The Interaction of Formal and Implicit Contracts with Adverse Selection *

Akifumi Ishihara†

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Abstract

To shed light on the effect of hidden information in multitasking agency problems, this paper studies a repeated agency model with two tasks where the agent has private information on the first task and there is no verifiable performance signal for the second task. Under the assumption that the principal has full bargaining power and imperfect commitment ability, the equilibrium level of the first task is determined so as to guarantee the credibility of the relational contracts to provide incentives for the second task. It implies that the total surplus is not monotonically increasing in the discount factor and the total surplus could be greater than when the second task is verifiable. When the principal can perform the second task by herself, the total surplus could be reduced. Since the information rent can be an incentive device for the unverifiable task, the principal never prefers to allocate the second task to a different agent.

1 Introduction

Problems of incentive provision are often observed in economic relationships such as procurement transaction, organization, and regulation of industry. The incentive problem often becomes more serious when there are multiple dimensions of tasks, all of which the right incentives must be provided for. As Holmström and Milgrom (1991) demonstrate in the principal-agent relationship, if the agent is faced with multi-dimensional tasks, then

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†Department of Economics and STICERD, London School of Economics and Political Science. E-mail: A.ishihara@lse.ac.uk

the principal must be concerned with balancing incentives among the tasks as well as the incentive provision. In the extreme case where some tasks do not generate any objective performance measure, legal contracts can never compensate him for the effort devoted to these tasks and then the agent would devote his effort only to a subset of the tasks. Thus the multitask property has suggested additional insights for contract design. It has also stimulated some issues in the theory of the firm. In particular, the task assignment problem to decide who has the responsibility of the task is definitely motivated by the multitasking incentive problem.

The aim of this paper is to find new insight into the multitasking incentive problem caused by *ex ante* asymmetric information and unverifiability of performance by courts. We argue that the interaction of information rent and reneging temptation of relational contracts causes several interesting results. Specifically, raising the discount factor and/or making the signal on the performance verifiable do not necessarily improve economic efficiency. This interaction provides new incentive perspective for the job design problem. The deriving force of these results are the information rent caused by *ex ante* asymmetric information, which affects the incentive provision in the task without verifiable signals.

In what follows, we analyse a repeated principal-agent relationship with two tasks. Each task can lead to a potential inefficiency. On the first task, the agent has *ex ante* private information on his cost in each period. On the second task, the cost is technologically independent of his private information. However, the performance measure is unverifiable so that it cannot be assessed by a third party and formal contracts contingent on it are not enforced by the court. We assume that both tasks are essential for the parties' payoff so that the principal must deal with these two incentive problems simultaneously.

One way to create incentives related to the second task is the use of relational contracts based on the unverifiable measurement. More specifically, the principal uses both formal contracts contingent only on verifiable signals and informal agreements contingent on unverifiable signals as well as verifiable ones. This is consistent with evidence showing that both formal and informal contracts are used as incentive devices.[1] In our model, the principal optimizes hew own payoff every period subject to two kinds of constraint. First, the principal must deal with the information rent caused by the agent's private information. Second, since informal agreements cannot be enforced by the court exogenously, both parties must be concerned with ensuring to honour the informal agreements voluntarily.

It is well known that there are many equilibria in repeated games even if they are restricted to the public perfect equilibria those are tractable in repeated games with public monitoring.[2] In order to obtain the prediction from our model, we make the additional assumption (Assumption 4) which requires that in each period the principal maximizes her own payoff as long as the players are on the equilibrium path. This restriction is analogous to the standard principal-agent models which presume the principal to make a take-or-leave-it offer. It also means the principal's imperfect ability at commitment. It implies that she cannot commit to the strategy which is not maximizing her own payoff in some period even if it can induce a preferable outcome *ex ante*.

The above assumption implies that the equilibrium is stationary; in equilibrium, the principal offers the same contract and the agent decision depends only on the current type.[3] Together with an additional assumption[4] on the equilibrium behaviour, this stationarity

[1] Gibbons (1998, 2005b) describes a number of cases in which bonus plans based on subjective assessments together with formal contracts were used within the firm. Asanuma (1989) investigates and concludes that in the Japanese automobile and electric machinery industries, the written contracts between a manufacturer and a supplier were relatively coarse, and that well-established informal practices supported their economic performances.

[2] The definition of the public perfect equilibrium is provided by Fudenberg et al. (1994).

[3] Without Assumption 4, under the strategy which maximizes the principal's payoff, she can exploit the agent in the first period and compensate for it in the future by maximizing the future aggregate benefit. See Section 7 for details.

[4] It is corresponding to Assumption 3 which we make later.

further implies that the equilibrium is characterized by the principal's static optimization problem as in the standard adverse selection model with an additional constraint requiring the players to honour the informal agreements. This additional constraint determines the scope of relational contracts and it is satisfied if the sum of the discounted future total surplus between the principal and the agent exceeds the agent's current deviation benefit on the second task. Thus, if this additional constraint is relevant, then the principal's concern of incentive provision in unverifiable tasks can alter the design of formal contracts from the optimal contract without the additional constraint. This effect causes the following results.

It is the comparative static analysis of our equilibrium characterization with respect to the discount factor that gives us our first main result. Clearly, the discount factor determines the scope of relational contracts since it affects the future discounted total surplus. It is important to distinguish three cases: high, intermediate, or low discount factor. If the discount factor is low, then relational contracts do not work at all and there is no value of the relationship between the principal and the agent since there is no way to provide the agent incentives for the second task. Conversely, if the discount factor is high, then relational contracts work well enough to provide the right incentive with respective to the second task. Nevertheless, the principal is still faced with hidden information with respect to the first task and hence she attempts to decrease the information rent to the agent. As in the standard adverse selection problem, this concern leads to the inefficient outcome with respect to the first task.[5]

When the discount factor is intermediate, the same equilibrium outcome as in the case of high discount factor cannot be supported since it does not create a sum of the discounted future total surplus that is large enough to sustain the relational contracts. Nevertheless,

[5]Specifically, the information rent problem causes the downward distortion on the first task.

the principal can increase the total surplus by mitigating the inefficiency caused by the information rent problem in the first task. In order to ensure the success of the relational contracts, she improves the efficiency relative to the case of a high discount factor. In other words, by decreasing the discount factor from a high level to an intermediate level, the principal's objective is altered from purely reducing the information rent to ensuring the functioning of the relational contract, which leads to a more efficient outcome.

The second main result shows that unverifiability could be socially desirable. Specifically, we compare our equilibrium characterization described above with one in which the performance measure on the second task is also verifiable under Assumption 4. In the latter, the principal's concern for ensuring the efficacy of relational contracts vanishes and then, regardless of the discount factor, the outcome is the same as the former for high discount factors.[6] It implies that, for intermediate discount factors, the aggregate benefit between the two parties is greater when the second task is unverifiable than when it is verifiable.[7] When the second task is also verifiable, the inefficiency caused by her concern about the information rent still remains even if the discount factor is not high.

In order to obtain insights for organizational design, we next consider the following task assignment problem. Before the game starts, the principal is allowed to allocate the second task to the agent or another party. We investigate three possibilities as follows: centralization where the principal performs the second task, task bundling where both tasks are assigned to a single agent, and task separation where the second task is allocated to a different agent . Clearly, task bundling is the same as the setting explained above.

The equilibrium under centralization is exactly the same as the one where the second

[6]Under Assumption 4, when the performance measure on the second task is also verifiable, the equilibrium results in the repetition of the equilibrium in the one shot game. The same outcome is implemented when the performance measure on the second task is unverifiable and the discount factor is high.

[7]The principal's payoff is greater when the second task is verifiable than when it is unverifiable.

task is verifiable. Under centralization, there is no agency problem in the second task and the principal's only concern is to reduce the information rent. Then, as in the discussion of verifiability, when the discount factor is intermediate, centralization generates lower social surplus than task bundling. Note that in contrast with centralization, task bundling is interpreted as delegation of the decision right to the agent. Thus it advocates the delegation of authority to the informed party; if the decision right can be sold to the agent, then the principal might prefer delegation since it can work as a commitment device to produce a more efficient outcome.

When only task bundling and task separation are available, the former is preferred to the latter from the social perspective. Intuitively, by bundling the tasks, the information rent obtained from the first task in the future can be an incentive device for the second task. By contrast, in task separation, the agent working on the second task no longer has opportunity to obtain the future information rent. Furthermore, the principal no longer has an incentive to improve the aggregate benefit among all the parties since doing so just increases the information rent for agent 1 and does not help to relax the self-enforcement constraint at all. These effects imply that task separation is undesirable not only from the social perspective but also the principal's perspective.

It should be emphasized that while the literature has shown the results similar to our main results, the logic behind our results is different from them. On desirability of unverifiability, the literature typically has argued that unverifiability can make a stronger punishment for deviation which can reinforce incentive provision through relational contracts. The literature of the job design problem in multitasking agency problems mainly has discussed complementarity of tasks and identified its conditions in terms of technological aspects such as cost structure and observability and verifiability of signals. Both of the literatures usually abstract *ex ante* asymmetric information which causes the information rent problem. By contrast, the present paper argues that the information rent problem causes the similar phenomena. Our explanation for desirability of unverifiability is that since the principal cares about the information rent, unverifiability can alter the her objective from her own payoff to total surplus for sustaining relational contracts which can improve the total surplus. The task assignment problem in our model suggests that since the information rent could be a useful instrument for the incentive provision on tasks without verifiable signals, tasks with hidden information and unverifiable measurement should be treated as complementary tasks from an incentive perspectives and allocated to a single agent.

The next section reviews the related literature and clarifies the contribution of the present paper. Section 3 describes the main model and defines the strategy and equilibrium used in the analysis. Section 4 characterizes the equilibrium conditions and provides comparative statistic analyses. Section 5 points out the possibility that unverifiability is socially desirable. Section 6 considers the task assignment problem. Section 7 discusses the robustness of the results. The final section concludes. The appendix contains proofs for some propositions and lemmas.

2 Related Literature

This paper is related to several strands of the literature on contract theory and the theory of the firm. In order to clarify the relation to the literature, we classify the related papers into the following three categories, relational contracts and screening, verifiability of performance measures, and multitasking incentive agency and job design.[8]

[8] MacLeod (2007) and Malcomson (2010) survey the various topics of self-enforcing relational contracts. Dewatripont et al. (2000) briefly summarize the topics of multitasking incentive and job design problems.

Relational Contracts and Screening Several papers study the design of relational contracts for screening. However, none of them include multitasking problems. Levin (2003) develops the design of relational contracts with asymmetric information.[9] Athey and Miller (2007) study repeated bilateral trading with a budget balancing constraint and two-sided private information. Alonso and Matouschek (2007) study a repeated coordination problem where monetary transfer is prohibited.

The following papers also study repeated principal-agent relationships with screening and are related to our results. Calzolari and Spagnolo (2010) study competitive bidding in procurement and suggest the importance of limiting the competition since it maintains information rent to the agent which is necessary for implementing unverifiable quality.[10] We partly share this logic and argue for a similar role of information rent in a bilateral relationship. Wolitzky (2010) considers a dynamic monopolist problem with price discrimination and unverifiable delivery decisions. Setting a price corresponds to our first task in the sense that it is related to private information about consumer value and the delivery decision corresponds to our second task in the sense that it is unverifiable. In our terminology, delivery is implemented by the principal and then his model is similar to centralization in our task assignment problem. He investigates both non-durable and durable goods monopolists and demonstrates that the so-called Coase conjecture, due to the ratchet effect in durable goods markets, is not observed when an unverifiable delivery cost is introduced. While his analysis of durable goods includes the dynamics of market rationing, our model essentially rules out such a dynamic path by excluding persistent state variables and then there is no issue of learning over the periods.[11] Dynamic flow of

[9] In his model the agent obtains private information after contracting while in our model he obtains private information before contracting.

[10] Board (2010) demonstrates the similar outcome in repeated competitive procurement. However, in his model, the agents have no private information and then the principal does not need to attempt to screen the agents' types.

[11] We briefly discuss the case of serially correlated type in Section 7.

information in repeated principal-agent relationships will be left for future research.[12]

Verifiable and Unverifiable Performance Measure As long as unverifiable performance is more informative on the task than verifiable performance, an interaction of formal and informal contracts could emerge. Baker et al. (1994), Pearce and Stacchetti (1998) and Itoh and Morita (2010) study this topic in an environment with biased measurement, risk-sharing, and relation-specific investments, respectively. However, these abstract from multitasking issues. The interaction of formal and informal contracts in multitasking environment is studied by Schmidt and Schnitzer (1995), Daido (2006), Schöttner (2008), and Iossa and Spagnolo (2009).[13] Neither of them includes hidden information.

Several papers have already found the possibility of welfare improvement by making a task unverifiable (Baker et al., 1994; Schmidt and Schnitzer, 1995; Bernheim and Whinston, 1998; Kovrijnykh, 2010). Their idea is basically that unverifiability can make a stronger punishment for deviation and then provide a larger incentive *via* relational contracts, which improves efficiency. As argued in Section 1, our idea is different from theirs.[14]

Kvaløy and Olsen (2009) introduce the principal's endogenous verification investment – interpreted as a cost of writing effective formal contracts – which stochastically determines the verifiability. They show that the discount factor and verifiability could have a negative impact on the quality of the transaction.[15] Our paper derives this phenomenon caused by hidden information instead of an endogenous verification decision.

[12]Halac (2009) studies repeated principal-agent relationships where each party holds private information on her/his own outside value. She assumes that the outside value is completely persistent over periods which induces dynamics of learning on the other's private information.

[13]Dewatripont et al. (1999) study a multitasking problem with implicit contracts. They focus on an agent with career concerns and the contract design problem is different from our model.

[14]We discuss the detail of the difference in Section 5.

[15]Their interest is in how a change of such factors affects the quality level implemented in the transaction and they do not mention the overall economic efficiency including the cost of the investment for verification. Actually, in their model, the overall economic efficiency is weakly increasing in the discount factor.

Multitasking Incentive Agency and Job Design Multitasking incentive problems and job design have been mainly discussed in environments without relational contracts. One of the main strands in the literature studies the environment with hidden action and identifies conditions on the verifiable measurement under which task bundling is preferable to task separation between multiple agents. (Holmström and Milgrom, 1991; Itoh, 1994; Meyer et al., 1996).[16] In the environment with hidden information, Jackson and Sonnenschein (2007) and Matsushima et al. (2010) investigate the linking mechanism in which many identical tasks are allocated to a single agent and the agent's messages on his preference are rationed so as to approximate the *ex ante* belief on the preference. Both of their papers demonstrate that when the number of the tasks are sufficiently large, the linking mechanism can almost beat the incentive problem without any transfer. The research interest in the present paper is different from them in the sense that we consider the effect of a small number of the tasks with hidden information on the multitasking incentive problem.

Schöttner (2008) is one of the most related papers to the present paper. She studies the effect of job design on relational contracts and points out that task bundling is often preferred to task separation. In her paper, however, hidden information is excluded. The implication for the job design problem with hidden information is one of our contributions to this literature.

[16] A number of papers study the vertical job design problem, i.e. centralization or delegation. See Mookherjee (2006) for a survey of this literature.

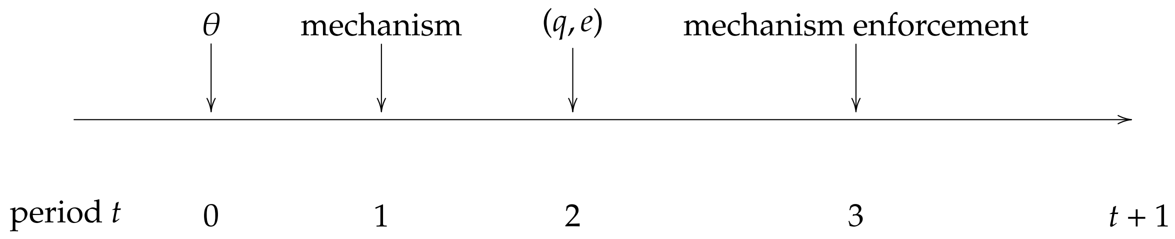

Figure 1: Timing within a Period

3 The Model

3.1 Environment

There are two parties, a principal (denoted by P) and an agent (denoted by A). Both live in periods $t = 0, 1, \ldots$ until infinity. Their common discount factor is $\delta \in [0, 1)$. Assume that they are risk-neutral. In each period t, they have a business opportunity and in period t the game proceeds as in Figure 1. At stage 0, A privately observes the cost parameter θ_t drawn from $[\underline{\theta}, \overline{\theta}]$ according to the cumulative distribution function $F(\theta)$ with the density function $f(\theta)$. Assume that θ_t is independently drawn in each period. At stage 1, P offers A a mechanism, which is to be defined formally later. Assume that P can choose to abstain from offering mechanisms. If P does not offer a mechanism or A rejects the mechanism, then the period ends and both players obtain zero payoff. If A accepts the mechanism, then A works on two tasks at stage 2. In the first task, he chooses $q_t \in [0, \overline{q}] \equiv Q$ where $\overline{q} > 0$ and in the second one he faces a binary choice $e_t \in \{0, \overline{e}\}$ where $\overline{e} > 0$. These decisions yield P the benefit $y(q_t, e_t)$ and A the cost $c(q_t, \theta_t) + e_t$. Finally, at stage 3 the parties make a decision on the enforcement of the mechanism. Later we will explain this stage precisely.

