Subprime Mortgages and the Housing Bubble

by†

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Abstract

This paper explores the link between the house-price expectations of mortgage lenders and the extent of subprime lending. It argues that bubble conditions in the housing market are likely to spur subprime lending, with favorable price expectations easing the default concerns of lenders and thus increasing their willingness to extend loans to risky borrowers. Since the demand created by subprime lending feeds back onto house prices, such lending also helps to fuel an emerging housing bubble. The paper, however, focuses on the reverse causal linkage, where subprime lending is a consequence rather than a cause of bubble conditions. These ideas are illustrated in a theoretical model, and empirical work tests for a connection between price expectations and the extent of subprime lending.

† *The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia, the Federal Reserve Board of Governors, or the Federal Reserve System. This paper is available free of charge at http://www.philadelphiafed.org/research-and-data/publications/working-papers./*

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1. Introduction

The spectacular run-up in US housing prices over the first years of the new century, along with the subsequent price collapse, are watershed events in real-estate history. No previous brief period witnessed such dramatic price escalation (on the order of 50 percent), and the rapid 30 percent price drop that ensued was also unprecedented.

Researchers have expended considerable effort in trying to understand these events, but the understanding is thus far incomplete. Investigators have asked whether expansionary monetary policy early in the decade, and the resulting sustained period of low mortgage interest rates, fueled the demand for housing, driving up prices. They have asked whether the growth in subprime lending, occurring partly in response to affordable-housing goals, was the culprit, amplifying the effect of low interest rates on demand. Researchers have also asked whether high loan-to-value ratios along with irresponsibly lax mortgage underwriting were the sources of price escalation. Finally, investigators have tried to link house-price patterns to market fundamentals such as population and income growth.

A recent paper by Glaeser, Gottlieb and Gyourko (2010) conveniently tests a number of these competing explanations in the context of a single study, and the results are not particularly affirmative. Relying on a multi-decade regression relating house prices to interest rates, the authors find that low interest rates can explain at most half of the price escalation through 2008. To measure the effect of lax underwriting standards (and the consequent easing of mortgage access), the authors use a regression of house prices on mortgage approval rates, finding little effect on prices. They also show that the increase in loan-to-value ratios over the period cannot explain the price explosion.

Similarly, although Coleman, LaCour-Little and Vandell (2008) show some role for market fundamentals in the price escalation through 2003, they cannot find a link to the expansion in subprime lending, low interest rates, or high LTV ratios.[1] But they argue that price increases through 2003 laid the foundation for a "bubble" psychology, where market participants grew to expect continuing future price increases.[2] Glaeser, Gottlieb and Gyourko (2010) reach a similar conclusion, arguing that, in the absence of measured links to the various factors they investigate, responsibility for the price explosion may lie in irrational expectations, as suggested by Case and Shiller (2003).

The role of expectations is studied by Mian and Sufi (2009). But instead of asking whether optimistic expectations drove the escalation of house prices, they ask the narrower question of whether favorable price expectations helped spur the growth of subprime lending. In particular, the authors provide a test of what they call the "expectations hypothesis," which asserts that expectations of future house-price growth, by easing default concerns, spurred subprime lending. They test the hypothesis in a highly indirect fashion. The approach compares price growth in zip codes with low credit scores at the beginning of the period (potential subprime areas) to price growth in non-subprime zip codes. The authors find no difference in price escalation between potential subprime areas, where most subprime lending occurred, and non-subprime zip codes, focusing on areas with elastic housing supply. They conclude that the faster expansion of mortgage credit in subprime areas could not have been driven by favorable price expectations, given that the actual price path was no different in these areas than in non-subprime zip codes where credit expanded less. Instead, Mian and Sufi conclude that subprime expansion was driven by supply factors, including relaxation of underwriting standards and the growth of mortgage securitization.

As Glaeser et al. (2010) and Coleman et al. (2008) have argued, expectations likely played a significant role in generating the housing "bubble." Mian and Sufi (2009) explore this channel through their indirect test for an expectations effect in subprime lending, implicitly viewing such lending as a driver of the bubble. However, given the likely importance of price expectations and the literature's limited exploration of their effects, additional research is clearly needed. The present paper fills this need by further exploring the role of lender price expectations in the operation of real-estate markets. The paper investigates a causal channel like that posited by Mian and Sufi, arguing that a favorable shift in lender expectations regarding future prices spurs subprime lending by easing default concerns.[3] The resulting increase in the demand for housing then feeds back into the market, driving up today's housing prices. So expectations of future price growth generate current price escalation, feeding a housing bubble.[4] Thus, the paper is built around the notion that subprime lending is *both a consequence and a cause of bubble conditions in the housing market*. Most of the effort is devoted to exploring the first channel, where the presence of a bubble spurs subprime lending, but the other causal direction is also kept in focus.[5]

These ideas are developed in a detailed theoretical model, and the paper then offers a more direct test of the link between price expectations and subprime lending than the one offered by Mian and Sufi (2009). In the model, subprime lending is portrayed as an extension of credit to borrowers with low "default costs." These costs represent the penalty incurred by a borrower who defaults on a mortgage, which include the cost of credit impairment, moving costs, and possibly the costs associated with guilt. With their poor credit ratings, subprime borrowers presumably have low default costs since default will not greatly worsen a credit standing that is already bad.

Although a previous paper by Brueckner (2000) analyzed mortgage lending when default costs are private information, the present analysis assumes that these costs (denoted C) are observable. The model generates a minimum value of C below which mortgage lenders cannot profitably offer a mortgage, and market-level changes that reduce this minimum will generate an expansion in subprime lending.

One such change is a shift in lender expectations regarding future house prices. With a more optimist view of future prices, default is less of a concern for lenders, allowing them to extend mortgages to borrowers with even lower C's, thus leading to an expansion of subprime lending. While the basic analysis in section 2 below makes this point, an extension of the model allows the expansion of lending to feed back onto current house prices, bidding them up. Section 3 of the paper goes further by embedding the model in an explicitly dynamic setting, where a shift in expectations generates an intertemporal adjustment process.

To test for the effect of price expectations on subprime lending, the paper uses the Consumer Credit Panel database from the Federal Reserve Bank of New York, aggregated up to the state level over the 2001-2008 period. The borrower-quality measure is the credit "risk score," which is analogous to the more familiar FICO score, being a measure of the borrower's creditworthiness. The dependent variable in the regressions is a state-level risk-score measure, equal to the mean risk score for new borrowers in the state or, alternatively, the 10th or 25th percentile of the state's risk-score distribution. The key explanatory variable, which is intended to capture price expectations, is the lagged annual rate of house-price appreciation, which relies on the state-level CoreLogic price index. The regressions include additional covariates as well as state and quarter fixed effects.

While the regression is meant to test for the causal path running from price expectations to subprime lending, the reverse path (by which subprime lending causes price escalation) is a potential source of simultaneity bias. Appropriate steps are taken to deal with this bias. The empirical results, presented in section 4 of the paper, provide support for the main hypothesis by suggesting a causal link between prior price appreciation and subprime lending at the state level.

2. The Model

2.1. The setup

The model, which draws on the framework of Brueckner (2000), is simple and stylized along a number of dimensions. The first simplifying assumption is that the mortgage term consists of a single period. The borrower arranges a loan to purchase a standardized house of value P_0, and without loss of generality, the mortgage is assumed to be a 100 percent loan, so that the amount borrowed equals P_0. Repayment is required in the subsequent period. The balance-due, denoted B, includes the principal P_0 as well as interest, with the interest rate r implicitly defined by $1 + r = B/P_0$.

The value of the house in the next period is denoted P. Since P may be lower than P_0, with the house value falling over the period, the borrower may default on the mortgage. Default occurs when P is low relative to B, in which case the borrower is better off surrendering the house to the lender via foreclosure rather than paying off the mortgage. The resulting loss of the low-valued housing asset is more than offset by cancellation of a larger liability.

The critical value of P, below which default occurs, depends on the magnitude of default costs. These costs, denoted C, include the cost of credit impairment, moving costs, and any psychic costs from failing to honor the mortgage contract, as noted above. The role of default costs, which are sometimes called transaction costs, has been analyzed previously by Kau, Keenan and Kim (1993, 1994), Riddiough and Thompson (1993), Brueckner (2000), Foote, Gerardi, and Willen (2008) and others. In the model, default costs are heterogeneous across borrowers, ranging from a minimum value $\underline{C}$ to a maximum value of $\overline{C}$.

The magnitude of some elements of C, and thus the outcome of the default decision, can be affected by "trigger events," as discussed by Foote, Gerardi, and Willen (2008).[6] Thus, default costs provide a way in which trigger events can play a role in a model that retains the spirit of the home-equity approach to default decisions.

Taking account of default costs, the borrower defaults on the mortgage when P satisfies

$$P \; - \; B \;\; < \;\; - \, C. \tag{1}$$

$P - B$ in (1) equals housing equity, which is retained by the borrower if he repays the mortgage and thus represents the (possibly negative) increment to wealth in the absence of default. Conversely, $-C$ in (1) gives the (negative) increment to wealth if default occurs. Therefore, wealth is larger with default when housing equity is less than $-C < 0$. Note that the presence of default costs implies that equity must fall well below zero before default is desirable. Rearranging, (1) shows that default is optimal when

$$P \;\; < \;\; B \; - \; C. \tag{2}$$

Thus, default occurs when the house price is less than the mortgage balance-due less default costs.

In contrast to the analysis of Brueckner (2000), where default costs are private information, the current model assumes that C is observable. In reality, the borrower's credit rating appears to give a good picture of the default propensity, with C small (and default more likely) when the credit score is low. The model assumes that there are no other unobserved influences on C, which is perfectly proxied by the credit score.

With default costs thus observable, the lender tailors the mortgage contract offered to the borrower to reflect the observed value of C. Competition among lenders ensures that the resulting contract generates zero lender profit.

Computation of profit involves a key element: lender expectations regarding the next period's house prices, which help determine the perceived likelihood of default. Expectations are represented by the continuous density function $f(P)$, which is the same for all lenders. Generally, the density f will depend on the entire past history of house prices, which will affect both the mean and variance of the distribution of anticipated prices, perhaps through a Bayesian updating process. The basic analysis that follows, however, does not require a description of exactly how expectations are formed. Instead, the main focus is on the effect of an *exogenous shift* in expectations, conditional on a general form for f. The formation of expectations must be specified, however, when the goal is to analyze the intertemporal adjustment process generated by an expectations shift, as seen in section 3 below.

Suppose the lender is risk neutral, focusing on expected profit, and that he incurs a mortgage origination cost of k for each loan issued. Then, incorporating f and letting $\eta < 1$ denote the lender's discount factor, the expected present value of profit from offering a loan with a balance-due of B to a borrower with default costs C is given by

$$\pi \;\equiv\; -\,k\,-\,P_0\,+\,\eta\int_0^{B-C} Pf(P)dP\,+\,\eta\int_{B-C}^{\infty} Bf(P)dP. \qquad (3)$$

Note that the lender transfers P_0 to the borrower when the loan is made. In the next period, he receives the loan balance B from the borrower over the range of house values where $P \geq B-C$ holds and default does not occur. He takes possession of the house through foreclosure, earning P from resale of the property, over the range of values where $P < B - C$ and default does occur. Without loss of generality, the foreclosure costs incurred by the lender are assumed to be zero.[7]

Although this formulation portrays the lender as using his own money to extend the loan, the analysis would be unaffected if he relied instead on borrowed funds (bank deposits). Then the outlay of P_0 would disappear from (3), being replaced by the discounted next-period interest payment $-\eta z P_0$, where z is the lender's cost of funds. Note that under either formulation, the supply of mortgage funds is viewed as perfectly elastic, unaffected by the total volume of lending.

The zero-profit condition for lenders requires that π in (3) equals zero. The resulting equation then implicitly defines a value of B that yields zero expected profit for a given value of C. Recalling that $1 + r = B/P_0$, the equation also implicitly defines the interest rate that generates zero profit from lending to a borrower with the given default costs.

2.2. The relation between B and C

Setting (3) equal to zero and totally differentiating, the effect of C on B is given by

$$\frac{\partial B}{\partial C} \quad = \quad -\frac{\pi_C}{\pi_B}, \tag{4}$$

where π_C and π_B are the derivatives of π with respect to these variables. Using Leibniz's rule,

$$\pi_C \quad = \quad \eta[Bf(B-C) - (B-C)f(B-C)] \quad = \quad \eta C f(B-C) \quad > \quad 0. \tag{5}$$

By making default less likely for a given B, higher default costs thus raise profit.

