Dynamic Competition with Network Externalities: Why History Matters*

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Abstract

This paper considers dynamic platform competition in a market with network externalities. A platform that dominated the market in the previous period becomes "focal" in the current period, in that agents play the equilibrium in which they join the focal platform whenever such equilibrium exists. We ask whether a low-quality but focal platform can maintain its focal position along time, when it faces a higher quality competitor. We find that when platforms are patient enough and with infinite horizon, there are multiple equilibria in which either the low or the high quality platform dominates. If qualities are stochastic, the more platforms care about the future, the platform with the better average quality wins more often than the other. As a result, social welfare can decrease when platforms become more forward-looking.

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1 Introduction

Platform competition typically involves repeated interaction. In each of the markets for smartphones, tablets, video-game consoles, etc., a small set of firms compete with each other repeatedly over time. Platforms should therefore take into account how their strategies today affect their future profits. How the competition in dynamic setting plays out may have an important effect on platforms' profits. Microsoft's Windows wins the market for computer operating system over Apple's OS many generations in a row. It has been often suggested that the Apple's OS is of a better quality, but Windows wins because Microsoft gained the dominant position in the past, and the network effects allow this advantage to carry over time, despite inferior product. In other markets, like video-game consoles and smartphones, market leaders seem to be changing every few generations. So the platforms in those markets cannot count on the same future advantage from winning the market as in the market for computer operating systems.

In repeated interaction between firms, dynamic considerations play a new role when the firms are "platforms," because in such markets, the firms operate in environments with network effects. A platform offers users a product which has some stand-alone value, but the value of the product increases if other users also join the same platform. The benefit may come directly from the presence of other users, or through endogenous provision of complementary goods (e.g., apps are more likely to be developed for a popular platform). The difficulty, however, is that users need to form beliefs before purchase about which platform will offer most network benefits. In many markets we observe that the platform that became the dominant in the recent past has the advantage of users' expectations that it will attract other users and/or complementor providers (e.g., app developers). That is, it becomes the *focal* platform. But despite this beliefs advantage, the platform that won in the past not necessarily will win in the future.

This paper considers repeated platform competition in a market with network externalities. We explore the implications of history dependency, when past success raises the chance to become focal, regarding two related research questions. First, in some cases, a platform that benefits from the focal position, can dominate the market even if it offers a product of lower stand-alone quality than a new platform. In such a case, the focal platform uses its focal advantage to overcome its quality disadvantage. In a dynamic environment, however, when platforms have an infinite horizon, we may expect that the platform with the highest quality will have the strongest incentive to compete aggressively in order to gain and than maintain a focal position. We therefore ask whether a low-quality platform can maintain its focal position along time, when facing a higher-quality platform.

The first research question has an important implication for social welfare. If a dynamic environment makes it more profitable for a high-quality platform to gain a focal position from a lower-quality platform and maintain it along time, then social welfare should increase the more platforms care about future profits. Our second, related, research question is therefore whether social welfare increases or decreases the more patient are the platforms.

To investigate these two research questions, we consider dynamic competition between two platforms. In each period, one of the platforms wins the market. In order to focus on the dynamic aspects of the model, we assume homogeneous consumers. Hence the winning platform captures the whole market. The dynamic set-up allows consumers to base their behavior in the current period on the observation of the past outcomes. Specifically, the platform that won the market in the previous period becomes focal in the current period. In such a case, winning the market in one period gives the platform an advantage in the future periods. Hence, a non-focal platform may be willing to sacrifice current profit to gain future market position.

In our base model, we assume that each platform has stand-alone quality which is constant for all periods. We show that when the platforms are short-sighted (i.e., their discount factor is below a threshold value), the focal platform maintains its focal position even though it offers a lower quality than the non-focal platform, as long as the quality gap is sufficiently small. But when platforms have a high discount factor and the horizon is finite but long enough, the superior quality platform wins the market at the start of the game and maintains its leadership. The reason is that as it expects higher gains from wining a focal position today (as opposed to "wait and compete tomorrow") than the lowest quality platform, and the gap increases with the horizon. However, under infinite horizon with high discount factor and a quality gap sufficiently small, there are multiple Markov equilibria: in some the focal and low-quality platform can maintain its leadership infinitely, but in another Markov equilibrium the non-focal and high-quality platform wins the focal position in the first period and then maintains it infinitely. By contrast, when platforms have intermediate discount factor, there is a unique equilibrium in which the high-quality platform wins the market with finite or infinite horizon.

The intuition for these results is that when platforms are very short-sighted, a non-focal platform does not have incentive to compete aggressively and incur loses to gain a focal position in the future, making it possible for the focal platform to maintain its leadership even with a lower quality. In contrast, when both platforms have a high discount factor, they have strong incentive to win and maintain a focal position. Intuitively the benefit from wining a focal position is larger for superior quality which is why when firms are patient, there is an equilibrium where the superior quality platform wins the market and maintains the focal position. In this equilibrium, the low-quality platform expects that the non-focal and high-quality platform will compete very aggressively in every period in order to gain the focal position, making it not worthwhile for the low-quality platform maintain its leadership. Still the low-quality platform can win the market if the high-quality platform expects that even if it were to gain the focal position, the low-quality platform will compete aggressively in every period, making it not worthwhile for the high-quality platform to be very aggressive itself. Moreover, such aggressive strategy for winning the market back is rational for the low quality platform if it expects accommodating behavior of its competitor. Such an equilibrium arises solely with infinite horizon as it relies on self-supporting beliefs that would vanish at the end game with finite horizon. For intermediate discount factor or finite horizon, the former equilibrium is the a unique equilibrium so that the high-quality platform wins, even if it starts in a non-focal position.

For social welfare, these results indicate that when the platforms' discount factor increases from a low level to an intermediate level, social welfare will increase because the market moves from the equilibrium in which the low-quality platform wins to the one in which the high-quality platform overcomes its non-focal position and then maintain the focal position infinitely. However, the effect of a further increase in the discount factor on welfare is ambiguous because for a high discount factor there are multiple equilibria.

In our base model, the same platform wins in all periods. In some cases, platforms "take turns" in being the dominant platform. In the market for smartphones, for example, Nokia dominated the early stage, along with RIM, with smartphones based on physical keyboard. Apple then revolutionized the industry by betting on the new touch screen technology and its new operating system. Nokia and RIM stuck to their physical keyboard technology and operating systems in the subsequent updates of their products, and eventually lost the leadership position to Apple. Few years later, Samsung, managed to gain a substantial market share (though not strict dominance) by betting on smartphones with large screens, while Apple continues to stick to its 3.5 inch screen. Only recently, when it became evident that there is a high demand for smartphones with large screens, Apple decided to increase the screen size for iPhone 5. Today, Nokia is trying to regain its dominant position by betting on the new Windows Phone technology. In a bid to win back their position in the smartphone market, RIM (now simply called BlackBerry) introduced new phones, Q10 and Z10, with new operating system and many innovative features. Both Apple and Samsung choose to remain with their previous operating systems, and only offer periodical upgrades.

Such industry leader changes were also common in the history of the video-game consoles market. Nintendo, Sony and Microsoft alternated in being the market leader—none of them winning more than two generations in a row. While the technology significantly improved with each generation of video-game consoles, some generations were marked by radical innovation, e.g. Nintendo's Wii.[1]

To study such markets, we then consider the more realistic setting in which the platforms' qualities are stochastic: they vary in each period. Platforms observe the qualities at the beginning of each period, but are uncertain about the potential qualities in future periods. One of the platforms has a higher expected quality then the other. In equilibrium, each platform can win the market in each period with some probability. In particular, there is a threshold in the quality gap between the two platforms such that in each period, each platform wins the market if its quality is sufficiently higher in comparison with the quality of other platform.

We find that unlike the case of constant qualities, the expected social welfare under stochastic qualities can decrease the more platforms are long-sighted. In particular, social welfare when both platforms are substantially long-sighted (i.e., the discount factor is close to 1) is lower than in the case when both platforms are substantially short-sighted (i.e., the discount factor is close to 0). Supportingly, a platform can lose even if it is focal and offers a higher quality than the non-focal platform. This result can never emerge in the case of constant qualities.

The intuition for these results is that with stochastic qualities, a higher discount factor implies that a platform that expects to have high-quality realizations in future periods will have more of an incentive to compete aggressively to win the market in the current period, even if in the current period its quality is substantially inferior to that of the competing platform. This platform will win the market today more than it should from the viewpoint of maximizing expected social welfare, and is likely to maintain a focal position in future

[1]Hagiu and Halaburda (2009)

periods more than it should.

In a generalization to the concept of focal position, we consider intrinsic equilibrium uncertainty captured by assuming a public correlated equilibrium. Although being focal provides an advantage, it is limited in the sense that for a small value adjusted price differential the non-focal platform wins with positive probability. We find that while there may still be equilibria where the focal platform maintains forever its dominant position, existence conditions are more restrictive. Moreover there will be equilibria where alternation occurs in any period with positive probability.

Most theoretical analyses of platform competition focus on static games. Caillaud and Jullien (2001, 2003) consider competition between undifferentiated platforms, where one of them benefits from favorable beliefs. Hagiu (2006) considers undifferentiated platform competition in a setting where sellers join the platform first, and only then buyers. Lopez and Rey (2009) consider competition between two telecommunication networks when one of them benefits from customers' "inertia," such that in the case of multiple responses to the networks' prices, consumers choose a response which favors one of the networks. Halaburda and Yehezkel (forthcoming) consider competition between platforms when one of them has only partial beliefs advantage. While all those papers acknowledge the dynamic nature of the platform competition, they aim at approximating the characteristics of the market in static models. Halaburda and Yehezkel (forthcoming) explore how platform's strategies affect their future profits in a simple multi-period setup where the beliefs advantage depends on the history of the market. Markovich (2008) analyzes hardware standardization in a dynamic market where software firms invest in new product innovation. But the dynamics of platform competition is still underexplored. Cabral (2012) develops a dynamic model of competition with forward looking consumers but where only one consumer chooses at a time avoiding the coordination issue we focus on.

