Integration, Delegation, and Management

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October 26, 2015

Abstract

We present a competitive industry equilibrium model in which managerial scarcity affects the joint determination of firm boundaries and internal decision-making structures. Integration through asset sale grants authority to a manager, who may choose to delegate decision making back to his subordinates, or to retain control for himself, depending on his comparative advantage in coordinating activities, which he learns after contracting. Exogenous productivity and industry price are key determinants of both integration and delegation: across a heterogeneous population of firms, or with variation in price, delegation and integration may co-vary. There is an inverted U relationship between integration and price levels. When managers are scarce, a lower proportion of managers increases total output in the industry when the product market demand is high, and decreases total output when demand is low. In this sense, there may be too much integration when prices are high and too little when prices are low.

Keywords: Integration, centralization, firm boundaries, product price, industry performance, managers, finance, OIO.
JEL codes: D2, L2, M1

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*We thank participants at the CEPR fourth workshop on Incentives, Organization and Management and at the MIT organizational seminar. Legros gratefully acknowledges support of the European Research Council (Advanced Grant 339950
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1 Introduction

Though it is widely agreed that management is essential to the functioning of firms and other organizations, it is fair to say that consensus has yet to emerge on precisely why it matters or even how to characterize it. Nevertheless, three aspects of management can already be identified as salient. First, managers allocate resources by authority. In private enterprise economies, authority emanates from property rights, and the scope of management is therefore constrained by ownership structure. Second, managers design the organization, particularly when and to whom to delegate decision rights. Owners will take account of the likely directions of delegation when choosing whether to sell their assets; the possibility that one might re-acquire decision-making power softens the blow of giving up control. Hence, integration and delegation decisions are interdependent; firm boundaries limit delegation decisions, but the anticipation of delegation encourages the broadening of firm boundaries. Third, management is a scarce resource. The costs and benefits of integrating are affected by the price one pays for a manager to exercise authority competently, and the scarcity of managerial talent affects that price: management is not only constrained by the firm boundaries, it also constrains them.

In this paper we focus on the interactions among these three aspects of management: we study the relationship between firm boundaries and internal organization choices, how they respond to changes in the economic environment, especially the market for managers, and how they affect industry performance. Since managerial markets differ radically across countries, our analysis will have something to say about how the differential performance of firms and industries in developed and developing countries is tied to their differential ownership, organizational design, and management practices.

Our setting also provides an opportunity to underscore the important differences between outsourcing, akin to non-integration in our model, and delegation (or decentralization); in fact we will show that non-integration and delegation may move in opposite directions as the price of output or productivity changes.

Following [Hart et al. (2010)] and [Legros and Newman (2013)] we consider environments in which integration facilitates alignment between suppliers. The precise mechanism for this is due to a change of ownership following integration that gives authority to a central party who values only revenues, and will therefore try to coordinate activities in order to maximize expected output, whereas the initial asset holders disagree on which decisions have to be made and fail to coordinate under non-integration because alignment generates private costs for both of them. It follows
that both integration and non integration involve costs and benefits: integration
tends to put too little weight on private costs which leads to over-alignment while
non integration tends to put too much weight on private costs which leads to too
little alignment.

Managers who have authority to make decisions in the firm may decide to delegate
decision making if they realize that they do not have a comparative advantage,
something they learn after contracting has taken place; this possibility of delegation
mitigates the cost of excessive alignment under integration. Delegation has to happen
in real time and is not governed by contracts; there is no time to sit down, renegotiate
output shares or even ownership.

The demand for managerial services is a function of the surplus gain from inte-
gration, something that we show to be an inverted U. When the price is low or is
high, there is little gain from integration: in the first case because revenues are small
anyway, and in the second case because non integration performs well in terms of
output.

Within this model, we can address some key questions concerning the interaction
between the product market, the managerial market and the performance of the
industry.

**Decentralization vs. non integration** With non integration asset holders effect-
ively commit to make decisions non cooperatively. Under integration, asset holders
relinquish their authority to another party and will be delegated back the right to
make decisions only if it is incentive compatible for this third party (e.g., Aghion and
Tirole (1997); Baker et al. (1999)); once decisions have to be made there is little scope
for sitting at a bargaining table and redraw a contract, or such renegotiation may
lead to significant delays in production. This message extends beyond the current
application which views integration as mitigating the conflict of interests between
$U, D$, and could be leveraged for alternative theories of integration, for instance as in
(?) where integrated structure in need of adaptation face a tradeoff between incen-
tive provision and delegation, or as in (Legros and Newman 1996) where integration
facilitates, at a cost, monitoring to solve moral hazard problems.

Because management by delegation is subject to incentive compatibility condi-
tions, the performance of agents under delegation is influenced by their contractual
shares of firm revenues, which in turns shapes the willingness of managers to delegate
authority, and also the willingness of asset holders to integrate in the first place.

As we argued in Legros and Newman (2013), the price level is a crucial determi-
nant of organizational decisions since it modifies the trade-off between alignment and private costs. As the price increases in our model, the suppliers have good incentives to coordinate and it is therefore less likely that the manager will have a comparative advantage; delegation is more likely when prices are high.