Similarly, the effect of B on profit is given by

$$\pi_B \quad = \quad \eta\left[\int_{B-C}^{\infty} f(P)dP \quad - \quad Cf(B-C)\right], \tag{6}$$

where the first term comes from differentiating under the integral in (1). The sign of (6) is ambiguous as a result of two opposing effects. First, an increase in B raises the lender's revenue in the event that default does not occur, an effect captured by the first term in brackets. However, an increase in B makes default more likely, which has the opposite effect and leads to the negative second term in (6). One would expect the positive effect to dominate the second one, so that a larger balance-due raises expected profit. This outcome, for example, is guaranteed in the case where f is uniform and given by $1/(\overline{P} - \underline{P})$ over the support $[\underline{P}, \overline{P}]$, provided that an additional assumption holds. In this case,

$$\pi_B = \eta \frac{\overline{P} - B}{\overline{P} - \underline{P}}, \tag{7}$$

which is positive under the assumption that the lender does not ask the borrower to pay back more than the maximal anticipated next-period house price ($B < \overline{P}$). More generally, with a unimodal f, it can be shown that π_B is positive provided that the density is not too steep over its descending range.

Assuming that $\pi_B > 0$, (4) is then negative, implying $\partial B/\partial C < 0$. Thus, an increase in C allows a reduction in B, with the lender setting the balance-due lower, or equivalently charging a lower interest rate, for a better quality borrower (one with a higher C). Since such a borrower is less likely to default, the amount B that the bank recovers when default does not occur can be set lower. Therefore, a curve relating B to C is downward sloping over the relevant range, as shown in Figure 1.[8]

That range is limited at the upper end by $\overline{C}$, the maximal value of default costs. A key additional assumption puts a lower limit on C. This assumption realistically imposes an upper bound on B via a stylized form of the mortgage payment-to-income constraint. Such a constraint has been a key feature of mortgage underwriting for decades, although income requirements were substantially loosened for subprime borrowers in recent years.

To see the effect of the payment-to-income constraint, note that the mortgage payment is a component of B, being equal to $rP_0 = B - P_0$. Assuming that borrower incomes are uniform and equal to y despite heterogeneous default costs, the payment-to-income constraint is written $rP_0/y \leq \alpha$ for some constant $\alpha < 1$. Substituting, the constraint becomes $(B - P_0)/y \leq \alpha$, or

$$B \leq \alpha y + P_0 \equiv \widehat{B}. \tag{8}$$

Let the value of C associated with $\widehat{B}$ in Figure 1 be denoted $\widehat{C}$. Then, referring to the figure, the $B \leq \widehat{B}$ requirement implies $C \geq \widehat{C}$, so that an upper limit on the balance-due puts a floor of $\widehat{C}$ under allowable default costs. The reason is that, in lending to a borrower with default costs below $\widehat{C}$, zero profit would require a balance-due above the limit fixed by the payment-to-income constraint. Thus, mortgages cannot be offered to borrowers of less than a certain quality, those with C values lying below $\widehat{C}$. As seen in Figure 1, $\underline{C}$ is assumed to be less than $\widehat{C}$, so that low-quality potential borrowers in the range $[\underline{C}, \widehat{C}]$ cannot get a mortgage.

The discussion so far has implicitly assumed that borrowers will always accept an offered mortgage, regardless of the magnitude of the balance-due. In effect, (owner-occupied) housing is assumed to be essential, so that a mortgage will be accepted regardless of its cost. Thus, the demand more mortgage funds is perfectly inelastic.

2.3. Subprime lending

In the model, subprime lending corresponds to a reduction in $\widehat{C}$. In other words, subprime lending gives some low-quality potential borrowers, who previously could not get a loan, access to mortgage funds. Parametric changes in the model can generate a reduction in $\widehat{C}$ and thus the emergence of subprime lending. Three changes in particular are of interest. The first is a relaxation of the payment-to-income constraint, as has occurred in recent years. The second change is a decline in the cost of lending, perhaps reflecting efficiency gains from greater reliance on the Internet. The third change, which is the main focus of the analysis, is a favorable shift in the lender's house-price expectations.

To analyze the effects of these changes, the explicit condition that determines $\widehat{C}$ is needed. This condition comes from setting π from (3) equal to zero, and then substituting $\widehat{B}$ in place of B. The resulting condition determines the associated C value, namely $\widehat{C}$, and it is written

$$-\,k \;-\; P_0 \;+\; \eta\int_0^{\widehat{B}-\widehat{C}} Pf(P,\delta)dP \;+\; \eta\int_{\widehat{B}-\widehat{C}}^{\infty} \widehat{B}f(P,\delta)dP \;=\; 0. \tag{9}$$

In (9), δ is a shift parameter for the house-price density. It is assumed that an increase in δ shifts the density toward higher values in the sense of stochastic dominance. In other words, with an increase in δ, the cumulative distribution function shifts downward at each value of P within the support of f. Thus, the δ derivative of $F(P,\delta) \equiv \int_0^P f(z,\delta)dz$ satisfies $F_\delta(P,\delta) < 0$ over this support, where the subscript denotes partial derivative.

Using (9), the effect on $\widehat{C}$ of an increase in α, the maximum payment-to-income ratio, is easily derived. Totally differentiating (9) with respect to α and $\widehat{C}$, recognizing that α determines $\widehat{B}$ via (8), yields

$$\frac{\partial \widehat{C}}{\partial \alpha} \;=\; -\frac{\pi_B(\partial \widehat{B}/\partial \alpha)}{\pi_C} \;<\; 0, \tag{10}$$

where the inequality follows because $\pi_C > 0$, π_B is assumed positive, and $\partial \widehat{B}/\partial \alpha > 0$ from (8). Thus, by raising the maximum possible B, relaxation of the payment-to-income constraint lowers the floor on C, allowing $\widehat{C}$ to fall and spurring subprime lending. In other words, by allowing lenders to recoup a larger balance-due when default does not occur, a higher α allows lenders to extend mortgages to borrowers of even lower quality without incurring a loss.[9]

Similarly, since $\pi_k = -1$, it follows that $\partial \widehat{C}/\partial k = 1/\pi_C > 0$. Thus, by raising profit, a decline in mortgage-origination costs allows lenders to extend loans to lower-quality borrowers.

The effect on $\widehat{C}$ of a shift in the lender's house-price expectations requires more extensive computation. The appendix shows that an increase in δ raises π holding $\widehat{C}$ fixed, with

$$\pi_\delta \;=\; -\eta \left[C F_\delta(\widehat{B} - \widehat{C}, \delta) \;+\; \int_0^{\widehat{B}-\widehat{C}} F_\delta(P, \delta) dP \right] \;>\; 0. \tag{11}$$

Therefore, differentiating (9),

$$\frac{\partial \widehat{C}}{\partial \delta} \;=\; -\frac{\pi_\delta}{\pi_C} \;<\; 0. \tag{12}$$

A favorable shift in the house-price density thus leads to a reduction in $\widehat{C}$, with more optimistic price expectations spurring subprime lending. With better expectations, the perceived likelihood of default declines, other things equal, allowing loans to be extended to even lower-quality borrowers without generating a loss for the lender.

Recognizing that a shift in expectations is characteristic of a housing bubble, the results in (10) and (12) are summarized as follows:

Proposition 1. *A favorable shift in the density of anticipated future house prices (as occurs under a housing bubble) spurs subprime lending, with the caliber of the* *lowest-quality borrower falling. This effect is reinforced by a relaxation of the payment-to-income constraint or a decline in mortgage-origination costs.*

2.4. Endogenizing P_0

So far, the initial house price P_0 has been viewed as fixed despite the increase in the number of home buyers that follows from a decline in $\widehat{C}$. A full analysis, however, should allow P_0 to be endogenous, so that it can change along with $\widehat{C}$ in response to changes in the model's parameters. The size of houses, however, remains fixed at some standardized value.

To carry out this extension, the first step is to relate the number of demanders of housing to $\widehat{C}$. Let $G(C)$ denote the cumulative distribution function for default costs over the $[\underline{C}, \overline{C}]$ range, with $G(\underline{C}) = 0$ and $G(\overline{C}) = N$, the number of potential mortgage borrowers. Then, the number of housing demanders as a function of $\widehat{C}$ is simply $N - G(\widehat{C})$, which gives the number of individuals with C values large enough to get a mortgage.

On the supply side, let $S(P_0)$ denote the supply function for new houses, which gives the number of houses supplied as an increasing function of price. Note that housing production is assumed to be instantaneous, so that producers have no need to consider prices outside of the current period. Taking account of the supply function, the number of houses available would then equal the stock passed on from the previous period, denoted H, plus $S(P_0)$. The condition that equates housing supply and demand is then

$$\Phi \;\equiv\; H \;+\; S(P_0) \;-\; [N \;-\; G(\widehat{C})] \;=\; 0. \tag{13}$$

While a high price generates an increase in the number of houses, so that $S(P_0) > 0$ holds for large values of P_0, units are removed from the housing stock when P_0 is low. This outcome reflects the existence of some alternate use for the housing land input, which is superior when housing commands a low price. Thus, the supply function satisfies $S(P_0) > (<)\ 0$ when $P_0 > (<)\ P^*$, with P^* giving the price where supply is zero.

The second equilibrium condition, which is based on (9), links P_0 and $\widehat{C}$ from the lender's side. The required modification comes from (8), which makes $\widehat{B}$ a function of the now-endogenous P_0 instead of simply a constant. This function is $\widehat{B}(P_0) \equiv \alpha y + P_0$, and it shows a positive relationship between $\widehat{B}$ and P_0.[10] Substituting $\widehat{B}(P_0)$ in place of $\widehat{B}$ in (9), the condition is rewritten as

$$-\,k \;-\; P_0 \;+\; \eta\int_0^{\widehat{B}(P_0)-\widehat{C}} Pf(P,\delta)dP \;+\; \eta\int_{\widehat{B}(P_0)-\widehat{C}}^{\infty} \widehat{B}(P_0)f(P,\delta)dP \;=\; 0. \qquad (14)$$

Conditions (13) and (14) jointly determine equilibrium values of $\widehat{C}$ and P_0.

The effect on these equilibrium values of a shift in house-price expectations can be analyzed diagrammatically. The equilibrium is the intersection of two locii in $(\widehat{C}, P_0)$ space, with one given by (13) and the other by (14). The locus given by (13), denoted the "supply-demand (s-d) locus," is downward sloping. Totally differentiating (13), the slope of the locus is given by

$$\frac{\partial P_0}{\partial \widehat{C}}_{\,|s-d} \;=\; -\frac{\Phi_{\widehat{C}}}{\Phi_{P_0}} \;=\; -\frac{G'(\widehat{C})}{S'(P_0)} \;<\; 0, \qquad (15)$$

where the sign follows from $G', S' > 0$.

The locus given by (14), denoted the "π locus," is upward sloping. Its slope is given by

$$\frac{\partial P_0}{\partial \widehat{C}}_{\,|\pi} \;=\; -\frac{\pi_C}{\pi_{P_0}} \;=\; \frac{-\pi_C}{\pi_B(\partial\widehat{B}/\partial P_0) - 1} \;>\; 0. \qquad (16)$$

Given $\pi_C > 0$, the inequality in (16) is a consequence of the negativity of the last denominator expression, which is easily demonstrated.[11]

Figure 2 illustrates the s-d and π locii and their intersection. When house-price expectations shift, with δ increasing, the π locus shifts, while the s-d locus (which is independent of δ) remains fixed. The shift in the π locus is leftward in the $\widehat{C}$ direction, as shown in the figure. This conclusion follows given that $\partial\widehat{C}/\partial\delta|_\pi < 0$ holds from (12), which indicates that $\widehat{C}$ falls as δ increases, holding P_0 fixed. As seen in Figure 2, this shift in the π locus reduces the equilibrium value of $\widehat{C}$ while raising P_0.

Since (10) shows that the π locus again shifts to the left when α increases, a relaxation of the payment-to-income constraint has the same impacts on $\widehat{C}$ and P_0 as a shift in expectations. These same effects also arise when origination costs k decline. Summarizing yields.

Proposition 2. *When the current house price P_0 is endogenous, a rightward shift in the density of anticipated future prices (as occurs under a housing bubble) raises P_0 while again spurring subprime lending, with the caliber of the lowest-quality borrower falling. A relaxation of the payment-to-income constraint or a decline in mortgage origination costs reinforces these effects.*

Thus, growth in subprime lending continues to be a consequence of a shift in future house-price expectations, a relaxation of the payment-to-income constraint or a decline in mortgage-origination costs. In addition, all three changes feed back onto current prices, making houses more expensive.