Bligaiser, Crémer and Dobos (2013) consider dynamic competition in the presence of switching costs. Our model share with theirs the feature that success provides an incumbency advantage. But the intertemporal linkage and the demand dynamics differ between the two models. While it is possible to think of network externalities as a type of switching costs, there is a qualitative difference between the two. With network externalities, consumers pay switching costs only of they join the "wrong" platform. If all consumers join the same platform, they do not pay switching costs even if they move from one platform to another.

Consequently, consumers in our model do not need to form beliefs about the identity of the focal platform in future periods. In particular, this distinction enables consumers to switch from one platform to another when qualities are stochastic regardless of probability that a platform will remain focal in the future. Our real-life examples (i.e., the markets for smartphones and videogames) may include both the traditional switching costs (coming from the need to adjust to a new operating system, for example) and network externalities. While Bligaiser, Crémer and Dobos (2013) focus on the former, our paper complements their paper by focusing on the latter type of costs. A second main difference between the two papers is that they consider switching cost heterogeneity, while we focus on quality differential.

Argenziano and Gilboa (2012) consider a repeated coordination game where players use history to form beliefs regarding the behavior of other players. Our paper adopts the same approach in the context of platform competition and study how platforms should compete given such belief formation by consumers. In our paper, platforms can alter beliefs by wining and shifting consumers' coordination in their favor. Our paper is related to ongoing work by Biglaiser and Crémer (2012) trying to define a notion of consumer inertia creating an history dependency. We do not try to model how history dependency emerges but its implications for competition.

2 The Model

Consider an homogeneous population of size 1 and two competing platforms, $i = A$, B, with the same cost normalized to 0. There are T periods, $t = 0, 1, ..., T-1$, where T may be finite or infinite. Each platform i offers a stand-alone value, q_i, which we call quality.[2] Additionally, consumers benefit from network effects. The value of other consumers joining the same platform is β.[3]

Every period each platform i sets a price p_{it}, and then consumers decide which platform to join for the current period. In what follows prices can be negative, interpreted as price below cost.[4]

[2] In the benchmark model, q_i's are fixed over time. In the extensions (Section 6) we consider qualities that change between periods.

[3] Since the consumers are homogeneous they all join the same platform.

[4] To allow for truly negative prices, we need to assume that agents who collect the subsidy indeed join the platform and provide the benefit to other users.

The issue with competition in an environment with network effects is that there is a multiplicity of equilibria. Indeed consider the allocation of consumers that emerges for given prices. If $q_i - p_{it} > q_j - p_{jt} + \beta$, then all consumers would join platform i. But if

$$|q_A - q_B + p_{Bt} - p_{At}| < \beta, \tag{1}$$

there are two possible allocations, all consumers join A or all join B. This multiplicity creates a major difficulty to discussing dynamic competition in environments with network effects, and several solutions have been proposed to address this issue. In this paper we rely on the idea of pessimistic beliefs and focal platform as developed in Caillaud-Jullien (2003), Hagiu (2006) and Jullien (2000). We say that platform i is *focal* if under condition (1), the consumers join platform i. We assume that at any date there is a focal platform.

Assumption: At any date there is a focal platform.

A dynamic model with $t = 0, \ldots, T-1$ allows to determine the identity of the focal platform in period $t > 0$ from the history. To simplify the matters, we focus on one period dynamics.

At every period t, let us summarize the market outcome by a pair (w_t, f_t), where $w_t \in \{A, B\}$ is the identity of the active platform, i.e., the platform who wins the market in t,[5] and $f_t \in \{A, B\}$ is the identity of the focal platform in t. It is possible for the non-focal platform to win the market, therefore those two do not need to be the same. Based on the observation of past outcome, consumers form conjectures about the platform most likely to win in the current period. These conjectures are assumed to converge to a single focal platform. In $t = 0$ one of the platforms is arbitrarily set as the focal platform. Call this platform A. At any date the focal platform f_t is common knowledge and it is the only payoff relevant variable. The dynamics of the platform focality is then given by transition probabilities, $\Pr(f_t = i \mid w_{t-1}, f_{t-1})$. We consider a deterministic rule where the last winner of the market becomes focal, i.e., $\Pr(f_t = w_{t-1} \mid w_{t-1}, f_{t-1}) = 1$.

[5] In this model there cannot be market sharing in equilibrium: at each date, a single platform attracts the whole population.

3 Longer time horizon mitigates inefficiency

3.1 Inefficiency due to inertia in a static model

Network externalities in a static game may give rise to inefficiency in equilibrium. Platform A is the focal platform, but it may be of higher or lower quality than platform B. When $q_A - q_B + \beta > 0$, in the equilibirium the platforms set $p_A = q_A - q_B + \beta, \quad p_B = 0$, and all customers join A. When $q_A - q_B + \beta < 0$, such strategy would yield platform A negative profits, so it is not an equilibrium. In such a case, in the equilibrium the platforms set $p_A = 0, \quad p_B = q_B - q_A - \beta > 0$, and all customers join platform B.

Thus, for q_A such that $q_A < q_B$ but $q_A > q_B + \beta$ platform A wins despite offering lower quality. It wins because it happens to be focal. This effect is called excess inertia and it creates inefficient outcome in equilibrium.

3.2 Inefficiency mitigated in dynamic interaction

When there are multiple periods, a non-focal platform may find it worthwhile to win the market by setting negative price in an earlier period. While it would yield negative profit one period, the focal position could allow to recover those loses in future periods. In the static market no platform finds it optimal to win the market at negative prices, as there is no way to recoup the loses. Thus, the focal platform has the upper hand even when it offers lower quality.

In the dynamic market, we could expect the higher quality non-focal platform to have an upper hand, because with the higher stand-alone quality it can earn higher profits as a focal platform than the lower quality one. Thus, it should be more worth the investment for the higher-quality platform to win the market than for the lower-quality platform to defend it.

3.2.1 Simple dynamics: two periods

We start the analysis of the dynamic game by considering a two-period case, i.e., $T = 2$. As earlier, platform A is initially focal. If B wins the first period, it becomes focal in the second. Otherwise, A stays focal. Since the subgame in the last period is the one-period game, the focal platform i earns $q_i - q_j + \beta$, and the non-focal earns 0. Thus, in $t = 0$ the platform that wins expects to earn additional $\delta(q_i - q_j + \beta)$.[6] To win in $t = 0$, platform A needs to set $p_{A0} \leq p_{B0} + q_A - q_B + \beta$. Platform B needs to set $p_{B0} < p_{A0} + q_B - q_A - \beta$ to win in $t = 0$. Platform B never sets p_{B0} lower than $-\delta(q_B - q_A + \beta)$, because it is the most it can recoup. Thus, platform A wins the market by setting such p_{A0} that would force $p_{B0} < -\delta(q_B - q_A + \beta)$, i.e.

$$p_{A0} = -\delta(q_B - q_A + \beta) + q_A - q_B + \beta\,, \tag{2}$$

and earns

$$\Pi_A^A(T = 2) = p_{A0} + \delta(q_A - q_B + \beta) = (1 + 2\delta)(q_A - q_B) + \beta\,. \tag{3}$$

It is only worth for platform A to win the market when $\Pi_A(T = 2) \geq 0$. Otherwise, when $(1 + 2\delta)(q_A - q_B) + \beta < 0$, platform B sets $p_{B0} = -\delta(q_A - q_B + \beta) + q_B - q_A - \beta$, wins the market in $t = 0$ and earns profits

$$\Pi_B^A(T = 2) = p_{B0} + \delta(q_B - q_A + \beta) = (1 + 2\delta)(q_B - q_A) - \beta > 0\,,$$

which are positive.

Thus, unlike in the one-period case, with multiple periods, it is possible for higher-quality non-focal platform to win the market despite network effects. But it is not enough just to have a higher quality — the quality differential must be large enough. Platform A wins the market despite $q_A < q_B$ when $q_A + \beta/(1 + 2\delta) > q_B$.

While under $q_B > q_A$ platform B gains more from the focal position in the last period, it is cheaper for platform A to defend the market than for platform B to win it. This is because of A's initially focal position and the advantage of customers expectations. Thus, for $0 < q_B - q_A < \beta/(1 + 2\delta)$ inertia due to network externalities still causes inefficiency in the equilibrium. But this is a smaller range of inefficiency than in the static model, i.e., $0 < q_B - q_A < \beta$.

3.2.2 Arbitrary finite Time Horizon: The longer the horizon, the less inefficiency

Under a longer time horizon, the winning platform may have a longer time to collect profits, and thus has stronger incentives to win the market in $t = 0$. Of course, the focal platform anticipates this and strives to prevent the non-focal platform from taking the market.

[6] This is for the case when $q_i - q_j < \beta$ for $i = A,\, B$. The more general analysis that follows includes larger differences between q's.

For a more general analysis, let us denote by $\Pi_i^f(T)$ the discounted profit of platform i when platform f is focal at date $t = 0$ and there are T periods. Then, in $t = 0$, platform i is willing to invest up to $\delta\Pi_i^i(T-1)$ to win the market and gain or maintain the focal position with $T-1$ horizon. It is possible for either of the platforms to win or maintain the focal position. But it may be more costly than the benefit of being focal in the future. To avoid loses in such a case, the platform lets the other platform win by setting the current price to 0 and receiving 0, and becomes non-focal in the next period. To win in $t = 0$, the focal platform A needs to set

$$p_{A0} \leq p_{B0} + q_A - q_B + \beta = -\delta\Pi_B^B(T-1) + q_A - q_B + \beta \,.$$

Then, platform A's profit is

$$\Pi_A^A(A \text{ wins in } t = 0) = q_A - q_B + \beta - \delta\Pi_B^B(T-1) + \delta\Pi_A^A(T-1)\,. \tag{4}$$

Similarly, to win in $t = 0$, platform B sets

$$p_{B0} \leq p_{A0} + q_B - q_A - \beta = -\delta\Pi_A^A(T-1) + q_B - q_A - \beta$$

and receives the profit of

$$\Pi_B^A(B \text{ wins in } t = 0) = q_B - q_A - \beta - \delta\Pi_A^A(T-1) + \delta\Pi_B^B(T-1) \equiv -\Pi_A^A(A \text{ wins in } t = 0)\,. \tag{5}$$

Notice that only for one of the platforms the payoff from winning the market, $\Pi_i^A(i \text{ wins in } t = 0)$, is positive. Therefore, for only one of the platforms it is worthwhile to win the market. To facilitate further analysis, let $\hat{\Pi}_i^f(T) = \Pi_i^f(i \text{ wins in } t = 0)$ be the discounted profit of platform i if it wins the market. It may be positive or negative. Then we can represent (5) as $\hat{\Pi}_i^f(T) = -\hat{\Pi}_j^f(T)$. When $\hat{\Pi}_i^i(T)$ is negative, it is not worth for the platform to win the market. The optimal action in such a case is to cede the market, by setting price to 0. Such strategy yields 0 profit and non-focal position in the next period. Thus, the profit from the optimal action is $\Pi_i^f(T) = \max\{\hat{\Pi}_i^f(T), 0\}$.

