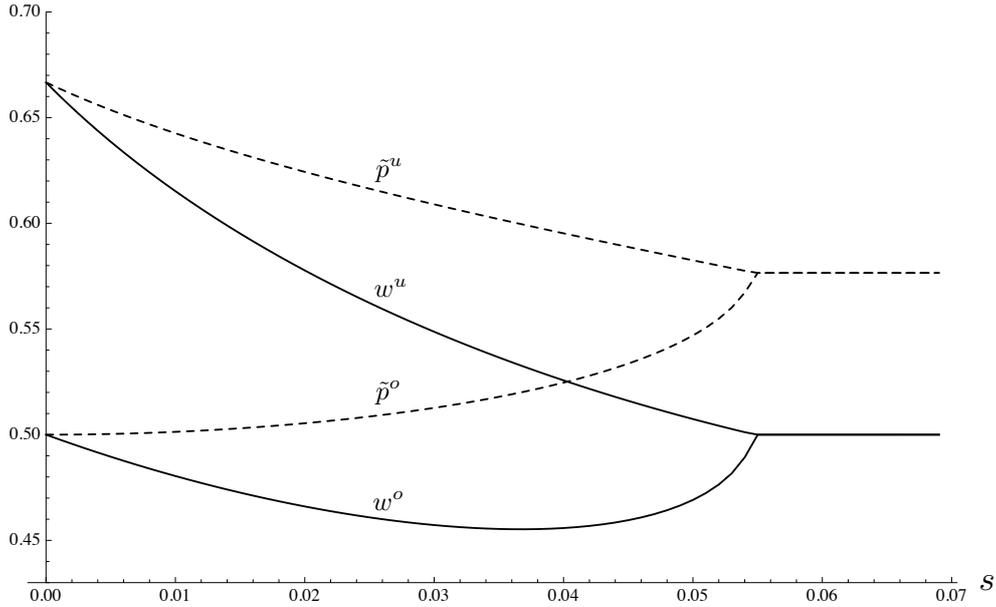


Figure 4: Upstream prices (solid lines) and average weighted downstream prices (dashed) for the two cases as functions of s for $\lambda = 0.5$.

retail margins for given cost. Second, the double marginalization problem becomes less severe as retailers are able to pass on upstream price increases and the upstream firm internalizes this effect and charges significantly lower prices in response (see the steep decrease in the w^u curve in Figure 4). As the second effect dominates in the case of linear demand, the total effect is that retail prices are decreasing. The fact that in the unobserved case the expected retail price has to be at least locally decreasing in s is a corollary to Propositions (3) and (4). Together these Propositions state that, in the observed case the wholesale price (and thus the retail price distributions) are identical for $s = 0$ and $s \geq \bar{s}_\lambda$. As the wholesale price for s approaching 0 in the unobserved case is larger than in the observed case, whereas the two cases coincide for $s \geq \bar{s}_\lambda$, it follows that at least locally retail prices must be decreasing in s in the unobserved case.

Corollary 2. *If λ is such that in the unobserved case a reservation price equilibrium exists for all $s < \bar{s}_\lambda$, then there must be values of s such that \tilde{p} is decreasing in s .*

This result sheds an alternative perspective on the effect of websites like www.edmunds.com that attempt to reduce search costs by making it easier for consumers to find lower car prices. Websites that intend to help consumers in getting better deals may in the end lead to higher prices, unless they also inform consumers about upstream prices.

The figures also clearly convey the idea that the search impact on the double marginalization problem may well be of quantitative significance: the average retail price \tilde{p} in the unobserved case may be significantly higher (up to 33% higher for $D(p) = 1 - p$) than in the observed case. Identifying the normal double marginalization problem with the extent to which this latter \tilde{p} is larger than the vertically integrated monopoly price of 0.5, the figures also show that the strengthening of the double marginalization problem by the consumers' inability to observe upstream prices may outweigh the normal double marginalization effect.

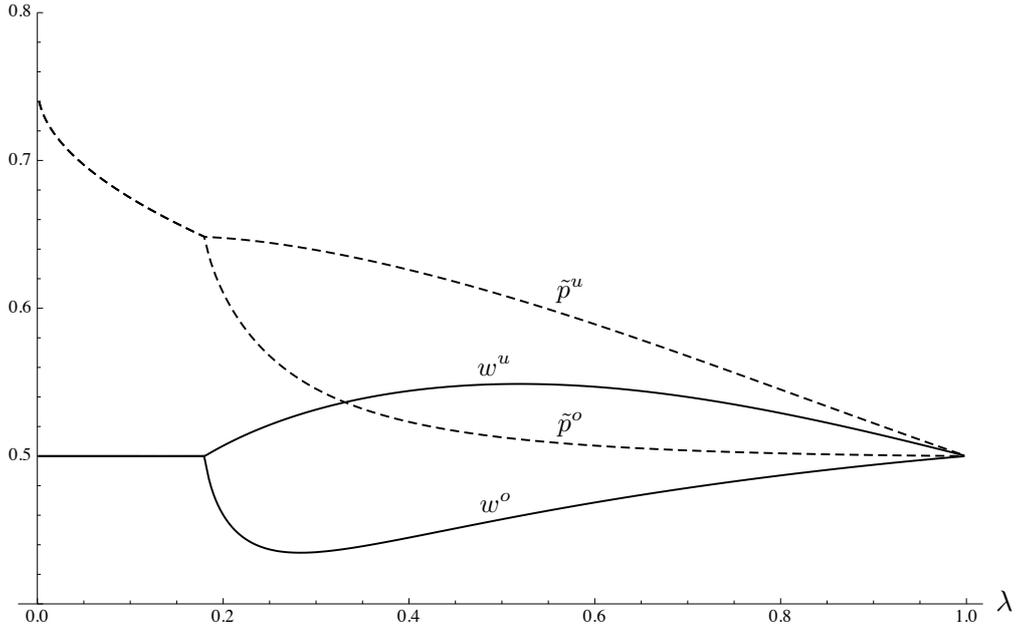


Figure 5: Upstream prices (solid lines) and average weighted downstream prices (dashed) for the two cases as functions of λ for $s = 0.03$.

Table 1: Equilibrium for observed w , $D(p) = 1 - p$ and $\lambda = 0.5$.

s	w	\underline{p}	\bar{p}	\tilde{p}	π	$2\pi_r$	$\pi + 2\pi_r$	CS	W
0.001	0.498	0.499	0.502	0.500	0.249	0.001	0.250	0.125	0.375
0.02	0.466	0.491	0.552	0.505	0.231	0.019	0.250	0.122	0.372
0.05	0.469	0.516	0.688	0.547	0.213	0.034	0.247	0.103	0.350
0.07	0.500	0.546	0.750	0.577	0.212	0.031	0.243	0.090	0.333

Table 2: Equilibrium for observed w , $D(p) = 1 - p$ and $s = 0.05$.