2.5. Eventual default

The actual default decisions of borrowers occur in the subsequent period, and they depend on the house price that emerges in that period. To analyze default, the realized value of P must be inserted into the default condition (2) along with the balance due B owed by a given borrower, which is shown by the curve in Figure 1. To write the balance-due as a function of the relevant variables, let (3) be rewritten to include the shift parameter δ. Then, setting (3) equal to zero, the condition determines B as a function of C and the other variables P_0 and δ, written as $B = \phi(C, P_0, \delta)$. This function is decreasing in C, as seen above, and it is also increasing in P_0 and decreasing in δ.[12]

Let $\widetilde{C}$ denote the critical value of C that makes a borrower indifferent in the subsequent period between defaulting and not doing so. Referring to (2), $\widetilde{C}$ satisfies

$$P \;=\; \phi(\widetilde{C}, P_0, \delta) \;-\; \widetilde{C}. \tag{17}$$

Since ϕ is decreasing in C, it follows that $\phi(C, P_0, \delta) - C$ is also decreasing in C, which means that $P < \phi(C, P_0, \delta) - C$ holds for $C < \widetilde{C}$. Thus, after taking account of B's dependence on C, actual defaulters are those borrowers with the lowest default costs.

For future reference, note that (17) determines the critical value $\widetilde{C}$ as function of the remaining variables in the equation. This function, which is written $\widetilde{C}(P, P_0, \delta)$, is decreasing in P given that a larger realized house value makes a borrower less prone to default, requiring C to fall below a lower critical value to make default desirable. In addition, since a larger P_0

raises B, higher default costs are required to forestall default, so that $\widetilde{C}$ is increasing in P_0. Finally, since a higher δ reduces the required B, lower default costs are required to induce default, making $\widetilde{C}$ a decreasing function of δ.[13]

While this discussion is silent about how the realized P value is determined, this omission is remedied below when the model is embedded in a dynamic setting.

3. Making the Model Dynamic: An Example

While the paper's empirical work relies only on Proposition 2, further insight can be gained by using the model to carry out a dynamic analysis. In a dynamic setting, a one-time shift in house-price expectations leads to an intemporal adjustment process. Analysis of this process can give insight into the evolution of housing and mortgage markets in the presence of a housing bubble.

The first step is to attach a time index t to P_0 and $\widehat{C}$, which are now written P_0^t and $\widehat{C}^t$. The realized house price in the period following t then equals the P_0 value for that period, or P_0^{t+1}. The next step is to specify exactly how the lender's house-price expectations are formed, a question that could be skirted in the analysis up to this point. While the expectations density f will depend on the entire past history of house prices, as mentioned above, this dependence is assumed to take a very simple form in order to permit a tractable analysis. Given this limitation, the ensuing discussion should be viewed as only providing an example of how the model might behave in a dynamic setting.

The simplifying assumption is that the position of the density of anticipated prices depends *on the previous period's house price*, with a change in that price shifting the density without changing its shape. Thus, the time-t density for the next period's anticipated price is written $f(P - P_0^{t-1})$, so that a given increase in P_0^{t-1} shifts the density to the right by the amount of the increase.[14] This assumption could correspond, for example, to a situation where f is symmetric and centered at P_0^{t-1}, so that a change in P_0^{t-1} alters the mean of the distribution while its variance is independent of the past history. In addition, a change in expectations, as reflected in an increase in δ, is assumed to shift the density in the same way as the past price, so that when δ is nonzero, the density is written as $f(P - P_0^{t-1} - \delta)$.

Incorporating these changes, the zero-profit condition (14) at time t is rewritten as

$$-k - P_0^t + \eta \int_0^{\widehat{B}(P_0^t)-\widehat{C}^t} Pf(P-P_0^{t-1}-\delta)dP + \eta \int_{\widehat{B}(P_0^t)-\widehat{C}^t}^{\infty} \widehat{B}(P_0^t)f(P-P_0^{t-1}-\delta)dP \;=\; 0. \tag{18}$$

To adapt the supply-demand condition in (13) to a dynamic setting, recall that H in (13) is the number of housing units inherited from the previous period. Assuming that housing is perfectly durable (unless intentionally removed from the stock), H simply equals the number of mortgage borrowers in the previous period. Therefore, (13) at time t becomes

$$N \;-\; G(\widehat{C}^{t-1}) \;+\; S(P_0^t) \;-\; [N \;-\; G(\widehat{C}^t)] \;=\; 0. \tag{19}$$

Finally, the dynamic setup involves a particular assumption on the fate of borrowers who default on their mortgages.[15]

Since P_0^{t-1} enters (18), the position of the π locus relating P_0^t and $\widehat{C}^t$ depends on P_0^{t-1}. An increase in P_0^{t-1} shifts that locus to the left, just like the increase in δ analyzed above. Similarly, the position of the s-d locus depends on $\widehat{C}^{t-1}$, with an increase in $\widehat{C}^{t-1}$ shifting the locus to the right.[16] Note that, with a higher $\widehat{C}^{t-1}$, a smaller housing stock is handed forward from time $t-1$, so that supply-demand equilibrium at t requires fewer current mortgage borrowers (a higher $\widehat{C}^t$) for any given P_0^t.

Conditions (18) and (19) constitute a system of difference equations that governs the evolution of the $\widehat{C}$ and P_0 variables over time. Suppose the economy is in a steady state up to time $\tau - 1$. The steady-state price equals the zero-supply price P^*, and the steady-state $\widehat{C}$ is denoted $\widehat{C}^*$.[17] Then, suppose that the equilibrium is perturbed at time τ by a shift in house-price expectations, with δ becoming positive. The π locus shifts as in Figure 2, with P rising and $\widehat{C}$ falling at time τ, as shown again in Figure 3. The equilibrium moves from the steady-state values P^* and $\widehat{C}^*$, which lie at the intersection of the steady-state π^* and s-d^* locii, to the values at the intersection of π^τ and s-d^* locii.

At time $\tau + 1$, the expectations shock is passed, with δ returning to zero. But the time-τ changes in P and $\widehat{C}$ affect the positions of the π and s-d locii at $\tau + 1$, following the above discussion. Since $\widehat{C}^{\tau+1} < \widehat{C}^*$, the *s-d* locus at time $\tau+1$ lies to the left of the steady-state locus, as seen in Figure 3. To locate the $\pi^{\tau+1}$ locus, it can be shown that the house price at time τ does not rise by the full amount of the expectations shift, so that $P_0^\tau < P^* + \delta$.[18] As a result, the π locus at $\tau+1$ (whose position depends on P_0^τ) is not as far to the left as the π^τ locus (whose position depends on $P^*+\delta$), as shown in Figure 3. Given these two shifts, it follows that *the house price at* $\tau+1$ *must be lower than at* τ, with $P_0^{\tau+1} < P_0^\tau$. Depending on the exact positions of the curves, $\widehat{C}$ could either rise or fall between times τ and $\tau+1$, with Figure 3 showing the latter case.[19]

This information can be used to investigate the time path of defaults. Letting $\widetilde{C}^t$ denote, for time t, the critical $\widetilde{C}$ value below which default occurs, the default rate at t is given by

$$D^t \quad = \quad \frac{G(\widetilde{C}^t) - G(\widehat{C}^{t-1})}{N - G(\widehat{C}^{t-1})}. \tag{20}$$

The denominator equals the number of consumers getting mortgages at $t-1$ (whose C values lie above $\widehat{C}^{t-1}$) and the numerator equals the number of these individuals who default at t (whose C values lies above $\widehat{C}^{t-1}$ and below $\widetilde{C}^t$). At time $\tau-1$, the G arguments in (20) equal the starred, steady-state values, yielding the steady-state default rate D^*.

The increase in the house price at τ reduces the incentive for default, pushing the critical $\widetilde{C}$ value below the steady-state level, so that $\widetilde{C}^\tau < \widetilde{C}^*$.[20] Since $\widehat{C}^{\tau-1}$ equals the steady value $\widehat{C}^*$, it follows from (20) that the default rate declines at time τ, with $D^\tau < D^*$.

In moving from time τ to $\tau+1$, the default rate is affected by changes in both $\widetilde{C}$ and the lagged $\widehat{C}$. Differentiation of (20) shows D^t rises when $\widehat{C}^{t-1}$ decreases, so that an expansion in subprime lending raises the subsequent default rate, holding $\widetilde{C}$ fixed.[21] Thus, the decline in the lagged $\widehat{C}$ from $\widehat{C}^{\tau-1} = \widehat{C}^*$ to $\widehat{C}^\tau < \widehat{C}^*$ tends to increase the default rate between times τ and $\tau+1$. However, the change in $\widetilde{C}$ between these periods is ambiguous.[22] Although the drop in P_0 raises the incentive to default, which tends to increase $\widetilde{C}$, the higher past price and the previous period's expectation shift have opposing effects on the balance-due owed at time $\tau+1$, which makes the direction of change in $\widetilde{C}$ unclear. However, if the $\widehat{C}$ effect dominates, the default rate will rise. Summarizing yields

Proposition 3. *Under the maintained assumptions, a house-price expectations shift at time τ raises the current house price P_0 above the steady-state level while reducing the default rate D at τ relative to its own steady-state level. In the subsequent period $(\tau+1)$, the house price falls relative to the level at time τ. The change in the default rate is ambiguous, but a large expansion in subprime lending at τ will tend to make it positive.*

The ambiguity regarding $D^{\tau+1}$ as well as the evolution beyond time $\tau+1$ can be explored numerically. The calculations assume that the density f at time t is uniform, with height $1/2\mu$ over the interval $[\underline{P}, \overline{P}] = [P_0^{t-1} + \delta - \mu, P_0^{t-1} + \delta + \mu]$. In addition, default costs are uniformly distributed, and the supply function is linear.

Under the parameter values listed in the appendix, $P^* = 5.0$ and $\widehat{C}^* = 2.2$. At time τ, P_0 rises to 5.36 and $\widehat{C}$ falls dramatically to 0.67, as seen in Figure 4. From a steady-state value of 1.7 percent, the default rate at time τ drops to zero, as seen in Figure 5. At time $\tau+1$, P_0 falls almost all the way back to the steady-state value, dropping to 5.01, while $\widehat{C}$ declines further to 0.57 (matching Figure 3). The previous period's expansion of subprime lending dominates in determining the change in the default rate, which surges to 40 percent at time $\tau+1$, a number that approximates actual experience.[23] At time $\tau+2$, the house price drops below the steady-state level while $\widehat{C}$ rises almost back to $\widehat{C}^*$, indicating a drop in subprime lending. The default rate declines only slightly.[24]

The subsequent evolution of P_0 and $\widehat{C}$ is depicted in Figure 4, which shows convergence back to the steady state.[25] Close inspection of the adjustment paths shows that they exhibit the cyclical convergence seen in Figure 3, an outcome that is robust to variations in the underlying parameter values. Figure 5 shows that the default rate also convergences cyclically back to its steady-state value, spending several periods at zero before returning to D^*.

Given the analytical and simulation results, several observations can be made about the intertemporal path generated by the shock to house-price expectations. In response to the initial shock, subprime lending surges as $\widehat{C}$ declines, and the resulting increase in demand pushes up the current house price and enlarges the housing stock. While this price increase partly sustains favorable price expectations for the subsequent period, the optimism is weaker than under the initial shock. This fact (captured in the π locus's rightward shift), together with the larger housing stock handed forward from time τ (reflected in the s-d locus's leftward shift), pushes down the housing price at time $\tau+1$. This drop, along with the presence of new low-C borrowers, leads to a spike in defaults. As the adjustment process continues, subprime lending eventually abates, with P_0, $\widehat{C}$ and the default rate ultimately returning to their steady-state values.[26]

4. Empirical Evidence

4.1. Data

The discussion now turns to a description of the empirical work designed to test the predictions contained in Proposition 2. Credit risk scores, drawn from the New York Fed's Consumer Credit Panel database, are used to measure the extent of subprime lending in a state. The database is a 5 percent sample of the entire US credit-bureau population (individuals with credit information), taken as a panel. The same set of social security numbers is drawn each quarter. The risk-score measures are scaled like the FICO scores that are also commonly used as a criterion for mortgage lending, and they are similarly based on credit-bureau data (the sample score range is from 280 to 850). In the data, the borrower's risk score is tabulated when a new mortgage is originated in a given quarter. Risk scores are then aggregated by state for each quarter, with the state's mean score, the 10th percentile score, and the 25th percentile score computed. The sample covers the 32 quarters running from 2001Q1 to 2008Q4.