**Is there too little or too much delegation?** From the point of view of industry supply, the degree of delegation is efficient: the manager compares his comparative advantage at coordinating activity to that of subordinates and therefore chooses optimally to delegate only if subordinates are better at coordinating. However, when we take into account the private costs of subordinates, there can be too much centralization or too much delegation depending on the level of price. When the price is low, the subordinates have low private cost when they decide, and value more delegation than the manager but when output is high, the private costs are high and the subordinates value more the commitment benefit of centralization than the manager. Hence while integration and managerial activity is always productivity enhancing at the firm level, this may not be the case at the industry level.

**Is there the right supply of managers?** In particular, could output be increased by increasing or decreasing the supply of managers? In the short run, when the proportion of managers is fixed but their compensation is endogenous, there is a trade off between the larger output in integrated firms and the smaller number of firms in the industry. When demand is low, the integration effect dominates, and increasing the proportion of managers is output enhancing. But when demand is high, the industry scale effect dominates and decreasing the proportion of managers will increase output. These results are preserved in the long run when we assume that entry into management is costly. By increasing this cost, the proportion of managers decreases and this is output enhancing only if demand is high.

**What is the Role of Finance?** In the model, the key role of integration is to facilitate alignment among suppliers. If individuals have large cash holdings and no limited liability, they can solve their alignment problem through contracting, and will not integrate\(^1\). Limited cash holdings is therefore a key determinant of integration. Assuming that the suppliers on the long side of the market have no cash holding, the degree of integration in the industry will be an increasing function of the cash holdings.

\(^1\)This is consistent with the view that managers or small entrepreneurs want a quiet life (Bertrand and Mullainathan, 2003) or more recently Hurst et al. (2011) who show by using a panel data of entrepreneurs, that most small businesses have little desire to grow.
holdings of producers (who are on the short side of the market), and integrated firms will also be less centralized. If there is borrowing to finance new capital, firms that are more leveraged will be more centralized: repayment is akin to a decrease in the price of output and makes alignment by subordinates less likely.

**Links to the Literature** Our model borrows from the literatures on firm boundaries and internal organization. These literatures have a lot in common but have evolved in parallel; papers focusing on firm boundary analyze the determinants and economic effects of integration versus non integration choices by asset holders, while papers on internal organization focus on the organization of communication flows and the optimal mix of centralization and decentralization. Both non integration and decentralization decisions tradeoff the benefit of having other parties, more informed or more able, in making decisions, versus the cost due to the loss of control when there is a conflict of interest among the parties.

However, non integration and decentralization decisions are quite different since in the former case owners and decision makers coincide, which is not the case in the case of decentralization. As Aghion and Tirole (1997), Baker et al. (1999) have articulated, decisions that are delegated within a firm are subject to an incentive problem on the part of the owner, or the person who has authority, since he can overturn decisions, something that does not happen under non integration. With non integration, there is commitment to decentralization, much less so in integrated firms.

In our model, this incentive problem shapes the way integration contracts are designed, and in particular who should have authority within the integrated firm. We show that it is crucial for the manager to have authority in equilibrium; if this is not the case the initial asset holders cannot commit to coordinate and may as well retain ownership and not bring in a manager and integrate. Hence, while it may be reasonable to look at the internal organization problem when one, or both, initial asset holder(s) have authority, these situations are inconsistent with the fact that there would be integration to begin with. This illustrates the benefit of looking jointly at the determination of firm boundary and internal organization. Another illustration is provided by our result that while delegation is always increased when the price increases, integration is non-monotonic in the price level. As far as we

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2The literature is too large to survey here; see Aghion et al. (2014); Bolton et al. (2011); Dessein (2014); Legros and Newman (2014) for recent surveys on firm boundaries, internal organization, authority and the interplay between organizations and markets.

3See also Harris and Raviv (2010) who apply these ideas in a corporate governance context with asymmetry of information and moral hazard.
know the first paper to analyze simultaneously the problem of firm boundary and
degregation is [Hart et al. (2010)](#), but they do so by looking at two parties while we
are also interested in the market determinants of firm boundaries and delegation
decisions.

Our paper is also part of the literature on the market determinants of contractual
or organizational forms[4] We bring to this literature the idea that the characteristics
of the managerial market and the joint determination of integration, and delegation
decisions, are important determinants of the productivity and efficiency of firms.

The rest of the paper is organized as follows. We first present the partial equi-
librium model of integration and delegation, taking the price of the product and of
the manager as given. We then embed this model into a market equilibrium and
investigate the determinants of integration and delegation, both in the short run and
in the long run, and develop along the way the answers to the three questions we
have sketched above.

## 2 A Model of Integration and Delegation

### 2.1 Basics

A supply chain is composed of assets $U, D$, and individuals, denoted also $U, D$, as-
associated with these assets and specialized in making decisions on them. Production
requires decisions $u, d$ to be made on each asset. When the decisions are made, there
are private costs $d^2$ and $(1 - u)^2$ imposed on individuals working on assets $U, D$
respectively.

If assets are owned by the individuals who work with them, we will call this non
integration, they choose $u, d$ non-cooperatively, and the probability of obtaining an
output of 2 is $\frac{1}{2}(1 - (u - d)^2)$. This probability is bounded above by 1/2, which is
attained when $u = d$, that is when the two decisions coincide, a situation we shall
refer to as perfect coordination.