Suppose that $\hat{\Pi}_i^i(k) > 0$ for both $i = A,\ B$ and $k = 1, \ldots, T-1$. Then from (4) we get[7]

$$\hat{\Pi}_i^i(T) = q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T-1) + \delta\hat{\Pi}_i^i(T-1) = (q_i - q_j)\sum_{k=0}^{T-1}(2\delta)^k + \beta = (q_i - q_j)\frac{1-(2\delta)^T}{1-2\delta} + \beta\,. \tag{6}$$

The fraction $\frac{1-(2\delta)^T}{1-2\delta}$ is positive and increasing with T. Therefore, $\hat{\Pi}_i^i(T)$ is also monotonic. When $q_i - q_j > 0$, then $\hat{\Pi}_i^i(T)$ is positive and increasing. Conversely, when $q_i - q_j < 0$, then $\hat{\Pi}_i^i(T)$ is decreasing, and when $q_i - q_j < -\beta\frac{1-2\delta}{1-(2\delta)^T}$ it may even be negative.[8] And once it is negative, it stays negative for all larger T's.

Let T_i be the smallest T for which $\hat{\Pi}_i^i(T)$ as calculated in (6) is negative, i.e. $\hat{\Pi}_i^i(T_i - 1) \geq 0$ and $\hat{\Pi}_i^i(T_i) < 0$.[9]

Lemma 1 *If $\hat{\Pi}_i^i(T) < 0$, then for all $T' > T$, $\hat{\Pi}_i^i(T') < 0$.*

Proof. Suppose $T_i > 1$. By definition of T_i, $\hat{\Pi}_i^i(T_i - 1) > 0$ (and given by (6)), and

$$\hat{\Pi}_i^i(T_i) = q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T_i - 1) + \delta\hat{\Pi}_i^i(T_i - 1) < 0\,. \tag{7}$$

$\hat{\Pi}_i^i(T)$ for $T > T_i$ can no longer be calculated using (6). We need to directly apply (5) directly:

$$\hat{\Pi}_i^i(T_i + 1) = q_i - q_j + \beta - \delta\Pi_j^j(T_i) + \delta\Pi_i^i(T_i) = q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T_i)$$

since $\Pi_j^j(T_i) = \hat{\Pi}_j^j(T_i)$ and $\Pi_i^i(T_i) = 0$.

By properties of (6), $\hat{\Pi}_j^j(T_i) > \hat{\Pi}_j^j(T_i - 1)$. By $\hat{\Pi}_i^i(T_i) < 0$, $\delta\Pi_j^j(T_i - 1) > q_i - q_j + \beta + \delta\hat{\Pi}_i^i(T_i - 1) > q_i - q_j + \beta$. Thus $\delta\hat{\Pi}_j^j(T_i) > q_i - q_j + \beta$ and $\hat{\Pi}_i^i(T_i + 1) < 0$. And so on for each $T > T_i$. ■

[7]Follows from applying the same formulas recursively in

$$\hat{\Pi}_i^i(T-1) - \hat{\Pi}_j^j(T-1) = 2(q_i - q_j) + 2\delta[\hat{\Pi}_i^i(T-2) - \hat{\Pi}_j^j(T-2)] = 2(q_i - q_j)\sum_{k=0}^{T-2}(2\delta)^k\,.$$

[8]This also implies that one of the $\Pi_i^i(T)$ must be positive. A negative $\hat{\Pi}_i^i(T)$ for some T implies $q_i - q_j < 0$, and $q_j - q_i > 0$ implies $\hat{\Pi}_j^j(T) > 0$ for all T.

[9]Note that when $q_i - q_j + \beta < 0$, formula in (6) is always negative. Then we say that $T_i = 1$. Otherwise, $T_i > 1$. Note also that it must be $q_i < q_j < 0$ for $\hat{\Pi}_i^i(T) < 0$ for some T.

A negative $\hat{\Pi}_i^i(T)$ for T larger than T_i means that when the time horizon is longer than T_i, it is not worth for platform i to win the market even if it is a focal platform. To avoid loses in such a case, i sets $p_{i0} = 0 = -\delta\Pi_i^i(T)$, which lets the other platform win the market and the focal position for the next period.

Notice that if the time horizon would be shorter than T_i, platform i would hold it for the whole time, if it were focal at the beginning of this period. However, if platform i loses the market because the time horizon is longer than T_i, it never wins the market back. Since for $\hat{\Pi}_i^i(T)$ to be negative, q_i must be lower than q_j, then it must be that $\hat{\Pi}_j^j(T) > 0$. That means that once j wins the market, it will hold it for all the remaining periods.

Thus the market leadership (in terms of focality) can only change in $t = 0$. If platform B succeeds in winning the market, because $T > T_A$, platform A cannot win it back. If platform B could not win the market profitably in $t = 0$, it will not succeed in later periods.

Lemma 2 *In equilibrium, the same platform wins all the periods. For a game with T periods, platform A wins when $\hat{\Pi}_A^A(T) \geq 0$, and platform B wins otherwise.*

Corollary 1 *When $T \leq T_A$, platform A wins every period. When $T > T_A$, platform B wins every period.*

Nonetheless, the presence of the "losing" platform affects the prices the winning platform can charge. The winning platform keeps winning in equilibrium because when focal, it avoids setting the price too high, lest the non-focal platform wins the market profitably. Every period the losing platform i sets $p_{it} = -\delta\Pi_i^i(T-1-t)$. The winning platform sets $p_{jt}^j = -\delta\Pi_i^i(T-1-t) + q_j - q_i + \beta$ when it is focal, and $p_{jt}^i = -\delta\Pi_i^i(T-1-t) + q_j - q_i - \beta$ when it is non-focal. Note that for $T > T_i$, $\Pi_i^i(T) = 0$.

The following lemma characterizes how the equilibrium outcome depends on the parameters.

Lemma 3 (Subgame perfect equilibrium for arbitrary finite T) *For an arbitrary finite T the equilibrium depends on the difference $q_A - q_B$:*

1. $|q_A - q_B| < \beta\frac{1-2\delta}{1-(2\delta)^T}$

   Then A wins every period because it is initially focal, and earns the total profit of

$$(q_A - q_B)\frac{1-(2\delta)^T}{1-2\delta} + \beta\,.$$

2. $q_A - q_B > \beta \frac{1-2\delta}{1-(2\delta)^T}$
   Then $T_B < T$. Platform A wins every period, because it has quality advantage.

   (a) *When $\beta \frac{1-2\delta}{1-(2\delta)^T} < q_A - q_B < \beta$, platform A earns*

$$(q_A - q_B + \beta)\frac{1-\delta^{T-T_B}}{1-\delta} + \delta^{T-T_B}\left((q_A - q_B)\frac{1-(2\delta)^{T_B}}{1-2\delta} + \beta\right).$$

   (b) *When $q_A - q_B > \beta$, it earns*

$$(q_A - q_B + \beta)\frac{1-\delta^T}{1-\delta}.$$

3. $q_B - q_A > \beta \frac{1-2\delta}{1-(2\delta)^T}$
   Then $T_A < T$. Platform B wins every period because it has sufficient quality advantage.

   (a) *When $\beta \frac{1-2\delta}{1-(2\delta)^T} < q_B - q_A < \beta$, platform B earns*

$$q_B - q_A - \beta + \delta\left[(q_B - q_A + \beta)\frac{1-\delta^{T-1-T_A}}{1-\delta} + \delta^{T-1-T_A}\left((q_B - q_A)\frac{1-(2\delta)^{T_A}}{1-2\delta} + \beta\right)\right].$$

   (b) *$q_B - q_A > \beta$, it earns*

$$q_B - q_A - \beta + \delta(q_B - q_A + \beta)\frac{1-\delta^{T-1}}{1-\delta}.$$

The losing platform's profits are 0 in all cases.

Platform B wins the market only if it has quality advantage. But platform A may win either because it has a quality advantage, or it can win despite offering lower quality, because it started with a focal position. The latter happens when $0 < q_B - q_A < \beta\frac{1-2\delta}{1-(2\delta)^T}$, and it is an inefficient outcome. In all other cases, the higher quality platform wins, so the equilibrium outcome is efficient. Notice that the set of parameters for which the equilibrium outcome is inefficient is decreasing as T and δ increase.

Thus, competition over multiple periods yields efficient equilibrium outcome for parameters that in a static model resulted in lower-quality platform winning. There is less inefficiency when the time horizon increases. In the next section we investigate whether the inefficiency would disappear altogether if the time horizon was extended to infinity.

3.2.3 Extending the time horizon to infinity: inefficiency is not completely eliminated

We can directly extrapolate the equilibrium outcome in Lemma 3 for $T \to \infty$. We need to recognize, however, that the ratio $\frac{1-(2\delta)^T}{1-2\delta}$ converges to $\frac{1}{1-2\delta}$ for $\delta < \frac{1}{2}$, and converges to ∞ for $\delta > \frac{1}{2}$.