λ	w	\underline{p}	\bar{p}	\tilde{p}	π	$2\pi_r$	$\pi + 2\pi_r$	CS	W
0.2	0.500	0.606	0.750	0.643	0.179	0.050	0.229	0.064	0.293
0.5	0.469	0.516	0.688	0.547	0.213	0.034	0.247	0.103	0.350
0.7	0.475	0.496	0.643	0.513	0.231	0.018	0.249	0.119	0.368

Table 3: Equilibrium for unobserved w , $D(p) = 1 - p$ and $\lambda = 0.5$.

s	w	\underline{p}	\bar{p}	\tilde{p}	π	$2\pi_r$	$\pi + 2\pi_r$	CS	W
0.001	0.661	0.663	0.667	0.664	0.222	0.001	0.223	0.057	0.280
0.02	0.578	0.607	0.688	0.624	0.217	0.017	0.234	0.071	0.305
0.05	0.507	0.552	0.740	0.582	0.212	0.030	0.242	0.088	0.330
0.07	0.500	0.546	0.750	0.577	0.212	0.031	0.243	0.090	0.333

Tables 1-4 summarize the findings of our numerical analysis, where we also study the impact of changes in s and λ on upstream and downstream profits, (weighted) consumer surplus and total welfare (which here is measured as the sum of total industry profit and consumer surplus).

Table 4: Equilibrium for unobserved w , $D(p) = 1 - p$ and $s = 0.05$.

λ	w	\underline{p}	\bar{p}	\tilde{p}	π	$2\pi_r$	$\pi + 2\pi_r$	CS	W
0.2	0.500	0.606	0.750	0.643	0.179	0.050	0.229	0.064	0.293
0.5	0.507	0.552	0.740	0.582	0.212	0.030	0.242	0.088	0.330
0.7	0.516	0.537	0.701	0.555	0.230	0.017	0.246	0.099	0.346

As retail prices are chosen according to a mixed strategy distribution, the table reports expected values for profits, consumer surplus and welfare. As the first search is free and consumers do not search beyond the first search, search cost is not incorporated into the measure of total welfare. Tables 1 and 2 provide the numerical analysis of the observed and unobserved case for a fixed value of $\lambda = 0.5$ and different values of s . Tables 3 and 4 provide the numerical analysis of the observed and unobserved case for a fixed value of $s = 0.05$ and different values of λ .

The tables convey two important messages.²⁸ The first message is that when consumers do not observe the upstream price, an increase in search cost is both good for total industry profit and for consumers. The reason is that as (i) \tilde{p} at which consumers buy are initially very high (*i.e.*, considerably higher than the vertically integrated monopoly price) and as (ii) an increase in search cost decreases this average retail price, total surplus is much higher with higher search cost and firms benefit from the increase in demand. As already explained above, it is the retail firms that really benefit from this increased search cost (as they can increase their margins) at the expense of the upstream firm. Again, the effects are quantitatively significant: Table 2 shows that an increase in search cost from a level close to 0 to 0.07 may increase welfare by 20%. (The effects on consumer surplus and welfare are so strong that even if we assume the first search comes at a cost s , consumer surplus and welfare are still increasing in s).²⁹

The second message is that comparing the observed case to the unobserved case (that is comparing Tables 1 and 2, and also Tables 3 and 4) reveals that all market participants benefit when the upstream price is observed by consumers. Again, the main difference between the two cases is that the weighted average of retail prices is much lower in the observed case, and as these prices are above the vertically integrated monopoly price, total surplus generated in the observed cost case is much higher. In this case, all market participants benefit and for some parameter values the total change in welfare can be as high as 35 percent.

5 Extensions

In this section we consider several extensions.³⁰ We first consider an extension with upstream competition where multiple manufacturers compete to be able to have all retailers sell their product. We show that upstream and retail prices continue to be larger in the unobserved case, resulting in lower welfare. However, the resulting retail prices may now be below the

²⁸As an aside, note that not only the expected price consumers pay is higher in the unobserved case, but also that the price spread, defined as the difference between upper and lower bound, is larger in the unobserved case.

²⁹Note that when $\lambda = 0.5$ only half of the consumers pays a search cost and in that case we have to deduct half of the difference in search cost to see the effect on welfare and consumer surplus of including a first costly search.

³⁰In the working paper version of this paper we also show that the unobservability effect leading to larger upstream prices continues to exist under two-part tariffs, but that the order of magnitude is much smaller.

vertically integrated monopoly price (due to the competition upstream), and manufacturers may now have an incentive to keep the wholesale arrangement hidden from consumers. In a second extension we numerically confirm that our results, established in the main body of the paper for a downstream duopoly market, can be generalized to more than two retailers. Finally, we show that a monopolistic manufacturer will never want to price discriminate as its profits will be lower than when it charges both retailers the same price.

5.1 Upstream competition

In this extension, we point out that with upstream competition, manufacturers may not have an incentive to truthfully reveal the wholesale arrangement. To illustrate that the manufacturers' incentive to reveal information depends on the severity of upstream competition, we model upstream competition in such a way that it does not affect the analysis of the downstream market. To do so, we have one manufacturer serving all retailers, but different manufacturers compete for the right to serve the market. The choice of the monopoly manufacturer depends on the upstream prices that are offered by the different manufacturers, but also on other factors that are beyond their control. With N manufacturers, we introduce a probability $\gamma(w_i, w_{-i})$ that firm i wins the competition and serves the downstream market at the price w_i if other firms charge w_{-i} . The function γ can take different forms. If $\gamma(w_i, w_{-i}) = 1/N$ for all $\{w_i, w_{-i}\}$, then effectively there is no price competition upstream. There is Bertrand competition in the upstream market if $\gamma(w_i, w_{-i})$ is such that $\gamma(w_i, w_{-i}) = 1/N$ if all firms choose the same upstream price and $\gamma(w_i, w_{-i}) = 0$ if $w_i > \min w_{-i}$ and $\gamma(w_i, w_{-i}) = 1$ if $w_i < \min w_{-i}$. The severity of upstream competition is measured by the derivative of $\gamma(w_i, w_{-i})$ with respect to w_i evaluated at the symmetric point where $w_i = w_j$ for all i, j .³¹

Depending on whether or not consumers observe the upstream price, we then model upstream competition by having manufacturers choose w_i by maximizing $\gamma(w_i, w_{-i})\pi(w_i, \rho(w_i))$ or $\gamma(w_i, w_{-i})\pi(w_i, \rho^*)$, for the observed and unobserved cases, respectively.

In both cases, in a symmetric equilibrium the first-order condition for firm i is

$$\frac{1}{N} \frac{\partial \pi(w_i, \bar{p}(w_i))}{\partial w_i} + \frac{\partial \gamma(w_i, w_{-i})}{\partial w_i} \pi(w_i, \bar{p}(w_i)) = 0. \quad (14)$$

If the probability γ is unaffected by w_i , in which case $\frac{\partial \gamma(w_i, w_{-i})}{\partial w_i} = 0$, then all manufacturers will choose the same price as the upstream monopolist would do in the baseline model. If, on the other hand, $\frac{\partial \gamma(w_i, w_{-i})}{\partial w_i}$ is negative, then upstream prices will be lower than in the monopoly model.

In this modified model, it is still the case that upstream prices are higher in the unobserved case than in the observed case. In the monopoly model, this means that the upstream firm always prefers to be in the observed case, as in the unobserved case it ends up distorting its price choice upward. Under upstream competition, however, if the equilibrium upstream price falls enough because of competitive pressure, manufacturers will prefer to be in the unobserved case where prices are higher.

³¹The advantage of modeling competition in this way is that the probability $(1 - \gamma(w_i, w_{-i}))$ of losing i 's monopoly puts downward pressure on w_i , as competition should do, but the assumption of monopoly supply to retailers is preserved and thus the model is kept tractable.