Alternatively, the assets could be purchased by a third party and the authority
to decide given to a professional manager $M$. This manager may be better aligning
decisions $u$ and $d$ ("coordinating") than $U$ and $D$ can do on their own, but is often an
outsider or at least less expert than $U$ or $D$ to decide on the use the assets. We model
these aspects by assuming that the manager has ability $\mu$ to coordinate activities but
that this ability is random, with continuous density $f$ and cumulative $F$ on $[0, 1]$.

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organization.
with the realization of $\mu$ occurring after contracting as private information to the manager\[5\].

If the manager decides on $u, d$ the probability of success is $\frac{\mu}{2}(1 - (u - d)^2)$. The manager can choose to centralize decision making, in which case he will be making decisions $u, d$, or to decentralize, delegate, the decisions to $U, D$.

To summarize, defining:

$$y(u, d) \equiv 1 - (u - d)^2,$$

if decisions are $u, d$ and the manager has ability $\mu$, the expected output is:

$$\begin{cases} 
    y(u, d) & \text{if } U, D \text{ decide} \\
    \mu \times y(u, d) & \text{if } M \text{ decides.}
\end{cases}$$

Enterprises behave competitively, facing a market price $P$ for their product and having to offer an expected payoff of at least $h$ to attract a manager.

**Contracts**

Only output is contractible; a contract stipulates in addition to the asset ownership, the share of output that each party will obtain. Following standard arguments, it is sufficient to define the shares of revenue when output is equal to 2 as well as lump sum transfers at the time of contracting.

If there is non integration, the transfers $t$ and shares $s$ involve $U, D$ only, and when there is integration they involve $U, D, M$\[6\]. The sequence of events evolves as in figure [1]. The grayed area represents the contracting stage; only $M$ observes the realization of $\mu$, and when there is integration $u, d$ is either chosen non-cooperatively by $U, D$ when there is delegation or by $M$ when there is centralization.

In terms of asset ownership, the other possibilities are that $D$ or that $U$ owns both $U, D$ assets, but these situations are dominated by non integration or integration.

**Endowments and Outside Options**

$U$ has a zero outside option, and $D$ has all the bargaining power. $M, D$ have large cash holdings, while $D$ has no cash holdings. Anticipating on the industry equilibrium,

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5We comment at the end on the general support case at the end of the paper.

6It is standard to show that it is not worth borrowing for making lump sum transfers; a change in the share of output being a better instrument for transferring surplus than costly borrowing.
since the $U$s are in excess supply, they will have a zero expected payoff. We will make this assumption below to avoid useless generality.

2.2 Non Integration

Under non integration, $U, D$ decide non-cooperatively on $u, d$. In this case, the two parties play a non-cooperative game and it is immediate to show (see [Legros and Newman 2013]) that the expected output in the enterprise depends only on the total revenue accruing to $U, D$ when output is equal to 2, but not on the way this revenue is allocated.

Precisely, under integration, $U, D$ share the total revenue in case of success of $2P$, and it is convenient to write their shares as $s_U = 1 - \alpha$, $s_D = \alpha$. The equilibrium decisions are $d = \frac{\alpha P}{1 + P}$ and $u = \frac{1 + \alpha P}{1 + P}$, implying that the expected output is equal to:

$$Q(P) \equiv 1 - \frac{1}{(1 + P)^2},$$

which is indeed independent of $\alpha$. However, the equilibrium payoffs depend on the way revenue is allocated. $D$’s expected payoff under non integration, is given by the function:

$$\pi(P, \alpha) = \alpha Q(P)P - \alpha^2 C(Q(P)), \ i = U, D,$$

where the cost function $C$ is:

$$C(Q) \equiv \left(1 - \sqrt{1 - Q} \right)^2.$$
Since $U$’s expected payoff is $\pi(P, 1 - \alpha)$, and $\pi(P, \alpha)$ is concave in $\alpha$, the sum of payoffs is maximum when $\alpha = 1/2$.

When the $U$s have a zero payoff, the optimal non integration contract specifies $\alpha = 0$, in which case the expected payoff to $D$ is equal to:

$$V^N(P) = Q(P)P - C(Q(P)).$$  \hspace{1cm} (5)$$

Since $D$ is effectively the unique decision maker ($u = 1$ for any $P$ since $U$ has a zero share of output), the envelop theorem implies that the marginal value of non integration is equal to the expected output:

$$V^N_P(P) = Q(P).$$  \hspace{1cm} (6)$$

### 2.3 Integration

With integration, $M$ decides on centralization or decentralization after having observed $\mu$. It is convenient to write $s_M = 1 - s$, and $s_U = (1 - \alpha)s, s_D = \alpha s$.

**M must have ownership under integration** While $M$ can decide on centralization or decentralization, if he does not have authority he must convince $U, D$ to follow his instructions, a form of leadership. By contrast, with authority, he will be able to implement his preferred decisions.

We show that when $\mu \in [0, 1]$, integration is beneficial to $U, D$, only if $M$ has authority. Indeed, suppose that $M$ does not have authority, a contract $(t, s, \alpha)$ is chosen. $M$ decides to centralize when $\mu \in \mathcal{C}$ by choosing decisions $u^*, d^*$. Since $U, D$ have authority they will be able to choose individually which decision to take, and therefore $U$ will choose his best response to $d^*$, and $D$ his best response to $u^*$. Since they make the decisions the expected output is given by $y(u, d)$ instead of $\int_{\mu \in \mathcal{E}} \mu dF(\mu)y(u^*, d^*)$. Since $\int_{\mu \in \mathcal{E}} \mu dF(\mu) < 1$, the best response of $U$ to $d^*$ leads to a larger payoff than what $U$ obtains by not overturning the decision. If there is an equilibrium it must therefore be the case than when there is centralization, $U, D$ overturn and because they have a revenue in case of success of $sP$, the expected output is $Q(sP)$ in each state, implying that there is no benefit to integration for $U, D$.