In period t, given θ_t, q_t, e_t, and a monetary transfer w_t from P to A, P and A obtain the (*ex post*) payoff $y(q_t, e_t) - w_t$ and $w_t - c(q_t, \theta_t) - e_t$ respectively. Denote the aggregated surplus by $s(q, e, \theta) := y(q, e) - c(q, \theta) - e$ and the first best decision by $(q^{FB}(\theta), e^{FB}(\theta)) \in$

$\arg\max_{q,e} s(q, e, \theta)$. We make the following assumptions.

Assumption 1 *For all $q \in Q$, $e \in \{0, \bar{e}\}$, and $\theta \in [\underline{\theta}, \overline{\theta}]$, the following conditions are satisfied;*

1. *$c(q, \theta)$ is three-times differentiable and bounded in each component, $c_q(q, \theta) > 0$, $c_\theta(q, \theta) > 0$, $c_{q\theta}(q, \theta) > 0$ and $c(0, \underline{\theta}) > 0$.*

2. *$s(q, e, \theta)$ is twice differentiable in q, bounded in each component, $s(q, 0, \theta) < 0$, $s(0, e, \theta) < 0$, $s_{qq}(q, \bar{e}, \theta) < 0$ and there exists $q' \in (0, \bar{q})$ such that $s_q(q', \bar{e}, \theta) = 0$.*

3. *Let $J(q, \bar{e}, \theta) \equiv s(q, \bar{e}, \theta) - c_\theta(q, \theta)F(\theta)/f(\theta)$. Then $J_{qq}(q, \bar{e}, \theta) < 0$, $J_{q\theta}(q, \bar{e}, \theta) < 0$, and $J(q^{FB}(\theta), \bar{e}, \theta) > 0$.*

If the second task e is absent, implying that it is a single task model, then these assumptions would be fairly standard in the adverse selection literature.[17] Nevertheless some remarks are useful here. Part 2 of Assumption 1 implies that both tasks are strict complements in the sense that to implement $e = \bar{e}$ and $q > 0$ is essential for the value of the relationship. If it is impossible to induce positive amounts of (q, e), then the business is less valued than the outside option.

$J(q, e, \theta)$ is known as the virtual-surplus, which becomes the consequent objective function for P in a typical adverse selection model. Part 3 guarantees that for each θ, $\arg\max_{q,e} J(q, e, \theta)$ is unique. Hereafter denote the maximand of $J(q, e, \theta)$ by $(e^{SB}(\theta), q^{SB}(\theta))$. It can be seen that $e^{SB}(\theta) = \bar{e}$ and $J_q(q^{SB}(\theta), \bar{e}, \theta) = 0$[18] for each $\theta \in [\underline{\theta}, \overline{\theta}]$ and $q^{SB}(\theta)$ is decreasing in θ. Part 2 and Part 3 further guarantee that $e^{FB}(\theta) = \bar{e}$[19] and $s_q(q^{FB}(\theta), \bar{e}, \theta) = 0$ for each $\theta \in [\underline{\theta}, \overline{\theta}]$, $q^{FB}(\theta)$ is decreasing in θ,[20] and $q^{FB}(\theta) \geq q^{SB}(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ with equality holding only if $\theta = \underline{\theta}$.

[17] The assumption that $c_\theta(0, \theta) > 0$ rules out a case where $c(q, \theta) \equiv \theta q + K$ for some $K > 0$ since it is not satisfied at $q = 0$. This assumption is only used for simplifying the proof of Lemma 1 and it can be shown even if $c_\theta(0, \theta) = 0$.

[18] Note that $\max_q J(q, 0, \theta) < 0$ for any q and θ by Part 2 of Assumption 1.

[19] It is because $\max_q s(q, \bar{e}, \theta) > \max_q J(q, \bar{e}, \theta) \geq J(q^{FB}(\theta), \bar{e}, \theta) > 0 > \max_q s(q, 0, \theta)$ for any q and θ.

[20] Note that $s_{q\theta}(q, e, \theta) < 0$ for all q and θ.

The assumption on the observability and verifiability is as follows. All variables except for θ are observable by both parties. P cannot observe θ and she believes that it is distributed according to $F(\theta)$ and $f(\theta)$. Both q_t and e_t are describable at stage 1 in period t whereas unless otherwise stated, only the first task q_t is verifiable.

A mechanism is defined as a pair of a formal contract and an informal agreement. A formal contract is a transfer schedule $p_t(q_t)$ from P to A contingent on q_t and enforced by the court.[21] We assume that $p_t(q_t)$ is continuous in q_t.[22] An informal agreement $b_t(q_t, e_t)$ is a transfer schedule from P to A which can be contingent on both q_t and e_t and must be enforced by themselves.[23] Both $p_t(q_t)$ and $b_t(q_t, e_t)$ can be positive or negative. Enforcement of informal agreements is defined as follows. At stage 3, each party chooses $I_t^i \in \Im \equiv \{H, R\}$ for $i = P, A$ where H denotes honouring the informal agreement, and R reneging. The informal agreement $b_t(q_t, e_t)$ is enforced if and only if both parties chose to honour it, i.e., $I_t \equiv (I_t^P, I_t^A) = (H, H)$. If it is not enforced, i.e., $I_t \neq (H, H)$, then no transfer is made by the informal agreements.

We later discuss the effect of verifiability of the second task e. We define that the second task e is verifiable if an informal agreement $b_t(q, e)$ is also enforced by the court for each t. It implies that when the second task is verifiable, $b_t(q, e)$ is successfully transferred regardless of I_t.

[21]Here we implicitly assume that monetary transfer is also verifiable.

[22]The assumption of continuity of $p_t(q_t)$ is required only for the proof of Lemma 6 which shows the existence of an equilibrium strategy used for the punishment. It means that this technical assumption is only for the construction of the off-path outcome and innocuous in the other parts of the analysis. Thus except for Lemma 6, we will not explicitly mention this assumption. See the proof of Lemma 6 and footnote 44 there for the detail.

[23]The assumption on the describability of q_t and e_t implies that neither p_t nor b_t can be contingent on q_τ and e_τ for $\tau > t$.

3.2 Economic Interpretation of the Setting

Throughout the paper, we treat this environment as a manufacturer-supplier relationship. The manufacturer offers a menu contract $p(q)$ contingent on units of the products q from the supplier. However, this product is valueless without the supplier's effort which can be interpreted as improvement of quality, careful delivery, or regular maintenance. For simplicity, in what follows, we use the term "quantity" for the first task and "effort" for the second task.

This framework can be applied to the analysis of other situations. On management of a production line of automobiles or or electric machineries, the product consists of hard parts q assembled in the expertized line and software e for controlling the system the performance of which is hard to be evaluated from the outside. In an relationship between a firm owner and a CEO, q might be interpreted as the size of a project and e as the effort to find and tailor the project for successful implementation. In the context of regulation, the regulation authority asks the regulated firm to meet several environmental standards and some of the standards might be hard to measure objectively. The analysis below would be applicable to these cases.

3.3 Strategy and Equilibrium

A pure strategy in the repeated game is defined as a mapping from a privately observable history to a decision variable. Formally, let $D \equiv Q \times \{0, \bar{e}\}$ be the set of the task levels and $\mathfrak{W} \equiv (\{(p(\cdot), b(\cdot, \cdot)) \mid p : Q \to \mathbb{R}, b : D \to \mathbb{R}\})$ be the set of feasible mechanisms. Denote $\overline{\mathfrak{W}} \equiv \mathfrak{W} \cup \{\phi\}$ where ϕ denotes no mechanism being offered. Denote also the decision of A's rejection of the mechanism by ω. Then the public history, a sequence of publicly observable variables[24], up to period $t \geq 1$ is defined as $h^t \equiv (h_\tau)_{\tau=0}^{t-1}$ where for each $\tau = 0, \ldots, t-1$, $h_\tau \in \{\phi\} \cup (\mathfrak{W} \times (\{\omega\} \cup (D \times \mathfrak{I}^2)))$.[25] Let h^0 be the null history and $\mathcal{H}$ be the set of the public histories. A's private history, the sequence of A's observable variables, up to period t is defined as $h^{A0} = h^0$ and for $t \geq 1$, $h^{At} \equiv (\theta_\tau, h_\tau)_{\tau=0}^{t-1}$. Let $\mathcal{H}^A$ be the set of A's private histories.

P's pure strategy is a pair of mappings $\sigma^P \equiv (\Gamma, \iota^P)$ where $\Gamma : \mathcal{H} \to \overline{\mathfrak{W}}$ and $\iota^P : \mathcal{H} \times \mathfrak{W} \times D \to \mathfrak{I}$. These define the mechanism offered by P and whether P enforces the informal agreement given that the mechanism is accepted, respectively. A's pure strategy is a pair of mappings $\sigma^A \equiv (\chi, \iota^A)$ where $\chi : \mathcal{H}^A \times [\underline{\theta}, \overline{\theta}] \times \mathfrak{W} \to \overline{D}$, $\iota^A : \mathcal{H}^A \times [\underline{\theta}, \overline{\theta}] \times \mathfrak{W} \times D \to \mathfrak{I}$, and $\overline{D} \equiv D \cup \{\omega\}$. χ stipulates A's response to an offered mechanism and ι^A whether to enforce the informal agreement. Denote the set of strategies by Σ^i for $i = P, A$. A pure strategy profile (σ^P, σ^A) is a perfect Bayesian equilibrium if after every history, there is no incentive to deviate to improve her/his own average payoff at any stage of the period and P's belief about θ is computed by Bayes' rule whenever it is possible and consistent with the strategy profile.[26]

Since the game has a recursive structure, we can decompose strategy (σ^P, σ^A) in the following way. For any public history up to stage 0 of period t, $h^t \in \mathcal{H}$, let

$$\sigma_+^P(h^t) \begin{cases} \Gamma(h^t) \in \overline{\mathfrak{W}} \\ \iota^P(\cdot \mid h^t) : \mathfrak{W} \times D \to \mathfrak{I} \\ \sigma_+^P(h^t, \cdot) : \{\phi\} \cup (\mathfrak{W} \times (\overline{D} \cup \{\omega\})) \to \Sigma^P \end{cases}$$

[24] We mean by "publicly observable" and "public history" that it is observable for both P and A but not necessarily observable for others. Then note that it does not necessarily coincide with "verifiable".

[25] It is implicitly presumed that if h_τ contains an element in D, then it means that A accepts the mechanism in period τ.

[26] Since the equilibrium mechanism seen later does not have bunching for the first task, the restriction to pure strategy profiles is without loss of generality for the optimum. See Strausz (2006) for details.

where $\sigma_+^P(h^0) \equiv \sigma^P$, and for any A's private history up to stage 0 of period t, $h^{At} \in \mathcal{H}^A$,

$$\sigma_+^A(h^{At}) \begin{cases} \chi(\cdot \mid h^{At}) : [\underline{\theta}, \overline{\theta}] \times \mathfrak{W} \to \overline{D} \\ \iota^A(\cdot \mid h^{At}) : [\underline{\theta}, \overline{\theta}] \times \mathfrak{W} \times D \to \mathfrak{I} \\ \sigma_+^A(h^{At}, \cdot) : [\underline{\theta}, \overline{\theta}] \times (\{\phi\} \cup (\mathfrak{W} \times (\overline{D} \cup \{\omega\}))) \to \Sigma^A. \end{cases}$$

where $\sigma_+^A(h^0) \equiv \sigma^A$. This formulation is interpreted as that the strategy stipulates players' actions in the current period and a strategy itself played in the repeated game starting from next period, that is the continuation strategy.

The repeated game typically has multiple equilibria. Thus we make some additional restrictions on the equilibrium strategy. First, we focus on the public strategy in the equilibrium.

Assumption 2 *σ^A satisfies that for all $t \geq 0$ and $h^{At} \in \mathcal{H}^A$, $\sigma_+^A(h^{At})$ is independent from $(\theta_0 \ldots, \theta_{t-1})$ and $\iota^A(\theta_t, W_t, d_t \mid h^{At})$ is independent from $(\theta_0 \ldots, \theta_{t-1}, \theta_t)$.*

It simply states that A's equilibrium strategy is independent from θs which are irrelevant to his current payoff. An equilibrium of a public strategy is known as the public perfect equilibrium (PPE) and every pure strategy equilibrium payoff can be achieved by PPE.[27] Instead of $\sigma_+^A(h^{At})$, hereafter denote A's public strategy by $\sigma_+^A(h^t)$ where h^t is a public history.

The next assumptions stipulate the bargaining power and the commitment ability.

Assumption 3 *A public strategy σ satisfies $\sigma_+(h^t, W, \omega) = \sigma_+(h^t, \phi) = \sigma_+(h^t)$ for all $h^t \in \mathcal{H}$.*

Assumption 4 *Let $\hat{\mathcal{H}}(\sigma) \subset \mathcal{H}$ be the set of public histories achieved with positive probability by public strategy σ. For any $h^t \in \hat{\mathcal{H}}(\sigma)$, $\sigma_+(h^t) \equiv (\sigma_+^P(h^t), \sigma_+^A(h^t))$ must attain the maximum average payoff for P in the PPE.*

[27] See Fudenberg et al. (1994) for details.

Assumption 3 means that the event of no contracting at the beginning of the period is not interpreted as serious misbehaviour and then the same strategy is taken in the continuation game. These could prevent the punishment for deviating party which can reinforce efficiency. Assumption 4 means that as long as the players are on the equilibrium path, P has absolute bargaining power at the stage of contracting in the sense that the equilibrium strategy will optimize P's utility subject to the equilibrium constraints. This is for consistency with the standard principal-agent models which presume the principal to make a take-or-leave-it offer. It also implies that P's commitment ability to the future play is not so strong that P may not voluntarily choose the worse outcome which could lead to more efficient outcome *ex ante*.

Assumption 3 and 4 cause some loss of generality. In particular, Assumption 4 is crucial if the type is independently drawn each period as in our model. In Section 7, we discuss how much loss of generality is caused by these assumptions.

Hereafter call the equilibrium satisfying Assumption 2, 3, and 4 the Optimal Public Perfect Equilibrium (OPPE). The following proposition shows that it is without loss of generality to focus on the following simple trigger strategy for finding the OPPE. To state the proposition, let $\mathcal{H}^R \equiv \{h^t \in \mathcal{H} \mid {}^\exists\tau < t, {}^\exists i, {}^\exists(d_\tau, I_\tau) \in D \times \mathfrak{I}^2, I^i_\tau = R\}$, that is the set of public histories in which there exists a party having reneged on an informal agreement and $\mathcal{H}^H \equiv \mathcal{H} \setminus \mathcal{H}^R$, the set of public histories in which informal agreements have been honoured on.

Proposition 1 *Let σ be an OPPE and (π, u) be the OPPE payoff. Then there exists an OPPE σ^* such that*

- *for all $h^t \in \mathcal{H}^H$,*
  - $\Gamma^*(h^t) = W^* \in \overline{\mathfrak{W}}$,

– *if* $W^* \in \mathfrak{W}$, *then* $\chi^*(W^* \mid h^t, \theta_t) = d^*(\theta_t) \in \overline{D} \cup \{\omega\}$, *and* $\iota^{P*}((W^*, d \mid h^t) = \iota^{A*}((W^*, d \mid h^t, \theta_t) = H$,

- *for* $h^t \in \mathcal{H}^R$, $\Gamma^*(h^t) = \phi$, *and*
- *the associated payoff* (π^*, u^*) *satisfies that* $\pi^* = \pi$ *and* $u^* \geq u$.

Proposition 1 drastically simplifies our analysis. Specifically, we obtain several notable features of σ^* that (i) on the equilibrium path, if $W^* \in \mathfrak{W}$ and $d^*(\theta_t) \in \overline{D}$, i.e., P actually offers a mechanism and A accepts it, then the informal agreement must always be honoured, (ii) P's mechanism and A's decision are independent of public histories as long as they have reneged on informal agreements, and (iii) after deviation to reneging on an informal agreement, both players obtain 0 payoff from the outside options. Furthermore if $W^* = \phi$, then it is straightforward that both parties obtain 0 as their average payoff. Thus our problem is to seek the mechanism $W^* \in \mathfrak{W}$ which maximizes P's average payoff. If there exists such a mechanism with P's average payoff $\pi^* \geq 0$, then W^* is offered on the equilibrium path and if not, then no mechanism is offered on the equilibrium path.

Hereafter we focus on the OPPE which satisfies Proposition 1.

4 Analysis

Thanks to Proposition 1, the OPPE can be derived by a stationary problem with some constraints. Furthermore, the following proposition shows that the OPPE is more simplified in that all types accept the equilibrium mechanism.

Lemma 1 *Let* $W^* \in \mathfrak{W}$ *be the OPPE mechanism and* $d^*(\theta)$ *be* A*'s OPPE decision given that mechanism* W^* *is offered. Then* $d^*(\theta) \in D$ *for all* $\theta \in [\underline{\theta}, \overline{\theta}]$.