For example, in the linear demand model with two upstream firms, $\lambda = 0.5$, $s = 0.02$ and $\gamma(w_i, w_j) = \frac{w_j^\gamma}{w_i^\gamma + w_j^\gamma}$, it is sufficient that $\gamma = 1.15$ for manufacturers to earn higher profits in the unobserved case. This is illustrated in Figure 6. The black solid curve depicts upstream monopoly profits for the observed case. As the manufacturer that wins the upstream competition is, in the end, a monopolist delivering the product to the retailers, the manufacturer's profit will be a point on this curve. Because of the upstream competition, a manufacturer does not choose the monopoly wholesale price, but (depending on the strength of the competition) a lower price. The dotted and dashed curves represent expected duopoly profits in the observed and unobserved case for a given equilibrium level of the wholesale price, respectively, scaled by a factor of 2 to make expected duopoly profit comparable with monopoly profits. It remains true that the equilibrium wholesale price under the unobserved case is larger than in the observed case, but as both prices are lower compared to the equilibrium values under monopoly, equilibrium wholesale profits are larger in the unobserved case if upstream competition is strong enough.

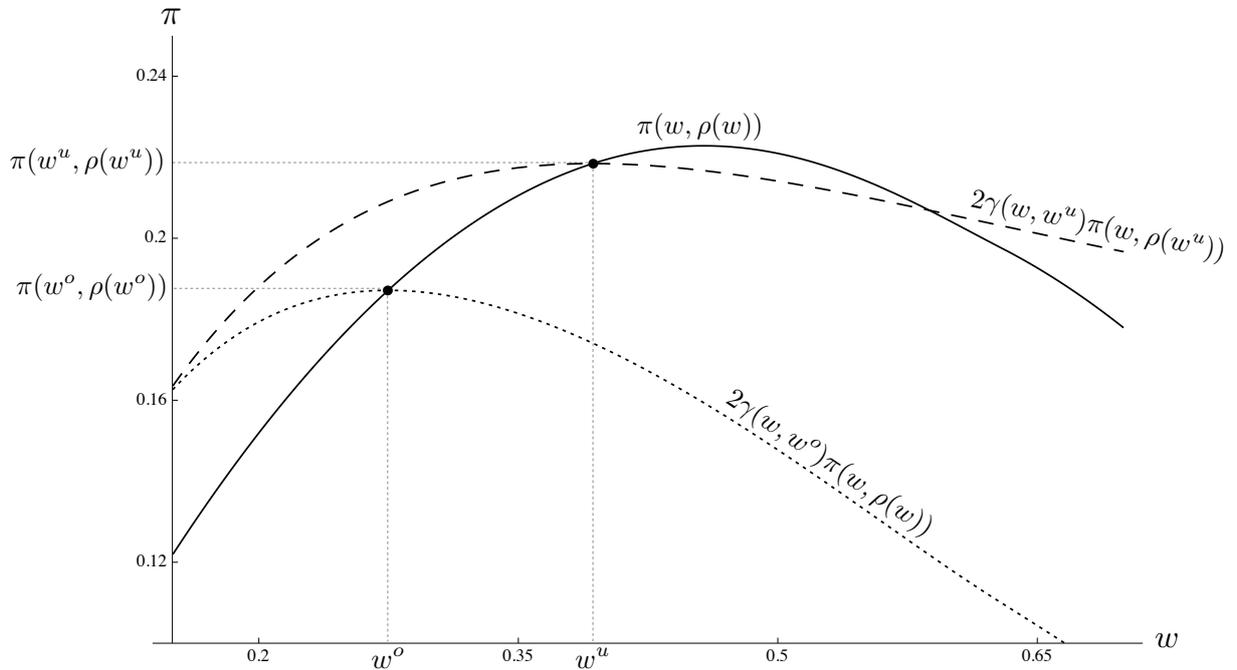


Figure 6: Upstream duopoly for $D(p) = 1 - p$ and $\gamma(w_i, w_j) = \frac{w_j^\gamma}{w_i^\gamma + w_j^\gamma}$ where $\gamma = 1.15$. The solid curve depicts upstream monopoly profit for w and the corresponding correct beliefs. The dotted and dashed curves represent expected duopoly profits in the observed and unobserved case for a given equilibrium level of the wholesale price, respectively, scaled by a factor of 2 to make expected duopoly profit comparable with monopoly profits.

To conclude, under upstream competition, manufacturers may not have an incentive to reveal information to consumers about the price they set. Apart from the fact that it may simply be too difficult credibly to convey information about upstream prices, upstream competition may be another reason why in real-world markets, firms do not reveal their wholesale price arrangements.

5.2 Retail Oligopoly

Next, we show that the qualitative properties of the equilibria under retail duopoly extend to retail oligopoly. The effects shown by the numerical analyses we present here are easily

interpreted. Roughly speaking, there are two effects. First, the region of parameters where the information consumers have on the upstream price makes a difference becomes smaller. This is quite intuitive if we recall the result by Stahl (1989) that the reservation price is increasing in the number of firms. In the context of our model, this implies that when the number of firms is larger the upper bound of the retail price distribution is given by the retail monopoly price for smaller values of s . As this is the region where the two models coincide, the region where there is a difference between the two information scenarios becomes smaller. Second, when the search cost is small, the price raising effect of unobservable upstream prices becomes stronger. That is, the larger the number of firms, the stronger is the effect that the equilibrium upstream price is decreasing in s . The reason is that with a larger number of firms, the retail price distribution gives more probability mass to higher prices, reducing the effect of an increase in the upstream price on the demand for the upstream firm.

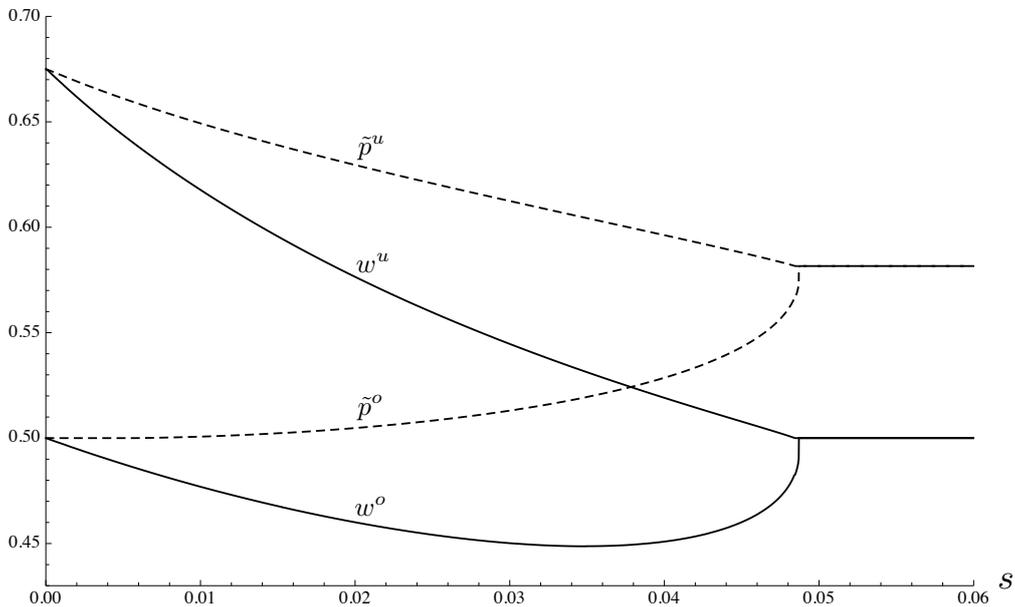


Figure 7: Upstream (solid) and weighted average downstream (dashed) prices for the two cases for $N = 3$.