By contrast if $M$ has authority and decides to delegate when $\mu \leq Q(sP)$, he will not want to overturn $U, D$’s decisions when there is delegation since $\mu \leq Q(sP)$. Therefore when $\mu$ has support on $[0, 1]$, giving authority to $M$ is the only way for $U, D$
to commit to an effective centralization of decisions and benefit from integration.

Other ownership structures, like $D$ owning both assets, or $M$ owning $U$ asset, are dominated by $M$ ownership. Indeed, if $D$ owns both assets, his dominant strategy is to force coordination on $u = d = 0$ and obtain $\alpha P$, and since $\bar{\mu} \leq 1$, centralization cannot bring an expected payoff greater than $\alpha P$.

**Proposition 1.** If there is integration, $M$ has ownership of the $U, D$ assets.

**The optimal contract under integration.** Since $M$ has authority on decisions, if he chooses centralization, he will choose to perfectly coordinate the decisions $u = d$, generating an expected output of $\mu$. Because the manager is indifferent among all these coordinated decisions, we assume that he chooses the cost minimizing decision $u = d = 1/2$.

If $M$ decides to delegate, he anticipates that $U, D$ will generate an expected output $Q(sP)$ since $U, D$ have the same incentives as in the non integration case when they face a revenue of $sP$ and have shares $\alpha, 1 - \alpha$. Therefore as long as $1 - s$ is positive, $M$ chooses to centralize whenever $\mu \geq Q(sP)$.

**Optimal ex-ante Contracting**

Assuming that $U$ have a zero outside option and no cash, the maximum surplus a $D$ asset holder can have under non integration is obtained when $s_D = 1$:

$$\pi(P, 1) = Q^N(P)P - C(Q^N(P)) = \frac{P^2}{1 + P} = V^N(P).$$

For integration, the optimal contracting problem reduces to choosing $(s, \alpha)$ and transfers $t_U, t_D$ to solve:

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7See Appendix A for an extension of the model that allows for biased manager.
\[
\max_{s,\alpha,t_U,t_D} \int_{Q(sP)}^{1} \left( \mu sP - \frac{1}{4} \right) dF(\mu) + F(Q(sP))\pi(sP,\alpha) + t_D \\
\text{s.t.} \int_{Q(sP)}^{1} \left( \mu(1-\alpha)sP - \frac{1}{4} \right) dF(\mu) + F(Q(sP))\pi(sP,1-\alpha) + t_U \geq 0 \quad (7) \\
\left( \int_{Q(sP)}^{1} \mu dF(\mu) + F(Q(sP))Q(sP) \right) (1-s)P - t_U - t_D \geq h \quad (8) \\
t_U \geq 0. \quad (9)
\]

The first constraint (7) is the participation constraint of $U$, the second constraint (8) is the participation constraint of $M$. The objective function is increasing in $s$ and in $\alpha$. In (7), setting $\alpha = 1$ requires $t_U = \frac{1-F(Q(sP))}{4}$. In (8), the constraint binds, and therefore the maximum payoff to $U$ when $\alpha = 1$ is

\[
\int_{Q(sP)}^{1} \left( \mu P - \frac{1}{2} \right) dF(\mu) + F(Q(sP))Q(sP)(Q(sP)P - C(Q(sP))) - h,
\]

which is increasing in $s$. It is therefore optimal to set $s$ as close to 1 as possible, and to have $t_D = -h - t_U = -h - \frac{1-F(Q(sP))}{4}$. This assumes that $D$ has enough cash to make this ex-ante transfer, and we will assume this for now. An interpretation of the optimal integration contract is that $D$ contributes to the purchase of $U$’s asset, but foregoes ownership in order to give authority to the manager (CEO).

The case $s = 1$ is a knife edge case since $M$ is indifferent between centralization and decentralization; however because his preferences are strict as long as $1-s$ is positive, we can consider the case $s = 1$ as a limit case when the cash holding of $D$ becomes large enough to make a transfer equal to $h + \frac{1-F(Q(P))}{4}$ to $M$.\footnote{In Legros and Newman (2013), the value of $s$ did not affect the total surplus since there was always centralization. In the current paper, a higher share to $M$ makes decentralization less likely and decreases the total surplus from integration. In order to have a strictly positive share, one could assume that a manager has to exert a small effort to make decisions, however this would complicate the analysis without changing the main qualitative results.}

**Proposition 2.** Suppose that $D$ has large cash holdings, then the surplus under integration is:

\[
V^I(P, h) = \int_{Q(P)}^{1} \left( \mu P - \frac{1}{2} \right) dF(\mu) + F(Q(P))V^N(P) - h,
\]

Integration leads to a lottery where with probability $F(Q(P))$ there will be delegation and the expected payoff in this case is the same as under non integration;
with the complementary probability there will be full alignment by \( M \). A necessary condition for integration to be beneficial is therefore that the expected value of alignment when \( \mu > Q(P) \) is greater than \( V^N(P) \).