The sample of borrowers is split into repeat home buyers, refinancers, investors and first-time buyers. A repeat buyer is a borrower who had a previously recorded mortgage and changed address upon receiving the new mortgage. A refinancer is a borrower with a previously recorded mortgage who did not change address in the quarter before or the two quarters after receiving the new mortgage. An investor is a borrower who has two additional first-lien mortgages with positive balances when the new mortgage was received.

Identification of first-time home buyers is less straightforward than for these other categories. A first-time buyer is a borrower who did not have a recorded mortgage in the prior four quarters, who was 40 years old or younger, and whose oldest account still active (as recorded by the credit bureau) was less than or equal to 240 months from its time of origination. Although some households could have paid off an unobserved previous mortgage well before borrowing again, thus looking like first-time buyers, the restriction to households no older than 40 with credit histories shorter than 20 years lessens the chance of such misidentification since few borrowers with these characteristics could have paid off a previous mortgage. Another potential problem is that some prior mortgages may not be recorded in credit-bureau records, although in recent years this oversight is likely to be uncommon.

Additional variables include quarterly state house price indexes from Core Logic, quarterly state unemployment rates from the Bureau of Labor Statistics, and the Conference Board's quarterly regional Consumer Confidence Index for the nine U.S. Census regions. Another variable is real quarterly state personal income, constructed by Haver Analytics from state personal income data deflated using the PCE chained price index, both from the Bureau of Economic Analysis. These latter three variables, which were downloaded from Haver DLX databases, control for demand-side effects, including the possibility that the risk composition of borrowers varies with the economic cycle.[27]

4.2. Empirical model

The empirical model portrays lender expectations somewhat differently from the theoretical model of section 3. For tractability, that model assumed that the location of the density of anticipated house prices depended on the *level* of the previous period's price, with an expectation shock having the same effect on the density as an exogenous increase in the previous price. A theoretically less tractable but more realistic approach for empirical purposes is to assume that rapid past price *appreciation*, rather than a high past price level, produces favorable expectations for the future price. In this setting, an expectations shock would be equivalent to an exogenous increase in the past appreciation rate. The empirical work does not provide an internal test of this assumption that lenders extrapolate past appreciation forward in predicting future house prices. But such behavior seems reasonable, and it is supported by indirect evidence.[28]

Any empirical specification relating borrower risk scores to past price appreciation must, however, confront a simultaneity problem. Past appreciation may encourage subprime lending, but the feedback effect from this lending will raise current house prices and thus the rate of appreciation through the current period. In other words, since housing demand at time t will depend on the volume of new subprime mortgages originated at t, it follows that price appreciation over the period ending at t will be affected by this lending volume and hence by the borrower risk-score measure at time t. To isolate the reverse causal path, where fast price appreciation signals high future prices and thus spurs subprime lending (depressing risk scores), the appreciation rate must be lagged, unlinking it from the current price. Accordingly, the empirical model relates the aggregate risk-score measure in state j in quarter t, denoted $RISKSCORE_{jt}$, to the one-year (4-quarter) lag of annual house-price appreciation, denoted $HPICHG_{jt-4}$. This variable is computed as $(HPI_{jt-4} - HPI_{jt-8})/HPI_{jt-8}$, where HPI is house-price index. A second version of the model instead uses the sixth-month lag of annual house-price appreciation, $HPICHG_{jt-2}$, computed as $(HPI_{jt-2} - HPI_{jt-6})/HPI_{jt-6}$.

$RISKSCORE_{jt}$ is assumed to also depend on a vector of additional covariates, denoted X_{jt}. These variables include the state unemployment rate for the given quarter (UNR_{jt}), per capita income (PC_{jt}), and the regional consumer confidence index (CC_{jt}). Also included in X_{jt} are state fixed effects, captured by a vector S of state dummy variables that takes the value S_j for state j, and quarterly fixed effects, captured by a vector T of quarter dummy variables that takes the value T_t for quarter t.

By capturing economic well-being at the household level, UNR_{jt} and PC_{jt} help determine borrower creditworthiness and thus individual risk scores, so that their presumed effects on the aggregate measure are respectively negative and positive.[29] The consumer-confidence variable may help determine lender optimism about the future, while also capturing economic well-being. Under the first interpretation, an increase in CC_{jt} should lead to a reduction in the aggregate risk-score measure as lenders serve riskier borrowers, but under the second interpretation, the effect of CC_{jt} could be positive. State and quarterly fixed effects are also elements of X_{jt}. The equation determining $RISKSCORE_{jt}$ is thus written

$$RISKSCORE_{jt} \;=\; \beta_0 \;+\; \beta_1 HPICHG_{jt-4} \;+\; \beta_2 X_{jt} \;+\; \epsilon_{jt}, \tag{21}$$

where ϵ_{jt} is an error term. Under the maintained hypothesis, β_1 is negative.

Even though use of the lagged appreciation rate eliminates a main source of potential simultaneity in the empirical model, a residual effect may remain as a consequence of serial correlation in the error terms in (21). To understand this point, observe that the feedback effect from subprime lending to prices can be captured by an equation relating $HPICHG_{jt-4}$ to $RISKSCORE_{jt-4}$ and additional covariates Z_{jt-4}, which is written

$$HPICHG_{jt-4} \;=\; \gamma_0 \;+\; \gamma_1 RISKSCORE_{jt-4} \;+\; \gamma_2 Z_{jt-4} \;+\; \eta_{jt-4}. \tag{22}$$

Note that since an expansion of subprime lending (a lower $RISKSCORE_{jt-4}$) raises price appreciation, $\gamma_1 < 0$ holds. The error term η in (22) is assumed to be uncorrelated with ϵ from (21), with η_{jr} and ϵ_{ks} being uncorrelated for any r, s, j and k. The Z_{jt-4} vector would include the X_{jt-4} variables UNR_{jt-4}, PCI_{jt-4} and CC_{jt-4}, all of which are shifters of housing demand, as well as additional demand shifters such as the annual rate of population change for the state.

To see that serial correlation in the ϵ's poses a threat to consistent estimation, let (21) be lagged four quarters to generate $RISKSCORE_{jt-4}$, with the result substituted into (22). After simplifying, the resulting equation can be written

$$HPICHG_{jt-4} \;=\; \theta_0 \;+\; \theta_1 HPICHG_{jt-8} \;+\; \theta_2 Z_{jt-4} \;+\; \gamma_1 \epsilon_{jt-4} \;+\; \eta_{jt-4}. \tag{23}$$

Because $HPICHG_{jt-4}$ depends on ϵ_{jt-4} from (23), $HPICHG_{jt-4}$ in (21) will be correlated with the error term ϵ_{jt} if the ϵ's are themselves serially correlated. This correlation will in turn bias the ordinary least-squares estimates of (21).

This source of bias can be eliminated by applying an autoregressive transformation to the model. Suppose that the ϵ follows an annual AR process, so that $\epsilon_{jt} = \rho\epsilon_{jt-4} + v_{jt}$, where ρ is the autoregressive parameter and the v_{jt} are i.i.d. error terms.[30] Then, lagging (21) by four quarters, multiplying by ρ and subtracting the result from (21) yields

$$\begin{aligned} RISKSCORE_{jt} \;&=\; (1-\rho)\beta_0 \;+\; \rho RISKSCORE_{jt-4} + \;\beta_1 HPICHG_{jt-4} \\ &\qquad -\; \rho\beta_1 HPICHG_{jt-8} \;+\; \beta_2 X_{jt} \;-\; \rho\beta_2 X_{jt-4} \;+\; v_{jt} \end{aligned} \tag{24}$$

Since $HPICHG$ depends on the contemporaneous ϵ from (23) and thus on the contemporaneous v, it is independent of later v's.[31] As a result, both $HPICHG_{jt-4}$ and $HPICHG_{jt-8}$ in (24) are independent of v_{jt}, eliminating the correlation between the right-hand variable and the error term that prevents consistent estimation of (21).

To estimate the model, the usual approach would rely on the OLS residuals from (21) to estimate ρ, with the result substituted in (24) and the β coefficients then estimated by OLS. However, since OLS estimation of (21) is inconsistent given the correlation between $HPICHG_{jt-4}$ and the error term, the resulting estimate of ρ is inconsistent as well. A different approach that circumvents this problem is to estimate (24) by nonlinear least squares.

The next section presents the results of this approach along with the inconsistent OLS estimates for comparison purposes. Nonlinear squares estimates are also presented for the alternative case of where (21) uses a two-quarter rather than four-quarter lag of annual house-price appreciation ($HPICHG_{jt-2}$ rather than $HPICHG_{jt-4}$).[32] In order to give bigger states (with their larger number of mortgage borrowers) more weight, (24) is estimated in weighted fashion, with quarterly observations for each state weighted by the average annual borrower count for the state. Significance tests are based on robust standard errors.[33]

Table 1 shows the summary statistics for the variables used in the model and gives their definitions. One noteworthy comparison is that each of the three $RISKSCORE$ measures for first-time buyers is lower than the corresponding measure for the other borrower groups.

Before presenting the empirical results, it is important to recognize that borrower risk scores and prior house-price appreciation may be linked for a reason different from the one envisioned in the model. In particular, since rapid prior appreciation will raise the home equity of repeat buyers and refinancers, a lower loan-to-value ratio becomes feasible on a new mortgage. The resulting reduction in default risk may in turn make lenders more willing to extend mortgages to borrowers with lower risk scores, generating an inverse relationship between prior appreciation and risk scores like that predicted by the model. This linkage, however, is not operative in the case of first-time buyers, for whom prior appreciation does not generate a wealth gain. Therefore, the regression results for first-time buyers provide a crucial way of distinguishing between these two sources of correlation between prior appreciation and borrower risk scores.[34]

4.3. Results

Table 2 shows the results for the subsample of repeat buyers. In the table's first three-column block, the aggregate risk score measure is the mean state risk score for this borrower group. The two remaining three-column blocks show results when the aggregate measures are the 10th and 25th percentile state risk scores, respectively. Within the first column block, the first regression shows the results of estimating (21) by OLS using the mean $RISKSCORE$ as dependent variable along with a four-quarter $HPICHG$ lag. The $HPICHG_{t-4}$ coefficient (β_1 in (21)) is negative and statistically significant, suggesting that more rapid past price appreciation lowers the mean risk score among current repeat buyers, reflecting an expansion of subprime lending. The estimates are thus consistent with the empirical hypothesis that subprime lending is spurred by favorable price expectations.

However, the OLS estimates are inconsistent, as explained above, and the next two regressions show the results from consistent nonlinear estimation of the transformed model. The second column, which is again based on a four-quarter lag, yields a significantly negative $HPICHG_{t-4}$ coefficient and a significant ρ estimate of about 0.2. The β_1 coefficient is smaller in absolute value than in the OLS regression, suggesting that the β_1 estimate under OLS is downward biased, a conclusion that also follows from the model. To see this point, note that since $HPICHG_{jt-4}$ is inversely related to ϵ_{jt-4} by (24) (recall $\gamma_1 < 0$), a negative correlation exists between $HPICHG_{jt-4}$ and the error term ϵ_{jt} in (21) when $\rho > 0$, leading to downward bias in the OLS estimate of β_1.

These more reliable estimates again support the hypothesis that subprime lending is spurred by favorable price expectations. But to see whether this conclusion is robust to the period over which past price appreciation is measured, the last regression in the first column block shows the nonlinear estimation results when the lag in (21) is two quarters rather than four, with $HPICHG_{jt-4}$ replaced by $HPICHG_{jt-2}$. The β_1 estimate is again significantly negative and similar in size to the previous values, and the ρ estimate is now larger at 0.26, a difference that makes sense given the shorter lag.

Among the other covariates, only the consumer confidence index has a significant effect on the mean risk score. The positive direction of the effect suggests that CC_t is capturing current economic well-being, which generates high borrower risk scores, rather than proxying favorable lender expectations about the future, which would encourage lending to riskier borrowers.

The regressions shown in the remaining column blocks of Table 2 use the 10th and 25th percentile state risk scores as dependent variables, and the estimates are similar to those based on the mean risk score. The β_1 coefficients are again signficantly negative, and an increase in the consumer confidence index again leads to higher risk scores. Note also that the positive expected effect of state per capita income emerges in the first two 25th-percentile regressions.

