

Are Market-Share Contracts a Poor Man's Exclusive Dealing?*

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Abstract

Contracts that reference rivals have long been a focus of antitrust law and the subject of intense scholarly debate. This paper compares two such contracts, exclusive-dealing contracts and market-share contracts, in a model of naked exclusion. We discuss the different mechanisms through which each works and identify the fundamental tradeoff that arises: market-share contracts are better at maximizing a seller's benefit from foreclosure whereas exclusive dealing is better at minimizing a seller's cost of foreclosure. We give settings in which each is the more profitable contract and show that welfare can be worse with market-share contracts.

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1 Introduction

Contracts that reference rivals, and in particular, contracts in which a seller puts restrictions on how much a buyer can buy from other sellers, have long been a focus of antitrust law and the subject of intense scholarly debate.¹ In an exclusive-dealing contract, the buyer agrees to purchase exclusively from the seller (i.e., you agree not to buy from my rivals). In a market-share contract, the buyer agrees to purchase some minimum share of its requirements from the seller (i.e., you agree that your purchases from my rivals will not exceed $x\%$ of your total purchases).² In both cases, the concern is whether such contracts might harm competition by discouraging efficient entry or by leaving existing rivals with insufficient alternatives to compete for sales.

Lost in this debate, however, is any sense of why some sellers prefer exclusive dealing while other sellers opt for the less restrictive market-share requirements. Indeed, the use of market-share contracts by would-be excluding sellers is puzzling. If a seller's intent is to exclude a potential entrant or a rival upstream seller, then isn't it obvious that exclusive dealing will be more effective? Under exclusive dealing, a seller leaves no room for other firms to sell to the buyer, whereas the door to sales is not fully closed to them under a market-share contract. That exclusive-dealing arrangements are presumed to be more powerful is also implicit in many court proceedings, where plaintiffs often seek to have defendants' market-share contracts declared to be "de facto" exclusive dealing.³ But this only begs the question, if they are so powerful, then why did the defendants in these court cases not opt to go with exclusive dealing in the first place?

One's views on this matter because if one believes that exclusive dealing will always be the seller's preferred choice when the seller's intent is to exclude, then one might also believe that if we observe a seller using a market-share contract, then it must be for non-exclusionary reasons (a pro-Chicago school view) because otherwise the seller would have used exclusive dealing. Taken to the extreme, the implication of this is that courts should adopt a *laissez-faire* attitude toward

¹See the discussion in Tom et al. (2000) and Whinston (2006). See also the transcripts of the FTC - DOJ Workshop on Conditional Pricing Practices: Economic Analysis & Policy Implications (June 23, 2014), available at www.ftc.gov/news-events/events-calendar/2014/06/conditional-pricing-practices-economic-analysis-legal-policy.

²Recent cases involving exclusive dealing include *Van den Bergh Foods v. Commission* [Case T-65/98 (2003)], *Conwood v. U. S. Tobacco* [290 F. 3d 768 (2002)], *U.S. v. Visa U.S.A.* [344 F. 3d 229 (2003)], *U.S. v. Dentsply* [399 F. 3d 181 (2005)], *FTC v. Transitions Optical* [File No. 091 0062 (2010)], and *FTC v. McWane, Inc* [2014]. Recent cases involving market-share contracts include *Masimo Corp. v. Tyco Healthcare Group*, [No. 02-CV-4770 (2005)], *Prokent v. Tomra* [Case COMP/E-1/38.113 (2006)], *FTC v. Intel* [File No. 061 0247 (2010)], *ZF Meritor v. Eaton* [696 F. 3d 254 (2012)], *Intel v. Commission* [Case T-286/09 (2014)], and *Eisai v. Sanofi Aventis* [2016].

³In *ZF Meritor v. Eaton* [2012], for example, ZF Meritor alleged that Eaton's contract, which required high minimum percentage purchases from Eaton, was a *de facto* exclusive-dealing arrangement and the court agreed. And similarly, in *Eisai v. Sanofi Aventis* [2016], Eisai argued that Sanofi Aventis' provision that buyers purchase at least 90% of their anticoagulant drug requirements from Sanfoi Aventis was the same as exclusive dealing.

market-share contracts, while continuing to subject exclusive dealing to scrutiny. The problem with this ‘quick and dirty’ solution, however, is that we might then expect to observe sellers using market-share contracts even when their intent really is to engage in exclusionary conduct.

In posing the question “Why do we observe both market-share contracts and exclusive-dealing contracts when the latter would seem to be the better choice for achieving exclusion?” to fellow economists and policymakers, we have encountered many responses. One is that only *some* sellers believe that by using market-share contracts they can fool authorities into believing their intent is benign. The rest are willing to take their chances with exclusive dealing. Others agree that exclusive dealing is likely to be more effective, but conjecture that the transactions costs of implementing it may sometimes be higher than the transactions costs of implementing a market-share contract.⁴ A third response is that market-share contracts are a poor man’s way of excluding – not as effective, but also not as costly. Expanding on this last point, we have also heard it argued that although market-share contracts may not be as effective or as powerful as exclusive dealing at foreclosing rival sellers, the compensation that would be needed to induce a buyer to agree to such a contract will be less — and therefore if one were to observe a seller opting to use a market-share contract (when its intent is to exclude), it must be because it believes that the relatively lower cost of exclusion would more than offset the relatively lower effectiveness.

Implicit in these responses is the notion that market-share contracts will indeed not be as effective or as powerful as exclusive dealing at achieving exclusion, and that given this, they are likely to be less harmful to competition all else being equal. We believe this common wisdom is both *ad hoc* and misleading. The fallacy is that it considers the benefit and cost of exclusion on a per-buyer basis, and implicitly takes the number of buyers who have agreed to the seller’s contract as given. However, the reality is that how many buyers to sign up, and thus how much of the market to foreclose, are choices of the seller that can vary depending on the market context.

When viewed in this light, the comparison between exclusive-dealing contracts and market-share contracts begins to look much different. Suppose, for instance, that the downstream market consists of five identical buyers, and that the upstream market consists of an incumbent seller and a potential entrant. If the incumbent can induce all five buyers to agree to exclusive dealing, the potential entrant will be excluded for sure. The problem is that signing this many buyers may be prohibitively costly (especially if the buyers are able to coordinate with each other before they sign). What then? Should the incumbent give up and not sign any buyers, knowing that the

⁴One might argue, however, that the opposite is true: in many instances, it may be less costly to verify whether a buyer is purchasing something from a rival than it is to verify how much a buyer is purchasing from the rival.

entrant will then enter for sure, or should it offer its contract to a subset of the buyers, knowing that the more buyers it signs, the more likely it is that the entrant will be deterred? And if the latter is called for, is it better to require the buyers to commit to buy all their purchases from the incumbent, or only a fraction of their purchases from the incumbent? More concretely, suppose that the costs and benefits are such that it is optimal for the incumbent to foreclose 60% of the market. Under exclusive dealing, the incumbent can achieve this by signing up three of the five buyers. But it can also achieve this by signing up four of the five buyers and imposing a minimum-share requirement of 75%, or by signing up all five buyers and imposing a minimum-share requirement of 60%. What should it do? In this simple setting, the incumbent's choice will likely come down to whether the cost to it of imposing exclusive dealing on three buyers will be higher or lower than the cost to it of imposing a minimum-share requirement of, say, 75% on four buyers. In more complicated settings, however, even the level of foreclosure that can be achieved will generally differ depending on which type of contract is used, and to complicate matters further, there is no reason to think that the seller's profit-maximizing level of foreclosure will be independent of whether it uses exclusive-dealing contracts or market-share contracts.

In this paper, we characterize the incumbent's optimal exclusionary contract using a framework that has the same timing and many of the same features as the "naked exclusion" literature put forward by Rasmusen, Ramseyer, and Wiley (1991) (hereafter RRW) and Segal and Whinston (2000) (hereafter SW). We also follow SW in allowing buyers to coordinate their acceptance decisions, thereby ruling out equilibria in which the seller is able to obtain exclusion costlessly. We differ from RRW-SW, however, in that the entrant's costs are assumed to be unknown at the time the incumbent makes its offers.⁵ We also differ from RRW-SW in that we allow the incumbent to choose between offering an exclusive-dealing contract or a market-share contract.

We find that although market-share contracts are profitable if and only if exclusive-dealing contracts are profitable, the mechanism through which the two types of contracts work differ. When a buyer signs one of these contracts, the attractiveness of the market for the entrant is reduced, which reduces the probability of entry. This reduction negatively impacts buyers in two ways. First, it reduces the unsigned buyers' expected surplus. This is due to the *inter-group externality* that signed buyers impose on unsigned buyers. Second, it reduces the expected surplus of the uncommitted purchases of each signed buyer. While each signed buyer must be compensated for the expected reduction in surplus that its signing causes for *its* own uncommitted

⁵In contrast to RRW-SW, where the entrant's fixed costs are known at the time of contracting, and thus where it is known how many buyers are needed to deter the entrant, the incumbent and the buyers in our model only know that the probability of entry is (weakly) decreasing in the number of buyers who sign the incumbent's contract.

purchases, the negative impact that its signing causes for the *other* signed buyers' uncommitted purchases is not compensated. Importantly, however, this *intra-group externality* arises only when market-share contracts are offered, not when exclusive-dealing contracts are offered, since the signing buyers have no uncommitted purchases when they sign an exclusive-dealing contract.

The profitability of exclusive dealing can thus be attributed solely to an inter-group externality, whereas the profitability of market-share contracts can be attributed to both an inter-group and an intra-group externality.⁶ Since the inter-group externality is stronger under exclusive dealing, and since the relative importance of the two externalities depends on the ratio of the number of signed to unsigned buyers, it follows that the relative profitability of market-share contracts will increase as more and more weight is placed on the intra-group as opposed to the inter-group externality. This can explain why market-share contracts may dominate exclusive dealing when the incumbent cannot selectively offer its contracts to a subset of buyers (because when all buyers are signed, there is only an intra-group externality), but it cannot yet explain why market-share contracts may dominate exclusive dealing even when selective offers are feasible.

For this, it is useful to recognize that the entrant's profit gross of any fixed costs depends only on the size of the market it can contest, which in turn means that the incumbent's expected benefit from foreclosure also depends only on this magnitude, not on how it is achieved. When viewed in this light, it can be seen that market-share contracts potentially offer a huge advantage over exclusive dealing in that they allow the incumbent to fine tune the level of foreclosure it can achieve. Suppose, for example, that the incumbent would ideally like to foreclose 60% of the market, but instead of there being five identical buyers, there are only four identical buyers. This poses a problem for exclusive dealing because under exclusive dealing the closest the incumbent can come to its ideal is either to sign two buyers and foreclose 50% of the market, or three buyers and foreclose 75% of the market, neither of which is optimal. By contrast, with market-share contracts, the incumbent can realize a foreclosure level of 60% simply by offering its contract to all four buyers and imposing a minimum-share requirement of 60% (alternatively, it could also do so by offering its contract to three buyers and imposing a minimum-share requirement of 80%).

Market-share contracts are thus seen to be weakly better than exclusive dealing at maximizing the incumbent's benefit from foreclosure. We will show, however, that market-share contracts will not always be preferred to exclusive dealing because the other side of the equation, the cost of foreclosure, is also higher when market-share contracts are used. This is because the incumbent needs to compensate each signed buyer not only for its expected loss on the purchases it commits

⁶The intra-group externality does not arise in SW because of their focus on exclusive contracts.

to the incumbent (say 75% of its total purchases) but also for the negative externality that its committed purchases imposes on its uncommitted purchases (say 25% of total purchases). It follows that with market-share contracts the incumbent must over-compensate each signed buyer for the contribution of its committed purchases. By contrast, there is no such over-compensation under exclusive dealing because then there are no uncommitted purchases for the signed buyers.

We find that there is thus a fundamental trade-off in the design of exclusionary contracts. Market-share contracts do better at maximizing the incumbent's benefit from foreclosure, whereas exclusive contracts do better at minimizing the incumbent's cost of foreclosure. We show that depending on how the entrant's costs are distributed, market-share contracts can be more profitable than exclusive dealing in some settings, while in other settings, the opposite holds. We also show that as the number of buyers increases, market-share contracts are more likely to dominate exclusive dealing and thus are more likely to be the incumbent's preferred means of exclusion.

These findings are in sharp contrast to the view that market-share contracts are a poor man's excluding dealing, and they are in sharp contrast to the conventional view that market-share contracts are a weaker version of exclusive dealing and thus by implication less powerful. Instead, we find that the incumbent must pay a higher per-unit cost of foreclosure under market-share contracts than it would have paid under exclusive dealing for the same number of committed purchases, and that market-share contracts can lead to higher foreclosure levels in equilibrium.

Literature review

Antitrust cases involving allegedly exclusionary contracts are controversial in part because of the "Chicago-School Critique" popularized by Director and Levi (1956), Posner (1976), and Bork (1978). These authors argue that such contracts are unlikely to be anticompetitive because buyers would not agree to them if their purpose was to exclude upstream sellers. They suggest that there must instead be some efficiency-enhancing reason that accounts for their usage.

The economic literature on exclusion, which has developed in response to the Chicago-School critique, has focused almost entirely on exclusive-dealing contracts.⁷ The two most prominent theories of harm in the exclusion literature are the aforementioned "naked exclusion" scenario put forward by RRW-SW, and the "rent-extraction" scenario of Aghion and Bolton (1987) (hereafter AB), in which exclusion is induced inadvertently as a result of rent extraction. RRW show that exclusive dealing imposes a negative externality on buyers, which the incumbent may be able to

⁷With the exception of Chen and Shaffer (2014), discussed below, the literature on market-share contracts has focused on other motives. In particular, market-share contracts have been alleged to facilitate rent shifting (Marx and Shaffer, 2004), screen buyers when demand is private information (Majumdar and Shaffer, 2009; Calzolari and Denicolo, 2013), induce more service provision (Mills, 2010), and soften competition (Inderst and Shaffer, 2010).

exploit when the buyers are unable to coordinate their acceptance decisions. SW show that by adopting a “divide-and-conquer” strategy in which it offers to fully compensate some buyers in order to earn monopoly profits on the remaining buyers, the incumbent may be able to exclude the entrant profitably even in the absence of a coordination failure by the buyers. In contrast, AB take a very different approach and show that an incumbent may be able to extract rents from a more efficient entrant by offering exclusive-dealing contracts that contain penalty clauses.

In addition to the work by RRW-SW, related literature includes: Innes and Sexton (1994), who look at a setting in which buyers can form coalitions with the entrant; Spector (2011), who allows the more efficient excluded firm to be present at the time of contracting; the papers by Landeo and Spier (2009, 2012), and Smith (2010), who look at naked exclusion using RRW-SW’s framework in experimental settings; and Miklós-Thal and Shaffer (2016), who consider the efficacy of divide-and conquer strategies when contracts are unobservable. The RRW-SW strand of literature has also been extended by several authors to settings in which buyers compete.⁸ In addition to the work by AB, related literature includes: Chen and Sappington (2011), who find that exclusive dealing generally reduces the entrant’s R&D and can reduce the incumbent’s R&D; Ide, Montero, and Figueroa (2016), who find that exclusionary contracts with all-unit discounts cannot be anticompetitive without upfront payments; and Chone and Linnemer (2015), who find that the supply of the rival good can be distorted downwards when the incumbent offers a nonlinear price-quantity schedule and the buyer opportunistically purchases from the incumbent. However, none of these papers considers the possibility of exclusion with market-share contracts.