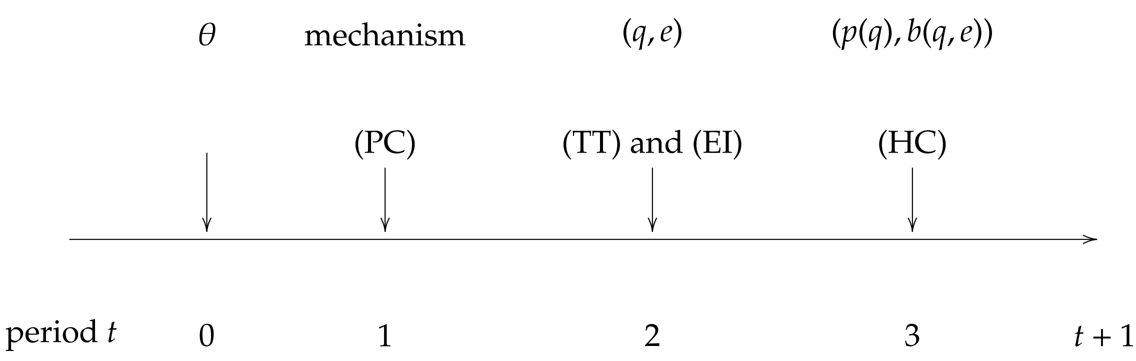

Figure 2: Equilibrium Constraints

Let $W^* = (p^*(q), b^*(e,q))_{q \in Q, e \in \{0, \bar{e}\}}$ and $d^*(\theta) = (q^*(\theta), e^*(\theta))$ be the equilibrium task level chosen by A given the mechanism W^*. Due to the stationarity of W^* and $d^*(\theta)$, P's average payoff is given by

$$\begin{aligned}
\pi^* &= (1-\delta)\sum_{\tau=0}^{\infty} \delta^{\tau} E_{\theta}\left[y(q^*(\theta), e^*(\theta)) - w^*(q^*(\theta), e^*(\theta))\right] \\
&= E_{\theta}\left[y(q^*(\theta), e^*(\theta)) - w^*(q^*(\theta), e^*(\theta))\right]
\end{aligned}$$

where $w^*(q,e) = p^*(q) + b^*(q,e)$. This is the objective to be maximized subject to the equilibrium conditions. In each period, the players are faced with four equilibrium conditions which are common among periods on the equilibrium path. Figure 2 describes them in order. At stage 1, the participation constraint (PC) is satisfied if and only if A with type $\theta \in [\underline{\theta}, \overline{\theta}]$ accepts W^* offered by P. At stage 2, A chooses $d^*(\theta) = (q^*(\theta), e^*(\theta))$. There are two concerns on this incentive compatibility constraint. First, the truth-telling constraint (TT) is satisfied if and only if type θ does not pretend to be any other type θ'. Second, the effort incentive constraint (EI) is satisfied if and only if type θ does not choose $e \neq e^*(\theta)$.[28] Finally at stage 3, the honouring constraint (HC) is satisfied if and only if both

[28] Strictly, we must also check the incentive constraint not to choose $q \notin Q^* \equiv \{q' \in Q \mid {}^{\exists}\theta \in [\underline{\theta}, \overline{\theta}], q' = q^*(\theta)\}$. However since we can show that Q^* is an closed interval and $p^*(q)$ is non-decreasing in $q \in Q^*$, the possibility to choose $q \notin Q^*$ can be excluded by constructing $p^*(q)$ such that for $q > \sup Q^*$, $p^*(q) = \sup_{q \in Q^*} p^*(q)$

parties choose to honour the informal agreement.

Let

$$\begin{aligned} U^*(\theta) &:= w^*(q^*(\theta), e^*(\theta)) - c(q^*(\theta), \theta) - e^*(\theta) \\ u^* &:= E_\theta \left[U^*(\theta)\right] \\ s^* &:= u^* + \pi^* \end{aligned}$$

be A's *ex post* and *ex ante* payoff and the *ex ante* total surplus within a period. In general, the players are concerned about their future continuation payoff as well as the payoff in the current period. Nevertheless, by construction of the strategy, the continuation payoff is not altered by any deviation on (TT), (EI), and (PC). It means that it is enough to consider these constraints without the future continuation payoff. On (HC), choosing $\iota^{i*} = H$ induces the continuation payoff, π^* and u^* for P and A respectively. Conversely, choosing $\iota^{i*} = R$ induces no mechanism in the future and then the continuation payoff is 0 for both parties. Thus, while the continuation payoff must be taken into account in (HC), the value of the continuation payoff is still history-independent. Therefore the optimization problem of characterizing the OPPE can be transformed to a static mechanism design problem.

First, look at (TT). For each $\theta, \theta \in [\underline{\theta}, \overline{\theta}]$, θ does not pretend $\theta' \in [\underline{\theta}, \overline{\theta}]$ if and only if

$$\begin{aligned} U^*(\theta) &\geq w^*(q^*(\theta'), e^*(\theta')) - c(q^*(\theta'), \theta) - e^*(\theta') \\ &= U^*(\theta') + c(q^*(\theta'), \theta') - c(q^*(\theta'), \theta). \end{aligned}$$

and for $q < \inf Q^*$, $p^*(q)$ is sufficiently small.

The envelope theorem by Milgrom and Segal (2002) implies that this is equivalent to that $q^*(\theta)$ is non-increasing in θ and

$$U^*(\theta) = U^*(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} c_\theta(q^*(z), z)dz. \quad (1)$$

Second, consider (EI). (EI) is satisfied if for each θ, θ' and $e' \neq e^*(\theta')$, θ does not choose $(q(\theta'), e')$. It is expressed as

$$\begin{aligned} U^*(\theta) &\geq w^*(q^*(\theta'), e') - c(q^*(\theta'), \theta) - e' \\ &= U^*(\theta') - b^*(q^*(\theta'), e^*(\theta')) + b^*(q^*(\theta'), e') + c(q^*(\theta'), \theta') - c(q^*(\theta'), \theta) + e^*(\theta') - e'. \end{aligned}$$

Substituting (1) implies that

$$\int_{\theta}^{\theta'} c_\theta(q^*(x), x)dx - c(q^*(\theta'), \theta') + c(q^*(\theta'), \theta) \geq -b^*(q^*(\theta'), e^*(\theta')) + b^*(q^*(\theta'), e') + e^*(\theta') - e'. \quad (2)$$

Third, (PC) means that every type accepts the mechanism. Since to reject generate zero payoff for A, the condition is described as

$$U^*(\theta) \geq 0$$

for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Since $U^*(\theta)$ is non-increasing due to (1), it is satisfied if and only if

$$U^*(\overline{\theta}) \geq 0. \quad (3)$$

Finally, consider (HC). Given that q and e were observed and A chooses $I^A = H$, if P chooses $I^P = H$, then $b^*(q, e)$ is transferred from P to A and the continuation payoff from the next period is π^*. Conversely if P chooses $I^P = R$, then $b^*(q,e)$ is not to be transferred and the continuation payoff from the next period is 0 because she does not offer any mechanism from the next period. Thus P chooses $I^P = H$ if and only if

$$-b^*(q,e) + \frac{\delta}{1-\delta}\pi^* \geq 0.$$

Similarly, A chooses $I^A = H$ if and only if

$$b^*(q,e) + \frac{\delta}{1-\delta}u^* \geq 0.$$

Combining them implies that

$$\frac{\delta}{1-\delta}\pi^* \geq b^*(q,e) \geq -\frac{\delta}{1-\delta}u^*. \tag{4}$$

Plugging (1) into π^* and integrating by parts yield that

$$\begin{aligned}
\pi^* &= E_\theta\left[y(q^*(\theta), e^*(\theta)) - c(q^*(\theta), \theta) - e^*(\theta) - U^*(\theta)\right] \\
&= \int_{\underline{\theta}}^{\overline{\theta}} J(q^*(\theta), e^*(\theta), \theta) f(\theta) d\theta - U^*(\overline{\theta}) \qquad (5)
\end{aligned}$$

where $J(q,e,\theta)$ is defined as in Assumption 1. In equilibrium, it is maximized by control variables $(q^*(\theta), e^*(\theta), U^*(\theta), b^*(q^*(\theta), e))$ subject to (2), (3), (4), and monotonicity of $q^*(\theta)$ in θ. This problem is simplified as follows.

Lemma 2 *The optimization problem described above is equivalent to the following;*

$$\max_{q^*(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} J(q^*(\theta), \bar{e}, \theta) f(\theta) d\theta$$
$$\textit{subject to } \frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\overline{\theta}} s(q^*(z), \bar{e}, z) f(z) dz \geq \bar{e} \tag{6}$$

and $q^*(\theta)$ is non-increasing in θ.

If constraint (6) is absent, then the Euler equation implies that $q^*(\theta) = q^{SB}(\theta)$ for all θ. Since $q^{SB}(\theta)$ is non-increasing in θ, it is the solution if $q^{SB}(\theta)$ satisfies (6), that is

$$\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\overline{\theta}} s(q^{SB}(z), \bar{e}, z) f(z) dz \geq \bar{e} \iff \delta \geq \bar{\delta} \equiv \frac{\bar{e}}{\bar{e} + \int_{\underline{\theta}}^{\overline{\theta}} s(q^{SB}(z), \bar{e}, z) f(z) dz}. \tag{7}$$

Suppose that (7) is not satisfied. Now consider the following Lagrangian with multiplier $\lambda^* \geq 0$:

$$\mathcal{L} \equiv \int_{\underline{\theta}}^{\overline{\theta}} J(q^*(\theta), \bar{e}, \theta) f(\theta) d\theta + \lambda^* \left[\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\overline{\theta}} s(q^*(\theta), \bar{e}, \theta) f(\theta) d\theta - \bar{e} \right].$$

The Euler equation implies that the solution satisfies that

$$s_q(q^*(\theta), \bar{e}, \theta) = \frac{1}{1 + \frac{\delta}{1-\delta}\lambda} \frac{F(\theta)}{f(\theta)} c_{q\theta}(q^*(\theta), \theta). \tag{8}$$

Recall that by definition of $q^{SB}(\theta)$, $q^*(\theta) = q^{SB}(\theta)$ if $\lambda = 0$. However since it does not satisfy (6), then λ^* is positive and determined to satisfy (6) with equality.

Under what conditions is it feasible? It is easy to see that $q^*(\theta)$ characterized by (8) is decreasing in θ by Assumption 1. The right hand side of (8) is greater then 0 and $q^{SB}(\theta)$

satisfies that $J_q(q^*(\theta), \bar{e}, \theta) = 0$ or equivalently

$$s_q(q^{SB}(\theta), \bar{e}, \theta) = \frac{F(\theta)}{f(\theta)} c_{q\theta}(q^{SB}(\theta), \theta),$$

the right hand side of which is greater that of (8). Thus we see that for $\delta < \bar{\delta}$, $s_q(q^{FB}(\theta), \bar{e}, \theta) < s_q(q^*(\theta), \bar{e}, \theta) < s_q(q^{SB}(\theta), \bar{e}, \theta)$ which implies that by concavity of $s(q, e, \theta)$

$$q^{FB}(\theta) > q^*(\theta) > q^{SB}(\theta)$$

for all $\theta \in (\underline{\theta}, \overline{\theta})$. Furthermore, (8) shows that $q^*(\theta)$ is increasing in λ^* (provided that $\delta > 0$) and approaches $q^{FB}(\theta)$ as λ^* goes to infinity. Then by concavity of $s(q, \bar{e}, \theta)$ in q, s^* is increasing as λ^* goes up and the supremum of it is $\int s(q^{FB}(z), \bar{e}, z) f(z) dz$. Thus the solution obtained by (8) is feasible if and only if

$$\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\overline{\theta}} s(q^{FB}(z), \bar{e}, z) f(z) dz > \bar{e} \iff \delta > \underline{\delta} \equiv \frac{\bar{e}}{\bar{e} + \int_{\underline{\theta}}^{\overline{\theta}} s(q^{FB}(z), \bar{e}, z) f(z) d\theta} \tag{9}$$

where strictly inequality must be satisfied because of Slater constraint qualification for constrained optimization problems. When $\delta = \underline{\delta}$, it is obvious that $q^{FB}(\theta)$ for each θ is the unique feasible q and then it is the solution.

Finally, if $\delta < \underline{\delta}$, then there is no $q^*(\theta)$ which satisfies (6). This implies that $e^*(\theta) = 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and then $\pi^* < 0$. In this case, P strictly prefers to offer no mechanism. The summary of the above analysis is as follows.

Proposition 2 *The OPPE satisfies the following.*

1. *For $\delta \geq \bar{\delta}$, $d^*(\theta) = (q^{SB}(\theta), \bar{e})$.*

2. *For $\delta \in (\underline{\delta}, \bar{\delta})$, $d^*(\theta) = (q^*(\theta), \bar{e})$ where $q^*(\theta)$ satisfies (6) and (8) with equality for some*

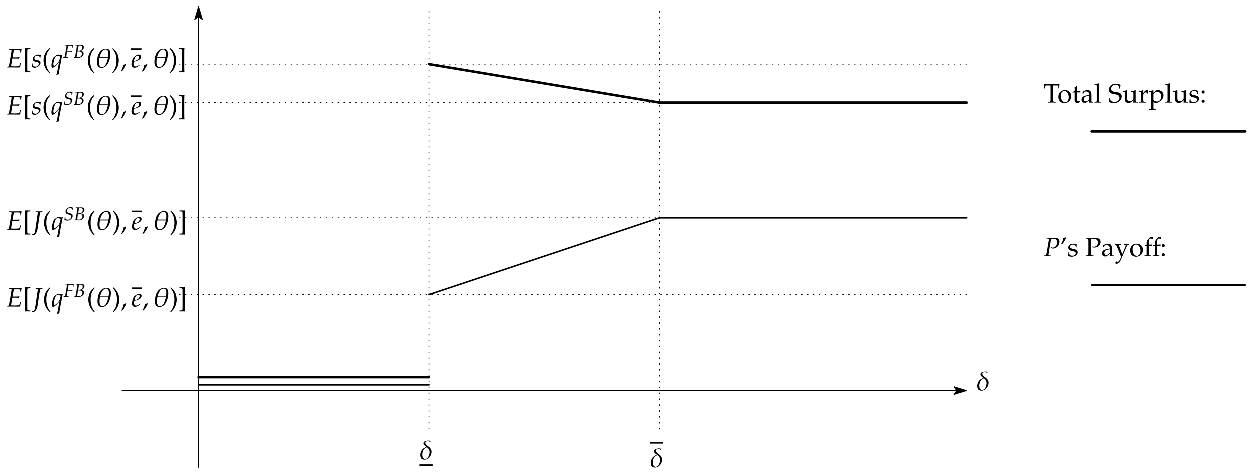

Figure 3: Total Surplus and P's payoff in the OPPE

$\lambda^* > 0$.

3. *For* $\delta = \underline{\delta}$, $d^*(\theta) = (q^{FB}(\theta), \bar{e})$.

4. *For* $\delta < \underline{\delta}$, $W^* = \phi$.

Proposition 2 implies the following corollary.

Corollary 1 *On the OPPE, the total surplus for* $\delta \in (\underline{\delta}, \overline{\delta})$ *is decreasing in* δ *and strictly greater than* $\delta \geq \overline{\delta}$. *P's payoff for* $\delta \in (\underline{\delta}, \overline{\delta})$ *is increasing in* δ *and strictly less than* $\delta \geq \overline{\delta}$.

Figure 3 illustrates the result in Proposition 2 and Corollary 1.

The intuition of Corollary 1 is as follows. Since effort is unverifiable, self-enforcement contracts are required to induce positive effort from A. The self-enforcement contracts are credible if the value of the relationship in the future is large enough. This is captured by constraint (6). For high discount factors such that $\delta \geq \overline{\delta}$, the players value the future surplus so much that P is allowed to optimize her utility with respect to the first task. As the discount factor is lower such that $\delta \in [\underline{\delta}, \overline{\delta})$, the optimal level of the first task does not ensure the future surplus enough to sustain the self-enforcing contracts. Nevertheless as (6) shows, it is possible to induce $\bar{e}$ by raising the total surplus for an intermediate value of the discount factor. Thus the total surplus is rather improved by mitigating inefficiency with respective to the first task caused by asymmetric information. In other words, P's objective is altered from optimizing her own payoff to caring about the total surplus to sustain the relational contracts.

5 Social Desirability of Unverifiability

Contrary to the previous assumption, assume now that e is also verifiable within a period. We show that verifiability could lead to a lower social surplus.

Even when e is verifiable, the basic procedure for deriving the equilibrium is similar to the previous section.[29] The difference is that informal contracts $b^*(q^*(\theta), e)$ do not have to satisfy (HC) which is corresponding to (4). Thus here the optimization problem to derive the OPPE is to maximize (5) by control variables $(q^*(\theta), e^*(\theta), U^*(\theta), b^*(q^*(\theta), e))$ subject to (2), (3), and monotonicity of $q^*(\theta)$ in θ.

Proposition 3 *When e is verifiable, the OPPE satisfies that $W^* \in \overline{\mathfrak{W}}$, $q^*(\theta) = q^{SB}(\theta)$, and $e^*(\theta) = \bar{e}$ for any $\delta \in [0, 1)$.*

Comparing it with the result in Proposition 2 immediately implies the following corollary.

Corollary 2 *For $\delta \geq \underline{\delta}$, unverifiability of e is weakly socially desirable. In particular, for $\delta \in [\underline{\delta}, \bar{\delta})$, unverifiability of e is strictly socially desirable.*

Figure 4 illustrates the comparison between the unverifiable case and the verifiable one. Actually the problem of the verifiable case is the same as that of the unverifiable

[29] The optimality of stationary contracts with all the types participating can be proven by a similar proof of Proposition 1 and Lemma 1.