Since the model predicts that more-favorable price expectations should spur lending to the riskiest borrowers, one might expect lagged $HPICHG$ to have a stronger effect at lower percentiles of the risk-score distribution. A comparison of the nonlinear four-quarter-lag results for the 10th and 25th percentiles confirms this expectation. The elasticity of the 10th percentile risk score with respect to the lagged $HPICHG$ is more negative than the 25th percentile elasticity, an outcome that can be seen by noting that the larger absolute value of the 25th percentile β_1 is more than offset by the larger 25th percentile riskscore in the ratio used to compute the elasticity (see Table 1). In contrast, the nonlinear results with a two-quarter lag yield elasticities that are about equal. This outcome suggests that the first elasticity pattern may not be robust, a conclusion that is reinforced below in the case of first-time buyers. The explanation could be that the 25th percentile risk score already embraces the bulk of subprime borrowers, with impacts at the 10th percentile governed by other considerations.

To gauge the quantitative implications of the results, note that the mean regressions in Table 2 imply that a 10 percentage-point increase in past annual house-price appreciation reduces the mean risk score among repeat buyers by about 4 points. According to the model, this reduction comes from a decrease in the minimum risk score ($\widehat{C}$) due to an expansion in subprime lending. Further analysis shows this drop in the lowest risk score will equal some multiple of the 4-point decrease in the mean score.[35] Thus, the results are consistent with an appreciable reduction in the risk score of the worst borrowers receiving mortgages in response to higher past price appreciation price.

Table 3 shows the results for the case of refinancers. The main patterns established in the repeat-buyer case again emerge. All the β_1 coefficients are significantly negative, with the OLS estimates larger in absolute value than the nonlinear estimates for the four-quarter-lag case. $HPICHG$ elasticities are larger for the 10th percentile case than at the 25th percentile, now for both the four and two-quarter-lag specifications. As for the other covariates, the positive consumer confidence effect disappears in the nonlinear regressions, while the positive per capita income effect now emerges more consistently.

Although the estimates in Tables 2 and 3 are consistent with the main hypothesis, they could also reflect favorable wealth effects from past price appreciation, which would allow riskier borrowers among the repeat-buyer and refinancer groups to secure mortgages. As explained above, first-time buyers experience no such wealth effect, which means that they offer a more stringent test of the main hypothesis. The first-time buyer results are shown in Table 4, and they mostly conform to previous patterns. When the dependent variable is the mean risk score or the 25th percentile score, the β_1 coefficients are again significantly negative, although the OLS and nonlinear 4-quarter-lag coefficients are now similar in size. In contrast, the OLS and nonlinear β_1 estimates are insignificant when the dependent variable is the 10th percentile risk score. While this result is unexpected, the fact that the anticipated negative effect of the lagged $HPICHG$ still emerges for first-time buyers near the bottom of the risk score distribution (at the 25th percentile), as well as emerging at the mean, supports the main hypothesis. Among the other covariates, the positive per capita income effect is again present, while the consumer confidence effect is more consistently positive than in the refinancer case, mirroring the repeat-buyer results.

Table 5 shows the regression for the investor case, and the results once again show negative β_1 coefficients. As in the case of first-time borrowers, the OLS and nonlinear four-quarter-lag estimates are similar in size. In contrast to previous results, the coefficients of the other covariates are mostly insignificant, with the few significant cases showing negative rather than positive signs.

Overall, the empirical results provide considerable support for the empirical hypothesis that faster past price appreciation, by raising lender optimism about future prices, spurs lending to riskier borrowers. Although a wealth effect could contribute to the observed association between past appreciation and borrower riskiness for previous homeowners, the evidence that this association is also present for first-time buyers lends substantial credence to the maintained hypothesis.

5. Conclusion

This paper has explored the link between the house-price expectations of mortgage lenders and the extent of subprime lending. It argues that bubble conditions in the housing market are likely to spur subprime lending, with favorable price expectations easing the default concerns of lenders and thus increasing their willingness to extend loans to risky borrowers. Since the demand created by this subprime lending feeds back onto house prices, the lending also helps to fuel an emerging housing bubble. The paper, however, focuses mostly on the reverse causal linkage, where subprime lending is a consequence rather than a cause of bubble conditions, further exploring what Mian and Sufi (2009) call the "expectations hypothesis."

The paper's theoretical model portrays subprime lending as the extension of loans to borrowers with low observable "default costs," and the analysis shows that a favorable shift in house-price expectations spurs such lending by encouraging loans to borrowers with even lower default costs (and thus higher default risks). The empirical work shows that rapid past appreciation, which is presumed to generate favorable expectations regarding future prices, does indeed lead lenders to extend mortgages to riskier borrowers (those with worse credit ratings).

By providing a deeper theoretical grounding for the "expectation hypothesis" than prior work as well as a more-direct empirical test, the paper advances our understanding of the housing crisis. A better understanding of this watershed economic event is crucial in formulating policies to prevent its repetition, and this paper may add some of the required insights.

Appendix

Derivation of (12)

Integrating by parts, the profit expression in (9) can be rewritten as

$$- k \; - \; P_0 + \eta \left((\widehat{B} - \widehat{C})F(\widehat{B} - \widehat{C}, \delta) \; - \; \int_0^{\widehat{B}-\widehat{C}} F(P, \delta)dP \; + \; \widehat{B}[1 - F(\widehat{B} - \widehat{C}, \delta)] \right) \; =$$

$$k - \; P_0 \; - \; \eta \left[\widehat{C}F(\widehat{B} - \widehat{C}, \delta) \; + \; \int_0^{\widehat{B}-\widehat{C}} F(P, \delta)dP \; - \; \widehat{B} \right]. \qquad (a1)$$

Differentiating $(a1)$ with respect to δ yields (11).

Simulation assumptions

The height of the uniform default-cost density is $1/g$, where $g = \overline{C} - \underline{C}$, and the cumulative distribution function in (19) is then $G(\widehat{C}) = (\widehat{C} - \underline{C})/g$ (the mass N of potential mortgage borrowers is normalized to unity). Furthermore, with linear housing supply, $S(P_0^t) \equiv nP_0^t - k$, implying $P^* = k/n$. Given these latter assumptions, (19) reduces to

$$\widehat{C}^t \; - \; \widehat{C}^{t-1} \; + \; g(nP_0^t - k) \quad = \quad 0. \qquad (a2)$$

Letting $a \equiv \alpha y$, so that $B(P_0^t) = a + P_0^t$, and assuming $\eta = 1$, (18) reduces to

$$(\widehat{C}^t)^2 \; - \; (a + P_0^t)^2 \; + \; (P_0^{t-1} + \delta - \mu)[2(a + P_0^t) - (P_0^{t-1} + \delta - \mu)] \; + \; 4a\mu \quad = \quad 0. \qquad (a3)$$

To carry out the simulation, $(a2)$ is used to eliminate $\widehat{C}^t$ in $(a3)$, so that P_0^t can be written as a function of the prior values P_0^{t-1} and $\widehat{C}^{t-1}$. With P_0^t then known given these values, substitution in $(a2)$ determines $\widehat{C}^t$ as a function of P_0^{t-1} and $\widehat{C}^{t-1}$. The time paths shown in Figure 4 reflect the following additional parameter values: $a = 2.28$, $\mu = 4.5$, $k = 10$, $n = 2$ and $g = 4$.

Under these parameter values, $B < \overline{P}$ holds at each point in time for all borrowers, so that π_B in (7) is positive as assumed (note that $B - C < \overline{P}$ then also holds). In addition, assuming $\underline{C} = 0$, the condition $B - C > \underline{P}$ is satisfied at each point in time for all borrowers.

To compute default rates, the uniformity assumption is imposed in (18) (lagged one period), and the condition is solved for B to yield

$$B \quad = \quad P_0^{t-2} + \mu + \delta - \left[C^2 + 4\delta\mu - 4\mu P_0^{t-1} + 4\mu P_0^{t-2} \right]^{1/2} \qquad (a5)$$

This solution with C replaced by $\widetilde{C}$ is then substituted into the condition $P_0^t = B - \widetilde{C}$ to yield $\widetilde{C}$ as a function of P_0^t, P_0^{t-1}, P_0^{t-2} and δ.