Recently, Calzolari and Denicolo (2015) find that exclusive-dealing contracts can impose contractual restrictions on buyers without necessarily compensating them, in a setting where buyers’ willingness to pay for the product is private information. Their setting follows the literature on “rent-shifting” put forward by O’Brien and Shaffer (1997) and Bernheim and Whinston (1998), which is different from the literature on naked exclusion. In Calzolari and Denicolo, exclusive dealing acts as a mechanism of rent-extraction under asymmetric information. The dominant firm with competitive advantages over its competitors can use exclusive dealing to distort the rival’s sales and makes the rival’s contracts less attractive. In our paper, by contrast, exclusionary contracts create contractual externalities among the buyers and act as a mechanism that transforms the buyers’ coordination game into a prisoners’ dilemma game. Moreover, their paper is mainly focused on exclusive-dealing contracts, whereas our paper studies the optimal design

⁸See, for example, Fumagalli and Motta (2006), Simpson and Wickelgren (2007), Abito and Wright (2008), Wright (2009), Argenton (2010), and Asker and Bar-Isaac (2014).

of exclusionary contracts when both exclusive dealing and market-share contracts are allowed.

Our paper also differs from Chen and Shaffer (2014), who focus on a setting in which the incumbent is restricted to offering exclusionary contracts to all buyers. In their model, exclusive dealing is never profitable (because there is no inter-group externality), but market-share contracts are profitable (because there remains the intra-group externality). In contrast, we focus on optimal contract design when the number of offers is endogenously determined. In doing so, we find that the two externalities interact in complex ways, and we identify the fundamental trade-off between the cost and the benefit of foreclosure that has not previously been shown.

The rest of the paper proceeds as follows. In Section 2, we set up the model and introduce notation. We discuss the inter and intra-group externalities in Section 3, and identify the fundamental trade-off between the cost and benefit of foreclosure in Section 4. In Section 5, we analyze the impact on the profitability of exclusionary contracts when the number of buyers increases. Section 6 characterizes the optimal market-share and exclusive-dealing contracts, and Section 7 concludes the paper. The proofs of the lemmas and propositions can be found in the appendix.

2 The Model

2.1 The Setting

We adopt the setup in RRW-SW and consider a setting with three types of players: an incumbent seller, a potential entrant, and a set of N homogeneous buyers. The seller(s) offer a single divisible good for sale. Each buyer demands a fixed amount of the good, which we normalize to one unit. The incumbent can produce at constant marginal cost c , while the potential entrant, if it decides to enter, can produce at constant marginal cost $c - \delta$, where $\delta \in (0, c)$. To ensure there are gains from trade, it is assumed that each buyer is willing to pay up to a maximum of $v > c$ for its unit.

Unlike the incumbent, who is already established in the market, we assume the entrant must incur a fixed cost of entry, f , before it can produce anything. Moreover, we assume it is common knowledge among all players that f is distributed with positive density $g(\cdot)$ between zero and $\bar{N}\delta$, where $\bar{N} \leq N$. The lower bound ensures that attempts by the incumbent to reduce the entrant's flow payoff will succeed in deterring entry if the reduction is large enough. The upper bound ensures that in the absence of such attempts, entry will occur with probability one.⁹

⁹Since the entrant can always set a per-unit price slightly lower than the incumbent's marginal cost, we assume, for simplicity, that each buyer will purchase from the entrant if the incumbent and the entrant charge the same price. The claim then follows because in the event of entry, and assuming all buyers are free to purchase from either seller, the entrant can always earn a net profit of $N\delta - f > 0$ by charging a per-unit price of c to each buyer.

As in RRW-SW, it is assumed that the incumbent has a first-mover advantage and can offer exclusionary, non-renegotiable contracts to buyers prior to the entrant's entry decision. Unlike RRW-SW, who restrict attention to exclusive-dealing contracts, however, we allow the incumbent to offer contracts that require only partial exclusivity. That is, we consider a class of contracts in which a signing buyer agrees only to purchase some minimum share $s \in [0, 1]$ of its total purchases from the incumbent. We also differ from RRW-SW in the kinds of inducements the incumbent can offer. In RRW-SW, the incumbent can offer a lump sum payment to each buyer who signs its contract, but it cannot commit in advance to the per-unit price it will charge. Here, we allow the incumbent to do both: offer a lump-sum payment *and* commit to a per-unit price.¹⁰

Timing of the game

The timing of the game proceeds as follows:

Period 1: The incumbent offers to each of K buyers an exclusionary contract $C = \{s, x, p\}$. In this contract, $s \in [0, 1]$ denotes the minimum share of the buyer's total purchases that must be purchased from the incumbent, x denotes the lump-sum payment to be paid, and p denotes the constant per-unit price at which the buyer can purchase the incumbent's good. For ease of exposition, it is convenient to restrict attention in the text to contracts in which $p \in [c, v]$.¹¹

Period 2: Buyers simultaneously decide whether to accept or reject the incumbent's offers. Acceptance implies agreement to the incumbent's terms and conditions. Rejection implies that the buyer can purchase as much as it wants from the entrant if the entrant enters the market.

Period 3: The potential entrant learns the value of f and, after observing the incumbent's offers and the buyers' accept-or-reject decisions, decides whether or not to enter the market.

Period 4: The incumbent and the entrant (if active) compete for the uncommitted purchases of each buyer by posting prices in a spot market.¹² Buyers who have agreed to the incumbent's exclusionary offer have an option to buy as much as they want from the incumbent at the price p , but must buy at least s share of their purchases at this price. For the rest of their purchases, they can either buy from the incumbent at the contract price of p , or they can buy from either firm in the spot market. Buyers who have not agreed to the incumbent's exclusionary offer can

¹⁰Price commitments are commonly observed in settings in which the nature of the good to be delivered in future periods is known to the incumbent and the buyers at the time the contracts are written and signed.

¹¹Offers of $p < c$ invite possible rent-shifting along the lines discussed in Aghion and Bolton (1987), and as such are likely to engender additional scrutiny from competition authorities on predatory pricing grounds. We rule out such offers in the spirit of RRW-SW in order to keep the focus on the relative foreclosure potential of exclusive dealing versus market-share contracts. It is easily verified in Appendix D that $p > v$ cannot arise in equilibrium.

¹²The uncommitted purchases consist of $1 - s$ share of the purchases of a buyer who has signed the incumbent's exclusionary contract and all the purchases of a buyer who has not signed the incumbent's exclusionary contract.

purchase as much as they want from either the incumbent or the entrant in the spot market.

We will refer to contracts in which buyers agree to purchase only from the incumbent, $s = 1$, as *exclusive-dealing* contracts (*ED*). We will refer to contracts in which buyers must make at least s share of their purchases from the incumbent, with $s < 1$, as *market-share* contracts (*MS*).

2.2 Characterization of Equilibria

Suppose the incumbent offers $C = \{s, x, p\}$ to each of K buyers in Period 1, and let $n \in \{0, 1, \dots, K\}$ denote the number of buyers who accept it.

Pricing decisions

We solve for the equilibrium of the game using backwards induction. Consider first the pricing game in Period 4. There are two cases to consider. If the entrant does not enter, it is optimal for the incumbent to charge the monopoly price v to all unsigned buyers. In this case each unsigned buyer will purchase one unit of the good and obtain a surplus of zero. Signed buyers on the other hand will exercise their right to fulfill their entire demand at the per-unit price p . Because $p \leq v$, each signed buyer will thus purchase one unit of the good and obtain a surplus of $v - p$.

If the entrant does enter, we assume that competition *a la* Bertrand for the uncommitted purchases of the signed and unsigned buyers will drive the entrant's price down to the incumbent's per-unit cost c . Unsigned buyers will thus purchase one unit of the good from the entrant and obtain a surplus of $v - c$. Signed buyers, however, can only purchase at most $1 - s$ share of their total purchases from the entrant at the entrant's price of c . The remaining s share of their total purchases must be purchased from the incumbent at the per-unit contract price of p . Each signed buyer thus faces an average price of $p_a = sp + (1 - s)c$ and obtains a surplus of $v - p_a$.

Entrant's entry decision

If the entrant enters in Period 3, the entrant incurs a fixed cost of entry f and earns a flow payoff of $n(1 - s)\delta$ from the signed buyers and $(N - n)\delta$ from the unsigned buyers, for a total flow payoff of $\Pi_E(n, s) \equiv n(1 - s)\delta + (N - n)\delta = (N - ns)\delta$. In contrast, the entrant earns zero if it does not enter. Thus, it is profitable for the entrant to enter in Period 3 if and only if

$$f \leq \Pi_E(n, s). \quad (1)$$

Here we see that the entrant's flow payoff is decreasing in the number of units that have been foreclosed. This implies that the incumbent has two ways of decreasing the entrant's flow payoff. First, it can induce more buyers to sign its contract (i.e., increase n) for a given s , or second, it can require a larger minimum share of each buyer's purchases (i.e., increase s) for a given n .

Buyers' acceptance decisions

Although f is known prior to the entrant's entry decision, the realization of f is not yet known at the time the buyers must accept or reject the incumbent's offer. When making their decisions, therefore, buyers must form expectations of the likelihood of entry. Recall that f is distributed with positive density $g(\cdot)$ between zero and $\bar{N}\delta$, and let $G(\cdot)$ denote the distribution function. Using condition (1) and the definition of $G(\cdot)$, the likelihood of entry is thus given by¹³

$$\alpha_n(s) \equiv G(\Pi_E(n, s)).$$

It follows that a buyer who accepts the incumbent's offer in Period 2 receives surplus $v - p_a + x$ with probability α_n and $v - p + x$ with probability $1 - \alpha_n$, whereas if it rejects the incumbent's offer, it receives surplus $v - c$ with probability α_{n-1} and no surplus with probability $1 - \alpha_{n-1}$.

Putting it all together, the expected surplus of a buyer who accepts the incumbent's offer in Period 2 when $n - 1$ other buyers are also accepting the incumbent's offer is given by

$$U_A(n) \equiv \alpha_n(v - p_a) + (1 - \alpha_n)(v - p) + x,$$

whereas its expected surplus if it rejects the incumbent's offer in Period 2 is given by

$$U_R(n - 1) \equiv \alpha_{n-1}(v - c).$$

It is thus optimal for the buyer to accept the incumbent's offer if and only if it receives a lump-sum payment of $x \geq x_n(s, p)$, where $x_n(s, p)$ is the value of x such that $U_A(n) = U_R(n - 1)$:¹⁴

$$x_n(s, p) \equiv \alpha_{n-1}(v - c) - \alpha_n(v - p_a) - (1 - \alpha_n)(v - p). \quad (2)$$

It should be clear from the definition of $x_n(s, p)$ that if $x \geq x_K(s, p)$, then all K buyers accepting the incumbent's offer is a Nash equilibrium because $U_A(K) \geq U_R(K - 1)$ implies that no unilateral deviation is profitable. However, there may also be other equilibria. To support an equilibrium in which only $k < K$ buyers accept, for example, it must be that $U_A(k) \geq U_R(k - 1)$ and $U_R(k) \geq U_A(k + 1)$. When $x = x_K(s, p)$, these inequalities can hold if the relationship among $x_K(s, p)$, $x_k(s, p)$, and $x_{k+1}(s, p)$ is such that $x_K(s, p) \geq x_k(s, p)$ and $x_K(s, p) \leq x_{k+1}(s, p)$.

To narrow the set of possible equilibria, we follow SW in allowing buyers to coordinate their decisions when choosing whether to accept or reject the incumbent's contract. Allowing buyers

¹³Since $G(\cdot)$ is non-decreasing and $\Pi_E(n, s)$ is decreasing in n and s , $\alpha_n(s)$ will also be decreasing in n and s .

¹⁴Here we see that the sign of $x_n(s, p)$ depends on p . When p is sufficiently small (e.g., $p = c$, $x_n(s, c) = (\alpha_{n-1} - 1)(v - c) \leq 0$), the payment flows from the buyer to the incumbent. However, when p is sufficiently large (e.g., $p = v$, $x_n(s, v) = \alpha_{n-1}(v - c) - \alpha_n(v - p_a) > 0$), the payment flows from the incumbent to the buyer.

to coordinate their decisions presents a formidable hurdle for the incumbent to overcome. The only restriction SW impose is that the buyers' collective actions must be self-enforcing. This idea is captured by Bernheim et al's (1987) concept of a perfectly coalition-proof Nash equilibrium (PCPNE), which requires that all equilibria be immune to self-enforcing coalitional deviations.¹⁵

In solving for PCPNE in our setting, note first that if the payment offered by the incumbent exceeds $x_{k+1}(s, p)$, then there cannot be an equilibrium in which only $k < K$ buyers accept the incumbent's offer because the payment would exceed the minimum payment that would be needed to induce a buyer to accept the incumbent's offer if k other buyers are also accepting the incumbent's offer. Since the same logic applies to any number of buyers, $k = 0, \dots, K - 1$, it follows that if the incumbent were to offer buyers a lump-sum payment of $x > x^*(s, p)$, where

$$x^*(s, p) \equiv \max_{n \leq K} \{x_n(s, p)\}, \quad (3)$$

all K buyers would accept the offer in the unique Nash equilibrium of the continuation game.

Note next that this unique Nash equilibrium is also coalition-proof because a lump-sum payment of $x > x^*(s, p)$ means that $U_A(n) > U_R(n - 1)$ for all $n \leq K$, from which it follows that there is no self-enforcing coalitional deviation that can benefit the buyers. The reasoning behind this is straightforward. Suppose a group of $k \geq 2$ buyers were to deviate and jointly reject the offer. Then, each buyer in the coalition would get $U_R(K - k)$. However, this would not be self-enforcing because a buyer in the coalition would be able to make itself better off by deviating from the coalition and accepting the incumbent's offer, thereby earning $U_A(K - k + 1) > U_R(K - k)$.

Note finally that if the incumbent were to offer $x < x^*(s, p)$, which implies that there exists some number $m \leq K$ such that $x < x_m(s, p)$, then the condition $U_R(m - 1) > U_A(m)$ implies that a coalition of $K - m + 1$ buyers would be better off jointly rejecting the incumbent's offer.

These findings, and those for the case of $x = x^*(s, p)$, are summarized in the following lemma.

Lemma 1 *Suppose the incumbent offers the contract $C = \{s, x, p\}$ to K buyers. Then, if*

- $x > x^*(s, p)$, all K buyers accept the offer in the unique PCPNE in the continuation game;
- $x < x^*(s, p)$, there is no PCPNE in the continuation game in which all K buyers accept;
- $x = x^*(s, p)$, there exists a PCPNE in the continuation game in which all K buyers accept.

¹⁵In other words, valid deviations are judged by the same criteria used to judge the candidate equilibrium. They must be self-enforcing in the sense that no proper sub-coalition can reach a mutually beneficial agreement to deviate from the deviation. Any potential deviation by a sub-coalition must also be self-enforcing, and so on.

Lemma 1 highlights the role of $x^*(s, p)$ in inducing all K buyers to accept the incumbent's contract in a PCPNE.¹⁶ It implies that (i) offering $x > x^*(s, p)$ is sufficient to induce all buyers to sign; and (ii) offering at least $x = x^*(s, p)$ is necessary to induce all buyers to sign. Given the importance of $x^*(s, p)$, therefore, it is useful to consider some of its properties before proceeding.