At \( \mu = Q(P) \), the difference between the expected payoff under centralization and decentralization is equal to \( C(Q(P)) - \frac{1}{2} \), which is positive only if \( P > \sqrt{2} + 1 \). Hence, when \( P < \sqrt{2} + 1 \), there are states leading to centralization where \( U, D \) end up worse off with alignment than if they had the authority to choose the decision. In this sense, there is too much centralization.

If \( P > \sqrt{2} + 1 \), it is always the case that alignment improves on decentralization, but there is too little centralization from the point of view of \( U, D \) who would like to have \( M \) coordinate for values of \( \mu \) smaller than \( Q(P) \).

Having too much centralization at low prices and too little centralization at high prices is reflected in the comparison between the marginal value of integration and the expected output under integration. Indeed, a corollary of Proposition 2 is that at low prices, the marginal value of integration is larger than the expected output, because when \( P \) increases, \( U, D \) avoid more often inefficient centralization. The opposite is true for large prices where price increases make it more likely that there is not sufficient centralization.

**Corollary 1.** If \( P < \sqrt{2} + 1 \), the marginal value of integration with respect to price \( V^I_P(P,h) \) is greater than the expected output, and if \( P > \sqrt{2} + 1 \) the marginal value of integration with respect to price is lower than the expected output.

**Proof.** By differentiating the Lagrangian with respect to \( P \) (details are in the Appendix), the marginal payoff from integration as \( P \) varies is equal to:

\[
V^I_P(P,h) = \int_{Q(P)}^1 \mu dF(\mu) + F(Q(P))Q(P) + Q'(Q(P)) f(Q(P)) \left( \frac{1}{2} - C(Q(P)) \right).
\]

The result follows since the last term on the right-hand sum is non-negative only if \( P \leq \sqrt{2} + 1 \).

**Comparing Integration and non integration**

A necessary condition for integration to be chosen is that it is chosen when \( h = 0 \), that is when

\[
\Delta(P) \equiv \int_{Q(P)}^1 \left( \mu P - \frac{1}{2} - V^N(P) \right) dF(\mu)
\] (10)
is non-negative. Since \( P - \frac{1}{2} = V^N(P) \) at \( P = 1 \), if \( P \leq 1 \), \( \Delta(P) \leq 0 \), and integration will not be chosen. If \( P > \sqrt{2} + 1 \), \( \Delta(P) \) is positive, and therefore there exists \( P \in (1, \sqrt{2} + 1) \) for which \( \Delta(P) = 0 \); we show that such a value is unique.

**Lemma 1.** There exists a unique price \( P_0 \in (1, \sqrt{2} + 1) \) such that \( \Delta(P) \) is positive if, and only if, the price is larger than \( P_0 \).

**Proof.** Let \( \delta(\mu, P) \equiv \mu P - \frac{1}{2} - V^N(P) \). From the arguments in the text, for all \( P \geq \sqrt{2} + 1 \), \( \delta(\mu, P) > 0 \) and we necessarily have \( \Delta(P) > 0 \) for \( P_0 > \sqrt{2} + 1 \). Now, for \( P \in (1, \sqrt{2} + 1) \), the sign of \( \delta(\mu, P) \) is “increasing” in \( P \) since by (6) \( \delta_P(\mu, P) = \mu - Q(P) \) is positive for \( \mu \geq Q(P) \); therefore the sign of \( \Delta(P) \) is increasing in \( P \). The result follows.

While \( \Delta(P) \) is positive for \( P > P_0 \), it converges to zero as \( P \to \infty \). Therefore, when the price of managers is positive, there are at least two price levels such that \( \Delta(P) = h \). To simplify the exposition, we restrict ourselves to distribution functions such that \( \Delta(P) \) is quasi-concave. For instance, power distributions \( F(\mu) = \mu^a \) where \( a \geq 1 \) have this property.

**Assumption 1.** \( \Delta(P) \) is quasi-concave in \( P \).

If the price of managers is exogenously fixed at \( h \), chains will be integrated only if \( \Delta(P) \geq h \), that is when the product price is neither too high nor too low, as illustrated in Figure 2 and in the proposition below. There will be an inverted U shaped relationship between the product market price an integration.

**Proposition 3.** (i) If \( h > \max_P \Delta(P) \), there is no integration.

(ii) If \( h \leq \max_P \Delta(P) \), there exist finite prices \( P(h), \overline{P}(h) \), with \( P(h) > 1 \), and \( \overline{P}(h) \geq \max(P(h), \sqrt{2} + 1) \) such that there is integration if, and only if, \( P \in [P(h), \overline{P}(h)] \).

(iii) \( P(h) \) is an increasing function of \( h \) and \( \overline{P}(h) \) is a decreasing function of \( h \), these two bounds coincide at the price maximizing \( \Delta(P) \), and \( P(0) = P_0, \overline{P}(0) = \infty \).

**First-Order Shifts in \( F \).** If managers are of better quality, in the sense that \( F(\mu) \) shifts in a first-order stochastic sense, one would expect that integration will be favored more often. While correct, this conclusion is not immediate because when \( P \) belongs to \( [1, \sqrt{2} + 1] \), there is too much centralization from \( U, D \)'s perspective, something that will be reinforced when \( F \) shifts in a first order stochastic way.
Lemma 2. Suppose that $\Delta(P)$ is positive for some distribution $F$. Then as $F$ increases in the first-order, $\Delta(P)$ increases.