We show these effects in two different ways. For $N = 3$, Figure 7 shows that the relationship between upstream and expected retail price as a function of s is similar to the corresponding figure for $N = 2$ (apart from the two differences noted above). Next, we also show the relationship between upstream price and the number of firms when the search cost is low. As explained above, Figure 8 shows that this relationship is increasing.

5.3 Price discrimination between retailers

In the main body of the paper, we considered environments where the upstream firm cannot price discriminate between different retailers. In this extension, we discuss the kind of considerations that apply when price discrimination is feasible and ask whether or not the symmetric equilibrium without price discrimination we characterized so far remains an equilibrium.

To consider this question, we have to distinguish two different cases, depending on whether or not rival retailers observe the upstream price the other retailer pays. In the main part of

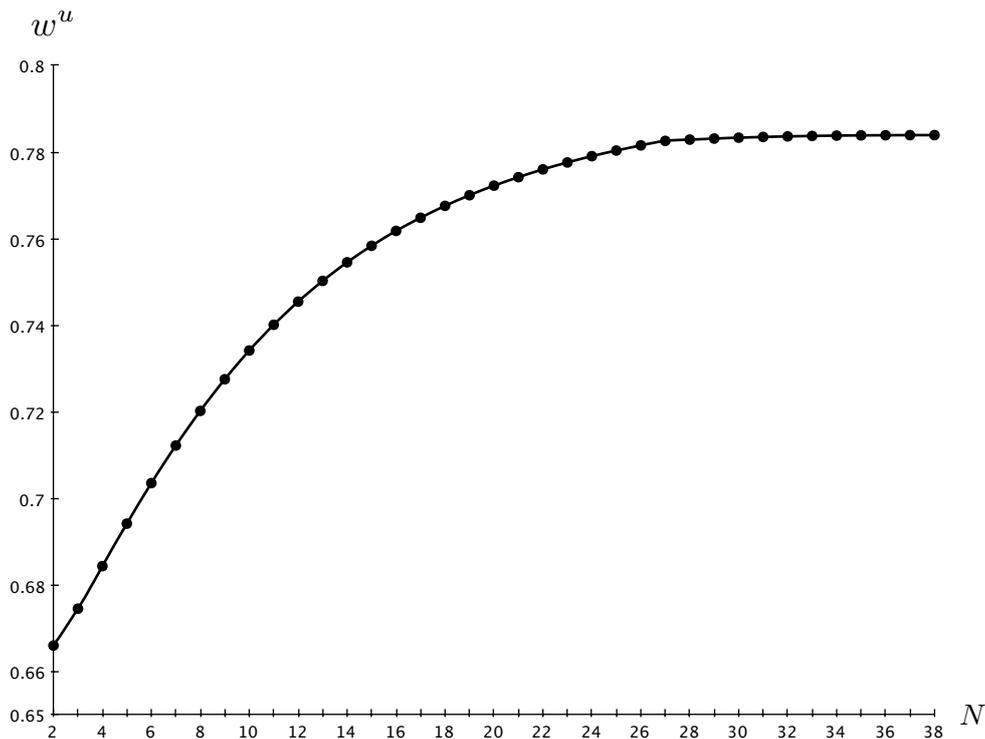


Figure 8: The upstream price in the unobserved case for $\lambda = 0.5$ and $s = 0.0001$ as function of N .

the analysis where retailers know the upstream firm cannot price discriminate, these two cases coincide as knowing your own cost level you also know the cost of your competitor. Here, we only consider the case where retailers do not observe the upstream price rival firms pay.³²

Whether or not it is optimal for the upstream firm to deviate from charging symmetric prices and introduce price discrimination depends on the beliefs retailers have about the deviation chosen by the upstream firm. Note that in this case of private information about their own upstream price, after observing a deviation by the upstream firm, retailers have to form beliefs about the upstream price the deviating upstream firm charges to the rival retailer, and they have to have beliefs about the rival's beliefs about their own upstream price, and so on. There is no standard refinement notion in game theory that restricts this type of beliefs, and one can easily construct beliefs such that the retailers react in such a way that the upstream firm's deviation is not profitable. For example, if after observing a deviation, a retailer believes that the upstream firm has deviated in such a way that both retailers have the same cost and believes that the other retailer has similar beliefs, then our analysis of no price discrimination applies.³³ Thus, one can support the symmetric equilibrium we have concentrated on so far, also when the upstream firm can price discriminate between retailers by appropriate beliefs of the retailers after a deviation of the upstream firm.

³²The case where rival retailers observe each other's costs is discussed in the working paper version of this paper (Janssen and Shelegia (2012)). There we show that if retailers observe each other's cost, local deviations are not profitable, but to show that global deviations are also unprofitable is beyond analytical tractability. Given that related literature (see, e.g., McAfee and Schwartz (1994)) has concentrated on secret discounts, the case where the price discrimination is kept secret seems more worthwhile to pursue.

³³This is only one set of beliefs that work. There are many other beliefs the retailers may hold, and depending on these beliefs the upstream firm either has or does not have an incentive to deviate.

Note that whether or not retailers observe each other's wholesale arrangement, the upstream firm does not have an incentive to charge one of the retailers such a high price that it effectively forecloses this firm from actively selling in the market. The reason is that given that the equilibrium upstream price is larger than the vertically integrated monopoly price, the upstream firm benefits from downstream competition, and even though it is imperfect because of consumer search, it is better than having no downstream competition.

6 Discussion and Conclusion

This paper has investigated the implications of vertical industry structure in markets where consumer search is important. In particular, we have focussed on the effect of consumers not observing the upstream price in markets where this price is endogenously determined by an upstream manufacturer. We find that consumers not observing the upstream price has both important qualitative and quantitative effects on market outcomes. The most important qualitative result is that when the wholesale price is not observed by consumers, consumers' reservation price is not affected by the actual choice of the upstream price. This makes the upstream demand less elastic and gives manufacturers an incentive to increase their prices, squeezing the profits of the retailers. In equilibrium, the actual choice of the upstream price has to be correctly anticipated by consumers, but this can only arise at levels of the wholesale price that are higher than the price a vertically integrated monopolist would set. In numerical examples with linear demand, welfare may be around 20% lower because upstream prices are not observed.

Another important difference concerns the implications of changes in search cost. In the observed case, expected retail prices are increasing in search cost, and upstream prices are non-monotonic. In contrast, in the unobserved case, expected retail prices and upstream prices are *decreasing* in search cost. The main reason for this latter result is that the upstream firm internalizes the fact that retailers can charge higher margins when search costs are higher, and this causes the upstream firm to reduce its price.

Overall, this paper draws attention to the importance of taking into account the incentives of upstream firms for the study of retail markets where consumer search is important. Given the findings in this paper, we expect that other findings in the consumer search literature may be exacerbated once the upstream perspective is taken into account. In addition, the paper opens up the question of whether other topics in the study of vertical market structures (such as exclusive dealing, retail price maintenance, etc.) should be reconsidered for markets where consumer search is important. Finally, the paper redirects the theoretical literature on consumer search to consider non-reservation price equilibria.