2.3 Properties of $x^*(s, p)$

We begin by noting that $x_1(s, p)$, and thus $x^*(s, p)$, is bounded below by $s(p - c)$. Intuitively, a buyer who signs the incumbent's contract is committing to buy s share of its purchases from the incumbent at a per-unit price of p . By not signing, the buyer reasons that it will be able to purchase these same units in the spot market at a per-unit price of c . The buyer will therefore need to be compensated with a lump-sum payment of *at least* $s(p - c)$ in order for it to sign.¹⁷

We say that the incumbent must offer at least $s(p - c)$ in compensation because there is an additional factor that comes into play when MS is offered that is not present with ED. To see this, note that our assumption that f is bounded above by $\bar{N}\delta$ implies that the incumbent must sign up more than $\Omega \equiv N - \bar{N}$ buyers if its contract is to have any effect (i.e., if it is to have any chance of reducing the probability of entry). So, suppose the incumbent offers its contract to exactly $\Omega + 1$ buyers, and consider the payment $x_{\Omega+1}(s, p)$. Using condition (2), we have:¹⁸

$$\begin{aligned} x_{\Omega+1}(s, p) &= \alpha_{\Omega}(v - c) - (1 - \alpha_{\Omega+1})(v - p) - \alpha_{\Omega+1}(v - p_a) & (4) \\ &= v - c - (1 - \alpha_{\Omega+1})(v - p) + (1 - \alpha_{\Omega+1})(v - p_a) - (v - p_a), \\ &= s(p - c) + (1 - \alpha_{\Omega+1})(1 - s)(p - c). \end{aligned}$$

The additional factor can be seen from the third line in (4), which implies that the incumbent must compensate a signing buyer not just for the surplus that the latter expects to lose on the share of the total purchases that it commits to the incumbent, i.e., $s(p - c)$, but also for the reduction in surplus that its signing causes for the $1 - s$ share of its purchases that are uncommitted, an amount that is equal to $(1 - \alpha_{\Omega+1})(1 - s)(p - c)$. This follows because given that it expects Ω other buyers to sign, the buyer will recognize that (i) its signing will reduce the

¹⁶There is another possibility to obtain K buyers: the incumbent could offer its contract to more than K buyers (say $\hat{K} > K$ buyers) and expect that only K buyers from among them will accept the offer. In this case, however, the incumbent would still have to offer $x > x^*(s, p)$ to the \hat{K} buyers in order to induce K buyers to sign.

¹⁷If the lump-sum compensation were less than $s(p - c)$, the buyers would have an incentive to form a coalition and jointly reject the incumbent's offer, thereby ensuring that the entrant would be able to enter profitably.

¹⁸The first line of the expression in (4) follows directly from the definition of $x_n(s, p)$. The second line uses the fact that $\alpha_{\Omega}(s) = 1$, for all s , and the third line uses the fact that $p_a - c = s(p - c)$ and $p - p_a = (1 - s)(p - c)$.

likelihood of entry from one to $\alpha_{\Omega+1}$, and (ii) this matters because if the entrant enters, the per-unit price of the uncommitted purchases in the spot market will be c , whereas if the entrant does not enter, these same units will only be available from the incumbent at the incumbent's per-unit price of p . The latter term is zero under ED because under ED, there are no uncommitted units.

We now characterize what can be said about the relation between $x^*(s, p)$ and $s(p - c)$.

Lemma 2 *Suppose the incumbent offers the contract $C = \{s, x, p\}$ to K buyers. Then, if*

- *the contract specifies exclusive dealing, $x^*(1, p) = p - c$;*
- *the contract specifies a market-share of $s < 1$ and $\alpha_K(s) = 1$, $x^*(s, p) = s(p - c)$;*
- *the contract specifies a market-share of $s < 1$ and $\alpha_K(s) < 1$, $x^*(s, p) > s(p - c)$.*

The characterization in Lemma 2 depends solely on whether the incumbent offers ED or MS, and if it offers MS, on whether the likelihood of entry would be reduced. Specifically, Lemma 2 implies that if the incumbent offers ED, or if the likelihood of entry would not be reduced, the incumbent will only have to compensate buyers by the amount $s(p - c)$ to induce them to sign,¹⁹ whereas if it offers MS and the likelihood of entry would be reduced, the incumbent will have to compensate buyers by more than $s(p - c)$ to induce them to sign. In the first instance, buyers have no uncommitted purchases and thus no stake in whether entry subsequently occurs. In the second instance, entry occurs with probability one, implying that any price $p > c$ represents a certain loss of $s(p - c)$ for these buyers. In the third instance, each buyer must be compensated not only for the expected loss in surplus on its committed purchases, but also for the expected loss in surplus that its signing causes for the $1 - s$ share of its purchases that are uncommitted.

3 Two externalities

We will focus on answering two main questions in this section: (i) when are ED and MS profitable, and (ii) how do they work. To answer these questions, it is useful to distinguish between the profit the incumbent can expect to earn from the K buyers with whom it has an agreement, and the profit it can expect to earn from the $N - K$ buyers with whom it does not have an agreement.

We know from Lemma 1 and the discussion thus far that if the incumbent is to induce K buyers to sign its contract, each must be offered a payment of at least $x \geq x^*(s, p)$. Since there

¹⁹The first result follows from the fact that $x_n(1, p) = p - c$ for all $n \leq \Omega + 1$ and $x_n(1, p) = \alpha_{n-1}(v - c) - (v - p)$ is decreasing in n for all $n \geq \Omega + 1$. The second result follows from the fact that $\alpha_n(s) \geq \alpha_K(s)$ for all $n \leq K$.

is no reason to offer any more than this amount, and since the incumbent's payoff is strictly decreasing in x , it follows that the incumbent will offer exactly $x = x^*(s, p)$ in any equilibrium. The incumbent's problem in Period 1 is thus to choose K , $s \in (0, 1]$, and $p \in [c, v]$ to maximize:

$$\Pi(K, s, p) = K\Pi^S(K, s, p) + (N - K)\Pi^U(K, s), \quad (5)$$

where

$$\Pi^S(K, s, p) = s(p - c) + (1 - \alpha_K(s))(1 - s)(p - c) - x^*(s, p),$$

$$\Pi^U(K, s) = (1 - \alpha_K(s))(v - c).$$

Here we see that the incumbent's expected profit is a weighted sum of its expected profit from each signed buyer, $\Pi^S(K, s, p)$, and its expected profit from each unsigned buyer, $\Pi^U(K, s)$, where the weights are N and $N - K$, respectively. Its actual profit, of course, will depend on whether the entrant is deterred. If it is not, the incumbent will earn $K(s(p - c) - x^*(s, p))$, which is what it obtains from the committed purchases of the signed buyers net of what it pays them. If it is (which occurs with probability $1 - \alpha_K(s)$), the incumbent will earn the aforementioned amount plus a further $(1 - s)(p - c)$ from each signed buyer and $v - c$ from each unsigned buyer.

Consider first the case of ED. Previous literature has shown that ED can be profitable for the incumbent because of the *inter-group externality* that signed buyers impose on unsigned buyers. This externality can also be seen in our setting by substituting $s = 1$ and $x^*(1, p) = p - c$ (which comes from Lemma 2) into the expressions for Π^S and Π^U above and noting that while the incumbent's payoff from each signed buyer is always zero, its expected payoff from each unsigned buyer is strictly positive for all $K < N$ such that $\alpha_K(1) < 1$. It is thus straightforward to see from this that ED will be profitable if and only if there exists a $K < N$ such that $\alpha_K(1) < 1$.

Now consider the case of MS. The inter-group externality, which is so crucial for the profitability of ED, is also present, of course, under MS (when there is a $K < N$ and $s < 1$ such that $\alpha_K(s) < 1$), but things are more nuanced under MS because there is also an *intra-group externality* which the incumbent can exploit. This intra-group externality can be seen most easily by supposing for now that the distribution of f is such that the maximum payment needed to ensure that all K buyers sign is given by $x^*(s, p) = x_{\Omega+1}(s, p)$.²⁰ In this case, PCPNE requires the incumbent to compensate each buyer as if the entrant would enter with probability one if the buyer did not sign. This is a stringent requirement because it means that the incumbent must

²⁰It is straightforward to show that a sufficient condition for $x_{\Omega+1}(s, p)$ to be the maximum payment among all x_n is that signing up the $\Omega+1$ 'st buyer results in the largest drop in the probability of entry (i.e., $\alpha_{\Omega} - \alpha_{\Omega+1} \geq \alpha_{n-1} - \alpha_n$ for all $n \leq K$). We show in Appendix B that this condition holds, for instance, when $G(\cdot)$ is weakly convex.

compensate each buyer not only for the loss of surplus the buyer will incur on its *committed* purchases when it signs, which is equal to $s(p - c)$, but also for the loss of surplus that the buyer's signing causes for the $1 - s$ share of its purchases that are uncommitted, which is equal to $(1 - \alpha_{\Omega+1})(1 - s)(p - c)$, for a total payment of $x^*(s, p) = s(p - c) + (1 - \alpha_{\Omega+1})(1 - s)(p - c)$.

Substituting this into the expression for Π^S above, and canceling common terms, we obtain

$$\Pi^S(K, s, p) = (\alpha_{\Omega+1}(s) - \alpha_K(s))(1 - s)(p - c). \quad (6)$$

It follows that the incumbent will earn strictly positive expected profit from the signed buyers for all $p > c$ and $s < 1$, such that $\alpha_{\Omega+1}(s) > \alpha_K(s)$. When this condition holds, each signing buyer imposes a negative externality not only on every unsigned buyer, but also on every other signing buyer, and this is so despite the fairly steep cost required to induce each buyer to sign.

As we did for ED, we now characterize when MS will be profitable. Although this is relatively straightforward to do, the implications may nevertheless be surprising. Despite the different mechanisms at work (two externalities with MS versus one externality with ED), it turns out that there exist profitable contracts with MS *if and only if* there exist profitable contracts with ED. The reason for this, loosely speaking, is because the intra-group externality, which arises only under MS, is not operative unless the inter-group externality is also operative, and the conditions for the inter-group externality to be operative are the same for both ED and MS.

To see this, note that if there exists some $K < N$ such that $\alpha_K(1) < 1$, then by continuity, it must be that $\alpha_K(s) < 1$ for some s sufficiently close to 1. And if there does not exist some $K < N$ such that $\alpha_K(1) < 1$, then there will also not exist some $K < N$ and $s < 1$ such that $\alpha_K(s) < 1$. It follows that the necessary and sufficient condition for the inter-group externality to hold is thus the same for both ED and MS. Moreover, to see that the intra-group externality cannot arise independently of the inter-group externality, note that if $\alpha_K(s) = 1$ for all $K < N$, then $x^*(s, p) = s(p - c)$ from Lemma 2, and the intra-group externality vanishes (the second term in the expression for Π^S vanishes, and the first plus the third terms cancel each other out).

It remains only to state the condition in terms of the primitives on the distribution of entry costs. This too is straightforward to do. The following proposition summarizes our results.

Proposition 1 *There exist profitable exclusionary contracts if and only if no one buyer is sufficient to support entry with probability one (i.e., if and only if $f > \delta$ with positive probability). The profitability of ED can be attributed to the inter-group externality that the signed buyers impose on the unsigned buyers. The profitability of MS can be attributed not only to this inter-group externality but also to the intra-group externality that the signed buyers impose on each other.*

Proposition 1 lays the groundwork for what follows. It implies that exclusion will be profitable for the incumbent if and only if no one buyer is sufficient to support entry with probability one. The sufficiency of this condition should be clear. The reason for the “only if” in the proposition is that if one buyer alone were sufficient to support entry, there would be no inter- or intra-group externalities for the incumbent to exploit. There would be no inter-group externalities because the incumbent would have to sign up all buyers. There would be no intra-group externalities because each signing buyer would have to be fully compensated for its expected loss. Any gain from the exclusion would thus be offset by an equal lump-sum cost to induce each buyer to sign.

Proposition 1 also implies that contracts with MS work somewhat differently from contracts with ED in the sense that MS imposes two types of externalities on buyers, whereas ED imposes only one. Alternatively, one can think of there being only one type of externality, the externality that *committed purchases* impose on *uncommitted purchases*. The difference between ED and MS would then be that under ED, the buyers who sign have no uncommitted purchases, whereas under MS, the buyers who sign have both committed purchases and uncommitted purchases.

Although the inter-group externality that signed buyers impose on unsigned buyers is well known from SW, the intra-group externality that signed buyers impose on each other is not. Its existence may even invite skepticism because it is difficult to understand how the incumbent can induce buyers who are able to coordinate to sign and yet still be able to profit from them. The insight that emerges, however, is that the incumbent only has to compensate each signing buyer for the loss that its own signing has on its own expected surplus. This leads to a prisoners’ dilemma. Even when all other buyers are expected to reject the incumbent’s offer, any one buyer can be induced to accept the incumbent’s offer as long as it is compensated not only for the expected loss on its committed purchases but also for the reduction in the probability of entry that its signing has on the expected surplus of its uncommitted purchases.²¹ The gain to the incumbent from these same purchases, however, is larger because of the larger reduction in the probability of entry that occurs when *all* K buyers sign (not just the marginal buyer). There is thus potentially an additional source of profit arising under MS that is not present under ED.

This additional source of profit has strong implications for the relative profitability of ED and MS. In its absence, if the only source of profit was from the unsigned buyers, ED would always dominate MS because $\Pi^U(K, 1) \geq \Pi^U(K, s)$ for all $K \leq N$ and all $s \leq 1$. With it, however, the profit the incumbent receives under MS from the signed buyers may be more than enough to

²¹In our example with $x^*(s, p) = x_{\Omega+1}$, this amount is equal to $(1 - \alpha_{\Omega+1}(s))(1 - s)(p - c)$. In contrast, the incumbent’s expected gain from each signed buyer’s uncommitted purchases is equal to $(1 - \alpha_K(s))(1 - s)(p - c)$.

offset MS's relative disadvantage on the unsigned buyers. It depends on how many signed buyers there are relative to the total number of buyers in the market. When the number of signed buyers is fixed at $K = 1$, we know that ED always weakly dominates MS because $\Pi^U(1, 1) \geq \Pi^U(1, s)$ for all $s \leq 1$ and there is no offsetting intra-group externality (because there needs to be at least two signed buyers for this externality to arise), whereas when the number of signed buyers is fixed at $K = N$, we know that MS always weakly dominates ED because there are no unsigned buyers in this case, and hence, the incumbent's profit under ED is therefore always zero. More generally, it is straightforward to show that there exists some $\hat{K} \in \{2, \dots, N\}$ such that MS weakly dominates ED if and only if the number of signed buyers is greater than or equal to \hat{K} . The reason is that as more and more buyers sign the incumbent's contract, more and more weight is placed on the intra-group externality relative to the inter-group externality, which favors MS.

The existence of the intra-group externality also has strong implications for the incumbent's optimal price p . It can explain, for instance, why this price might be expected to differ between ED and MS. Under ED, the incumbent's expected profit is independent of the particular price p that is specified in the contract because its profit comes solely from the unsigned buyers. Increases or decreases in p which would otherwise affect the signed buyers' surplus, and therefore the incumbent's profit, are simply offset one for one by corresponding increases or decreases in x . This is *not* the case under MS, however, where, because of the intra-group externality, increases in p do not have to be offset one for one by decreases in x . This can be seen formally by differentiating $\Pi^S(K, s, p)$ with respect to p and noting that it is strictly positive for all $p \leq v$.²² As a result, we would expect contract prices to be weakly higher under MS than under ED, all else being equal, and at the optimum, we would expect the incumbent to set $p = v$ under MS.

4 The fundamental trade-off

We now turn our attention to understanding the relative strengths and weaknesses of ED and MS. Our finding that it is optimal for the incumbent to set $p = v$ under ED and MS (it is strictly optimal under MS) is useful in this regard because it allows us to rewrite the incumbent's profit in (5), after grouping common terms, as the sum of the payoff the incumbent gets from the buyers' "uncommitted" purchases and the payoff it gets from the buyers' "committed" purchases:

$$\Pi(K, s, v) = (N - Ks)(1 - \alpha_K(s))(v - c) + K(s(v - c) - x^*(s, v)). \quad (7)$$

²²See Appendix D for the proof that the incumbent's profit under MS is increasing in p for all $p \leq v$.