Proof. We can write

$$\Delta(P) = (1 - F(Q(P))) \left[ \int_{Q(P)}^{1} (\mu P - 1/2) \frac{dF(\mu)}{1 - F(Q(P))} - V^N(P) \right]$$

If $\hat{F}$ dominates $F$ in the first order, then the conditional $\frac{\hat{F}(\mu)}{1 - F(Q(P))}$ dominates $\frac{F(\mu)}{1 - F(Q(P))}$ on $\mu \in [Q(P), 1]$. It follows that $\int_{Q(P)}^{1} (\mu P - 1/2) \frac{dF(\mu)}{1 - F(Q(P))}$ increases with $F$. Because $1 - F(Q)$ increases with $F$, the result follows since the overall expression increases when the term in brackets is positive.

While of interest, this result does not immediately imply that industries where managers are more able, in the sense that $F$ increases in the first-order, will have more integration. Indeed, the expected output of integrated firms increases with $F$, and therefore we should expect a decrease in the price, a force that will tend to favor non integration.

Exogenous Productivity.

If firms have different productivity levels $\theta$ and if the price in the industry is $P$, Proposition 3 implies that firms with $\theta \in \left[ \frac{\bar{\theta}(h)}{P}, \frac{\bar{\theta}(h)}{P} \right]$ will be integrated, and among
these firms, the higher $\theta$ is the less centralization there will be since the probability of delegation is $F(Q(\theta P)\theta)$, increasing in $\theta$.

**Corollary 2.** Among integrated firms, more productive firms are less centralized.

**The Role of Cash Endowments**  Let us continue to assume that $U$ has no cash holding, but suppose now that $D$ has limited cash holding $L_D < h$.

Under integration, $D$ benefits from transferring cash ex-ante to $M$: this has the effect of softening the constraint (8) and therefore to increase the value of $s$ that would bind (8). Hence, the optimal integration problem is similar to a problem where $U, D$ have no cash holdings and the right hand side of (8) is replaced by $h - L_D$, and where there is the addition constraint that $t_D \geq 0$. In this class of problems, the transfer constraints bind, $t_U = t_D = 0$ since for any transfer that $M$ would make to $U, D$, the share $s$ will have to decrease, implying a decrease in the total surplus. There is a maximum share $\bar{s}(h, P)$ that binds (8), and given this share, a value of $\alpha$ binding (7). It is immediate to show that these shares are decreasing in $h - L_D$, hence increasing in $L_D$.

Contrary to the case of large cash holdings, where $V^I(P, h) = V^I(P, 0) - h$, limited cash holdings of $D$ imply that $V^I(P, h) < V^I(P, 0) - h$: transferring one extra utility payoff to $M$ requires a decrease in $s$ or $\alpha$ and an extra loss of surplus for $D$. For the same reason, an increase in $D$’s cash holding has a multiplier effect on the surplus from integration. Since $D$’s cash holding plays no role under non integration, integration is more likely as $L_D$ increases.

**Corollary 3.** Suppose that $U$ has no cash holding. As the cash endowment of $D$ increases, the integration contract specifies a larger share $s$ and a larger relative share $\alpha$ to $D$. This is a force towards integration and delegation.

**Outside Financing**  If integration involves a capital cost $K$ that has to be financed from an outside lender, the integration contract must satisfy an additional repayment constraint to the outside lender. This will lead to a decrease in the share $s$ going to $U, D$ and therefore like in [Jensen (1986)] a cost of capital greater than $K$. As $K$
increases, the share $s$ decreases and therefore the probability of delegation $F(Q(sP))$ will decrease.

**Corollary 4.** More leveraged integrated firms are more centralized.

### 3 Industry Equilibrium

The industry equilibrium will be affected by the nature of the managerial market, in particular on whether enterprises face an elastic or inelastic supply of managerial services. We will consider two situations. In the short-run, $m$ is exogenous but $h$ is endogenous; in the long-run, both $m, h$ are endogenous.

To make the comparison between the short and long run easier, we will assume that the total measure of $Ds$ and $Ms$ is equal to 1, and derive the proportion $m$ of managers. In the short-run, this proportion is exogenous, while it is endogenous in the long run.

The output under non integration is equal to $Q(P)$. We denote the output under integration by $Y(P)$:

$$Y(P) = Q(P) + \int_{Q(P)}^{1} (\mu - Q(P))dF(\mu),$$

(11)

It is clear that $Y(P) > Q(P)$ for each value of $P$.

#### 3.1 Short-Run

We first consider a short-run situation where the measure of managers is exogenously given, and we assume that $m$ is strictly less than $1/2$, that is managers are scarce.

A *short-run equilibrium* is defined by prices $(P, h)$ and organizational (integration or non integration together with shares of output) decisions for each chain such that:

- For each chain, $U, D$ are better off choosing the equilibrium organization.
- Product market clearing: total industry supply equals demand.

There is also a supplier market clearing condition that there is no excess demand or supply of $Us$, but this reduces to the condition that the $Us$ get a zero equilibrium payoff.

Because the measure of managers is fixed, the market for managers may not clear; there can be excess demand if there is a positive value for integration since $m < 1/2$ and excess supply if there is a negative value for integration.
As \( P < P_0 \), there is no demand for integration and managerial services since the net payoff under integration is less than that under non-integration for any value of \( h \). For these prices, firms are non-integrated and total industry output is \((1 - m)Q(P)\).