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Appendix: Proofs

Theorem 1. (i) If $s \geq \bar{s}_\lambda$, then equilibrium always exists and the upstream price is identical in both cases and equal to w^m ; (ii) If $s < \bar{s}_\lambda$, then the equilibrium upstream price in the observed case equals w^o and, if equilibrium exists in the unobserved case, the upstream price equals w^u with $w^o < w^u$.

Proof. We first show that if $s = \bar{s}_\lambda$, then $w^o = w^u = w^m$.

The expected quantity sold by the upstream firm is given by

$$Q(w) = (1 - \lambda) \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) f(p) dp + 2\lambda \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) f(p) (1 - F(p)) dp.$$

As $\rho(w^m) = p^m(w^m)$ at $s = \bar{s}_\lambda$ demand is the same in all three relevant cases if the upstream firm charges w^m : (i) when the upper bound is the retail monopoly price, (ii) when the upper bound is the reservation price conditional on w^m (as in the observed case) and (iii) when the upper bound is the reservation price conditional on the non-shoppers' belief about the upstream price being w^m (as in the unobserved case). Thus, to determine whether the first-order condition for profit maximization yields the same w^m in all three cases, we only need to evaluate the derivative of the manufacturer's demand with respect to w and show that it is equal in all three cases.

This derivative equals

$$\begin{aligned} & (1 + \lambda) \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) \frac{\partial f(p)}{\partial w} dp - 2\lambda \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) \left(\frac{\partial f(p)}{\partial w} F(p) + \frac{\partial F(p)}{\partial w} f(p) \right) dp - \\ & (1 + \lambda - 2\lambda F(\underline{p}(w))) D(\underline{p}(w)) f(\underline{p}(w)) \frac{\partial \underline{p}(w)}{\partial w} + \\ & (1 + \lambda - 2\lambda F(\bar{p}(w))) D(\bar{p}(w)) f(\bar{p}(w)) \frac{\partial \bar{p}(w)}{\partial w}. \end{aligned}$$

From (7) we know that $f(p) = 0$ when $\bar{p}(w) = p^m(w)$ because $\pi_r'(p^m(w)) = 0$ by the definition of $p^m(w)$. Moreover, $F(\bar{p}(w)) = 1$ and $F(\underline{p}(w)) = 0$. Thus, we are left with an expression for the derivative as

$$\begin{aligned} & (1 + \lambda) \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) \frac{\partial f(p)}{\partial w} dp - 2\lambda \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) \left(\frac{\partial f(p)}{\partial w} F(p) + \frac{\partial F(p)}{\partial w} f(p) \right) dp - \\ & (1 + \lambda) D(\underline{p}(w)) f(\underline{p}(w)) \frac{\partial \underline{p}(w)}{\partial w}. \end{aligned}$$

We will first show that the two integrals reduce to identical expression in all three cases. This is certainly the case if $\frac{\partial F(p)}{\partial w}$ (and hence $\frac{\partial f(p)}{\partial w}$) is the same in all three cases. Using (6), if the upper bound equals the retail monopoly price

$$\frac{\partial F(p)}{\partial w} = - \frac{(1 - \lambda)(p^m(w) - p)D(p^m(w))}{2\lambda(p - w)^2 D(p)} - \frac{\pi_r(p^m(w))}{2\lambda\pi_r(p)} \frac{\partial p^m(w)}{\partial w},$$

which reduces to

$$\frac{\partial F(p)}{\partial w} = - \frac{(1 - \lambda)(p^m(w) - p)D(p^m(w))}{2\lambda(p - w)^2 D(p)}. \quad (15)$$

If the upper bound is given by $\rho(w)$, the derivative equals

$$\frac{\partial F(p)}{\partial w} = -\frac{(1-\lambda)\left((\rho(w)-p)D(\rho(w)) + (p-w)\pi'_r(\rho(w))\frac{\partial\rho(w)}{\partial w}\right)}{2\lambda(p-w)^2D(p)},$$

which at $s = \bar{s}_\lambda$, and thus $\rho(w) = p^m(w)$, reduces to the same expression as (15).

Finally, consider the upper bound is given by ρ as in the case where the wholesale sale price is not observed by consumers. In this case, as ρ does not depend on w , the derivative equals

$$\frac{\partial F(p)}{\partial w} = -\frac{(1-\lambda)(\rho-p)D(\rho)}{2\lambda(p-w)^2D(p)},$$

which at $s = \bar{s}_\lambda$ is again the same as (15).

Thus, the derivative of the manufacturer's demand with respect to his own decision variable w at $w = w^m$ when $s = \bar{s}_\lambda$ can only differ between these three cases if $\frac{\partial p(w)}{\partial w}$ differs between these cases. However, in all three cases the relation between upper and lower bound of the retail price distribution is given by $(1-\lambda)\pi_r(\bar{p}(w)) = (1+\lambda)\pi_r(\underline{p}(w))$. Taking the total differential and using the fact that at $s = \bar{s}_\lambda$, $\rho(w) = \rho = p^m(w)$ we have that in all three cases

$$\frac{d\underline{p}(w)}{dw} = \frac{(1+\lambda)D(\underline{p}(w)) - (1-\lambda)D(\bar{p}(w))}{(1+\lambda)\pi'_r(\underline{p}(w))}.$$

This completes the proof that if $s = \bar{s}_\lambda$, then $w^o = w^u = w^m$.

We next prove that in the observed case, when $s = \bar{s}_\lambda$, the upstream firm sets its price equal to w^m in equilibrium. Assume the opposite. Then there should exist w' such that $\pi(w') > \pi(w^m)$. By the definition of w^m , it cannot be that $\rho(w') \geq p^m(w')$. So it has to be the case that $\rho(w') < p^m(w')$. Then, the first order condition

$$\frac{\partial\pi(w, \rho(w))}{\partial w} \Big|_{w=w'} = 0$$

should hold. This is the condition that defines w^o , but as shown above, when $s = \bar{s}_\lambda$, this condition coincides with the condition that defines w^m . Since w^m is unique, we conclude that $w' = w^m$, a contradiction.

Now we prove that in the unobserved case, when $s = \bar{s}_\lambda$, the upstream firm also sets its price equal to w^m . Assume the opposite, so that some $w' \neq w^m$ is charged in equilibrium. If it is the case that $\rho(w') < p^m(w')$, then the necessary condition for the upstream firm's profit maximization is

$$\frac{\partial\pi(w, \rho^* = \rho(w'))}{\partial w} \Big|_{w=w'} = 0.$$

This implies that $w' = w^u$ which in turn leads to a contradiction because, as proven above, $w^u = w^m$. Now consider

$$\rho(w') > p^m(w').$$

If the upstream firm charges w^m instead of w' , its profit will be at least as large as $\pi(w^m, p^m(w^m))$, and by the definition of w^m , the profit will be larger than at w' . Thus w' cannot be charged in equilibrium. Finally, if w' is such that $\rho(w') = p^m(w')$, by deviating down the upstream firm

will ensure that the upper bound is the monopoly price, and by deviating up it will ensure that the upper bound is $\rho^* = \rho(w')$. Since when the monopoly price is equal to the reservation price, derivatives are equal on both sides (as proven above), the derivative of profit on both sides has to be zero, and thus $w' = w^m$, a contradiction. Thus, it has to be the case that the upstream firm charges w^m , and this is the unique equilibrium wholesale price because the upstream firm has no profitable deviation, and w^m is unique by Assumption 1.