The first term represents the incumbent's payoff from the buyers' $N - Ks$ uncommitted purchases (i.e., the units that are not contractually committed to it when the contracts are signed). It is weakly positive because the incumbent receives $v - c$ from each of these units with probability $(1 - \alpha_K(s))$. The second term in (7) represents the incumbent's payoff from the buyers' Ks committed units (i.e., the units that are foreclosed to the entrant when the contracts are signed). For these units, the incumbent pays $x^*(s, v)$ to each signed buyer and receives $s(v - c)$ in return. This term is weakly negative because Lemma 2 implies that $x^*(s, v) \geq s(v - c)$ for all $s \leq 1$.

It follows that (7) consists of a weakly positive term, which can be thought of as the incumbent's benefit from foreclosure, and a weakly negative term, which can be thought of as the incumbent's cost of foreclosure. Both terms depend on the total number of foreclosed units. Interestingly, however, only the second term depends on how the foreclosure is achieved (i.e., on whether ED or MS is offered). To see this, let $\theta \equiv Ks$ measure the total level of foreclosure and recall that $\alpha_K(s) = G((N - Ks)\delta)$. Then, the incumbent's profit in (7) can be re-written as:

$$\tilde{\Pi}(\theta, s) = \underbrace{(N - \theta)(1 - G((N - \theta)\delta))(v - c)}_{\text{benefit from foreclosure}} - \underbrace{\theta \frac{(x^*(s, v) - s(v - c))}{s}}_{\text{cost of foreclosure}}. \quad (8)$$

Maximizing $\tilde{\Pi}(\theta, s)$ is seemingly straightforward. The benefit from foreclosure, which depends only on θ , is maximized at $\theta = K^{ED}$, where $K^{ED} \equiv \arg \max (N - K)(1 - G((N - K)\delta))(v - c)$, while the cost of foreclosure, which depends on both θ and s , is minimized at $s = 1$ (in fact, it is zero under ED and positive under MS because we know from Lemma 2 that the compensation needed to induce buyers to give up their right to purchase from the entrant altogether is given by $x^* = v - c$, whereas the compensation needed to induce buyers to give up their right to purchase only *some* of their units from the entrant is given by $x^* > s(v - c)$ for all $s < 1$). It follows that if the incumbent could directly choose its desired foreclosure level θ , ED would always dominate.

The problem is that the incumbent cannot choose θ directly. Under ED, the incumbent can only choose the number of buyers K that will receive its contract. This creates a certain "lumpiness" to the maximization problem and may cause the incumbent to miss its mark, perhaps substantially. If, for instance, there are $N = 4$ buyers in the market and $K^{ED} = 2.4$ is the unique maximizer of $\tilde{\Pi}(\theta, 1)$, the incumbent would optimally want to foreclose 60% of the market. But because K must be a whole number, the incumbent would be forced instead to choose between signing either two buyers or three buyers (i.e., foreclosing either 50% or 75% of the market).

The situation is markedly different under MS. Under MS, the incumbent does not have to compromise on the level of foreclosure. It can achieve whatever level it wants merely by offering

its contract to more than the corresponding number of buyers and choosing its market-share requirements accordingly. And, it can typically achieve its desired level in more than one way. For example, to foreclose 60% of the market when there are $N = 4$ buyers, the incumbent can either offer MS to three buyers and set $s = .80$ or offer MS to all four buyers and set $s = .60$.

However, this embarrassment of riches comes at a cost. Unlike contracts with ED, contracts with MS do not minimize the cost of foreclosure. Under MS, the incumbent must over pay buyers for their committed units relative to what it would have had to pay them under ED in the sense that the average cost per unit is strictly higher under MS than under ED. Thus, whereas MS clearly does better than ED on the benefit side, it does strictly worse than ED on the cost side.

This highlights a fundamental trade-off between ED and MS, which we can express as follows:

Proposition 2 *ED and MS have different strengths and weaknesses. ED's strength is that it minimizes the cost of foreclosure. Its weakness is that it does not allow the incumbent to fine-tune the level. MS's strength is that it allows the incumbent to achieve whatever target level of foreclosure it wants. Its weakness is that the incumbent will have to over pay for the foreclosure on a per-unit basis. ED is thus better at minimizing costs. MS is better at maximizing benefits.*

Proposition 2 sheds light on aspects of ED and MS that have not been well understood. Policy makers tend to think of MS as a “poor man’s ED” in the sense that ED is thought to be the more effective of the two at deterring entry — but also more costly. It is thought to be the more effective of the two at deterring entry because the entrant is completely foreclosed from selling to the buyer under ED, whereas only $s < 1$ share of the buyer’s demand is foreclosed under MS. It is thought to be more costly because the buyer will almost surely require a higher compensation under ED than it will under MS.²³ According to this logic, if one observes a firm using MS for exclusionary purposes, then it must be that the firm believes that the relatively lower cost of exclusion under MS more than makes up for the relatively lower effectiveness.

We suggest that this reasoning is backwards. The fallacy in the logic is that it is not the cost per buyer, or how effective the foreclosure is per buyer, that matters. Rather, what is relevant for the incumbent’s purposes is the overall effectiveness of the foreclosure and how much it will cost. Proposition 2 implies that, relative to ED, foreclosed units will be purchased at a higher average cost per unit under MS (i.e., at a premium, not at a discount). This suggests that the reason we would expect to observe firms choosing MS over ED for exclusionary purposes, when

²³This can be verified in our setting by noting that under ED, the incumbent must compensate each buyer by the amount $p - c$, whereas under MS, $x^*(s, p) = s(p - c) + (1 - \alpha_{Q+1})(1 - s)(p - c)$, which is strictly less than $p - c$.

both are feasible, is because the benefits gross of costs are higher under MS than under ED. Unlike with ED, MS allows the incumbent to fine tune the level of foreclosure it can achieve. Although buyers may not be divisible, share requirements are. This fact gives MS a significant advantage over ED, one that in some settings can more than offset its relative cost disadvantage.

4.1 Numerical examples

To fix ideas, and to illustrate the fundamental tradeoff, we now work through two examples. In the first example, MS dominates. In the second example, ED dominates. In both examples, there are three buyers, $v - c = 1000$, and $\delta = 100$. Only the distribution of entry costs is different.

Example 1: MS dominates ED

Suppose that f can take on one of five values, with the listed probabilities:

$$\begin{array}{l} f : \quad 50, \quad 100, \quad 150, \quad 225, \quad 275 \\ g : \quad 0.1, \quad 0.1, \quad 0.6, \quad 0.1, \quad 0.1. \end{array}$$

In the absence of ED and MS, the entrant can earn a flow profit of 300 if it enters. Since this exceeds the maximum value of f , we would expect the entrant to enter with probability one in the absence of any exclusionary contracts. On the other hand, if, for instance, the incumbent can foreclose 1.5 units, then only 1.5 units will be available to the entrant, in which case the entrant's profit will be reduced to 150 and the probability of entry will be reduced to $\Pr\{f < 150\} = 0.2$.²⁴

Consider ED first. By signing up one buyer, the incumbent can reduce the probability of entry to $\alpha_1(1) = 0.8$, and earn an expected profit of $(3 - 1)(1 - .8)1000 = 400$. Signing up two buyers, however, does even better. The probability of entry is reduced to $\alpha_2(1) = 0.1$, and the incumbent earns an expected profit of $(3 - 2)(1 - .1)1000 = 900$. Signing up all three buyers yields zero. The incumbent will therefore sign up two buyers and earn an expected profit of 900.

Notice that the incumbent could have done even better if it could have signed up 1.5 buyers because then the probability of entry would have been reduced 0.2, and its expected profit would have been $(3 - 1.5)(1 - .2)1000 = 1200$. But this is not possible under ED. With MS, on the other hand, achieving this level of foreclosure is feasible. The incumbent can, for example, offer $s = .75$ to two buyers and achieve the same reduction in the probability of entry, and thus the same maximum benefit from foreclosure. In order to induce the two buyers to accept MS with $s = .75$, however, the incumbent will have to pay each buyer an amount equal to

²⁴ Assume, for simplicity, that the entrant will not enter unless its profit exceeds its fixed cost of entry.

$(1 - \alpha_1(s))(1 - s)(v - c) = 50$. Nevertheless, on net, this still leaves the incumbent with a profit of $1200 - 100 = 1100$, which is more than 20% higher than what it can earn under ED.²⁵

Example 2: ED dominates MS

Now suppose that the distribution of f is modified as follows:

$$\begin{aligned} f &: 50, & 100, & 150, & 225, & 275 \\ g &: 0.1, & 0.25, & 0.45, & 0.1, & 0.1. \end{aligned}$$

Notice that this modification does not affect the incumbent's maximum profit under exclusive dealing. It is still 900. The modification also does not affect the over pay under MS when $s = .75$. It is still 100. However, it does change the probability of entry when 1.5 units are foreclosed. Instead of 0.2, it is now 0.35, which implies that the incumbent's maximum benefit from foreclosure is now equal to $(3 - 1.5)(1 - .35)1000 = 975$. The incumbent's maximum profit under MS is therefore only $975 - 100 = 875$ in this case, which is less than it can earn under ED.

The examples illustrate that whether ED or MS dominates depends on whether the gain from being able to realize the optimal foreclosure level under MS is enough to offset the loss from having to overpay for the buyers' committed units. In the first example, where the benefit from realizing the foreclosure level of 1.5 is relatively greater, MS dominates. In the second example, where the benefit from realizing the foreclosure level of 1.5 is relatively less, ED dominates.

5 More buyers and the dilution effect

We have focused on a small number of buyers in our examples because the ability of the buyers to coordinate with each other is seemingly more plausible when N is small. Nevertheless, one might think that this focus biases outcomes in favor of MS. More specifically, one might think that the constraint that arises from K having to be a whole number would be decreasing in importance as the number of buyers increases, and that as a result, increases in N would tend to favor ED over MS because there would then be less of a need for the incumbent to fine tune the level of foreclosure (which is MS's relative advantage). This reasoning turns out to be incorrect.

Consider first the intuition that the importance of the constraint will be decreasing as N increases. The logic behind it is that the incumbent is more likely to come close to achieving its target level of foreclosure under ED when N is large than when N is small. In our case of $N = 4$ buyers, for example, where $K^{ED} = 2.4$ implies that the incumbent would like to foreclose

²⁵It is easy to check that $x^*(.75, v) = x_1(.75, v)$ in this case, where $x_1(.75, v) = .75(v - c) + (1 - \alpha_1(.75)).25(v - c)$.

60% of the market, the constraint would seem to be quite severe because at $K = 2$ or $K = 3$, the incumbent would be forced to choose between foreclosing either 50% of the market (which is 10% less than it would ideally like to foreclose) or 75% of the market (which is 15% more than it would ideally like to foreclose), respectively. In contrast, to keep the foreclosure level at roughly 60% when the number of buyers is $N = 8$, the incumbent can offer its contract to $K = 5$ buyers, thereby realizing a foreclosure level of 62.5%. In this case, the gap between the desired and the obtainable foreclosure level is only 2.5%, which is significantly less than what it is when $N = 4$.

The problem with this logic, however, is that it presumes the percentage of the market the incumbent would like to foreclose is fixed. What matters to the incumbent, however, is not what percentage of the market to foreclose, but rather how many units to foreclose and thus how many units to leave uncommitted. To see this, note that the incumbent's profit under ED is given by

$$\begin{aligned}\tilde{\Pi}(\theta, 1) &= (N - \theta)(1 - G((N - \theta)\delta))(v - c) \\ &= (N - K)(1 - G((N - K)\delta))(v - c),\end{aligned}\tag{9}$$

which, for a given δ and $v - c$, depends only on $N - K$. It follows that maximizing $\tilde{\Pi}(\theta, 1)$ with respect to K is equivalent to maximizing it with respect to $N - K$, and thus, from the definition of K^{ED} , it is optimal for the incumbent to leave $N - K^{ED}$ units uncommitted. It also follows that this optimum does not change when N changes. If, for example, N were to increase by ΔN , the unconstrained optimum would simply increase from K^{ED} to $K^{ED} + \Delta N$ and the incumbent would realize the same profit as before. It cannot do better because if it could increase its profit by making a different number of offers, then K^{ED} would not have been optimal in the first place.

This implies that increasing N does not affect the incumbent's maximized profit under ED. It also implies that if \tilde{K} denotes the optimal number of signed buyers before the increase in N , then $\tilde{K} + \Delta N$ will be the optimal number of signed buyers after the increase in N , and thus $N - \tilde{K}$ and the incumbent's constrained maximized profit under ED will also be unaffected. The intuition for this is simple. Under ED, the incumbent only earns profit from the unsigned buyers. After K optimally adjusts to the increase in N , the number of unsigned buyers does not change, nor is there any change in the probability of entry given that the number of uncommitted units is also unchanged. It follows that the incumbent will not benefit (or lose) from an increase in N .

Proposition 3 *For a given δ , $v - c$, and distribution of entry costs, the incumbent's profit under ED depends only on the number of uncommitted units. Increases in N therefore have no effect on the incumbent's maximized profit under ED. As N increases, the incumbent simply adjusts the number of offers it makes in order to leave the number of uncommitted units unchanged.*

Although this increases the number of signed buyers, and thus the percentage of the market that is foreclosed to the entrant, it does not change the probability that the entrant will be deterred.

Several implications follow from Proposition 3. First, and most importantly, Proposition 3 implies that the constraint that arises from K having to be a whole number does *not* decrease in importance as the number of buyers increases, contrary to what one might have thought. To see this, suppose N increases from $N = 4$ to $N = 10$. If the flawed reasoning were correct (i.e., if the foreclosure percentage were fixed at 60%), the incumbent would offer ED to exactly six buyers and the integer constraint would no longer be binding. This is, however, clearly not optimal. Proposition 3 implies that the incumbent should instead optimally foreclose 8.4 units in order to keep the number of uncommitted units at 1.6 units. The gap between the desired foreclosure level and the obtainable foreclosure level is therefore exactly the same as before. That is, it is equal to 0.4 units if ED is offered to $K = 8$ buyers, or 0.6 units if ED is offered to $K = 9$ buyers.

Second, the fact that the gap between the incumbent's desired foreclosure level and its obtainable foreclosure level under ED is independent of N implies that there is just as much of a need to fine tune the level of foreclosure after an increase in N as there was before the increase. This implies that from the benefit side of foreclosure, increases in N favor neither ED nor MS.

Third, from a policy perspective, Proposition 3 implies that judging the relative harm from exclusion in any given case by focusing on the percentage of the market that is foreclosed, as courts are often inclined to do, is at best misleading.²⁶ In our stylized examples, the incumbent would ideally like to foreclose 60% of market when $N = 4$, leaving the entrant with only 1.6 uncommitted units, whereas, when $N = 8$, the incumbent would ideally like to foreclose 80% of market (because then $K^{ED} = 6.4$). Although the latter percentage is significantly higher than the former percentage, the number of uncommitted units is the same in the two cases, and therefore it follows that the entrant will neither be better or worse off in one case or the other.

5.1 Cost savings and the dilution effect

Our finding that increases in N need not effect the number of uncommitted units under ED also applies to MS. Under MS, the incumbent controls both K and s , and it can always adjust them in such a way as to keep the benefit from foreclosure the same. To see this, note that if $N - \hat{K}\hat{s}$ is the initial number of uncommitted units under MS before the increase in N , and N increases by ΔN , then the incumbent can always choose $K = \hat{K} + \Delta N$ and $s \equiv s^* = \frac{\hat{K}\hat{s} + \Delta N}{\hat{K} + \Delta N} > \hat{s}$ after the

²⁶Wright (2012) calls it “naive foreclosure analysis”, while recognizing that courts overwhelmingly do it.

increase to achieve the same number of uncommitted units as before.²⁷ Since the benefit from foreclosure under MS, as it was under ED, depends only on the number of uncommitted units, it follows that the benefit from foreclosure need not change under MS when N increases.