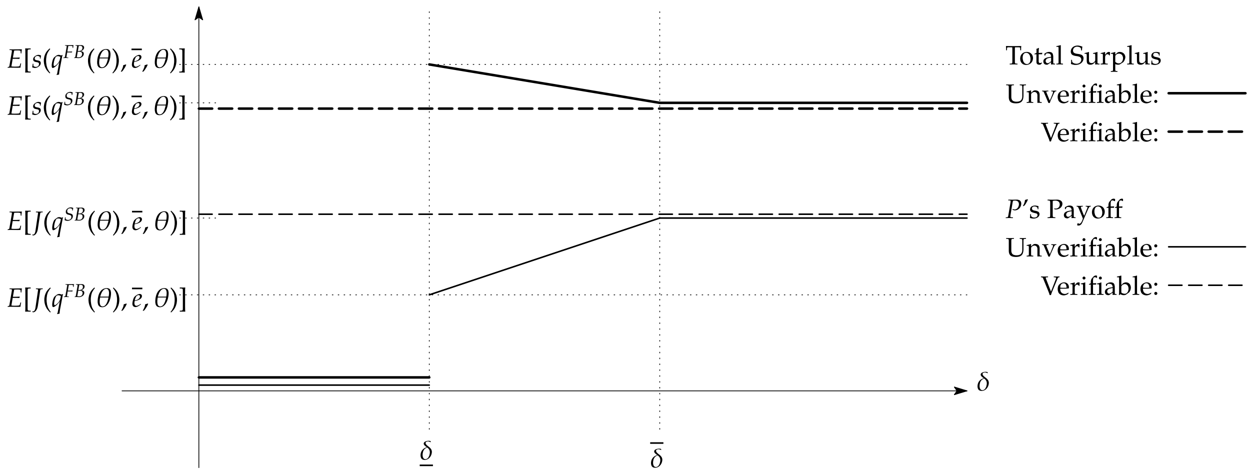

Figure 4: Comparison between the Verifiable and Unverifiable Cases

case without (6). Thus the equilibrium achieved in the verifiable case is the same as in the unverifiable case with $\delta \geq \bar{\delta}$. However, it is less socially efficient than the unverifiable case with $\delta \in [\underline{\delta}, \bar{\delta})$.

The intuition is similar to the comparison between the cases of high and intermediate discount factor in the previous section. Even for intermediate δ, if e is verifiable, then P has no problem about implementing $\bar{e}$. Unverifiability of e urges P to consider the implementation of $\bar{e}$ more seriously which induces more social efficiency as discussed before.

Note that unverifiability is not desirable from P's view. Actually, regardless of verifiability of e, P's objective is the expected value of $J(q^*(\theta), \bar{e}, \theta)$ and the unverifiability could change the quantity level from $q^{SB}(\theta)$ to a more socially efficient level. Since $q^{SB}(\theta)$ is the optimum of $J(q^*(\theta), \bar{e}, \theta)$, the unverifiability is obviously less preferred by P. Thus P does not strategically leave the contract incomplete even if she can.[30]

[30]Nevertheless we can demonstrate the strategic contractual incompleteness. by extending the model. One way of extension is just introducing a positive cost of writing a contract on e. Another extension is assuming that at the beginning of the repeated game there is an option to invest in a technology which makes the effort verifiable (e.g., introducing the objective measurement related to the effort). Then P does not necessarily choose to invest in the technology by receiving a transfer from A as a consideration for leaving the unverifiability.

We can also obtain an economic implication for the role of courts and contract law. The standard contract theory typically tells us that if the court could void a contract, a party anticipating that action would lose the incentive to implement better economic performance due to the lack of the commitment to the *ex post* compensation. This view makes it difficult to explain why courts sometimes intervene to void contractual terms in reality. One of the explanations from our model is that when the long-term relationship is working and the party with bargaining power are faced with the party with private information, the court should sometimes void the terms of the contract. Intuitively, contract design by the party without information causes inefficiency due to the concern of the information rent. If verifiability is not assured by the court, then the contracting party cannot control the information rent and then the inefficiency caused by it can be mitigated.

Several papers in the literature point out the desirability of unverifiability. Nevertheless it should be emphasized that the logic here is quite different from that of the literature. To understand the difference, consider a strategy which implements the same decision every period in equilibrium. Then it can be a public perfect equilibrium if and only if the following inequality is satisfied:

$$\frac{\delta}{1-\delta}\left[s^{*}-\underline{s}\right] \geq g^{*} \tag{10}$$

where s^{*} denotes the equilibrium total surplus, $\underline{s}$ does the total surplus without relational contracts and g^{*} the agent's deviation benefit from unverifiable tasks. It expresses that the self-enforcing contract can work if and only if the future total surplus from relational contracts exceeds the one-shot benefit from deviation in the unverifiable tasks. By substituting $\underline{s} = 0$ and $g^{*} = \bar{e}$, it is the same as (6) in our model.

The typical argument is that unverifiability can make for a stronger punishment on deviation, which improves efficiency. For instance, Schmidt and Schnitzer (1995) study a relational contracting model where, as in our model, the agent's tasks are two dimensional. They show that the above inequality is necessary and sufficient for the equilibrium condition and compare the cases where both tasks are unverifiable and one of the tasks is verifiable. Then in the latter, since the verifiable task can be implemented by formal contracts, the deviation incentive g^* is smaller. At the same time, however, the total surplus without relational contracts $\underline{s}$ could be larger since formal contracts can always give incentives for the verifiable task. Overall, whether the constraint is relaxed or not is ambiguous. In particular, if the effect on $\underline{s}$ dominates that of g, then verifiability is not necessarily desirable. Here the payoff $\underline{s}$ is the total surplus after informal contracts are reneged on. Thus $\underline{s}$ expresses the degree of effectiveness of the punishment. Making one task verifiable can weaken the punishment necessary for sustaining relational contracts, which can worsen efficiency.[31]

By contrast, our argument starts with the case where both tasks are verifiable. Then the formal contract can completely eliminate the deviation incentive meaning that $g^* = 0$. Furthermore, since there is no unverifiable task, the principal will implement the decision as she likes. Now suppose that one of the tasks becomes unverifiable. Since the same strategy as in the verifiable case is most preferred to the principal, it would be implemented as long as it satisfies (10). If not, then the principal would attempt to improve the total surplus to satisfy (10). In other words, by increasing g, the principal's objective is changed from maximizing her own payoff distorted from the total surplus to improving the total surplus to sustain the relational contracts. In this sense, a more important effect of

[31] A similar argument is developed by Baker et al. (1994), Bernheim and Whinston (1998), and Kovrijnykh (2010).

unverifiability on the efficiency in our argument is increasing g rather than decreasing $\underline{s}$.

6 Task Assignment Problem

In the main analysis, we assume that a single agent dedicates to the two tasks. The literature on the multitasking agency problem demonstrates that the distribution of the tasks to multiple agents could mitigate or reinforce the inefficiency caused by the multitasking incentive problem. The effect of the task allocation suggests whether the tasks should be allocated to one agent as a bundle or two or more agents separately and gives us implications on organizational design.

In this section, we develop a discussion of the task assignment problem in the following manner. The first task is highly expertized so that only the original agent can work on it and he still holds private information on that technology. By contrast, the second task can be delegated to another party. However the performance of it is still unverifiable so that a self-enforcing mechanism is required.

Formally, e is unverifiable but the decision right of e is contractible for infinite periods. Thus we modify the repeated game by introducing period -1 in which P chooses a party to whom the second task is allocated. Then from period 0, they play the repeated game in which in each period the party with the decision right of e makes a decision and incurs the cost of e.

We consider the following three modes.

- Task Bundling (B): the decision right is allocated to the agent deciding q
- Centralization (C): P keeps the decision right of e
- Task Separation (S): the decision right is allocated to an agent different from one who

decides q.

Recall that (B) is exactly the same as in the analysis of section 4. Here we investigate (C) and (S).[32]

6.1 Centralization

First suppose that P keeps the decision right of e and denote the OPPE under (C) by superscript C. If she offers a mechanism, among the equilibrium conditions, the constraint (EI) can be ignored because she no longer has to care about A's incentive to exert effort. Instead, she must care about her own incentive to choose e^C.[33] The constraint would be

$$\int_{\underline{\theta}}^{\overline{\theta}} [y(q^C(\theta), e^C) - w^C(q^C(\theta), e^C)]d\theta \geq \int_{\underline{\theta}}^{\overline{\theta}} [y(q^C(\theta), e') - w^C(q^C(\theta), e')]d\theta \tag{11}$$

for $e^C \neq e'$. The other constraints are unchanged. Then, given that P offers some mechanism $W^C \in \mathfrak{W}$, the problem of deriving the OPPE under centralization is to maximize (5) subject to (1), (3), (4), (11) and monotonicity of $q^C(\theta)$. The equilibrium under (C) is characterized as follows.

Lemma 3 *The optimization problem described above is equivalent to the following one;*

$$\max_{q^C(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} J(q^C(\theta), \bar{e}, \theta) f(\theta) d\theta$$

subject to monotonicity of $q^C(\theta)$.

[32]While depending on the allocation the strategy space is changed, a similar proof of Proposition 1 and Lemma 1 shows that we can still focus on the stationary strategy where all the types accept if a mechanism is offered.

[33]Here e^C must be independent from θ because we assume that P chooses e before observing A's choice of q. Nevertheless, the same result is established even if P chooses e after observing A's choice of q so that e^C can be type-dependent.

It is easy to see that Assumption 1 assures the following result.

Proposition 4 *For any $\delta \in [0,1)$, the OPPE under (C) satisfies that $W^C \in \overline{\mathfrak{W}}$, $q^C(\theta) = q^{SB}(\theta)$, and $e^C = \bar{e}$.*

6.2 Task Separation

Next consider (S) in which P delegates the decision right of e to another agent. Denote the equilibrium by superscript S and let us call the agent to choose q $A1$ and one to choose e $A2$, respectively. The timing of the game within a period is modified as follows. At stage 0, only $A1$ knows his type θ. At stage 1, P offers a mechanism $p^S(q)$ to $A1$ and $b^S(e)$ to $A2$ simultaneously and publicly in the sense that every party observes this offer.[34] If at least either of the agents rejects the mechanism offered by P, then the period ends and all the parties obtain zero payoff as their outside option. If both accept, then at stage 2, $A1$ and $A2$ make a decision on their respective tasks simultaneously. Finally at stage 3, P and $A2$ decide whether to enforce the informal agreement or not.

Suppose that P offers a mechanism. Since $A1$ is not involved in the second task or relational contracts, the constraints (EI) and (HC) are ignored and then the constraints on $A1$ are characterized by (1), (3), and monotonicity of $q^S(\theta)$. Instead, P must care about $A2$'s incentive to choose e^S, $A2$'s participation constraint, and the self-enforcing constraint between P and $A2$. Let $u^{2S} := b^S(e^S) - e^S$ be $A2$'s payoff within a period. The constraints

[34] The result is not changed even if P can offer an informal agreements to $A1$, an informal agreement can depend on q, or the offers are sequential.

are respectively described as

$$\begin{aligned} u^{2S} &\geq b^S(e') - e' \text{ for } e^S \neq e' && (12) \\ u^{2S} &\geq 0 && (13) \\ \frac{\delta}{1-\delta}\pi^S &\geq b^S(e) \geq \frac{\delta}{1-\delta}u^{2S} \text{ for } e = \{0, \bar{e}\}. && (14) \end{aligned}$$

Although these additional constraints must be taken into account, the procedure for deriving the optimal solution can be similarly simplified.

Lemma 4 *The optimization problem described above is equivalent to the following;*

$$\max_{q^S(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} J(q^S(\theta), \bar{e}, \theta) f(\theta) d\theta$$

$$\text{subject to } \frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\overline{\theta}} J(q^S(z), \bar{e}, z) f(z) dz \geq \bar{e} \quad (15)$$

and $q^S(\theta)$ is non-increasing in θ.

By using this simplified problem, the equilibrium can be characterized as follows.

Proposition 5 *Let $\underline{\delta}^S \equiv \bar{e}/(\bar{e} + \int_{\underline{\theta}}^{\overline{\theta}} J(q^{SB}(z), \bar{e}, z) f(z) dz)$. The OPPE under (S) satisfies the following.*

1. *For $\delta \geq \underline{\delta}^S$, $q^S(\theta) = q^{SB}(\theta)$ and $e^S = \bar{e}$.*

2. *For $\delta < \underline{\delta}^S$, $W^S = \phi$.*

6.3 Implications

Note that given that P offers a mechanism in equilibrium, P's objective is $\int_{\underline{\theta}}^{\overline{\theta}} J(q(\theta), \bar{e}, \theta) f(\theta) d\theta$ which is common among (B), (C), and (S). Then if P can choose among those, she chooses

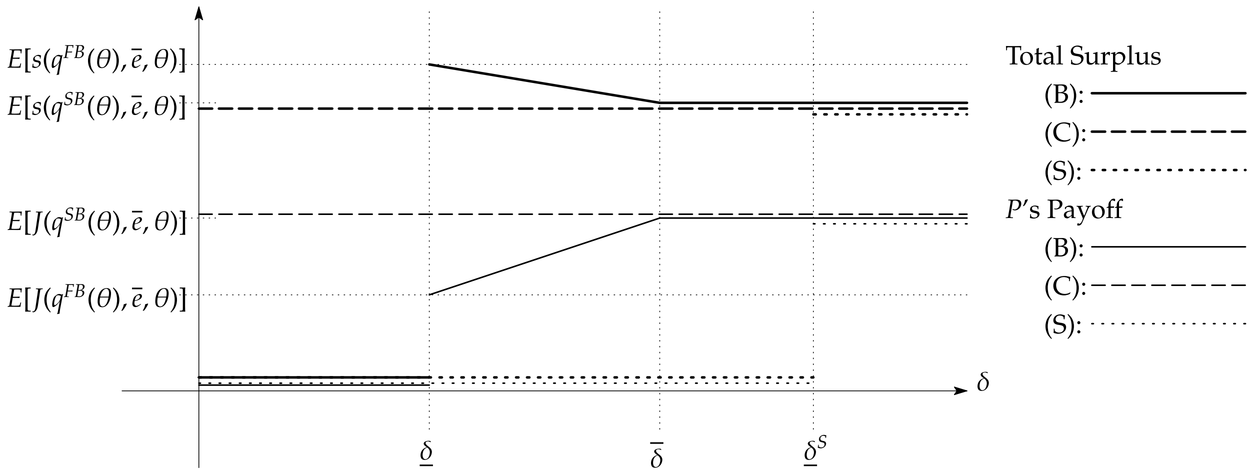

Figure 5: Comparison among (B), (C), and (S)

the one attaining the maximum equilibrium payoff. The corollary below directly follows from Proposition 2, 4, and 5.

Corollary 3 1. *Suppose that P can choose (C). Then she weakly prefers (C) to the others. She strictly prefers (C) to (B) if $\delta < \overline{\delta}$ and to (S) if $\delta < \underline{\delta}^S$.*

2. *For $\delta \in [\underline{\delta}, \overline{\delta})$, when (B) and (C) are available, the total surplus is increased by preventing (C) from being chosen.*

3. *Suppose that P can choose only (B) and (S). Then she weakly prefers (B) to (S) for any $\delta \in [0,1)$ and strictly prefers (B) for $\delta \in [\underline{\delta}, \underline{\delta}^S)$.*

4. *The total surplus in (B) is weakly greater than (S) for any $\delta \in [0,1)$ and strictly greater for $\delta \in [\underline{\delta}, \underline{\delta}^S)$.*

Corollary 3 can be easily obtained from Figure 5, which illustrates the result of the three modes. Comparison between (B) and another mode gives us interesting implications. First, centralization is always preferred by P but could be less socially efficient than task bundling. Since e generates direct benefit for P, P is willing to choose $e = \bar{e}$ in (C) and then she can simply optimize $\int_{\underline{\theta}}^{\overline{\theta}} J(q,\bar{e},\theta)f(\theta)d\theta$ by formal contracts. This is very similar to the situation in which e is verifiable as in Section 5. It eliminates the effect of improving social efficiency caused by the concern for implementing the unverifiable task when $\delta \in [\underline{\delta}, \overline{\delta})$.

This analysis provides a rationale for delegation in organizations. Specifically, if monetary transfer is allowed in period -1 in which the allocation of decision right on e is determined, P should "sell" the decision right to A unless δ is so small that relational contracts do not work at all. Interestingly, this is somewhat counter-intuitive in the sense that since e has a direct benefit for P, P has a right incentive to work on the unverifiable task and then the inefficiency caused by incomplete contracts disappears under centralization. Here, delegation to the party with small bargaining power can be a commitment device to avoid the inefficiency caused by the asymmetric information.

Second, task separation is generally less preferred from both P's and the social perspective. It is obvious from the comparison between the constraints of (B) and that of (S). Contrary to (6) in (B), the consequent constraint in (S) becomes (15). The key feature of (S) different from (B) is that whether relational contracts work or not is independent from $A1$'s payoff. Actually the left hand side is the aggregate surplus between P and $A2$ which does not include $A1$'s payoff. Since $s(q, \bar{e}, \theta) > J(q, \bar{e}, \theta)$ for all $q \in Q$ and $\theta \in (\underline{\theta}, \overline{\theta}]$, if $q^S(\theta)$ is feasible in the optimization problem of (S), then it is also feasible in the optimization problem of (B). Then P does not prefer (S) to (B) because of the feasibility. In addition, P does not have any incentives to mitigate the social inefficiency caused by $A1$'s information rent under (S). This implies that (S) is also socially less desirable.

Comparison between (B) and (S) suggests that while the tasks have no technological complementarity in the sense that the total costs are additively separable, they are complements due to the incentive problem. Intuitively, it is because the future information rent can be an incentive device for the task with unverifiable performance. Bundling the tasks provides information rent caused by the first task for the agent who also works on the second task and then the incentive constraint for the second task can be relaxed in comparison with task separation. In this sense, tasks with hidden information and unverifiable measurement would be complements when relational contracts are used for providing incentives.