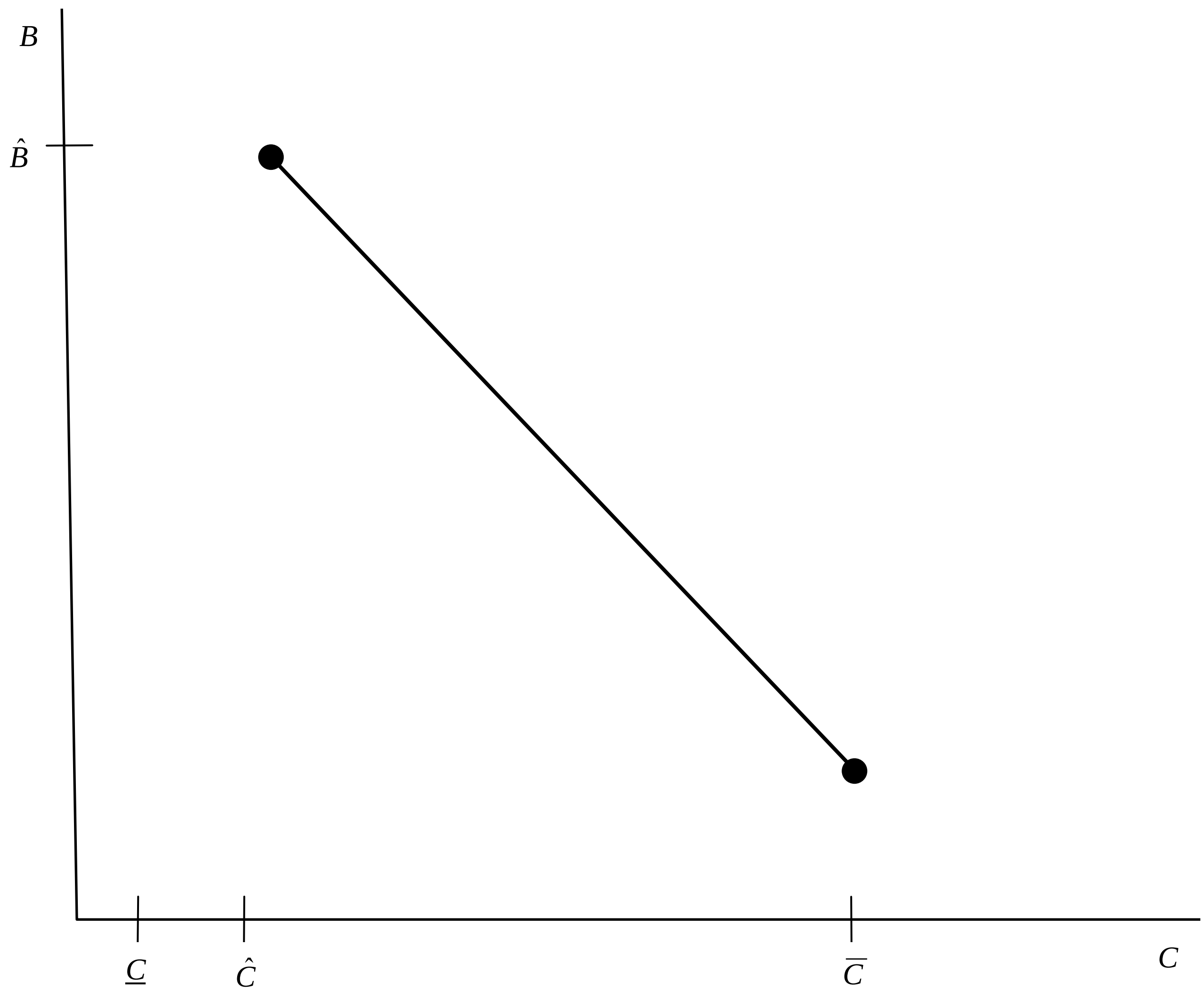

Figure 1: The relationship between B and C

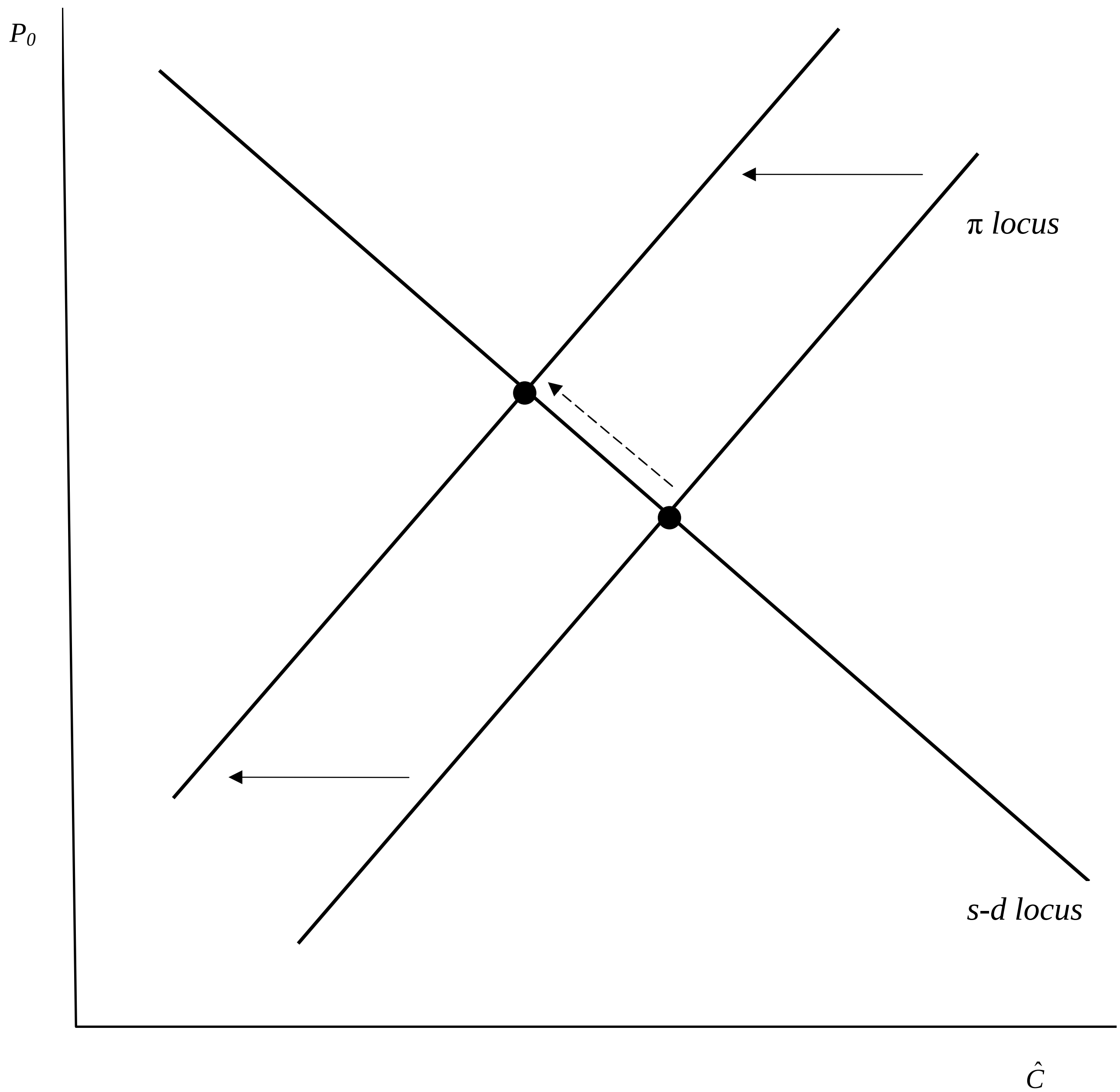

Figure 2: The effect of a shift in house-price expectations

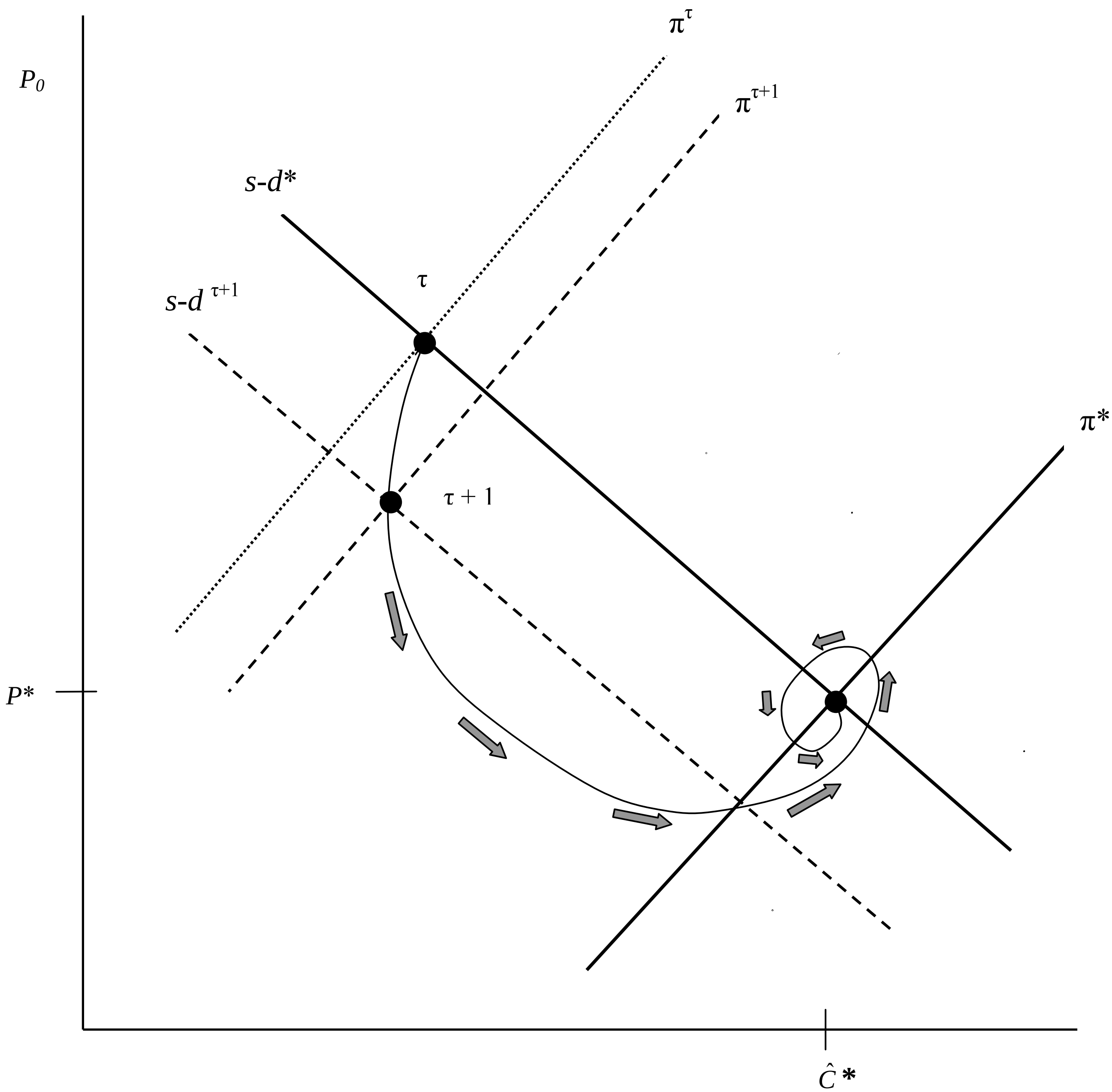

Figure 3: Adjustment to a house-price-expectation shock

Figure 4: Simulation Results

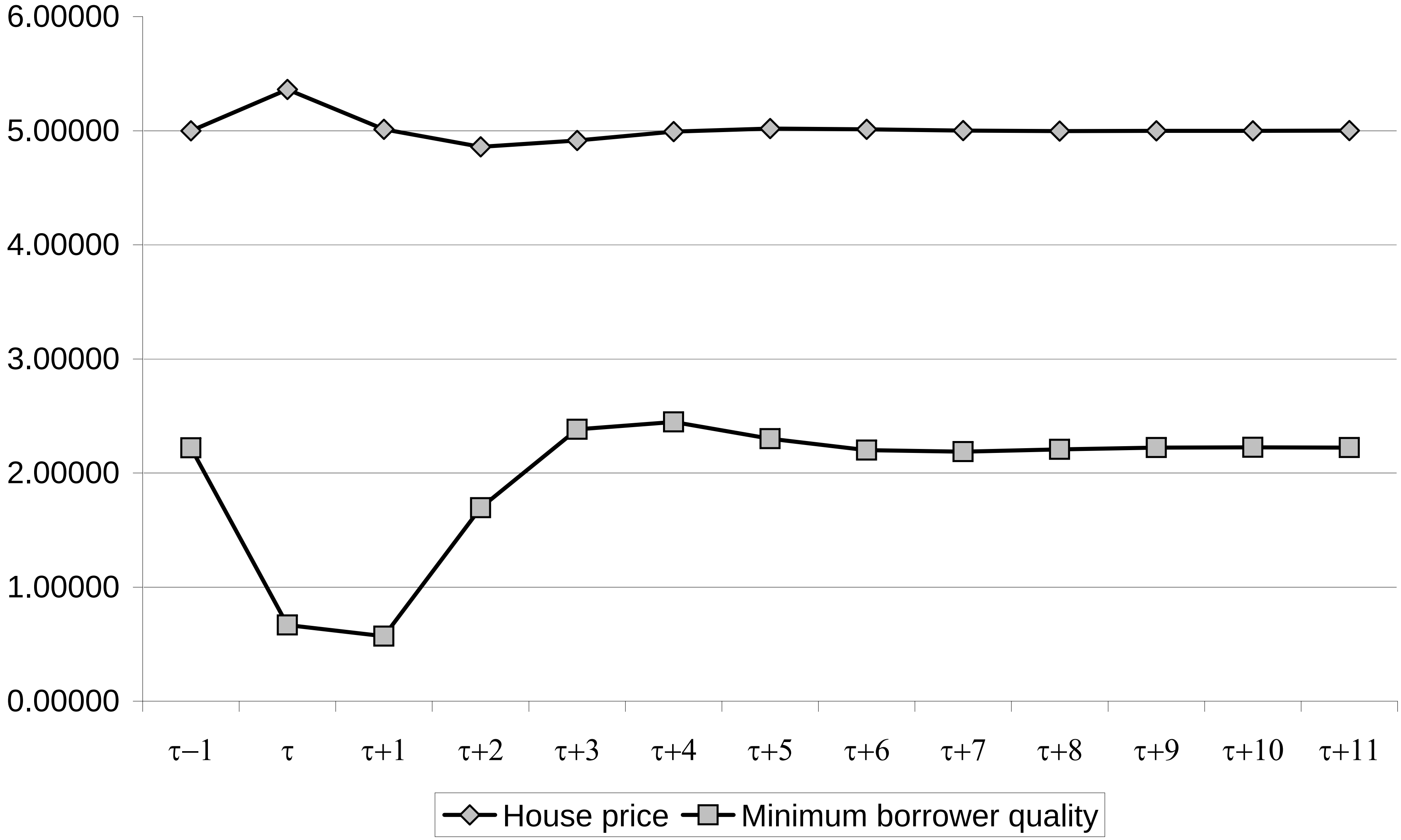

Figure 5: Default rate

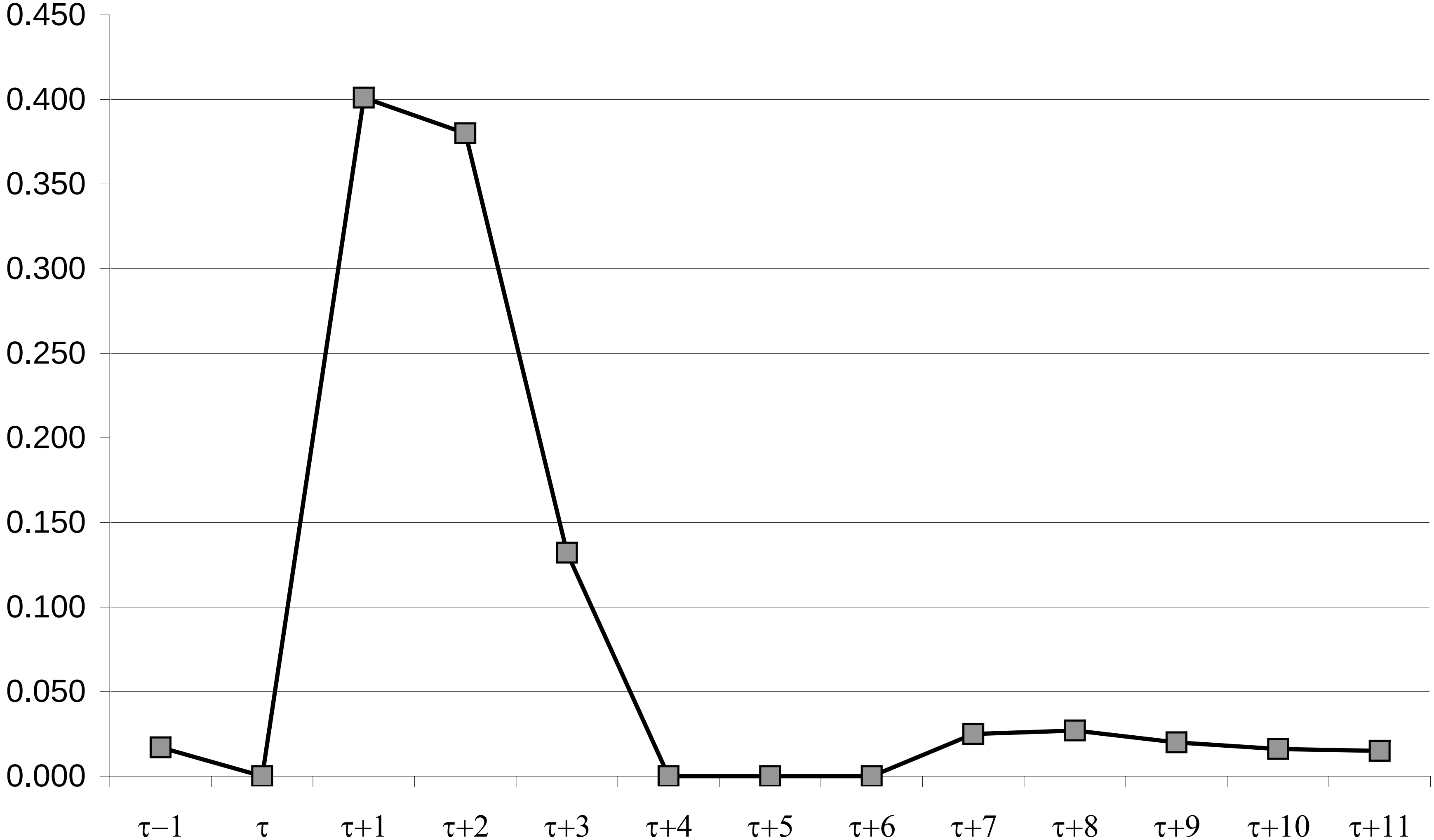

Table 1. Summary statistics									
		First time buyers		Repeat buyers		Refinancers		Investors	
	N	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Risk score – mean *RISKSCOREM*	1600	689.0	15.47	722.62	12.96	715.6	16.36	735.3	17.44
Risk score – 10th percentile *RISKSCORE10*	1600	585.4	20.52	627.66	20.89	610.4	25.61	649.0	29.39
Risk score – 25th percentile *RISKSCORE25*	1600	636.4	20.26	683.70	18.21	669.8	23.90	698.9	25.52
Unemployment rate *UNR*	1600	4.92	1.17	4.92	1.17	4.92	1.17	4.92	1.17
Consumer confidence ind. *CC*	1600	95.52	21.90	95.52	21.90	95.52	21.90	95.52	21.90
State house price inflation *HPICHG*	1600	5.23	8.07	5.23	8.07	5.23	8.07	5.23	8.07
Per capita income *PCI*	1600	35.25	11.54	35.25	11.54	35.25	11.54	35.25	11.54

Table 2. RISKSCORE regressions for repeat buyers – 2001Q1 – 2008Q4									
	Mean			10th Percentile			25th Percentile		
	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag
Constant	467.779** (4.822)	714.933** (7.551)	716.652** (8.067)	407.462** (10.211)	611.468** (14.959)	610.444** (15.635)	437.864 (8.029)	660.194** (12.205)	662.026** (13.036)
ρ		0.186** (0.033)	0.264** (0.033)		0.104** (0.029)	0.142** (0.032)		0.139** (0.032)	0.211** (0.034)
$HPICHG_{t-4}$ (β_1)	-0.469** (0.047)	-0.436** (0.046)		-0.647** (0.087)	-0.625** (0.086)		-0.682** (0.072)	-0.637** (0.071)	
$HPICHG_{t-2}$ (β_1)			-0.395** (0.048)			-0.501** (0.086)			-0.553** (0.074)
CC_t (β_2)	0.109** (0.018)	0.082** (0.020)	0.073** (0.021)	0.238** (0.035)	0.219** (0.037)	0.196** (0.038)	0.176** (0.028)	0.145** (0.030)	0.129** (0.033)
UNR_t (β_2)	0.002 (0.332)	0.246 (0.346)	0.143 (0.377)	-0.910 (0.657)	-0.724 (0.672)	-0.461 (0.696)	-0.345 (0.523)	-0.002 (0.546)	-0.004 (0.585)
PCI_t (β_2)	0.275 (0.171)	0.388 (0.194)	0.223 (0.200)	0.534 (0.352)	0.634 (0.382)	0.490 (0.388)	0.657* (0.283)	0.822* (0.317)	0.605 (0.323)

Standard errors in parentheses.

Significance: **1%, *5%

N=1600 for each regression. The regressions use robust standard errors, and coefficients of the quarter and state dummy variables are not reported.

Table 3. RISKSCORE regressions for refinancers – 2001Q1 – 2008Q4									
	Mean			10th Percentile			25th Percentile		
	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag
Constant	839.825** (13.426)	696.735** (12.117)	671.574** (16.270)	654.739** (25.945)	521.663** (24.433)	498.200** (30.094)	742.111** (22.600)	620.935** (20.992)	581.994** (27.638)
ρ		0.505** (0.056)	0.612** (0.058)		0.412** (0.060)	0.557** (0.079)		0.555** (0.068)	0.663** (0.070)
$HPICHG_{t-4}$ (β_1)	-0.631** (0.084)	-0.454** (0.051)		-0.891** (0.167)	-0.687** (0.118)		-0.916** (0.146)	-0.608** (0.082)	
$HPICHG_{t-2}$ (β_1)			-0.464** (0.064)			-0.679** (0.131)			-0.595** (0.102)
CC_t (β_2)	0.151** (0.022)	0.026 (0.036)	-0.017 (0.034)	0.300** (0.039)	0.110 (0.065)	0.033 (0.067)	0.268** (0.037)	0.049 (0.064)	-0.018 (0.058)
UNR_t (β_2)	0.089 (0.468)	0.991* (0.500)	0.866 (0.483)	0.376 (0.898)	1.748 (1.060)	1.644 (0.993)	0.171 (0.797)	1.411 (0.832)	1.386 (0.829)
PCI_t (β_2)	0.413 (0.256)	0.847** (0.309)	0.840* (0.354)	1.646** (0.502)	2.406** (0.632)	2.227** (0.660)	1.246** (0.433)	1.741** (0.492)	1.519** (0.566)

Standard errors in parentheses.

Significance: **1%, *5%

N=1600 for each regression. The regressions use robust standard errors, and coefficients of the quarter and state dummy variables are not reported.

Table 4. RISKSCORE regressions for first-time buyers – 2001Q1 – 2008Q4									
	Mean			10th Percentile			25th Percentile		
	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag
Constant	685.443** (7.656)	663.657** (8.140)	658.563** (9.104)	589.630** (10.872)	564.056** (10.899)	561.001** (12.103)	613.835** (10.632)	592.554** (11.176)	583.233** (12.331)
ρ		0.264** (0.031)	0.382** (0.036)		0.450** (0.030)	0.277** (0.035)		0.266** (0.033)	0.380** (0.037)
$HPICHG_{t-4}$ (β_1)	-0.152** (0.046)	-0.149** (0.044)		-0.085 (0.059)	0.100 (0.060)		-0.170* (0.067)	-0.176** (0.065)	