Under ED, this finding, along with showing that the incumbent cannot do better, was sufficient to establish that the incumbent's maximized profit under ED is independent of N . It is not the end of the story under MS, however, because an increase in N impacts MS differently from ED. In addition to the benefit from foreclosure, there is also the cost of foreclosure to consider. Under ED, this cost is always zero. Under MS, however, this cost depends on both K and s .

Continuing with our example, where $K = \hat{K}$, $s = \hat{s}$, and f is distributed such that $x^*(\hat{s}, v) = x_{\Omega+1}(\hat{s}, v)$, the incumbent's cost of foreclosure under MS before the increase in N is

$$\begin{aligned} & \hat{K} (x^*(\hat{s}, v) - s(v - c)) \\ &= \hat{K} (1 - \hat{s}) (1 - \alpha_{\Omega+1}(\hat{s})) (v - c). \end{aligned} \quad (10)$$

After the increase in N , and after K and s adjust to keep the benefit from foreclosure the same (i.e., $K = \hat{K} + \Delta N$ and $s = s^*$), the incumbent's cost of foreclosure under MS is

$$(\hat{K} + \Delta N) (1 - s^*) (1 - \hat{\alpha}_{\Omega+1+\Delta N}(s^*)) (v - c),$$

where $\hat{\alpha}_n(s) \equiv G((N + \Delta N - ns)\delta)$ denotes the probability of entry when n out of $N + \Delta N$ buyers sign the incumbent's contract. Substituting in for s^* , and rearranging terms, yields

$$\hat{K} (1 - \hat{s}) (1 - \hat{\alpha}_{\Omega+1+\Delta N}(s^*)) (v - c). \quad (11)$$

Comparing the cost of foreclosure in (11) with the cost of foreclosure in (10), we can see that the cost of foreclosure in (11) will be lower if and only if the probability of entry when the incumbent signs up the first effective buyer is higher after the increase in N than it was before. That is, the incumbent's cost of foreclosure will be lower in (11) than in (10) if and only if

$$\hat{\alpha}_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s}). \quad (12)$$

Fortunately, this relationship turns out to be relatively easy to establish because the same adjustments in K and s that keep the benefit from foreclosure the same imply that the actual probability of entry before and after the increase in N will be the same, and thus we know that

$$\hat{\alpha}_{\hat{K}+\Delta N}(s^*) = \alpha_{\hat{K}}(\hat{s}). \quad (13)$$

²⁷Note that $N + \Delta N - (\hat{K} + \Delta N) s^* = N + \Delta N - (\hat{K} \hat{s} + \Delta N) = N - \hat{K} \hat{s}$.

Going from the equality in (13) to establishing that the inequality in (12) holds then follows straightforwardly once it is recognized that after the increase in N , the difference between the actual number of uncommitted units available to the entrant and the number of uncommitted units that are available to the entrant after the first effective buyer is signed is greater than the corresponding difference before the increase in N . After the increase in N , the difference is $s^*(\hat{K} - (\Omega + 1))$ units, whereas before the increase, the difference is only $\hat{s}(\hat{K} - (\Omega + 1))$ units.

The intuition is that the incumbent must sign at least $\Omega + 1$ buyers before the increase in N if it is to have any effect on lowering the probability of entry, whereas after the increase in N , it must sign this many plus an additional ΔN more buyers if it is to have any effect. Each signing buyer's contribution to the initial reduction in the probability of entry is thereby effectively diluted when the number of buyers increases. We call this the *dilution effect*, and it implies that signing buyers do not have to be compensated as much for their contribution toward exclusion.

We can illustrate this effect with the help of our example in which $N = 4$ and $K^{ED} = 2.4$. Under MS, the incumbent can realize the optimal foreclosure level of 2.4 units by offering MS to three buyers with $\hat{s} = 0.8$. Signing the first buyer in this case reduces the number of uncommitted units to $4 - 0.8 = 3.2$ units. Suppose now that $N = 7$. Here, the incumbent can realize the optimal foreclosure level of 5.4 units by offering MS to six buyers with $s^* = 0.9$ (so that the number of uncommitted units remains at 1.6). Signing the fourth buyer reduces the number of uncommitted units to $7 - 4 \times 0.9 = 3.4$, which is greater than it was before the increase in N .

We have just shown that while an increase in the number of buyers need not affect the incumbent's benefit from foreclosure, it would be expected to reduce its cost of foreclosure. It follows that we would expect the incumbent to strictly gain from an increase in N under MS.

Proposition 4 *Because of the dilution effect, the incumbent need not compensate each signing buyer as much for its contribution toward exclusion after an increase in N as it did before the increase in N . The incumbent's maximized profit under MS is thus strictly increasing in N .*

Proposition 4 in conjunction with our earlier finding that the incumbent's maximized profit under ED is independent of N , implies that an increase in N expands the number of settings in which the incumbent will choose MS over ED. This is the exact opposite of what one might have expected, and it implies that an increase in N benefits MS relative to ED in the sense that (i) if initial conditions are such that MS is more profitable than ED, then MS will continue to be more profitable when the number of buyers increases; and (ii) if initial conditions are such that ED is more profitable than MS, then for a given increase in the number of buyers, the gap will

narrow, and it is possible that MS could even overtake ED ex-post and become more profitable.

As we have seen, one way to think about why increases in N favor MS over ED is that, for a given benefit from foreclosure, the cost of foreclosure under ED is independent of N , whereas the cost of foreclosure under MS is decreasing in N . However, another way to think about the relative effects of an increase in N is to note that while an increase in N leads to a corresponding increase in the number of signed buyers under both ED and MS, this increase does not help the incumbent under ED because full compensation must be offered to all signed buyers and therefore the incumbent cannot profit from them. But the increase in the number of signed buyers does help the incumbent under MS because of the intra-group externality that signed buyers impose on each other. In fact, one can show that this externality only gets stronger as N increases, which is what generates the cost savings and thus allows increases in N to favor MS.²⁸

5.2 Numerical example

We now construct an example in which ED dominates MS when there are only three buyers, but in which MS dominates ED when the number of buyers increases to five or more.

Example 3: MS dominates ED

Consider the same set-up as in Example 2, but suppose now that $N = 5$. In this case, the incumbent earns its maximized expected profit of 900 under ED by increasing the number of signed buyers from two to four. Under MS, the incumbent can achieve its desired foreclosure level of 3.5 units by offering $s^* = .875$ to four buyers. This reduces the actual probability of entry to $\hat{\alpha}_4(s^*) = .35$, but increases the probability of entry when the $\Omega + 1$ 'st buyer signs to²⁹

$$\hat{\alpha}_3(s^*) = G((5 - 3 \times .875) 100) = G(237.5) = 0.9.$$

The cost of foreclosure when N increases to $N = 5$ is thus reduced from 100 when $N = 3$ to

$$4(1 - s^*)(1 - \hat{\alpha}_3(s^*))(v - c) = 500 \times 0.1 = 50.$$

Thus, the profit under MS is now equal to $975 - 50 = 925$, which is higher than that under ED.

6 Characterizing optimal contracts

It remains to characterize the optimal ED and MS contracts and to compare foreclosure levels. To set the stage, recall that $\tilde{\Pi}(\theta, s)$ is maximized at $\theta = K^{ED}$ and $s = 1$. The problem that

²⁸The proof of this claim, as well as the proof of Proposition 4, is given in the Appendix.

²⁹In contrast, when $N = 3$, this probability was $\alpha_1(.75) = .8$. It is easy to check that $x_3(s^*) > x_4(s^*)$.

arises for the incumbent, however, is that it would have to offer its contract to $K = K^{ED}$ buyers to implement this solution, and K^{ED} in generic is not an integer. The first-best is therefore not generally attainable, and thus the incumbent will often have to resort to a second-best solution.

Figure 1 illustrates possible outcomes when the incumbent must settle for second-best. Depicted in Figure 1 are three iso-foreclosure loci. Each locus represents combinations of K and s that yield the same foreclosure level. The first-best solution occurs at Point A, which corresponds to the foreclosure level $\theta = K^{ED}$ and $s = 1$. If this point is not attainable, and the incumbent wants to continue with ED to minimize costs, it can opt for a point such as Point B, which has a foreclosure level of $\theta = \underline{K}^{ED}$, or for a point such as Point C, which has a foreclosure level of $\theta = \overline{K}^{ED}$, where \underline{K}^{ED} and \overline{K}^{ED} are the two closest integers to K^{ED} . Both points are clearly compromises, which, as we have seen, may result in substantial losses in profit relative to the first best (in our first example in Section 4.2, the loss accounts for 33% of the incumbent's expected profit under ED). Alternatively, the incumbent can switch to MS when Point A is not attainable and obtain any foreclosure level it wants, albeit at a relatively higher cost per-unit of foreclosure. The possibilities are endless. To achieve the same foreclosure level as in Point A, for example, the incumbent can offer MS to (i) $K = \overline{K}^{ED}$ buyers and set $s = \frac{K^{ED}}{\overline{K}^{ED}}$ (Point D), (ii) $K = \overline{K}^{ED} + 1$ buyers and set $s = \frac{K^{ED}}{(\overline{K}^{ED} + 1)}$ (Point E), or (iii) all N buyers and set $s = \frac{K^{ED}}{N}$ (Point F). And similarly, it can achieve other foreclosure levels as well (for example, it can implement Point G by offering MS to \overline{K}^{ED} buyers). Once again, however, depending on the distribution of f , the losses in profits that are associated with these departures from Point A can be very significant.

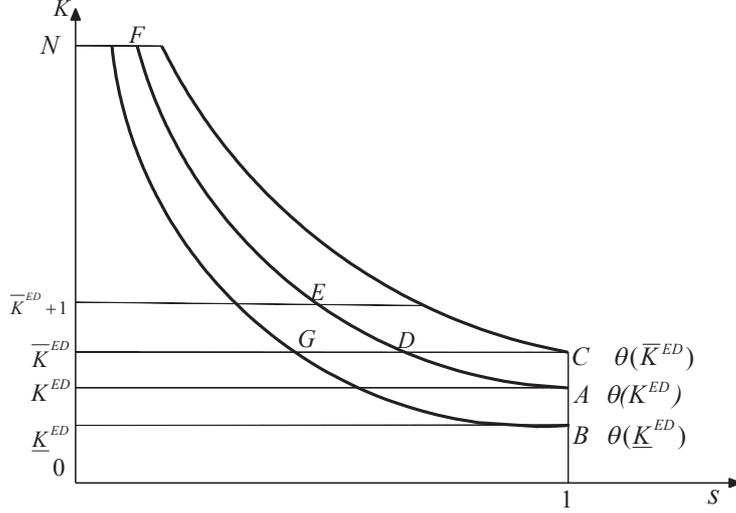


Figure 1

We have shown from our examples that whether the losses relative to the first best are higher under ED or MS depend on the distribution of f , and unfortunately, not much can be said in general. We know that for all levels of foreclosure that are attainable under ED, ED dominates MS (this is because for any given θ , ED minimizes the total cost of foreclosure). Thus, for example, we know that Point B dominates all other points on the iso-foreclosure curve with total foreclosure $\theta = \underline{K}^{ED}$, and we know that Point C dominates all other points on the iso-foreclosure curve with total foreclosure $\theta = \overline{K}^{ED}$. What are not as easy to compare are the MS combinations that lie on iso-foreclosure curves that are *unattainable* under ED. Thus, for example, one cannot easily compare the incumbent's profit under MS at point F to its profit under ED at points B and C. Even a comparison among points D, E, and F is not as straightforward as one might think because although all three points lie on the same iso-foreclosure curve, and thus yield the same level of foreclosure, one cannot say in general which will be associated with the lowest cost.

To gain further insights, therefore, we now make the following two-part assumption on $G(\cdot)$.

Assumption 1 *The distribution function $G(\cdot)$ is (i) differentiable, and (ii) either everywhere weakly convex, or everywhere weakly concave, such that $zG''(z) > -2G'(z)$ for all $z \in [0, \bar{N}\delta]$.*

Assumption 1 is useful because it has important implications for the benefit and cost of foreclosure, as given in the profit expression in (8). We summarize these implications as follows:

Lemma 3 *When the distribution function $G(\cdot)$ is such that Assumption 1 holds,*

- *the benefit from foreclosure, $\tilde{\Pi}(\theta, 1)$, is concave in θ ;*
- *for any given θ , the cost of foreclosure is decreasing in s .*

Using the result that $\tilde{\Pi}(\theta, 1)$ is concave in θ , and that the cost of foreclosure is uniquely minimized at $s = 1$, it follows from the definition of K^{ED} as the argmax of $\tilde{\Pi}(\theta, 1)$ that

$$\tilde{\Pi}(\underline{K}^{ED}, 1) > \tilde{\Pi}(\theta, 1) > \tilde{\Pi}(\theta, s), \quad \text{for all } \theta < \underline{K}^{ED} \text{ and } s < 1,$$

and

$$\tilde{\Pi}(\overline{K}^{ED}, 1) > \tilde{\Pi}(\theta, 1) > \tilde{\Pi}(\theta, s), \quad \text{for all } \theta > \overline{K}^{ED} \text{ and } s < 1.$$

These conditions in turn imply that the optimal foreclosure level under ED will either be at $\theta = \underline{K}^{ED}$ (the integer just below K^{ED}) or $\theta = \overline{K}^{ED}$ (the integer just above K^{ED}), as any other foreclosure level will be farther away from the unconstrained maximum at $\theta = K^{ED}$. They also imply that in any setting in which MS is more profitable than ED, the optimal foreclosure level under MS must be such that $\underline{K}^{ED} < \theta < \overline{K}^{ED}$ (otherwise, ED would do better on both maximizing the benefits and minimizing the costs). In terms of Figure 1, it follows that the optimal foreclosure level under ED will either be at Point B or Point C. And it follows that the optimal foreclosure level under MS, whenever MS is more profitable than ED, will be above the iso-foreclosure locus ending in Point B and below the iso-foreclosure locus ending in Point C.

Using the result that the benefit from foreclosure is independent of s , and that, for any given θ , the cost of foreclosure is decreasing in s , it follows that, for any given θ , an increase in s (equivalently, a decrease in K) will increase the incumbent's profit. This means that the incumbent's profit will be increasing from left to right along the iso-foreclosure loci in Figure 1 (Points E and F will be dominated by Point D). It also means that since $\theta \in (\underline{K}^{ED}, \overline{K}^{ED})$ when MS is more profitable than ED, the incumbent's profit will be maximized when MS is given to \overline{K}^{ED} buyers. If MS is given to fewer buyers, then $\theta \in (\underline{K}^{ED}, \overline{K}^{ED})$ cannot be achieved. If MS is given to more buyers, then $\theta \in (\underline{K}^{ED}, \overline{K}^{ED})$ can be achieved, but not in the least cost way.

It remains to show that when the conditions in Assumption 1 hold, settings exist in which ED dominates MS, and settings exist in which MS dominates ED. Such examples are easy to construct, as we show in the Appendix, for the case of three buyers and assuming $v - c = 1000$.