The benefit of task bundling from an incentive perspective has been already discussed in the literature. In static settings, it has been shown that, given that tasks have no complementarity in the agent's cost, the tasks should be bundled and allocated to one agent if the difficulty of measuring performance is similar among those tasks (Holmström and Milgrom, 1991) or if there is only an aggregate measurement for the tasks available for terms in contracts (Itoh, 1994). In dynamic settings, this would also be the case if there is a component which causes a ratchet effect (Meyer et al., 1996). All of these studies rule out private information held by the agent and the possibility of relational contracting.[35]

Recently, as argued by Gibbons (2005a), there has been an increasing interest in the effect of organizational design on relational contracts. Schöttner (2008) studies a relational contracting model with multiple tasks and discusses the effect of job design on relational contracts. She focuses on moral hazard environments and shows that task bundling can reinforce relational contracts due to the malfunction of formal contracts.[36] Our model suggests another perspective for job design problems with relational contracts; task bundling is beneficial since the task with private information can provide an incentive for the unverifiable task *via* information rent.

[35] In case of hidden information with a large number of tasks, task bundling could also be beneficial by using the linking mechanism. See Jackson and Sonnenschein (2007) and Matsushima et al. (2010) for the detail.

[36] This argument is similar to the discussion of verifiability in the sense that both verifiability and job design can be instruments that change the effectiveness of punishment for cheating in relational contracts.

7 Extension and Robustness

7.1 Extension to a General Setting

For tractability, our model is simplified in some aspects which might seem not to be reasonable. Here we discuss how these assumptions are innocuous to the results derived above.

Binary Choice of Effort First, we assume that the second task e is a binary decision. This is not a crucial assumption as long as both q and e are assumed to be essential for the value of the relationship. For instance, suppose that e is chosen from interval $[0, E]$ where $E \geq \bar{e}$. Then the same result is obtained if $y(q, e)$ is a step function in e which jumps from 0 to positive value at $e = \bar{e}$. Or even if $y(q, e)$ is increasing and differentiable everywhere in e, a similar equilibrium is attained if $y_e(q, e)$ is sufficiently high. The crucial assumption for our result is that e has a large marginal effect on the aggregate benefit so that providing an incentive for e is a priority. It forces P to improve the total surplus to induce high effort. The extreme form of this idea is a binary decision.

Measurement Error Second, our model does not contain any error in measuring the outcome of the task. It can be relaxed as follows. Suppose that both q and e are unobservable for P and then unverifiable. Instead, a stochastic signal $k \in \{1, 0\}$ correlated with q is verifiable and both parties can observe an unverifiable signal $y \in \{1, 0\}$ correlated with e. Finally P's *ex post* benefit is given by Bky where B is constant positive. In this setting, according to Levin (2003), without loss of generality, the equilibrium is still characterized by the stationary strategy without any possibilities of terminating the relationship and the result is qualitatively similar to the case without measurement errors.

While we abstract from measurement errors for focusing on the information rent problem, this extension would be valuable to investigate. One promising topic for future research is the usefulness of less informative verifiable signals as discussed in Baker et al. (1994) for single task agency. They study whether and how a noisy verifiable signal is useful when the parties intend to use relational contracts based on a more informative but unverifiable signal. In our model, the verifiable signal k does not contain any information on the unverifiable task e. By introducing correlation between k and e, we may discuss which kind of verifiable signal is desirable for relational contracts. Another topic is further investigation into task assignment problem. Our model generates no trade-off between task bundling and task separation; task bundling is weakly dominant from an incentive perspective. However, as Holmström and Milgrom (1991) point out, heterogeneity in the degree of informativeness of the signals can lead to task separation since task separation can avoid misallocation of effort across tasks and Schöttner (2008) points out that it has the opposite effect if relational contracts are available. Thus, incorporating measurement noise into our model can stimulate discussion on the job design problem.

Additive Separability of Cost Function The third simplification is about additive separability of A's cost function. We should admit that this assumption is for tractability of the model. Nevertheless, the result seems to be robust as long as a marginal change of θ and q has little effect on that of e.

Let us briefly review the technical difficulty. Suppose that A's cost function is generalized to $c(q, e, \theta)$ and denote $\bar{c}(q, \theta) \equiv c(q, \bar{e}, \theta)$ and $\underline{c}(q, \theta) \equiv c(q, 0, \theta)$. Assume that $\bar{c}_q(q, \theta) \geq \underline{c}_q(q, \theta)$ and $\bar{c}_\theta(q, \theta) \geq \underline{c}_\theta(q, \theta)$. When the analysis proceeds as in Section 4, (6) becomes

$$\frac{\delta}{1-\delta}\int_{\underline{\theta}}^{\overline{\theta}} s(q^*(z),\overline{e},z)f(z)dz \geq \overline{c}(q^*(\theta),\theta) - \underline{c}(q^*(\theta),\theta)$$

for all θ who accept the mechanism. This inequality says as before that the cost of the unverifiable task must be no more than the future discounted sum of the expected total surplus. Here the cost of the unverifiable task depends on the type.[37] Then the optimization problem must take into consideration the constraints for every θ. Since there are infinitely many θ, it makes the problem much more complex.[38] It further suggests that it could be the case that some types do not accept the mechanism in equilibrium even under Assumption 1. Thus we might have to consider the set of types which reject the mechanism, which makes the analysis more complex.

7.2 Assumptions on the Behaviour

Punishment upon Rejection of Mechanisms Assumption 3 imposes that if no mechanism is offered or an offered mechanism is rejected, then the players play the same strategy in the continuation game. It implies that if the equilibrium stipulates that P offers a mechanism in period 0, then the same mechanism must be offered in the next period if it was rejected in period 0.

For $\delta \geq \underline{\delta}$, on the OPPE the same mechanism is offered each period and all types accept it. Then rejection is an unexpected deviation in the sense that it leads off the equilibrium path. According to Abreu (1988), allowing the punishment for deviations leading off the equilibrium path could make the players better off. Thus, allowing termination of the

[37]It would be the case even if $\overline{c}_\theta(q,\theta) = \underline{c}_\theta(q,\theta)$ because the type affects the optimal quantity which further affects the effort as long as $\overline{c}_q(q,\theta) > \underline{c}_q(q,\theta)$.

[38]If we make an assumption that $\overline{c}_\theta(q,\theta) = \underline{c}_\theta(q,\theta)$, then it can be shown that the right hand side achieves the maximum at $\theta = \underline{\theta}$.

relationship after rejecting the mechanism could change the equilibrium.

In order to look at this possibility, consider the following situation. After A rejects the equilibrium mechanism, instead of the same mechanism being offered, P does not offer any mechanism and both parties obtain zero payoff. Denote this equilibrium by superscript $**$. Now consider the participation constraint for A with type θ. If he accepts the mechanism, then he obtains $U^{**}(\theta)$ in the current period and u^{**} as his continuation payoff. Conversely, If he rejects it, then he obtains 0 in the current period and also 0 as his continuation payoff. Then instead of (3), the participation constraint becomes

$$U^{**}(\theta) + \frac{\delta}{1-\delta} u^{**} \geq 0$$

for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

By the same procedure as in Section 4, we see that the equilibrium satisfies that for high δ,

$$s_q(q^{**}(\theta), \bar{e}, \theta) \quad = \quad (1-\delta) c_{q\theta}(q^{**}(\theta), \theta) \frac{F(\theta)}{f(\theta)}$$

and for intermediate δ,

$$s_q(q^{**}(\theta), \bar{e}, \theta) \quad = \quad \frac{1-\delta}{1 + \frac{\delta}{1-\delta}\lambda^{**}} c_{q\theta}(q^{**}(\theta), \theta) \frac{F(\theta)}{f(\theta)}$$

where $\lambda^{**} > 0$. It immediately implies that for high δ, $q^{FB}(\theta) > q^{**}(\theta) > q^{*}(\theta) = q^{SB}(\theta)$ for $\theta \in (\underline{\theta}, \overline{\theta}]$ meaning that total efficiency is improved by imposing punishment for A's rejection.[39] Intuitively, by rejecting the mechanism, A loses two benefits: the current

[39] For intermediate δ, direct comparison between $q^{*}(\theta)$ and $q^{**}(\theta)$ is impossible in general since the threshold of the discount factor and the Lagrange multiplier are different. Nevertheless the same inequality is established.

information rent and the expected payoff from future trade. It means that relative to the case without punishments for the rejection, the current payoff is less important for A and then the incentive to mimic another type to obtain the information rent is also less important. It leads to a more efficient outcome since P's concern about the reduction of information rent is less serious. Imposing punishment on A's rejection also alters A's payoff level. Specifically,

$$U^{**}(\overline{\theta}) = -\delta \int_{\underline{\theta}}^{\overline{\theta}} c_{\theta}(q^{**}(\theta), \theta) F(\theta) d\theta$$

which is obviously negative for $\delta > 0$. Then even if the current payoff is lower than his outside option, A could choose to participate because present loss can be compensated by future trade.

Whereas imposing punishment on rejection can improve efficiency, we rule out this possibility in the main analysis for two reasons. First, it is not robust to some aspects. For instance, the result that every type participates in equilibrium is due to Assumption 1 on $J(q, \bar{e}, \theta)$. If this assumption is relaxed and there are some types rejecting the mechanism in equilibrium, rejection is no longer an unexpected observable deviation and then the punishment is not necessarily optimal. Second, while the equilibrium is altered by imposing punishment on rejection, our main results derived above are qualitatively the same. In particular, the social desirability of unverifiability and the issues in job design can be demonstrated similarly even if we allow punishment for rejection and then our message from the analysis is basically unchanged.

Principal's Commitment Ability The important restriction made by Assumption 4 is that in each period the strategy must maximize the principal's payoff subject to the equilibrium conditions as long as the players are on the equilibrium path. In other words, the principal cannot commit themselves to a worse equilibrium in the continuation game. While this assumption seems to be reasonable in the case where we presume that P has always full bargaining power, we should review the case where the principal can commit themselves to a worse outcome.

If both Assumption 3 and 4 are absent, then P's optimal equilibrium strategy is no longer stationary. Nevertheless, it is still simply characterized as follows.[40]

- if $\delta \geq \underline{\delta}$ or e is verifiable, then the equilibrium decision in period t, $(q_t(\theta), e_t(\theta))$ is independent of the past history and satisfies

$$s_q(q_0(\theta), \bar{e}, \theta) = (1-\delta)c_{q\theta}(q_0(\theta), \theta)\frac{F(\theta)}{f(\theta)},$$

  $q_t(\theta) = q^{FB}(\theta)$ for $t \geq 1$, and $e_t(\theta) = \bar{e}$ for $t \geq 0$, and

- otherwise, the principal offers no contract every period.

When $\delta \geq \underline{\delta}$ or e is verifiable, only the decision in the first period is distorted in equilibrium. When the principal implements the first best, she must give more information rent to the agent than the second best level. While it is costly for her in the one shot relation, the dynamic structure allows that this loss for her can be compensated by transfer from the agent in the first period.

It is not hard to confirm that these results are not consistent with what we have argued so far. For instance, raising δ and verifiability of e never worsen efficiency. Then it seems that our argument is vulnerable without Assumption 4. However, we argue that it is not true because of the following reason. The essentiality of Assumption 4 in our model is

[40]The proof is available upon request from the author.

due to the assumption that θ is independently drawn over time, which is required for tractability. If θ is serial correlated, then our argument would be valid even without Assumption 4 while we do not formally show the result here.

The above conjecture can be obtained from the result by Battaglini (2005) who studies a model of dynamic price discrimination where the consumer has private information about the valuation of the good and the valuation evolves in a Markovian way. Whereas he considers a single task model with binary types, the first task level in the above equilibrium is very similar to his equilibrium characterization where the type is independently drawn over time. As Battaglini (2005)'s result suggests, if the type evolves in a Markovian way, then the decision optimal for the principal still remains inefficient distortion in the future. It implies that if in our two task model the type evolves in a Markovian way and Assumption 3 and 4 are absent, then the principal attempts to implement the quantity similar to Battaglini (2005)'s characterization as long as it can provide an incentive for the second task. We have argued that when the constraint to honour relational contracts are stringent, the future inefficiency can be mitigated for ensuring the relational contract to work. Thus our guess is that even if Assumption 3 and 4 are dropped, our argument is still valid if the type is serially correlated.

Incentives through Efficiency Wages Throughout the paper, we assume that *ex post* transfer stipulated by informal agreements is available every period as long as the players have no incentive to deviate. Nevertheless, it might be presumably infeasible from some reasons.[41] If *ex post* transfer is unavailable, the efficiency wage can be a substitute for

[41]For instance, if we consider a sequence of short-lived principals instead of one principal playing in infinite periods, then each principal cannot commit to honour any informal agreements. Another example is a situation where there are multiple agents competing each period to get the monopolistic transaction opportunity with the principal. In this case, the principal has an opportunity to find and transact with another agent next period. If the parties outside the transaction have no way of knowing if cheating was taken place in the informal agreement, then punishing such cheating becomes impossible. It makes it

providing incentives for unverifiable tasks. As Shapiro and Stiglitz (1984) demonstrate, in the efficiency wage scheme, the players terminate their relationship after observing that the agent makes an unexpected deviation.

Let us describe the stationary equilibrium with efficiency wages. Denote the equilibrium with efficiency wage by superscript EW. Since they do not use any informal agreements, (HC) is no longer relevant. It is straightforward that (TT) and (PC) are the same as before. The constraint is changed in (EI). If A with type θ chooses $q = q^{EW}(\theta)$ and $e = \bar{e}$, then he can obtain $U^{EW}(\theta)$ in the current period and u^{EW} as the continuation payoff. Conversely if he chooses $q = q^{EW}(\theta')$ and $e = 0$, then his payoff in the current period is

$$p^{EW}(q^{EW}(\theta')) - c(q^{EW}(\theta'), \theta) = U^{EW}(\theta') + c(q^{EW}(\theta'), \theta') - c(q^{EW}(\theta'), \theta) + \bar{e}$$

and the continuation payoff is 0 because the relationship with the principal ends. Hence the equilibrium condition is

$$U^{EW}(\theta) + \frac{\delta}{1-\delta} u^{EW} \geq U^{EW}(\theta') + c(q^{EW}(\theta'), \theta') - c(q^{EW}(\theta'), \theta) + \bar{e}.$$

As Lemma 2, the equilibrium is characterized by the optimization problem to maximize $\int_{\underline{\theta}}^{\overline{\theta}} J(q^{EW}(\theta), \bar{e}, \theta) F(\theta) d\theta$ subject to

$$\frac{\delta}{1-\delta} u^{EW} \geq \bar{e} \tag{16}$$

and monotonicity of $q^{EW}(\theta)$ where $u^{EW} = \int_{\underline{\theta}}^{\overline{\theta}} c_\theta(q^{EW}(\theta), \theta) F(\theta) d\theta$. It is again straightforward that if δ is so high that $q^{EW}(\theta) = q^{SB}(\theta)$ satisfies (16), then it is the solution. Otherwise,

impossible for the principal to credibly commit herself to informal agreements no matter how patient she is. See MacLeod and Malcomson (1989, 1998) and Calzolari and Spagnolo (2010) for an analysis of the competitive situation.

it is still valid to solve the Lagrangian if $c_{\theta qq} \leq 0$. In this case, $q^{EW}(\theta)$ satisfies that

$$s_q(q^{EW}(\theta), \bar{e}, \theta) = \left(1 - \frac{\delta \lambda^{EW}}{1-\delta}\right) \frac{F(\theta)}{f(\theta)} c_{\theta q}(q^{EW}(\theta), \theta)$$

with $\lambda^{EW} \geq 0$ which satisfies (16) with equality provided that $\lambda^{EW} \leq (1-\delta)/\delta$.[42] If these qualifications are satisfied, then a similar result is obtained to the case with informal agreements; the total surplus for intermediate discount factors is greater than for high discount factors.

If the above characterization by the Lagrangian is not valid, then the quantity could be distorted upward from $q^{FB}(\theta)$. Whereas we admit it to be an open question, our guess is reasonable since (16) shows that to induce $e^{EW} = \bar{e}$ is possible if A's (future) information rent is sufficiently large. Since the information rent is assured by increasing q monotonically, P would distort q upward as long as such large q is still beneficial to her.

8 Conclusion

This paper studied the interaction of formal and informal contracts in multitasking agency problem with adverse selection. We investigated the multitasking incentive problem to show how the incentive problems due to hidden information and unverifiability interact. The analysis provided various insights on contract and organization design.

We have shown that unverifiability can improve social efficiency. If a performance measure of some task becomes unverifiable, then the principal needs to use relational contracts to provide incentives to undertake the task. Then the principal's main concern might shift from the reduction of the agent's rent to assuring credible relational contracts.

[42] If $\lambda^{EW} > (1-\delta)/\delta$, then $q^{EW}(\theta)$ does not satisfy monotonicity.

This shift could improve social efficiency since relational contracts can be sustained by increasing the future total surplus.

Second, the task assignment problem implies that those tasks should be bundled and allocated to a single agent. Specifically, whereas leaving the decision rights on the unverifiable task in the hands of the principal seems to mitigate the agency problem, as in the discussion of the verifiable task, it negatively affects the relational contracts. Furthermore, separating those tasks between two agents is totally undesirable. This suggests that those tasks should be treated as complements from the incentive perspective.