Now consider the observed case when $s > \bar{s}_\lambda$. If the upstream firm charges w^m , then its profit is the same as for $s = \bar{s}_\lambda$ because the upper bound on retail prices is $p^m(w^m) < \rho(w^m)$. Since for any given w , $\pi(w, \rho(w))$ decreases in w , no $w \neq w^m$ can increase upstream profits. Thus w^m is the equilibrium price for all $s > \bar{s}_\lambda$.

Next consider $s < \bar{s}_\lambda$. In this case $\rho(w^m) < p^m(w^m)$. At the w that maximizes upstream profits, $\rho(w) \geq p^m(w)$ cannot hold, or otherwise the upstream firm can earn higher profits by charging w^m (in this case the profit will be at least as large as $\pi(w^m, p^m(w^m))$ and thus larger than profit if w is charged). Thus, at the new maximizer $\rho(w) < p^m(w)$ holds and so the upstream profit is maximized at w^o .

Thus we showed that in the observed case for $s \geq \bar{s}_\lambda$ the upstream profit is maximized at w^m , and for $s < \bar{s}_\lambda$ it is maximized at w^o .

Finally consider the unobserved case when s is not equal to \bar{s}_λ . For $s > \bar{s}_\lambda$, w^m is an equilibrium because consumer beliefs are as in Definition 1, so for any deviation to $w \neq w^m$, the upper bound of retail prices is still $p^m(w)$, and thus profit can only be lower. For $s < \bar{s}_\lambda$, $\rho(w^m) < p^m(w^m)$. At the w that maximizes upstream profits, $\rho^* \geq p^m(w)$ cannot hold, or otherwise the upstream firm can earn higher profits by charging w^m , in which case the profit will be larger than $\pi(w, p^m(w^m))$ and thus larger than $\pi(w, \rho(w))$. Thus, for the equilibrium upstream price $\rho^* < p^m(w)$ holds and so the upstream firm will set its price equal to w^u . Equilibrium may not exist, but if it does, the upstream firm has to charge w^u .

So, for all $s \geq \bar{s}_\lambda$, in both the observed and unobserved cases the equilibrium exists, and the upstream price equals w^m . If $s < \bar{s}_\lambda$, the equilibrium upstream price in the observed case is given by w^o , and in the unobserved case, if equilibrium exists, the upstream price equals w^u . \square

Lemma 1. The reservation price is increasing in the expected upstream price: $\rho'(w^e) > 0$.

Proof. Note first that (9) is defined for consumer's conjectured upstream price w^e . Thus the condition for the reservation price is $ECS(\rho, w^e) = s$. From this, we have

$$\frac{\partial ECS(\rho, w^e)}{\partial \rho} \frac{\partial \rho}{\partial w^e} + \frac{\partial ECS(\rho, w^e)}{\partial w^e} = 0.$$

As expected consumer surplus decreases with w^e when ρ is held constant, $\rho'(w^e) > 0$ immediately follows if we can show that $\frac{\partial ECS(\rho, w^e)}{\partial \rho} > 0$. As

$$\frac{\partial F(p)}{\partial \rho} = -\frac{1-\lambda}{2\lambda} \left(\frac{\pi'_r(\rho)}{\pi_r(p)} \right)$$

and

$$\frac{\partial F(p)}{\partial p} = \frac{1-\lambda}{2\lambda} \left(\frac{\pi_r(\rho)\pi'_r(p)}{\pi_r(p)^2} \right)$$

we can write

$$\frac{D(p) \frac{\partial F(p)}{\partial \rho}}{D(\rho) \frac{\partial F(\rho)}{\partial \rho}} = - \frac{D(p) \frac{\pi_r'(\rho)}{\pi_r(p)}}{D(\rho) \frac{\pi_r(\rho) \pi_r'(p)}{\pi_r(p)^2}} = - \frac{(\rho - w) \pi_r'(\rho) / \pi_r^2(\rho)}{(p - w) \pi_r'(p) / \pi_r(p)^2},$$

which, given the assumption that $(p - w) \pi_r'(p) / \pi_r(p)^2$ is increasing in p , lies in the interval $[-1, 0)$. Thus, we can rewrite

$$\begin{aligned} \frac{\partial ECS(\rho, w^e)}{\partial \rho} &= \frac{\int_{\underline{p}}^{\rho} D(p) F(p) dp}{\partial \rho} = D(\rho) + \int_{\underline{p}}^{\rho} D(p) \frac{\partial F(p)}{\partial \rho} dp \\ &> D(\rho) - \int_{\underline{p}}^{\rho} D(p) \frac{\partial F(p)}{\partial p} dp = D(\rho) \left(1 - \int_{\underline{p}}^{\rho} \frac{\partial F(p)}{\partial p} dp \right) = D(\rho) (1 - F(\rho)) = 0. \end{aligned}$$

□

Lemma 2. For a given w , the manufacturer's demand and profit are decreasing in ρ for $w \leq \rho < p^m(w)$.

Proof. Take the derivative of $F(p)$ from (6) for $\bar{p}(w) = \rho$ with respect to ρ . This gives

$$-\frac{(1 - \lambda) \pi_r'(\rho)}{2\lambda \pi_r(p)}.$$

This expression is negative given that $\rho < p^m(w)$ and $\pi_r'(\rho) > 0$. So for a larger ρ the distribution of retail prices first order stochastically dominates the one for a smaller ρ . Given that the downstream demand is downward-sloping, for a given w , the upstream profit decreases in ρ . □

Proposition 2. The manufacturer and consumers are worse off in the unobserved case compared to the observed case. There exist $\tilde{\lambda}$ and \tilde{s} such that this is also true for retailers for $\tilde{s} < s < \hat{s}_\lambda$ or $\tilde{\lambda} < \lambda < 1$.

Proof. The first part is trivial. Since $w^u > w^o$, and beliefs are correct in equilibrium, upstream profits are lower in the unobserved case. As for consumers, the price distribution in the unobserved case first order stochastically dominates the distribution in the observed case. This means that consumers are worse off in the unobserved case.

In order to prove that retail profits are lower in the unobserved than in the observed case, it is sufficient to show that retail profits are everywhere decreasing in w . Since retail profits are equal to

$$(1 - \lambda) \pi_r(\rho(w))$$

we need to show that this expression is decreasing in w . This is equivalent to

$$\rho'(w) < \frac{D(\rho(w))}{\pi_r'(\rho(w))}.$$

Using the definition of $\rho(w)$ we get

$$\begin{aligned}\rho'(w) &= \frac{-\int_{\underline{p}(w)}^{\rho(w)} D(p) \frac{\partial F(p)}{\partial w} dp}{D(\rho(w)) + \int_{\underline{p}(w)}^{\rho(w)} D(p) \frac{\partial F(p)}{\partial p} dp} = \frac{-\int_{\underline{p}(w)}^{\rho(w)} D(p) \frac{(p-\rho(w))(1-\lambda)D(\rho(w))}{2\lambda(p-w)^2 D(p)} dp}{D(\rho(w)) + \int_{\underline{p}(w)}^{\rho(w)} D(p) \frac{-(1-\lambda)\pi'_r(\rho(w))}{2\lambda(p-w)D(p)} dp} \\ &= \frac{\frac{(1-\lambda)D(\rho(w))}{2\lambda} \int_{\underline{p}(w)}^{\rho(w)} \frac{(\rho(w)-p)}{(p-w)^2} dp}{D(\rho(w)) - \frac{(1-\lambda)\pi'_r(\rho(w))}{2\lambda} \int_{\underline{p}(w)}^{\rho(w)} \frac{1}{(p-w)} dp}.\end{aligned}$$