Summarizing the discussion in this section thus leads to the following proposition:

Proposition 5 *When Assumption 1 holds, settings exist in which ED and MS can dominate. When ED is more profitable than MS, the optimal exclusive contract is such that $s = 1$ and the contract is offered to either \underline{K}^{ED} or \overline{K}^{ED} buyers. When MS is more profitable than ED, the optimal exclusive contract is such that $s \in \left(\frac{\underline{K}^{ED}}{\overline{K}^{ED}}, 1 \right)$ and the contract is offered to exactly \overline{K}^{ED} buyers. In the former case, the optimal foreclosure level is either $\theta = \underline{K}^{ED}$ or $\theta = \overline{K}^{ED}$. In the latter case, the optimal foreclosure level lies in the interval between $\theta = \underline{K}^{ED}$ and $\theta = \overline{K}^{ED}$. The profit-maximizing foreclosure level under MS can thus sometimes be higher than under ED.*

In characterizing the optimal contracts under both ED and MS, we are now able to address many outstanding questions. One of the most basic questions is how should the incumbent use MS to achieve its desired foreclosure level when there is more than one way to do it? For example, in foreclosing 60% of a four-buyer market, is it better for the incumbent to offer $s = .80$ to three buyers or $s = .60$ to four buyers? And similarly, in foreclosing 70% of an eight-buyer market, is it better for the incumbent to offer $s = 93.33$ to six buyers, $s = .80$ to seven buyers, or $s = 70$ to eight buyers, etc.? Does it even matter for the incumbent's profits? The answer, according to Proposition 5 is that it does matter, for cost-minimizing reasons, and that the least costly way for the incumbent to do it is to offer MS to the minimum number of buyers. It follows that in the first example, with four buyers, the incumbent should offer $s = .80$ to three buyers, and in the second example, with eight buyers, the incumbent should offer $s = 93.33$ to six buyers.

We can also get a sense from Proposition 5 as to how large one would expect the share requirements to be when MS is offered. The prescription that MS should be offered to the minimum number of buyers necessary to achieve a given foreclosure level suggests that s will be on the high side, and this is made more precise by noting that the optimal s is indeed such that

$$s > \frac{\underline{K}^{ED}}{\overline{K}^{ED}} = \frac{\underline{K}^{ED}}{\underline{K}^{ED} + 1}.$$

This provides a lower bound on s , and it implies, for example, that $s > 1/2$ for all $\underline{K}^{ED} \geq 1$ and $s > 3/4$ for all $\underline{K}^{ED} \geq 3$. In Examples 1 and 2 in the text, $s = .75$. In Example 3, $s = .875$.³⁰

Notice finally that the optimal number of signed buyers under MS is weakly higher than the optimal number of signed buyers under ED. This is an important finding because if it were not true, ED would always give rise to higher foreclosure levels and thus would always be worse for welfare. As it turns out, however, Proposition 5 implies that MS can sometimes be worse. In

³⁰This accords with minimum-share requirements that one observes in practice. For example, in the Intel case, *NEC* was required to purchase no less than 80% of its CPU needs from Intel, and *HP* was required to purchase no less than 95% of its needs from Intel. See the European Commission's Decision C(2009) 3726, (May 13, 2009).

particular, it shows that simple rules of thumb such as “ED is likely to be more powerful than MS because it forecloses a greater fraction of the market to the entrant” are not only misleading, but can often be wrong. When MS is more profitable than ED, the optimal exclusive contract is given to \overline{K}^{ED} buyers, and the optimal foreclosure level is strictly greater than \underline{K}^{ED} but less than \overline{K}^{ED} . By contrast, under ED, the optimal exclusive contract is given to either \underline{K}^{ED} or \overline{K}^{ED} buyers, with corresponding foreclosure levels of \underline{K}^{ED} and \overline{K}^{ED} , respectively. MS thus results in a higher foreclosure level relative to the former case, and a lower foreclosure level relative to the latter case. This suggests that welfare under MS can indeed be worse than welfare under ED.

6.1 Numerical example

We conclude with a simple example to show that, depending on the distribution of entry costs, MS can indeed be more profitable than ED and result in a higher foreclosure level. Consider the same set-up as in Examples 1 and 2, except now suppose that the distribution of f is given by

f	50	100	150	225	275
g	0.15,	0.05,	0.35,	0.25,	0.20.

It is easy to check that in the absence of ED and MS, the entrant can earn a profit of 300 if it enters. It is also easy to check that the optimal foreclosure level is $K^{ED} = 1.5$ units, the same as before, and that the incumbent’s profit in this case is $\Pi^{ED}(K^{ED}) = (3 - 1.5)(1 - .2)1000 = 1200$.

When the incumbent signs up one buyer under ED, it can reduce the probability of entry to entry to $\alpha_1(1) = 0.55$ and earn an expected profit of $(3 - 1)(1 - .55)1000 = 900$. When the incumbent signs up two buyers, it can reduce the probability of entry to $\alpha_2(1) = 0.15$ and earn an expected profit of $(3 - 2)(1 - .15)1000 = 850$. Signing up three buyers yields zero. The best the incumbent can do under ED is thus to sign up one buyer ($K = \underline{K}^{ED}$) and foreclose one unit.

If instead the incumbent offers MS to two buyers with $s = 0.75$, it can foreclose exactly 1.5 units and achieve the same reduction in the probability of entry as if it could set $K = K^{ED}$ directly. It can therefore achieve the same maximum benefit from foreclosure as before, 1200. Since the cost of doing this is only $2(1 - .75)(1 - \alpha_1(.75)(v - c)) = 500 \times (1 - .55) = 225$, it follows that incumbent’s maximized profit under MS exceeds its maximized profit under ED.

7 Conclusion

This paper uses the naked-exclusion framework of RRW-SW to study the optimal design of exclusionary contracts for an incumbent seller when both market-share contracts (MS) and exclusive-

dealing contracts (ED) are feasible. It is well known in this framework — which has hitherto focused on ED — that when buyers can coordinate their acceptance decisions, offering ED to every buyer is prohibitively costly, and a divide-and-conquer strategy in which ED is offered to some buyers but not to others must be used instead. We take this as a starting point of our analysis and ask whether in this setting of partial exclusion the incumbent can do even better. In particular, we ask whether it is better for the incumbent to require some of the buyers to buy all their purchases from the incumbent, or some or all of the buyers to buy only a fraction of their purchases from the incumbent. Consistent with observing both types of contracts in practice, we show that sometimes the former is better, and sometimes the latter is better, as each has its strengths and weaknesses. Contrary to conventional wisdom, we find that MS can be more effective than ED in the sense that the incumbent’s benefit from foreclosure will be weakly higher under the former. However, and also contrary to what one might have thought, the cost of compensation will be higher with MS than with ED in the sense that on a per-unit basis, the incumbent will have to pay buyers more under MS to achieve the same level of foreclosure as under ED. The fundamental tradeoff then is that ED is better at minimizing the incumbent’s costs of foreclosure, whereas MS is better at maximizing the incumbent’s benefit from foreclosure.

The model thus offers strong implications for how the incumbent should exclude, even among contracts that might otherwise look similar. In our motivating example in the introduction with five buyers, we found that the seller could achieve the assumed optimal level of foreclosure of 60% in three ways: offer ED to three of the five buyers; offer MS to four of the five buyers and set the minimum-share requirement at 75%; or offer MS to all five buyers and set the minimum-share requirement at 60%. Our findings suggest that the first of these will be the most profitable, as it is the least costly way of doing so. Now consider the example with four buyers. In this case, depending on how the entrant’s fixed costs are distributed, we know there are settings in which ED is more profitable and settings in which MS is more profitable. Whether or not MS dominates ED, however, we can unambiguously say that offering MS to three buyers and setting the minimum-share requirement at 80% will be more profitable for the seller than offering MS to all four buyers and setting the minimum-share requirement at 60%. The reason is that although the same foreclosure level will be realized, the latter will cost the seller more in compensation.

We have also shown that the profitability of exclusionary contracts comes from two sources. There is an inter-group externality that signed buyers impose on unsigned buyers, and an intra-group externality that signed buyers impose on each other. Whereas the profitability of ED arises solely from the former, the profitability of MS stems from both types of externalities. This

difference in how the contracts work gives rise to some useful comparative-static implications. Because the incumbent does not earn profit from the signed buyers under ED, the per-unit price p given to these buyers plays no role. Increases or decreases in p are simply offset by corresponding increases or decreases in lump-sum compensation. However, under MS, the incumbent does care, and in general prefers that p be as high as possible. This suggests that MS would be expected to have relatively higher per-unit prices all else equal. Another comparative-static implication relates to the effect of an increase in the number of buyers on the incumbent's profitability. Under ED, there is no effect. Increases in the number of buyers are offset by increases in the number of signed buyers, with no change in profit (albeit there is a change in the share of the market that is foreclosed). In contrast, increases in the number of buyers increase the profitability of MS due to the dilution effect. With more signed buyers, the amount that has to be paid to any one buyer for the reduction it causes in the probability of entry decreases, and hence, an increase in the number of buyers reduces the amount by which each buyer has to be overcompensated.

We believe that our findings have important implications for how public policy should view MS. Our experience has overwhelming been that policy makers focus on the adverse effects of foreclosure on a buyer per-buyer basis, giving rise to the view that when MS is observed, it will not be as effective, or as powerful, at foreclosing an entrant as ED would have been (hence, the perceived need by plaintiffs to argue that the minimum-share required by the incumbent in their given setting was so large that it was effectively the same as exclusive dealing). We have spent much of the paper arguing that that this view is misleading and ignores the endogeneity of the incumbent's decisions. We have shown that what matters for the incumbent's decisions are the overall level of foreclosure (which determines the likelihood of entry) relative to the overall cost of foreclosure (i.e., the cost of compensating the buyers to accept the seller's exclusionary terms), and that what when viewed through this lens, market-share contracts are not the poor-man's exclusive dealing that they have been made out to be. Moreover, we have shown that because the incumbent will sign up weakly more buyers under MS than would have been the case under ED, overall levels of foreclosure may well be higher under MS than under ED. This suggests that MS, not ED, may well be the more powerful of the two at inducing exclusion in certain settings.

Lastly, our analysis suggests that while much can be learned from examining in any given setting how much of the downstream market is foreclosed, and comparing it to what the counterfactual would have been in the absence of the alleged exclusionary conduct, it does not support the use of simple market-share screens as a definitive indicator of likely harm. As we have shown, a growth in the overall foreclosure level in a given market over time may simply reflect a growth

in the number of buyers (size of the market) independent of the effect on a potential entrant.

We conclude by highlighting two assumptions that are critical for our results. Our assumption that the incumbent can commit to a per-unit price in its contract is important because if this were not the case, the incumbent would be able to extract all the surplus *ex post* from its signed buyers, implying that these buyers would have to be given full compensation. Similarly, our assumption that entry costs are stochastic is also essential for our results. If the entrant's fixed costs were known *ex ante*, there would always exist a coalition k of buyers who could induce entry by jointly rejecting the incumbent's offer. These buyers would therefore also have to be given full compensation for signing up. In both cases, the intra-group externality that we have identified as being an important driver of MS would not arise and thus ED would always dominate MS.

Appendix

A: Proof of Lemma 1

(1): Suppose the incumbent offers the contract to K buyers with $x > x^*(s, p)$. Notice that $x^*(s, p) \equiv \max_{n \leq K} \{x_n(s, p)\}$ is the maximum inducement among all x_n for all $n \leq K$. Then, for all $n \in \{1, 2, \dots, K\}$, $x > x^*(s, p)$ implies $U_A(n) > U_R(n - 1)$. To show that all K buyers accept the incumbent's contract in the unique PCPNE of the continuation game, we first show that all buyers accepting the incumbent's contract forms a PCPNE. Suppose all buyers accept the incumbent's contract. Then, $U_A(K) > U_R(K - 1)$ implies that a unilateral deviation is not profitable. Suppose now a coalition of $k \geq 2$ buyers deviates and jointly rejects the incumbent's contract. Then each buyer in the coalition earns payoff $U_R(K - k)$. However, this joint deviation is not self-enforcing given that a buyer in this coalition can make itself better off by deviating from the coalitional action and accepting the incumbent's contract, since, in this case, $U_A(n) > U_R(n - 1)$ for any $n \in \{1, 2, \dots, K\}$ implies $U_A(K - k + 1) > U_R(K - k)$. Therefore, in this case, there is no self-enforcing coalitional deviation that can benefit buyers.

Secondly, we must show that no other PCPNE of the continuation game exists. Suppose to the contrary that there exists a PCPNE where k buyers, $k \neq K$, accept the contract while the other $K - k$ buyers reject the contract. Then each buyer in the coalition $K - k$ who jointly rejects the contract can benefit from a unilateral deviation to accept the contract; doing so will bring more surplus, $U_A(k + 1) > U_R(k)$, and therefore this cannot form an equilibrium.

(2): Suppose the incumbent offers a contract to K buyers with $x < x^*(s, p)$, and K buyers accept the contract. We show this cannot form a PCPNE. Let $x_m(s, p) = x^*(s, p)$, for some $1 \leq m \leq K$, then $x < x_m(s, p)$ implies that $U_A(m) < U_R(m - 1)$. Since $U_A(K) \leq U_A(K - 1) \leq \dots \leq U_A(m) < U_R(m - 1)$, this implies that $K - m + 1$ buyers could jointly deviate by rejecting the contract offer and make themselves better (each buyer now earns $U_R(m - 1)$). To show that such coalitional deviation is self-enforcing, first note that unilateral deviation from this coalition is not profitable since $U_R(m - 1) > U_A(m)$. Suppose now there exist $j \geq 2$ buyers in this coalition that want to deviate and jointly accept the incumbent's offer, by which each member can earn payoff $U_A(m - 1 + j)$. However, this joint deviation from the coalition is not profitable since $U_A(m - 1 + j) \leq U_A(m) < U_R(m - 1)$.

(3): Suppose the incumbent offers a contract to K buyers with $x = x^*(s, p) = x_m(s, p)$, and K buyers accept the contract, then each signed buyer gets $U_A(K)$. Consider two cases:

Case A: $x = x^*(s, p) = x_m(s, p)$ and $p > c$. As $x = x^*(s, p) \geq x_K(s, p)$ implies $U_A(K) \geq U_R(K - 1)$, the unilateral deviation is not profitable. Suppose now a subset of k buyers jointly

reject the offer, in which each deviated buyer obtains $U_R(K - k)$. But $x = x^*(s, p) \geq x_{K-k+1}$ implies $U_R(K - k) \leq U_A(K - k + 1)$, and thus such coalition deviation is not self-enforcing.

Case B: $x = x^*(s, c) = 0$ and $p = c$. Then $p_a = p = c$ implies $U_A(n) = \alpha_n(v - p_a) + (1 - \alpha_n)(v - p) + x = v - c$ for all $n \leq K$, that is, each buyer is secured a payoff $v - c$ in this case. On the other hand, $U_R(n) = \alpha_n(v - c) \leq v - c$. To show that K buyers accepting the incumbent's contract forms a PCPNE, note that $U_A(K) \geq U_R(K - 1)$ implies that a unilateral deviation is not profitable. Moreover, there is no coalitional deviation of $k \geq 2$ buyers that is profitable because $U_R(0) = U_A(n) = v - c$ for all n , and $U_A(n) \geq U_R(n - 1)$ implies $U_A(K) \geq U_R(K - k)$.

Q.E.D.

B. Proof of Lemma 2 and Characterization of $x^*(s, p)$

Suppose the incumbent offers the contract $C = \{s, x, p\}$ to K buyers. Consider three cases as follows:

(1) If the contract specifies $s = 1$, then, $p_a = p$, and

$$x_n(1, p) = \alpha_{n-1}(v - c) - (v - p).$$

It follows that $x_n(1, p)$ decreases with the number of signed buyers n and thus $x^*(1, p) = x_{\Omega+1}(1, p) = p - c$ (note that $\alpha_{\Omega} = 1$).