We conclude by suggesting a future research agenda. First, our model does not include any measurement error. According to Levin (2003), an extension in this direction seems to be somewhat easy since the optimal equilibrium would still be stationary and, as mentioned in Section 7, it can provide additional insights on contract and organization design. Second and more broadly, we guess that a similar result can be obtained without hidden information. In particular, the key idea behind the assumption on the first task is that the decision on the first task is distorted from the efficient level due to the principal's cost of implementing the efficient decision. This phenomenon also emerges for instance in moral hazard environments with limited liability. However if there is another task whose performance measure is unverifiable, the principal again alters her priority from incentivizing the first task according to her payoff to ensuring credible relational contracts. Thus a formal analysis that investigates what kind of multitasking agency problems generates an outcome similar to ours is a future research topic for examining the robustness of our prediction.[43]

[43]The difficulty of the model of relational contracts with limited liability is that the optimal equilibrium is in general non-stationary and then the dynamics must be taken into account. For instance, see Thomas and Worrall (1994) and Fong and Li (2010).

A Proofs

A.1 Proof of Proposition 1

First we show two lemmas.

Lemma 5 *Let (π, u) be a PPE payoff. Then $\pi \geq 0$ and $u \geq 0$.*

Proof *P can achieve at least zero payoff by abstaining from offering mechanisms regardless of the public history. A can obtain at least zero payoff by rejecting any mechanism regardless of the public history.* $\square$

Lemma 6 *There exists a PPE σ such that $\Gamma(h^t) = \phi$ for all $h^t \in \mathcal{H}$.*

Proof *Consider the strategy satisfying the following. For all $h^t \in \mathcal{H}$, $\Gamma(h^t) = \phi$ and for any $\tilde{W} = (\tilde{p}, \tilde{b}) \in \mathfrak{W}$ and $d \in D$*

$$\begin{aligned} \chi(\tilde{W} \mid h^t, \theta) &= \begin{cases} (\tilde{q}, 0) \in D & \text{if } \tilde{q} \in \arg\max_{q \in Q}[\tilde{p}(q) - c(q, \theta)] \text{ and } \tilde{p}(\tilde{q}) - c(\tilde{q}, \theta) \geq 0 \\ \omega & \text{otherwise,} \end{cases} \\ \iota^i(\tilde{W}, d \mid h^t) &= R. \end{aligned}$$

Namely, P does not offer any mechanism and A does not choose $e = \bar{e}$ after every history as long as it assures non-negative payoff in the current period. Given history h^t, type θ, and mechanism $\tilde{W}$, A has no incentive to deviate from $\chi(\tilde{W} \mid h^t, \theta)$ since deviation does not improve his current payoff nor change the continuation payoff.[44] *Given history h^t, type θ, mechanism $\tilde{W} \in \mathfrak{W}$, and decision $d \in D$, to change from $\iota^A(\tilde{W}, d \mid h^t) = R$ to H does not alter A's payoff since $\iota^H(\tilde{W}, d \mid h^t) = R$ and*

[44]We have assumed that an formal contract $p(\cdot)$ must be continuous. When we allow P to offer non-continuous $p(\cdot)$, this strategy is not necessarily sequentially rational. Specifically, for some non-continuous $p(\cdot)$, $\arg\max_{q \in Q}[p(q) - c(q, \theta)]$ might be empty. It means that in the subgame after such $p(\cdot)$ is offered, there is no optimal decision for A and then this strategy does not satisfy the conditions of PPE off the equilibrium path.

the continuation payoff is not changed. Thus the strategy is sequentially rational for A. Suppose that changing P's strategy alters her expected payoff. Note that due to the same argument, to change from $\iota^P(\tilde{W}, d \mid h^t) = R$ to $I^P = H$ does not alter P's payoff. Then it could be the case only when she deviates at stage 0 in some period and A must accept the mechanism. However because of the construction of A's strategy, if A accepts the mechanism, then the total benefit must be strictly lower than 0 *by Assumption 1. It implies that either P or A's payoff must be strictly lower than* 0 *which contradicts Lemma 5. Thus changing P's strategy does not alter her expected payoff implying that P has no incentive to change her strategy. Then this strategy is a PPE.* □

Note that Lemma 6 implies that there is a PPE in which the expected payoff vector is $(0, 0)$.

Suppose that $\Gamma(h^0) = \phi$. Then by assumption 4, $\Gamma(h^t) = \phi$ for any h^t on the equilibrium path. The strategy described in the proof of Lemma 6 generates the equilibrium payoff $(0, 0)$. Since Lemma 5 assures that the OPPE payoff must be no less than 0 for both parties, the statement in Proposition 1 is satisfied.

In the rest of the proof, suppose that $\Gamma(h^0) \in \mathfrak{W}$. Denote the set of public strategies by $\hat{\Sigma}^i$ for $i = P, A$. As mentioned in Section 3.3, σ^P and σ^A can be respectively rewritten as

$$\sigma^P \begin{cases} W \in \mathfrak{W} \\ \iota^P : \mathfrak{W} \times D \to \mathfrak{I} \\ \sigma^P_+ : \{\phi\} \cup (\mathfrak{W} \times ((D \times \mathfrak{I}^2) \cup \{\omega\})) \to \hat{\Sigma}^P, \end{cases} \quad \text{and} \quad \sigma^A \begin{cases} \chi : [\underline{\theta}, \overline{\theta}] \times \mathfrak{W} \to \overline{D} \\ \iota^A : \mathfrak{W} \times D \to \mathfrak{I} \\ \sigma^A_+ : \{\phi\} \cup (\mathfrak{W} \times ((D \times \mathfrak{I}^2) \cup \{\omega\})) \to \hat{\Sigma}^A. \end{cases}$$

For notational simplicity, let $\ell(W, d) = \mathbf{1}\{\iota^i(W, d) = H \text{ for } i = P, A\}$ and $(\pi_+(h^1), u_+(h^1))$ be the continuation payoff where the public history after period 0 was h^1. Let (π, u) be the corresponding payoff. By applying the one-shot deviation principle, it is shown that σ satisfies if the following five conditions (PPC), (IC), (PHC), (AHC), and (CE) are satisfied;

(PPC)

$$
\begin{aligned}
&\int_{\theta:d(\theta,W)\in D}\Big[y(\chi(\theta,W))-p(q(\theta,W))-\ell(W,\chi(\theta,W))b(\chi(\theta,W))\\
&+\frac{\delta}{1-\delta}\pi_+(W,\chi(\theta,W),\iota^P(W,\chi(\theta,W)),\iota^A(W,\chi(\theta,W)))\Big]f(\theta)d\theta+\int_{\theta:d(\theta,W)\notin D}\frac{\delta}{1-\delta}\pi f(\theta)d\theta\\
\geq\ &\max\Big\{\sup_{W'\in\mathfrak{W}}\Big[\int_{\theta:d(\theta,W')\in D}\Big[y(\chi(\theta,W'))-p(q(\theta,W'))-\ell(W',\chi(\theta,W'))b(\chi(\theta,W'))\\
&+\frac{\delta}{1-\delta}\pi_+(W',\chi(\theta,W'),\iota^P(W',\chi(\theta,W')),\iota^A(W',\chi(\theta,W')))\Big]f(\theta)d\theta\\
&+\int_{\theta:d(\theta,W')\notin D}\frac{\delta}{1-\delta}\pi f(\theta)d\theta\Big],\frac{\delta}{1-\delta}\pi\Big\}
\end{aligned}
$$

(IC) when $\chi(\theta,W)=(q(\theta,W),e(\theta,W))\in D$,

$$
\begin{aligned}
&p(q(\theta,W))-c(q(\theta,W),\theta)-e(\theta,W)+\ell(W,\chi(\theta,W))b(\chi(\theta,W))\\
&+\frac{\delta}{1-\delta}u_+(W,\chi(\theta,W),\iota^P(W,\chi(\theta,W)),\iota^A(W,\chi(\theta,W)))\\
\geq\ &\max\left\{\sup_{d'\equiv(q',e')\in D}\Big[p(q')-c(q',\theta)-e'+\ell(W,d')b(d')+\frac{\delta}{1-\delta}u_+(W,d',\iota^P(W,d'),\iota^A(W,d'))\Big],\frac{\delta}{1-\delta}u\right\}
\end{aligned}
$$

and when $\chi(\theta,W)=\omega$,

$$
\frac{\delta}{1-\delta}u\ \geq\ \sup_{d'\equiv(q',e')\in D}\Big[p(q')-c(q',\theta)-e'+\ell(W,d')b(d')+\frac{\delta}{1-\delta}u_+(W,d',\iota^P(W,d'),\iota^A(W,d'))\Big],
$$

(PHC) when $\iota^P(W,d)=H$,

$$
-b(d)+\frac{\delta}{1-\delta}\pi_+(W,d,H,\iota^A(W,d))\geq\frac{\delta}{1-\delta}\pi_+(W,d,R,\iota^A(W,d)),
$$

and when $\iota^P(W,d)=R$,

$$
-b(d)+\frac{\delta}{1-\delta}\pi_+(W,d,H,\iota^A(W,d))\leq\frac{\delta}{1-\delta}\pi_+(W,d,R,\iota^A(W,d)),
$$

(AHC) when $\iota^A(W,d) = H$,

$$b(d) + \frac{\delta}{1-\delta}u_+(W,d,\iota^P(W,d),H) \geq \frac{\delta}{1-\delta}u_+(W,d,\iota^P(W,d),R),$$

and when $\iota^A(W,d) = R$,

$$b(d) + \frac{\delta}{1-\delta}u_+(W,d,\iota^P(W,d),H) \leq \frac{\delta}{1-\delta}u_+(W,d,\iota^P(W,d),R),$$

and

(CE) for all $z = \phi$ or $(W,d) \in \mathfrak{W} \times (\{\omega\} \cup (D \times \mathfrak{I}^2))$, $(\pi_+(z), u_+(z))$ is a PPE and it is also an OPPE if z is on the equilibrium path.

Let $\Sigma^* \equiv \{\sigma \in \hat{\Sigma} \mid \sigma$ satisfies (PPC), (IC), (PHC), (AHC), and (CE)$\}$. The OPPE is the strategy which maximizes P's payoff in Σ^*. Let Ψ^* be the set of payoff vectors attained by $\sigma \in \Sigma^*$.

Lemma 7 *If $\sigma \in \Sigma^*$, its associated mechanism in period 0 is $W \in \mathfrak{W}$, and W is accepted with positive probability, then there exists an OPPE $\tilde{\sigma}$ with its associated mechanism in period 0 is $\tilde{W}$ such that the payoff is the same as σ and the informal agreements are honoured whenever $\tilde{W}$ is offered and A accepts it.*

Proof *Construct strategy $\tilde{\sigma}$ as follows; for $d \in D$ such that $\iota(W,d) \neq (H,H)$,*

$$\begin{aligned}
\tilde{b}(d) &= 0 \\
(\tilde{\pi}_+(\tilde{W},d,H,H), \tilde{u}_+(\tilde{W},d,H,H)) &= (\pi_+(W,d,\iota(W,d)), u_+(W,d,\iota(W,d))) \\
(\tilde{\pi}_+(\tilde{W},d,I), \tilde{u}_+(\tilde{W},d,I)) &= (0,0) \textit{ for } I \neq (H,H) \\
\tilde{\iota}^P(\tilde{W},d) = \tilde{\iota}^A(\tilde{W},d) &= H
\end{aligned}$$

and the others are the same as σ*, i.e.,* $\tilde{\chi}(\theta, \tilde{W}) = \chi(\theta, W)$*,* $\tilde{p}(q) = p(q)$*,* $\tilde{b}(d) = b(q)$*,* $\tilde{\iota}^i(\tilde{W}, d) = \iota^i(W, d)$ *for* $i = P, A$*,* $(\tilde{\pi}_+(\tilde{W}, d, I), \tilde{u}_+(\tilde{W}, d, I)) = (\pi_+(W, d, I), u_+(W, d, I))$*, and* $(\tilde{\pi}_+(\tilde{W}, \omega), \tilde{u}_+(\tilde{W}, \omega)) = (\pi_+(W, \omega), u_+(W, \omega))$*. Note that* $(\tilde{\pi}_+(\tilde{W}, d, H, H), \tilde{u}_+(\tilde{W}, d, H, H)) \in \Psi^*$ *and* $(\tilde{\pi}_+(\tilde{W}, d, I), \tilde{u}_+(\tilde{W}, d, I))$ *is a PPE for* $I = (H, R), (R, H), (R, R)$ *thanks to Lemma 5. Then (CE) is satisfied. By construction, (PHC) and (AHC) with* $\tilde{\iota}^P(\tilde{W}, d') = \tilde{\iota}^A(\tilde{W}, d') = H$ *are satisfied for any* $d' \in D$ *and*

$$\begin{aligned} & \tilde{p}(q) + \tilde{\ell}(\tilde{W}, d')b(d') + \frac{\delta}{1-\delta}\tilde{u}_+(\tilde{W}, d', H, H) \\ = \; & p(q) + \ell(W, d')b(d') + \frac{\delta}{1-\delta}u_+(W, d', \iota^P(W, d'), \iota^A(W, d')) \end{aligned} \tag{17}$$

that assures (PPC) and (IC). Hence $\tilde{\sigma}$ *is a PPE. Since* $\tilde{\chi}(\tilde{W}, \theta) = \chi(W, \theta)$ *and (17) holds, A's payoff in* $\tilde{\sigma}$ *is the same as* σ*, generating the same payoff as* σ*.* □

Then without loss of generality, (PHC) and (AHC) are written as

(PHC′) for any $d \in D$,

$$-b(d) + \frac{\delta}{1-\delta}\pi_+(W, d, H, H) \;\; \geq \;\; \frac{\delta}{1-\delta}\pi_+(W, d, R, H)$$

(AHC′) for any $d \in D$,

$$b(d) + \frac{\delta}{1-\delta}u_+(W, d, H, H) \;\; \geq \;\; \frac{\delta}{1-\delta}u_+(W, d, H, R).$$

Lemma 8 Ψ^* *is compact.*

Proof *For all* $(\pi, u) \in \Psi^*$*, Lemma 5 assures* $(\pi, u) \geq (0, 0)$ *and since* $s(q, e, \theta)$ *is bounded from above,* $\pi + u$ *is also bounded above. Then* Ψ^* *is bounded.*

To show that it is closed, consider an arbitrary converging sequence $\{(\pi^n, u^n)\}_{n=0}^{\infty}$ *where* $(\pi^n, u^n) \in \Psi^*$ *for all* $n \geq 0$ *and* $\lim_{n\to\infty}(\pi, u)$ *and let* σ^n *be the corresponding strategy pro-*

file. The proof is completed if $(\pi, u) \in \Psi^*$. *Let* $\underline{\mathfrak{W}} \equiv \{p : Q \to [-K, K], b : D \to [-K, K]\}$ *where* $K \equiv \sup_{q' \in Q, e' \in \{0, \bar{e}\}, \theta' \in [\underline{\theta}, \bar{\theta}]} s(q', e', \theta')\delta/(1-\delta)$. *Then the mechanism offered under* σ^n *is without loss of generality in* $\underline{\mathfrak{W}}$ *which is compact. Then we can construct a converging subsequence* $\{\tilde{\sigma}^n\}_{n=0}^{\infty}$. *Let* $\tilde{\sigma}$ *be its limit. Note that* $\tilde{\sigma}$ *can be decomposed to* $\tilde{W}$, $\tilde{d}(\theta, W)$, $\tilde{\iota}^i(W, d)$, *and* $\tilde{\sigma}^i_+(h^1)$ *for* $i = P, A$. *Now suppose that* $(\pi, u) \notin \Psi^*$. *Then either (PPC), (IC), (PHC'), (AHC'), or (CE) is violated. If (PPC) is violated, then there exists* $W' \in \mathfrak{W}$ *such that*

$$
\begin{aligned}
& \int_{\theta:\tilde{d}(\theta,\tilde{W})\in D} \Big[y(\tilde{\chi}(\theta,\tilde{W})) - \tilde{p}(\tilde{q}(\theta,\tilde{W})) - \tilde{\ell}(\tilde{W},\tilde{\chi}(\theta,\tilde{W}))\tilde{b}(\tilde{\chi}(\theta,\tilde{W})) \\
& + \frac{\delta}{1-\delta}\tilde{\pi}_+(\tilde{W},\tilde{\chi}(\theta,\tilde{W}),\tilde{\iota}^P(\tilde{W},\tilde{\chi}(\theta,\tilde{W})),\tilde{\iota}^A(\tilde{W},\tilde{\chi}(\theta,\tilde{W})))\Big] f(\theta)d\theta + \frac{\delta}{1-\delta}\int_{\theta:\tilde{d}(\theta,\tilde{W})\notin D} \tilde{\pi} f(\theta)d\theta + \varepsilon \\
< \; & \max\Big\{ \int_{\theta:\tilde{d}(\theta,W')\in D} \Big[y(\tilde{\chi}(\theta,W')) - p'(\tilde{q}(\theta,W')) - \tilde{\ell}(W',\tilde{\chi}(\theta,W'))b'(\tilde{\chi}(\theta,W')) \\
& + \frac{\delta}{1-\delta}\tilde{\pi}_+(W',\tilde{\chi}(\theta,W'),\tilde{\iota}^P(W',\tilde{\chi}(\theta,W')),\tilde{\iota}^A(W',\tilde{\chi}(\theta,W')))\Big] f(\theta)d\theta + \frac{\delta}{1-\delta}\int_{\theta:\tilde{d}(\theta,W')\notin D} \tilde{\pi} f(\theta)d\theta, \\
& \frac{\delta}{1-\delta}\tilde{\pi}\Big\}
\end{aligned}
$$

for some $\varepsilon > 0$. *Since* $\tilde{\sigma}$ *is a limit of a converging sequences* $\{\tilde{\sigma}^n\}_{n=0}^{\infty}$, *There exists* $N > 0$ *such that*