Proposition 1 (Subgame perfect equilibrium extrapolated for $T \to \infty$) *For $T \to \infty$:*

1. $|q_A - q_B| < \beta(1-2\delta)$ *or* $q_A = q_B$
   Then platform A wins every period because it is initially focal, and earns the total profit of
   $$\Pi_A^A = \frac{q_A - q_B}{1-2\delta} + \beta\,.$$
2. $q_A - q_B > \max\{\beta(1-2\delta), 0\}$
   Then platform A wins every period, because it has quality advantage. And the platform earns
   $$\Pi_A^A = \frac{q_A - q_B + \beta}{1-\delta}\,.$$
3. $q_B - q_A > \max\{\beta(1-2\delta), 0\}$
   Then platform B wins every period because it has sufficient quality advantage. And the platform earns
   $$\Pi_B^A = q_B - q_A - \beta + \delta\frac{q_B - q_A + \beta}{1-\delta} = \frac{q_B - q_A + \beta}{1-\delta} - 2\beta\,.$$

The losing platform's profits are 0 in all cases.

Even under infinite time horizon the equilibrium outcome may be inefficient. When $0 < q_B - q_A < \beta(1-2\delta)$, platform A wins despite lower quality. But the problem of inefficiency due to excessive inertia occurs less often in with longer time horizons. And the inefficiency disappears altogether when platforms care about future more than about the present, i.e., $\delta > \frac{1}{2}$.

In following sections, we explore other ways why inefficient outcome may occur in equilibrium.

4 Other Markov perfect equilibria under infinite time horizon

In Proposition 1, we characterized an equilibrium in the infinite game by extrapolating the subgame perfect equilibrium of an arbitrary finite game. Infinite time horizon, however, may give rise to other equilibria as well. In this section we identify Markov perfect equilibria in the infinite game. The subgame perfect equilibrium identified in Proposition 1 is a Markov perfect equilibrium. But we find that there are also other Markov perfect equilibrium that cannot arise from extrapolating any finite-game solution. Those new equilibria often result in inefficient outcomes for parameters where Proposition 1 equilibrium is efficient.

Every period t of the infinite game is characterized by the state variable at time t, f_t. A Markov perfect equilibrium is characterized by the strategies of both platforms in all possible states, and the outcome in each state. We will consider three pure strategy equilibria outcomes: (i) platform A wins in both states, (ii) platform B wins in both states, and (iii) the focal platform wins.[10]

In what follows we characterize the strategies supporting those equilibria outcomes, and parameter conditions under which each equilibrium exists. We define the value function V_i^f as the equilibrium expected discounted profit of platform i when platform f is focal.

Consider first the equilibrium outcome where platform A wins in both states. In this equilibrium the value functions for platform B are $V_B^B = V_B^A = 0$, because platform B never sell. We assume that platform B sets price $p_B = 0$, because in no situation platform B would like to win with price $p_B < 0$, given that it cannot count on future profits to justify the "investment" in taking over the market. When A is focal, it optimally sets $p_A^A = q_A - q_B + \beta$. Similarly, were B focal, platform A sets $p_A^B = q_A - q_B - \beta$, and platform B sets $p_B^B = 0$. Were A to set a higher price, platform B would keep the market and make non-negative profits. In such a case

$$V_A^A = q_A - q_B + \beta + \delta V_A^A \qquad \text{and} \qquad V_A^B = q_A - q_B - \beta + \delta V_A^A \,.$$

Moreover, incentive compatibility for platform A requires that

$$V_A^A \geq \delta V_A^B \qquad \text{and} \qquad V_A^B \geq 0 \,.$$

[10]The fourth possibility of a pure strategy equilibrium outcome, that non-focal platform wins, cannot be supported by any strategy.

Therefore, this equilibrium exists when $q_A - q_B \geq \beta(1-2\delta)$. With a similar analysis for platform B, we arrive at the following result.

Lemma 4 *There is an equilibrium where platform i wins in both states if $q_i - q_j \geq \beta(1-2\delta)$.*

Lemma 4 shows that a non-focal platform B can win a focal position and maintain it in all future periods in the following cases. First, when its quality is substantially superior than the quality of platform A. Second, when platforms are very forward-looking, such that δ is high. Third, when β is low. Notice, however, that for $\delta > 1/2$, a focal A can hold the market every period even if $q_A < q_B$. Similarly, platform B can win and hold the market for ever even if $q_B < q_A$ for such high δ.

The remaining equilibrium to consider is one where the focal platform wins. Recall that p_i^f denotes the price of platform i when f is focal in such an equilibrium. Since the wining platform anticipates it will stay active and focal from the new period on, we have values function

$$V_i^i = \frac{p_i^f}{1-\delta}, \quad V_i^j = 0$$

The benefits of selling at a given date is $p_{it} + \delta V_i^i$. It follows that the minimal profit that platform i is willing to sacrifice today to gain the market is $-\delta V_i^i$. In such an equilibrium it must be the case that the focal platform sets a price $p_i^i \leq q_i - q_j + \beta - \delta V_j^j$, otherwise the competing platform would set a price above $-\delta V_i^i$ and wins the market. Ruling out cases where $p_j < -\delta V_j^j$ because wining at this price would not be profitable for firm j, we obtain equilibrium prices[11]

$$p_i^i = q_i - q_j + \beta - \delta V_j^j, \; p_j^i = -\delta V_j^j.$$

This leads to values function in such an equilibrium solutions of

$$\begin{aligned} (1-\delta) V_A^A + \delta V_B^B &= q_A - q_B + \beta \\ (1-\delta) V_B^B + \delta V_A^A &= q_B - q_A + \beta \end{aligned}$$

yielding

$$V_A^A = \frac{q_A - q_B}{1-2\delta} + \beta; \quad V_B^B = \frac{q_B - q_A}{1-2\delta} + \beta.$$

We then conclude that:

[11] This is innocuous for existence argument

Lemma 5 *There is an equilibrium where the focal platform wins in every state if* $\beta |1 - 2\delta| > |q_B - q_A|$.

Proof. For this to be an equilibrium it is necessary and sufficient that $V_A^A > 0$ and $V_B^B > 0$. ■

Lemma 5 shows that a focal platform A can maintain its focal position in all future periods even when it offers a lower quality than platform B. To see the intuition for this result, consider first the case of $\delta \leq 1/2$. As Lemma 5 shows, the equilibrium holds in this case if δ and the quality gap, $q_B - q_A$, are sufficiently low. Intuitively, suppose that q_B increases. This has two effects on the equilibrium V_B^B. First, a *direct* effect – since $p_B^B = q_B - q_A + \beta - \delta p_A^B$, taking V_A^A as given, platform B can now attract agents with a higher p_B^B, implying that V_B^B will increase. Second, a *strategic* effect – since $p_B^A = -\delta V_B^B$, platform A will now know that even if it is focal, it will compete against a more aggressive platform B, because platform B has more to gain by becoming focal. This reduces V_A^A, which in turn increases V_B^B because platform A will not compete aggressively to gain a focal position when it is not focal. Both the direct and the strategic effects work in the same direction of increasing V_B^B and decreasing V_A^A. If the gap $q_B - q_A$ is sufficiently wide, $V_A^A < 0$, implying that platform A cannot maintain its focal position when competing against a superior quality platform. As δ increases, platform B cares more about future profit so it will have a stronger incentive to win the market when it is not focal, and maintain its focal position when it if focal.

Now suppose that $\delta > 1/2$. As Lemma 5 reveals, in this case the equilibrium is completely reversed. Now, if $q_B > q_A$, then $V_A^A > V_B^B$, and as q_B increases, V_B^B decreases while V_A^A increases. However, the equilibrium in the case of $\delta > 1/2$ relies on the somewhat strong assumption that platforms "overreact", such that as q_B increases, while the direct effect increases V_B^B (as in the case of $\delta < 1/2$), the strategic effect works in the opposite direction and is stronger than the direct effect. To see how, suppose that platform B is focal and q_B increases. The equilibrium holds when platform B expects that as a response to the increase in q_B, platform A will over-react in the opposite direction than in the case of $\delta < 1/2$, by becoming very aggressive and decreasing its price when it is not focal, p_A^B. In this case, V_B^B increases since q_B increases (direct effect), but decreases since p_A^B decreases (strategic effect). The strategic effect outweighs the direct effect, and the overall effect is to decrease V_B^B and to increase V_A^A. In this equilibrium however, platform A reduces its price p_A^B because it anticipates that it will benefit once focal from competing with a more efficient rival, a rather peculiar feature.

Notice that if we rule out the possibility of overreaction, then the equilibrium in which the focal platform A always wins fails when $q_B > q_A$ and $\delta > 1/2$. The equilibrium in which the non-focal platform B wins in the first period and maintains its focal position infinitely, as we characterized is Lemma 4, always holds when $q_B > q_A$ and $\delta > 1/2$, and does not relies on platforms' overreactions.

Proposition 2 below summarizes the results of Lemma 4 and Lemma 5.

Proposition 2 (Markov perfect equilibria) *Suppose that platform A is focal at period t=0. Then,*

(i) for $q_B - q_A > \beta|1-2\delta|$ there exists a unique equilibrium, and in that equilibrium platform B wins;

(ii) for $\beta(1-2\delta) < q_B - q_A < \beta|1-2\delta|$, which occurs only for $\delta > 1/2$, there exist multiple equilibria, and in one of those equilibria platform B wins;

(iii) for $q_B - q_A < \beta(1-2\delta)$, platform A wins in all equilibria.