(2) If the contract specifies $s < 1$, but signing up K buyers does not reduce the probability of entry, i.e., $\alpha_K(s) = 1$, then $\alpha_n(s) = 1$ for all $n \leq K$. This implies $x_n(s, p) = p_a - c = s(p - c)$, for all $n \leq K$, and thus $x^*(s, p) = s(p - c)$.

(3) If the contract specifies $s < 1$, and signing up K buyers reduces effectively the probability of entry, i.e., $\alpha_K(s) < 1$. There exists a cut-off number \hat{n} with $\Omega < \hat{n} < K$ such that $\alpha_n(s) = 1$ for all $n \leq \hat{n}$ and $\alpha_n(s) < 1$ for all $n > \hat{n}$. Then

$$\begin{aligned} x^*(s, p) &\geq x_{\hat{n}+1}(s, p) = s(p - c) + (1 - \alpha_{\hat{n}+1}(s))(1 - s)(p - c) - (1 - \alpha_{\hat{n}}(s))(v - c) \\ &= s(p - c) + (1 - \alpha_{\hat{n}+1}(s))(1 - s)(p - c) > s(p - c), \end{aligned}$$

where we used the fact that $\alpha_{\hat{n}}(s) = 1$ and $\alpha_{\hat{n}+1}(s) < 1$.

We now characterize the sufficient conditions for $x^*(s, p) = x_{\Omega+1}(s, p)$. Using equation (2) for $x_n(s, p)$, we obtain

$$x_{\Omega+1}(s, p) - x_n(s, p) = (\alpha_{\Omega}(s) - \alpha_{n-1}(s))(v - c) - (\alpha_{\Omega+1}(s) - \alpha_n(s))(p - p_a).$$

Consider two cases:

(i) If $s = 1$, $p = p_a$, then $x_{\Omega+1}(s, p) \geq x_n(s, p)$ for all $n \geq \Omega + 1$, thus $x^*(1, p) = x_{\Omega+1}(1, p)$ under ED.

(ii) If $s < 1$ and $\alpha_{\Omega}(s) - \alpha_{\Omega+1}(s) \geq \alpha_{n-1}(s) - \alpha_n(s)$ for all $n \leq K$, then $\alpha_{\Omega}(s) - \alpha_{n-1}(s) \geq \alpha_{\Omega+1}(s) - \alpha_n(s)$ and $v - c \geq p - p_a$ imply $x_{\Omega+1}(s, p) \geq x_n(s, p)$ for all $n \leq K$. Note that this condition holds when $G(\cdot)$ is weakly convex, as $\alpha_{\Omega}(s) - \alpha_{\Omega+1}(s) \geq \alpha_{\Omega+1}(s) - \alpha_{\Omega+2}(s) \geq \dots \geq \alpha_n(s) - \alpha_{n+1}(s)$ for all n .

Thus, $x^*(s, p) = x_{\Omega+1}(s, p)$ if ED is offered, or if $G(\cdot)$ is weakly convex under MS. **Q.E.D.**

C. Proof of Proposition 1

Suppose the incumbent offers an exclusionary contract $C = \{s, x, p\}$ to K buyers and all K buyers have accepted the contract in the PCPNE of the continuation game. We show first that $f > \delta$ with positive probability is a necessary condition for profitable exclusionary contracts. Suppose instead that $f \leq \delta$ with probability one. Then, the entrant's profit when facing only one free buyer is sufficiently high to cover the entry cost, that is, $\Pi_E(N-1, s) = (N-1)(1-s)\delta + \delta \geq \delta$, and the entrant will enter with probability one as long as there is one unsigned buyer. This implies that, for all $K < N$ and $s \leq 1$, $\alpha_K(s) = 1$, and by Lemma 2 $x^*(s, p) = s(p - c)$, thus

$$\Pi^S(K, s, p) = s(p - c) + (1 - \alpha_K(s))(1 - s)(p - c) - x^*(s, p) = 0.$$

Hence, offering exclusionary contracts to $K < N$, whatever ED or MS, is not profitable.

Obviously, signing up all buyers with ED is not profitable. If instead the incumbent signs up $K = N$ buyers with MS, it must pay each signed buyer at least $x^*(s, p) \geq x_N(s, p) = s(p - c) + (1 - \alpha_N(s))(1 - s)(p - c)$, and the incumbent ends up with non-positive profit since

$$\begin{aligned} \Pi^S(N, s, p) &= s(p - c) + (1 - \alpha_N)(1 - s)(p - c) - x^*(s, p) \\ &\leq s(p - c) + (1 - \alpha_N)(1 - s)(p - c) - x_N(s, p) = 0. \end{aligned}$$

We show second that $f > \delta$ with positive probability is also a sufficient condition. Suppose that $f > \delta$ with positive probability. There exists some $\gamma > 0$ such that $\Pr\{f \geq \delta + \varepsilon\} = \gamma$, where ε can be an arbitrarily small number. Then, signing up $N - 1$ buyers with ED reduces effectively the probability of entry, that is, $\alpha_K(1) = \alpha_{N-1}(1) = \Pr\{f < \delta\} \leq 1 - \gamma < 1$, and thus $\Pi^U(N - 1, 1) > 0$.

To see that the incumbent can make a positive profit by offering a market share contract, it suffices to show that there exists some $s < 1$ such that $\Pi^U(N - 1, s) > 0$, since the profit from

signed buyers is always non-negative. Note that the entrant's profit when $N - 1$ buyers have signed the contract is given by $\Pi_E(N - 1, s) = (N - 1)(1 - s)\delta + \delta$, then choosing any market share such that

$$s > \hat{s} \equiv 1 - \frac{\varepsilon}{(N - 1)\delta}$$

ensures that $\Pi_E < \delta + \varepsilon$, which reduces effectively the probability of entry to $\alpha_{N-1}(s) < 1 - \gamma < 1$. Thus, $\Pi^U(N - 1, s) > 0$ for all $s > \hat{s}$, and the incumbent earns a positive profit from the unsigned buyer. **Q.E.D.**

D. Optimal Pricing under MS

We show that the optimal per-unit price under MS is equal to v . Suppose the incumbent offers a contract $C = \{s, x, p\}$ to K buyers with $s < 1$ and all K buyers accept the contract in the PCPNE of the continuation game. We restrict to the scenario with $p \geq c$, as any price below its unit cost is allegable for predatory pricing and could be prohibited by antitrust laws. Consider two cases as follows:

Case (1): The incumbent charges a price $p \leq v$. In this case, the signed buyers will purchase all of their demands from the incumbent when the entrant does not enter. Note that the profit from unsigned buyers is independent of the committed price p , whereas the profit from signed buyers depends on p . To see further the impact of price change on the incumbent's profit, let $x^*(s, p) = x_m(s, p) = \alpha_{m-1}(s)(v - c) - (1 - \alpha_m(s))(v - p) - \alpha_m(s)(v - p_a)$ and substitute into the profit from each signed buyer, we have

$$\begin{aligned} \Pi^S(K, s, p) &= s(p - c) + (1 - \alpha_K(s))(1 - s)(p - c) - x^*(s, p) \\ &= (1 - \alpha_{m-1}(s))(v - c) + (1 - s)(\alpha_m(s) - \alpha_K(s))(p - c) \end{aligned}$$

Differentiating $\Pi^S(K, s, p)$ with respect to p , we obtain

$$\frac{\partial \Pi^S(K, s, p)}{\partial p} = (1 - s)(\alpha_m(s) - \alpha_K(s)).$$

The above derivative is strictly positive for any $s < 1$ and $\alpha_m > \alpha_K$, thus, the profit from signed buyers is always increasing in p , and the optimal price under MS is equal to v . If $\alpha_m = \alpha_K$, then $\Pi^S(K, s, p) = (1 - \alpha_{m-1}(s))(v - c)$ and the incumbent's profit is independent of p .

Case (2): Suppose the incumbent charges a price $p > v$. Then each signed buyer will only purchase s units regardless whether the entrant enters or not, as buying more units incurs more loss to the signed buyer. When the entrant enters, each signed buyer will purchase the residual demand of $1 - s$ units from the entrant at a per unit price c , and obtains a surplus $v - p_a$.

When the entrant stays out, each signed buyer will only purchase s units from the incumbent and obtains a (negative) surplus $s(v - p)$. Thus, the expected surplus of a buyer who agrees to the incumbent's offer when $n - 1$ other buyers are also agreeing to the incumbent's offer is given by

$$\begin{aligned} U_A(n) &= (1 - \alpha_n) s(v - p) + \alpha_n(v - p_a) + x \\ &= s(v - p) + \alpha_n(1 - s)(v - c) + x. \end{aligned}$$

The expected surplus of this same buyer if it rejects the incumbent's offer is given by

$$U_R(n - 1) = \alpha_{n-1}(v - c).$$

Thus, the exact compensation that makes this buyer indifferent between accepting or rejecting is equal to

$$\tilde{x}_n(s, p) = \alpha_{n-1}(v - c) - s(v - p) - \alpha_n(1 - s)(v - c).$$

Let $\tilde{x}^*(s, p) = \tilde{x}_m(s, p)$ be the maximum payment, the incumbent's profit from each signed buyer is then given by

$$\begin{aligned} \tilde{\Pi}^S(K, s, p) &= s(p - c) - \tilde{x}^*(s, p) \\ &= s(p - c) - \alpha_{m-1}(v - c) + s(v - p) + \alpha_m(1 - s)(v - c) \\ &= (s + \alpha_m(1 - s) - \alpha_{m-1})(v - c). \end{aligned}$$

It follows that the incumbent's profit is independent of p for any $p > v$. Since $s + \alpha_m(1 - s) - \alpha_{m-1} < 1 - \alpha_{m-1}$, we have

$$\tilde{\Pi}^S(K, s, p) < (1 - \alpha_{m-1})(v - c) + (1 - s)(\alpha_m - \alpha_K)(v - c) = \Pi^S(K, s, v).$$

Therefore, charging $p > v$ is strictly dominated by setting $p = v$. **Q.E.D.**

E. Proof of Proposition 4

We need to show that $\alpha_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s})$, and thus the cost of foreclosure is decreasing when N increases. The rest of the proposition has been shown in the main context. Note that

$$\begin{aligned} \alpha_{\Omega+1+\Delta N}(s^*) &= G((N + \Delta N - (\Omega + 1 + \Delta N)s^*)\delta), \\ \alpha_{\Omega+1}(\hat{s}) &= G((N - (\Omega + 1)\hat{s})\delta), \end{aligned}$$

then $\alpha_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s})$ if and only if $N + \Delta N - (\Omega + 1 + \Delta N) s^* > N - (\Omega + 1) \hat{s}$. To see this, note that the incumbent chooses s^* such that the optimal amount of uncommitted purchases is exactly the same as before N increases, that is,

$$N + \Delta N - \left(\hat{K} + \Delta N \right) s^* = N - \hat{K} \hat{s}. \quad (\text{A.1})$$

When N increases by ΔN , the incumbent must sign up $(\Omega + 1 + \Delta N) s^*$ units to reduce the probability of entry effectively, and sign up further $\left(\hat{K} - (\Omega + 1) \right) s^*$ units to reduce the uncommitted purchases to the optimal level. Thus,

$$\begin{aligned} & N + \Delta N - (\Omega + 1 + \Delta N) s^* \\ &= N + \Delta N - \left(\hat{K} + \Delta N \right) s^* + \left(\hat{K} - (\Omega + 1) \right) s^* \\ &= N - \hat{K} \hat{s} + \left(\hat{K} - (\Omega + 1) \right) s^* \\ &> N - \hat{K} \hat{s} + \left(\hat{K} - (\Omega + 1) \right) \hat{s} \\ &= N - (\Omega + 1) \hat{s}, \end{aligned}$$

where we have used equation (A.1) to get the third line and the inequality comes from the fact that $s^* > \hat{s}$. **Q.E.D.**

F. Proof of Lemma 3

We show first the profit function $\tilde{\Pi}(\theta, 1)$ is concave under Assumption 1. Recall that

$$\tilde{\Pi}(\theta, 1) = (1 - G((N - \theta)\delta))(N - \theta)(v - c).$$

Differentiating $\tilde{\Pi}(\theta, 1)$ with respect to θ , we obtain the first-order and the second-order derivatives respectively

$$\begin{aligned} \frac{\partial \tilde{\Pi}(\theta, 1)}{\partial \theta} &= [(N - \theta)\delta g((N - \theta)\delta) - (1 - G((N - \theta)\delta))](v - c), \\ \frac{\partial^2 \tilde{\Pi}(\theta, 1)}{\partial \theta^2} &= -[2g((N - \theta)\delta) + (N - \theta)\delta g'((N - \theta)\delta)]\delta(v - c). \end{aligned}$$

Thus, the profit function is concave if and only if

$$2g((N - \theta)\delta) + (N - \theta)\delta g'((N - \theta)\delta) > 0.$$

The above condition holds if $G(\cdot)$ is weakly convex, or if it is weakly concave such that $zG''(z) > -2G'(z)$ for $z = (N - \theta)\delta$.

We show second that the per-unit cost of foreclosure under MS is decreasing in s , and thus the total cost of foreclosure is decreasing in s given any foreclosure θ . For this purpose, we denote by $C(s) \equiv x^*(s) - s(v - c)$ the per-buyer cost and $AC(s) = C(s)/s$ the per-unit cost of foreclosure. Since

$$\frac{dAC(s)}{ds} = \frac{sC'(s) - C(s)}{s^2},$$

then $AC(s)$ is decreasing in s if and only if

$$\Phi(s) \equiv C(s) - sC'(s) > 0.$$

Consider two cases as follows:

Case (1): Suppose $G(\cdot)$ is weakly convex. From Appendix B, we know that $x^*(s) = x_{\Omega+1}(s)$, which implies

$$C(s) = (1 - s)(1 - \alpha_{\Omega+1}(s))(v - c).$$

Differentiating $C(s)$ with respect to s , we have

$$C'(s) = [-(1 - \alpha_{\Omega+1}(s)) - (1 - s)\alpha'_{\Omega+1}(s)](v - c),$$

and thus

$$\Phi(s) = C(s) - sC'(s) = [1 - \alpha_{\Omega+1}(s) + s(1 - s)\alpha'_{\Omega+1}(s)](v - c).$$

Note that when $s = 0$, $\Phi(0) = 0$, and when $s = 1$, $\Phi(1) = (1 - \alpha_{\Omega+1}(1))(v - c) > 0$. Moreover, $\Phi(s)$ is increasing in s since

$$\begin{aligned} \Phi'(s) &= -\alpha'_{\Omega+1}(s) + (1 - 2s)\alpha'_{\Omega+1}(s) + s(1 - s)\alpha''_{\Omega+1}(s) \\ &= -2s\alpha'_{\Omega+1}(s) + s(1 - s)\alpha''_{\Omega+1}(s) \\ &= 2s(\Omega + 1)\delta g((N - (\Omega + 1)s)\delta) + s(1 - s)(\Omega + 1)^2\delta^2 g'((N - (\Omega + 1)s)\delta) \\ &> 0, \end{aligned}$$

where the inequality comes from the fact that $G(\cdot)$ is weakly convex. It thus follows that $\Phi(s) > 0$ for any $s \in (0, 1]$.