$$
\begin{aligned}
& \int_{\theta:d^N(\theta,W^N)\in D} \Big[y(\chi^N(\theta,W^N)) - p^N(q^N(\theta,W^N)) - \ell^N(W^N,\chi^N(\theta,W^N))b^N(\chi^N(\theta,W^N)) \\
& + \frac{\delta}{1-\delta}\pi^N_+(W^N,\chi^N(\theta,W^N),\iota^{PN}(W^N,\chi^N(\theta,W^N)),\iota^{AN}(W^N,\chi^N(\theta,W^N)))\Big] f(\theta)d\theta \\
& + \frac{\delta}{1-\delta}\int_{\theta:d^N(\theta,W^N)\notin D} \tilde{\pi} f(\theta)d\theta \\
< \; & \max\Big\{ \int_{\theta:d^N(\theta,W')\in D} \Big[y(\chi^N(\theta,W')) - p'(q^N(\theta,W')) - \ell^N(W',\chi^N(\theta,W'))b'(\chi^N(\theta,W')) \\
& + \frac{\delta}{1-\delta}\pi^N_+(W',\chi^N(\theta,W'),\iota^{PN}(W',\chi^N(\theta,W')),\iota^{AN}(W',\chi^N(\theta,W')))\Big] f(\theta)d\theta \\
& + \frac{\delta}{1-\delta}\int_{\theta:d^N(\theta,W')\notin D} \pi^N f(\theta)d\theta, \frac{\delta}{1-\delta}\pi^N\Big\}
\end{aligned}
$$

which contradicts that $\tilde{\sigma}^N$ *is a PPE. Thus it could not be the case that (PPC) is violated. Similar* *arguments show that neither (IC), (PHC′), nor (AHC′) is not violated. Finally, suppose that (CE) is violated. Then there exists a strategy profile $\tilde{\sigma}_+(h^1)$ for some public history up to period 1, h^1, such that party i can increase his payoff by changing to another strategy $\sigma^{i\prime} \in \Sigma^i$. If $i = P$, then it states that $\pi(\tilde{\sigma}_+(h^1)) + \varepsilon < \pi(\sigma^{P\prime}, \tilde{\sigma}_+^A(h^1))$ for some $\varepsilon > 0$ where $\pi(\sigma)$ is P's average payoff given strategy profile σ. Note that since $\pi(\sigma)$ is a sum of the discounted payoff, if $\{\tilde{\sigma}_+^n(h^1)\}_{n=0}^\infty$ is a converging sequence, then $\{\pi(\tilde{\sigma}_+^n(h^1))\}_{n=0}^\infty$ and $\{\pi(\sigma^{P\prime}, \tilde{\sigma}_+^n(h^1))\}_{n=0}^\infty$ are converging sequences too. Then there exists $N > 0$ such that $\pi(\tilde{\sigma}_+^N(h^1)) < \pi(\sigma^{P\prime}, \tilde{\sigma}_+^{NA}(h^1))$ for some $\sigma^{P\prime} \in \Sigma^P$. It contradicts that $\tilde{\sigma}_+^N(h^1)$ is a PPE. The same argument can be applied for $i = A$.* □

Suppose that σ is an OPPE. By Lemma 8, σ can be restricted to one such that the payoff $(\pi, u) \in \Psi^*$ satisfies that for all $(\pi', u') \in \Psi^*$, $\pi \geq \pi'$ and $u \geq u'$ if $\pi = \pi'$. That is, (π, u) is on the Pareto frontier of Ψ^* subject to π attaining the maximum. Now we construct a strategy σ^* which is in Ψ^* and generates a payoff which weakly Pareto-dominates that of σ. For $d \in D$, let $W^* = (p^*(\cdot), b^*(\cdot))$ be such that

$$
\begin{aligned}
b^*(d) &= b(d) + \frac{\delta}{1-\delta}\left[u_+(W, d, H, H) - u\right] - \inf_{d \in D}\left[b(d) + \frac{\delta}{1-\delta} u_+(W, d, H, H)\right] \\
p^*(q) &= p(q) + \inf_{d \in D}\left[b(d) + \frac{\delta}{1-\delta} u_+(W, d, H, H)\right].
\end{aligned}
$$

In each period t, P chooses $W_t = \phi$ if each of the players has chosen $I_s^i = R$ for some $s < t$ and $W_t = W^*$ otherwise. If W^* is offered, then A chooses $\chi^*(\theta, W^*) = \chi(\theta.W)$ for all θ.

Note that Lemma 6 ensures that there is a PPE such that P does not offer any mechanism. Since P offers W^* repeatedly on the equilibrium path of σ^*, if (IC), (PHC′), and (AHC′) under σ^* are satisfied, there is no incentive to deviate at any information set implying that σ^* is a PPE. Note that the continuation payoff is u^* unless the implicit contracts were reneged on.

We first confirm that (IC) is satisfied under W^*. Note that since σ is an OPPE, (IC) under σ implies that for $\chi(\theta, W) = (q(\theta, W), e(\theta, W)) \in D$,

$$\begin{aligned}
&p(q(\theta, W)) - c(q(\theta, W), \theta) - e(\theta, W) + b(\chi(\theta, W)) + \frac{\delta}{1-\delta} u_+(W, \chi(\theta, W), H, H)) \\
&\geq \max\left\{ \sup_{d' \equiv (q', e') \in D} \left[p(q') - c(q', \theta) - e' + b(d') + \frac{\delta}{1-\delta} u_+(W, d', H, H) \right], \frac{\delta}{1-\delta} u_+(W, \omega) \right\}
\end{aligned}$$

and for $\chi(\theta, W) = \omega$,

$$\frac{\delta}{1-\delta} u_+(W, \omega) \geq \sup_{d' \equiv (q', e') \in D} \left[p(q') - c(q', \theta) - e' + b(d') + \frac{\delta}{1-\delta} u_+(W, d', H, H) \right].$$

These imply (IC) under σ^* by the following argument. First, note that

$$\begin{aligned}
u^* &= (1-\delta) \int_{\theta: d(W,\theta) \in D} \left[p^*(q(\theta, W)) - c(q(\theta, W), \theta) - e(\theta, W) + b^*(\chi(\theta, W)) \right] f(\theta) d\theta + \delta u^* \\
&= (1-\delta) \int_{\theta: d(W,\theta) \in D} \Big[p(q(\theta, W)) - c(q(\theta, W), \theta) - e(\theta, W) + b(\chi(\theta, W)) \\
&\quad + \frac{\delta}{1-\delta} \left[u_+(W, \chi(\theta, W), H, H) - u \right] \Big] f(\theta) d\theta + \delta u^* \\
\Longleftrightarrow u^* &= \int_{\theta: d(W,\theta) \in D} \Big[p(q(\theta, W)) - c(q(\theta, W), \theta) - e(\theta, W) + b(\chi(\theta, W)) \\
&\quad + \frac{\delta}{1-\delta} \left[u_+(W, \chi(\theta, W), H, H) - u \right] \Big] f(\theta) d\theta.
\end{aligned}$$

and

$$\begin{aligned}
u &= \int_{\theta: d(W,\theta) \in D} \left[(1-\delta) \left(p(q(\theta, W)) - c(q(\theta, W), \theta) - e(\theta, W) + b(\chi(\theta, W)) \right) \right. \\
&\quad \left. + \delta u_+(W, \chi(\theta, W), H, H) \right] f(\theta) d\theta + \delta \int_{\theta: d(W,\theta) \notin D} u_+(W, \omega) f(\theta) d\theta.
\end{aligned}$$

Since (π, u) is on the Pareto frontier of Ψ^*, Assumption 4 ensures that $u \geq u_+(W, \omega)$. It implies that

$$
\begin{aligned}
u^* &= \frac{1}{1-\delta}u - \frac{\delta}{1-\delta}\int_{\theta:d(W,\theta)\notin D} u_+(W,\omega)f(\theta)d\theta - \frac{\delta}{1-\delta}\int_{\theta:d(W,\theta)\notin D} uf(\theta)d\theta \\
&\geq \frac{1}{1-\delta}u - \frac{\delta}{1-\delta}\int_{\theta:d(W,\theta)\notin D} uf(\theta)d\theta - \frac{\delta}{1-\delta}\int_{\theta:d(W,\theta)\notin D} uf(\theta)d\theta = u
\end{aligned}
$$

Then, for $\chi(\theta, W) = (q(\theta, W), e(\theta, W)) \in D$,

$$
\begin{aligned}
& p^*(q(\theta, W)) - c(q(\theta, W), \theta) - e(\theta, W) + b^*(\chi(\theta, W)) + \frac{\delta}{1-\delta}u^* \\
= \;& p(q(\theta, W)) - c(q(\theta, W), \theta) - e(\theta, W) + b(\chi(\theta, W)) + \frac{\delta}{1-\delta}u_+(W, \chi(\theta, W), H, H) \\
\geq \;& \max\left\{\sup_{d'\equiv(q',e')\in D}\left[p(q') - c(q',\theta) - e' + b(d') + \frac{\delta}{1-\delta}u_+(W, d', H, H)\right], \frac{\delta}{1-\delta}u_+(W,\omega)\right\} \\
= \;& \max\left\{\sup_{d'\equiv(q',e')\in D}\left[p^*(q') - c(q',\theta) - e' + b^*(d') + \frac{\delta}{1-\delta}u\right], \frac{\delta}{1-\delta}u_+(W,\omega)\right\} \\
= \;& \max\left\{\sup_{d'\equiv(q',e')\in D}\left[p^*(q') - c(q',\theta) - e' + b^*(d') + \frac{\delta}{1-\delta}u^*\right], \frac{\delta}{1-\delta}u^*\right\}
\end{aligned}
$$

and for $\chi(\theta, W) = \omega$,

$$
\begin{aligned}
\frac{\delta}{1-\delta}u^* = \frac{\delta}{1-\delta}u_+(W,\omega) &\geq \sup_{d'\equiv(q',e')\in D}\left[p(q') - c(q',\theta) - e' + b(d') + \frac{\delta}{1-\delta}u_+(W, d', H, H)\right] \\
&= \sup_{d'\equiv(q',e')\in D}\left[p^*(q') - c(q',\theta) - e' + b^*(d') + \frac{\delta}{1-\delta}u\right] \\
&= \sup_{d'\equiv(q',e')\in D}\left[p^*(q') - c(q',\theta) - e' + b^*(d') + \frac{\delta}{1-\delta}u^*\right].
\end{aligned}
$$

These inequalities are equivalent to (IC) under σ^*.

Next we confirm (AHC′). By construction of $b^*(d)$,

$$
\begin{aligned}
b^*(d) + \frac{\delta}{1-\delta}u^* &= b(d) + \frac{\delta}{1-\delta}\left[u_+(W, d, H, H) - u\right] - \inf_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W, d, H, H)\right] + \frac{\delta}{1-\delta}u^* \\
&\geq b(d) + \frac{\delta}{1-\delta}u_+(W, d, H, H) - \inf_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W, d, H, H)\right] \geq 0
\end{aligned}
$$

which implies (AHC′) for σ^*.

Finally, we confirm (PHC′). Note that

$$\begin{aligned}
\pi^* &= (1-\delta)\int_{\theta:d(W,\theta)\in D}\left[y(\chi(\theta,W)) - p^*(q(\theta,W)) - b^*(\chi(\theta,W))\right]f(\theta)d\theta + \delta\pi^* \\
&= (1-\delta)\int_{\theta:d(W,\theta)\in D}\Big[y(\chi(\theta,W)) - p(q(\theta,W)) - b(\chi(\theta,W)) \\
&\quad -\frac{\delta}{1-\delta}\left[u_+(W,\chi(\theta,W),H,H) - u\right]\Big]f(\theta)d\theta + \delta\pi^* \\
\iff \pi^* &= \int_{\theta:d(W,\theta)\in D}\Big[y(\chi(\theta,W)) - p(q(\theta,W)) - b(\chi(\theta,W)) \\
&\quad -\frac{\delta}{1-\delta}\left[u_+(W,\chi(\theta,W),H,H) - u\right]\Big]f(\theta)d\theta.
\end{aligned}$$

and

$$\begin{aligned}
\pi &= \int_{\theta:d(W,\theta)\in D}\left[(1-\delta)\left(y(\chi(\theta,W)) - p(q(\theta,W)) - b(\chi(\theta,W))\right)\right. \\
&\quad \left.+\delta\pi_+(W,\chi(\theta,W),H,H)\right]f(\theta)d\theta + \delta\int_{\theta:d(W,\theta)\notin D}\pi_+(W,\omega)f(\theta)d\theta.
\end{aligned}$$

Since Assumption 4 ensures that $\pi_+(W,\omega) = \pi$,

$$\begin{aligned}
\pi^* &= \frac{1}{1-\delta}\pi - \int_{\theta:d(W,\theta)\in D}\frac{\delta}{1-\delta}\pi_+(W,\chi(\theta,W),H,H)f(\theta)d\theta - \frac{\delta}{1-\delta}\int_{\theta:d(W,\theta)\notin D}\pi_+(W,\omega)f(\theta)d\theta \\
&\quad -\frac{\delta}{1-\delta}\int_{\theta:d(W,\theta)\in D}\left[u_+(W,\chi(\theta,W),H,H) - u\right]f(\theta)d\theta \\
&= \pi + \frac{\delta}{1-\delta}\left[\pi - \int_{\theta:d(W,\theta)\in D}\left(s_+(W,\chi(\theta,W),H,H) - u\right)f(\theta)d\theta - \int_{\theta:d(W,\theta)\notin D}\pi_+(W,\omega)f(\theta)d\theta\right] \\
&= \pi + \frac{\delta}{1-\delta}\left[\int_{\theta:d(W,\theta)\in D}\left(s - s_+(W,\chi(\theta,W),H,H)\right)f(\theta)d\theta\right].
\end{aligned}$$

Note that by construction of σ, $s \geq s_+(W,\chi(\theta,W),H,H)$ for all θ. Thus we obtain $\pi^* \geq \pi$.

(PHC′) is satisfied if and only if

$$-b^*(d) + \frac{\delta}{1-\delta}\pi^* \quad = \quad -b(d) - \frac{\delta}{1-\delta}\left[u_+(W,d,H,H) - u\right] + \inf_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right] + \frac{\delta}{1-\delta}\pi^*$$

is no less than 0 for all d. It is equivalent to that the following is no less than 0;

$$\begin{aligned}
& -\sup_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right] + \inf_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right] + \frac{\delta}{1-\delta}[\pi^* + u] \\
= \quad & -\sup_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right] + \inf_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right] + \frac{\delta}{1-\delta}s^* \geq 0 \\
\iff \quad & \frac{\delta}{1-\delta}s^* \geq \sup_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right] - \inf_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right]. \qquad (18)
\end{aligned}$$

Recall from (PHC′) and (AHC′) under σ that

$$b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H) \geq \frac{\delta}{1-\delta}u_+(W,d,H,R) \geq 0$$

implying that

$$-\inf_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right] \leq 0$$

and

$$\begin{aligned}
\sup_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}u_+(W,d,H,H)\right] \quad & = \quad \sup_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}\left(s_+(W,d,H,H) - \pi_+(W,d,H,H)\right)\right] \\
& \leq \quad \sup_{d\in D}\left[b(d) + \frac{\delta}{1-\delta}\left(s^* - \pi_+(W,d,H,H)\right)\right] \\
& = \quad -\inf_{d\in D}\left[-b(d) + \frac{\delta}{1-\delta}\pi_+(W,d,H,H)\right] + \frac{\delta}{1-\delta}s^* \\
& \leq \quad -\frac{\delta}{1-\delta}\pi_+(W,d,R,H) + \frac{\delta}{1-\delta}s^* \leq \frac{\delta}{1-\delta}s^*
\end{aligned}$$

Summing these inequalities implies (18).

A.2 Proof of Lemma 1 and 2

Let Θ^* be the set of types for which $d^*(\theta) \in D$, i.e. the set of types who accept the mechanism in the OPPE, and $\Theta^{*C} \equiv [\underline{\theta}, \overline{\theta}] \setminus \Theta^*$. We first establish Lemma 2 by showing that given $\Theta^* = [\underline{\theta}, \overline{\theta}]$, the equilibrium can be characterized by the optimization problem in Lemma 2. We next establish Lemma 1 by showing the equilibrium must satisfy either $\Theta^* = [\underline{\theta}, \overline{\theta}]$ or $\Theta^* = \emptyset$.

Proof of Lemma 2 Note that because of (1),

$$u^* = U^*(\overline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} c_\theta(q^*(\theta), \theta) F(\theta) d\theta$$

and then (4) can be written as

$$\frac{\delta}{1-\delta}\left[\int_{\underline{\theta}}^{\overline{\theta}} J(q^*(\theta), e^*(\theta), \theta) f(\theta) d\theta - U^*(\overline{\theta})\right] \geq b^*(q, e) \geq -\frac{\delta}{1-\delta}\left[U^*(\overline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} c_\theta(q^*(\theta), \theta) F(\theta) d\theta\right]. \quad (19)$$

Now $b^*(q, e)$ appears only in constraints (2) and (19). By observing (2), we see that (2) is relaxed by increasing $b^*(q^*(\theta'), e^*(\theta'))$ and by decreasing $b^*(q^*(\theta'), e')$. It implies that without loss of generality

$$b^*(q^*(\theta), e) = \begin{cases} \frac{\delta}{1-\delta}\left[\int_{\underline{\theta}}^{\overline{\theta}} J(q^*(z), e^*(z), z) f(z) dz - U^*(\overline{\theta})\right] & \text{if } e = e^*(\theta) \\ -\frac{\delta}{1-\delta}\left[U^*(\overline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} c_\theta(q^*(z), z) F(z) dz\right] & \text{if } e = e' \end{cases} \quad (20)$$

and then (2) becomes

$$
\begin{aligned}
& \frac{\delta}{1-\delta}\int_{\underline{\theta}}^{\overline{\theta}} s(q^*(z), e^*(z), z) f(z) dz + \int_{\theta}^{\theta'} c_\theta(q^*(x), x) dx - c(q^*(\theta'), \theta') + c(q^*(\theta'), \theta) \geq e^*(\theta') - e' \\
\Longleftrightarrow \quad & \frac{\delta}{1-\delta}\int_{\underline{\theta}}^{\overline{\theta}} s(q^*(z), e^*(z), z) f(z) dz + \int_{\theta}^{\theta'} \left[c_\theta(q^*(x), x) - c_\theta(q^*(\theta'), x)\right] dx \geq e^*(\theta') - e'. \qquad (21)
\end{aligned}
$$

Now (19) is redundant and $U^*(\theta)$ appears only in the objective and (3). Then (3) is obviously binding; $U^*(\overline{\theta}) = 0$. Thus the rest of the constraints are (21) and monotonicity of $q^*(\theta)$.