Proof. This follows from the assumption that A is initially focal and from Lemma 4 and Lemma 5. ■

Equilibrium active platform is depicted in Figure 1. The figure shows that for low discount factor and low quality differential, there is a unique equilibrium in which focal platform A wins. Intuitively, in this case the same qualitative results of a static game follows to the dynamic game. For positive quality differential $q_B - q_A$ and intermediate values of δ, there is a unique equilibrium in which the most efficient platform B takes over the market and maintain its position infinitely. But for high discount factors and low quality differential, there are multiple equilibria in which either platform A or B win. Notice that disregarding the equilibria of Lemma 5 (for being unlikely to emerge) would not restore efficiency of the equilibrium in this parameter region as there are also two equilibria — including one where the low-quality platform wins — arising from Lemma 4. In both of these equilibria, one platform expects low competitive pressure while the other renounces wining the market because it expects high competitive pressure, and these expectations are self-fulfilling. Thus

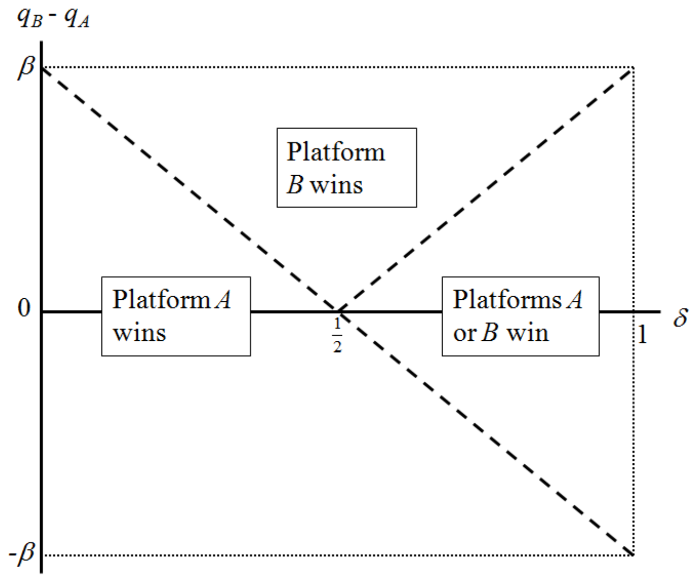

Figure 1: Equilibrium configuration

at high discount factors, the prospect of gaining a focal position is not sufficient to outweigh firms' self-fulfilling expectations about the competitive pressure they will face.

5 Investment in Qualities

6 Stochastic qualities

The previous section focused on the case where the qualities of the two platforms are constant for infinity. Consequently, in any equilibrium the same platform wins the market in all periods. In many markets for platforms there is a shift in leadership every few years, parallel to technology improvements. In this section we consider the more realistic case in which qualities are stochastic. We show that there is an equilibrium in which each platform can win in each period with some probability. The main conclusion of this section is that unlike the constant-qualities case, social welfare under stochastic qualities may decrease with δ.

Suppose that qualities change randomly in each period. At the beginning of each period, both platforms observe the realization of their qualities for this particular period. Then, the two platforms compete by setting prices.

The results of the previous sections showed that the equilibrium depends on the difference between the qualities of the two platforms, and not their absolute values. Suppose then, without loss of generality, that $q \equiv q_B - q_A$ change randomly in each period with full support on the real line according to a probability function $f(q)$, with a cumulative distribution function $F(q)$. Our assumption that the support is infinite ensures that there is an equilibrium in which each platform can win the market with a positive probability.[12] Suppose that q has a mean $\mu > 0$ such that on average, platform B is of superior quality than platform A. The case of $\mu < 0$ is symmetric.

Let $\bar{q}^A$ and $\bar{q}^B$ denote equilibrium cutoffs such that if platform A is focal in time t, it wins if $q \leq \bar{q}^A$ and platform B wins otherwise. Likewise, if platform B is focal in time t, it wins if $q \geq \bar{q}^B$ and platform A wins otherwise.[13] This equilibrium has the feature that when platform A is the focal platform, it will win in every period as long as $q < \bar{q}^A$. Then, once there is a realization with $q > \bar{q}^A$, platform B takes over the market and becomes focal. Platform B will maintain its focal position in future periods as along as $q \geq \bar{q}^B$, until eventually in a certain period there is a realization of q with $q < \bar{q}^B$, and platform A wins back its focal position. The game then repeats itself infinitely, with platforms "taking turns" in winning depending on the realization of q.

Let V_i^f denote the expected value function of platform i when platform f is focal. To solve for the equilibrium, suppose that platform A is focal in time t and the quality difference has some realization, q. The lowest price platform B is willing to charge in order to win the market is $-\delta V_B^B + \delta V_B^A$. This is because platform B will earn the expected value of V_B^B from becoming focal in the next period, and earn the expected value of V_B^A from remaining non-focal. To win the market faced to this price, the focal platform A will need to set $p_A = \beta - q - \delta V_B^B + \delta V_B^A$. Platform A earns $p_A + \delta V_A^A$ if indeed it wins (if $q \leq \bar{q}^A$) and earns $0 + \delta V_A^B$ if it loses (if $q > \bar{q}^A$). Therefore:

$$V_A^A = \int_{-\infty}^{\bar{q}^A} (\beta - q - \delta V_B^B + \delta V_B^A + \delta V_A^A) f(q) dq + \int_{\bar{q}^A}^{\infty} \delta V_A^B f(q) dq.$$

Suppose now that platform A is non-focal. The lowest price platform B is willing to charge to maintain its focal position is $p_B^B = -\delta V_B^B + \delta V_B^A$. Again, if platform A wins, it sets p_A^B that ensures that $-p_A^B \geq \beta - p_B^B + q$, or $p_A^B = -\beta - q - \delta V_B^B + \delta V_B^A$. Platform A

[12] We should note that this is a stronger assumption than what we need, as our results hold even with a finite support, as long as it is wide enough. Our assumption of infinite support facilitates the analysis and enables us to avoid corner solutions.

[13] It is straigtforward to see that any Markov equilibrium must have this form.

earns $p_A^B + \delta V_A^A$ if indeed it wins the market (when $q \leq \bar{q}^B$), and earns $0 + \delta V_A^B$ if it loses the market (when $q > \bar{q}^B$). Therefore:

$$V_A^B = \int_{-\infty}^{\bar{q}^B} (-\beta - q - \delta V_B^B + \delta V_B^A + \delta V_A^A) f(q) dq + \int_{\bar{q}^B}^{\infty} \delta V_A^B f(q) dq.$$

The cases of V_B^B and V_B^A are symmetric by recalling that platform B wins the market if $q \geq \bar{q}^B$ when it is focal, and if $q > \bar{q}^A$ when it is not. Moreover, q positively affects the profit of platform B. Therefore:

$$V_B^B = \int_{\bar{q}^B}^{\infty} (\beta + q - \delta V_A^A + \delta V_A^B + \delta V_B^B) f(q) dq + \int_{-\infty}^{\bar{q}^B} \delta V_B^A f(q) dq,$$

$$V_B^A = \int_{\bar{q}^A}^{\infty} (-\beta + q - \delta V_A^A + \delta V_A^B + \delta V_B^B) f(q) dq + \int_{-\infty}^{\bar{q}^A} \delta V_B^A f(q) dq.$$

Next consider the equilibrium $\bar{q}^A$ and $\bar{q}^B$. The equilibrium $\bar{q}^A$ is such that for $q = \bar{q}^A$, a focal platform A is exactly indifferent between wining the market or not, taking the equilibrium future value functions and the price of platform B as given. That is:

$$\beta - \bar{q}^A - \delta V_B^B + \delta V_B^A + \delta V_A^A = \delta V_A^B.$$

Notice that the condition for making the non-focal platform B indifferent between winning and not is equivalent to the condition above. Turning to $\bar{q}^B$, the equilibrium $\bar{q}^B$ should be such that for $q = \bar{q}^B$, a non-focal platform A is exactly indifferent between wining the market or not, taking the equilibrium future value functions and the price of platform B as given. That is:

$$-\beta - \bar{q}^B - \delta V_B^B + \delta V_B^A + \delta V_A^A = \delta V_A^B.$$

Again notice that the condition for making the focal platform B indifferent between winning and not is equivalent to the condition above.

The set of the six equations above define the equilibrium V_A^A, V_A^B, V_B^B, V_B^A, $\bar{q}^A$ and $\bar{q}^B$. Using the above equations, the following proposition provides a sufficient condition for unique equilibrium values of $\bar{q}^A$ and $\bar{q}^B$:

Proposition 3 (Unique solutions to $\bar{q}^A$ and $\bar{q}^B$) *Suppose that $4\beta \max f(q) < 1$. There are unique equilibrium values of $\bar{q}^A$ and $\bar{q}^B$, with the following features:*

(i) for $\delta = 0$, $\bar{q}^A = \beta$ *and* $\bar{q}^B = -\beta$*;*

(ii) $\bar{q}^A - \bar{q}^B = 2\beta$ *for all* δ*;*

Proof. Directly from the formulas for V_A^A, V_A^B, V_B^B, V_B^A, and conditions for $\bar{q}^A$ and $\bar{q}^B$, we obtain

$$\bar{q}^A - \bar{q}^B = 2\beta\,.$$

Moreover,

$$\begin{aligned} V_A^A &= \int_{-\infty}^{\bar{q}^A} \left(\bar{q}^A - q\right) f(q)\, dq + \delta V_A^B, \\ V_A^B &= \int_{-\infty}^{\bar{q}^B} \left(\bar{q}^B - q\right) f(q)\, dq + \delta V_A^B = \frac{1}{1-\delta} \int_{-\infty}^{\bar{q}^B} \left(\bar{q}^B - q\right) f(q)\, dq, \end{aligned}$$

and

$$\begin{aligned} V_B^B &= \int_{\bar{q}^B}^{+\infty} \left(q - \bar{q}^B\right) f(q)\, dq + \delta V_B^A, \\ V_B^A &= \frac{1}{1-\delta} \int_{\bar{q}^A}^{+\infty} \left(q - \bar{q}^A\right) f(q)\, dq, \end{aligned}$$

The optimality condition is then

$$\bar{q}^A = \beta - \delta V_B^B + \delta V_B^A + \delta V_A^A - \delta V_A^B$$

which can be written

$$\bar{q}^A = \beta + \delta \phi\left(\bar{q}^A\right) \tag{8}$$

where

$$\phi\left(\bar{q}^A\right) = \int_{\bar{q}^A}^{+\infty} \left(q - \bar{q}^A\right) f(q)\, dq + \int_{-\infty}^{\bar{q}^A} \left(\bar{q}^A - q\right) f(q)\, dq - \int_{-\infty}^{\bar{q}^B} \left(\bar{q}^B - q\right) f(q)\, dq - \int_{\bar{q}^B}^{+\infty} \left(q - \bar{q}^B\right) f(q)\, dq.$$

Integrating by parts:

$$\phi\left(\bar{q}^A\right) = -2\beta + 2\int_{\bar{q}^A - 2\beta}^{\bar{q}^A} F(q) dq. \tag{9}$$

We have

$$\begin{aligned} \phi'\left(\bar{q}^A\right) &= 2\left(F\left(\bar{q}^A\right) - F\left(\bar{q}^A - 2\beta\right)\right) \\ \phi(-\infty) &= -2\beta \\ \phi(+\infty) &= 2\beta \end{aligned}$$

This implies that for $\bar{q}^A = \infty$, $\bar{q}^A > \beta + \delta\phi(\bar{q}^A)$ and for $\bar{q}^A = -\infty$, $\bar{q}^A < \beta + \delta\phi(\bar{q}^A)$. Therefore, there is a unique solution to $\bar{q}^A$ if $\bar{q}^A - \beta - \delta\phi(\bar{q}^A)$ is increasing with $\bar{q}^A$, or $\delta\phi'\left(\bar{q}^A\right) < 1$. We notice that $\delta\phi'\left(\bar{q}^A\right) < 1$ when

$$2\delta \max\left(F\left(q\right) - F\left(q - 2\beta\right)\right) < 1.$$

In this case the equilibrium is unique. This is the case for all δ and if $4\beta \max f\left(q\right) < 1$.