Case (2). Suppose $G(\cdot)$ is weakly concave such that $zG''(z) > -2G'(z)$. Let $x^*(s) \equiv x_m(s) = (\alpha_{m-1}(s) - (1 - s)\alpha_m(s))(v - c)$, then

$$C(s) = x^*(s) - s(v - c) = (\alpha_{m-1}(s) - (1 - s)\alpha_m(s) - s)(v - c),$$

and

$$C'(s) = [\alpha'_{m-1}(s) + \alpha_m(s) - (1 - s)\alpha'_m(s) - 1](v - c),$$

it thus follows that

$$\Phi(s) = C(s) - sC'(s) = [\alpha_{m-1}(s) - \alpha_m(s) - s\alpha'_{m-1}(s) + s(1-s)\alpha'_m(s)](v-c).$$

Note that $\Phi(0) = 0$ when $s = 0$. Note also that $x^*(s) = x_{\Omega+1}(s)$ when $s = 1$ (i.e., $m = \Omega + 1$), thus $\Phi(1) = (1 - \alpha_{\Omega+1}(1))(v-c) > 0$. We show now $\Phi(s)$ is increasing in s , and thus $\Phi(s) > 0$ for all $s \in [0, 1]$.

To see this, differentiating $\Phi(s)$ with respect to s , we obtain

$$\begin{aligned} \Phi'(s) &= \alpha'_{m-1}(s) - \alpha'_m(s) - \alpha'_{m-1}(s) - s\alpha''_{m-1}(s) + s(1-s)\alpha''_m(s) + (1-2s)\alpha'_m(s) \\ &= s(1-s)\alpha''_m(s) - s\alpha''_{m-1}(s) - 2s\alpha'_m(s) \\ &= s \left[2m\delta g((N-ms)\delta) + (1-s)m^2\delta^2 g'((N-ms)\delta) - (m-1)^2\delta^2 g'((N-(m-1)s)\delta) \right] \\ &> s \left[-m\delta^2 g'((N-ms)\delta)(N-ms - (1-s)m) - (m-1)^2\delta^2 g'((N-(m-1)s)\delta) \right] \\ &= -s \left[m\delta^2 g'((N-ms)\delta)(N-m) + (m-1)^2\delta^2 g'((N-(m-1)s)\delta) \right] \\ &> 0, \end{aligned}$$

where we have used the assumption $2g(z) > -zg'(z)$ for $z = (N-ms)\delta$ to derive the first inequality, and the second inequality comes from the fact that $G(\cdot)$ is weakly concave. **Q.E.D.**

G. Proof of Proposition 3

Suppose Assumption 1 holds. We show that ED can be more profitable than MS and *vice versa* with two illustrative examples. In both examples the distribution function $G(\cdot)$ is differentiable and weakly concave.

Example 1: ED is more profitable than MS

Consider a market with three identical buyers. Each buyer desires a fixed amount of the good and the gains from trade are given by $v - c = 1000$. For simplicity, we assume here that the entry cost is distributed between 0 and 3δ , according to the following distribution function

$$\begin{aligned} G(x) &= \frac{0.4x}{\delta}, \text{ for } 0 < x < \delta, \\ &= 0.4 + \frac{0.3(x-\delta)}{\delta}, \text{ for } \delta < x < 3\delta. \end{aligned}$$

That is, with probability 0.4, f is uniformly distributed between 0 and δ , and with probability 0.3, f is uniformly distributed between δ and 3δ . It follows that the probability of entry is given by $G((3-\theta)\delta) = 0.4(3-\theta)$ for $\theta \geq 2$ and $G((3-\theta)\delta) = 0.4 + 0.3(2-\theta)$ for $\theta \leq 2$.

Suppose the incumbent offers a contract $C = \{s, v, x\}$ to K buyers and all these buyers accepted the contract in the equilibrium. Using (8) and noting that $x^*(s) = x_1(s) = s(v - c) + (1 - s)(1 - \alpha_1(s))(v - c)$,³¹ we can rewrite the incumbent's profit as

$$\tilde{\Pi}(\theta, s) = 1000(3 - \theta)(1 - G((3 - \theta)\delta)) - \theta \frac{(x^*(s, v) - 1000s)}{s} \quad (\text{A.2})$$

$$= 1000(3 - \theta)(1 - G((3 - \theta)\delta)) - 1000\theta \frac{(1 - s)(1 - G((3 - s)\delta))}{s}, \quad (\text{A.3})$$

where the first term is the benefit from foreclosure whereas the second term is the cost of foreclosure. Maximizing the first term yields the optimal level of foreclosure $\theta^* = K^{ED} = 1.75$,³² and the associated expected benefit is equal to 625. Suppose the incumbent offers ED contract, it is optimal to offer to $\bar{K}^{ED} = 2$ buyers which reduces the cost of foreclosure to zero, and the incumbent's profit is $\tilde{\Pi}(2, 1) = 600$.

Suppose instead the incumbent offers MS to two buyers, its profit is then given by

$$\begin{aligned} \Pi(3, s) &= 1000(3 - 2s)(1 - G((3 - 2s)\delta)) - 2000(1 - s)(1 - G((3 - s)\delta)) \\ &= 600(3 - 2s)s - 600(1 - s)s, \end{aligned}$$

where the second line is derived by substituting the probability of entry $G((3 - \theta)\delta) = 0.4 + 0.3(2 - \theta)$. Suppose the incumbent offers $s^* = K^{ED}/2$ to maximize the benefit (which is equal to 625), then the cost of foreclosure is equal to 65.6, and the incumbent's profit is equal to 559.4, which is about 7% less than the profit under ED. It appears that the incentive of minimizing the cost dominates the incentive of maximizing the benefit, thus offering ED is optimal in this case.³³

Example 2: MS is more profitable than ED

Consider the same setup as above, but let the distribution function now be given by

$$\begin{aligned} G(x) &= \frac{0.8x}{\delta}, \text{ for } 0 < x < \delta, \\ &= 0.8 + \frac{0.1(x - \delta)}{\delta}, \text{ for } \delta < x < 3\delta. \end{aligned}$$

That is, with probability 0.8, f is uniformly distributed between 0 and δ , and with probability 0.1, f is uniformly distributed between δ and 3δ . The incumbent's expect profit is still given

³¹It is easy to check that $x_1(s) \geq x_2(s) \geq x_3(s)$ in this example.

³²We need to compare the benefit in two cases regarding on $\theta \geq 2$ and $\theta \leq 2$ respectively with different probability of entry. It is straightforward to see that the benefit function is maximized in the second case at $\theta^* = 1.75$.

³³Indeed, the profit function can be reduced to $\Pi(3, s) = 600s(2 - s)$, which is increasing in s , and it is thus optimal to set $s = 1$.

by (A.2), but the probability of entry is different: $G((3 - \theta)\delta) = 0.8(3 - \theta)$ for $\theta \geq 2$ and $G((3 - \theta)\delta) = 0.8 + 0.1(2 - \theta)$ for $\theta \leq 2$. Maximizing the benefit then yields $\theta^* = K^{ED} = 2.375$ and the associate profit is equal to 312.5.

Suppose the incumbent offers ED, it is optimal to offer ED to $\underline{K}^{ED} = 2$ buyers (giving ED to all buyers yields zero profit), which yields an expected profit

$$\Pi^{ED}(2, 1) = 1000(3 - 2)(1 - G(2\delta)) = 200.$$

While offering ED reduces the cost of foreclosure to zero, the resulting loss of the benefit is equal to 112.5, which accounts for more than 56% of its actual profit.

This suggests that offering MS that maximizes the benefit from foreclosure could do better. Suppose the incumbent offers $\hat{s} = K^{ED}/3$ to three buyers, which achieves the maximum benefit of 312.5. The resulting cost of foreclosure is equal to (using $G((3 - \hat{s})\delta) = 0.8 + 0.1(2 - \hat{s})$)

$$3000(1 - \hat{s})(1 - G((3 - \hat{s})\delta)) = 300(1 - \hat{s})\hat{s} \simeq 49.5,$$

and the incumbent earns a net profit of 263. Thus, the incentive of maximizing benefit from foreclosure dominates the incentive of minimizing the cost. Note that offering MS can increase the incumbent's profit by more than 30% in this case.

Indeed, in this example the optimal MS contract leads to higher foreclosure than θ^* . To see this, we can rewrite the incumbent's expected payoff as (given $s > 2/3$)

$$\begin{aligned} \Pi(3, s) &= 1000[(3 - 3s)(1 - G((3 - 3s)\delta)) - 3(1 - s)G((3 - s)\delta)] \\ &= 3000(1 - s)(2.3s - 1.4). \end{aligned}$$

Maximizing the above profit leads to the optimal market share

$$s^* = \frac{37}{46} \simeq 0.8,$$

and the corresponding expected profit is equal to 264.1.³⁴ Thus, the equilibrium foreclosure, $3s^*$, is greater than θ^* , and thus higher than the foreclosure under ED.

The property that $\tilde{\Pi}(\theta, 1)$ is weakly concave implies that the exclusive dealing contracts must be offered to either \overline{K}^{ED} or \underline{K}^{ED} buyers. We now characterize the optimal market-share contract when it is more profitable than ED. The property that $\tilde{\Pi}(\theta, 1)$ is concave also implies that the optimal foreclosure θ^* is an interior solution satisfying $\Omega\delta < \theta^* < N\delta$, and that the profit

³⁴Notice that the inducement is given by $x_n(s) = (\alpha_{n-1}(s) - (1 - s)\alpha_n(s))(v - c)$. Evaluating at s^* , we have $x_1(s^*) = 820$, $x_2(s^*) = 755$, and $x_3(s^*) = 747$. It thus follows that $x_1(s^*)$ is the largest compensation among all.

function $\tilde{\Pi}(\theta, 1)$ increases in θ for all $\theta < \theta^*$ and decreases in θ for all $\theta > \theta^*$. Thus, the optimal foreclosure under MS must satisfy $\theta \in (\underline{K}^{ED}, \overline{K}^{ED})$. If a market-share contract results in $\theta \leq \underline{K}^{ED}$ (respectively $\theta \geq \overline{K}^{ED}$), then it is strictly dominated by the exclusive dealing contract giving to \underline{K}^{ED} buyers (respectively \overline{K}^{ED} buyers), as the latter results in foreclosure closer to the optimum and at the same time minimizes the cost of foreclosure as $x^*(1) - (v - c) = 0$.

The optimal market-share contract must give to $K \geq \overline{K}^{ED}$ buyers, and by Lemma 3, the cost of foreclosure is decreasing in s , which ensures that offering MS to exactly \overline{K}^{ED} buyers is optimal. Thus, the optimal market share, $s^* = \theta/\overline{K}^{ED}$, must satisfy $s^* \in (\frac{\underline{K}^{ED}}{\overline{K}^{ED}}, 1)$. **Q.E.D.**

References

- Abito, J.M. and J. Wright (2008), “Exclusive Dealing with Imperfect Downstream Competition”, *International Journal of Industrial Organization*, 26: 227-246.
- Aghion, P. and P. Bolton (1987), “Contracts as a Barrier to Entry”, *American Economic Review*, 77: 388-401.
- Argenton, C. (2010), “Exclusive Quality”, *Journal of Industrial Economics*, 58: 690-716.
- Asker, J. and H. Bar-Isaac (2014), “Raising Retailers’ Profits: On Vertical Practices and the Exclusion of Rivals”, *American Economic Review*, 104(2): 672-686.
- Bernheim, B.D., Peleg, B., and M. Whinston (1987), “Coalition-Proof Nash Equilibria: Concepts”, *Journal of Economic Theory*, 42: 1-12.
- Bernheim, B.D. and M. Whinston (1998), “Exclusive Dealing”, *Journal of Political Economy*, 106: 64-103.
- Bork, R. (1978) *The Antitrust Paradox: A Policy At War With Itself*, New York: Basic Books.
- Calzolari, G. and V. Denicolo (2013), “Competition with Exclusive Contracts and Market-Share Discounts”, *American Economic Review*, 103(6): 2384-2411.
- Calzolari, G. and V. Denicolo (2015), “Exclusive Contracts and Market Dominance.” *American Economic Review*, 105(11): 3321-3351.
- Chen, Y. and D. Sappington (2011), “Exclusive Contracts, Innovation, and Welfare.” *American Economic Journal: Microeconomics*, 3: 194-220.
- Chen, Z. and G. Shaffer (2014), “Naked Exclusion with Minimum-Share Requirements.” *Rand Journal of Economics*, 45: 64-91
- Chone, P. and L. Linnemer (2015), “Nonlinear Pricing and Exclusion: I. Buyer Opportunism”, *Rand Journal of Economics*, 46(2):217-240.
- Director, A. and E. Levi (1956), “Law and the Future: Trade Regulation.” *Northwestern University Law Review*, 51: 281-296.
- Fumagalli, C. and M. Motta (2006), “Exclusive Dealing and Entry, When Buyers Compete”, *American Economic Review*, 96: 785-795.
- Ide, E., J.P. Montero, and N. Figueroa (2016), “Discounts as a Barrier to Entry”, *American Economic Review*, 106: 1849-1877.
- Inderst, R. and G. Shaffer (2010), “Market-Share Contracts as Facilitating Practices”, *Rand Journal of Economics*, 41: 709-729.
- Innes, R. and R. Sexton (1994), “Strategic Buyers and Exclusionary Contracts”, *American Economic Review*, 84: 566-584.
- Landeo, C. and K. Spier (2009), “Naked Exclusion: An Experimental Study of Contracts with Externalities”, *American Economic Review*, 99: 1850-1877.

- Landeo, C. and K. Spier (2012), “Exclusive Dealing and Market Foreclosure: Further Experimental Results”, *Journal of Institutional and Theoretical Economics*, 168: 150-170.
- Majumdar, A. and G. Shaffer (2009), “Market-Share Contracts with Asymmetric Information”, *Journal of Economics & Management Strategy*, 18: 393-421.
- Marx, L. and G. Shaffer (2004), “Rent-Shifting, Exclusion, and Market-Share Contracts”, mimeo.
- Miklós-Thal, J. and G. Shaffer (2016), “Naked Exclusion with Private Offers”, forthcoming in *American Economic Journal: Microeconomics*.
- Mills, D. (2010), “Inducing Downstream Selling Effort With Market-Share Discounts”, *International Journal of Economics and Business*, 17: 129-146.
- O’Brien, D.P. and G. Shaffer (1997), “Nonlinear Supply Contracts, Exclusive Dealing, and Equilibrium Market Foreclosure”, *Journal of Economics & Management Strategy*, 6: 755-785.
- Posner, R. (1976). *Antitrust Law: An Economic Perspective*. University of Chicago Press.
- Rasmusen, E., Ramseyer, J.M, and J. Wiley (1991), “Naked Exclusion”, *American Economic Review*, 81: 1137-45.
- Segal, I. and M. Whinston (2000), “Naked Exclusion: Comment”, *American Economic Review*, 90: 296-309.
- Smith, A. (2010), “An Experimental Study of Exclusive Contracts”, *International Journal of Industrial Organization*, 29: 4-13.
- Simpson, J. and A. Wickelgren (2007), “Naked Exclusion, Efficient Breach, and Downstream Competition”, *American Economic Review*, 97: 1305-1320.
- Spector, D. (2011), “Exclusive Contracts and Demand Foreclosure”, *Rand Journal of Economics*, 42: 629-638.
- Tom, W., Balto, D., and N. Averitt (2000), “Anticompetitive Aspects of Market-Share Discounts and Other Incentives to Exclusive Dealing”, *Antitrust Law Journal*, 67: 615-640.
- Whinston, M.D. (2006), *Lectures on Antitrust Economics*. MIT Press.
- Wright, J. (2009), “Exclusive Dealing and Entry, When Buyers Compete: Comment”, *American Economic Review*, 99: 1070-1081.
- Wright, J. (2012), “Moving Beyond Naive Foreclosure Analysis”, *George Mason Law Review*, 1163-1198.