As \( P > P_0 \), there is a demand for integration and managerial services as long as \( h \leq \Delta(P) \). Since the total supply of managerial services at price \( h < \Delta(P) \) is less than the demand \( 1 - m \), managers must have compensation \( h = \Delta(P) \) in the short run. It follows that there is a measure \( m \) of integrated firms and a measure \( 1 - 2m \) of non-integrated firms; the short run industry supply is equal to

\[
S(P, m) := Q(P) + m(Y(P) - 2Q(P)).
\]  

While \( Y(P) \geq Q(P) \) for all \( P \), as \( P \) is large \( Y(P) - Q(P) \) becomes small and therefore \( Y(P) < 2Q(P) \), implying that the industry supply would be larger if integration is prevented. There exists a unique value of \( P^* \) such that \( Y(P) = 2Q(P) \) and industry supply increases when \( m \) increases if \( P < P^* \), and when \( m \) decreases if \( P > P^* \).

**Proposition 4.** Let \( P^* \) be the price level solving \( Y(P) = 2Q(P) \). Then, if the equilibrium product price is \( P < P^* \), increasing the proportion of managers will increase industry output, and if the equilibrium product price is \( P > P^* \), decreasing the proportion of managers will increase output.

**Proof.** Since demand decreases with \( m \) when the equilibrium price is \( P > P^* \) and supply increases with \( m \) when \( P < P^* \), as \( m \) increases the supply curve rotates to the left around \((P^*, Q(P^*))\). The result follows. 

This result shows that scarcity of managerial talent is not necessarily detrimental to achieving high levels of output in an industry. The opportunity cost of having managers is the reduction in the number of chains that can be formed—an industry scale effect. But managers facilitate integration of production chains and increase their output—a firm scale effect. When the price of output is low, the firm scale effect dominates; reducing scarcity of managerial talent is output enhancing. However when prices are high, the industry scale effect dominates; increasing scarcity of talent is output enhancing.

This begs however the question of whether such effects would arise when individuals can choose their occupation anticipating the demand for integration by chains. We turn to this case now.

\[10\] Indeed, the variation of \( Y(P) - Q(P) \) is \(-(1 - F(Q(P))Q'(P)) < 0 \) while \( Q(P) \) is an increasing function of \( P \).
3.2 Long Run OAS

We model the long run by assuming that out of a measure 1 of agents, a proportion \( m \) choose to become managers at cost \( E \), and the remaining proportion become producers, holding assets \( D \). There is a measure larger than 1 of \( U \) suppliers.

When the managerial market is active, and a manager has a compensation of \( h - E \), the payoff of a \( D \) producer is \( V^I(P, 0) - h \) when there is integration. Therefore if there are integrated firms, it must be the case that the following occupational indifference condition holds:

\[
h^o(P) := \frac{V^I(P, 0) + E}{2}.
\]

Now, there is a demand for managerial services only if \( h \leq \Delta(P) \). Therefore, a necessary and sufficient condition for an active managerial market is:

\[
\Delta(P) \geq \frac{V^I(P, 0) + E}{2}.
\]  \hspace{1cm} (13)

Consider the case \( E = 0 \). From Corollary 2 at \( P = 0 \) the variation of \( V^I(P, 0) \) is greater than \( Y(0) = \int_0^1 \mu dF(\mu) \). Since the variation of \( V^N(0) \) is equal to \( Q(0) = 0 \), it follows that the variation of \( V^I(P, 0) \) is smaller than the variation of \( 2\Delta(P) \) at \( P = 0 \). Hence, the occupational indifference locus (13) and \( \Delta(P) \) are as in figure 3; there is integration in the long run only if the final price is in the interval \([P_L, P_H]\).

![Figure 3: Occupational and Organizational Indifference Conditions in the Long Run \((E' > E)\)](image-url)
As $E$ increases, the occupational indifference locus shifts upwards. Note that as $E$ first increases beyond $E = 0$, the price interval consistent with integration is such that both bounds decrease, and there exists $E_0 > 0$ such that $P_L(E_0) = P_0$. Then, for values of $E$ larger than $E_0$, the lower bound increases while the upper bound decreases; see such a case in figure [3].

When $P \in (P_L, P_H)$, all chains want to be integrated and therefore it is necessary that $m = 1/2$. It follows that the long-run OAS of the industry is:

$$S(P; E) = \begin{cases} 
Q(P) & \text{if } P \notin [P_L(E), P_H(E)] \\
Y(P)/2 & \text{if } P \in (P_L(E), P_H(E)) \\
mY(P) + (1 - 2m)Q(P) & \text{where } m \in [0, 1], \text{ if } P \in \{P_L(E), P_H(E)\}.
\end{cases}$$  \hspace{1cm} (14)

\[ \text{Figure 4: Long run OAS; } E' > E \]

**Is there too much or too little management in the long-run?** Suppose that $E$ is larger than $E_0$ solving $P_L(E_0) = P_0$. Then, as $E' > E$, we have $P_L(E') > P_L(E)$ and $P_H(E') < P_H(E)$.

Let $P^*(E)$ denote the long run equilibrium price when the cost of becoming a manager is $E$. 