Finally, notice that evaluated at $\delta = 0$, the solution to $\bar{q}^A = \beta + \delta\phi(\bar{q}^A)$ is $\bar{q}^A = \beta$.

■

The condition $4\beta \max f(q) < 1$ requires that the quality gap is sufficiently disperse and network effects are not too high. The intuition for these conditions is that they ensure that a non-focal platform can always overcome its competitive disadvantage if its quality is sufficiently high and therefore there are unique equilibrium values of $\bar{q}^A$ and $\bar{q}^B$. Proposition 3 also shows that evaluated at $\delta = 0$, $\bar{q}^A = \beta$ and $\bar{q}^B = -\beta$. Intuitively, at $\delta = 0$, the equilibrium is identical to the one-period benchmark in which a focal platform wins as long as its quality gap is higher than the network effects.

Next, we turn to study the effect of δ, β and μ on the equilibrium values of $\bar{q}^A$ and $\bar{q}^B$. To this end, we make the simplifying assumption that $f(q)$ is symmetric and unimodal ($S - U$ thereafter) around μ. That is, $f(\mu + x) = f(\mu - x)$ and $f(q)$ is weakly increasing (decreasing) with q for $q < \mu$ ($q > \mu$). This is a sufficient condition – the results below may hold even when $f(q)$ is not strictly $S - U$ if $f(q)$ places higher weights on positive values of q then negative values, such that platform B has higher probability to be focal in future periods. We also assume the uniqueness condition of Proposition 3 that $4\beta f(\mu) < 1$. With this assumption, we have:

Proposition 4 (The effect of δ, β and μ on $\bar{q}^A$ and $\bar{q}^B$) *Suppose that $f(q)$ is symmetric and unimodal around μ and $4\beta f(\mu) < 1$. Then:*

(i) $\bar{q}^A$ and $\bar{q}^B$ are decreasing with δ. If $F(0) < 1/4$ then $\bar{q}^A < 0$ when δ is sufficiently high;

(ii) $\bar{q}^A$ and $\bar{q}^B$ are decreasing with μ (holding constant the distribution of $q - \mu$);

(iii) $\bar{q}^A$ is increasing with β and $\bar{q}^B$ is decreasing with β if $\delta < 1/2$. If $F(0) < 1/4$ and δ is close to 1, then $\bar{q}^A$ is decreasing with β.

Proof.

Proof of part (i): Since $\bar{q}^A = \beta + \delta\phi(\bar{q}^A)$,

$$\frac{\partial \bar{q}^A}{\partial \delta} = \frac{\phi(\bar{q}^A)}{1-\delta\phi'(\bar{q}^A)}.$$

From the proof of Proposition 3, if $4\beta f(\mu) < 1$ then $1-\delta\phi'(\bar{q}^A) > 0$. To see that $\phi(\bar{q}^A) < 0$ for all $\bar{q}^A \leq \beta$, suppose first that $\bar{q}^A < \mu$. Then:

$$\phi(\bar{q}^A) = -2\int_{\bar{q}^A-2\beta}^{\bar{q}^A}\left(\frac{1}{2}-F(q)\right)dq < 0,$$

where the inequality follows because $S-U$ implies that for all $q<\mu$, $F(q) < 1/2$. Next, consider $\mu < \bar{q}^A \leq \beta$. Then:

$$\phi(\bar{q}^A) = -2\int_{\bar{q}^A-2\beta}^{\mu-(\bar{q}^A-\mu)}\left(\frac{1}{2}-F(q)\right)dq - 2\int_{\mu-(\bar{q}^A-\mu)}^{\mu+(\bar{q}^A+\mu)}\left(\frac{1}{2}-F(q)\right)dq < 0,$$

where the first term is negative because $\bar{q}^A > \mu > 0$ and $S-U$ implies that $F(\mu-(\bar{q}^A-\mu)) < F(\mu) = \frac{1}{2}$ and the second term equals to 0 because $S-U$ implies that $F(\mu+q)-\frac{1}{2} = \frac{1}{2}-F(\mu+q)$. Since $\phi(\bar{q}^A) < 0$, $\frac{\partial \bar{q}^A}{\partial \delta} < 0$ and since $\bar{q}^B = \bar{q}^A - 2\beta$, $\frac{\partial \bar{q}^B}{\partial \delta} < 0$.

Next, $\bar{q}^A$ is less than 0 if

$$0 > \beta + \delta\phi(0),$$

which holds for δ large if

$$-\beta > \phi(0) = -2\beta(1-2F(-2\beta)) + \int_{-2\beta}^{0}(-2q)f(q)\,dq = -2\beta + 2\int_{-2\beta}^{0}F(q)\,dq,$$

or if

$$\beta > 2\int_{-2\beta}^{0}F(q)\,dq.$$

This is true for all β if $F(0) < 1/4$.

Proof of part (ii): Let $F(q;\mu)$ denote the $F(q)$ given μ. We have:

$$\frac{\partial \bar{q}^A}{\partial \mu} = \frac{2\int_{\bar{q}^A-2\beta}^{\bar{q}^A}\left(\frac{\partial F(q;\mu)}{\partial \mu}\right)dq}{1-\delta\phi'(\bar{q}^A)} < 0,$$

where the inequality follows because $S-U$ implies that $F(q;\mu)$ is decreasing with μ.

Proof of part (iii): We have:

$$\frac{\partial \bar{q}^A}{\partial \beta} = \frac{1 - 2\delta + 4\delta F(\bar{q}^A - 2\beta)}{1 - \delta\phi'(\bar{q}^A)} > 0,$$

where the inequity follows because $1-2\delta+4\delta F(\bar{q}^A-2\beta) > 0$ if $\delta < \frac{1}{2}$. Since $\bar{q}^B = \bar{q}^A - 2\beta$, we have:

$$\frac{\partial \bar{q}^B}{\partial \beta} = -\left[\frac{1 + \delta(2 - 4F(\bar{q}^A))}{1 - 2\delta(F(\bar{q}^A) - F(\bar{q}^A - 2\beta))}\right] < 0,$$

where the inequality follows because the numerator in the squared brackets is positive when $\delta < \frac{1}{2}$ because $F(\bar{q}^A) < 1$ and the denominator is positive when $\delta < \frac{1}{2}$ because $F(\bar{q}^A) - F(\bar{q}^A - 2\beta) < 1$. When $F(0) < 1/4$ and $\delta = 1$, we have:

$$\frac{\partial \bar{q}^A}{\partial \beta}|_{\delta=1} = \frac{-1 + 4F(\bar{q}^B)}{1 - \phi'(\bar{q}^A)} < \frac{-1 + 4\frac{1}{4}}{1 - \phi'(\bar{q}^A)} = 0,$$

where the inequality follows because $F(\bar{q}^B) < F(0) < 1/4$.

■

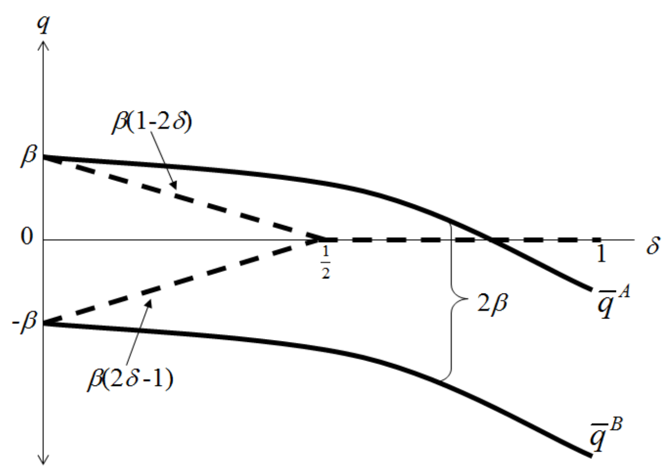

Figure 2: The effect of δ on $\bar{q}^A$ and $\bar{q}^B$ (when $F(0) < 1/4$)

Figure 2 illustrates part (i) of Proposition 4. The figure reveals that an increase in δ does not necessarily increase the probability that the highest quality platform wins. To see why, consider first the case where platform B is focal. Then, if $\delta = 0$, $\bar{q}^B = -\beta$ and $\bar{q}^B$ decreases with δ. Therefore, as δ increases, a focal platform B is more likely to win the market with a lower quality than platform A, implying that the probability that the "wrong" platform wins increases with δ. Next, consider the case where platform A is focal. Then, when δ is low, an increase in δ makes it less likely that a focal platform A will be able to maintain it's focal position with a lower quality than platform B, in that $\bar{q}^A$ decreases with δ. However, when δ is sufficiently high and $F(0) < 1/4$, $\bar{q}^A$ crosses the 0 line, becomes negative and decreases further below 0 as δ increases. Now, platform A can lose the market even if it is focal and of superior quality than platform B (for realizations $\bar{q}^A < q < 0$). Therefore, the probability that the lower-quality platform wins increases with δ when either platform A or B are focal.