$HPICHG_{t-2}$ (β_1)			-0.162** (0.044)			-0.082 (0.057)			-0.153* (0.065)
CC_t (β_2)	0.048* (0.020)	0.041 (0.022)	0.034 (0.023)	0.0163** (0.025)	0.159** (0.027)	0.143** (0.028)	0.122** (0.030)	0.118** (0.032)	0.101** (0.035)
UNR_t (β_2)	-0.487 (0.274)	-0.315 (0.279)	-0.339 (0.327)	0.060 (0.383)	0.056 (0.389)	0.074 (0.455)	-0.611 (0.366)	-0.407 (0.375)	-0.175 (0.431)
PCI_t (β_2)	0.769** (0.163)	0.790** (0.194)	0.796** (0.221)	0.508* (0.237)	0.525* (0.263)	0.533 (0.292)	1.245** (0.228)	1.224** (0.269)	1.283** (0.299)

Standard errors in parentheses.

Significance: **1%, *5%

N=1600 for each regression. The regressions use robust standard errors, and coefficients of the quarter and state dummy variables are not reported.

Table 5. RISKSCORE regressions for investors – 2001Q1 – 2008Q4									
	Mean			10th Percentile			25th Percentile		
	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag	OLS: 4Q lag	NL: 4Q lag	NL: 2Q lag
Constant	273.698** (3.495)	765.342** (9.861)	755.881** (10.987)	241.608** (7.664)	670.811** (20.458)	657.788** (22.343)	263.383** (5.292)	734.584** (14.626)	721.917** (16.191)
ρ		0.236** (0.028)	0.309** (0.031)		0.073* (0.029)	0.169** (0.031)		0.155** (0.029)	0.255** (0.032)
$HPICHG_{t-4}$ (β_1)	-0.283** (0.042)	-0.292** (0.040)		-0.556** (0.101)	-0.555** (0.100)		-0.464** (0.065)	-0.466** (0.064)	
$HPICHG_{t-2}$ (β_1)			-0.140** (0.047)			-0.315** (0.107)			-0.250** (0.073)
CC_t (β_2)	0.031 (0.028)	0.003 (0.030)	-0.019 (0.031)	0.095 (0.052)	0.089 (0.053)	0.044 (0.057)	0.067 (0.042)	0.042 (0.044)	-0.001 (0.046)
UNR_t (β_2)	-0.669 (0.399)	-0.637 (0.422)	-0.078 (0.466)	-2.754** (0.923)	-2.649** (0.949)	-1.571 (1.023)	-0.781 (0.630)	-0.769 (0.657)	0.075 (0.726)
PCI_t (β_2)	-0.516* (0.222)	-0.382 (0.247)	-0.297 (0.270)	0.033 (0.492)	0.094 (0.509)	0.232 (0.547)	-0.465 (0.333)	-0.409 (0.362)	-0.262 (0.397)

Standard errors in parentheses.

Significance: **1%, *5%

N=1600 for each regression. The regressions use robust standard errors, and coefficients of the quarter and state dummy variables are not reported.

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Footnotes

[*]We thank Rainald Borck, Bill Branch, Kangoh Lee, and Pierre Picard for helpful comments, and we are especially grateful to David Brownstone for econometric advice. Errors or shortcomings, however, are our responsibility.

[1]Wheaton and Nechayev (2008), by contrast, show a connection between subprime lending and house-price escalation. They start by estimating a regression relating prices to economic fundamentals through 1998 and then show that the regression underpredicts prices during the subsequent 1998-2005 period. The authors then demonstrate that the forecast errors from this regression are larger in MSAs with substantial subprime lending activity, establishing a connection between such lending and price growth.

[2]Himmelberg, Mayer and Sinai (2005) argue against the presence of bubble conditions as of 2004 by showing that the user-cost of owner-occupied housing (based on historical appreciation rates) was similar to rent levels across different MSAs. Under bubble conditions, user-cost would be far below current rents.

[3]Coleman et al. (2010) acknowledge the existence of this channel, and they test for it by regressing subprime lending on prior price growth (reversing their main regression). However, despite the possible existence of two-way causation, they do not use simultaneous-equations methods.

[4]The term "bubble" in this paper is used in its popular sense, not in the technical fashion seen in the theoretical macro literature.

[5]Recent papers in macroeconomics explore the factors that generate price volatility and bubbles in the housing market. Lambertini, Mendicino and Punzi (2010) present an empirical analysis showing a link between expectations and boom-bust housing cycles. Arce and López-Salido (2011) view a housing bubble as an equilibrium where houses are held only for resale (not generating rent or utility) and show that a low downpayment requirement is required to sustain such an equilibrium. Iacoveillo and Neri (2010) use a calibrated DSGE model to explore the sources of volatility in house prices. All this work draws on the classic paper of Kiyotaki and Moore (1997), which demonstrated the effect of collateral constraints on the dynamic behavior of the economy.