So the sufficient condition is

$$\begin{aligned}\frac{\frac{(1-\lambda)D(\rho(w))}{2\lambda} \int_{\underline{p}(w)}^{\rho(w)} \frac{(\rho(w)-p)}{(p-w)^2} dp}{D(\rho(w)) - \frac{(1-\lambda)\pi'_r(\rho(w))}{2\lambda} \int_{\underline{p}(w)}^{\rho(w)} \frac{1}{(p-w)} dp} &< \frac{D(\rho(w))}{\pi'_r(\rho(w))} \iff \\ \pi'_r(\rho(w)) \frac{(1-\lambda)}{2\lambda} \int_{\underline{p}(w)}^{\rho(w)} \left(\frac{\rho(w)-w}{(p-w)^2} \right) dp &< D(\rho(w)) \iff \\ (D(\rho(w)) + (\rho(w)-w)D'(\rho(w))) \frac{(1-\lambda)(\rho(w)-\underline{p}(w))}{2\lambda(\underline{p}(w)-w)} &< D(\rho(w)).\end{aligned}$$

This condition is satisfied for s sufficiently close to \bar{s} because as s approaches \bar{s} , $\rho(w)$ approaches $p^m(w)$ and so the LHS approaches 0, while the RHS is bound away from zero. Rewriting $\frac{(\rho(w)-\underline{p}(w))}{(\underline{p}(w)-w)}$ as $\frac{(\rho(w)-w)}{(\underline{p}(w)-w)} - 1$ and then as $\frac{(1+\lambda)D(\underline{p}(w))}{(1-\lambda)D(\rho(w))} - 1 = \frac{(1+\lambda)D(\underline{p}(w)) - (1-\lambda)D(\rho(w))}{(1-\lambda)D(\rho(w))}$ gives the other sufficient condition. For λ sufficiently close to 1, $\frac{(1-\lambda)(\rho(w)-\underline{p}(w))}{2\lambda(\underline{p}(w)-w)}$ converges to 1, and so the condition is satisfied due to the negativity of $D'(\rho(w))$. \square

Proposition 3. Suppose $D(p) = 1 - p$. For all λ , if $s \geq \bar{s}_\lambda$, $w^m = 1/2$.

Proof. To prove the first part, note that as

$$1 - F(p) = -\frac{1-\lambda}{2\lambda} + \frac{1-\lambda}{2\lambda} \frac{\pi_r(\bar{p}(w))}{\pi_r(p)}$$

upstream profits can be rewritten as

$$\begin{aligned}\pi(w, \bar{p}(w)) &= \left((1-\lambda) \int_{\underline{p}(w)}^{\bar{p}(w)} f(p) \frac{\pi_r(\bar{p}(w))}{(p-w)} dp \right) w = \left(\frac{(1-\lambda)^2}{2\lambda} \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) \frac{\pi_r(\bar{p}(w))^2 \pi'_r(p)}{\pi_r(p)^3} dp \right) w \\ &= \left(\frac{(1-\lambda)^2 (1-w)^4}{32\lambda} \int_{\underline{p}(w)}^{\bar{p}(w)} \frac{1+w-2p}{(p-w)^3 (1-p)^2} dp \right) w \\ &= \left(\frac{(1-\lambda)^2 (1-w)^4}{32\lambda (1-w)^3} \left[-\frac{(1-w)^2}{2(p-w)^2} + \frac{2(1-w)}{p-w} - \frac{1-w}{1-p} - \ln(p-w)(1-p) \right]_{\underline{p}(w)}^{\bar{p}(w)} \right) w,\end{aligned}$$

where we used $\bar{p}(w) = p^m(w) = \frac{1+w}{2}$ and $1 - \bar{p}(w) = \bar{p}(w) - w = \frac{1-w}{2}$. We can write

$$\underline{p}(w) = \frac{1+w - (1-w)\sqrt{\frac{2\lambda}{1+\lambda}}}{2}$$

so that $1 - \underline{p}(w) = \underline{p}(w) - w = \frac{1-w}{2} \left(1 - \sqrt{\frac{2\lambda}{1+\lambda}} \right)$. We will argue that the term in square brackets

is therefore only a function of λ and not of w . To see this, note that for $k = 1, 2$

$$\frac{\left(\frac{1-w}{2}\right)^k}{(\bar{p}(w) - w)^k} - \frac{\left(\frac{1-w}{2}\right)^k}{(\underline{p}(w) - w)^k} = \frac{\left(\frac{1-w}{2}\right)^k}{(1 - \bar{p}(w))^k} - \frac{\left(\frac{1-w}{2}\right)^k}{(1 - \underline{p}(w))^k} = \frac{\left(1 - \sqrt{\frac{2\lambda}{1+\lambda}}\right)^k - 1}{\left(1 - \sqrt{\frac{2\lambda}{1+\lambda}}\right)^k}.$$

Moreover,

$$\frac{(1 - \bar{p}(w))(\bar{p}(w) - w)}{(1 - \underline{p}(w))(\underline{p}(w) - w)} = \left(\frac{1}{1 - \sqrt{\frac{2\lambda}{1+\lambda}}}\right)^2.$$

Thus, all four terms in square brackets evaluated at $p = \bar{p}(w)$ and $p = \underline{p}(w)$ as the upper and lower bound respectively are functions of λ only and can therefore be expressed as some function $g(\lambda)$. Upstream profits can thus be written as

$$\pi(w) = \left(\frac{(1 - \lambda)^2(1 - w)}{32\lambda}g(\lambda)\right)w,$$

for some function $g(\lambda)$. It is easy to see that this expression has a maximum at $w^* = 1/2$. \square

Proposition 4. Suppose demand is given by $D(p) = 1 - p$. If s approaches 0, then

- (a) the equilibrium w^o approaches $1/2$,
- (b) the equilibrium w^u approaches $1/(1 + \lambda) > 1/2$ in case a reservation price equilibrium exists.

Proof. Let us first consider the limit when s goes to 0 for the observed case. In this case $\bar{p}(w) = \rho(w)$ and both $\rho(w)$ and $\underline{p}(w)$ go to w as s goes to 0. Thus, the demand for both shoppers and non-shoppers equals $1 - w$ and the upstream firm maximizes $w(1 - w)$, which is maximized at $w^* = 1/2$. Consider then the unobserved case. As $\bar{s}_\lambda > 0$, it will be the case that the upper bound $\bar{p}(w) = \rho$ and thus independent of w . The search cost s does not directly enter the profit function $\pi(w, \rho)$. For a fixed w , $\underline{p}(w)$ is a function of w and ρ , and thus $\lim_{s \rightarrow 0} \rho = w$, we have

$$\lim_{s \rightarrow 0} \pi(w, \rho) = \lim_{\rho \rightarrow w} \pi(w, \rho).$$