The intuition for these results is the following. Recall that a platform's expected profit includes current profit and the probability of maintaining its focal position in future periods. Since $\mu > 0$, platform B is more likely to have in future periods a higher quality than platform A. As δ increases, platform A internalizes that it is less likely to win in future periods and will therefore have less of an incentive to compete aggressively in the current period. Platform B internalizes that it is more likely to win the market in future periods and will therefore have more of an incentive to compete aggressively to win the market in the current period. This in turn provides platform B with a stronger competitive advantage over platform A, even when the current quality of platform B is inferior than the quality of platform A. If $F(0) < 1/4$, then μ is sufficiently high and platform B's competitive advantage is strong enough to deter platform A from winning the market even when it is focal and offer a higher quality than platform B.

The intuition above also explains the intuition behind part (ii) of Proposition 4. As μ increases, platform B is more likely to have higher quality in future periods. This provides platform B with a higher incentive to win in the current period, and as a result $\bar{q}^A$ and $\bar{q}^B$ decrease.

Part (iii) of Proposition 4 shows that if δ is not too high, then an increase in the degree of the network effect makes it more likely that a focal platform wins. This result is similar to the myopic case. An increase in the network effects increases the strategic advantage of being focal, as it becomes easier for the focal platform to attract consumers. However, when δ is sufficiently high and $F(0) < 1/4$, then an increase in network effects decreases the ability of a focal platform A to win the market. Intuitively, in such a case an increase in the network effects increases the incentives of the non-focal platform B to take over the market because of its superior expected quality which implies that platform B has a higher probability of maintaining its focal position.

Next we turn to social welfare. We first ask whether social welfare is higher when platform B is focal than when platform A if focal. Since platform B has a higher expected quality than platform A, it is intuitive to expect that social welfare is higher when platform B is focal. However, Proposition 4 showed that when δ increases, the probability that platform B wins when platform A has a superior quality increases, which may offset the first effect.

To this end, we normalize $q_A = 0$ and therefore $q_B = q$. Let $\bar{W}^i$, $i = A, B$, denote the recursive expected social welfare when platform i is focal in period t, where:

$$\bar{W}^A = \int_{-\infty}^{\bar{q}^A} (\beta + \delta \bar{W}^A) f(q) dq + \int_{\bar{q}^A}^{\infty} (\beta + q + \delta \bar{W}^B) f(q) dq,$$

$$\bar{W}^B = \int_{\bar{q}^B}^{\infty} (\beta + q + \delta \bar{W}^B) f(q) dq + \int_{-\infty}^{\bar{q}^B} (\beta + \delta \bar{W}^A) f(q) dq,$$

and let $W^i = (1 - \delta)\bar{W}^i$ denote the one-period expected welfare. Comparing W^A with W^B, we obtain the following:

Proposition 5 (The effect of δ on social welfare) *Suppose that $f(q)$ is symmetric and unimodal around μ and $4\beta f(\mu) < 1$. Then,*

(i) evaluated at $\delta = 0$, $W^B \geq W^A$ and W^A is increasing with δ while W^B is decreasing with δ ;

(ii) There is a cutoff of δ'', $0 \leq \delta'' \leq 1$, such that $W^B > W^A$ for $\delta \in (0, \delta'')$ and $W^A > W^B$ for $\delta \in (\delta'', 1)$. A sufficient condition for $\delta'' < 1$ is $F(0) < \frac{1}{4}$;

(iii) evaluated at $\delta = 1$, $W^A = W^B$.

Remark: the case where q is distributed uniformly along a finite interval is a special case of $S - U$ in which $f(q)$ is constant. In this case $\delta'' = 0$ such that $W_A > W_B$ for $\delta \in (0, 1)$ and $W_A = W_B$ for $\delta = 0, 1$.

Proof.

Solving for W^A and W^B:

$$W^A = \beta + \frac{(1 - \delta + \delta F(\bar{q}^B) \int_{\bar{q}^A}^{\infty} q f(q) dq + \delta(1 - F(\bar{q}^A)) \int_{\bar{q}^B}^{\infty} q f(q) dq}{1 - \delta F(\bar{q}^A) + \delta F(\bar{q}^B)},$$

$$W^B = \beta + \frac{\delta F(\bar{q}^B) \int_{\bar{q}^A}^{\infty} q f(q) dq + (1 - \delta F(\bar{q}^A)) \int_{\bar{q}^B}^{\infty} q f(q) dq}{1 - \delta F(\bar{q}^A) + \delta F(\bar{q}^B)}.$$

Consider first W^A. Solving the derivative of W^A with respect to δ and then evaluating at $\delta = 0$ yields:

$$\frac{\partial W^A}{\partial \delta}|_{\delta=0} = (1 - F(\bar{q}^A)) \left(\int_{\bar{q}^B}^{\infty} qf(q)dq - \int_{\bar{q}^A}^{\infty} qf(q)dq \right) - f(\bar{q}^A)\bar{q}^A \frac{\partial \bar{q}^A}{\partial \delta}$$

$$= (1 - F(\beta)) \int_{-\beta}^{\beta} qf(q)dq - f(\beta)\beta \frac{\partial \bar{q}^A}{\partial \delta},$$

where the equality follows from substituting $\bar{q}^A = \beta$ and $\bar{q}^B = -\beta$. By our assumption of $S - U$, $\int_{-\beta}^{\beta} qf(q)dq \geq 0$ (proof available upon request, implying the the first term is non-negative. Since Proposition 2 shows that $\bar{q}^A$ is decreasing in δ, the second term is positive implying that $\frac{\partial W^A}{\partial \delta}|_{\delta=0} > 0$.

Next, consider W^B. Solving the derivative of W^B with respect to δ and then evaluating at $\delta = 0$ yields:

$$\frac{\partial W^B}{\partial \delta}|_{\delta=0} = -F(\bar{q}^B)) \left(\int_{\bar{q}^B}^{\infty} qf(q)dq - \int_{\bar{q}^A}^{\infty} qf(q)dq \right) - f(\bar{q}^B)\bar{q}^B \frac{\partial \bar{q}^B}{\partial \delta}$$

$$= -F(-\beta) \int_{-\beta}^{\beta} qf(q)dq + f(-\beta)\beta \frac{\partial \bar{q}^B}{\partial \delta},$$

where the equality follows from substituting $\bar{q}^B = -\beta$ and $\bar{q}^A = \beta$. By our assumption of $S - U$, $\int_{-\beta}^{\beta} qf(q)dq \geq 0$, implying the the first term is non-positive. Since Proposition 2 shows that $\bar{q}^B$ is decreasing in δ, the second term is also negative implying that $\frac{\partial W^B}{\partial \delta}|_{\delta=0} < 0$.

Next, consider the gap $W^B - W^A$:

$$W^B - W^A = \frac{(1 - \delta)(\int_{\bar{q}^B}^{\infty} qf(q)dq - \int_{\bar{q}^A}^{\infty} qf(q)dq)}{1 - \delta F(\bar{q}^A) + \delta F(\bar{q}^B)} = \frac{(1 - \delta)}{1 - \delta F(\bar{q}^A) + \delta F(\bar{q}^B)} M(\bar{q}^A),$$

where

$$M(\bar{q}^A) = \int_{\bar{q}^A - 2\beta}^{\bar{q}^A} qf(q)dq.$$

Since $1 \geq F(q) \geq 0$ and $0 \leq \delta \leq 1$, $\text{sgn}\big(W^B - W^A\big) = \text{sgn}\big(M(\bar{q}^A)\big)$.

Consider first $\delta = 0$ such that $\bar{q}^A = \beta$. Then, $S - U$ implies $M(\beta) = \int_{-\beta}^{\beta} qf(q)dq \geq 0$ and $W^B - W^A \geq 0$. Second, consider $\delta = 1$. Then, $W^B - W^A = \frac{0}{1} M(\bar{q}^A)$, where $M(\bar{q}^A)$ is finite thus $W^B - W^A = 0$.

Next, we turn to $1 > \delta''$. We distinguish between two case, $F(0) < 1/4$ and $F(0) > 1/4$ that we analyze in turn.

Case 1: $F(0) < 1/4$. In this case, Proposition 2 implies that there is a cutoff, δ' where δ' is the solution to $\bar{q}^A = 0$, such that $\bar{q}^A > 0$ for $\delta \in [0, \delta']$ and $\bar{q}^A < 0$ for $\delta \in [\delta', 1]$. For all $\delta \in [\delta', 1]$, $M(\bar{q}^A) < 0$ because $q < 0$ for all $q \in [\bar{q}^A - 2\beta, \bar{q}^A]$. For $\delta \in [0, \delta']$, $M(\bar{q}^A)$ is decreasing with δ. To see why:

$$\frac{\partial M(\bar{q}^A)}{\partial \delta} = \left[\bar{q}^A f(\bar{q}^A) - (\bar{q}^A - 2\beta) f(\bar{q}^A - 2\beta)\right] \frac{\partial \bar{q}^A}{\partial \delta}.$$

The term inside the squared brackets is positive for all $\delta \in [0, \delta']$ because $\bar{q}^A \geq 0$, $f(q) > 0$ and because $\bar{q}^A \leq \beta$ implies that $\bar{q}^A - 2\beta \leq \beta - 2\beta = -\beta < 0$. Since $\bar{q}^A$ is decreasing with δ, $\frac{\partial M(\bar{q}^A)}{\partial \delta} < 0$

To summarize, $M(\bar{q}^A) \geq 0$ for $\delta = 0$, $M(\bar{q}^A)$ is decreasing with δ for $\delta \in [0, \delta']$ and $M(\bar{q}^A) < 0$ for $\delta \in [\delta', 1]$. Therefore, there is a unique cutoff $\delta'' < \delta'$ such that $M(\bar{q}^A) > 0$ for $\delta \in [0, \delta'']$ and $M(\bar{q}^A) < 0$ for $\delta \in [\delta'', 1]$. Since sgn$(W^B - W^A)$= sgn $M(\bar{q}^A)$, this implies that $W_B > W_A$ for $\delta \in [0, \delta'']$ and $W_B < W_A$ for $\delta \in [\delta'', 1)$.