[6]For example, moving costs may be an element of C in the absence of a trigger event, while disappearing from C when a trigger event is present, as follows. Without a trigger event that requires a move, moving costs will be zero in the absence of default (in which case the borrower stays in the house), but the cost will be positive with default, in which case a move

is required. In this case, moving cost is an element of C. Suppose instead that a trigger event such as unemployment occurs. Then, assuming the house becomes unaffordable, the borrower must move regardless of whether default occurs, so that moving costs no longer represent part of default costs. Thus, the trigger event depresses C, making default more likely, as seen in the ensuing discussion.

[7]The density $f(P)$ is presumably positive only over a subinterval within $[0, \infty]$, which can be denoted $[\underline{P}, \overline{P}]$. For (3) to be relevant, $B - C$ must lie within this interval. Otherwise, the probability of default is either zero or one.

[8]The curve is drawn as linear even though other shapes are possible. The same point applies to Figure 2 below.

[9]Given that α and y are multiplied in (8), the effect of an increase in income y is identical to the impact of a higher α.

[10]The reason is that, since P_0 is a component of B, with the remainder equal to the interest payment, a larger P_0 would shrink that payment if $\widehat{B}$ were held fixed, pushing it below αy. For the interest payment to remain at its maximal level, $\widehat{B}$ must then increase with P_0.

[11]Negativity follows because $\partial\widehat{B}/\partial P_0 = 1$ and $\pi_B < 1$. Referring to (6), this latter inequality follows because $\eta \leq 1$ and the integral in (6) is less than unity.

[12]These conclusions follow because the modified version of (3) is increasing in B, decreasing in P_0, and increasing in δ.

[13]These properties follow from differentiation of (17) and use of the properties of the ϕ function.

[14]This property follows because, if P_0^{t-1} is increased by an amount ϵ, then P must also be increased by ϵ to keep the height of the density at its original value.

[15]These borrowers are assumed to reenter the pool of potential mortgage borrowers without penalty. The potential-borrower pool thus consists of the same individuals from period to period, who need a mortgage to repurchase the housing they just relinquished either through sale or default. In reality, borrowers who default enter the rental market for housing, which is suppressed under the current setup. This suppression is not completely unrealistic given that renter status for defaulters is typical temporary, with new mortgages available to them after a few years. Another point is that any capital gains earned upon sale of a house do not affect a borrower's ability to secure another mortgage. By affecting wealth, not income, these gains do not loosen the payment-to-income constraint, and they cannot be used for a

downpayment given the assumption of 100 percent loans.

[16]When $\widehat{C}^{t-1}$ increases, $\widehat{C}^{t}$ must increase holding P_0^t fixed to maintain the equality in (19).

[17]$\widehat{C}^{t-1} = \widehat{C}^{t}$ must hold in the steady state, so that the number of borrowers $1 - G(\widehat{C})$ and thus the number of houses is constant over time. From (19), this requirement implies $P_0^t = P^*$, ensuring that housing supply equals zero and that the stock is thus constant. Then, substituting $P_0^t = P_0^{t-1} = P^*$ into (18) (along with $\delta = 0$), the condition yields a steady-state value for $\widehat{C}$, denoted $\widehat{C}^*$.

[18]This conclusion follows from computing $\partial P_0/\partial\delta = -\pi_\delta/\pi_{P_0}$ from (18) and showing that it is less than unity. Since the upward shift in the π locus is thus less than δ, it follows from Figure 2 that the increase in P_0 must also be less than δ. In other words, given the negative slope of the s-d locus, the increase in P_0 must be smaller than the vertical shift of the π locus, which is itself less than δ.

[19]The relation between $\widehat{C}^{\tau+1}$ and $\widehat{C}^{\tau}$ depends on whether $P_0^{\tau+1}$ is above or below P^*. If $P_0^{\tau+1} > P^*$ holds, then the housing stock is growing, and the number of mortgage borrowers must be rising, implying $\widehat{C}^{\tau+1} < \widehat{C}^{\tau}$, as in Figure 3. If $P_0^{\tau+1} < P^*$, then the housing stock is shrinking, implying $\widehat{C}^{\tau+1} > \widehat{C}^{\tau}$.

[20]To see this point, recall that the function $\widetilde{C}(P, P_0, \delta)$ from above indicates that the critical C value below which default occurs depends on the realized price (P), last period's price (P_0), and the position of last period's anticipated house-price density, as captured by δ. In the dynamic setting, the critical value $\widetilde{C}^{\tau}$ at time τ is found by replacing P in this function by P_0^{τ}, P_0 by P^* (the price at $\tau - 1$), and δ by P^*, recognizing that the density position in the previous period is now represented by the lagged price (in this case $P_0^{\tau-2} = P^*$) plus the expectations shift, which is zero at time $\tau - 1$. Thus, $\widetilde{C}^{\tau} = \widetilde{C}(P_0^{\tau}, P^*, P^*)$. By contrast, the steady-state value $\widetilde{C}$ is given by $\widetilde{C}^* = \widetilde{C}(P^*, P^*, P^*)$. Since, from above, $\widetilde{C}$ is decreasing in its first argument, it follows that $\widetilde{C}^{\tau} < \widetilde{C}^*$, so that the critical value declines at τ.

[21]The derivative of D^t with respect to $\widehat{C}^{t-1}$ has the sign of $G'(\widehat{C}^{t-1})[G(\widetilde{C}^t) - N] < 0$.

[22]The critical $\widetilde{C}$ value at time $\tau + 1$ is given by $\widetilde{C}^{\tau+1} = \widetilde{C}(P_0^{\tau+1}, P_0^{\tau}, P^* + \delta)$, recognizing that the position of the density at τ is captured by the lagged price $P_0^{\tau-1} = P^*$ plus the time-τ expectations shift. Relative to $\widetilde{C}^{\tau} = \widetilde{C}(P_0^{\tau}, P^*, P^*)$, the first argument of $\widetilde{C}(P_0^{\tau+1}, P_0^{\tau}, P^* + \delta) = \widetilde{C}^{\tau+1}$ is smaller while the second and third are larger. Since $\widetilde{C}$ is increasing in the second argument and decreasing in the others, it follows that the relationship between $\widetilde{C}^{\tau+1}$ and $\widetilde{C}^{\tau}$ is ambiguous.

[23] $\widetilde{C}$ declines slightly between τ and $\tau+1$, tending to reduce the default rate, but the much-larger change in $\widehat{C}$ dominates.

[24] A further slight decline in $\widetilde{C}$ is offset by a continuing decline in $\widehat{C}$.

[25] If the lender's price expectations are myopic, with f independent of the past price history, it can be shown analytically that convergence back to the steady state is guaranteed. We are indebted to Pierre Picard for this demonstration, which holds in the general version of the model.

[26] A relaxation of the payment-to-income constraint (an increase in α) or a decline in origination costs k leads to a similar adjustment process. But if either of these changes is permanent, the steady-state is altered, in contrast to the case of a one-time expectations shock. Either change shifts the π locus upward in Figure 3, leading to an initial expansion of subprime lending and an increase in the house price. With the steady state altered in each case, however, convergence to new equilibrium values occurs. The steady-state house price remains at P^* given that neither change affects the supply-demand condition (19), but the steady-state value of $\widehat{C}$ falls when α increases or k declines. Therefore, the new steady-state equilibrium reflects a permanent increase in subprime lending and a larger housing stock. Note that since the adjustment process generated by either of these changes involves falling house prices, it will exhibit a temporary surge in defaults like that in Figure 5.

[27] All these quarterly numbers are averages of monthly values.

[28] This evidence comes from regressions of building permits on past price appreciation. Presumably, a large volume of building permits signals optimism about future prices. In a regression of the permit volume in state j and quarter t on past annual price appreciation along with state and quarter fixed effects, the appreciation coefficient is significantly positive. This outcome emerges regardless of whether appreciation is measured through the current quarter or whether the annual appreciation rate is lagged by two or four quarters (being measured through quarter $t-2$ or quarter $t-4$). Thus, optimism about future prices, as reflected in building permits, appears to be linked to past appreciation.

[29] This presumption could be reversed by other subtler effects. For example, high unemployment could keep riskier borrowers from even entering the housing market, leading to a positive association between risk scores and this variable. As will be seen below, the unemployment coefficient is almost always insignificant in the regressions, suggesting that such offsetting effects may be at work.

[30] This AR structure can be derived from an underlying process where the autoregressive lag is a single quarter, with $\epsilon_{jt} = \lambda\epsilon_{jt-1} + u_{jt}$, where the u_{jt} are i.i.d. error terms with variance

σ^2. Successive substitution yields $\epsilon_{jt} = \rho\epsilon_{jt-4} + v_{jt}$, where $\rho = \lambda^4$ and $v_{jt} = u_{jt} + \lambda u_{jt-1} + \lambda^2 u_{jt-2} + \lambda^3 u_{jt-3}$. In this case, the v_{jt}'s are correlated, with $E(v_{jt}, v_{jt-1}) = (\lambda + \lambda^3 + \lambda^5)\sigma^2$, $E(v_{jt}, v_{jt-2}) = (\lambda^2 + \lambda^4)\sigma^2$, $E(v_{jt}, v_{jt-3}) = \lambda^3\sigma^2$, and $E(v_{jt}, v_{jt-k}) = 0$ for $k > 3$. In the case where the $HPICHG$ lag in (21) is two quarters rather than four, $\epsilon_{jt} = \rho\epsilon_{jt-2} + v_{jt}$, where $v_{jt} = u_{jt} + \lambda u_{jt-1}$, $E(v_{jt}, v_{jt-1}) = \lambda\sigma^2$, and $E(v_{jt}, v_{jt-k}) = 0$ for $k > 1$.

[31]Another way to see this point is by successive substitution for the $HPICHG_j$'s and ϵ_j's on the right-hand side of (23), which shows that $HPICHG_{jt-4}$ depends on v_{jt-4} and all the previous v_j values.

[32]The state and quarter dummies are introduced directly into the nonlinear model rather than relying on the usual de-meaning of the data used to produce fixed effects estimates in OLS. Although ρ will be embedded in the dummy coefficients in (24), the fact that the estimation of the underlying fixed effects is not a goal means that this presence can be ignored in the nonlinear procedure. In contrast, ρ and the remaining β coefficients in (24) must be disentangled in order to measure the effects of interest.

[33]Since the main sources of within-state error correlation are removed by the inclusion of state dummies and the use of an autoregressive transformation, the alternative approach of clustering at the state level does not appear necessary. Note that when the autoregressive structure is generated by an underlying process with a one-quarter lag, then the error terms v_{jt} in (24) are correlated within each state but in a complex fashion. As seen in footnote 28, when the $HPICHG$ lag in (21) is four quarters, the error correlation is positive when the time index differs by three or less and equals zero otherwise. When the $HPICHG$ lag is two quarters, the error correlation is positive when the time index differs by one and equals zero otherwise. As a result, the error structure does not have the constant error correlation within states that justifies clustering, making robust standard errors more appropriate.

[34]Note that reduced affordability from faster house-price appreciation could actually mask an expectations-driven relationship between the risk score and prior appreciation for first-time borrowers. The reason is that rapid prior appreciation would prompt households to seek loans with higher LTVs, which would require a better risk score to gain approval.

[35]Letting $g(c)$ denote the density of default costs, the mean C among borrowers getting mortgages is $C_m \equiv \int_{\widehat{C}}^{\infty}[Cg(C)/(N - G(\widehat{C}))]dC$. In the case of a uniform g with support $[0, \overline{C}]$, $C_m = (\widehat{C} + \overline{C})/2$, so that the mean drops at half the rate at which $\widehat{C}$ declines, with $\widehat{C}$ conversely dropping at double the rate of the mean. Generally, $\partial C_m/\partial\widehat{C}$ equals $(C_m - \widehat{C})$ times the hazard-rate expression $g(\widehat{C})/[N - G(\widehat{C})]$. This hazard rate equals $1/(\overline{C} - \widehat{C})$ in the uniform case, yielding $\partial C_m/\partial\widehat{C} < 1$, a result that will be strengthened when the hazard rate at $\widehat{C}$ is smaller, as will happen with a unimodal density where $\widehat{C}$ lies below the mode. In such cases, $\partial C_m/\partial\widehat{C}$ will be well below one, implying that $\widehat{C}$ drops by a multiple of any

measured decline in C_m.