A necessary condition for the optimal choice of w^* is that it satisfies the first-order condition. Before deriving the first-order condition, we rewrite profits as

$$\frac{w(1-\rho)^2(1-\lambda)^2}{2\lambda(1-w)} \left[\left(\frac{\rho-w}{\underline{p}(w)-w}\right)^2 - 1 + \frac{(\rho-w)^2}{(1-w)} \left(\frac{2}{1-\underline{p}(w)} - \frac{2}{1-\rho} - \frac{1}{(1-w)} 2 \log \left(\left(\frac{1-\underline{p}(w)}{1-\rho}\right)^2 \frac{1+\lambda}{1-\lambda} \right) \right) \right].$$

Using $(1 + \lambda)(1 - \underline{p}(w))(\underline{p}(w) - w) = (1 - \lambda)(1 - \rho)(\rho - w)$ and opening the big bracket we get

$$\begin{aligned} & \frac{w(1-\rho)^2(1-\lambda)^2}{2\lambda(1-w)} \left(\left(\frac{1+\lambda}{1-\lambda} \frac{1-\underline{p}(w)}{1-\rho} \right)^2 - 1 \right) \\ & + \frac{w(1-\rho)^2(1-\lambda)^2(\rho-w)^2}{2\lambda(1-w)^2} \left(\frac{2}{1-\underline{p}(w)} - \frac{2}{1-\rho} - \frac{2}{(1-w)} \log \left(\left(\frac{1-\underline{p}(w)}{1-\rho} \right)^2 \frac{1+\lambda}{1-\lambda} \right) \right). \end{aligned} \quad (16)$$

Here recall that ρ is not a function of w when w is unobserved. The derivative of the second term is zero in the limit because all of its parts include multiples of $(\rho - w)$. This leaves the first term, the derivative of which is

$$\frac{(1-\lambda)^2(1-\rho)^2}{2\lambda(1-w)^2} \left(\left(\frac{1+\lambda}{1-\lambda} \frac{1-\underline{p}(w)}{1-\rho} \right)^2 - 1 \right) - \frac{w(1+\lambda)^2(1-\underline{p}(w))\underline{p}'(w)}{\lambda(1-w)}.$$

From (6) we can derive $\underline{p}'(w)$ by implicit differentiation

$$(1+\lambda)(1+w-2\underline{p}(w))\underline{p}'(w) = (1+\lambda)(1-\underline{p}(w)) - (1-\lambda)(1-\rho).$$

Using the fact that when s approaches 0, ρ and $\underline{p}(w)$ approach w we have that $\lim_{\rho \rightarrow w} \underline{p}'(w) = \frac{2\lambda}{1+\lambda}$. Thus, the limit of the derivative of profits can be written as

$$\begin{aligned} \lim_{\rho \rightarrow w} \frac{\partial \pi(w, \rho)}{\partial w} &= \lim_{\rho \rightarrow w} \left[\frac{(1-\lambda)^2(1-\rho)^2}{2\lambda(1-w)^2} \left(\left(\frac{1+\lambda}{1-\lambda} \frac{1-\underline{p}(w)}{1-\rho} \right)^2 - 1 \right) - \frac{w(1+\lambda)^2(1-\underline{p}(w))\underline{p}'(w)}{\lambda(1-w)} \right] \\ &= \frac{(1-\lambda)^2}{2\lambda} \left(\left(\frac{1+\lambda}{1-\lambda} \right)^2 - 1 \right) - 2(1+\lambda)w \\ &= 2 - 2(1+\lambda)w. \end{aligned}$$

Equating the derivative to zero gives the equilibrium upstream price in the limit

$$w^u = \frac{1}{1+\lambda}.$$

□

Proposition 5. In the unobserved case where demand is given by $D(p) = 1 - p$, there exists a critical value of λ , denoted by λ^* , such that for all $\lambda < \lambda^*$ there exists a $\tilde{s}(\lambda)$ such that for all $s < \tilde{s}(\lambda)$ a reservation price equilibrium does not exist, where $\lambda^* \approx 0.47$ solves $\frac{2(1-\lambda(1+\lambda))}{\lambda} - \frac{(1-\lambda)^2}{\lambda^2} \log \left(\frac{1+\lambda}{1-\lambda} \right) = 0$.

Proof. The limit of the second-order condition with respect to w can be found in a similar way as the limit of the first-order condition of the proof of Proposition 6. Using expression (16), taking the second derivative and eliminating all the terms with zero limit we are left with

$$\begin{aligned} &\frac{2w(1-\lambda)^2(1-\rho)^2}{\lambda(1-w)^2} \left[\frac{1}{1-\underline{p}(w)} - \frac{1}{1-\rho} - \frac{\log \left(\left(\frac{1-\underline{p}(w)}{1-\rho} \right)^2 \frac{(1+\lambda)}{(1-\lambda)} \right)}{1-w} \right] + \\ &\frac{(1-\lambda)^2}{\lambda} \frac{(1-\rho)^2}{(1-w)^3} \left(\left(\frac{1+\lambda}{1-\lambda} \frac{1-\underline{p}(w)}{1-\rho} \right)^2 - 1 \right) - \frac{2(1+\lambda)^2(1-\underline{p}(w))\underline{p}'(w)}{\lambda(1-w)^2} + \\ &\frac{w(1+\lambda)^2 \left(2\underline{p}'(w)^2 - 2(1-\underline{p}(w))\underline{p}''(w) \right)}{2\lambda(1-w)} \end{aligned}$$

Using implicit differentiation on (6) we have

$$(1 + w - 2b)\underline{p}''(w) + 2\underline{p}'(w) - 2(\underline{p}'(w))^2 = 0.$$

Using the fact that in the limit $w^* = \frac{1}{1+\lambda}$ and $\underline{p}'(w) = \frac{2\lambda}{1+\lambda}$ one arrives at $\underline{p}''(w) = -4\left(1 - \frac{2}{1+\lambda}\right)$. Thus, the limit of the second derivative of profits evaluated at the equilibrium upstream price gives

$$-\frac{2(1-\lambda)^2}{\lambda^2} \log\left(\frac{1+\lambda}{1-\lambda}\right) + \frac{(1-\lambda)^2}{\lambda} \frac{1+\lambda}{\lambda} \frac{4\lambda}{(1-\lambda)^2} - \frac{4(1+\lambda)^2}{\lambda} + 4 + \frac{4(1-\lambda)}{\lambda}.$$

So,

$$\lim_{\rho \rightarrow w} \frac{\partial^2 \pi(w, \rho)}{\partial w^2} \Big|_{w=w^*} = 2 \left(\frac{2(1-\lambda(1+\lambda))}{\lambda} - \frac{(1-\lambda)^2}{\lambda^2} \log\left(\frac{1+\lambda}{1-\lambda}\right) \right).$$

By inquiring when

$$\frac{2\lambda(1-\lambda-\lambda^2)}{(1-\lambda)^2} > \log\left(\frac{1+\lambda}{1-\lambda}\right)$$

it can be verified when the limit of the second order condition is negative. As (i) at $\lambda = 0$, the LHS = RHS, as (ii) the derivative of the LHS is first (for small values of λ) larger than the derivative of the RHS and then smaller and (iii) at $\lambda = 1$, the LHS < RHS, there exist a critical value of λ , denoted by λ^* such that for $\lambda < \lambda^*$ the limit of the second-order condition is positive, while it is negative for $\lambda > \lambda^*$. Thus, for small values of λ there exist values of s small enough such that a necessary condition for an equilibrium to exist (namely that the upstream firm chooses an optimal price and that the SOC is fulfilled) is not satisfied. Numerically, one can verify that $\lambda^* \approx 0.475$. \square