Case 2: $F(0) > 1/4$. In this case, $\bar{q}^A > 0$ at $\delta = 1$. Notice that $M(\bar{q}^A)$ is decreasing with δ for all $\delta \in [0, 1]$ (the proof that $\frac{\partial M(\bar{q}^A)}{\partial \delta} < 0$ requires only that $\bar{q}^A > 0$ which holds in case 2 for all $\delta \in [0, 1]$). However, unlike case 1, now $M(\bar{q}^A)$ at $\delta = 1$ can be either positive or negative. It will be positive if $\bar{q}^A$ at $\delta = 1$ is sufficiently higher than 0, in which case for all $\delta \in [0, 1]$, $M(\bar{q}^A) > 0$ and consequently $W_B > W_A$ for all $\delta \in [0, 1)$. In this case $\delta'' = 1$. $M(\bar{q}^A)$ can be negative at $\delta = 1$ if $\bar{q}^A$ at $\delta = 1$ is sufficiently close to 0, in which case at $\delta = 1$, $M(\bar{q}^A) < 0$ and consequently $W_B > W_A$ for $\delta \in [0, \delta'']$ and $W_B < W_A$ for $\delta \in [\delta'', 1)$, as in case 1.

Remark on uniform distribution: with uniform distribution, $M(\bar{q}^A) = 0$ at $\delta = 0$ and $M(\bar{q}^A) < 0$ otherwise. This implies that $W_A > W_B$ for all $\delta \in (0, 1)$ and $W_A = W_B$ otherwise. ■

Part (i) of Proposition 5 shows that at low values of δ, W_B is larger than W_A. Intuitively, in this case the two cutoffs, $\bar{q}^A$ and $\bar{q}^B$, are close to their myopic levels: a focal platform A wins if $q < \beta$ and a focal platform B wins if $q > -\beta$. Since it is more likely that q will be positive, it is welfare-maximizing when platform B starts as a focal. However, part (i) of Proposition 5 also shows that for low values of δ, W_B is decreasing with δ while W_A is increasing with δ. This is because when platforms become more patients it becomes more likely that a focal platform B will win when its quality is inferior to platform A, which reduces welfare. Part (ii) finds that when platforms are sufficiently patients (δ is sufficiently high) and it is most likely that platform B has a superior quality ($F(0) < 1/4$), social welfare is higher when platform A starts as focal because the focal position provides platform B with a too strong competitive advantage such that platform B wins more than it should. Notice that the results above do not argue that social welfare is maximized when platform A is focal in all periods. They imply that in the first stage only, it is welfare maximizing to start the dynamic game with platform A as focal, even though platform B has on average higher quality.

The results above suggest that social welfare in the myopic case might be higher than when platforms are very patients. The comparison between social welfare evaluated at $\delta = 0$ and $\delta = 1$ for the general distribution function is inconclusive. We therefore make the simplifying assumption that q is uniformly distributed.

Corollary 2 (Welfare under uniform distribution) *Suppose that q is uniformly distributed along the interval $[\mu - \sigma, \mu + \sigma]$ and $\sigma > \frac{1}{2}(\mu + 3\beta) + \frac{1}{2}\sqrt{(\mu^2 + 6\mu\beta + \beta^2}$. Then,*

$$\bar{q}^A = \beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}, \quad \bar{q}^B = -\beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}. \tag{10}$$

Moreover, $W^A|_{\delta=0} = W^B|_{\delta=0} > W^A|_{\delta=1} = W^B|_{\delta=1}$.

Proof. Substituting $F(q) = \frac{q+\sigma}{2\sigma}$ into (9), yields (10). To ensure that $\bar{q}^B > \mu - \sigma$, we need that σ is high enough such that $\sigma > \frac{1}{2}(\mu + 3\beta) + \frac{1}{2}\sqrt{(\mu^2 + 6\mu\beta + \beta^2}$. Notice that this assumption implies that $\sigma > 2\beta$. The recursive expected social welfare functions are:

$$\bar{W}^A = \int_{\mu-\sigma}^{\beta - \frac{2\delta\mu\beta}{\sigma-2\delta\beta}} (\beta + \delta\bar{W}^A)\frac{1}{2\sigma}dq + \int_{\beta - \frac{2\delta\mu\beta}{\sigma-2\delta\beta}}^{\mu+\sigma} (\beta + q + \delta\bar{W}^B)\frac{1}{2\sigma}dq,$$

$$\bar{W}^B = \int_{-\beta - \frac{2\delta\mu\beta}{\sigma-2\delta\beta}}^{\mu+\sigma} (\beta + q + \delta\bar{W}^B)\frac{1}{2\sigma}dq + \int_{\mu-\sigma}^{-\beta - \frac{2\delta\mu\beta}{\sigma-2\delta\beta}} (\beta + \delta\bar{W}^A)\frac{1}{2\sigma}dq.$$

Hence,

$$W^A = (1-\delta)\bar{W}^A = \frac{1}{4}\left(4\beta - \frac{\beta^2}{\sigma} + \sigma + \frac{\mu(4\delta^2\beta^2(2\beta - 3\sigma) - \sigma^2(\mu + 2\sigma) + \delta\beta\sigma(5\mu - 4\beta + 10\sigma)))}{(\delta\beta - \sigma)(\sigma - 2\delta\beta)^2}\right)$$

$$W^B = (1-\delta)\bar{W}^B = \frac{1}{4}\left(4\beta - \frac{\beta^2}{\sigma} + \sigma + 2\mu + \frac{(\mu(8(-1+\delta)\delta^2\beta^3 + \delta\beta(5\mu - 4(-1+\delta)\beta)\sigma - \mu\sigma^2))}{(\delta\beta - \sigma)(\sigma - 2\delta\beta)^2}\right)$$

The gap $W^A - W^B$ is:

$$W^A - W^B = \frac{2(1-\delta)\delta\mu\beta^2}{(\sigma - \delta\beta)(\sigma - 2\delta\beta)}.$$

Since by assumption $\sigma > 2\beta$, $W^A - W^B > 0$ for all $0 < \delta < 1$ and $W^A - W^B = 0$ for $\delta = 0$ and $\delta = 1$. Moreover:

$$W^A|_{\delta=0} - W^A|_{\delta=1} = \frac{\mu^2\beta^2(2\sigma - \beta)}{\sigma(\sigma - \beta)(\sigma - 2\beta)^2} > 0.$$

where the inequality follows because by assumption $\sigma > 2\beta$ and $\mu > 0$. ■

The result is illustrated in the next figure.

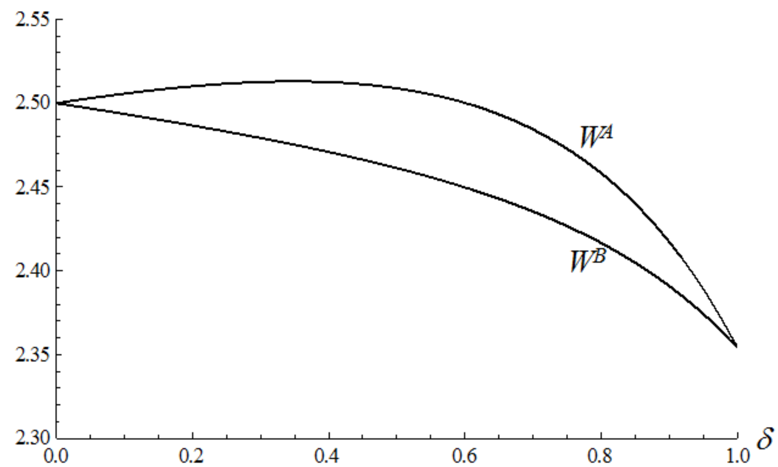

Figure 3: The effect of δ on welfare for a uniform distribution

7 Conclusions

In platform competition, having a superior quality over a competing platform may not be enough to dominate the market. In the presence of strong network externalities, a platform needs to convince consumers that other consumers will join it. A platform can benefit from a focal position such that when consumers decide which platform to join, and multiple equilibrium decisions exist, all consumers play the equilibrium in which they join the focal platform. Network externalities provide the focal platform with a competitive advantage, while it is a disadvantage for a non-focal platform. In a static model a platform can use its focal position for dominating the market even when it offers lower quality than the non-focal platform. The aim of our paper is to study whether this static advantage carries over to a dynamic environment.

In this paper we consider the two sources of platforms' competitive advantage: quality and focal position, in a dynamic, infinite-horizon model. Two platforms differ in their qualities. In the first period, the inferior quality platform is focal, but platforms take into account that dominating the market in a current period provides the platform with a focal position in the next period.

We first consider a finite game. We find that the more platforms are forward looking, it is more likely that a high-quality but non-focal platform will be able to win the market and then maintain its focal position infinitely. As a result, social welfare (weakly) increases the more platforms are forward-looking because then the better quality platform serves all consumers. Intuitively, a high-quality platform has more to gain by being focal in the final period and will therefore have more of an incentive to compete aggressively to gain a focal position in early periods when competing against the current focal platform.

We then turn to offer two explanations for inefficient market leadership that may arise even when platforms are forward-looking. The first explanation involves an infinitely repeated interaction. In this case, we find that as in the finite case, if platforms are sufficiently forward looking, a high-quality but non-focal platform can overcome its competitive disadvantage and win the market. However, when platforms are substantially forward-looking there are multiple equilibria because there is also an equilibrium in which the low-quality focal platform maintains its focal position infinitely.

The second explanation involves an environment with stochastic qualities which allows each platform win every period with some probability. Now, the more platforms are forward looking, it is less likely that a high-quality platform will be able to overcome a non-focal position. Intuitively, if one of the platforms has a higher average quality than the other platform, dynamic consideration will provide it with a stronger incentive to win the market and maintain a focal position in the current period, even with an inferior quality. At an extreme case, a focal and high-quality platform can still lose the market, if platforms are sufficiently forward looking. This result indicates that with stochastic qualities, social welfare can decrease the more platforms are forward looking.

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