The objective function shows that $e^*(\theta')$ must be $\bar{e}$ as long as it is feasible. If $e^*(\theta') = \bar{e}$, then the right hand side is $\bar{e}$. However recall that given $\theta' \in [\underline{\theta}, \overline{\theta}]$ fixed, (21) must be satisfied for any $\theta \in [\underline{\theta}, \overline{\theta}]$. Now the following lemma is useful.

Lemma 9 *Suppose that $q^*(\theta)$ is non-increasing in θ. Then*

$$
\min_{\theta, \theta' \in [\underline{\theta}, \overline{\theta}]} \left[\int_{\theta}^{\theta'} \left[c_\theta(q^*(x), x) - c_\theta(q^*(\theta'), x)\right] dx\right] = 0.
$$

Proof (Lemma 9) *Suppose that $q^*(\theta)$ is non-increasing in θ. Given arbitrary $\theta' \in [\underline{\theta}, \overline{\theta}]$ fixed,*

$$
\frac{d}{d\theta}\left[\int_{\theta}^{\theta'} \left[c_\theta(q^*(z), z) - c_\theta(q^*(\theta'), z)\right] dz\right] = -c_\theta(q^*(\theta), \theta) + c_\theta(q^*(\theta'), \theta) \begin{cases} \geq 0 & \text{if } \theta > \theta' \\ = 0 & \text{if } \theta = \theta' \\ \leq 0 & \text{if } \theta < \theta' \end{cases}
$$

meaning that the term $\int_{\theta}^{\theta'} [c_\theta(q^*(z), z) - c_\theta(q^*(\theta'), z)] dz$ is non-increasing in $\theta < \theta'$ and non-decreasing in $\theta > \theta'$. This implies that it attains the minimum at $\theta = \theta'$ the value of which is obviously zero. □

Lemma 9 implies that $e^*(\theta') = \bar{e}$ if and only if

$$\frac{\delta}{1-\delta}\int_{\underline{\theta}}^{\overline{\theta}} s(q^*(z), e^*(z), z) f(z) dz \geq \bar{e}$$

which is independent from θ'. This further implies that if $e^*(\theta) = \bar{e}$ for some θ, then $e^*(\theta) = \bar{e}$ for all θ. If $e^*(\theta) = 0$ for all θ, then Assumption 1 implies that $\pi^* < 0$ and P should choose to offer no mechanism. Thus the equilibrium in which some mechanism is offered must satisfy that $e^*(\theta) = \bar{e}$ for all θ and the condition becomes (6).

Proof of Lemma 1 Note that $U^*(\theta) = 0$ for all $\theta \in \Theta^{*C}$. If Θ^* has zero probability measure, then π^* is also 0. In this case, to offer no mechanism is indifferent. Thus without loss of generality, suppose that Θ^* has positive probability measure.

Furthermore, suppose that for $\theta < \theta'$, $\theta' \in \Theta^*$ and $\theta \in \Theta^{*C}$. For type θ', there is no incentive to mimic θ. It is satisfied if and only if

$$U^*(\theta') \geq 0$$

Conversely, type θ has no incentive to mimic θ' if and only if

$$\begin{aligned} 0 &\geq w^*(q^*(\theta'), e^*(\theta')) - c(q^*(\theta'), \theta) - e^*(\theta') \\ &= U^*(\theta') + c(q^*(\theta'), \theta') - c(q^*(\theta'), \theta). \end{aligned}$$

Combining them implies that $c(q^*(\theta'), \theta') \leq c(q^*(\theta'), \theta)$, which contradicts $\theta < \theta'$. It implies that if $\theta \in \Theta^{*C}$, then $\theta' \in \Theta^{*C}$ for all $\theta' > \theta$. It also means that there exists $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that $[\underline{\theta}, \hat{\theta}) \subset \Theta^*$ and $(\hat{\theta}, \overline{\theta}] \subset \Theta^{*C}$.

Given $\hat{\theta}$ fixed, the equilibrium conditions can be characterized in the same way as in the proof of Lemma 2 by replacing $\overline{\theta}$ with $\hat{\theta}$. Then it can be characterized by the following optimization problem;

$$\max \int_{\underline{\theta}}^{\hat{\theta}} J(q^*(z), \overline{e}, z) f(z) dz \quad \text{subject to} \quad \frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\hat{\theta}} s(q^*(z), \overline{e}, z) f(z) dz \geq \overline{e}$$

and monotonicity of $q^*(\theta)$. The Euler equation provides the characterization of the equilibrium as follows.

1. If

$$\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\hat{\theta}} s(q^{SB}(z), \overline{e}, z) f(z) dz \geq \overline{e} \iff \delta \geq \overline{\delta}(\hat{\theta}) \equiv \frac{\overline{e}}{\overline{e} + \int_{\underline{\theta}}^{\hat{\theta}} s(q^{SB}(z), \overline{e}, z) f(z) dz},$$

then the equilibrium satisfies that $d^*(\theta) = (q^{SB}(\theta), \overline{e})$ for $\theta > \hat{\theta}$.

2. If $\delta < \overline{\delta}(\hat{\theta})$ and

$$\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\hat{\theta}} s(q^{FB}(z), \overline{e}, z) f(z) dz > \overline{e} \iff \delta > \underline{\delta}(\hat{\theta}) \equiv \frac{\overline{e}}{\overline{e} + \int_{\underline{\theta}}^{\hat{\theta}} s(q^{FB}(z), \overline{e}, z) f(z) dz},$$

then the equilibrium satisfies that $e^*(\theta) = \overline{e}$,

$$\begin{aligned} s_q(q^*(\theta), \overline{e}, \theta) &= \frac{1}{1 + \frac{\delta}{1-\delta} \lambda^*(\hat{\theta})} \frac{F(\theta)}{f(\theta)} c_{q\theta}(q^*(\theta), \theta), \\ \frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\hat{\theta}} s(q^*(z), \overline{e}, z) f(z) dz &= \overline{e} \end{aligned}$$

with some $\lambda^*(\hat{\theta}) > 0$.

3. For $\delta = \underline{\delta}(\hat{\theta})$, $q^*(\theta) = q^{FB}(\theta)$ and $e^*(\theta) = \overline{e}$.

4. For $\delta < \underline{\delta}(\hat{\theta})$, $W^* = \phi$.

Note that the value of P's payoff is positive if $\delta \geq \underline{\delta}(\hat{\theta})$ while it is zero if $\delta < \underline{\delta}(\hat{\theta})$.

Now fix $\delta \in [0, 1)$ and suppose that $\hat{\theta} < \overline{\theta}$. In the following, we show that P's payoff is weakly increasing in $\hat{\theta}$, implying that choosing $\hat{\theta} < \overline{\theta}$ is never optimal.

First, suppose that $\delta < \underline{\delta}(\hat{\theta})$. Note that $\underline{\delta}(\hat{\theta})$ is decreasing in $\hat{\theta}$. When $\delta < \underline{\delta}(\overline{\theta})$, P's payoff is still 0 since $\delta < \underline{\delta}(\theta)$ for any θ. However when $\delta \geq \underline{\delta}(\overline{\theta})$, it become positive for $\hat{\hat{\theta}}$ such that $\delta \geq \underline{\delta}(\hat{\hat{\theta}})$. Then choosing $\hat{\theta} < \overline{\theta}$ is weakly dominated by another $\hat{\hat{\theta}}$ such that $\delta \geq \underline{\delta}(\hat{\hat{\theta}})$.

Second suppose that $\delta = \underline{\delta}(\hat{\theta})$. Then $q^*(\theta) = q^{FB}(\theta)$. Now consider $\hat{\hat{\theta}}$ which is slightly greater than $\hat{\theta}$. Since

$$
\begin{aligned}
\frac{d}{d\theta} \int_{\underline{\theta}}^{\theta} s(q^{FB}(z), \overline{e}, z) f(z) dz &= s(q^{FB}(\hat{\theta}), \overline{e}, \hat{\theta}) f(\hat{\theta}) > 0 \\
\frac{d}{d\theta} \int_{\underline{\theta}}^{\theta} J(q^{FB}(z), \overline{e}, z) f(z) dz &= s(q^{FB}(\hat{\theta}), \overline{e}, \hat{\theta}) f(\hat{\theta}) > 0
\end{aligned}
$$

for all $\theta \in [\hat{\theta}, \hat{\hat{\theta}}]$,

$$
\int_{\underline{\theta}}^{\hat{\hat{\theta}}} J(q^{FB}(z), \overline{e}, z) f(z) dz > \int_{\underline{\theta}}^{\hat{\theta}} J(q^{FB}(z), \overline{e}, z) f(z) dz
$$

and

$$
\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\hat{\hat{\theta}}} s(q^{FB}(z), \overline{e}, z) f(z) dz \geq \overline{e}
$$

those imply that by changing $\hat{\theta}$ to $\hat{\hat{\theta}}$, P can achieve the payoff greater than the optimal value under $\hat{\theta}$.

Suppose next that $\delta \in (\underline{\delta}(\hat{\theta}), \overline{\delta}(\hat{\theta}))$. In this case, $q^*(\theta)$ satisfies the first order condition of the following Lagrangian;

$$\mathcal{L}(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}} J(q^*(z), \bar{e}, z) f(z) dz + \lambda^*(\hat{\theta}) \left[\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\hat{\theta}} s(q^*(z), \bar{e}, z) f(z) dz - \bar{e} \right].$$

Note that $\int_{\underline{\theta}}^{\hat{\theta}} J(q^*(z), \bar{e}, z) f(z) dz = \mathcal{L}(\hat{\theta})$ in the neighbourhood of $\hat{\theta}$ and the envelope theorem implies that

$$\frac{d}{d\hat{\theta}} \left[\int_{\underline{\theta}}^{\hat{\theta}} J(q^*(z), \bar{e}, z) f(z) dz \right] = \mathcal{L}'(\hat{\theta}) = \frac{\partial \mathcal{L}}{\partial \hat{\theta}}(\hat{\theta}) = J(q^*(\hat{\theta}), \bar{e}, \hat{\theta}) f(\hat{\theta}) + \lambda^*(\hat{\theta}) \frac{\delta}{1-\delta} s(q^*(\hat{\theta}), \bar{e}, \hat{\theta}) f(\hat{\theta}) > 0$$

meaning that slightly increasing $\hat{\theta}$ improves the objective functions. Then such $\hat{\theta} < \overline{\theta}$ is never optimal.

Finally suppose that $\delta \geq \overline{\delta}(\hat{\theta})$. Since $\overline{\delta}(\hat{\theta})$ is decreasing in $\hat{\theta}$, when $\hat{\theta}$ is increasing, the value of the objective function is still $\int_{\underline{\theta}}^{\hat{\theta}} J(q^{SB}(z), \bar{e}, z) f(z) dz$ and it is increasing in $\hat{\theta}$. Then slightly increasing $\hat{\theta}$ improves the objective functions. Then again such $\hat{\theta} < \overline{\theta}$ is never optimal.

So far we have shown that $\Theta^* = [\underline{\theta}, \overline{\theta}]$ or $[\underline{\theta}, \overline{\theta})$. In the case of $\Theta^* = [\underline{\theta}, \overline{\theta})$, the optimization problem achieves P's payoff which is the same as in the case of $\Theta^* = [\underline{\theta}, \overline{\theta}]$ demonstrated in Section 4. Thus, both cases are indifferent for P and it is without loss of generality to focus on the case of $\Theta^* = [\underline{\theta}, \overline{\theta}]$.

A.3 Proof of Corollary 1

It is enough to demonstrate that the total surplus is decreasing and P's expected payoff is increasing in $\delta \in [\underline{\delta}, \overline{\delta})$. For $\delta \in [\underline{\delta}, \overline{\delta})$, $q^*(\theta)$ satisfies that

$$\int_{\underline{\theta}}^{\overline{\theta}} s(q^*(\theta), \bar{e}, \theta) f(\theta) d\theta = \frac{1-\delta}{\delta} \bar{e}.$$

The left hand side is the total surplus and the right hand side is decreasing in δ. On P's payoff, since it is characterized by the Lagrangian and the constraint is binding, the envelope theorem implies that

$$\frac{d}{d\delta}\int_{\underline{\theta}}^{\overline{\theta}} J(q^*(\theta),\bar{e},\theta)f(\theta)d\theta = \frac{d\mathcal{L}}{d\delta} = \frac{\partial\mathcal{L}}{\partial\delta} = \frac{1}{(1-\delta)^2}\int_{\underline{\theta}}^{\overline{\theta}} s(q^*(\theta),\bar{e},\theta)f(\theta)d\theta > 0.$$

A.4 Proof of Proposition 3

Note that $b^*(q^*(\theta),e)$ appears only in (2). Then by letting $b^*(q^*(\theta),e) = e$ for all θ and e, Lemma 9 implies that (2) is satisfied. Since $U^*(\theta)$ appears only in (5) and (3), (3) is obviously binding; $U^*(\overline{\theta}) = 0$. Then we obtain the problem that maximizes $\int J(q(\theta),e^*(\theta),\theta)d\theta$ subject to monotonicity of $q^*(\theta)$. By Assumption 1, the solution is $e^*(\theta) = \bar{e}$ and $q^*(\theta) = q^{SB}(\theta)$ for all θ.

A.5 Proof of Lemma 3

By substituting $U^C(\theta)$ and $b^C(q^C(\theta),e)$, (11) is written as

$$\int_{\underline{\theta}}^{\overline{\theta}} J(q^C(\theta),e^C,\theta)f(\theta)d\theta \geq \int_{\underline{\theta}}^{\overline{\theta}} [J(q^C(\theta),e^C,\theta) - b(q^C(\theta),e^C) + b(q^C(\theta),e')]f(\theta)d\theta.$$

Now $b^C(q^C(\theta),e)$ appears only in this inequality and (4). Then by substituting $b^C(q^C(\theta),e) = 0$ for any θ and e, these inequalities are satisfied. Since $U^C(\theta)$ appears only in (5) and (3), (3) is obviously binding; $U^C(\overline{\theta}) = 0$. Then we obtain the problem that maximizes $\int J(q(\theta),e^C,\theta)f(\theta)d\theta$ subject to monotonicity of $q^C(\theta)$. Assumption 1 implies that $e^C = \bar{e}$.

A.6 Proof of Lemma 4

By using the notation $U^{1S}(\theta) = p^S(\theta) - c(q^S(\theta), \theta)$ and $u^{2S} = b^S(e^S) - e^S$, the optimization problem for characterizing the OPPE under task separation is as follows;

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \left[s(q^S(\theta), e^S, \theta) - U^{1S}(\theta) - u^{2S} \right] f(\theta) d\theta$$

subject to (1), (3), (12), (13), (14), and monotonicity of $q^S(\theta)$. If $e^S = 0$, the the objective is less than 0 meaning that P prefers to abstain from offering any mechanism. Then we focus on $e^S = \bar{e}$. Now (12) and (13) are reduced to $u^{2s} \geq \max\{b^S(0), 0\}$ and since $b^S(0)$ appears only in this and (14), it should be lower as long as it is possible. It implies that $b^S(0) = -\delta u^{2S}/(1-\delta)$ and $u^{2S} \geq 0$. Since u^{2S} should also be lower, it implies that $u^{2S} = 0$ (and then $b^S(0) = 0$). Substituting (1) shows that (3) is binding and the objective becomes

$$\int_{\underline{\theta}}^{\overline{\theta}} J(q^S(\theta), \bar{e}, \theta) f(\theta) d\theta$$

and since $b^S(\bar{e}) = u^{2S} + \bar{e} = \bar{e}$, (14) becomes (15).

$$\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\overline{\theta}} J(q^S(\theta), \bar{e}, \theta) f(\theta) d\theta \geq \bar{e}.$$

A.7 Proof of Proposition 5

Since $q^{SB}(\theta)$ is monotone and maximizes the objective, it is the solution if it is feasible, i.e.,

$$\frac{\delta}{1-\delta} \int_{\underline{\theta}}^{\overline{\theta}} J(q^{SB}(\theta), \bar{e}, \theta) f(\theta) d\theta \geq \bar{e} \iff \delta \geq \underline{\delta}^S \equiv \frac{\bar{e}}{\bar{e} + \int_{\underline{\theta}}^{\overline{\theta}} J(q^{SB}(\theta), \bar{e}, \theta) f(\theta) d\theta}.$$

If it is not feasible, then there is no feasible $q^S(\theta)$ and then P chooses not to offer any mechanism.

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