Proposition 5. Increasing the cost of becoming a manager increases the long run equilibrium output if, and only if, the initial equilibrium price is greater than $P^*$. 

Proof. Suppose that $P^e(E) > P^*$. By the arguments in the text, for any $P > P^*$, $S(P; E') \geq S(P; E)$ where the inequality is strict when $P \in (P_H(E'), P_H(E))$; the result follows since for any demand function the equilibrium price $D(P) = S(P; E)$ is a decreasing function of $E$. If $P < P^*$, $S(P; E') \leq S(P; E)$ with a strict inequality if $P \in (P_L(E), P_L(E'))$; the result follows. □

4 Conclusion

A benefit of developing a single framework for analyzing firm boundaries and internal organization is to clarify the difference between outsourcing decisions and delegation decisions. In our framework, if integration is prevented, all chains outsource part of their supply, but the degree of delegation in integrated firms is a function of the level of price because it correlates with the efficiency of delegation. Since the price level also determines the desire of asset holders to integrate, there may be a covariation between integration and delegation.

References


A Biased Managers

In the model, managers are indifferent among all decisions that maximize expected revenue. Suppose however that a manager can be biased in the following sense. He has lexicographic preferences, with first element the revenue, and with second element the average cost to $U, D$ from decisions, where the weight on $U$’s cost is $\beta$ and the weight on $D$’s cost is $1 - \beta$. $\beta \in [0, 1]$ is realized after contracting takes place.

Hence when the manager makes a decision, he internalizes with probability $\beta$ the cost of $U$ and with probability $1 - \beta$ the cost of $D$. Our basic model is similar to a situation where $\beta = 1/2$ for then when the manager makes a decision, he chooses a unique decision $u = d$ in order to maximize revenues, but then chooses $u = d = 1/2$ in order to minimize $1/2(1 - u)^2 + 1/2d^2$.

Decentralization choices are not affected by the bias. Indeed, the manager makes this choice on the basis of the expected revenue that he will get. For the resulting equilibrium choice of decisions $(u, d)$ under decentralization, the manager does not want to overturn these decisions since by doing so he can get at most a revenue of $\mu$.

Hence biases will change the value of integration for $U, D$ since it will modify the expected cost under centralization.

The decision under centralization is to minimize the total cost, that is $d^*(\beta) := \arg \min \beta(1 - d)^2 + (1 - \beta)d^2$, that is $d^*(\beta) = \beta$. Therefore the total expected cost under bias is

$$\int_0^1 [(1 - \beta)^2 + \beta^2] dG(\beta).$$

(15)

which is greater than $1/2$ for any non trivial distribution $G$. Hence, biased managers decrease the likelihood of integration with respect to our basic model.

B Proof of Corollary 2 (i)

Let us denote the Lagrange coefficients of the constraints (7), (8) and (9) by $\lambda, \phi, \xi$, the functions on the right hand sides of (7) and (8) by $v_U(s, \alpha, t_U, P)$ and $v_M(s, \alpha, t_U, t_D, P)$, the objective function by $v_D(s, \alpha, t_D, P)$. Then the Lagrangian is:

$$\mathcal{L}(s, \alpha, t, P) = v_D(s, \alpha, t_D, P) + \lambda v_U(s, \alpha, t_U, P) + \phi v_M(s, \alpha, t_U, t_D, P) + \xi t_U.$$  (16)
It follows that:

\[
\frac{d\mathcal{L}}{dP} = \frac{\partial \alpha}{\partial P} \mathcal{L}_\alpha + \frac{\partial s}{\partial P} \mathcal{L}_s + \frac{\partial t_U}{\partial P} \mathcal{L}_{t_U} + \frac{\partial t_D}{\partial P} \mathcal{L}_{t_D} \\
+ \frac{\partial \lambda}{\partial P} \mathcal{L}_\lambda + \frac{\partial \phi}{\partial P} \mathcal{L}_\phi + \frac{\partial \xi}{\partial P} \mathcal{L}_\xi \\
= \mathcal{L}_P.
\]

The third equality is due to the fact that \(\alpha, s\) are constant and therefore have zero variation, that \(\mathcal{L}_{t_U} = \mathcal{L}_{t_D} = 0\) by optimality and interiority of \(t_U, t_D\), and that \(\mathcal{L}_\lambda = \mathcal{L}_\phi = 0\) since the two constraints bind. Finally, since \(t_U > 0, \xi = 0\). Note that \(\mathcal{L}_{t_D} = 1 - \phi\) and therefore \(\phi = 1\). Now, \(\mathcal{L}_{t_U} = \lambda - \phi - \xi\), and since \(\xi = 0\) it follows that \(\lambda = \gamma = 1\). Now,

\[
\mathcal{L}_P = \frac{\partial v_D(s, \alpha, t_D, P)}{\partial P} + \lambda \frac{\partial v_U(s, \alpha, t_U, P)}{\partial P} + \frac{\partial v_M(s, \alpha, t_U, t_D, P)}{\partial P}
\]

Hence,

\[
V^I_P(P, h) = \frac{d}{dP} \left[ \int_{Q(P)}^1 (\mu P - \frac{1}{2}) \right] dF(\mu) + F(Q(P))V^N(P) \\
= Q(P) + \int_{Q(P)}^1 (\mu - Q(P))dF(\mu)
\]

as claimed since \(\frac{dV^N(P)}{dP} = Q(P)\).